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DEVELOPMENT OF A CONSTRUCTION-BASED PEDAGOGY FOR A GRADUATE ENGINEERING REVIEW COURSE IN SCHOOL MATHEMATICS

THESIS

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DEVELOPMENT OF A CONSTRUCTION-BASED PEDAGOGY FOR A GRADUATE ENGINEERING REVIEW COURSE IN SCHOOL MATHEMATICS

THESIS

Presented to the Faculty of the School of Systems and Logistics of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Systems Management

Jerry D. Edwards, B.S.

Captain, USAF

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Preface

After 20 years of formal education, I have finally come to realize why I never really enjoyed school: methods of evaluation did not measure what I really knew about a subject. This thesis presents ideas, tools and methodologies that, if used, would have made me happier. However, their usefulness extends far beyond that.

The ideas, tools and methodologies presented in this thesis foster a meaningful learning environment: an environment in which learning is of the utmost importance, where the goal of teaching is to do as much as possible to facilitate the learning process, and where students are evaluated on what they know.

For all their efforts in making this thesis come to life, I must thank my advisors, Professor Dan Reynolds and Dr. Ted Luke. But most of all, my advisors and I must thank Richard Lamb. Rich, blind since birth, possesses an incredible understanding of mathematics. Rich devoted many hours teaching Dan and me everything we never learned about mathematics to prepare us for teaching the review. His uncanny ability to use "cane mathematics" to get his point across in the classroom made learning mathematics fun for all. Without him, none of this would have been possible.

Jerry D. Edwards

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Abstract

This thesis proposes a pedagogy and an experimental curriculum and instructional system designed to enhance the ability of engineering students to construct and manipulate mathematical representations of real-world phenomena and to create the new knowledge they would need to solve unprecedented engineering problems. Developing such a pedagogy required a theory of learning and a viable model of education. The learning theory employed is Ausubel's, artfully extended by Gowin and Novak through their invention of two heuristics for learning: the Concept Map and Vee. The model of education used is Gowin's description of Schwab's four commonplaces of education.

The need for development of this pedagogy, curriculum and instructional system stems from the current rule-based approach to the teaching and learning of mathematics that promotes rote memorization of rules and techniques rather than a conceptual orientation to mathematics. Because of the need for students to understand and employ mathematical concepts and use those concepts to create mathematical representations of the real world, a construction-based curriculum was developed. The construction-based curriculum emphasizes the connection between concepts and the real world, how those concepts are used in the creation of the

mathematical technologies, and how the concepts, along with the technologies, can be used to generate solutions to real-world problems.

This construction-based curriculum was taught to students in a special Engineering Math Review. The ability of the review to shift the focus of students to a more conceptual understanding of mathematics was evaluated. The results clearly show that the review was successful in accomplishing this task.

The ideas presented in this thesis apply not only to the teaching and learning of mathematics, but any educative event. Recommendations are provided on how these ideas can be extrapolated and used to enhance various curricula throughout the Air Force Institute of Technology.

DEVELOPMENT OF A CONSTRUCTION FOCUSED PEDAGOGY FOR A GRADUATE ENGINEERING REVIEW COURSE IN SCHOOL MATHEMATICS

I. Introduction

Since 1970, the world's technical knowledge has more than doubled. Coupled with enormous advances in science and engineering, this explosion of knowledge has challenged the Department of Defense (DOD), and its many branches, to completely rethink traditional ways of doing business. Indeed, the requirement to incorporate such changes, and cope with the rate of change itself, may be the most critical task the DOD must carry out, whatever the external threat posed by the enemies of freedom.

The Air Force's Engineering community, which is responsible for developing and maintaining a vast fleet of aircraft and missiles, has been challenged as never before. In particular, organizations like the Air Force Institute of Technology (AFIT), who are charged with educating Air Force Engineers, have had to reorient and question the "traditional way of doing business" to meet the educational and training needs of their students and faculty. This has forced them to make a constant, and sometimes painful, reevaluation of curriculum and instructional paradigms.

The greatest challenges and requirements for engineering curriculum innovation have centered around the development and deployment of a pedagogy concerned with the language of engineering: mathematics. New technologies require new mathematical tools to solve the unprecedented problems of modern engineering practices. Traditional mathematical curricula, which were designed for an earlier time, and which tend to emphasize paper-and-pencil exercises giving little acknowledgment to the role computers can play in the employment of mathematics, no longer suffice. Models of learning that emphasize the transfer of knowledge rather than its creation, de novo, simply cannot motivate the use of mathematics toward the facilitation and construction of new knowledge.

In former times, the practice of engineering usually required a master's-level student to study and use mathematical algorithms dealing with familiar problems. Aside from an occasional inspiration in times of extraordinary stress, learning how to use and apply the algorithms invented by the geniuses of old was enough. Today, such a pedagogy no longer adequately prepares the student for his encounters with the real world. Modern engineering students need to gain a solid conceptual understanding of mathematics and to learn how to create new representations of the world using the language of mathematics.

Most well-known solution procedures of engineering mathematics were constructed in the pre-computer age, an age characterized by a level of activity far less complex than that produced by modern military weapon systems. The teaching of mathematics as a "bag of tricks" or a well-defined set of algorithms, the intelligent selection of which is equated with the useful application of mathematics, can no longer be viewed as a responsible pedagogical act. Why not? Because such an educational act misrepresents mathematics as a store of knowledge, rather than the active and creative process of constructing new models of reality that meet the requirements of novel problem solving.

In past years, AFIT's Engineering Math Review has taken a rule-based orientation to the teaching and learning of mathematics. In four short weeks, vast numbers of algorithms have been paraded before new students who, it is presumed, have seen them all before. Such a review was consciously built to bring students "up to speed" in skills assumed to have been previously acquired during their undergraduate studies. Apparently, no conscious attempt has ever been made to acquaint students with the creative side of mathematical work, or with the experimental nature of any practical application of mathematics within engineering disciplines. Consequently, most AFIT students view mathematics as a cleverly designed set of algorithms that one only needs to memorize and intelligently select from whenever a problem lending itself to mathematical resolution is encountered. The need to view mathematics as a constructive activity, an activity of the mind that generates new knowledge, is, at best, ignored. Perhaps this is because there is so little time; perhaps it happens because many instructors do not view mathematics in this way; or, perhaps it occurs because many students find such an orientation incredibly taxing and very different from the orientation to mathematics they received in traditional curricula.

Whatever the reason for such avoidance, this orientation to the review of mathematical knowledge is no longer viable. Indeed, an entirely new pedagogy for teaching and learning mathematics during the Engineering Math Review must be evolved. This thesis is a first attempt at creating such a pedagogy. Before getting to the heart of the problem description, some terms and concepts dealing with education in general need to be discussed.

Some Pedagogical Definitions

Consider a coach and a player going over a playbook together, or a writer reading a new manuscript to an editor. On these occasions, people are sharing ideas and trying to make sense of human experience. The aim of the interaction is a mutual understanding of a set of meanings and distinctions often code-like in brevity. The one who uses the document to help the other to understand is the teacher, the one who needs the teacher to "crack the code" is the student. The document itself represents the curriculum because it is a record of some prior event that is used as a guide to making subsequent events happen (3:24).

Note that both exchanges occur in a social setting. In this case, two people are using language to share the meaning contained in a document (3:24). The social setting can be oppressive, demeaning or mutually supportive, but whatever the social order, getting things done in this setting requires administration (3:25).

It follows that education is a social event that occurs when curriculum materials are taught to, and learned by, a student under reasonable conditions of control (3:28). Four distinct commonplaces which Schwab describes as teaching, learning, curriculum and milieu are involved. None of these is reducible to any other and each must be given full consideration in any educational environment (6:6). This thesis adopts Gowin's convention of replacing the word milieu with the term governance (3:25). Like Gowin and Schwab, it considers the interaction of these four commonplaces: learning, teaching, curriculum and governance, as the event-filled process that makes education happen (3:47).

The curriculum consists of the material taken from the general body of knowledge that is deemed appropriate as well as the structure of that material: the concepts, facts, methods, algorithms and their relationship to the real world and each other (3:109). Teaching is the activity involved in sharing meaning of the curriculum material with the students (3:62). Learning occurs when the student integrates the meaning of the curriculum materials with his or her past knowledge (3:124-125). It is important to note that the interaction among these three commonplaces is reciprocal. That is, the curriculum materials being taught may change, based upon what the student already knows (3:124). Further, the meaning of the curriculum material being taught may change due to errors found during interactions between teacher and students when sharing those meanings (3:75).

The interaction of curriculum, teaching and learning is controlled by governance (3:153). Governance is employed when the curriculum is constructed. The meaning of the curriculum materials is decided when students choose what they want to learn (3:186-187). Governance controls the meaning that controls the effort (3:154).

For example, testing is a form of governance. If students are being taught concepts, but are tested on techniques and algorithms, students may choose to focus only on what is testable because the grade received on the tests determines their success or failure in the class (3:155). Thus, tests govern what the students will choose to learn and hence, determine the meaning of the curriculum materials. The interaction of governance with the other three commonplaces and their local embedding within governance is depicted in Figure 1-1.

The Origin of a Pedagogical Dilemma

When a mathematical curriculum is rule-based and presented with the goal of teaching a set of algorithms, the temptation exists to make the curriculum no more than a list

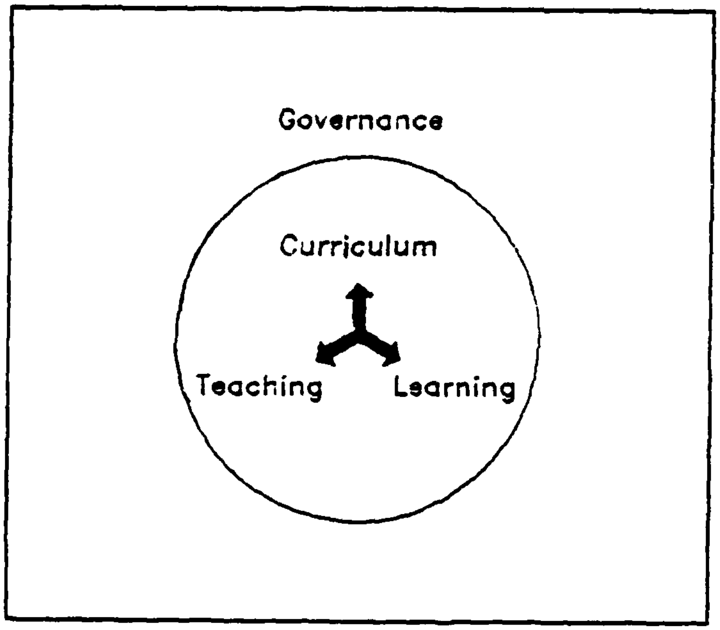


Figure 1-1. Interaction of the Four Commonplaces of Education

of topics to be covered. Usually, such topics are presented to the student in a sequence dictated by the order of their introduction in the course text. Then, little or no effort is made, either by the instructor or student, to interrelate the concepts of mathematics covered during the course. Consequently, the topics selected by the author of the text become the curriculum of the course. Little, if any, planning is devoted to curriculum development or instructional design.

Additionally, most mathematical texts are dominated by extreme formalism. Definitions and theorems are stated precisely, and many results are proved at a level of rigor that is acceptable to a working mathematician (1:227). Students find this formalism distracting at best, and at worst, completely unrelatable to what they already have learned at any point in the course. This causes most students to ignore such formalism and to concentrate on solving problems they believe are representative of the kind of problems they will encounter in upcoming tests (1:227). Such an orientation is reasonable when survival under a rule-based curriculum is required. Unfortunately, very little thought is given to the conceptual foundations of what is being learned. Further, such an approach precludes students from experiencing the usefulness of mathematical concepts and techniques in the real world.

Even more devastating is the clear correlation students observe between a particular type of problem and the algorithms proposed as the solution to the problem either by the author of the course text or by their teacher. Because of this, the use of mathematics starts to appear as the process of finding the right match between problem and extant algorithms stored away in some handbook. Thus, students are led to believe algorithm A should always be applied when problem type B is encountered. If algorithm A does not work, students often feel they have been taught the "wrong" material. If it does work, students seldom understand why. As long as the problem-solving exercises they experience in class are such "exercises of good fit," there is little or no motivation to understand why the mathematics work. After all, they reason, it is the teacher's responsibility to show them the right algorithm to apply in any situation they may be expected to encounter. This encourages students to establish a dependent relationship with their teacher and fosters the belief that "doing mathematics" is simply a matter of selecting an appropriate algorithm for a particular type of problem. There is virtually no evolving awareness of their need or responsibility for constructing new knowledge to solve a new problem, or even why this is necessary. They fail to appreciate that all known algorithms were constructed for a particular purpose and usually cannot be expected to work in the novel situations routinely encountered in their work as engineers.

In terms of the four commonplaces, a rule-based approach to any review of mathematics leads students to accept a passive and dependent mode of learning and posits blind application of algorithms to any problem as a legitimate way to use mathematics in the real world. Instructors reinforce this orientation by slavish adherence to textbook topics and explanations, by failing to demand a rigorous reconceptualization of the relevant mathematical material by students and by refusing to offer challenging, novel problems for students to solve. Encouraging problem solving by construction of new knowledge, rather than running to a book to find an algorithm that will provide a solution, is necessary if active, responsible learning is to be restored to the classrooms of the Engineering Math Review.

In addition, governance must create and support an active learning environment. It must be understood as the prime motivator of a milieu in which teaching, learning and curriculum function and actualize. Governance in all its forms--test procedures, class scheduling, class constitution, student classification and its many other forms--must ask students to manage their own learning and ensure each student understands the importance of learning and how to learn.

Textbooks and topic lists have a place in any classroom, but they must serve the greater need of challenging students to evolve a viable cognitive structure on their own initiative. The most crucial act of enlightened governance is ensuring students have an opportunity to share this process with their classmates and with their teacher. This is a call for effective management of the educational system. Administrators must realize students may find such demands overwhelming at first because they have become so habituated to requests for passive behavior in the classroom. Students and teachers will both need time to adjust to, and slowly move through, the levels of frustration any genuine learning experience creates. Compassionate support and consistent demands for objective displays of participants' (teachers' and students') current conceptual understanding by governance will greatly enhance the probability of success of such an educational undertaking.

Unfortunately, a great deal of evidence exists that shows such governance and such an environment for meaningful learning does not exist at AFIT, nor that any orchestrated movement exists to bring such an environment about. In fact, little thought appears to have been given to what must be done to create such an environment and generate a curriculum that will allow the concepts of mathematics to be rediscovered and reconstructed in a way that can instill deep and lasting appreciation for its usefulness. Therefore, the problem that motivated this thesis effort can now be stated.

Problem Statement

How can mathematics be taught so the student can meaningfully learn, and effectively use, mathematics to represent physical phenomena and, at the same time, introduce the student to practical methods for the self-development of learning problem-solving skills?

Research Objectives

To obtain a solution to this problem, three objectives were established. The following section introduces each objective and discusses its relationship to the problem under study.

Objective 1: To encourage students to take a more conceptual orientation toward mathematics by attempting to shift the focus of students from memorizing rules to mastering concepts, from getting a number to studying functional relationships, and from "plugging and chugging" with formulae to creating and experimenting with models.

Traditionally, engineering students have viewed mathematics as a tool kit that contains algorithms which will produce an answer to some problem, usually in the form of a number. The general requirement that real-world behavior be captured in terms of a mathematical model and that such models be amenable to repeated manipulation and experimentation never seems to cross many students' minds. Course work is viewed as just a series of hurdles to jump and the general use of mathematics is seen as a series of "plug-and-chug" exercises to be endured stoically and accomplished as correctly as possible. Unless a student understands that the engineer's application of mathematics involves representing real-world phemonena to study its behavior, the student cannot see the true power of mathematics. To restore meaning to mathematics and ensure a meaningful learning environment for mathematics exists, a change in emphasis and a shift to a more conceptual orientation toward mathematics is required.

Objective 2: To review the important mathematical rules and techniques of Calculus, Linear Algebra and Ordinary Differential Equations.

Adequate coverage of the fundamental rules and techniques of school mathematics is important for several reasons. First, the AFIT engineering community believes that basic mastery of the algorithms of Calculus, Linear Algebra and Ordinary Differential Equations is prerequisite to competent application of engineering technologies studied at AFIT. Second, meaningful learning of mathematics in a construction-based curriculum necessarily involves a solid understanding of basic concepts and algorithms before such construction is feasible or can be efficiently carried out in the time allotted for a review. Third, and perhaps most important, students who are being asked to study in a different way than they are used to will experience enormous anxiety unless they encounter familiar material and are asked to perform, at least in part, as they are used to performing. In order to meet all these demands and to ensure students are adequately prepared to encounter novel situations during the review, a thorough and structured presentation of the most basic rules and procedures of school mathematics was deemed to be a vital part of a construction-based curriculum.

Objective 3: To familiarize the student with AFIT's computing facilities and specialized mathematical software: MathCAD 2.0, Matrix-X and MacSyma.

Incoming students normally take a math review and a computer review as separate four-week courses during the summer review term. Experience has indicated that when these two courses are treated independently, students rarely perceive the computer as a source of assistance during mathematical work. Indeed, the computer is often seen as an additional burden and simply too difficult to master on top of all the demands for mathematical problem solving. In spite of this negative perception of their relationship, there were strong intuitions that an integration of the two courses would provide a synergistic mix of mathematical problem solving and computing that would serve to benefit the student in each domain and help demonstrate the complementary nature of the two domains.

The software chosen for emphasis was selected because of its graphical and computational power. MathCAD 2.0 allows students to visually represent and demonstrate difficult mathematical concepts. Matrix-X possesses incredible computational capabilities for matrix and linear algebraic manipulations. MacSyma, because of its symbolic orientation and ability to take over many of the tedious and memory-intensive acts of integration, differentiation and series construction, provides an ideal complement to the numerical facilities of MathCAD 2.0 and Matrix-X. Together, all three packages provide students with a toolbox of computer software that significantly aids them in their use and learning of mathematics. Using computers to facilitate their mathematical experimentation, students find more time is available to mentally conceptualize and construct mathematical representations.

By meeting this third objective, students would be given maximum encouragement to share their knowledge in a meaningful and interactive way. Templates and programs created by teachers and students to demonstrate functional behavior or concepts could be shipped and traded routinely among class members using telecommunication facilities provided by the AFIT network. This would, in turn, foster a sense of mutual concern for the learning efforts of all involved.

In short, there was every reason to believe that integrating the math and computer reviews into one review would enhance the possibility for the successful implementation of a construction-based curriculum.

Scope of Review

A special combined Math and Computer Review, whose aim was to fulfill the three objectives described above, was developed over a period of several months for presentation to the 21 GEO/GEP Engineering Physics students entering AFIT during June 1988. It was designed as a four-week review and consisted of two weeks devoted to Calculus, one week devoted to Linear Algebra, and one week of concentrated review of Ordinary Differential Equations. Approximately four years of undergraduate mathematical material was condensed into 18 two-hour presentations, with one hour a day devoted to an introduction to AFIT computing facilities and techniques for using MathCAD 2.0, Matrix-X and MacSyma. Some effort was made to address the problems of modeling real-world phenomena and a limited number of discussions was held concerning heuristics for learning. Because of the intense nature of the review and severe time constraints, the bulk of the lecture material focused on mathematical concepts and their application to real-world problems. Following the review, a questionnaire was administered in an attempt to obtain a descriptive evaluation of the success of the new pedagogy.

Limitations. The most significant constraint imposed on this review course was time. Meeting three hours a day for four weeks, the review required coverage of four years of mathematics along with an introduction to AFIT's computing facilities. This meant a great deal of material associated with Calculus, Linear Algebra and Differential Equations could not be covered. Careful selection of concepts to be addressed by the curriculum was accomplished by course instructors using personal selection criteria and by taking into consideration the stated preferences of faculty with AFIT's Department of Physics. Students' backgrounds and previous job experiences were given a great deal of consideration to establish what students could be expected to know at the start of the course.

