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**FUNDAMENTALS OF FATIGUE & FRACTURE MECHANICS**

**-Final Scientific Report On AFOSR - 86-0113**

G.B. Sinclair

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# FUNDAMENTALS OF FATIGUE & FRACTURE MECHANICS -FINAL SCIENTIFIC REPORT ON AFOSR - 86-0113

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The basic tenet of LEFM is that the stress intensity factor,  $K$ , is the key controlling parameter for fatigue crack growth and fast brittle fracture under monotonic loading. This research program examined whether or not this fundamental assumption formed an effective basis for an engineering technology. And, as needed, suggested directions for an improved technology.

First we consider the simpler of the two types of loading - monotonic loading. Here the critical value of the stress intensity factor, the plane strain fracture toughness,  $K_{Ic}$ , is required to be a material property under certain restricted circumstances. Indeed these circumstances are so restricted by standards (ASTM E399) as to raise serious questions about the applicability of resulting fracture toughness values in practice in far less controlled situations. Nonetheless, a prerequisite for even limited success in such situations is that  $K_{Ic}$  be a material property. Unfortunately, it does not appear to be one. More precisely, [1] found the variations in  $K_{Ic}$  values for a steel (AISI 4340) and an aluminum alloy (7075-T6) were about 100% of their respective mean values (cf. yield strengths which are about 20% of corresponding mean values). Following suggestions prompted by [1], the heat treatment of the steel is taken into account in [2]\* which results in four classes of 4340 steel. The average variation in  $K_{Ic}$  remains about 100%. Also in [2], further aluminum alloys are treated reducing variation somewhat but still maintaining an overall average of 90%, and a titanium alloy is examined and found to also average a variation of 90%. In all, it would seem unreasonable to attempt to continue to regard  $K_{Ic}$  as a material property at this time.

Part of the variability in fracture toughness may well stem from size effects. Variations due to thickness have long been recognized and E399 attempts to control these by determining values for thick or plane strain specimens which are supposed to be low and conservative. Unfortunately, this is not always the case; [3]\* demonstrates experimentally that a specimen which is 10× thinner than a thick, plane strain, specimen, can have a fracture toughness which is only 20% of the supposedly conservative value for the thick specimen. So there is a clear need to understand these effects better and we cannot guarantee a conservative approach with respect to them at this time. Turning to variations with in-plane scaling, the notion that  $K_{Ic}$  is a material property prohibits any such variations. Again unfortunately, the physical facts do not fit the requirements of the approach; [4] finds that when specimens are scaled by more than a factor of 3, toughness values differ by greater than 10% in more than 80% of reported testing found, while if the scale factor exceeds 7, no toughness value remains within 10% of its original value and some differ by up to a factor of three. Such inability to scale properly under carefully controlled laboratory conditions would seem to underscore a fundamental short coming in the approach and offer little hope for viably applying it in the more challenging and complex configurations encountered in engineering.

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\* References with stars have not been supplied to AFOSR previously and are therefore appended at this time.

Next we consider the role of K in estimating fatigue crack life. Here the essence of the approach remains, even today, the Paris law fit for crack growth per cycle, da/dN, viz

$$\frac{da}{dN} = C (\Delta K)^n \text{ for } \Delta K_l \leq \Delta K \leq \Delta K_u \quad (1)$$

In the above,  $\Delta K$  is the fluctuation in the stress intensity factor accompanying cyclic loading, C and n are constants, and  $\Delta K_l$ ,  $\Delta K_u$  the range of steady crack growth in which the fit can be applied. For this fit to be useful, C and n must approach being material constants under suitably controlled circumstances, as must  $\Delta K_l$ ,  $\Delta K_u$ . Unfortunately, the earlier shortcomings of the K-approach make their presence felt here too. To expand, [5] demonstrates that  $\Delta K_l$  and  $\Delta K_u$  can both shift by a factor of two with a change in size alone of a factor of four, so one cannot determine when the fit applies with any great confidence. Further, even when a consensus of when (1) applies is formed by drawing on all available sources from the literature for a given material under fairly tightly controlled laboratory conditions, n is found to vary by more than 100% of its mean value while C varies by three orders of magnitude [6]\*. If anything then, the approach is less satisfactory for fatigue crack growth than monotonic loading.

In summary, the current approach is inadequate as a truly predictive engineering technology and failure to recognize this may lead to dangerously erroneous predictions. While some trends are in agreement with those indicated by regarding K as the controlling parameter, the phenomena of fatigue and fracture at cracks are just too complicated to admit to such a simplistic approach. Needed is one which recognizes more of the material features. This is especially so since the highly stressed regions around cracks are highly localized, therefore small, and thus sensitive to microstructural size scales. As a first step in such a new approach, it is necessary to imbed these microstructural features in a physically realistic and realizable stress field. Consequently we need to relinquish the simplifying assumption of a zero root radius at the crack tip - the assumption which produces singular stresses and hence the one K owes its very existence to. Then, with a sufficiently careful stress analysis which incorporates material aspects as appropriate, we have a chance of a meaningful mechanistic approach to fracture and even fatigue. While this necessarily complicates the technology, it is necessary to accommodate the physical complexity of fatigue and fracture. Without a significantly different approach (and regrettably a more difficult one), we do not really have a viable engineering technology for fatigue and fracture nor are we likely to have.



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## APPENDIX

Copies of references [2], [3] and [6] follow.

## FURTHER REMARKS ON OBTAINING FRACTURE TOUGHNESS VALUES FROM THE LITERATURE

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A recent note by Chambers and Sinclair [1] draws on compendiums compiled by Hudson and Seward [2,3] to address the issue of the variability encountered when an engineer attempts to obtain toughness values from the literature. The note compares the variations observed in the plane strain fracture toughness ( $K_{Ic}$ ) and yield stress ( $\sigma_Y$ ) for two materials with large listings in the open literature, namely AISI 4340 steel and 7075-T6 aluminum alloy. The present work aims to extend and improve on that effort to gauge the reliability of treating  $K_{Ic}$  as a material property.

One suggestion elicited by [1] is that the heat treatment should be taken into account when examining the scatter in  $K_{Ic}$  for the 4340 steel. Another suggestion is that a comparison with the variations in ultimate stress values ( $\sigma_U$ ), rather than with yield stress ( $\sigma_Y$ ), is more appropriate. As a possible side benefit to such a comparison, the ultimate stress values so gathered could help distinguish heat treatments. And on our part, we wish to determine whether the variability reported in [1] is representative of the general scatter in  $K_{Ic}$  by considering more materials.

The major differences in the final condition of 4340 steel arise from variations in tempering temperature ( $T_T$ ); for a given temper, the other phases of the processing are essentially equivalent for our data set. Thus we classify based on temper, choosing the following ranges and designations:

<u>Temper</u>	<u>Referred to as -</u>
As quenched or $T_T < 150^\circ\text{C}$	As-quenched group, S1
$150^\circ\text{C} \leq T_T < 230^\circ\text{C}$	High strength group, S2
$230^\circ\text{C} \leq T_T < 350^\circ\text{C}$	Medium strength group, S3
$350^\circ\text{C} \leq T_T < 500^\circ\text{C}$	Low strength group, S4
$500^\circ\text{C} \leq T_T$ or as annealed	Annealed group, S5

With this classification, the ranges of ultimate stress are essentially distinct, overlapping by less than one MPa, confirming our earlier expectations. Thus while there are other ways of distinguishing processing for 4340 steel, this approach would seem sensible at least as regards  $\sigma_U$ .

With regard to our objective of comparing  $K_{Ic}$  with  $\sigma_U$ , ultimate tensile stress data for 7075-T6 aluminum are analyzed for scatter to complement the study in [1]. Turning to extending the scope of the study, aluminum alloys are the obvious first choice since they have readily distinguishable heat treatments. Two alloys with sizable listings, apparently large enough to assess scatter, are 7079-T6, chosen as a second representative of the 7000 aluminum alloy series, and 6061-T651, which extends the study to a different series and different final condition. Finally, in an attempt to examine a totally different material, the titanium alloy Ti-6Al-4V is included. For this material, and titanium alloys in general, the heat treatment and complexity of the resulting microstructures defy easy classification. We classify Ti-6Al-4V based on the similarity of final microstructure (after Williams [4]) as follows:

Processing	Referred to as -
Recrystallize annealed or diffusion bonded	RA/DB group, T1
Mill annealed	MA group, T2
General $\alpha+\beta$ processed	$\alpha+\beta$ group, T3
Solution treated around 950°C	ST950 group, T4
Beta processed	$\beta$ group, T5

While this is certainly not the only or most refined classification possible, in lieu of anything obviously markedly superior we use it here.

