An analysis of the normal and in-plane acceleration sensitivity of acoustic surface wave resonators supported uniformly at the base has been performed. The calculations were not performed over the actual surface wave mode shape, but only over a typical wavelength because a convenient representation of the mode shape was not yet available. In order to obtain information on the acceleration sensitivity of surface wave resonators supported along rectangular edges relatively quickly, somewhat crude analyses of the normal and in-plane acceleration sensitivities of ST-cut quartz surface wave resonators were performed. A variational analysis of the reflection of surface waves by arrays of reflecting grooves has been performed and the transmission matrix for the grooved array has been obtained. An analysis of the normal acceleration sensitivity of ST-cut quartz surface wave resonators with the substrate extending beyond the supports has been performed.
FINAL REPORT

1. ARO PROPOSAL NUMBER: 22485-PH

2. PERIOD COVERED BY REPORT: April 15, 1985 - July 14, 1988

3. TITLE OF PROPOSAL: Analytical Investigations of the Acceleration Sensitivity of Acoustic Surface Wave Resonators

4. CONTRACT OR GRANT NUMBER: DAAG29-85-K-0075

5. NAME OF INSTITUTION: Rensselaer Polytechnic Institute

6. AUTHOR(S) OF REPORT: Harry F. Tiersten

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP:


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BRIEF OUTLINE OF RESEARCH FINDINGS

An analysis of the normal and in-plane acceleration sensitivity of acoustic surface wave resonators supported uniformly at the base has been performed. The calculations were not performed over the actual surface wave mode shape, but only over a typical wavelength because a convenient representation of the mode shape was not yet available. Among other things, this work indicated that uniform base support had not been realized in practice and, in fact, may be impossible to achieve. Subsequent to this work, as a result of a propitious experimental accident at Raytheon, we realized that for an ST-cut quartz substrate supported around all four edges and subject to normal acceleration, which produces flexure of the plate, the very large changes in frequency caused by cylindrical flexure in the two orthogonal spanning directions are of opposite sign and, hence, can be made to cancel. Since for this type of support configuration in-plane acceleration produces extension which results in relatively small changes in frequency, this realization significantly influenced the direction of the further work.

In order to obtain information on the acceleration sensitivity of surface wave resonators supported along rectangular edges relatively quickly, somewhat crude analyses of the normal and in-plane acceleration sensitivities of ST-cut quartz surface wave resonators were performed. In order to simplify the treatment simple supports were assumed for the case of normal acceleration and an approximation to the surface wave mode shape was assumed. Although both simplifying assumptions result in overestimates of the normal acceleration sensitivity, for an appropriate planar spanning aspect ratio a zero-crossing was found. However, the slope of the normal acceleration sensitivity versus aspect ratio curve was quite steep. For the case of in-plane acceleration a new variational approximation procedure, which is very accurate, computationally efficient and convenient for use in the perturbation integral, was employed for the first time. The calculated in-plane acceleration sensitivities were of the order of parts in $10^{-11}$ per $g$, which were somewhat overestimated because of the assumed mode shape. Since the actual acoustic surface wave mode shape along the transmission path is needed to perform truly accurate calculations and a continuous representation, rather than a cascaded one, would be very convenient for use in the perturbation integral, a continuous representation of the mode shape in acoustic surface wave resonators was obtained. Although grooved reflectors are of actual interest, the analysis was performed for the case of reflecting strips because the transmission matrix was available for that case and we did not want to wait for the analysis of reflecting grooves because of some difficulties anticipated in that work.

A variational analysis of the reflection of surface waves by arrays of reflecting grooves has been performed and the transmission matrix for the grooved array has been obtained. The continuous representation of the mode shape for the grooved array, which is the case of primary interest to us, was obtained in that work. An analysis of the normal acceleration sensitivity of grooved ST-cut quartz surface wave resonators rigidly supported along rectangular edges has been performed. The variational principle for anisotropic static flexure in which both constraint- and natural-type edge conditions arise as natural conditions, which is required for the new approximation procedure used in the calculation of the biasing state, was obtained in that work. The proper continuous representation of the mode shape for the
grooved array was employed in that very accurate calculation. As in the case of simple supports, the normal acceleration sensitivity exhibits a zero-crossing for a certain planar aspect ratio, but for the case of rigid supports, the slope of the sensitivity versus aspect ratio curve is about 1/4 of that for the case of simple supports. A comparison of the in-plane acceleration sensitivity of ST- and SST-cut quartz surface wave resonators has been performed using the accurate mode shape and the accurate, convenient variational approximation procedure for the determination of the biasing state, and it has been shown that the sensitivity is somewhat less for the ST- than the SST-cut.

An analysis of the normal acceleration sensitivity of ST-cut quartz surface wave resonators with the substrate extending beyond the supports has been performed. This is the support configuration employed in the experimental work at Raytheon, and it has additional parameters to control the sensitivity, which can be used to advantage. As in the case of the edge supported configurations, the calculated results show that the sensitivity vanishes for certain planar aspect ratios, but that, as a consequence of the overhangs, the slope of the sensitivity versus aspect ratio curve is much shallower than in the case of the edge supported configurations. This means that in the support configuration with the overhangs, very low normal acceleration sensitivity can be achieved with relative ease. This has been verified experimentally at Raytheon. However, they cannot make sensitivity measurements as low as might be possible because their equipment has too high a noise floor. An analysis of the in-plane acceleration sensitivity of ST-cut quartz surface wave resonators with the substrate extending beyond the supports has been performed. Although no zero-crossings are found, the calculated in-plane acceleration sensitivities are of the order of a few parts in $10^{-4}$ per g. Although the in-plane measurements at Raytheon verify our findings for acceleration in the propagation direction, there might be some conflict for acceleration normal to the propagation direction. This is unclear and we do not consider their measurements to be definitive at this time because there is not enough data. We have still to perform calculations of the acceleration sensitivity for a revised support configuration with overhangs, which we believe will lower the sensitivity further.