It should be clear that whatever results accrue from this review, they only apply to this small test group. Any extrapolations and suggestions for future reviews will have to take into account new students' backgrounds and entering cognitive structures.

Finally, assessing the true benefits provided by such a pedagogy can only come after students have used what they learned in the engineering curriculum. Thus, evaluation of the success or failure of this pedagogy is limited to a descriptive statistical review and a subjective analysis of student reaction to the review.

Assumptions. Several assumptions were made about the entering of students' backgrounds. First, it was assumed that all the students would have undergraduate degrees in some area of engineering and that they would have previously encountered all the concepts to be covered in the math review. It was also assumed students would expect a broad coverage of standard algorithms and want to see how they could be applied to the real world.

Second, it was believed that most students would be coming to AFIT from a job where they would not have used mathematics for several years. However, some of the students in the class had recently graduated from school and were "up to speed" on most of the mathematics. It was believed that extended office hours and unlimited access to

AFIT's electronic mail facilities would provide ample opportunity to seek extra help and strike a balance between the two groups.

Finally, it was assumed that most students would be familiar with MS-DOS, the basic operation system of IBM personal computers. This assumption was based on the belief that students would have had access to IBM machines currently installed in most Air Force offices. However, it was believed students would not be competent with the application software and mainframe facilities to be used during the review.

Thesis Overview

This thesis proposes a pedagogy and an experimental curriculum and instructional system designed to enhance engineering students' abilities to construct and manipulate mathematical representations of real-world phenomena and to create the new knowledge they need to solve unprecedented engineering problems. Chapter I has been devoted to presenting and providing justification for the study of the problem that motivated the need for, and creation of, such a pedagogy. Chapter II offers a brief introduction to the learning theory that served as the foundation for constructing such a pedagogy. It also documents the model used to guide the curriculum development and instructional design of the Math Review. In Chapter III, this framework is used to outline the methodology employed to create the curriculum and formulate instructional methods for the Math Review. It documents the hypotheses and questions posed by this thesis and discusses the structure of the instrument used to evaluate the effectiveness of the new pedagogy. Chapter IV presents the results of the evaluation and their interpretation. Finally, in Chapter V, the conclusions and recommendations of this thesis are outlined and suggestions are made for future research.

II. Theory

The problem exposition in Chapter I proposed that a meaningful learning environment needs to be created in the AFIT short-term math review so students can acquire the ability to effectively use mathematics when solving real-world problems. It was assumed such an environment would foster adequate assimilation of mathematical concepts, their connection to the real world, their use in the creation of mathematical representations of physical phenomenon, and their manipulation by mathematical algorithms. Developing a pedagogy for such an environment required a theory of learning and a viable model of education. The purpose of this chapter is to introduce the theory which served as a basis for the pedagogy of the Engineering Math Review and to describe the model that provided a framework for construction and evaluation of this pedagogy.

Gowin's description of the interaction of the four commonplaces provides the basic context for the model of education that motivated this thesis. The learning theory employed is Dr. David Ausubel's, a theory which has been artfully extended and whose practicality has been enhanced by Gowin and Novak through their invention of two heuristics for learning: the Concept Map and Vee.

Understanding the interaction of the four commonplaces and how Ausubel's learning theory can facilitate the creation of a meaningful learning environment requires a thorough study concerning how the four commonplaces can support such a theory. Thus, in this chapter, I attempt to clarify how each of the commonplaces can contribute toward the production of a meaningful learning environment. The model that was used to guide the development of the curriculum and instructional design used during the Engineering Math Review is also introduced. To start, each commonplace, and how it can employ Ausubelian learning theory for this purpose of creating a truly meaningful learning environment, will be discussed.

Learning

In any educative event, the primary concern is with learning (4:21). Learning changes the meaning of human experience through an active reorganization of meaning (3:124). This implies then, that the problem of learning is to make connections between new material to be learned and what the learner already knows (3:124). This is the basic premise on which Ausubel's theory of meaningful learning is based.

According to Ausubel's theory, learning is accomplished using concepts which describe some regularity or relationship within a group of facts and designated by some sign or symbol (4:18). The primary contrast in Ausubel's theory is meaningful learning versus rote learning (6:7). Rote learning occurs when new knowledge is arbitrarily incorporated into a person's knowledge structure without interacting with what is already there (6:7). Meaningful learning, on the other hand, requires the integration of what needs to be known with what the learner already knows (3:124).

Meaningful learning involves a conscious effort on the part of the learner to relate new knowledge in a substantive, nonarbitrary way to relevant existing concepts or propositions in the learner's cognitive structure (5:456). This type of learning does not merely involve adding new concepts, but rather calls for new knowledge to be assimilated with existing knowledge (5:456). Assimilation is not simply a matter of associating new concepts with existing concepts, but requires the process of subsumption (5:456).

Subsumption is the continuous differentiation and integration of new concepts with existing concepts held in the learners cognitive structure. In other words, new concepts are added to existing concepts in terms of their differences and similarities. Differences and similarities between concepts are made clear by the use of propositions--a phrase, rule or principle that explicitly relates the concepts being learned (6:15).

The meaningful learning process is complete when integrative reconciliation occurs--when all the general concepts and their respective subsumers are differentiated and integrated with each other (5:457). An example might help clarify what is meant by integrative reconciliation.

Suppose students are studying both calculus and economics. The two subjects may appear to be unrelated. However, economics does make use of calculus, specifically derivatives, when discussing the concept of marginal cost. If meaningful learning in economics is to occur, the calculus concept of derivatives, along with its subsuming concepts, must be integrated with the economic concept of marginal cost. Also, the derivative concept should be subsumed by the application of calculus to economics. The meaning created by the integration of both calculus and economics concepts illustrates the process of integrative reconciliation that is required by meaningful learning theory (5:457).

Although meaningful learning is the desired outcome in most cases, rote learning does play a role in the educative process (4:26). Sometimes it is necessary for students to remember a concept or idea in its exact form. For example, when studying complex algebra, the number i, the square root of negative 1, must be rotely memorized because it is typically used without derivation. However, once memorized, the meaning of i can be modified by subsumption into the concept of complex variables, allowing meaningful learning to occur. The real point to be made here is that rote learning and meaningful learning are not mutually exclusive, but rather represent the extremes of a continuum. That is, meaningful learning of some new concept may require the rote memorization of the specific concept prior to the occurrence of subsumption and integrative reconciliation, before meaningful learning can occur (4:101).

Learning is not a passive activity because the subsumption and integrative reconciliation processes require a great deal of effort on the part of the learner. Putting forth this effort is the sole responsibility of the learner (3:124). However, learning cannot occur without a teacher to encourage and aid the learner in clarifying the relationships between concepts (3:133).

Teaching

The role of the teacher is to bring about a common understanding between student and teacher of the phrase, rule or principle needed to make the relationships between critical concepts explicit (3:62). That is, the teacher should seek to help the learner see what relationships exist and what the significance of those relationships is. But, this should not simply entail a transfer of ideas from teacher to student (7:16). Rather, the teacher must encourage students to put forth sufficient effort to discover the relationships for themselves (7:6). By doing this, an instructor fosters the meaningful learning process, keeping the onus on the learner to assimilate new knowledge with existing knowledge.

The accomplishment of common understanding between teacher and student of the concepts and their relationships depends on knowing the concepts and their relationships. The subtlety here is the difference between learning and knowing. Learning is personal and idiosyncratic (6:5). That is, what was chosen to be learned and how it was learned is a matter of personal preference. But most of all, it is internal to the learner. Knowing, on the other hand, is public and shared (6:5). It requires the externalization of what has been learned. Thus, both teacher and student must be able to publicly share what has been learned if the goal of teaching is to be met.

It follows that knowing is preceded by learning. That is, knowledge cannot be shared with others until it has first been learned. But where do the concepts and their relationships that are to be learned, and eventually known, come from? They come from the curriculum.

Curriculum

Curriculum is defined as a logically connected set of conceptually and pedagogically analyzed knowledge and value claims (3:109). This definition implies that the concepts to be taught and learned must logically follow from each other. This connectedness is required to facilitate the meaningful learning process (3:109). Further, the concepts to be learned are chosen as a result of consideration by the teacher of what the student already knows and what needs to be known (3:124). Finally, the definition implies that the concepts and their relationships have been externalized and are known (3:109). Thus, the curriculum represents a body of material that can be taught in a manner that facilitates meaningful learning.

Suppose the instructor chooses to teach the curriculum in a manner such that the information being received by learners could be admitted into their cognitive structure. That is to say, the concepts and their relationships were presented to the students by the teacher. The students then could store that information as it was presented. This type of an approach is referred to as reception learning. The teacher defines the structure of the material and students admit it into their cognitive structure in a similar form. Thus, the teacher provides ample guidance in directing the student's learning efforts (4:100).

But the teacher could also approach teaching the curriculum from another angle--teaching in a manner that provides minimum guidance in directing the learning efforts of students. This type of approach is referred to as discovery learning (3:100). In discovery learning the learner chooses what to learn and goes about learning it.

The first point to be made here is that the instructional approach used to present the curriculum affects learning. Second, the type of learning precipitated lies on a continuum between reception learning and discovery learning, and finally, that the continuum of reception and discovery learning is distinct from that of meaningful and rote discussed earlier (4:100). The difference lies in the fact that the former represents the effect on learning due to the instructional approach being used, and the latter represents the form in which information is acquired in cognitive structure (4:100). Figure 2-1 presents the interaction of these two distinct dimensions of learning.

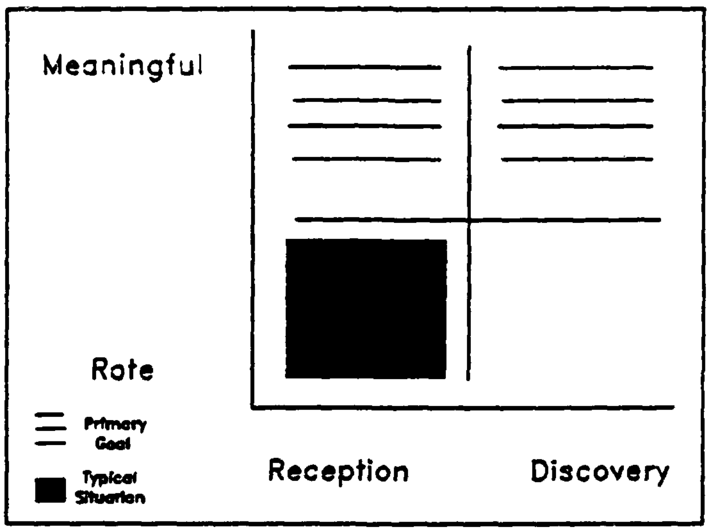


Figure 2-1. Distinct Learning Continuums and Their Interaction
(Adapted from 6:8)

Previously, it was established that meaningful learning was preferred over rote learning. However, it was pointed out that some rote learning may have to occur prior to the occurrence of meaningful learning. Preference, in terms of reception learning versus discovery learning, is not so clear-cut. However, few educators today would go so far as to make no effort to guide students in the selection of study material. Because of this feeling, some form of reception learning is the most common type of approach used in education (3:100). It follows then, that the primary goal would be to implement some form of meaningful reception learning when teaching the curriculum to students. Unfortunately, the current math review appears to support and promote rote reception learning: the darkened region of Figure 2-1. Getting meaningful reception learning to occur requires appropriate teaching methods and proper application of governance.

Governance

Governance is the exercise of power in a social setting and is required to orchestrate the administration of learning, teaching and curriculum (3:153). Governance controls the meaning that controls the effort (3:154). It plays a role in what the learner chooses to learn and what the teacher decides to teach. But most importantly, governance determines whether or not the educative event can occur.

Proper governance will focus the efforts of the teacher on sharing meaning. Further, it will foster students and teachers working together over the curriculum until congruence of meaning is achieved. Also, it will motivate students to take responsibility to integrate new knowledge being learned with existing knowledge. Finally, proper governance will precipitate the development of a curriculum that reveals the structure of the knowledge to be shared and learned, and also focus the efforts of students on learning the concepts and their relationships presented in the curriculum. In other words, students will learn what was intended for them to learn, assuming the students are willing to put forth the required effort and can see the value in doing so (3:154).

A Meaningful Learning Environment

A meaningful learning environment is an environment in which the four commonplaces interact in a manner which allows students to learn the curriculum materials being taught meaningfully. In such an environment, learners admit information into their cognitive structure via subsumption and integrative reconciliation. The role of teaching is to provide direction and seek congruence of meaning of curriculum materials between student and teacher. The curriculum consists of the concepts to be learned and their relationships, and the governance should try to focus the efforts of all parties involved in the educative event to accomplish these tasks.

The Achilles' heel of mathematics education at AFIT is the oppressive role played by governance. Time constraints and a testing environment that pay high premiums for rote memorization and blind application of algorithms tend to discourage an active and aggressive quest for a meaningful learning environment. Indeed, pressures for increasing the quantity, rather than the quality, of concept coverage often overwhelm sincere attempts to make learning more meaningful and a more thoughtful exercise at AFIT. One has a hard time pointing fingers because every individual student and instructor generally supports such a move. It is clear we are dealing with a systemically induced pathology and not some conspiracy on the part of students or faculty to prohibit the emergence of a more meaningful learning experience. The problem is that such systemic pressures, which are the natural outcome of jam-packed programs and an all-too-short 15- or 18-month time frame for completion of all course work and an individual thesis, make any attempt to move to a more conceptual presentation of mathematics difficult, if not impossible. Fortunately, by introducing the two learning heuristics: the Concept Map and the Vee, Gowin and Novak have provided concrete tools that have found application in this governance-dictated environment. While these heuristics were designed to encourage meaningful reception learning as well as greater self-management of student learning, they also serve quite well when rote learning becomes the only option. Because of their power and utilitarian nature, a discussion about their purposeful application in a meaningful learning environment is in order and follows.

Concept Mapping

Meaningful learning requires the process of subsumption and integrative reconciliation. As described earlier, subsumption involves modifying a general concept with subsuming concepts via a proposition. Concept mapping is a technique to document the meaningful learning process. It provides a means to externalize concepts and propositions that link those concepts (6:17). An example of a concept map is given in Figure 2-2.

In this example, the mathematical concept functions is the subsumer. The subsumed concepts are: relations, elementary, inverses and composite. Each of these is linked to the subsumer by the phrases: can be derived from, can be and may be. Such links objectify the propositions in the mind of the learner. The example points out how a subsuming concept can represent a more general notion with links to concepts that can, in turn, serve as subsumers. It also points out how subsumed concepts serve to modify and differentiate the subsuming concept. Such integrative reconciliation is what meaningful learning really involves.

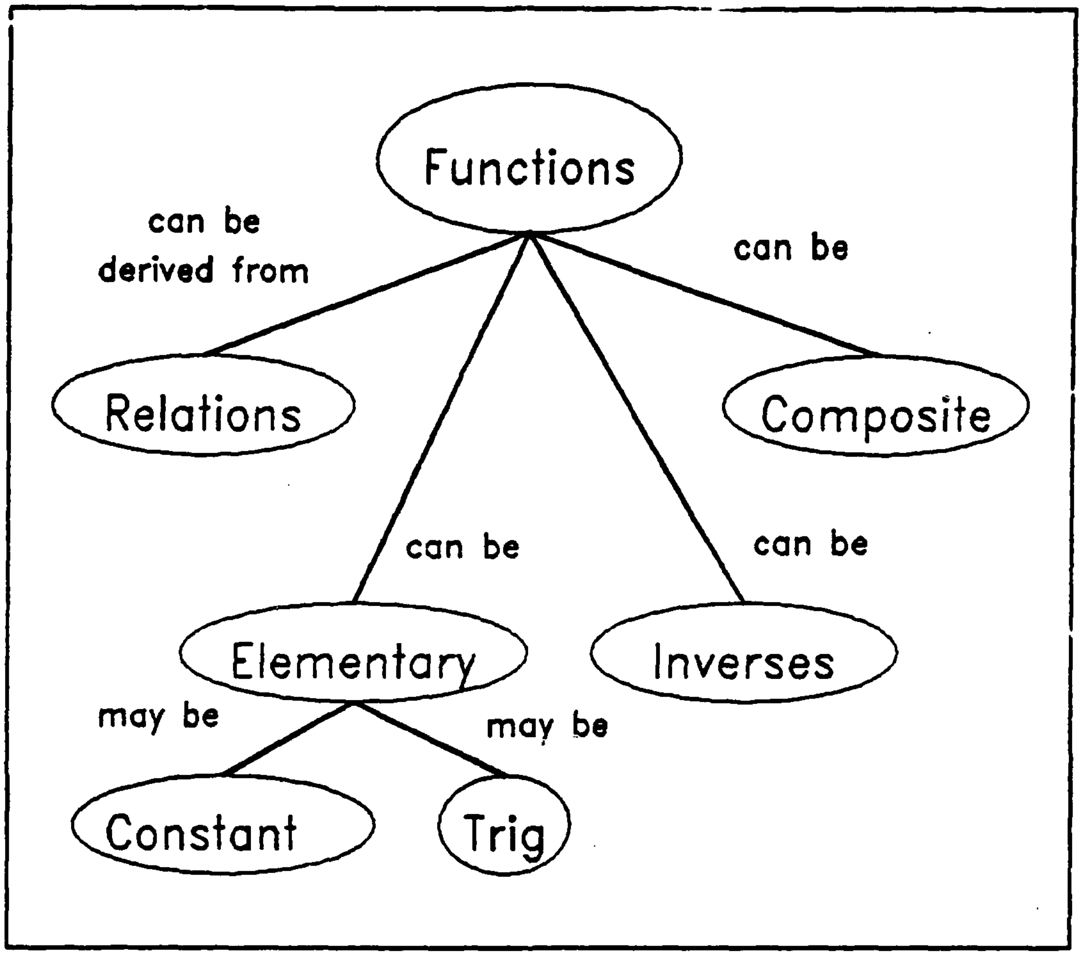


Figure 2-2. Example Concept Map (Adapted from 6:16)

Consider the usefulness of this tool. If this concept map was produced by a student, it would reflect the learner's current conceptualization of functions. Gowin and Novak point out that the accuracy of the representation in terms of the concepts possessed by the learner and their relationships is only conjecture at this time (6:17). But, suppose the student claimed that this map accurately represented his or her understanding. The teacher could then use the map to determine whether any misconceptions exist. For instance, in this case, the teacher could point to the fact that the student has accounted for only two of the six elementary functions.

Note, however, just because the concept map is missing four of the six elementary functions, it does not necessarily mean a misconception exists. The student may have chosen not to show them although he or she understood they were there. Clearing this up requires dialogue between teacher and student about the concept map. Through conversation, the map can be evolved to better reflect what the learner knows.

As with the learner, the teacher can also construct a concept map. This externalizes the knowledge of the teacher and facilitates the sharing of meanings: an act cited earlier as the sine qua non of a meaningful learning environment. From a different perspective, the teacher might also use this concept map to determine what needs to be taught. Thus, the concept map may represent the curriculum over which congruence of meaning should be reached between teacher and student.

The concept map may also be used to ensure proper governance is applied to the learning environment. If the map is employed to represent the curriculum, the learning efforts of the students and the teaching efforts of the teacher can all be focused on the sharing of meaning and the learning of that material. Focusing the efforts of all involved to ensure students learn what was intended for them to learn is the result of properly applied governance (3:154).

The concept map provides a means to document how the subsumption process occurs and reflects what the student understands to be true. However, it does not reflect the steps involved in assimilating new knowledge with existing knowledge, nor does it demonstrate how new knowledge is created. This is the province of the Vee heuristic (6:57). The generic form of the Vee is presented in Figure 2-3.