As in [1], the focus is on  $K_{Ic}$  as governed by ASTM E399 [5], and values are included if their contributors claim compliance with the standard. We do not fully verify this compliance ourselves because most of the references reviewed do not provide sufficient information to enable a check. In all instances, significant effort is made to locate the data sources listed in the two compendiums, yet we still remain unable to obtain a few. In processing the data, we report the means from each source for every distinct material composition and heat treatment, as well as for every distinguishable test specimen type. The results for all materials are summarized in appended histograms, wherein we define the relative frequency as the ratio of the frequency  $f$  of a class to the number  $N$  of data in the set, the normalized toughness as the ratio  $K_{Ic}/\bar{K}_{Ic}$ ,  $\bar{K}_{Ic}$  denoting the mean of the  $K_{Ic}$  data, and the normalized yield stress  $\sigma_Y/\bar{\sigma}_Y$  similarly. Individual plane strain fracture toughness values are presented in appended Tables 1 through 4, wherein, as in [1], single

numbers in brackets denote original sources, hyphenated numbers the corresponding reference in [2] or [3], and a virgule between the two implies as reported in the latter, e.g. [6]/[3-291] is [6]'s data as drawn from [291] in [3]\*.

As a measure of the variability in fracture toughness, we take  $\Delta K_{Ic}/\bar{K}_{Ic}$  where  $\Delta K_{Ic}$  is the range of the toughness values. We choose the range of the data rather than an approximate 95% confidence interval (i.e.  $\pm 1.96 s$  where  $s$  is the standard deviation) because it gives values of  $\Delta K_{Ic}/\bar{K}_{Ic}$  which are typically 16% lower. Further, we choose to report a comparison of  $\Delta K_{Ic}/\bar{K}_{Ic}$  with the corresponding quantity for the yield stress,  $\Delta\sigma_Y/\bar{\sigma}_Y$ , because the variability in yield stress is found to be *greater* than that in ultimate stress, though the two are highly comparable. The number of data points used to calculate  $\Delta K_{Ic}/\bar{K}_{Ic}$  is on average 27 and is always more than 10. Similar but not identical size samples are used for  $\Delta\sigma_Y/\bar{\sigma}_Y$ . The data for 4340 steel Group S5 has only five  $K_{Ic}$  values reported and is judged insufficient to reliably assess scatter, and is therefore excluded from the analysis. The results of the comparison can be summarized as follows:

Material and Designation	$\Delta\sigma_Y/\bar{\sigma}_Y$ (%)	$\Delta K_{Ic}/\bar{K}_{Ic}$ (%)
4340 steel, S1	13	87
S2	12	119
S3	11	102
S4	23	89
Al 7075-T6**	22	162
Al 7079-T6	17	72
Al 6061-T651	14	36
Ti-6Al-4V, T1	30	55
T2	32	117
T3	34	88
T4	43	110
T5	24	81

From the above, we see that the scatter in the plane strain fracture toughness for 4340 steel, represented by the quotient  $\Delta K_{Ic}/\bar{K}_{Ic}$ , is on average the same as the 100% reported for the aggregate of 4340 steel data in [1]. Note however that  $\Delta\sigma_Y/\bar{\sigma}_Y$  decreases from 17% to 15%, so that the ratio of scatter in toughness to scatter in yield stress increases roughly from a factor of about six to one of about seven. As for the aluminum alloys, the variation exhibited by

\* For the sake of brevity, we do not relist the references contained in [2], [3].

\*\* After [1].

Al 7075-T6 [1] turns out to be the highest of the three alloys considered. However, the toughness variation remains on average five times that of the yield stress, and is at least two and a half times as high. Finally for the titanium alloy, the scatter in  $K_{Ic}$  is on average about three times higher than that of the yield stress. Note that the relatively high scatter observed for  $\sigma_Y$  may very well be caused by an inadequacy of our definition of titanium groups, especially the one designated T4.

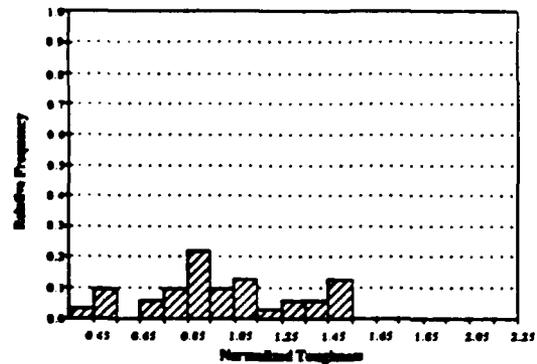
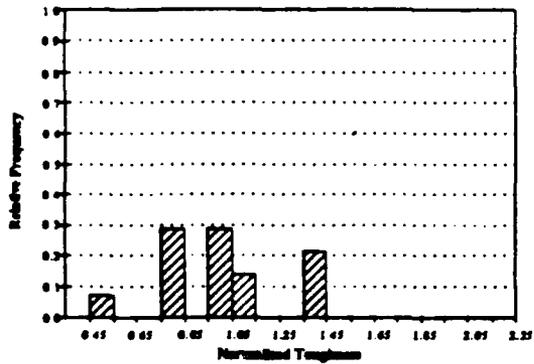
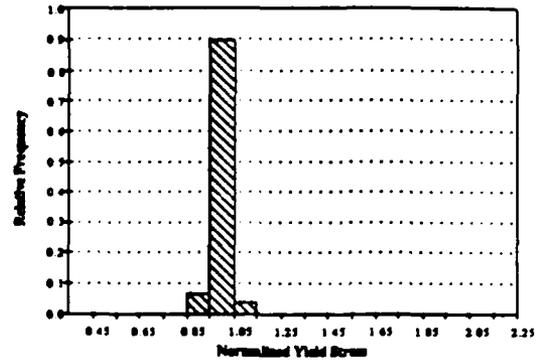
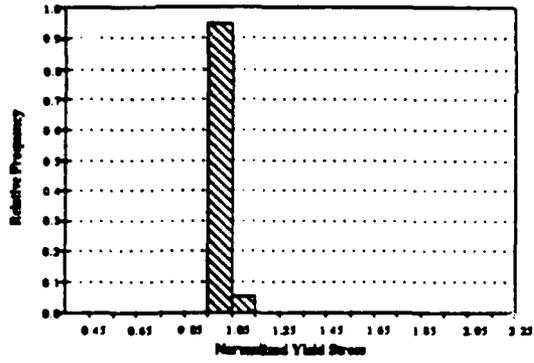
For the instance of 4340 steel, taking some account of heat treatment does not appear to reduce the scatter in toughness, at least on average: it is quite possible, however, that even tighter restrictions on composition and heat treatment would do so. It is not obvious how one would implement such a strategy for the aluminum alloys, though certainly alternatives exist for the titanium alloy. In all, more restrictive definitions of "material" should tend to reduce scatter in  $K_{Ic}$  and make it appear more of a "material property". It should be pointed out that the present tight testing standards already preclude the determination of  $K_{Ic}$  for a number of materials and that further restrictions can only further limit the applicability of this approach in engineering practice. Moreover, it does not seem probable that even the most refined definition of "material" would completely alleviate the problem. Indeed a recent investigation by Sinclair and Chambers [22] has shown that even when we measure fracture toughness for essentially the same material using the same specimen type and same test procedure, but with different sized specimens, apparently valid values of plane strain fracture toughness vary typically by 30% and can vary by a factor as high as three for significant size differences. Thus  $K_{Ic}$  lacks the size independence essential in a material property. Accordingly any engineer continuing to take such a simplistic approach to fracture faces the real possibility of serious errors, conservative and otherwise.

