Representation and Transfer in Problem Solving

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**Representation and Transfer in Problem Solving**

The work reported here consists of two series of experiments that investigate the role of problem representation in transfer of skill from one problem to another. The first series showed that increasing the inter-problem similarity of important representational features increased transfer, and allowed us to localize the transfer effect in accord with earlier work the solution process was found to consist of two distinct phases: a fairly lengthy exploratory phase, that takes most of the solution time and results in no net progress toward the goal, and a final path phase in which the subjects rapidly close on the goal. The results confirmed our prediction that the locus of the differing amounts of transfer would be in the exploratory part of the solution process, the transferred skill effectively substituting for something that occurs during problem exploration.

The second series of experiments compared the effects on transfer of the subjects' internal representation of the problem and of the external task domain or stimulus situation presented to them. The problems were constructed so as to be isomorphic. The subjects were given different interpretations or representations of the problems in some cases and identical ones in other cases. The amount of transfer was shown to be dependent on the similarity of the internal representation rather than on external stimulus similarities.

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**Subject Terms**
- Problem Solving
- Analogy
- Transfer
- Representation

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**Supplementary Notation**
Introduction

The work reported here consists of two series of experiments that investigate the role of problem representation in transfer of skill from one problem to another. We will first consider some previous work that resulted in the development of a two-phase model of the problem solving process, and demonstrated the centrality of the problem move operator in the transition from the first to the second phase. Our first series of experiments is focused on the specific locus of transfer, and shows that transfer affects the learning of the move operator that normally takes place in the first phase of problem solving. A second series of experiments examines the relative influence of problem stimulus qualities and problem representation on transfer, and shows that representation is the controlling factor in transfer.

The investigation of the amount of transfer that is obtained in problem solving has been of great interest, both historically, (Thorndike & Woodworth, 1901, Judd, 1908, Ruger, 1910, Katona, 1940), and more recently as well, (Hayes & Simon, 1977, Papert, 1980, Gentner, 1983, Singley & Anderson, 1985, Carver, 1986, Klahr & Carver, 1988, Gick & Holyoak, 1987). The issue has been approached in recent work in a number of ways. It has been viewed as a task of finding and mapping higher level analogies, (Gentner, 1983), a task of noticing problem similarity and abstracting a schema, (Gick & Holyoak, 1983), a task of mapping productions from source to target problem (Kieras & Bovair, 1986, Gray & Orasanu, 1987), a task dependent on surface feature "remindings" (Ross, 1985), a task heavily influenced by problem difficulty, (Hayes & Simon, 1977, Bhaskar & Simon, 1977), a task involving interesting asymmetries in source and target problem transfer, (Bassock & Holyoak, in press), a task that is very sensitive to the amount of training on the source problem, (Smith, 1986), and a task that is dependent on move operator compatibility (Kotovsky, Hayes, & Simon, 1985).

The attention that transfer has received is due not only to the theoretical issues surrounding transfer, but also to its practical importance. The pedagogical significance of transfer is that in many or most domains, the extent to which learning is generalizable (i.e. that positive transfer is attainable) is the extent to which learning is useful. Experimental and pedagogical experience suggests that it is hard (and often seemingly impossible) to teach/learn skills that are both specific enough to accomplish a given task, and general enough to be useful across the range of similar tasks the learner is likely to encounter. The importance of understanding transfer is thus based on both the richness of its theoretical
conceptions and implications, and the practical importance of its utilization and control.

Given the value and importance of transfer, and the venerability of some of the inquiries into it, we might question the need to continue the investigation of such a basic phenomenon at this late date. We might reasonably expect that it would be a thoroughly understood phenomenon by now, a phenomenon that at most needs a cleaning up at the fringes. We take the position that there is still much to learn about transfer, and that our experiments demonstrate that to understand transfer we must first understand the effects of problem representation in both problem solving and transfer.

One embarrassment that any worker in the vineyard of problem solving faces is that of referencing work associated with one author, or more correctly perhaps, one pair of authors. It is sometimes hard to escape the belief that people will think you only had enough money for one book in 1972, or never did learn how to get past S in the author index of Psychological Abstracts. Modern research on problem solving is work on issues, and within conceptions, and utilizing research paradigms, that were generated by this symposium's honoree. Whether it is the idea of means ends analysis, the task environment and problem search space, the importance of move operators, the use of verbal protocols, the value of computer simulation, and most importantly for the work we wish to discuss, the ideas of limited processing capacity and internal problem representation or problem space, (and thus the information processing approach in all its richness and fertility), the vintage is varietal Simon.