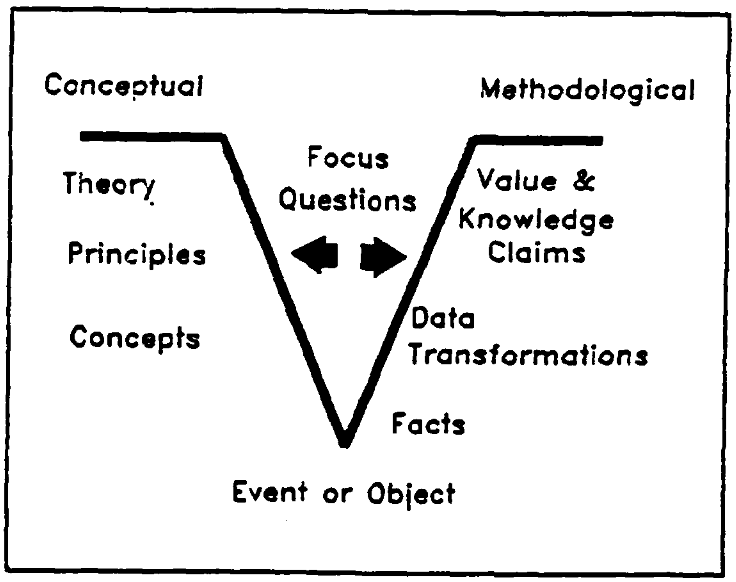


Figure 2-3. Generic Form of the Vee Heuristic (Adapted from 6:56)

As the name implies, this heuristic is in the shape of a V. Gowin and Novak point out that there is nothing sacred or absolute about the shape, but have found it valuable for two reasons. First, the Vee points to the object or event under study, which they claim to be the root of knowledge production (6:57). That is, new knowledge cannot be created without an object or event about which knowledge is sought. The point of the Vee signal makes the learner less vulnerable to collecting information irrelevant to the problem or failing to see the meaning of the data being generated (6:58). In other words, it keeps knowledge creation focused on the event or object under study. Second, the shape helps students see the interplay between existing knowledge constructed and modified over time, and expands their appreciation for how extant knowledge can be used to create new knowledge.

The Vee heuristic consists of four basic parts: the left side, the event or object under study, the focus question and the right side. The left side of the Vee is the conceptual side. It contains all the theories, principles and concepts the learner uses to study a particular event or object. This side of the Vee may contain a concept map developed by the learner such as the one in Figure 2-2. All these concepts are brought to bear upon the event or object under study. A focus question must be asked in order for those concepts to be useful and to determine the appropriate event or object to be studied. The right side represents the actual construction of new knowledge by the transformation of facts into data. These, in turn, are transformed into new propositions that provide an answer to the focus question. It also documents the value of any knowledge claim.

Suppose a student is using the Vee to determine the temperature at which water boils under normal atmospheric conditions. The first step in using the Vee would be the development of a focus question. In this case, the focus question is: at what temperature does water boil under normal atmospheric conditions? Next, an appropriate event would have to be chosen to study, and subsequently reach a solution to the focus question. The appropriate event in this case may be observing a pan of water, with a thermometer in the water, being heated on a stove (6:63).

The left side of the Vee contains the theory, principles and concepts about the effects of temperature on water. For instance, the concept of water as a solid, gas or liquid is presented on the left side of the Vee. The left side contains an expectation of what will occur when the event is studied. This is important because of what was said earlier about meaningful learning. That is, that learning is the making sense of human experience for oneself. Even though the student has been told that water boils around 100 degrees Celsius, it cannot be meaningfully learned until verified by the student. With these concepts in hand, a move can be made through the event to the right side of the Vee (6:63).

The right side proposes records will be made of what happened during the event under study. Temperatures will be taken when bubbles start to form and boiling begins to occur. From these records, the knowledge claim that water boils around 100 degrees Celsius can be made (6:63). Figure 2-4 shows the Vee used for this example.

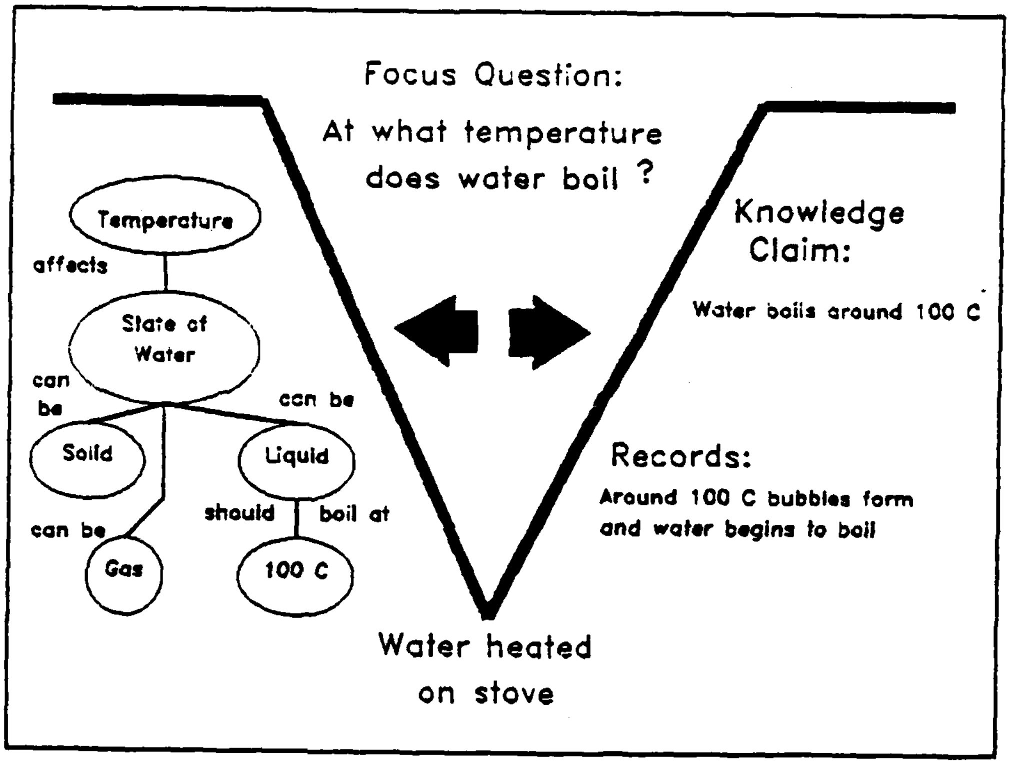


Figure 2-4. Vee Used for Boiling Water Example
(Adapted from 6:63)

Once this knowledge claim is made, the new knowledge can be related back into the left side of the Vee. This may precipitate new focus questions about the same event, leading to further knowledge claims about the states of water, effects of pressure on water or how the temperature of boiling water changes over time. The point is, the left and right side of the Vee interact with each other in the creation of new knowledge. Thus, the student learns there is always a mutual interplay between concept and method.

For the learner, then, the Vee is a way of visualizing how new knowledge can be created from, and assimilated to, existing knowledge. It provides a means for objectively picturing the process involved in the construction of new knowledge from an existing knowledge base. This same tool can help teachers externalize the process used to create knowledge in their chosen disciplines. The Vee provides a means of relating concepts and new knowledge in light of the real-world event from which such knowledge is derived. By using the Vee as an evaluation tool, students can be encouraged to concentrate on constructing new knowledge. In short, the Vee is an effective means for implementing governance that supports students taking charge of their own learning.

To reiterate, a meaningful learning environment is an environment in which the four commonplaces interact in a manner which encourages students' self-management of their learning of a curriculum presented by a teacher. Concept mapping and the Vee heuristic are, therefore, effective tools for fostering the maintenance of a meaningful learning process. The discussion will now focus on the framework that was used to create the curriculum and instructional design for the short-term math review.

Creation of a Meaningful Learning Environment

Figure 2-5 shows a simplified version of Johnson's model for curriculum and instruction design that was used to provide a framework to create the curriculum and instructional

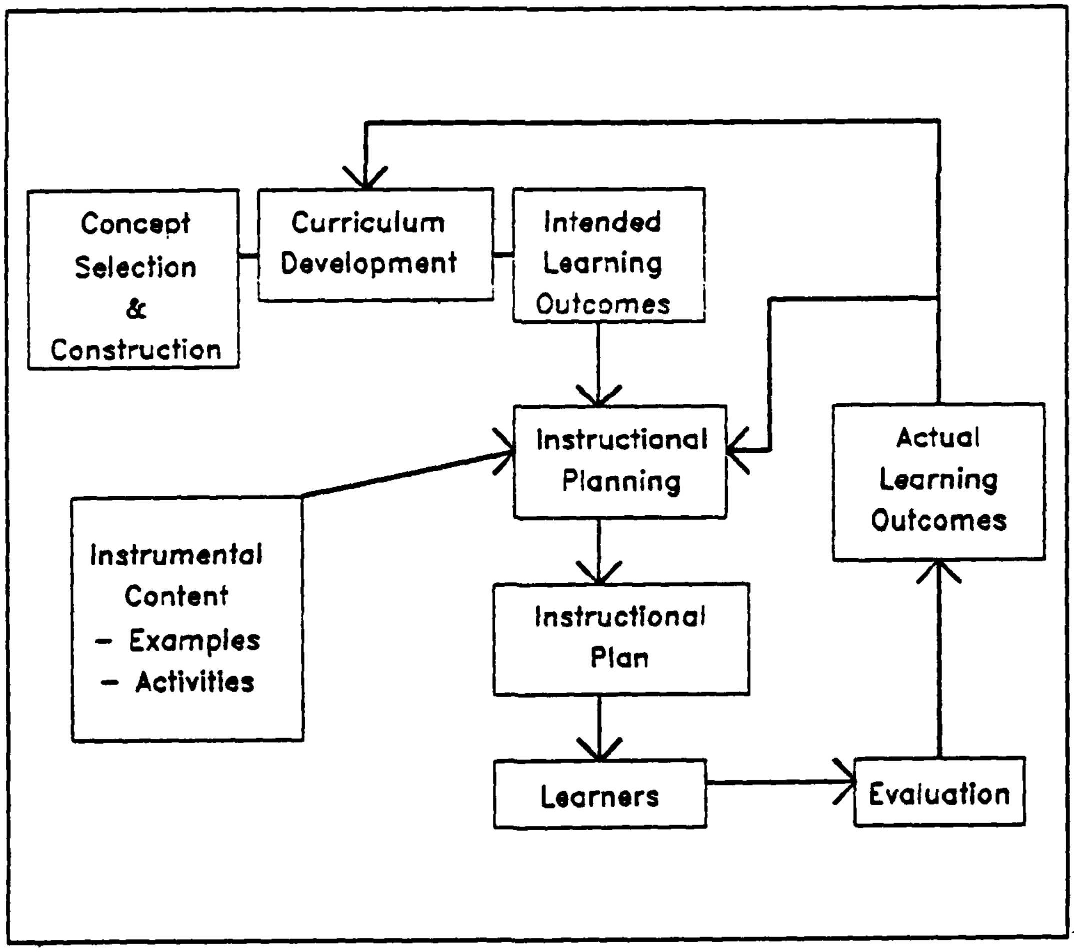


Figure 2-5. Simplified Version of Johnson's Model for Curriculum and Instructional Design (Adapted from 4:132)

techniques of the math review. The heart of this model is the box labeled Curriculum Development. Such development is strictly based on selecting and organizing a hierarchy of concepts. Instructor-produced concept maps facilitate this process. The Vee can be used to determine when concepts should be introduced during the course, and as an explanatory device when the origin and construction process that led to the invention of key mathematical concepts is discussed in class. The cognitive structure that emerges from such dialogue generates a curriculum that fosters meaningful learning (4:137).

Once the curriculum is established, the model proposes that intended learning outcomes be defined; that is, the matrix of concepts and relationships students should learn. Following this step, planning can begin to determine what instructional methods should be used to encourage students to self-manage their learning activities. This will involve them actively mastering material constituting the course curriculum. Such planning should include the construction of class examples and involve discussions concerning what will be used to clarify the meaning of the curriculum materials. The textbook for the class must be selected with great deliberation. Orchestrating the products of this planning process into some final form constitutes instructional planning (4:138).

An instructional plan is then developed that will be used to teach the curriculum materials to the student. The model proposes that concurrent with, and upon completion of, any educational event, an evaluation be performed to determine what students actually learned. Typically, this evaluation is in the form of an exam. During the math review no tests were given. Instead, students were encouraged to develop concept maps, use these maps along with the Vee heuristic to provide objective evidence of their growing conceptual mastery of course material, and to work at verbally explaining and justifying their maps to fellow students or faculty. Also, students were encouraged to develop computer programs and MathCAD 2.0 templates to illustrate their mastery of the mathematical concepts being taught. All of these were used to evaluate students' actual learning (5:139).

The goal of the educative process is for the actual learning outcomes to meet or exceed the intended learning outcomes. Failure to meet this goal may be caused by curriculum problems, poor selection of examples, or time constraints that lead to rote learning because the pace is too fast. Note that such failures may not include all the class members. Some may learn what was intended for them to learn while others may not. This may be due to differences in learning rates of students. Whatever the case, any failures to meet or exceed intended learning outcomes require changes to be made to the instructional plan (4:139).

It is important to note that planning should be continuous; indeed, it must be continuous whether understood to be so or not. Changes to the instructional plan must be made on a daily basis using feedback that, ideally, is actively solicited from students. If given a chance, students will be happy to provide input to the teacher about the process they are undergoing. Suggestions for improvements in any of the four commonplaces ought to be welcomed and creatively incorporated into the instructional plan whenever possible (4:139).

Chapter Conclusion

In this chapter, it has been shown that the pedagogy and experimental curriculum and instructional system designed by this thesis had to be based on an adequate theory of meaningful learning. It has been demonstrated that the Ausubel, Novak and Gowin theories and heuristics provide necessary and sufficient principles and concepts to satisfy such a requirement. Additionally, this chapter has devoted a good deal of space to proposing what should be considered during any attempt to create a meaningful learning environment of the teaching of mathematics. Finally, a framework to guide the construction of both the curriculum and instructional design was discussed. Chapter III will use this framework to present the methodology employed by this thesis to meet its objectives and to seek and evaluate hypotheses that were used to generate questions required for the evaluation of the entire effort.

III. Methodology

Chapter II pointed out that any educative event occurs within the auspices of the four commonplaces of education. If the learning environment in which these educative events occur is to be meaningful, the interaction of these commonplaces must promote students' self-management of their learning with respect to a curriculum presented by a teacher. Accomplishing this requires a curriculum that is organized so that, when taught, students can assimilate required concepts into their cognitive structures. This means the curriculum should be a matrix of concepts arranged in terms of their relationships with each other and be organized so knowledge can be gainfully employed to create new knowledge. Novak states that developing a curriculum consistent with Ausubelian learning theory can be accomplished by implementing Johnson's model (4:129).

The beauty of Johnson's model is that curriculum and instructional design development are formulated in terms of the four commonplaces. That is, what students are expected to learn, how to teach this material to the students, and evaluation of what students actually learned are given due consideration. This model encourages course designers to think in terms of appropriate governance. It implies that governance should be involved in choosing concepts to be taught and assessing what students actually learn during development of evaluation methods. These features led to the adoption of Johnson's model as a guide for developing the curriculum and instructional design of the Engineering Math Review.

This chapter discusses how Johnson's model was implemented and describes the resulting curriculum and instructional design employed in the math review. For the most part, the discussion will track the developmental sequence recommended by the model. To facilitate this discussion, the model is presented once again in Figure 3-1.

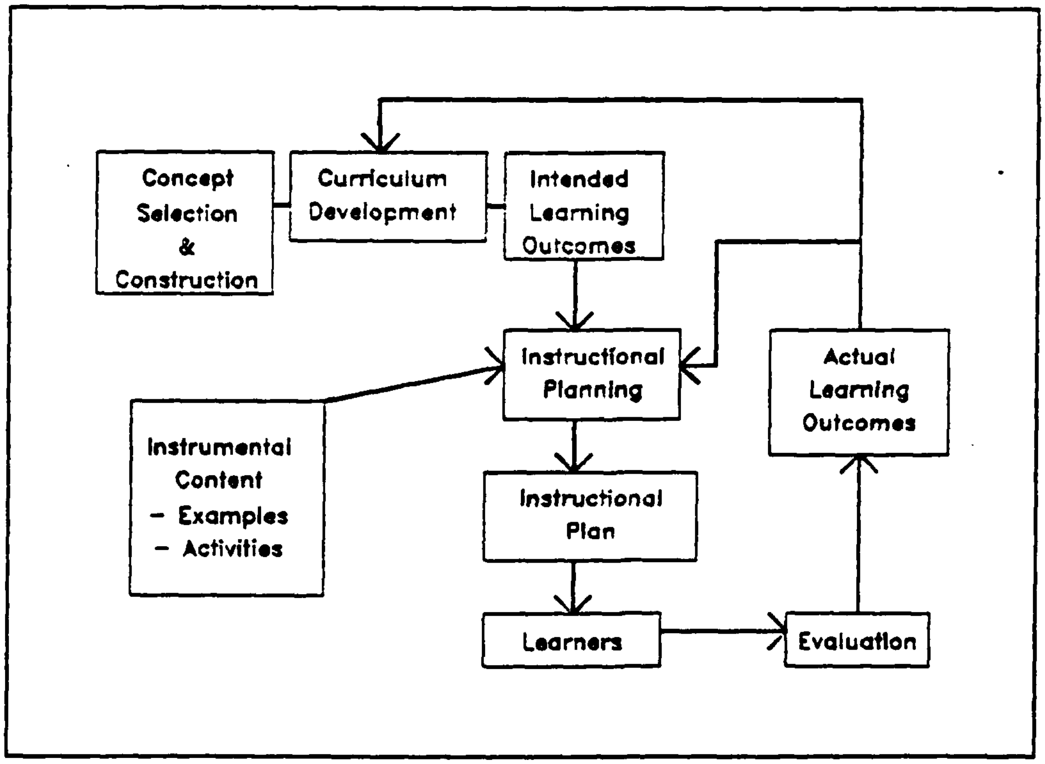


Figure 3-1. Simplified Version of Johnson's Model for Curriculum and Instructional Design (Adapted from N:132)

Creating the Curriculum and Instructional Design

The curriculum and instructional design for the math review was created by a four-member team. The team consisted of Professor Dan Reynolds, Dr. Ted Luke, Richard Lamb and Captain Jerry Edwards. Professor Reynolds, for all intents and purposes, was the team leader. He was very heavily involved in all aspects of the curriculum and instructional design. Dr. Luke's primary role was to provide inputs from the Physics department to ensure the team's efforts kept in line with the needs of the Physics department. The roles of Rich Lamb and Jerry Edwards were more specific.

Rich Lamb's role was to provide detailed insight into the mathematical concepts and their relationships that needed to be taught. Further, he provided background material on how and why key mathematical concepts were developed in the first place. He also helped teach the course with Professor Reynolds. Captain Edwards was responsible for developing computer examples and managing computer-facilitated learning activities that were carried out by students to reinforce the concepts being taught. He also provided student input to the team about ways the curriculum and instructional design should relate to the computer portion of the review.

Development of the curriculum and instructional design by the team began with the selection of concepts to be taught and concept mapping their relationship. The concept map detailing the hierarchical relationship of these mathematical concepts took several weeks to accomplish. The general topics to be presented, Calculus, Linear Algebra and Differential Equations, were determined to be the topics necessary to adequately prepare students for their future course work based on input from the Physics department. The need for covering these topics was also confirmed by means of a questionnaire given to upper-class students completing the Physics program. The reason these three topics were chosen is that the real world is comprised of systems that change. Calculus provides the language needed to represent change, Linear Algebra provides the tools needed to work with complex systems, and Differential Equations provides the language required to represent systems that change. The concept map for the course was discussed in several meetings with selected members of the Physics and Math departments to ensure its relevance. After completing these steps, the concept map became the foundation for curriculum development and is presented in Figure 3-2 on the following page.