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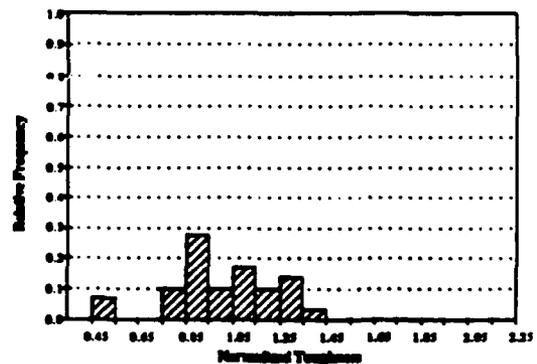
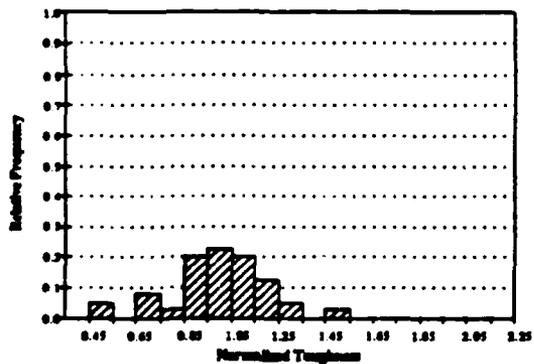
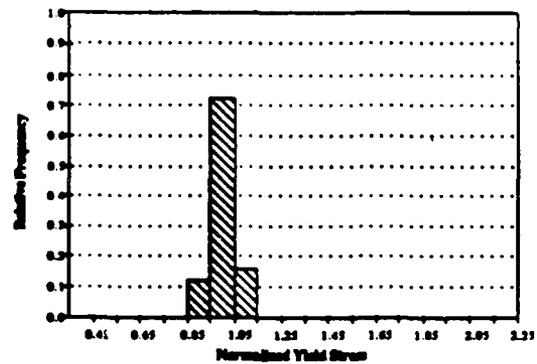
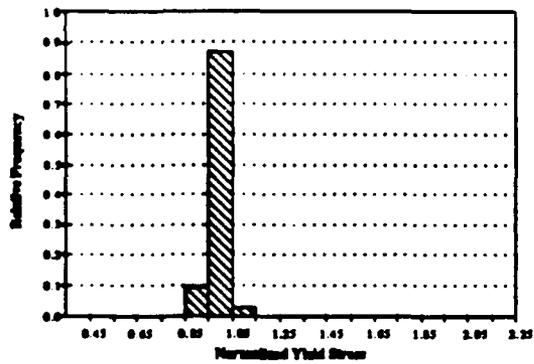
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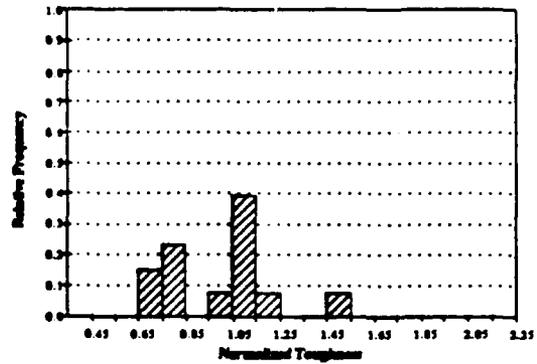
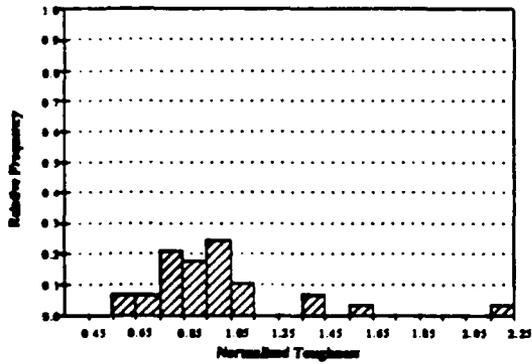
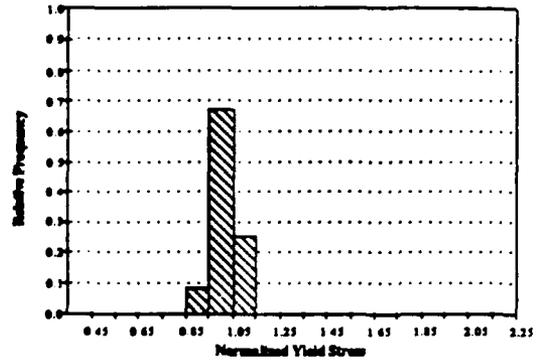
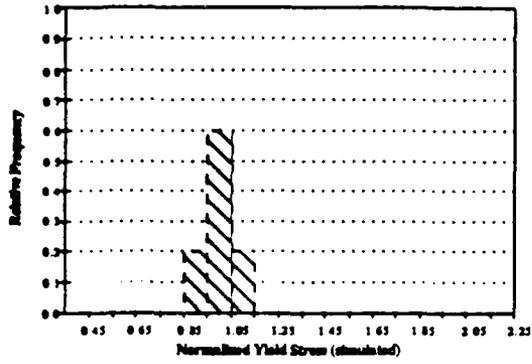
4340 Steel - Group S1

4340 Steel - Group S2



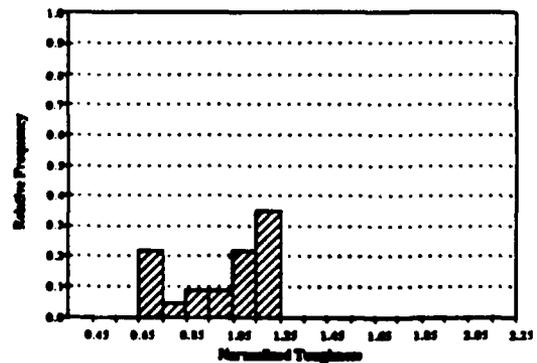
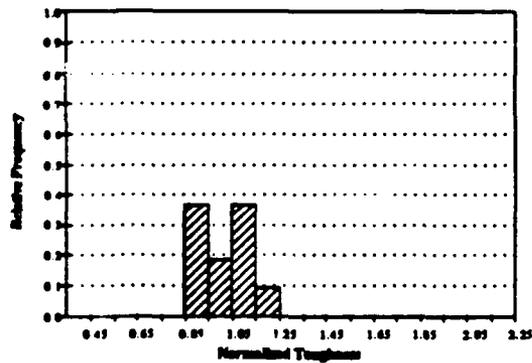
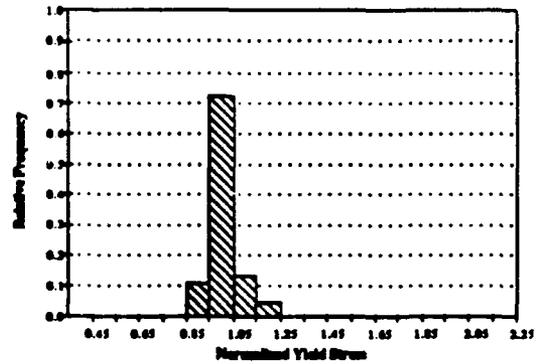
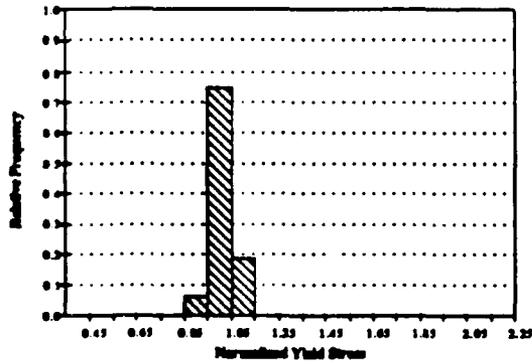
4340 Steel - Group S3

4340 Steel - Group S4



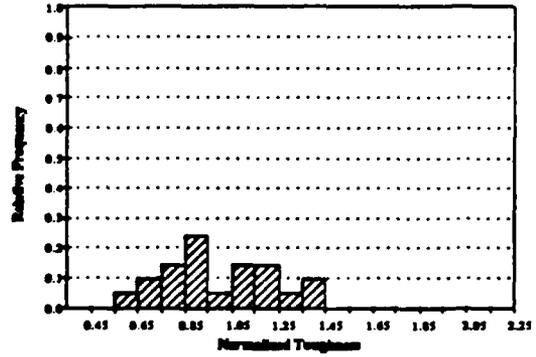
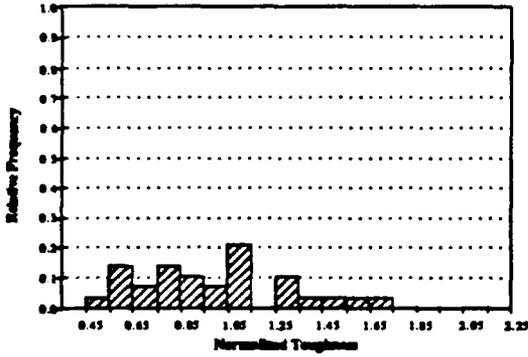
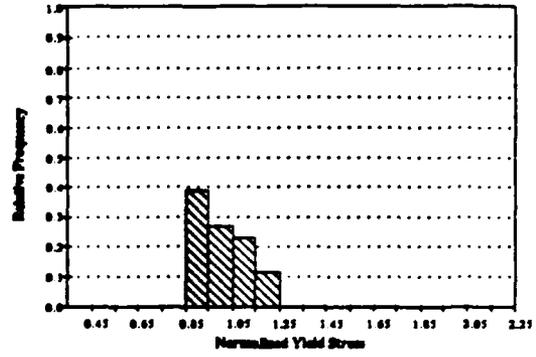
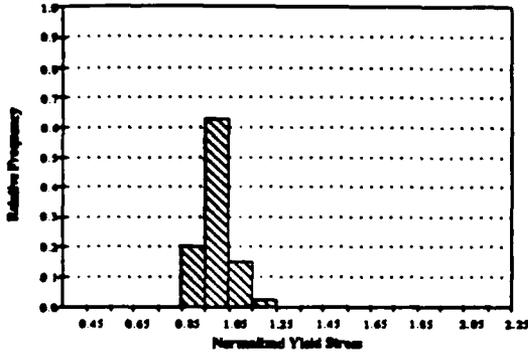
Al 7075-T6 (after [1])

Al 7079-T6



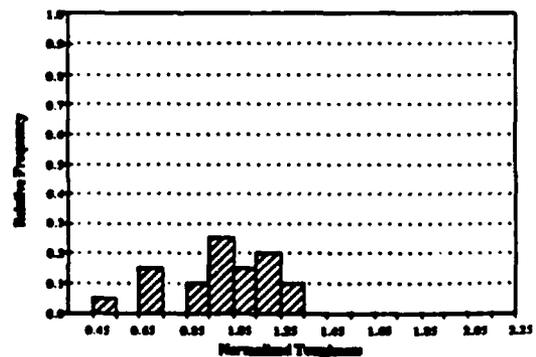
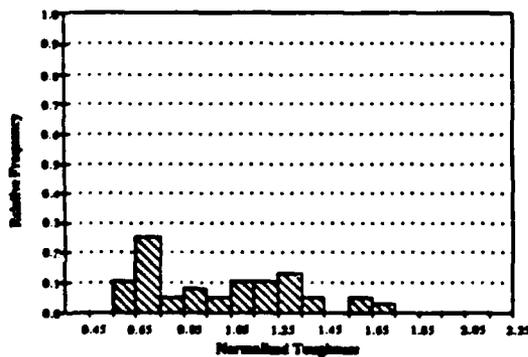
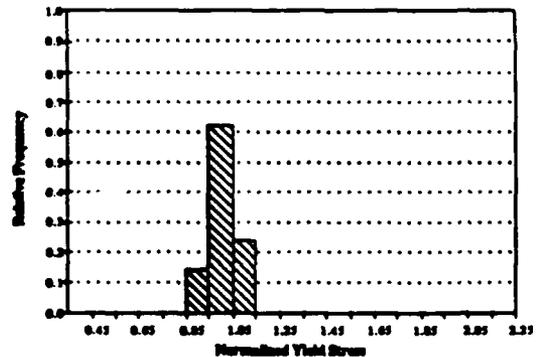
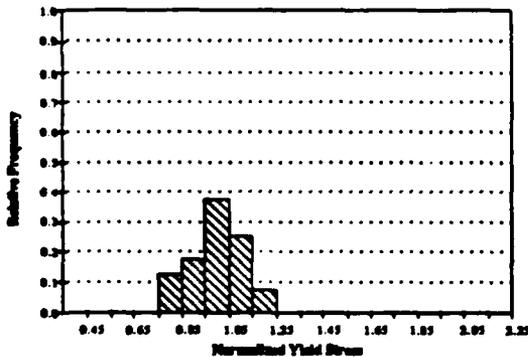
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TI-6Al-4V - Group T3



TI-6Al-4V - Group T4

TI-6Al-4V - Group T5

Table 1-a. Plane strain fracture toughness values for AISI 4340 steel - Group S1

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
37.6	[2-152]	38.7	[3-333]	48.0	[3-362]
67.5		69.7		52.7	
69.7		41.1	[3-362]	52.7	
26.5	[6]/[3-291]	41.1		50.4	
		48.0		50.4	

Table 1-b. Plane strain fracture toughness values for AISI 4340 steel - Group S2

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
88.6	[2-25]	56.1	[7]/[3-291]	68.6	[3-362]
88.6				83.0	
65.9	[2-152]	61.4	[8]/[3-291]	68.2	
85.1		52.5		68.2	
		43.7		75.6	
		49.7		75.6	
30.2	[6]/[3-291]	30.8		92.8	
39.0		27.0		56.8	
49.6				56.8	
61.4		50.4	[9]/[3-291]	92.8	
52.0		54.3			
21.0		57.8			
69.5		52.4			

Table 1-c. Plane strain fracture toughness values for AISI 4340 steel - Group S3

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
58.1	[2-12]	43.7	[6]/[3-291]	46.3	[8]/[3-291]
58.7		51.9		65.7	
90.2		56.2		52.1	
		50.9		29.4	
74.2	[2-25]			77.5	
74.2		57.9	[7]/[3-291]		
				61.9	[9]/[3-291]
66.4	[2-152]	57.8	[8]/[3-291]	68.4	
		73.8		54.5	
57.9	[2-160]	67.6		65.9	
55.7		42.1		68.4	
60.7		50.8		55.9	
		44.3			
58.7	[3-3]	30.7		57.4	[10]/[3-291]
		63.9			
71.1	[3-20]	76.3			
71.1		70.1			

Table 1-d. Plane strain fracture toughness values for AISI 4340 steel - Group S4

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
107.6	[2-7]	64.8	[6]/[3-291]	99.9	[7]/[3-291]
75.2		69.1		69.5	
		78.4			
77.5	[2-12]			79.5	[9]/[3-291]
		44.8	[7]/[3-291]	88.0	
88.6	[2-152]	83.6		74.0	
		91.0		90.6	
109.1	[2-156]	72.5		105.8	
		87.2		104.9	
116.2	[3-3]	74.0		88.0	
97.4		41.7			
77.5		102.8		75.3	[10]/[3-291]

Table 2. Plane strain fracture toughness values for Al 7079-T6

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
34.0	[2-2]	33.4	[3-130]/[2-12]	32.1	[2-41]
36.5	[2-3]	24.4	[11]/[2-15]	23.9	[3-39]
22.8	[2-5]	34.3	[3-130]/[2-15]	24.4	[3-109]
22.1	[2-9]	34.4	[2-24]		
44.5	[2-11]	33.2	[2-40]		

Table 3. Plane strain fracture toughness values for Al 6061-T651

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
29.1	[12]/[2-3]	28.8	[2-15]	33.5	[3-3]
26.1	[13]/[2-3]	23.6	[2-32]	24.4	[3-76]
30.6	[3-33]/[2-3]	29.4	[2-19]/[2-128]	31.0	[3-109]
23.8	[3-37]/[2-3]	23.6	[3-2]		

Table 4-a. Plane strain fracture toughness values for Ti-6Al-4V - Group T1

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
92.9	[2-3]	75.1	[2-3]	63.6	[3-3]
96.8		87.0		90.8	[3-654]
97.4		83.1		89.0	[3-662]
92.9		56.6		53.9	[3-663]
89.8		56.6			
90.7		54.2			
92.5		54.2			
72.8		67.5			
76.6		85.2			
97.4					

Table 4-b. Plane strain fracture toughness values for Ti-6Al-4V - Group T2

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	
97.0	[2-3]	76.9	[2-3]	66.4	[3-654]	
92.8		59.7		49.8		
124.0		39.2		81.9		
61.8		43.2		79.7		
60.9		45.4		72.0		
42.3		43.2		60.9		
105.9		118.7		52.0		[3-656]
102.2				79.7		
76.8						
93.0						
67.1		68.4	[3-3]			
76.5		59.2				