Once the curriculum was established, intended learning outcomes could be identified. To facilitate this process, a syllabus was created to show what would be covered each day and what was intended for the student to learn. For the math review, the intended learning outcomes focused on the conceptual nature of mathematics, the relationship between

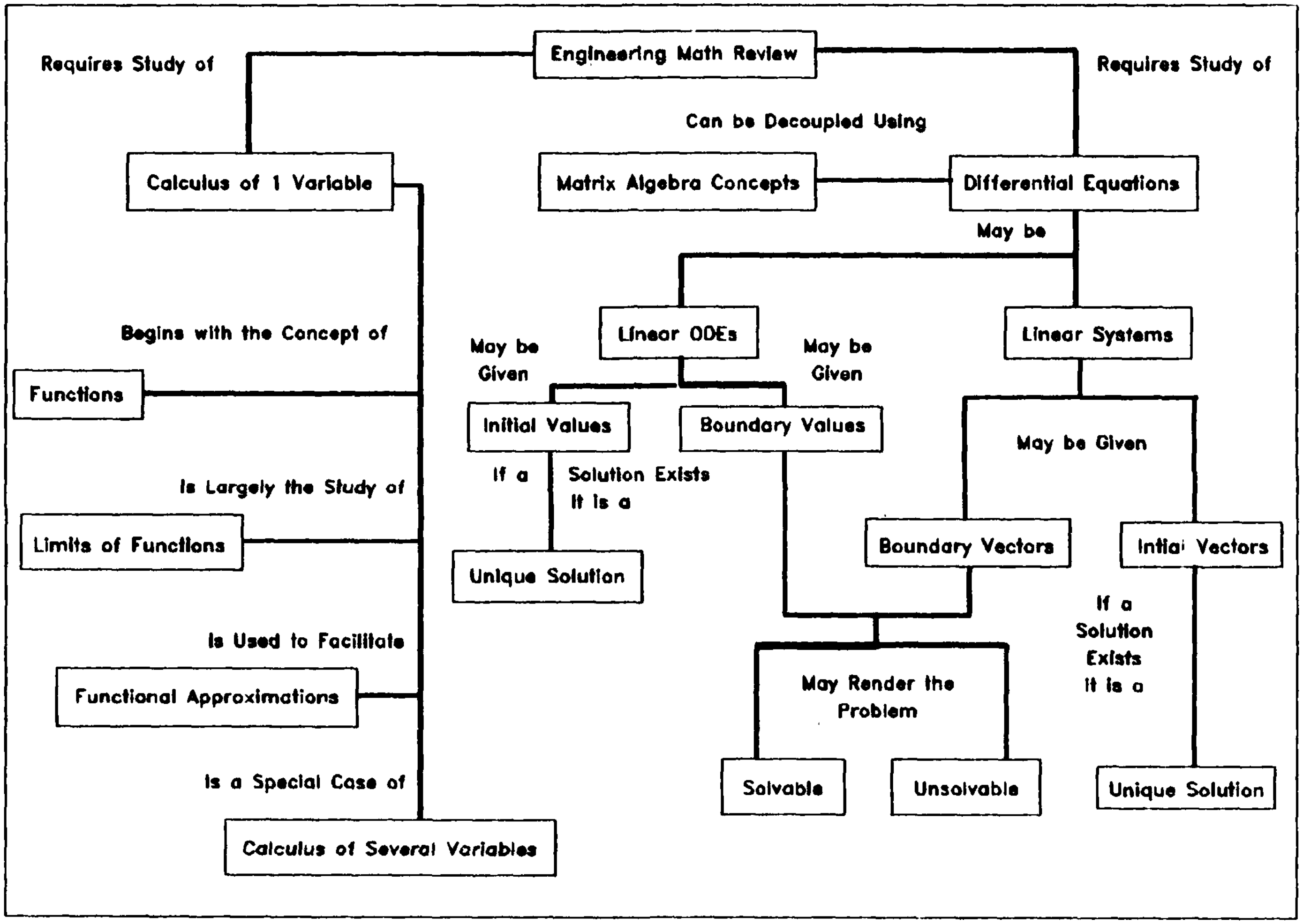


Figure 3-2. Concept Map Created for Math Review

the concepts and the real world, and how those concepts were used in the development of the methods of mathematics. These ideas were spelled out for each day of the review for each topic area.

The syllabus included problems students should work to reinforce the concepts being taught. It also suggested computer templates and programs to be developed by students. Each day students were provided detailed class notes containing the material to be covered for that day. Examples to be used in class to reinforce what was being taught were included in these notes. These notes, along with the syllabus, constituted the material portion of the instructional plan for the course. A copy of the syllabus and selected MathCAD templates can be found in Appendices A and B, respectively.

A mathematics book especially designed for engineers was chosen as the course text for the Calculus review. Notes were provided to students in lieu of a course textbook for the review of Linear Algebra and Differential Equations. A sufficient amount of derivation and adequate number of examples were provided, giving the students plenty of reference material to support their learning efforts.

The book used for the Calculus review, The Calculus Tutoring Book, was written by Robert and Carol Ash and published by IEEE Press. This book was chosen for use in the review because it uses informal language and stresses the underlying physical and geometric ideas of mathematical representations. In the book, Ash and Ash provide explanations for the ideas and procedures of engineering mathematics that reflect the way engineers actually think, then verbally and visually focus on the construction of mathematical representations that should be used to solve problems. They provide intuitive explanations concerning why a result is true and encourage students to think about mathematics in a way that facilitates student responsibility for their own learning of mathematics. The book's emphasis on construction and genuine encouragement for self-management of learning made it possible to present a construction-based approach to the teaching and learning of mathematics and ensured the objectives of this thesis had a chance of being met (1:230).

The final part of instructional planning considered ways governance could be employed to foster meaningful learning of curriculum materials. Remember, governance is the power that controls the meaning that controls the effort (3:154). From the outset of the review, the instructors emphasized the knowing of mathematical concepts, their relationship, and how such concepts could be used to construct new mathematical knowledge. Further, it was stressed that the sole responsibility for learning the materials presented in class rested with the students and that the instructors would do as much as possible to facilitate that process. Facilitating the learning process was accomplished by a construction-based teaching approach and the extensive use of mathematical software to help students discover on their own, how and why key concepts of mathematics served the purposes of engineering problem solvers.

The goal of the teaching approach was to encourage meaningful reception learning. This was accomplished by requests for concept maps and by promoting the use of the Vee by students to help them create and assimilate new knowledge. Class examples dealing with real-world phenomena were presented in the manner recommended by the Vee heuristic, so students could see how mathematical concepts served to represent the real world and can be dynamically used to gain new knowledge about real-world systems. In many instances, selected students were asked to work problems on the blackboard with the assistance of the rest of the class. Free and open dialogue was encouraged to clear up any misconceptions anyone might have concerning the examples or the concepts being discussed. For those needing extra help, extended office hours and unlimited access to AFIT's electronic mail facilities provided ample opportunity to seek help if required. Throughout the review, students were encouraged to construct concept maps to represent their current cognitive orientation and to display their understanding of the subject. These objectifications of knowledge helped clear up many conceptual misunderstandings.

Students were also provided with a copy of MathCAD 2.0 and had access to MacSyma and Matrix-X. These software packages gave students the opportunity to dynamically experiment with, and study the behavior of, the mathematical representations being taught. Using MathCAD's extensive graphics capability, students could actually verify the theory of limits and could see that the derivative of a function truly represents the instantaneous rate of change of some physical phenomena. Using the symbolic and computational power of MacSyma or Matrix-X, students could quickly derive the rules and algorithms applied to a variety of real-world problems. These packages allowed students to make changes to a representation and see the impact of such changes on model behavior. The use of computers in this fashion promoted meaningful learning of mathematical concepts and motivated self-management of learning in and outside the classroom.

Johnson's model requires an evaluation plan be constructed to assess what was actually learned by the students. Evaluation for this review was not accomplished by giving exams. Rather, students were asked to construct concept maps, Vee heuristics and computer templates to demonstrate their knowledge of mathematics. Throughout the course, feedback related to this submitted work was provided to the students by the instructors to clear us misconceptions, if any existed. When a student produced exceptional work, the work was shared with the class to further explicate the concepts and topics being discussed. This type of evaluation allowed instructors to identify the concepts that had been learned and those that had not.

The next-to-last step recommended by Johnson's model is to compare what was actually learned with what instructors intended students to learn. To do this effectively requires complete specification of intended learning outcomes. As stated earlier, the intended learning outcomes included the relevant concepts of Calculus, Linear Algebra and Differential Equations, the relationship between the concepts and the real world, and how those concepts were used in the development of mathematical technologies. Also, it was intended for students to discover for themselves that concept maps and the Vee heuristic were effective problem-solving tools, not just diagrams required by class instructors. Instructors hoped students would come to see them as tools for representing what is already known and how current concepts are used to construct new knowledge. Further, it was intended that students perceive these heuristics as tools designed to help them focus on the event under study and ask the questions needing answers if a solution to a problem was to be found. Also, it was intended that students should recognize the power of the computer, not only for its ability to perform mathematics, but also for its ability to help the student learn mathematics. But most of all, the team hoped students would come to view mathematics as a language representing the real world and that its competent use could enhance their problem-solving skills rather than simply add a collection of algorithms to their bag of "problem-solving tricks."

The final step required by Johnson's model is that changes be made to the curriculum and instructional plan to compensate for any failure to match intended learning outcomes with actual learning outcomes. This evaluation was accomplished by means of an end-of-class questionnaire.

Development of an Evaluation Instrument

The questionnaire was developed to determine two things. First, it provided input concerning student perception of how well intended learning outcomes were met. Second, it offered the team student assessments of the relevance of the curriculum and effectiveness of the instructional approach used in the review. In both cases, answers were provided that will allow changes to be made to the curriculum and instructional system of future offerings of the Engineering Math Review.

The questionnaire involved rating certain aspects of the review and also asked for written explanations of those ratings. Other questions asked for written descriptions only. The questions were derived from the objectives set for the review. From the objectives, hypotheses were developed. Answers to some of the questions asked in the questionnaire provided the data used to test and validate these hypotheses. The objectives, associated hypotheses and specific questions used to collect data to test the hypotheses will be discussed in the following section.

Objectives, Hypotheses, Questions and Statistical Evaluation

In this section, the objectives of the review will be stated and the hypotheses (derived from the objectives) will be presented. The specific questions asked to collect data to be used to evaluate the hypotheses will be addressed. Finally, the specific statistical techniques used to assess the hypotheses will be described.

A convenience sample of 21 students represented the potential group who would generate the data base for this evaluation. Two of the students were excused from the class. Of the 19 students who went through the course, 16 responded to the questionnaire. This represented a response rate of approximately 85%.

Objective 1: Related Hypotheses, Questions and Tests

Objective 1: To encourage students to take a more conceptual orientation toward mathematics by attempting to shift the focus of students from memorizing rules to mastering concepts, from getting a number to studying functional relationships, and from "plugging and chugging" with formulae to creating and experimenting with models.

Hypothesis 1: A construction-based approach to the teaching and learning of mathematics will help the students acquire a conceptually-oriented view of mathematics.

A rule-based orientation to mathematics in which students view mathematics as a cleverly designed set of algorithms that one only needs to memorize and intelligently select from whenever a problem lending itself to mathematical resolution is encountered is no longer viable. Rather, modern engineering students need to gain a solid conceptual understanding of mathematics. This hypothesis states that a construction-based approach will provide the modern engineering student with the conceptual base required to effectively use mathematics in solving real-world problems. The following question was used to collect data to assess the validity of this hypothesis:

Did we meet Objective 1 by helping you shift your focus

	No Change				Some Change				Major Change	
From memorizing rules to mastering concepts	1	2	3	4	5	6	7	8	9	10
From getting a number to studying functional relationships	1	2	3	4	5	6	7	8	9	10
From plugging and chugging with formulae to creating and experimenting with models	1	2	3	4	5	6	7	8	9	10

It was expected that the answer to this question would reflect at least some change in the views of students. This expectation is the result of the governance being applied to the learning environment, which stressed the importance of knowing the concepts and the usefulness of experimentation.

This expectation was evaluated using frequency histograms. The data were classified into three separate groups. The classes represented no change, some change and major change. Ratings of 1 to 3 were designated as no change, 4 to 7 as some change and 8 to 10 as major change. The three groups referred to were 1) all members of the class, 2) the Engineering Physics (EP) classmembers and 3) the Electro-optics (EO) class members.

Breakdown by class was accomplished to assess the impact of Dr. Luke's presence in the classroom. He is the program advisor for the EO students. On many occasions, Dr. Luke stressed the importance of understanding the concepts of mathematics and experimentation in future class work. By doing this, a message was sent to the EO students telling them that what was being covered was important and would be useful throughout their stay at AFIT. The EP students were not routinely provided with such assurance. Thus, it was believed that a greater positive shift would be made by the EO students than the EP students because of Dr. Luke's presence in the classroom.

Hypothesis 2. Students will welcome and readily adapt to a conceptually-oriented presentation of mathematics.

This hypothesis was derived from a belief that students would rather be subjected to a conceptual treatment of mathematics than a laundry list of algorithms, rules and techniques. Further, it was assumed that students welcome the mental challenge of thinking, since it presumably reflects what will be required of them at AFIT and on the job.

The following two questions were used to collect data needed to validate this hypothesis:

1. What is your opinion about the worthiness of Objective 1? Briefly support your position.

2. This review was designed to help students gain insight into ways basic concepts of mathematics can be used to model physical phenomena and to study the impact of changing various inputs and parameters fed to such models. Do you believe such a focus is appropriate for the GEO/GEP Math and Computer Review? Explain.

It was expected that the answers to both of these questions would be positive. Again, this expectation was based on the students' perception of the need to do this type of work at AFIT and when they go back to work.

This expectation was also evaluated using frequency histograms. Again, three classes were used, as well as three groups. The groups are the same as before; however, this time the classes are negative, positive with comment and positive.

The classes were driven by the expectation that students would answer either yes or no to the questions. However, it was believed some would answer yes and provide a comment regarding the ability to accomplish such an objective during the limited time frame of the review.

Hypothesis 3: The Vee heuristic and concept mapping will be recognized by students as effective learning and problem-solving tools.

The Vee and concept mapping have been presented as tools that can be used to promote meaningful learning and enhance problem-solving skills. Concept mapping provides a means to externalize what has been learned and can be used as input to the left side of the Vee. The Vee uses this existing knowledge to focus on an event about which questions are asked to create new knowledge. This hypothesis proposes that students will recognize these as tools to promote the self-development of learning and problem-solving skills.

The following question was asked to collect the data necessary to validate this hypothesis:

> What role should concept mapping and the Vee heuristic play in the math and computer review?

Because the review is so short and an overwhelming amount of material needs to be covered, instructors were unsure whether students would recognize the utility of these problem-solving tools. Consequently, it was impossible to determine what the results were expected to be.

Frequency histograms were again used to evaluate their reaction. The responses of three groups (All, EO and EP) were placed in two classes, yes or no.

Objective 2: Related Hypotheses, Questions and Tests

Objective 2: To review the important mathematical rules and techniques of Calculus, Linear Algebra and Differential Equations.

Hypothesis 1: Students will be satisfied that the math review adequately covered the important mathematical rules and techniques of school mathematics.

The purpose for stating this hypothesis was to provide a way to determine whether or not students were satisfied with the review of the rules and techniques in each of the three topic areas.

The following was asked in order to collect data for this hypothesis:

> Register your level of satisfaction concerning our review of the rules and techniques by rating the three topic areas.

	Totally Dissatisfied				Reasonably Satisfied				Completely Satisfied	
Calculus	1	2	3	4	5	6	7	8	9	10
Linear Algebra	1	2	3	4	5	6	7	8	9	10
Diff. Eqs.	1	2	3	4	5	6	7	8	9	10

Please share with us suggestions for improvement. It was anticipated that the ratings for the review would all be above 5. Because a number cannot provide qualitative feedback on why students who take a particular position do not feel completely satisfied with all three areas, suggestions for improvement were solicited.

Evaluation of this hypothesis was once again accomplished through the use of frequency histograms. The ratings served as the classes which were broken down by All, EO and EP.

> Hypothesis 2: The construction-based review will provide a math review that is rated at least as well as previous reviews.

The purpose of this hypothesis was to determine whether this review was at least as good as previous math reviews. Since this was the first time the review had been taught this way, doing no worse than previous reviews and obtaining feedback on ways to make it better for next year was considered an adequate outcome.

To obtain data to test this hypothesis, two questions needed to be asked. The first question used was the same as the one used in the previous hypothesis. However, this question was only addressed to the present students of the review. A second question polling a different group was required to get a rating for the review as it was previously taught. This was accomplished by means of a questionnaire given to previous year's GEO/GEP classes. The following was asked of the previous year's students

> Rate the usefulness of the math review on a scale of 1 to 10 (10 being the most useful). Give the basis for your rating.

The ratings provided by the previous year's students were low. However, to determine whether any significant increase in the ratings had occurred, the Wilcoxen Rank-Sum test was used.

Because of small sample sizes, invoking the central limit theorem to claim normality of the underlying probability distribution was not possible. Thus, a nonparametric test was required. The Wilcoxen Rank-Sum test is a nonparametric test that only assumes the random sample was taken from a continuous population characterized by a symmetric probability distribution.

The rankings provided by each class were compared to determine whether a significant difference between means existed. The null hypothesis of this test is that the difference between the means of the two sets of rankings will be zero. The alternative hypothesis is that the mean of the new students' ratings will be higher than that of previous students'. The test statistic for this test is the sum of the ranks with positive differences between pairs (2:164).

Both samples were of equal size. Also, both samples were greater than 10, which allowed the use of a normally-approximated test statistic (2:164). If it was determined that a large number of ties were encountered during the test, a correction factor would be added to the denominator of the standard deviation term to compensate (2:165).

Objective 3: Related Hypotheses, Questions and Tests

Objective 3: To familiarize the student with AFIT's computing facilities, specifically, mathematics software such as MathCAD 2.0, Matrix-X and MacSyma.

Hypothesis 1: Students will be unfamiliar with AFIT computing facilities and PC-based computing hardware and software used by the AFIT community.

This hypothesis validates the need to teach the computer review to familiarize students with the resources available at AFIT. Further, it demonstrates the need to set aside time to teach students how to use the software and hardware if it is going to be used effectively by students to perform mathematical computation and reinforce learning.

The question used to collect data for this hypothesis consists of several parts. Each part asks about a different type of software package that was used in the review. The assumption is that to use the software, some understanding of how to use the hardware must also be present. Therefore, the ability to use the hardware was not addressed directly in the question. The question used is as follows:

> Register your sense of computing competency, keeping in mind that we are asking about ability to meet objectives and homework requirements of this course, by rating your ability to use the following software packages:

VAX EDT Editor	None		Some			Full	
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7
Matrix-X	None		Some			Full	
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7
MacSyma	None		Some			Full	
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7
Procomm	None		Some			Full	
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7

MathCAD 2.0	None			Some			Full
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7

MS-DOS	None			Some			Full
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7

Dial Out Facilities	None			Some			Full
Before the review	1	2	3	4	5	6	7
After the review	1	2	3	4	5	6	7

The EDT editor is a full-screen editor that runs on AFIT mainframes. It is used to write code for input into MacSyma and Matrix-X, allowing batch operations to be run. Procomm is an emulator package that runs on the PC and is the medium through which the PC communicates with the mainframe machines. The dial out facility is an AFIT-provided resource that affords students the ability to communicate with the outside world. It allows calls to anywhere in the world, giving students full access to an enormous amount of information developed by other students or researchers.

The data used to validate this hypothesis were the responses to the "before-the-review" part of each question. It was expected that the responses would be between 1 and 3 for most students because they had never been exposed to these specific packages before. Frequency counts of the number of each response by all students were used to validate the hypothesis.

Hypothesis 2: Students will be significantly more familiar with AFIT computing facilities and PC-based computing hardware and software after the math review than before the math review.

This hypothesis provides feedback to verify that the students actually learned how to use the computer resources made available to them. The same question asked by the previous hypothesis was used to collect data to evaluate this hypothesis.

The Wilcoxen Signed-Rank test was used to determine whether a significant increase in ability to use the software and hardware had occurred. This test uses paired rankings for each student. Each data pair consists of a student's before-the-review rating and after-the-review rating for each subquestion. The null hypothesis for this test is that the difference in means will be zero. The alternative hypothesis is that the mean after the review will be significantly higher than before the review. The test statistic is given by the sum of the positive rankings (2:131).

Hypothesis 3: Students will find learning several mathematical packages attractive and will benefit from their concurrent introduction during the math review to 1) manipulate expressions and compute results, and 2) learn math.

The computer was introduced in the math review for two reasons. First, the computer could handle all the routine computational work required to be performed by students. Second, it was hoped that the learning of mathematics could be facilitated by mathematical software through the use of dynamic templates. That is, that inputs to representations constructed on the computer could be changed to study the behavior of the function and reinforce the concepts being learned.

The question used to validate whether or not students recognized this potential use of computers was as follows:

> This was the first time the AFIT math and computer reviews were treated as one subject. Did you find such treatment beneficial during the review to 1) manipulate mathematical expressions and compute results, and 2) learn math? Explain why or why not.

The responses to this question were expected to lean heavily toward the use side. It was expected that students would recognize the benefits of using the computer to perform routine operations that were previously done by hand. However, it was not expected that students would see the computer as an effective tool for studying the conceptual behavior of mathematical representations. Without a doubt, the computer has the potential to play this role. However, getting students to recognize this ability is not easy when time constraints and demands of other class work limit students' opportunity to think about a problem. Therefore, the responses were expected to emphasize the benefits associated with using the computer to perform fast manipulations rather than with helping them learn mathematics. A summary of student responses was used to validate the hypothesis in a conceptual manner.

Chapter Conclusion

This chapter has been devoted to discussing the development of the curriculum and instructional design system using the framework provided by Johnson's model. The model provides a step-by-step process whose output is an instructional plan built around the interaction of the four commonplaces of education and properly applied governance. The final step proposed by the model is the evaluation of the success of the instructional plan to meet intended learning outcomes. Indeed, the ultimate task of this thesis effort was to evaluate how well the review met its intended learning outcomes. The objectives, hypothesis and specific questions asked of the students to aid in this effort were addressed in this chapter. Chapter IV will present the results of the evaluation program.

IV. Analysis of Results

This chapter presents an analysis of the results garnered through the evaluation program outlined in Chapter III. The format of the analysis will be to present the objectives and hypotheses under test, and display survey data in pictorial fashion. An analysis of results will then be made including: a discussion of what is implied by the graphics, how the obtained results differ from what was expected, and the significance of the findings. the discussion will start with an analysis of Objective 1 and its respective hypotheses.

Objective 1: Evaluation of Results

Objective 1: To encourage students to take a more conceptual orientation towards mathematics by attempting to shift the focus of students from memorizing rules to mastering concepts, from getting a number to studying functional relationships, and from "plugging and chugging" with formulae to creating and experimenting with models.

Hypothesis 1: A construction-based approach to the teaching and learning of mathematics will help the students acquire a conceptually-oriented view of mathematics.

The frequency histograms for each shift addressed in Objective 1 are presented in Figures 4-1, 4-2 and 4-3.

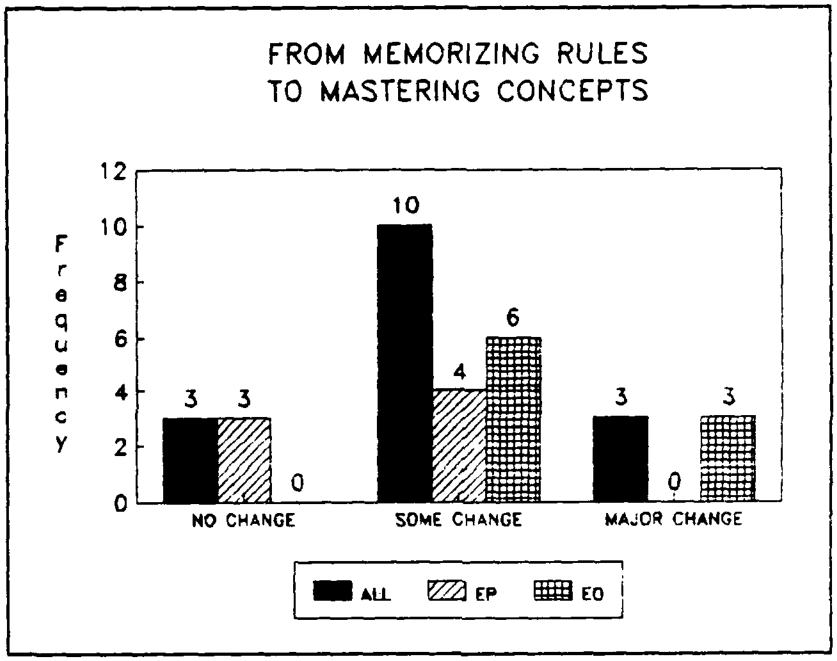


Figure 4-1. Frequency Counts for Objective 1, Hypothesis 1: Shift From Memorizing Rules to Mastering Concepts

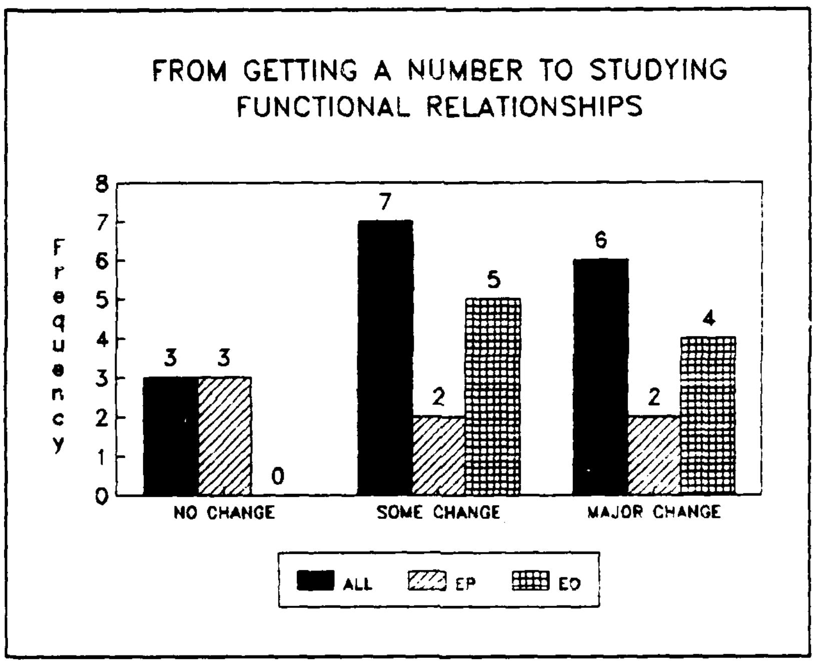


Figure 4-2. Frequency Counts For Objective 1, Hypothesis 1: Shift From Getting a Number to Studying Functional Relationships

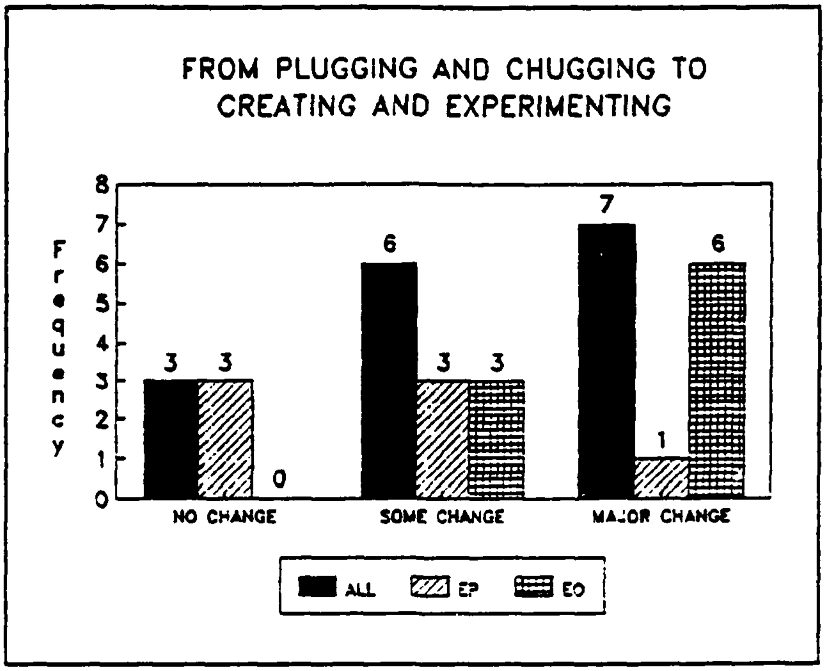


Figure 4-3. Frequency Counts for Objective 1, Hypothesis 1: Shift From Plugging and Chugging with Formulae to Creating and Experimenting with Models

From Figures 4-1, 4-2 and 4-3, it can be seen that a shift to a more conceptual and experimental orientation did occur among the students. The predominant indication was that some change rather than any major change in orientation did occur. One reason for this could be that students already think conceptually about mathematics, or think they do. Another could be reluctance on the part of the students to change the way they view mathematics based on previous experiences; knowing that after this class they will be subjected to the same rule-based orientation this class was designed to avoid.

The latter claim can be substantiated somewhat by noticing the differences between the EOs and the EPs. The EPs are grouped in the no change and some change classes, while the EOs are grouped in the some change and major change classes. Dr. Luke's emphasis on the need to view mathematics in a more conceptual manner may have been a positive influence on the EOs. Because the EPs will not take a class from Dr. Luke while here at AFIT, there was no apparent need for them to take this approach to the study of mathematics. This reinforces the role played by governance in an educational environment.

Clearly, there was a reasonable change in the way the students view mathematics. It appears this change is attributable to the approach used to teach the mathematics during the review. Therefore, it is claimed that this hypothesis has been supported.

Hypothesis 2: Students will welcome, and readily adapt to, a conceptually-oriented presentation of mathematics.

Validation of this hypothesis was accomplished using the responses of two questions. The first asked whether or not students felt that Objective 1 was a worthy objective. The second asked whether or not the focus of the review was appropriate. Frequency histograms showing the number of responses are provided in Figures 4-4 and 4-5 on the following page.

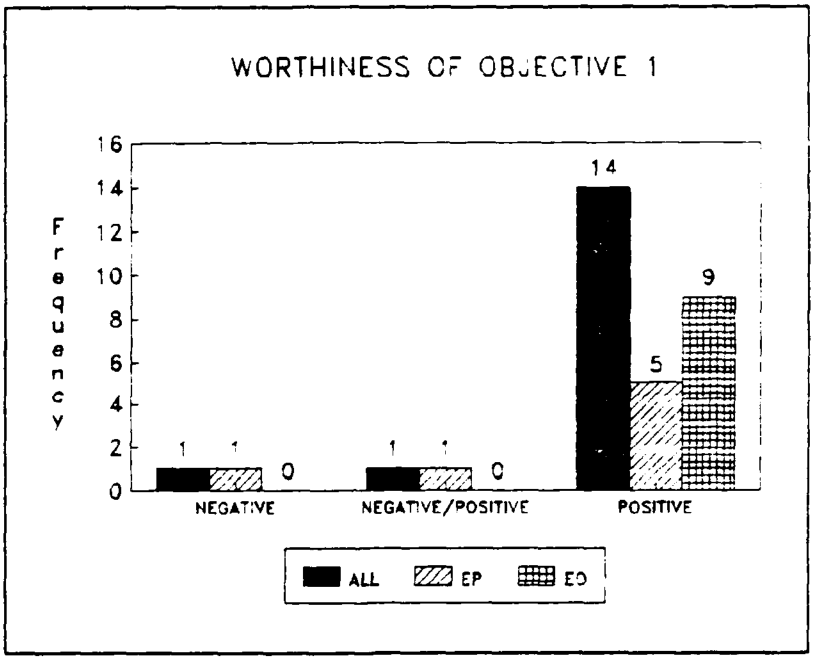


Figure 4-4. Frequency Counts for Objective 1, Hypothesis 2: Worthiness of Objective 1

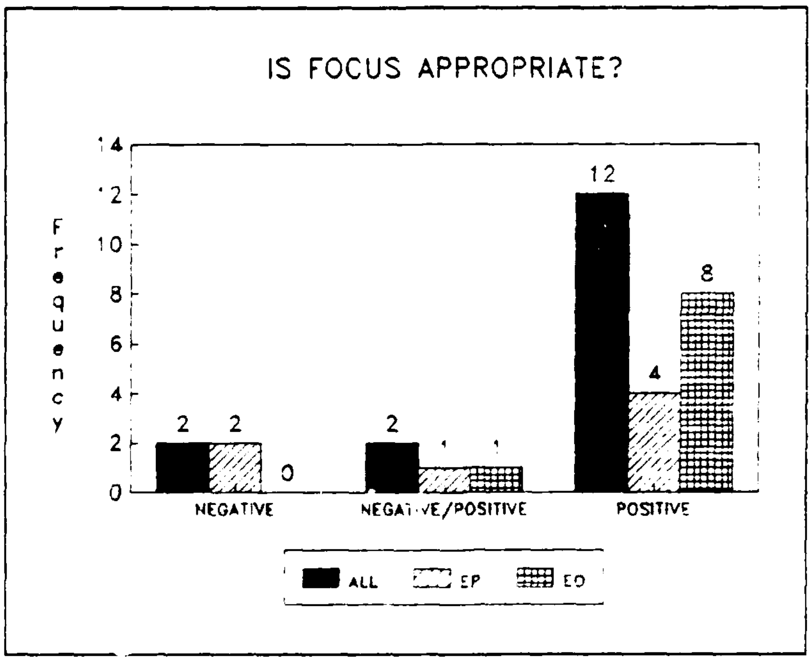


Figure 4-5. Frequency Counts for Objective 1, Hypothesis 2: Is Focus Appropriate for the Review

In both cases, the majority response was positive. For the category of negative/positive, the predominant response was that the objective and focus were exactly what they should be; however, many felt that there was not enough time in the review to accomplish these goals. Some felt that although it is a good idea, modeling and problem solving should be covered in a separate course. So, in some cases, seeing that modeling, problem solving and mathematics are related was totally missed. However, the number of such occurrences was small. Again, it should be pointed out that all the negative responses came from the EPs, again reinforcing the role governance plays in education. Overall, the responses seem to support the hypothesis.

Hypothesis 3: The Vee heuristic and concept mapping will be recognized by students as effective learning and problem-solving tools.

A frequency histogram showing the number of responses in the no or yes categories is presented in Figure 4-6. The results show that a majority of students did not see the Vee heuristic and concept mapping as effective learning and problem-solving tools. This result was not unexpected.

An abundance of information is presented to the student during the review. Also, along with the review, students are participating in other review courses. Decisions must be made by students on what is important and how best to utilize their time. Students focused on the mathematics

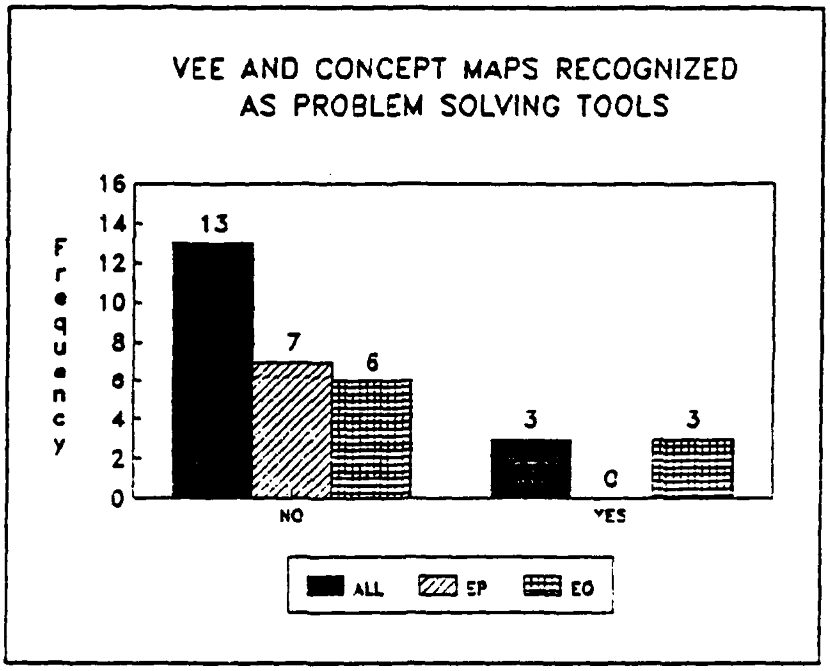


Figure 4-6. Frequency Counts for Objective 1, Hypothesis 3: Vee and Concept Maps Recognized as Problem-Solving Tools

rather than attempting to understand the usefulness of the Vee and concept mapping. Indeed, this is understandable because using these tools for learning requires a great deal of thought and effort. Also, a person must adopt the constructionist philosophy of learning and problem solving if one is truly to appreciate the power of these heuristics. Students were generally opposed to having philosophic discussions in the classroom. Without such conversations, however, changing the learning philosophy of students is probably not possible.

Overall, the data indicate students did not come to recognize the Vee and concept mapping as effective learning and problem-solving tools. This is probably due to the time constraints of the review and the abundance of other intellectual activities required of students, including several concurrent courses.

Objective 2: Evaluation of Results

Objective 2. To review the important mathematical rules and techniques of Calculus, Linear Algebra and Differential Equations.

Hypothesis 1. Students will be satisfied that the math review adequately covered the important mathematical rules and techniques of school mathematics.

Three histograms showing the number of responses in each of the ten classes for each topic area of the review are shown in Figures 4-7, 4-8 and 4-9. The results show that all students were reasonably satisfied with the coverage provided in the review.