Table 4-c. Plane strain fracture toughness values for Ti-6Al-4V - Group T3

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
102.8	[2-3]	48.7	[2-3]	73.4	[17]/[2-341]
88.1		47.2			
102.2				94.9	[2-666]
86.6		75.9	[2-6]		
87.5				56.5	[3-3]
66.9		64.2	[15]/[2-15]		
67.5				81.5	[3-91]
80.2		64.2	[16]/[2-15]		
65.7				59.9	[3-109]
56.4		39.3	[17]/[2-341]		

Table 4-d. Plane strain fracture toughness values for Ti-6Al-4V - Group T4

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
90.4	[2-3]	60.9	[18]/[2-15]	50.6	[2-343]
95.8		47.6	[2-61]	119.3	[2-347]
45.8					
94.3		50.7	[2-160]	77.9	[3-3]
80.9		52.1			
92.4		56.8		70.9	[3-654]
90.4				81.9	
65.8		77.5	[2-341]	88.6	
	[19]/[2-12]			93.0	
83.9		43.2	[21]/[2-341]	84.1	
	[20]/[2-12]	43.1			
49.0		43.1		71.4	[3-659]
46.7		42.0		77.7	
56.8		48.3			
	[15]/[2-15]	46.4		64.2	[3-660]
78.6		48.7			
111.8					
110.7					

Table 4-e. Plane strain fracture toughness values for Ti-6Al-4V - Group T5

$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source	$K_{Ic}$ (MPa m <sup>1/2</sup> )	Source
81.1	[2-3]	90.8	[2-61]	66.8	[2-343]
95.3		90.8			
105.1		110.7		88.2	[3-130]
83.4		110.7			
47.2		99.6		90.8	[3-654]
62.2		104.1			
63.7		116.2			

## THICKNESS EFFECTS MAY NOT DO WHAT YOU THINK THEY DO

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### INTRODUCTION

A BASIC tenet of linear elastic fracture mechanics (LEFM) is that a critical value of the stress intensity factor controls fracture, at least in brittle instances. For a given material, this critical value is termed the fracture toughness. Accordingly, for LEFM to prove useful in predicting failure for different configurations with a given material, that material's toughness needs to be largely independent of geometry or, at worst, have any geometry dependence fully understood. Unfortunately fracture toughness values usually are dependent on the thickness of the specimen furnishing them. Fortunately such *thickness effects* are well recognized, and current LEFM includes an allowance for this phenomena.

Typically, physical data on thickness effects exhibit a decrease in toughness with increasing thickness, if not monotonically for small thickness variations then virtually invariably overall for large changes. The usual argument advanced to explain such trends is as follows: increasing thickness increases constraint on any plastic flow present in the interior by virtue of the increased magnitude of the through-thickness normal stress component; with greater constraint, fracture can be expected to be more brittle and occur at lower loads with correspondingly reduced values of toughness; ultimately, though, such through-thickness stresses ought to be asymptotic to a state of generalized plane strain with an attendant limiting lower value for toughness. The usual strategy adopted in view of the physical evidence and companion explanation is to take the apparent limiting lower bound on toughness as the parameter governing fracture. This choice is quite naturally called the plane strain fracture toughness ( $K_{IC}$ ), and is hoped to give rise to conservative designs. Exactly how the selection is made and other aspects of determining  $K_{IC}$  are detailed in the ASTM E399 standard[1], and the considerable body of testing attempting to apply[1] attests to the wide acceptance of plane strain fracture toughness by the fracture mechanics community.

In the light of the foregoing, it is interesting to ask the question as to what, if any, thickness effects are present for a material which essentially displays no plastic flow prior to fracture irrespective of its thickness—the ideally brittle material in an engineering sense. Such a question aims to check the completeness of the usual rationalization of thickness effects. That is, if the explanation offered is the entire story, one would anticipate no effect on toughness due to thickness changes. The question is particularly important in the event that this does not transpire to be the case, since then our current understanding would have been demonstrated to be less than complete, thereby raising the possibility that our technology may not necessarily be conservative. We therefore seek to investigate this issue here.

There are numerous experimental studies on thickness effects, in general, reported in the literature: a fairly extensive recent bibliography[2] cites some 101 related references while more recent extensions to [2] by Pieri[3] provide a further 36 articles. In contrast, if one uses accepted estimators of yield region extent  $r_Y$ , in conjunction with the definition of limited plastic flow implicit in the standards for  $K_{IC}$ [1] (namely  $r_Y$  less than 2% of the crack length), then relatively few of these investigations can be classified as involving brittle response throughout any marked variation in thickness. Some studies which do contribute under these restrictions are: Kinloch and Gledhill[4], Mindess and Nadeau[5], and Smith and Chowdary[6]. The picture emerging from these studies is less than completely clear: while [5] is consistent with no thickness dependence for brittle behavior, [4] typically shows a reduction like that found for ductile

response and [6] even indicates an increase. Consequently, [4-6] are somewhat inconclusive regarding the issue of present concern, possibly in part because the range of thickness ratios is less than or equal to about five. As a result we seek to investigate the matter further here.

In what follows we outline an experimental study, undertaken with a material that can reasonably be regarded as extremely brittle, and involving a wide range of thicknesses. We begin by describing the experimental procedure adopted, then discuss how the results obtained are processed. The note closes with some brief concluding remarks.

### EXPERIMENTAL PROCEDURE AND RESULTS

The specimens for the experiments are made of 99.5% alumina, a material that exhibits no ductility at room temperature to all intents and purposes. The specimens are of the three-point-bend type with a span,  $L$ , of 27.8 mm, a width,  $W$ , of 12.7 mm, and a crack length,  $a$ , of 3.2 mm (Fig. 1): these inplane dimensions are compatible with the available test rig. The minimum

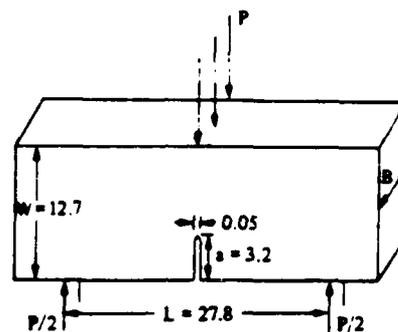


Fig. 1. Three point bend specimen (all dimensions in mm).

thickness,  $B$ , is 1.27 mm with subsequent thicknesses being factors of approximately 2, 4 and 16 times bigger. The specimens are notched using a diamond-coated copper wheel running at 2000 rpm with the width of the notch being controlled to be 0.05 mm. For each of the four different thickness geometries, a total of five specimens is made, five being regarded as an economical number to furnish an estimate of variability. The specimens are broken using a MTS servo-hydraulic machine operated in the stroke control mode to give a load point displacement of 4 mm/s, and all results recorded and included. The load cell is rated at 90 kN and is calibrated for load values comparable to the expected fracture loads to within 3½% typically and a maximum of 5½%. Finally, as a check on any variation in bulk strength from specimen to specimen, following the three-point bend testing the broken halves are fractured in simple bending.

The mean value of the peak loads, together with the standard deviation for each set, are presented in Table 1 alongside the precise thicknesses. The variation in the mean values for the bulk bend strengths are within  $\pm 7\frac{1}{2}\%$  of the overall mean value and the scatter in bulk strength within the different sets is comparable to this figure.

Table 1. Fracture loads and scatter for various thicknesses

Thickness, $B$ (mm)	Load, $P$ (N)	Standard deviation, $s$ (N)
1.27	27.2	1.2
2.31	39.7	2.3
5.10	206.9	8.7
19.81	1713.3	101.5

## CONVENTIONAL ANALYSIS

With a view to tracking any variations in toughness with thickness, we first calculate the stress intensity factor for our specimens. From [1], we have

$$K_1 = \frac{PL}{BW^{3/2}} f(a/w).$$

wherein  $K_1$  is the mode I stress intensity factor,  $P$  the load, and  $f(a/w)$  a finite width correction factor.† Substituting the peak loads of Table 1 then furnishes our toughness estimates: we denote these by  $K_1^\dagger$  rather than  $K_{IC}$  since our tests do not comply with all of the plane strain testing requirements in [1]. These values are then plotted in Fig. 2 along with the approximate 95% confidence limits ( $\pm 2s$ ). There is about a fourfold increase in  $K_1^\dagger$  with increasing thickness, an increase that cannot be attributed to scatter and/or any variations in bulk strength alone, these last possibly attributing for at most 25% of the increase.