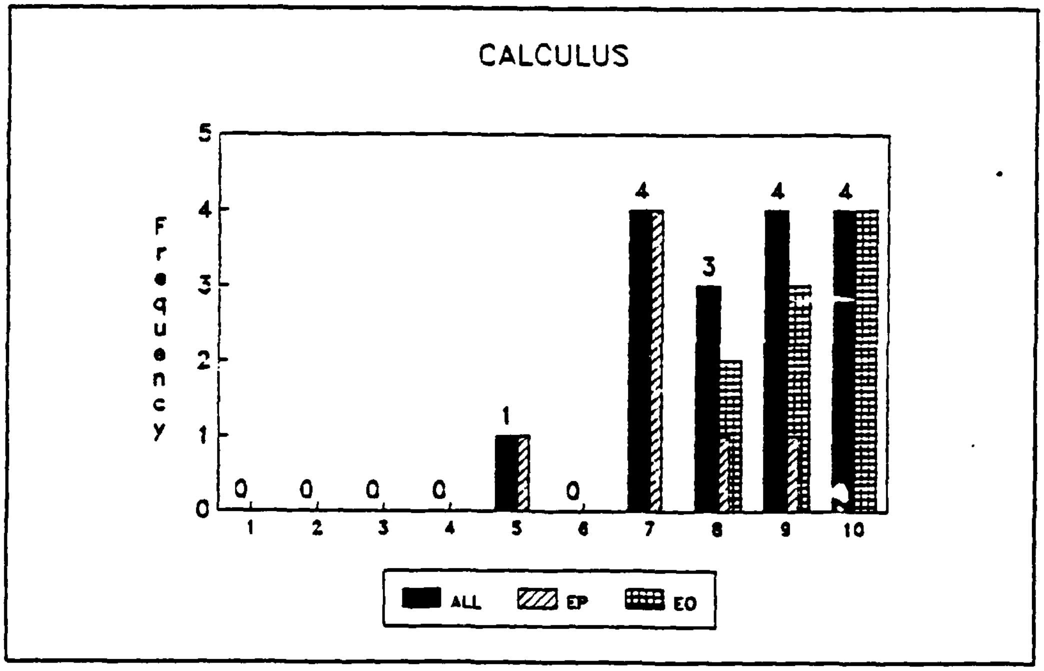


Figure 4-7. Frequency Counts for Objective 2, Hypothesis 1: Satisfaction with Calculus Review

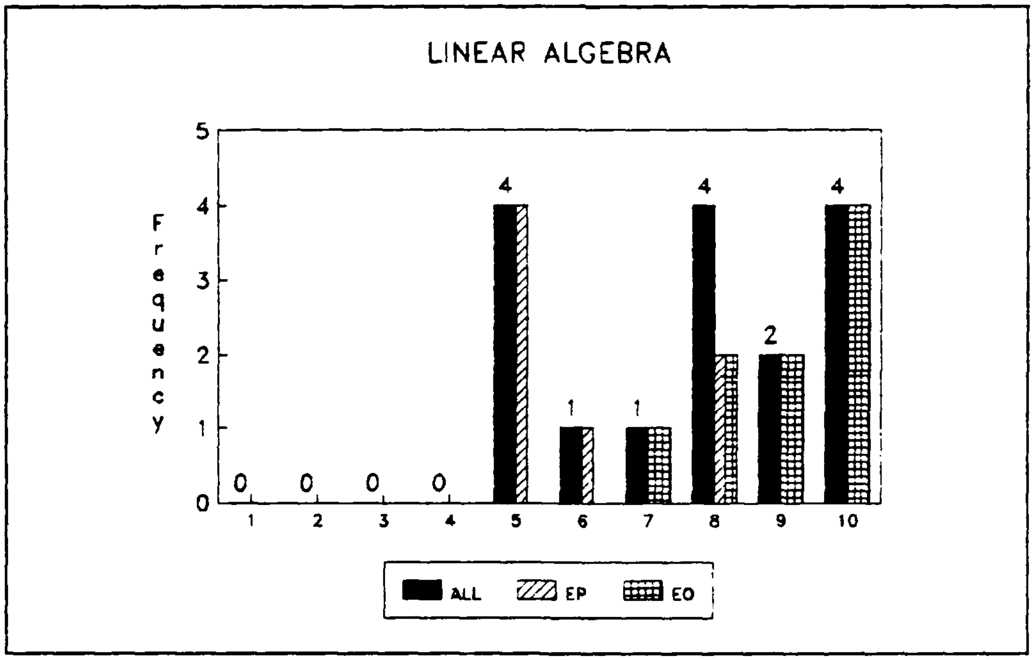


Figure 4-8. Frequency Counts for Objective 2, Hypothesis 1: Satisfaction with Linear Algebra

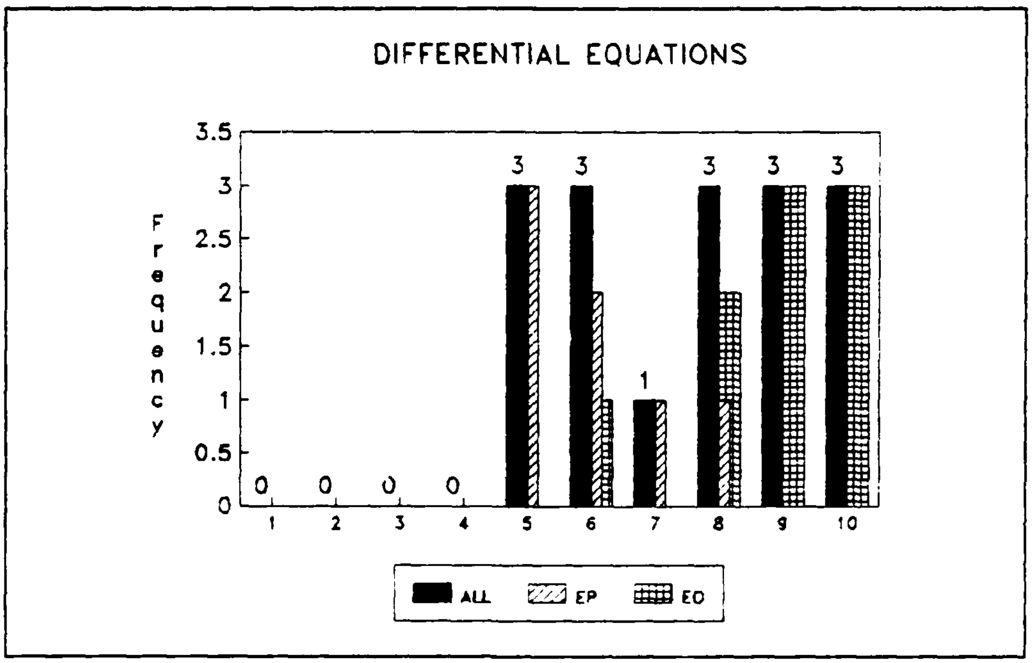


Figure 4-9. Frequency Counts for Objective 2, Hypothesis 1: Satisfaction with Differential Equations

Most of the comments provided by students to justify their ratings pointed out that time was a critical factor. That is, four weeks is just not enough time to cover many of the things they would have liked to cover. Others pointed out that the Vee and concept-mapping ideas should have been tossed out. Again, the motivation for making this comment is probably their reluctance to engage in the extraordinary amount of philosophizing that must be carried out if one is to truly understand the usefulness of these tools. Students stated early in the course they preferred to spend their time working problems and learning mathematics in ways more in line with their philosophic way of thinking.

Students provided a few suggestions on different topics that should be added to the review. However, for the most part, students were reasonably satisfied they were prepared for the upcoming curriculum. Thus, this hypothesis appears to have been supported.

Hypothesis 2. The construction-based approach will provide a math review that is rated at least as well as previous reviews.

Testing this hypothesis was done using the Wilcoxen Rank-Sum test. Two independent samples of ratings, one each from last year's class and this class, were compared to determine whether a difference in mean rating exists. The test was run with a significance level of .05. Since both sample sizes were greater than 10, 16 to be exact, normality was assumed and a z test statistic determined. The critical value for z in this case was -1.65. The test statistic, corrected for ties, was found to be -3.088. Thus, the null hypothesis was rejected in favor of a significant difference in the two sets of ratings. By this result, the claim that this review would be rated at least as well as previous reviews is supported.

Objective 3: Evaluation of Results

Objective 3. To familiarize the student with AFIT's computing facilities, specifically mathematics software such as MathCAD 2.0, Matrix-X and MacSyma.

Hypothesis 1. Students will be unfamiliar with AFIT computing facilities and PC-based computing hardware and software used by the AFIT community.

Instead of presenting seven histograms showing the number of responses in each category for each type of software or facility, the responses were totaled together and broken down into percentages. The percentages associated with the responses related to ability to use computing resources before the review are presented in pie-chart fashion in Figure 4-10.

The figure shows that 76% of the responses were ones reflecting no ability on the part of the student to use the software available to them before the review. It is interesting to note that eight students felt they were fully competent in the use of MS-DOS. Thus, the assumption stated in Chapter I that said most of the students would be familiar with MS-DOS because they had access to IBM PCs currently installed at most Air Force bases was not an unreasonable assumption. The data which showed the assumption that students would be unfamiliar with AFIT's computing resources were correct.

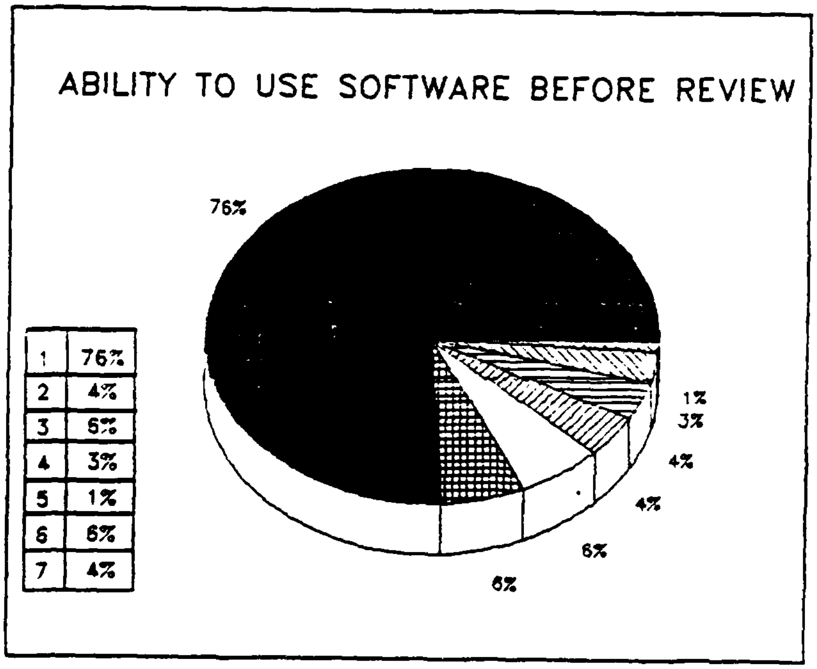


Figure 4-10. Percentage of Responses in Each Class for Objective 3, Hypothesis 1: Familiarity with Computing Facilities

<u>Hypothesis 2</u>. Students will be significantly more familiar with AFIT computing facilities and PC-based computing hardware and software after the math review than before the math review.

The test used to validate this claim was the Wilcoxen Signed-Rank test. Using paired observations of competency before and after the review, the test showed that in all cases, a significant increase in ability occurred. The test statistic, the sum of the positive ranks, was found to be zero, which at a significance level of .05, favored the rejection of the null hypothesis. Another pie chart is presented in Figure 4-11 to show the response percentages after the review. Clearly, a shift to more competency can be seen. The number of ones after the review was found to be 3%, down from the original 76%. In fact, 73% of the responses were ratings of 4 or higher. Thus, the claim that students would be more familiar with AFIT's computing resources after the review is substantiated.

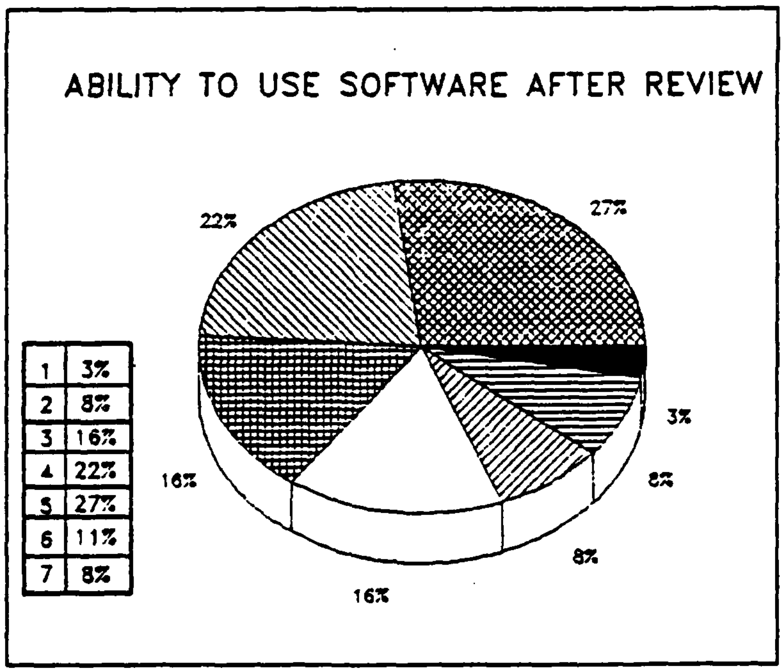


Figure 4-11. Percentage of Responses in Each Class for Objective 3, Hypothesis 2: Competency After the Review

Hypothesis 3. Students will find learning several mathematical packages attractive and will benefit from their concurrent introduction during the math review to 1) manipulate expressions and computer results, and 2) learn math.

A summary of student comments and in-class observations will be used to validate this hypothesis. For the most part, students reported that the computer was extremely helpful in reducing the tedium of calculation. But most of all, students were interested in how to use the software. They felt they could learn to use it to its full potential as long as they were presented with the basic commands they needed to get started.

The computer lab was always an interesting sight. An occasional question would come up about how to do a particular task, then off they would go continuing their work. Students were switching between packages to do different tasks.

But it is surprising that not many responded that they found the computer useful in learning mathematics. It appears they were using it for that purpose, but did not realize it. During their work, students were observed changing parameters and watching the change on the screen. They were heard explaining to each other what was occurring and why. This is excellent, although unconscious, implementation of the Vee heuristic.

In the questionnaire, students were asked to state what software packages appealed to them most. Although answers differed slightly, almost all of them commended the choice of mathematical software. MathCAD was well received because of its graphics capability and ability to produce templates with equations in a format similar to a mathematics textbook. MacSyma and Matrix-X were attractive because of their superb capabilities for performing symbolic and matrix manipulations.

From the responses and in-class observations, it appeared that students were extremely pleased with the opportunity to become familiar with, and use, several different math packages, each with its own unique powers. In fact, many students pointed out that other students taking the normal reviews were jealous of the fact that they were taking a combined review.

Although the use of the computer to enhance the learning of mathematics was not consciously recognized by students, there was considerable evidence that this did go on. However, there was no doubt that students were using the computer to manipulate expressions and compute results. So, based on the student responses and in-class observation, this hypothesis was well supported.

Chapter Conclusion

This chapter has been devoted to analyzing the results obtained through a questionnaire given to the students participating in the review. Each objective and its respective hypotheses were given, along with an analysis of results. Each of the hypotheses was validated, allowing the claim that the objectives of the review were met. Chapter V will present the conclusions and recommendations of this thesis and focus on several suggested areas for future research.

V. Conclusions and Recommendations

If AFIT's Engineering students are to learn mathematics meaningfully and receive adequate preparation for solving novel problems, an approach to teaching and learning mathematics that emphasizes the personal construction of mathematical representations must be implemented. Such an approach should be characterized by responsible involvement of the learner in every phase of knowledge creation, as well as the introduction of heuristics for learning that illustrate, through their use, the process of knowledge creation.

The goal of this thesis was to show how such a construction-based approach could transform the Engineering Math Review at AFIT from a primarily rote review of rules and algorithms of school mathematics into a meaningful learning experience. In fact, a posteriori evaluation of the review indicated a modest but significant change in students' attitudes toward the learning of mathematics. In particular, students strongly supported our construction-based orientation to teaching and learning mathematics--an approach that, by definition, requires the personal development of mathematical representations for novel problem scenarios.

In spite of their wariness toward the heuristics of Gowin and Novak--the Concept Map and VEE--students seemed quite content to work in an environment governed, to the extent instructors could manage it, in a manner compatible with the learning theory espoused by Ausubel, Gowin and Novak. Perhaps most gratifying was the evidence that this "new approach" was as acceptable, if not more acceptable, as any approach taken in past reviews, and that the rules and algorithms that play a major role in active problem solving were covered and mastered in a satisfactory manner. Such consistent and solid support for more involvement with a conceptual orientation to mathematics assured the teaching team that the three objectives set forth in Chapter I had been met and that a new pedagogy for the teaching and learning of mathematics had been created.

On the other hand, instructors' ambitious dreams for a computer-facilitated interactive learning environment in which students would create MathCAD templates to display and explore their current understanding of the concepts of mathematics could not be fully realized. This was primarily due to administrative dictation of too many topics to be covered, too little time to teach all the topics and an imposition of future course requirements for which students believed they were cognitively unprepared. Each of these factors generated extraordinary levels of student anxiety. Even more critical, each factor represented an output of an educational system designed and built in a way that is antithetical to recommendations for the establishment of a meaningful learning environment. Finally, and note this carefully, all these factors were predictable systemic outputs of the current system of governance.

Adding to the burden imposed by administrative constraints on self-management of learning were the inordinate amounts of time required to construct and explore the conceptual implications stimulated by MathCAD displays. During course preparations, instructors routinely discovered that interactive sessions with intense verbal exchanges of six to eight hours' duration were required if the benefits of construction-based learning were to be experienced by users of this new pedagogy. When confronted with the time limitations imposed by classroom learning and a simultaneous requirement to dialogue with 19 students, as well as other colleagues, instructors simply had to concede that their ambitious plans for fully implementing construction-based learning could not be realized in the standard short-term review.

Far from being discouraged by such findings, the team's enthusiasm for a construction-based learning approach has been fired all the more. Why? Because there is every evidence that a drastic change in governance, coupled with full implementation of a MathCAD template and Expert System/ Hypertext-orchestrated concept mapping of key mathematical concepts, will all but guarantee full manifestation of a meaningful learning environment. This will, in turn, allow computer-facilitated learning and exploration of mathematical concepts outside the confines of a standard classroom to become completely operational. In short, there is every reason to believe a viable educational system for self-management of learning can, and should be, instituted at AFIT.

Confident that the marriage of computer-assisted learning of quantitative studies will enhance attempts to foster the meaningful learning of mathematics, and that such technical assistance will motivate students to take full responsibility for their own learning, this thesis recommends the following research activities be undertaken as soon as possible.

Recommendations for Future Research

1) Expert System/Hypertext facilitation of multidimensional concept mapping should be explored.

The KnowledgePro language should be used to design a process for creating a knowledge base that can be applied by students in any of AFIT's engineering courses. Topics and concepts are always multifaceted. Their explication must be supported by recursively accessible definitions, flowcharts illustrating their implementation and animated simulations displaying their practical application in various fields of study. Selected students ought to be given an opportunity to help invent the key elements of the process of cognitive structure mapping. The primary goal of such research should be to demonstrate and evaluate the limitations of concept mapping via pen-and-pencil, two-dimensional displays and explore the synergistic benefits of complementary use of maps and "executable displays", especially as these enhance a student's ability to learn and fire up his or her motivation for learning. Subject areas that are ready to take advantage of such a development are mathematics, statistics, physics, computer science and management.

2) Mathematica, a system for doing mathematics by computer, should be procured and used to explore the pedagogical benefits of facilitating student access to a graphically rich and integrated computational environment.

Mathematica allows numerical, symbolic and graphical computations to take place within a common framework and facilitates their mutually supportive use. Using Mathematica, the teaching and learning of mathematics can stimulate involvement with both numeric and functional analyses. Once the package is procured, selected course materials and handouts can be transformed into dynamic and executable textbooks in the "hypercell environment" provided by Mathematica. Such materials could be developed and evaluated for their ability to enhance meaningful and active learning of concepts from simple arithmetic to the most elegant of mathematical disciplines. Active investigation of the practical application of theorems and proofs could be carried out. Graphical representations of difficult concepts could be attempted. Special computational laboratories could be established that would allow students to experience the experimental nature of real mathematical work and do this in ways that the standard classroom cannot support. The goal of such research should be to discover the optimum ways computer-facilitated learning can complement traditional educational technology that has been limited in standard classrooms to a piece of chalk, a blackboard and viewgraphs.

3) The pedagogy of the Engineering Math Review should be modified and research initiated to determine what would be involved in helping other Math and Computer Reviews at AFIT adopt the approach taken by the Engineering Math Review.

Integration of the Math and Computer reviews presented to incoming students of AFIT's School of Systems and Logistics should be the number one priority of those planning the curriculum for the 1989 reviews. Packages such as Quattro, MathCAD and the resident MacSyma system should be viewed as one tool kit and be routinely introduced to AFIT students who can then experience the benefits of a synergistic combination of premier mathematical software from day one of their stay at AFIT. They can learn first-hand how such packages can assist in the learning of subjects that lend themselves to computer assistance. Textbooks, special MathCAD projects and a concept map for the Graduate Engineering Management (GEM) and Graduate Systems Management (GSM) Math Review should be developed, and research undertaken to ensure a curriculum is presented that a) is responsive to what students can be expected to know when they first arrive at AFIT, and b) provides an adequate review of the mathematical and computing concepts they will need during their stay at AFIT. The efficacy of concept visualization should be explored and evaluated. Attempts should be made to see how such visualization can calm an audience that is so often stymied by math anxiety cultivated during years of miseducation and uninspired introductions to the quantitative sciences.

4) A study concerning ways to introduce governance that supports Self-Management of Learning should be undertaken.

It is clear that governance plays the key role and creates the milieu in which curriculum, teaching and learning function. Educational system governance must foster true autonomy and inspire each learner to take full responsibility for his or her learning, requiring the various dimensions of self-management be explored. Also, in lieu of system outputs that are antithetical to Ausubelian learning theory, the governance of the educational system must be designed to produce curricula that ask students to manage their own learning.

All these research activities can be built on the foundation laid by this thesis. While the initiation of a construction-based approach proved to be difficult, this thesis has demonstrated it can be accomplished at AFIT, but only if the educational system is organized to produce such output!

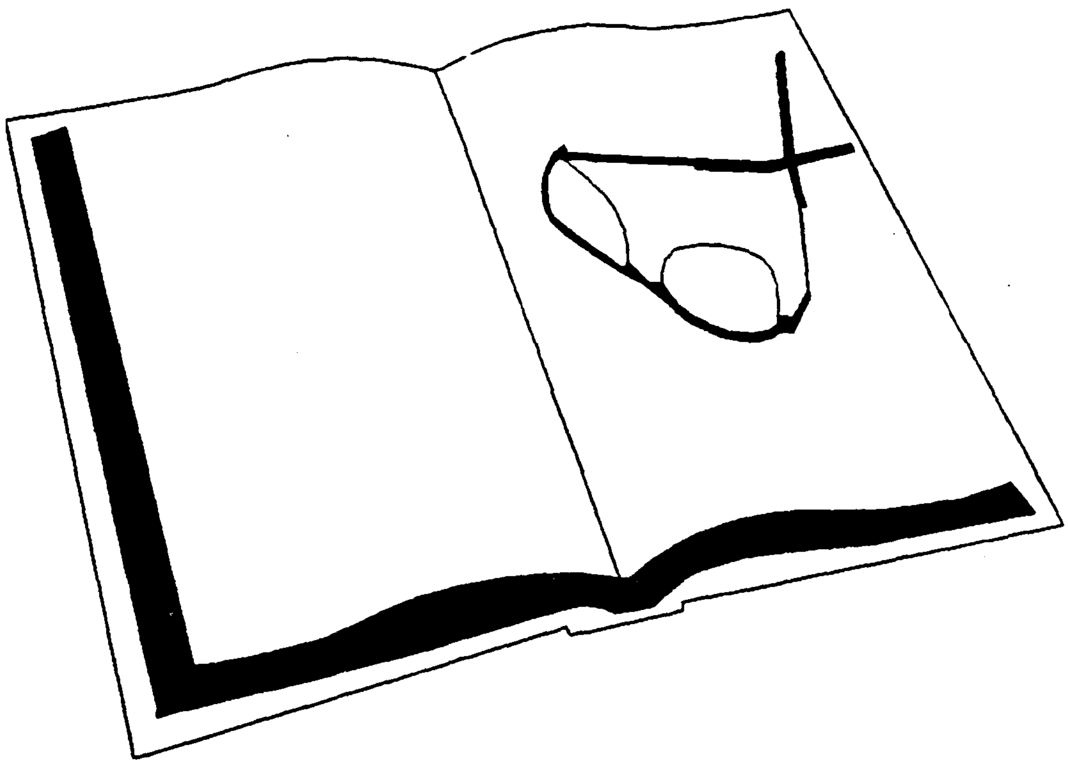
If real change in the educating system is to occur, all those who possess the skills required to make systemic change a reality in education must act! Let there be no illusion, this is a call for a revolution, a new structure for educating and a new form of governance. If and when this occurs, all those who are trying to create the curricula and devise the methods for teaching and learning required under such governance will be delighted with the results. Until then, hope and persistence must carry the day. Major change never comes all at once, although recognition of such change can come at any moment in time. A lot of little steps must be taken with humility, sincerity and integrity to make change in the educational system of AFIT a reality. It is hoped that this thesis represents the first of many steps in that direction.

Appendix A: Math and Computer Review Syllabi

This appendix contains the syllabus used for the mathematics portion of the review and the syllabus used for the computer portion of the review. For each day of the review, the math syllabus identifies the topic that was covered, the concepts used to motivate and develop the topic, how the concepts were used to develop their relationship to the real world and computer applications related to the concepts. Also, intended learning outcomes, relevant questions and suggestions for computer templates are provided.

The computer review syllabus is slightly different. It lays out three areas that were covered during the review. Each area was covered when students needed to use the information to complete their work. Intended learning outcomes and what these outcomes will enable the student to do are included in this syllabus.

Math Review



Dan Reynolds & Richard Lamb
June 1988

Math Review

The following daily topics will be covered during the Engineering Math Review to be held June-July 1988.

SECTION I: Calculus of a Single Variable
(7 days x 3 hours = 21 hours)

Day 1: Functions
Day 2: Limits
Days 3-4: The Derivative
Day 5: The Integral (An Introduction)
Day 6: Integration Methods
Day 7: Series

SECTION II: Linear Systems and Matrix Algebra
(3 days x 3 hours = 9 hours)

Day 8: Matrix Algebra: The Fundamental Entities and Operations
Day 9: Solving Linear Systems of Equations
Day 10: Vector Spaces and the Matrix Representation of Linear Transformations

SECTION III: Calculus of Several Variables
(4 days x 4 hours = 16 hours)

Day 11: Vectors and Scalar Fields
Day 12: 3D Analytic Geometry and Scalar Fields
Day 13: Partial Derivatives of Scalar Fields
Day 14: Multiple Integrals of Scalar Fields

SECTION IV: Ordinary Differential Equations
(4 days x 4 hours = 16 hours)

Day 15: Classification of Ordinary Differential Equations
Introduction to Solving Nonlinear First Order Differential Equations
Day 16: Solving First Order Linear Differential Equations
Day 17: Modeling Dynamical Systems
Using Systems of Differential Equations
Day 18: Solving nth Order Differential Equations

Each day of the review will consist of three major phases
sometimes run separately
sometimes run simultaneously.

PHASE 1: [M.D.] Motivation and Development of The Topic
PHASE 2: [S.I.] Socratic Investigation of the Use Features of Mathematics [Mathematical and Physical Applications]
PHASE 3: [C.A.] Computer Applications

The following is a daily list of concepts and activities that will constitute PHASE 1, 2 and 3 for that day.

=====================================
DAY 1: Topic of the Day: FUNCTIONS
=====================================

M.D. -- Basic Functions
Elementary Functions
Graphing Functions
Inequalities
Solutions of Inequalities

S.I. -- Modeling Using Functions

C.A. -- Plotting Functions w/MathCAD
Solving Equations and Inequalities w/MathCAD

:::INTENDED LEARNING OUTCOMES

1) To be able to explain how functions can be used to model physical phenomena

Relevant Questions:

What is a function?
What is not a function?
What can functions be used for?

2) To be able to graph basic and elementary functions and describe their unique and common behaviors

Relevant Questions:

What kids of basic functions are there?
What do such functions look like?
What are some of their properties?

3) To be able to solve and graph inequalities

Relevant Questions:

What are some reasonable approaches (some heuristics) to solving linear and nonlinear inequalities?

What are some common ways to solve inequalities on the computer?

What kinds of problems can occur when numerical packages are used?

4) To be able to translate, reflect, expand and sum functions

Relevant Question:

Given one representation of functions, how can we create another representation that will be simpler to deal with?

:::SUPPORTING MATHCAD TEMPLATES

-Graphing Functions
-Solving Inequalities and Graphing Inequalities

=================================
DAY 2: Topic of the Day: LIMITS
=================================

M.D. -- Definition of a Limit
One-Sided Limits
Limits of Continuous Functions
Limits of Combinations of Functions
Indeterminate Limits

S.I. -- Computing Velocity Given a Position Vector

C.A. -- Graphing the Limit of a Function w/MathCAD Using MacSyma to Compute Limits of Arbitrary Functions

:::INTENDED LEARNING OUTCOMES

1) To be able to verbalize and picture the intimate relationship between integration (as a limiting process) and differential (as a limiting process)

2) To be able to use the definition of a limit to construct the fundamental definitions of a derivative and an integral of scalar field

3) To be able to identify the type of discontinuities that can occur (point-and-jump discontinuities)

Relevant Questions:

How do the graphs of such discontinuities look?

What physical situation in a drag race could create such discontinuities in any one or all three critical functions (position, velocity, acceleration)?

What are the possible limits when such discontinuities occur?

4) To be able to obtain limits (if they exist) when an indeterminate form occurs

Relevant Questions:

What are the various indeterminate forms?

When can they arise?

:::SUPPORTING MATHCAD TEMPLATES

-Pictorialize One-sided and Two-sided Limits
-Display the Properties of Limits
-Display Cases if Indeterminate Limits

==
DAY 3-4: Topic of the Days: THE DERIVATIVE
==

M.D. -- Definition of a Derivative
Derivative of Basic Functions
Derivative of Elementary Functions
Special Techniques

S.I. -- Calculating Rates of Change of Position Functions
Problems Involving Related Rates

C.A. -- Numerical Differential w/MathCAD
Symbolic Differentiation w/MacSyma
Plotting Derivative w/MathCAD

:::INTENDED LEARNING OUTCOMES

1) To be able to apply the Algebra of Derivatives

Relevant Questions:

When and why should the following rules of differentiation be used?

Derivative of a sum
Derivative of a product
Derivative of a quotient
Derivative of a composite function
Derivative of an inverse function
Derivative of a logarithmic function

2) To be able to use derivatives and study and model physical scenarios involving related rates

Relevant Questions:

Given that we know the velocity function of at least two phenomena, what can be said about the rate of change of a resultant combination of these functions?

3) To be able to verbally and pictorially describe what the differential is and is used for

Relevant Question:

How can I approximate the value of a function near a point using a linear approximation of this function?

:::SUPPORTING MATHCAD TEMPLATES

-Studying the relationship between the derivative of a function and the differential dy = f'(x)*Delx

===================================
DAY 5: Topic of the Day: INTEGRAL
===================================

M.D. -- Definition of the Integral
Fundamental Theorem of Calculus
Improper Integral
Special Techniques

S.I. -- Selecting an Appropriate Integration Method
..Substitution
..Partial Fraction Decomposition
..Integration by Parts

C.A. -- Numerical Integration w/MathCAD
Symbolic Integration w/MacSyma

:::INTENDED LEARNING OUTCOMES

1) To be able to verbally and pictorially discuss what the definition of an integral implies and to relate its fundamental properties

Relevant Questions:

Is there an association between a function [f(x)] and its antiderivative [F(x)]?

How can the answer to the above question be used to compute the definite integral of f(x) over some interval?

What properties of the definite integral can be used to simplify its computation?

2) To be able to numerically integrate

Relevant Questions:

What type of numerical integration procedures exist?

What type of errors can you expect to encounter when these methods are computerized?

3) To be able to compute improper integrals

Relevant Questions:

What are the two basic types of improper integrals?

How can a person determine if he/she is destined or likely to encounter problems while numerically integrating a proper or improper integral? (A survey of the most important considerations)

Is there a relationship between a method of numerical integration and the errors incurred by using that method?

How do I choose an appropriate interval length to compute an improper integral?

:::SUPPORTING MATHCAD TEMPLATES

-The fundamental theorem of calculus...and its graphical explanation: the complementary relationship of the integral and derivative

===

DAY 6: Topic of the Day: MODELING PHENOMENA WITH THE INTEGRAL

===

M.D. -- Constructing Models Using Integrals
Various Types of Applications of the Integral

S.I. -- Heuristics for Modeling Real-World Phenomena and for Solving Real-World Problems Involving the Computation of Infinite Sums

C.A. -- Numeric Integration with MathCAD
Symbolic Integration with MacSyma

:::INTENDED LEARNING OUTCOMES

1) To be able to discuss the purpose of antidifferentiation

2) To be able to model different types of real-world phenomena using integral expressions

3) To be able to select an appropriate antidifferentiation technique from a menu of such techniques once the integral model has been constructed

Relevant Questions:

Is the function given immediately recognizable as the derivative of another function?

if not,

Is it a linear combination of immediately recognizable functions?

if not,

Can a variable substitution be made and used to obtain an immediately recognizable function?

if not,

Is there a product of two functions which will allow integration by parts to be used to obtain an immediately recognizable function? (Employ recursively, if necessary.)

if not,

Can one make an algebraic manipulation such as partial fraction decomposition, completing the square of expanding the polynomial to obtain an immediately recognizable function? (Be creative!)

if not,

Is numerical integration possible?

NOTE: Sometimes integration can be performed easily when a coordinate transformation is employed (e.g., Cartesian to polar coordinates). This type of integration will be discussed when subjects such as Coordinate Transformation are covered later in this course.

=================================

DAY 7: Topic of the Day: SERIES

=================================

M.D. -- The Mean Value Theorem
Taylor's Formula
Taylor Remainder Formula
Power Series
Topics in Convergence

S.I. -- Using the Taylor Series for Numerical Computation of Derivatives, Integrals and Solutions of Equations

C.A. -- Studying the Error Due to Truncation of the Taylor Series w/MathCAD

:::INTENDED LEARNING OUTCOMES

1) To be able to define what a series is

Relevant Questions:

What is a series?

How do we assess whether a series is summable, i.e., how can one determine if a Series converges or diverges?

2) To be able to define what a power series is

Relevant Questions:

How can power series be used to approximate elementary functions?

What forms can power series take?

How can a convergent power series be constructed for a given elementary function?

3) To be able to define what a power series can be used for

Relevant Questions:

What are some ways power series can be used in mathematics?

..to obtain numerical solutions of equations
..to numerically integrate
..to be able to integrate and differentiate functions that are not closed form (i.e., e**(x**2))

::SUPPORTING MATHCAD TEMPLATES

-Construction of the Taylor series
-Error estimation of series: the impact of the number of terms in a given series

==
Day 8: Topic of the Day: MATRIX ALGEBRA
==

M.D. -- Basic Concepts and Terminology of Matrix Algebra
Matrix Operations
Special Matrices

S.I. -- The Fundamental Entities and Operations of Matrix Algebra

C.A. -- Using MathCAD's Matrix Capabilities to Operate on Entities of Matrix Algebra

:::INTENDED LEARNING OUTCOMES

1) To be able to state the fundamental entities that are used in matrix algebra to represent and model various phenomena.

2) To be able to construct matrices and to set up systems of linear equations in matrix form.

3) To appreciate the value and problems that accrue computationally when matrices are handled by computer software packages.

Relevant Questions:

What is a matrix?

What are the basic operations that can be performed on matrices?

What are some of the basic rules related to the various matrix operations?

:::SUPPORTING MATHCAD TEMPLATES

-Basic matrix operations: working with matrices under MathCAD control

==

DAY 9: Topic of the Day: SOLVING LINEAR SYSTEM OF EQUATIONS

==

M.D. -- Coefficient Matrix
Right-Hand Side
Consistent/Inconsistent System
Homogeneous System
Augmented Matrix
Equivalent System
Row-Reduced Echelon Form (RREF)
Gauss-Jorden Elimination

S.I. -- The Importance of Linear Systems in the Modeling of Dynamical Systems

C.A. -- Using Matrix-X to Facilitate the Study and Solution of Linear Systems of Equations

:::INTENDED LEARNING OUTCOMES

To be able to use matrix methods to solve systems of linear equations

Relevant Questions:

How is a system of linear equations represented in matrix form?

What type of linear systems do we encounter?

How can we determine the characteristics associated with a given set of linear equations?

What is involved in exercising the Gaussian Elimination Method for solving linear systems of equations?

What computational strategies need to be considered when the Gaussian Elimination Method is used?

==

DAY 10: Topic of the Day: VECTOR SPACES AND LINEAR TRANSFORMATIONS

==

M.D. -- Linear Combination
Linear Independence and Dependence
Rank of a Matrix
Inverse of a Matrix
The Determinant
Vector Spaces
Linear Transformation and their matrix Representation

S.I. -- Why is the Concept of a Vector Space so Crucial to Someone Who Wants to Use Mathematics to Solve Real-world Problems?

:::INTENDED LEARNING OUTCOME

To be able to explain what a linear transformation is and how linear transformations are represented mathematically.

Relevant Questions:

What is a vector space?

What is the relationship between vector spaces and functions?

Why is the concept of a vector space required to competently study linear transformation?

What matrix concepts are required to carry out a linear transformation?

What motivates the use of linear transformations in science?

How can linear transformations be represented mathematically?

==

DAY 11: Topic of the Day: VECTOR AND SCALAR FIELDS

==

M.D. -- Vectors in Euclidean Space
Vector Operations
The Name of a Vector
The Dot Product and Its Properties
Components of a Vector
The Cross Product and its Properties

S.I. -- Velocity and Acceleration Vectors: What are Some of their Applications?

C.A. -- Implementing Vector Graphs on a Computer

:::INTENDED LEARNING OUTCOME

To be able to generalize the concept of a geometric vector

Relevant Questions:

What am I doing when I do vector addition and multiply a vector by a scalar?

Are there other quantities, such as functions or solution sets that, in general, perform just as geometric vectors do?

What are some examples of such quantities?

:::INTENDED LEARNING OUTCOME

To be able to model real-world phenomena in terms of geometric vectors

Relevant Questions:

How do we model physical phenomena such as velocity, position, acceleration via geometric vectors?

What goes on when we apply vector operations such as the dot product, cross product and scalar product?

To what extent can these vector operations be used in model creation and manipulation?

==

DAY 12: Topic of the Day: 3D ANALYTIC GEOMETRY AND SCALAR FIELDS

==

M.D. -- Spheres
Planes
Lines
Cylindrical and Quadric Surfaces
Cylindrical and Special Spherical Coordinates

S.I. -- How Can Various Coordinate Systems be Used to Simplify Mathematical Representations of Physical Phenomena?

C.A. -- Plotting Mathematical Representations of Cartesian and Polar Coordinates with MathCAD

:::INTENDED LEARNING OUTCOMES

1) To be able to mathematically represent planes, spheres, ellipses, paraboloids, etc.

2) To be able to pictorialize tangent planes and normals to a surface

Relevant Question:

How are these quantities used to represent Newtonian motion?

3) To acquire an ability to select a coordinate system that facilitates the simplest possible representation of a real-world event

===

DAY 13: Topic of the Day: PARTIAL DERIVATIVES OF SCALAR FIELDS

===

M.D. -- Graphs and Level Sets
Partials of n Order and their Connection With Graphs and Level Sets
Chain Rules for First and Second Order Partials

S.I. -- Optimization Methodology: How can we find Maxima and Minima of Scalar Fields (Gradients, Normal Vectors, and Direction of Steepest Ascent)?

C.A. -- Finding Optimal Points in Scalar Fields via Computer

:::INTENDED LEARNING OUTCOMES

1) To be able to use partial derivatives to deal with problems such as optimization

   Relevant Question:

   How can the maxima and minima of a function be determined?

2) To be able to compute and interpret direction derivatives connection with physical reality

   Relevant Questions:

   How can gradients be used in physics?

   How can we solve optimization problems involving constraints?

===

DAY 14: Topic of the Day: MULTIPLE INTEGRALS OF SCALAR FIELDS

===

M.D. -- Double Integrals within Cartesian and Polar Coordinate Systems Triple Integration in Cylindrical and Spherical Coordinate Systems

S.I. -- How can Various Coordinate Systems be Used to Simplify Mathematical Integration of Multidimensional Integrals?

C.A. -- Using MacSyma to Integrate Functions of the Cartesian, Polar Spherical and Cylindrical Coordinate Systems

:::INTENDED LEARNING OUTCOMES

1) To be able to perform multiple integration in various coordinate systems

   Relevant Questions:

   What is the definition of a multiple integral?

   How do we compute values for multiple integrals?

2) To be able to apply change of variable technology (Jacobians) in obtaining a solution to a multiple integral

   Relevant Question:

   How can we use multiple integrals to calculate area, volume and regions?

==

DAY 15: Topic of the Day: CLASSIFICATION OF ODEs

==

M.D. -- Classification of ODEs Using the V-Heuristic
-Conceptual Classifications

S.I. -- Learning to Recognize
Separable Differential Equations
Exact Differential Equations

C.A. -- Introduction to MacSyma's and Matrix-X's Capabilities for Solving ODEs.