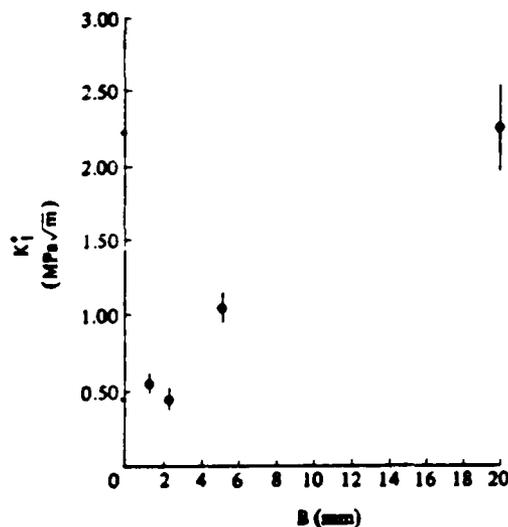


Fig. 2. Toughness variation with thickness.

## CONCLUDING REMARKS

Clearly the data do not comply with the usual rationalization of thickness effects in LEFM, indeed are completely opposite to it in the event there was some ductility present and certainly are in marked disagreement if the response was, in fact, perfectly brittle. One possible explanation is that the crack propagates by first running across the crack front, then becoming unstable and running through the ligament in front of the crack; under these circumstances, one might expect the energy requirement to increase linearly with thickness, hence the toughness as the square root of the thickness. Independent of the validity of this explanation, the data clearly demonstrate the incompleteness of the present understanding of thickness effects. Furthermore, the current strategy adopted in the belief that usual rationalization is substantially complete, namely that of taking toughness values from thicker specimens, is shown to be significantly *non-conservative* here. In all, then, our understanding of thickness effects needs to be improved in order to arrive at a reliable and safe technology.

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†Strictly the  $f(a/w)$  given in [1] does not apply to our specimens since  $L \sim 2W$  here instead of  $4W$ ; however, any errors so introduced can be expected to be small and, of course, are of no consequence in a relative sense since  $a/w$  remains fixed in our study.

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# ON OBTAINING FATIGUE CRACK GROWTH PARAMETERS FROM THE LITERATURE

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Recently, extensive compendiums of sources of fracture toughness and fatigue crack growth data have been compiled by Hudson and Seward [1, 2]. These lists of references prompted a note (Chambers and Sinclair [3]) investigating what an engineer would find on consulting them in terms of actual values of fracture toughness. The present note is a companion study to [3], and seeks to indicate what an engineer would find on consulting [1, 2] in terms of fatigue crack growth parameters.

At this time, the basis of fitting data from tests monitoring crack growth, under cyclic loading, is the accompanying oscillation in stress intensity factor,  $\Delta K$ . Probably the simplest approach is that stemming from the original work of Paris [4]. This recognizes three regimes of crack propagation: Regime I wherein cracks grow very slowly but typically with rapidly changing rates, Regime II where crack growth is stable with a steady increase in rate, and Regime III wherein cracks grow very rapidly for relatively few cycles until unstable crack propagation occurs (Fig. 1). In the stable crack propagation regime, the growth per cycle,  $da/dN$ , is fitted by the following:

$$\frac{da}{dN} = C(\Delta K)^n \quad (\Delta K_l \leq \Delta K \leq \Delta K_u). \quad (1)$$

Here, the coefficient  $C$  and exponent  $n$  are fitted constants, and  $\Delta K_l$ ,  $\Delta K_u$  are respectively the lower and upper limits on  $\Delta K$  defining Regime II in which (1) applies. Under suitably controlled conditions on the loading and the environment, the experimentally determined parameters in (1) are expected to be geometry independent for a given material. Thus (1)

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offers the possibility of predicting crack growth rates, and hence cyclic lives, in other instances for the same material under similar loading. The primary question of concern here is what are the sorts of values of  $C$  and  $N$  (and  $\Delta K_{\rho}$ ,  $\Delta K_u$ ) one might obtain from the literature for a given material under carefully controlled loading conditions.\*

In selecting materials for reviewing fatigue crack growth parameters, we first look to aluminum alloys because their heat treatment is specified in addition to their composition. Then we see alloys with a sufficient number of references in [1, 2] to estimate any variability in  $n$  and  $C$ . The two aluminum alloys that have the largest total listings of sources of fatigue crack growth data in [1] and [2] are 2024 T3 and 7075 T6 aluminum, and we focus on this pair in what follows. Regarding loading, we first require this to be of constant amplitude since this is the least complex of loading cycles and presumably leads to the most predictable results. Next we restrict the mean value of the applied stresses during cycling to be such that  $0 \leq R \leq 0.2$ , where  $R$  is defined as the minimum applied stress divided by the maximum: in this way we hope to limit mean stress effects yet still admit as much data as possible. We also restrict the frequency of the loading to be in the fairly common range of 1-120Hz, again with a view to limiting varying frequency effects yet permitting as many test results as possible. Concerning environment, we seek data gathered at room temperature and pressure, and in a noncorrosive atmosphere. We exclude data from any instances where test temperatures outside the range 15-30 °C (59-86 °F) are reported, and where any pressure other than atmospheric are described. Our first choice concerning atmosphere is an inert gas such as Argon. However, relatively little testing is performed under such conditions. Hence we relax our requirement in this regard to admit laboratory air, the most prevalent testing atmosphere and one which can reasonably be expected not to lead to significant corrosive effects in the frequency range considered.

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\* Since Paris [4], a number of alternatives to (1) have been put forward, e.g. Erdogan [5], Foreman *et al.*, [6], Collipriest [7], Annis *et al.*, [8]. While these fits attempt to capture other aspects of fatigue crack propagation, all of them feature a regime in which they expect crack growth to in essence be governed by (1).

Finally, regarding test specimen geometry, we merely require that it be a standard specimen so that its  $K$  is well calibrated. All of the above simply constitutes one set of restrictions that attempts to control fatigue test loading and environment while admitting sufficient data to obtain representative values of  $n$  and  $C$  together with their distributions. Other sets could be developed equally well to this end - the present choice is but a reasonable one.

In reducing the data, we proceed as follows. We combine data from a single source even if testing is repeated, provided there are not distinguishing features between tests. If, on the other hand, some of the tests are performed with one type of specimen while others are undertaken with a distinct type, we obtain separate fits. Too, if the data are all from a single class of test specimen but different specimen dimensions are involved, we obtain separate fits. In fitting the data, our preferred choice is to use any fit already provided. If this is of the form of (1), then  $n$  and  $C$  are noted along with the range,  $\Delta K_l$  to  $\Delta K_u$ , in which (1) is judged to fit the data well. If the form of the fit differs from (1), straight line segments over  $\Delta K$  intervals on a  $\log(da/dN)$  versus  $\log(\Delta K)$  plot are matched to the given fit, usually sufficiently well so that any gaps between the given curve and the piecewise linear do not exceed the width of the line being drawn. This results in different  $n$ ,  $C$  values for different  $\Delta K$  ranges, all of which are recorded. In doing this we are not suggesting that the particular author(s) of a source would regard all the  $\Delta K$  ranges so determined as regimes of stable crack propagation - rather we are just trying to establish a consensus from the physical evidence of what this regime is. For data with no fit drawn in, we prefer to use least squares regression of (1) over  $\Delta K$  intervals if actual data points are tabulated. If data are only given in graphical form, then we simply draw a straight line or line segments on log-log plots. In this last activity some scatter is introduced, as well as in the replotting necessary for quite a number of sources which had  $\log(da/dN)$  versus  $\Delta K$  instead of versus  $\log(\Delta K)$ . In simulations of the graphical inspection approach tried on data sets in fact having least squares fits, the deviation introduced in the exponent  $n$  was found to be less than 20% of its least squares value, while the coefficient  $C$  was found to be within an

order of magnitude of its least squares value. Any subsequent scatter found in results for  $n$  and  $C$  should therefore be viewed with these sort of discrepancies in data processing in mind. The results are summarized in Table 1 and detailed values and sources are given in Tables 2, 3 for 2024 T3, 7075 T6 aluminum respectively. All tables use SI units. In Tables 2, 3, single numbers in brackets denote original sources, hyphenated numbers the corresponding reference in [1] or [2], and a virgule between the two implies as reported in the latter, e.g. [9]/[1-74] is [9]'s data as drawn from [74] in [1]\* A serious effort was made to obtain all the pertinent references cited in [1, 2], but nonetheless some remain unavailable to us. Any other sources encountered during the study were also reviewed. In all to date, 70 odd references for the two aluminum alloys have been processed. Excluding references that duplicate data and/or describe testing that does not comply with our restrictions, 22 and 20 independent sources contribute for 2024 T3 and 7075 T6 aluminum, respectively. Any further data is welcomed for inclusion in the future. It is not thought, however, that there exists a sufficient body of untapped data to significantly alter the findings here.