==

DAY 16: Topic of the Day: SOLVING 1ST ORDER ODEs

==

M.D. -- Methodological Classifications
Separable Equations
Exact Equations
Equations that Can be Made Exact
Equations Requiring Numerical Integration

S.I. -- Heuristics for Selecting an Appropriate Method for Solving First Order ODEs

C.A. -- Using MacSyma to Solve First Order ODEs

:::INTENDED LEARNING OUTCOMES

1) To be able to use the concept of a second order differential equation to model real-world phenomena

2) To be able to select an appropriate equation form and computing procedure to symbolically or numerically solve a first order differential equation.

==

DAY 17: MODELING DYNAMICAL SYSTEMS USING SYSTEMS OF 1ST ORDER DIFFERENTIAL EQUATIONS

==

M.D. -- The V-Heuristic and Knowledge Construction
Problem Solving via Mathematical Modeling
State Variables and Dynamical Systems
Initial and Boundary Value Problems
Classifying ODEs and Systems of ODEs
Checking for Existence and Uniqueness of a Solution

S.I. -- Using the V-Heuristic to Formulate Mathematical Models of Dynamical Systems

:::INTENDED LEARNING OUTCOMES

1) To be able to describe the fundamental elements involved in the application of the V-heuristic

Relevant Questions:

What is the nature and purpose of laboratory work?

What is involved in knowledge production?

How can we distinguish between theories, principles, concepts, records, facts, data, information, noise, knowledge claims and value claims?

2) To be able to discuss the relationship between modeling as a process; the model, as a product of the modeling process; and the role mathematical representation plays in the modeling process

Relevant Questions:

What is a model?

Is a mathematical representation of real-world phenomena a model?

3) To be able to classify dynamical systems as amenable to modeling by ordinary, partial or systems of differential equations

Relevant Questions:

When are ordinary or partial differential equations needed to model dynamic systems?

How can systems of ODEs model situations involving dependent state variables?

==

DAY 18: Topic of the Day: SOLVING NTH ORDER LINEAR ODEs

==

M.D. -- Methodological Classifications
Linear-Homogeneous (with Constant Coefficients)
Linear-Homogeneous (with Variable Coefficients)
Linear-Nonhomogeneous (with Constant Coefficients)
Linear-Nonhomogeneous (with Variable Coefficients)
Euler's Formula

S.I. -- Heuristics for Selecting an Appropriate Method for Solving nth Order ODEs

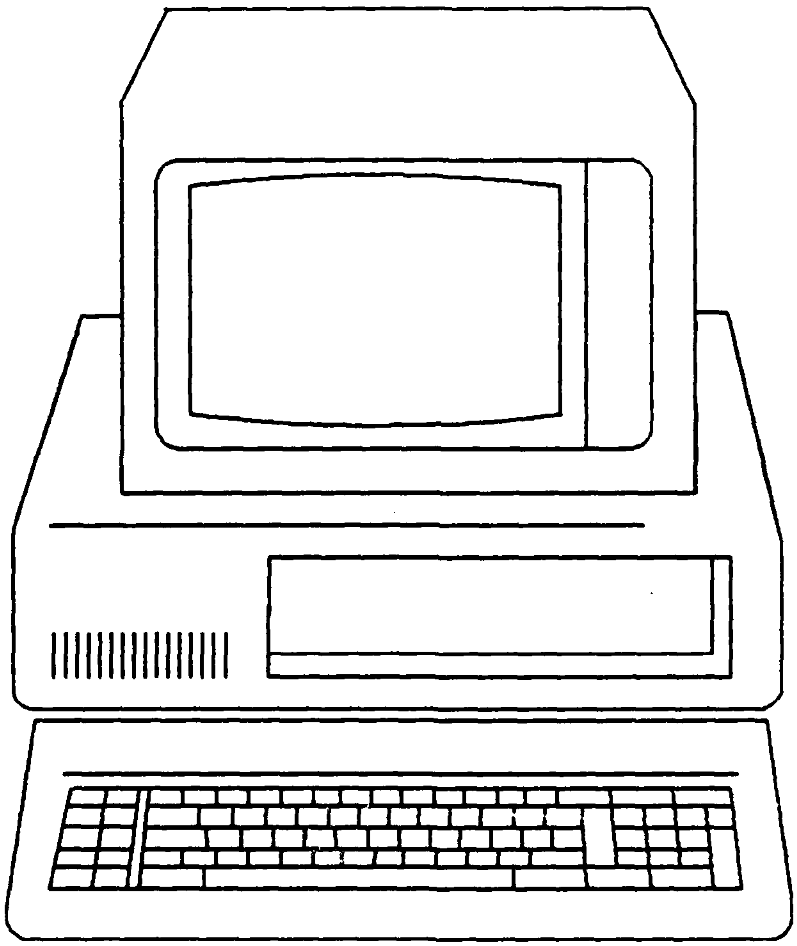
C.A. -- Using MacSyma to solve Second Order ODEs

:::INTENDED LEARNING OUTCOMES

1) To be able to use the Concept of an nth order differential equation to model real-world phenomena

2) To be able to select an appropriate equation form and computing procedure to symbolically or numerically solve a second order differential equation

Introduction to AFIT Computing



Dan Reynolds & Jerry Edwards
June 1988

Computer Review Syllabus

The purpose of the Computer Review Course is to help students gain familiarity with AFIT's total spectrum of computing services. The goal of the Computer Review is to empower students with the ability to use AFIT's computing resources in support of course work and research activities.

Three general areas that will be covered during this review are:

1) Using IBM-Compatible PCs
2) Using VAX Mainframes (VMS Systems)
3) Using AFIT's Dial Out facility to call bulletin boards and other computer systems around the world.

The first area, Using PCs, will deal with helping students learn how to use MathCAD and the standard operating system for IBM and IBM-Compatible PCs: MS-DOS 3.2. Editors and other more exotic MS-DOS capabilities will be covered as required, especially as they lend support to the use of MathCAD.

The second area, Using VAX mainframes, will involve an introduction to two software packages: MacSyma and Matrix-X. These are two of many mathematical packages that are resident on one or more of AFIT's mainframe systems. VMS Utilities, such as MAIL and PHONE, along with file transfers and editing, will be introduced early in the course.

Finally, discussions concerning the third area, using AFIT's Dial Out facility, will attempt to ensure students develop the necessary skills to dial PC bulletin boards nationwide. By establishing such links, students will be able to take advantage of vast amounts of free information and software that they can use to support their studies at AFIT.

Five computer reference books are recommended to students who want manuals to support their studies during the review:

1. Introduction to VAX/VMS, Terry C. Shannln.
2. Running MS-DOS, 3rd ed., Van Wolverton
3. MacSyma Handbook, Available in the AFIT/EN Library.
4. Matrix-X Manual, Available in Rm 133, Bldg 640.
5. MathCAD Manual.

Obviously, the material covered above does not encompass everything you may need or want to know about AFIT's computers and software. For example, setting up an autoexec.bat file and fixing up a config.sys file for your home system are not covered. These topics may become important as more students purchase home systems. If the need arises, these subjects can be discussed in class. Other topics can be added anytime. If you have a topic you would like to have discussed, feel free to bring it up.

Note: The computer review is being taught in conjunction with the math review. We realize that there are only four weeks in the short term and quite a bit of mathematical material to be covered. The computer areas will be covered as needed during the four-week session to complement the mathematics instruction. However, we will make time to answer any questions that arise relative to the computer.

Areas to be covered during the four-week session:

AREA 1: The PC and MathCAD

:::INTENDED LEARNING OUTCOMES

The student will

1) prepare a disk for data storage
2) load MathCAD
3) write, save and retrieve MathCAD templates using basic commands
4) determine what files are stored on a given disk

Enabling the student to

- Format a new disk
- Make a backup copy of disk using diskcopy command
- Use directory command with /w qualifier
- Change directory to MathCAD directory
- Run mcad.exe
- In MathCAD:
    - Enter text using <shift> " command
    - Enter text using <cntrl> t command
    - Use subscripted variables
    - Use discrete variable values
    - Enter equations and functions
    - Generate plots
    - Save templates to disk
    - Retrieve templates
    - Exit Program
- Change back to root directory

AREA 2: Mainframes

:::INTENDED LEARNING OUTCOMES

The student will

1) logon to the CSC
2) become familiar with the VAX menu
3) edit the login.com file to make it user-specific
4) read, reply and send mail
5) use the VAX phone utility
6) upload and download a file
7) unprotect and protect a file
8) copy an unprotected file from another student's directory
9) set a new password
10) access MacSyma and perform differentiation and integration of elementary mathematical functions
11) access Matrix-X to perform matrix operations

Enabling student to

- connect to CSC using Procomm (Zstem, if necessary)
- enter password
- talk about menu
- enter mail utility
  - read message
  - reply (using editor)
  - send (using editor)
  - exit mail utility
- enter phone utility
  - phone another user
  - answer phone
  - clear the screen
  - hang up
- copy @mrreview.dis from gsm88s:[jedwards] to student directory (distribution list for mail)
- download a file to PC
- upload file to VAX
- unprotect uploaded file
- send a note to classmates via distribution list (using editor)
- access MacSyma
  - differentiate and integrate elementary functions
- access Matrix-X
  - perform matrix operations
- set personal password

AREA 3: Dial Out Facility

:::INTENDED LEARNING OUTCOMES

Using the dial out facility, the student will

1) call up a bulletin board
2) logon
3) scan file listings
4) download a file
5) logoff

Enabling the student to

connect to dial out facility using Procomm
enter class
enter password
enter phone number
connect to BBS
logon
scan files
download a file
disconnect
if required
    de-archive the downloaded file
    archive a file to upload to BBS or VAX

SEE YOU IN CLASS!

Appendix B: Selected MathCAD Templates

This appendix contains two examples of templates used in the review. The first template deals with the concept of a function and the second deals with the concept of a derivative. In class, the computer was used in conjunction with a Datashow to project templates on a screen for the whole class to see.

The function template contains five frames. Each frame consists of 25 vertical lines, the number of lines on a typical computer screen and also a Datashow device. Developing templates this way allowed instructors to quickly page down without having to use the cursor keys. Setting up the templates this way made the process of presenting the templates in class extremely efficient. The function template used in class is presented on the following pages, one frame at a time. The derivative template follows. Note that references provided in the templates refer to page numbers in the MathCAD Users Guide. These references were provided so students would know where to look in the book to find out how equations were set up or how text was entered into the template.

FUNCTIONS

Purpose: To explore the idea of a function.

Terms: Function: a relationship between two or more variables such that for each value of the independent variable there corresponds exactly one value of the dependent variable.

Independent and Dependent Variable: Consider the relationship between the circumference of a circle and its radius. This relationship can be expressed as $C = 2\pi r$, where C depends on the choice of r. Thus, C is called the dependent variable and r the independent variable.

Domain: The collection of all values assumed by the independent variable is called the domain of the function.

Range: The collection of all values assumed by the dependent variable is called the range of the function.

Frame 1: Function Template

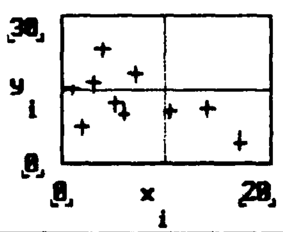
Functions map inputs to outputs and can be plotted. Often, functions are specified by formulas, however, they can also be specified by discrete values of the independent and dependent variable. Below is an example of both, starting with the latter case.

$i := 1 .. 10$

Here i is used as a subscript variable to identify the value of each of the independent variables and its associated dependent variable.
(Ref: p. 136)

$x_i :=$	$y_i :=$
1	15
3	16
5	12
6	10
7	18
4	23
17	4
10	10
14	11
2	7

As many values of the independent and dependent variable may be entered as you wish.
(Ref: p. 130)



Each value of x can be mapped to its corresponding value of y using the plotting function. Because each value of x corresponds to only one value of y, this represents a function.
(Ref: p. 199)

Frame 2: Function Template

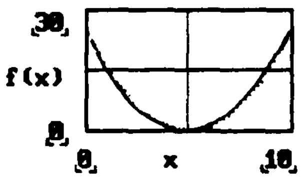
As stated earlier, functions are often specified by formulas. Consider the following:

$x := 0,.5\,..10$

Here x represents values of the independent variable. The values will start with 0, increment by .5 and stop at 10. (Ref: p. 136)

$f(x) := (x - 5)^2$

This is the formula under consideration. Because the dependent variable is a function of x, the (x) is necessary in order for all the values of x to be considered. (Ref: pg. 94)



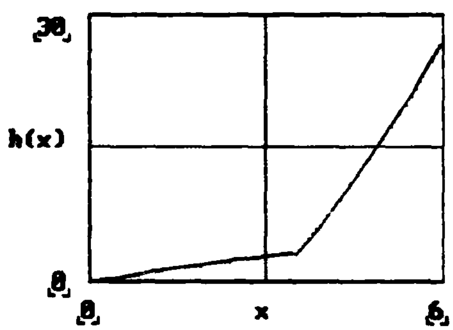
Because each value of the independent variable results in only one value of the dependent variable, this formula represents a function.

Frame 3: Function Template

Functions can also be described piecewise, where different functions are used in different intervals to define the overall function. Below is an example of a piecewise function.

$x := 0,.5\,..6$ $\qquad f(x) := x$ $\qquad g(x) := x^2 - 9$

$h(x) := f(x)\cdot \mathrm{if}(x \le 3,1,0) + g(x)\cdot \mathrm{if}(x > 3,1,0)$
(Ref: p. 160)



Again, because each value of the independent has only one dependent value it is a function. Further, because each value of the independent variable results in one unique value of the dependent variable, the function is also one to one.

Frame 4: Function Template

When determining the domain and range of functions specified by formulas, we exclude all the real numbers for which the formula is undefined. For example, consider the following:

$$f(x) := \frac{4}{x^2 - 9}$$

The domain would be all real numbers except for 3 and -3, which would cause the denominator to go to zero. Another example would be

$$f(x) := \sqrt{x + 2}$$

where the domain would be real numbers greater than or equal to -2 if complex numbers are not considered.

Frame 5: Function Template

The next template to be presented is the derivative template. This template consists of a single frame; however, three frames are shown, each with different input values. By varying the input values, the student can study the behavior of a particular mathematical concept. This template demonstrates that the derivative of a function is in fact the slope of the tangent line to the curve at a given point. It also points out the difference between the average rate of change and the instantaneous rate of change. Because this template is a bit more complicated than the first, an explanation of how the template works will be provided after the first frame is shown on the next page.

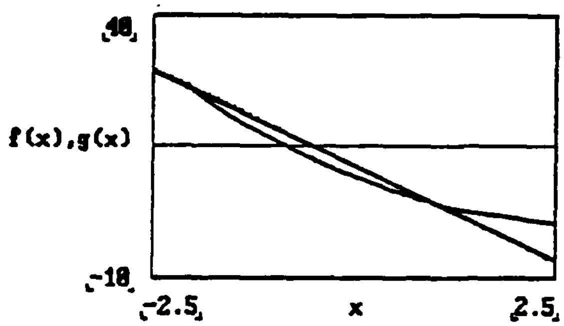
$pt1 := 1 \qquad pt2 := -2.4$

$x := -2.5, -1.5 \,..\, 2.5 \qquad f(x) := (x-3)^2$

$h := pt2 - pt1$

$$f' := \frac{f(pt1 + h) - f(pt1)}{h} \qquad g(x) := f' \cdot (x - pt1) + f(pt1)$$

$f' = -7.4$



$$slope := \frac{g(pt2) - g(pt1)}{pt2 - pt1}$$

$slope = -7.4$

Frame 1: Derivative Template

The definition of a derivative uses f(x) and f(x+h) where h is a small delta x. The variable Ptl represents the point at which the derivative of f(x) is being considered, and Pt2 represents the second point used to calculate h. Once h is determined, f', the derivative computed using the definition, can be found. This value of f' is then used in the function g(x), which is the equation of the line passing through points Ptl and Pt2. Indeed, the derivative f' is equal to the slope of the line determined by g(x).

It is important to note that when h is large, as in this case, f' is not truly a derivative. Rather, f' is the average rate of change of f(x) in the interval Ptl to Pt2. Changing the value of Pt2 to a value close, but not equal, to Pt1 will provide a close approximation to the true derivative. In essence, changing the value of Pt2 so it approaches Pt1 is the same as taking the limit as h approaches zero as called for by the definition of the derivative. The result of this process can be seen in the two frames presented on the next page.

The beauty of this template is that the student can change anything he or she wishes and experiment with the derivative concept. For example, a different f(x) could be looked at, the derivative at a different point could be found, or the average rate of change could be determined for a larger value of h. Similar templates can be created for almost any mathematical concept, providing the student with unlimited ability to explore, experiment and learn the concepts.

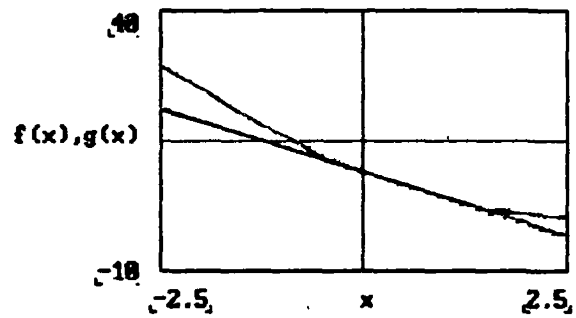
pt1 := 1   pt2 := 0

$x := -2.5, -1.5 \,..\, 2.5$ $f(x) := (x-3)^2$

$h := pt2 - pt1$

$$f' := \frac{f(pt1 + h) - f(pt1)}{h} \qquad g(x) := f' \cdot (x - pt1) + f(pt1)$$

$f' = -5$



$$slope := \frac{g(pt2) - g(pt1)}{pt2 - pt1}$$

$slope = -5$

Frame 2: Derivative Template

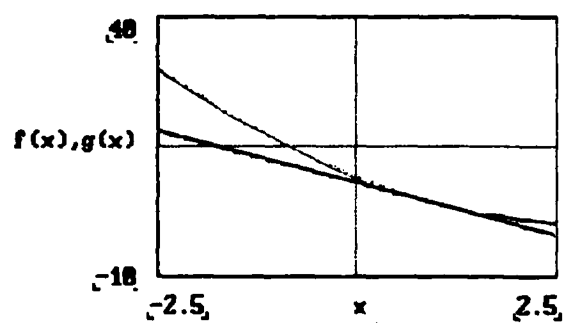
pt1 := 1   pt2 := .9999999

$x := -2.5, -1.5 \,..\, 2.5$ $f(x) := (x-3)^2$

$h := pt2 - pt1$

$$f' := \frac{f(pt1 + h) - f(pt1)}{h} \qquad g(x) := f' \cdot (x - pt1) + f(pt1)$$

$f' = -4$



$$slope := \frac{g(pt2) - g(pt1)}{pt2 - pt1}$$

$slope = -4$

Frame 3: Derivative Template

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VITA

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This thesis proposes a pedagogy and an experimental curriculum and instructional system designed to enhance the ability of engineering students to construct and manipulate mathematical representations of real world phenomena and to create the new knowledge they would need to solve unprecedented engineering problems. Developing such a pedagogy required a theory of learning and a viable model of education. The learning theory employed is Ausubel's, artfully extended by Gowin and Novak through their invention of two heuristics for learning: the Concept Map and Vee. The model of education used is Gowin's description of Schwab's four commonplaces of education.

The need for development of this pedagogy and curriculum and instructional system stems from the current rule-based approach to the teaching and learning of mathematics that promotes rote memorization of rules and techniques rather than a conceptual orientation to mathematics. Because of the need for students to understand and employ mathematical concepts and use those concepts to create mathematical representations of the real world, a construction-based curriculum was developed. The construction-based curriculum emphasizes the connection between concepts and the real world, how those concepts are used in the creation of the mathematical technologies, and how the concepts, along with the technologies, can be used to generate solutions to real world problems.

This construction-based curriculum was taught to students in a special Engineering Math Review. The ability of the review to shift the focus of students to a more conceptual understanding of mathematics was evaluated. The results clearly show that the review was successful in accomplishing this task.

The ideas presented in this thesis apply not only to the teaching and learning of mathematics, but any educative event. Recommendations are provided on how these ideas can be extrapolated and used to enhance various curricula throughout AFIT.