In constructing Table 1, we start with  $\Delta K$  of  $4\text{MPa m}^{1/2}$  as the common lowest value for which (1) is fitted in Tables 2, 3. Thereafter we increment  $\Delta K$  by  $3\text{MPa m}^{1/2}$ , and take  $n$ ,  $C$  pairs from Tables 2, 3 if the  $\Delta K$  ranges there overlap with two thirds or more of the increment. We then pick the  $\Delta K$  ranges for Table 1 as the ones having the greatest number of such  $n$ ,  $C$  pairs - none of the increments so selected have fewer than 17. Within these increments, we determine the mean exponent,  $\bar{n}$ , as a representative value of  $n$ . As a measure of the variability in  $n$ , we take the difference between the approximate 95% confidence limits ( $\pm 1.96s$  where  $s$  is the standard deviation in  $n$ ), normalized by  $\bar{n}$  and expressed as a percentage. For a representative value of the coefficient, we take the antilog of the mean of  $\log(C)$  (referred to as the "log mean for  $C$ "), while as a measure of its variability

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\* In the interests of brevity we do not relist references given in [1, 2].

we take the ratio of the antilogs of the approximate 95% confidence limits for  $\log C$ .

In the  $\Delta K$  ranges in Table 1, the mean,  $\bar{n}$ , is fairly constant for each alloy (within 21%), and the respective representative values of  $C$  consistent to within an order of magnitude. At higher  $\Delta K$  levels,  $\bar{n}$  increases for both alloys in keeping with approaching unstable crack propagation and entering Regime III. Too, if the mid-points of the  $\Delta K$  intervals are substituted into (1) with the corresponding representative values of  $n$  and  $C$ , the  $da/dN$  so calculated essentially lie on a straight line on a log-log plot of  $da/dN$  versus  $\Delta K$ .<sup>\*</sup> Hence the intervals in Table 1 would appear to be consistent with the stable crack growth of Regime II, and consequently we look to examine the values of  $n$  and  $C$  within these  $\Delta K$  ranges.

The examination reveals that the exponent varies nearly the same amount for both alloys, namely about 105% of its mean value on average, while the coefficient varies in a similar way for each alloy, changing by three or more orders of magnitude. These fluctuations do not seem to be that sensitive to the measure chosen: if the variability of  $n$  is based on range instead of confidence limits, the percentages are similarly distributed and 1% lower on average, while if simply the extreme values of  $C$  are used to calculate ratios instead of the procedure in Table 1, the ratios are within a factor of 6 or more in Table 1 and typically higher. Nor can these fluctuations reasonably be attributed just to the scatter in underlying data reproduction techniques or testing procedures - recall that, in the least preferred situation, processing scatter is only as high as 20% of the mean of  $n$  and an order of magnitude in  $C$ , while scatter from repeated tests for the same conditions in a single laboratory is basically removed here by taking the means of such testing.<sup>\*\*</sup> Such variations would therefore appear to be inherent in  $n$ ,  $C$  under the conditions specified.

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<sup>\*</sup> Such calculations yield  $da/dN$  that are uniformly higher for 7075 T6 than for 2024 T3, as observed elsewhere, e.g. [13].

<sup>\*\*</sup> In any event, such experimental scatter is quite comparable to that stated as possible due to data reduction (see, e.g. [17]).

These variations are far greater than that occurring in an accepted material property such as yield stress (for example, the variation in  $n$  is about a factor of five times greater than that reported in [3] for the yield stress of 7075 T6 aluminum). Perhaps a more appropriate comparison is with like quantities in the fatigue testing of smooth, uncracked, specimens. Though not usual practice, to this end we fit life to stress amplitude (or equivalent strain) raised to a power on S-N curves - the analogue of (1) in effect. This fit works quite well over an appreciable range of lives. Applying it to medium strength steels in the atlas of fatigue curves compiled by Boyer [20], we find that the exponents for the extremes are less than 25% of their mean value apart, while the coefficients associated with the extremes are within a factor of 20. Applying it to the collection of S-N data for 2024 T3 and 7075 T6 assembled by Ruff and Hyler [2-200], we find that the exponents for the 90% confidence limit given in [2-200] and its inferred symmetric counterpart are less than 20% and 30% of their means apart, respectively, while the associated coefficients are within factors of 25 and 55, respectively. The variability in  $n$  and  $C$  of (1) for the A1 alloys treated here is markedly higher, then, than in these corresponding quantities for S-N curves.\*

By further restricting our test conditions, it may be possible to reduce the variability in  $n$  and  $C$ . Accordingly we next attempt to quantify the likely effects of more constrained conditions. We begin by considering the influence of mean stress or R-value, an aspect of fatigue crack growth which has attracted considerable attention in the literature. Removing the two sources wherein R is only implicitly small and thereafter halving the allowable range by removing two other sources does reduce the variation in  $n$  and  $C$ , though not by much. Explicitly, for  $0 \leq R \leq 0.1$ , the 95% confidence interval for  $n$  as a percentage of  $\bar{n}$  is reduced by about 6% for 2024 T3 but is left essentially unchanged for 7075 T6, and the ratios of limiting C's for both alloys are typically reduced but by less than a factor of 5. The outcome of adopting a similar tactic for frequency is less discernible because too many

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\* A limited survey for 4340 steel, which does not attempt to distinguish between heat treatments and is based on only 9 references drawn from [1, 2], indicates similar greater variation in  $n$ ,  $C$  for this material, with  $n$  varying by more than 100% of its mean and  $C$  ranging over four orders of magnitude.

references are only implicitly in the common range of 1-120Hz. Removing [1-77], the one source which is explicit in exceeding the half range, 1-60Hz, has no real effect, but is less than a complete assessment of the influence of tighter frequency requirements. However, Donaldson and Anderson [9] demonstrates that there is little difference between 0.8Hz and 20Hz for 2024 T3 (their results correspond to, at most, a 10% change in  $n$  and a factor of 5 for  $C$ ), while Wanhill [2-194] indicates only a little more sensitivity for 2024 T3 for 2Hz and 57Hz, and for 7075 T6 for 0.4Hz and 57Hz provided  $\Delta K \geq 10 \text{ MPa m}^{1/2}$  (less than a 20% change in  $n$ ). Regarding temperature and pressure, we are unable to gauge the results of very tight constraints on our data set because of an absence of precise information. In the light of Mackay [2-205] and Hudson [1-49], it would not seem likely that more stringent requirements on either temperature or pressure would really reduce the variation in  $n$ ,  $C$ . Concerning atmosphere, several investigators note that dry air is effectively the same as an inert gas (e.g. Selines *et al* [2-259]), so the issue is really one of relative humidity. Again, there is insufficient explicit information to permit a fair evaluation from our data set. A survey of the effects of dry and saturated air can be found in Hahn and Simon [13]. For the  $\Delta K$  of Table 1, the upper and lower bounds of  $da/dN$  response represent changes of less than 20% in  $n$  and an order of magnitude in  $C$ . Finally, although not part of our testing conditions, we can consider more restrictive constraints on materials. The obvious candidate here is to separate out grain orientation. While again there is not really sufficient information provided in our data set to apply this segregation, its effects for the  $\Delta K$  ranges of Table 1 can be inferred for 2024 T3 from Senijve in [2-207], and for 7075 T6 from Brussat *et al* [2-258]. These references imply differences in  $n$  of less than 5% and in  $C$  of less than a factor of 2. In all then, the further restrictions entertained do reduce the fluctuations in  $n$  and  $C$  somewhat. However, even in combination, they would not seem to control variability to the point that one engineer could reliably use another's exponent and coefficient in an application. In sum, the basic approach of fitting crack propagation rates with  $\Delta K$  appears to be too simple for the complex phenomena it attempts to model, and

thus does not result in a reliable engineering technology for predicting fatigue crack growth.

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Table 1. Mean values and variability of some fatigue crack growth parameters

ΔK range	2024 T3					7075 T6				
	Number of data	Mean $\frac{n}{\bar{n}}$	Variation in $n + \bar{n}$ (%)	Log mean for C	Ratio of upper to lower C	Number of data	Mean $\frac{n}{\bar{n}}$	Variation in $n + \bar{n}$ (%)	Log mean for C	Ratio of upper to lower C
10-13	21	2.93	113	$8.4 \times 10^{-8}$	$1 \times 10^4$	21	2.97	112	$2.9 \times 10^{-7}$	$8 \times 10^3$
13-16	21	2.89	98	$9.5 \times 10^{-8}$	$2 \times 10^3$	21	3.01	107	$2.4 \times 10^{-7}$	$6 \times 10^3$
16-19	22	2.94	94	$8.5 \times 10^{-8}$	$1 \times 10^3$	21	3.42	101	$7.7 \times 10^{-8}$	$9 \times 10^3$
19-22	18	3.46	114	$2.6 \times 10^{-8}$	$7 \times 10^4$	20	3.40	95	$6.7 \times 10^{-8}$	$1 \times 10^4$
22-25	17	3.50	115	$2.3 \times 10^{-8}$	$9 \times 10^4$	18	3.66	103	$3.0 \times 10^{-8}$	$1 \times 10^5$

Note: Units for ΔK are MPa m<sup>1/2</sup>, units for C are consistent with ΔK and da/dN in mm/cycle

Table 2. Fatigue crack growth parameters for 2024 T3 aluminum

$\Delta K$ range	n	c	Source	$\Delta K$ range	n	c	Source
				10-20	2.28	$6.1 \times 10^{-7}$	[2-198]
9-29	4.03	$2.6 \times 10^{-8}$	[1-73]	6-30	3.13	$4.6 \times 10^{-8}$	[2-199]
				6-11	5.00	$6.5 \times 10^{-10}$	[2-204]
				11-24	2.37	$3.2 \times 10^{-7}$	
6-9	5.48	$3.6 \times 10^{-10}$	[9]/[1-74]	30-38	6.95	$5.8 \times 10^{-13}$	
10-33	2.77	$1.7 \times 10^{-7}$		44-61	12.80	$3.6 \times 10^{-23}$	
				44-98	5.71	$5.3 \times 10^{-12}$	
4-20	2.61	$1.9 \times 10^{-7}$	[10,11]/[1-77]	47-76	4.60	$1.0 \times 10^{-9}$	
7-30	2.62	$2.9 \times 10^{-7}$	[12]/[1-36]	4-7	1.82	$2.4 \times 10^{-7}$	[2-205]
4-10	4.10	$9.1 \times 10^{-9}$	[13]/[2-36]				
10-19	2.91	$9.7 \times 10^{-8}$					
20-40	5.02	$2.6 \times 10^{-9}$					
				7-9	7.85	$2.8 \times 10^{-12}$	
7-24	3.21	$5.3 \times 10^{-8}$	[2-52]	9-22	2.71	$1.4 \times 10^{-7}$	
6-40	4.06	$7.7 \times 10^{-9}$	[2-53]	7-20	3.29	$5.0 \times 10^{-8}$	[2-206]
17-38	3.45	$2.4 \times 10^{-8}$	[2-55]	4-6	3.08	$3.5 \times 10^{-7}$	[15]/[2-207]
				6-12	1.99	$2.2 \times 10^{-6}$	
11-30	3.38	$2.9 \times 10^{-8}$	[2-71]	12-25	4.84	$2.1 \times 10^{-9}$	
10-24	1.90	$1.8 \times 10^{-6}$	[2-164]	10-18	2.28	$4.6 \times 10^{-7}$	[2-208]
24-34	5.98	$3.2 \times 10^{-12}$					
				10-20	2.39	$1.3 \times 10^{-9}$	[2-209]
11-19	2.60	$8.7 \times 10^{-8}$	[14]/[2-194]				
19-28	4.94	$1.2 \times 10^{-10}$		6-14	5.66	$8.4 \times 10^{-11}$	[16]/[2-210]

$\Delta K$ range	n	c	Source	$\Delta K$ range	n	c	Source
9-19	2.54	$2.5 \times 10^{-7}$	[2-197]	14-50	2.04	$1.0 \times 10^{-6}$	[17]
20-30	4.97	$1.4 \times 10^{-10}$		50-100	4.22	$5.4 \times 10^{-10}$	
				8-15	2.73	$3.4 \times 10^{-7}$	
			15-33	3.24	$1.2 \times 10^{-7}$		
20-31	3.55	$1.4 \times 10^{-8}$					

Note: Units for  $\Delta K$  are  $\text{Mpa m}^{1/2}$ , units for C are consistent with  $\Delta K$  and  $da/dN$  in mm/cycle.

Table 3. Fatigue crack growth parameters for 7075 T6 aluminum

$\Delta K$ range	n	c	Source	$\Delta K$ range	n	c	Source
10-16	2.26	$3.2 \times 10^{-6}$	[1-48]	7-26	3.19	$1.2 \times 10^{-7}$	[2-198]
17-27	4.19	$1.4 \times 10^{-8}$					
10-27	3.63	$9.8 \times 10^{-8}$		7-27	2.82	$4.3 \times 10^{-7}$	[2-199]
10-25	3.23	$2.3 \times 10^{-7}$					
8-20	2.86	$5.2 \times 10^{-7}$		11-21	2.67	$4.6 \times 10^{-7}$	[2-204]
				22-43	4.33	$2.5 \times 10^{-9}$	
7-26	2.92	$3.6 \times 10^{-7}$	[1-49]				
				3-6	2.24	$2.1 \times 10^{-7}$	[2-205]
4-16	2.89	$1.1 \times 10^{-6}$	[1-73]	6-9	5.86	$6.1 \times 10^{-10}$	
16-31	4.38	$1.8 \times 10^{-8}$					
				4-9	2.46	$2.7 \times 10^{-6}$	[15]/[2-207]
8-25	1.64	$6.6 \times 10^{-6}$	[1-90]	9-18	5.06	$8.4 \times 10^{-9}$	
25-48	4.06	$2.5 \times 10^{-9}$					
				7-16	2.47	$5.9 \times 10^{-7}$	[2-212]
4-7	4.65	$8.4 \times 10^{-9}$	[13]/[2-36]	16-28	3.68	$2.0 \times 10^{-8}$	
8-15	2.45	$1.0 \times 10^{-6}$					
16-40	4.30	$4.9 \times 10^{-9}$					
6-40	3.59	$5.8 \times 10^{-8}$	[2-53]	8-22	3.02	$2.9 \times 10^{-7}$	[2-257]
10-24	2.22	$2.1 \times 10^{-6}$	[2-7]	14-23	2.98	$1.8 \times 10^{-7}$	[2-258]
24-36	4.08	$5.7 \times 10^{-9}$		24-42	5.83	$2.2 \times 10^{-11}$	
				43-51	8.70	$5.6 \times 10^{-16}$	
4-13	2.17	$5.8 \times 10^{-6}$	[18]/[2-191]	19-48	4.23	$3.6 \times 10^{-9}$	
				49-64	6.92	$1.0 \times 10^{-13}$	

$\Delta K$ range	n	c	Source	$\Delta K$ range	n	c	Source
11-26	5.03	$2.6 \times 10^{-10}$	[14]/[2-194]	9-25	2.85	$1.3 \times 10^{-7}$	[2-259]
				25-36	4.46	$7.3 \times 10^{-10}$	
8-16	2.23	$1.3 \times 10^{-6}$	[2-197]	6-22	3.22	$4.2 \times 10^{-8}$	[19]
16-28	4.29	$3.5 \times 10^{-9}$		22-31	5.44	$3.8 \times 10^{-11}$	

Note: Units for  $\Delta K$  are MPa m<sup>1/2</sup>, units for C are consistent with  $\Delta K$  and da/dN in mm/cycle.