In their seminal 1972 book, Human Problem Solving, Allan Newell and Herbert Simon introduced a theoretical framework for describing problem solving. Their theory describes problem solving as taking place in an external task environment with its associated objective search space. Out of the set of possible internal representations of that external task environment, the problem solver generates one (or more) problem spaces within which he or she operates. The problem space is the problem solver's internal representation of the problem. It includes the move operators together with the restrictions on their application (legality tests), and the set of knowledge states that he or she occupies on the way from start to goal. The examination of representational issues we undertake in this chapter builds on Newell and Simon's concept of the problem space, and focuses on the internal representation as the determinant of transfer. The experiments we have done can only have been conceived and understood in the context of their approach. We would argue that the
Figure 1: Problem search space. Tower of Hanoi isomorphs
development of that approach was necessary before any reasonably complete understanding of transfer could be attained. Of course, the development of the "modern information processing approach" to problem solving has a rather direct relationship to the reason for the 1987 Carnegie Symposium: the honoring of the work of Herbert A. Simon. We now turn to a set of experiments that are an instantiation of that approach, and that form the immediate foundation for the new work that we report here.

Experimental Background: Problem Difficulty

In 1974, Hayes and Simon published the first of a series of articles on isomorphs of the Tower of Hanoi problem. The large array of isomorphs they generated and investigated in that and later work consisted of problems that had the same problem search space as the three disk Tower of Hanoi problem, the same number of move operators, the same starting position, and the same goal. A major class of such problems were termed "Monster" problems because they involved monsters passing globes back and forth, or changing the sizes of the globes they were holding. These two types of Monster problem were labelled the "Monster Move" and the "Monster Change" problems respectively. A major finding was that isomorphic problems could differ significantly in difficulty. In particular, they found that difficulty (solution time) ratios of 2:1 were obtained between Monster Change and Monster Move problems. Kotovsky, Hayes, and Simon (1985), extended the investigation to a broader array of problems, and obtained difficulty ratios of up to 16:1 for their hardest/easiest pair of problem isomorphs. Two findings that emerged from the latter work are the starting points for the research reported here. Those are: (a) the role of the move operator in determining problem difficulty and transfer, and (b) the discovery of a dichotomous pattern of moves as people moved through the problem space to reach a solution to the problem.

The problem search space\(^1\) of most of the problems we used is identical to the problem search space of the problems used in the work of Kotovsky, Hayes, and Simon (1985), as well as the earlier work of Simon and Hayes (1976) and Hayes and Simon (1974, 1977). That search space is shown in Figure 1 (after Nilsson, 1971).

\[\text{Insert Figure 1 About Here}\]

\[^{1}\text{Throughout the chapter we differentiate between the external problem search space and the internal representation by designating the former as the task environment or search space, and the latter as the problem space or representation.}\]
The search space shown in Figure 1 consists of 27 possible states joined by links that represent legal moves. The labels on the states represent configurations of disks on pegs in the Tower of Hanoi problem, or configurations of globes held by monsters in the Monster problems. Each move involves transferring a disk from one peg to another. All but three of the states have three possible legal moves associated with them. The three moves consist of a return to the previous state, or a move to one of two new states. The three states from which only two legal moves are possible represent the cases where all three disks are stacked on one peg. The different problems that share this common problem space are defined by the internal problem representation including the move operators that define move legality. The internal representation is engendered by either an external representation of important features of the problem such as the physical pegs and discs of the Tower of Hanoi problem (Figure 2), or a "cover story" such as the one that describes the monsters and their globes in the Monster problems described Table 1.

One of the major findings to emerge from Kotovsky, Hayes, and Simon's work was that problems differed greatly in difficulty. The hardest isomorph, the Monster Change Problem, took about 16 times longer to solve than the easiest isomorph, the Tower of Hanoi problem. The differences in problem difficulty were due to differences in the move operators. The more difficult problems employed move operators that imposed more of a processing load. The processing loads of the different move operators could be ranked in terms of the number of entities (globes, monster loci) that had to be imaged in order to test the legality of a move. This ranking was predictive of both the difficulty of making individual moves and also of overall problem difficulty. Thus when subjects were asked to judge the legality of single moves that were presented tachistoscopically, their response latencies were correlated with the number of entities that had to be imaged in order to make the judgement. For example, in the Move Problem, subjects' judgements were relatively fast when they compared the sizes of two globes held by the same monster, and relatively slow when they compared the sizes of two globes held by different monsters. An even harder comparison occurs in the Change Problem where subjects had to imagine changing the size of a globe.
Figure 2: Tower of Hanoi Problem. The goal is to move the three disks to the rightmost peg.
Table 1: Monster Problem isomorphs (a) The rules for a Change problem and (b) the rules for a Move problem

(a) Monster Change Problem

Three five-handed extra-terrestrial monsters were holding three crystal globes. Because of the quantum-mechanical peculiarities of their neighborhood, both monsters and globes come in exactly three sizes with no others permitted: small, medium, and large. The small monster was holding the medium-sized globe; the medium-sized monster was holding the large globe; and the large monster was holding the small globe. Since this situation offended their keenly developed sense of symmetry, they proceeded to shrink and expand the globes so that each monster would have a globe proportionate to its own size.

Monster etiquette complicated the solution of the problem since it requires that:

1. only one globe may be changed at a time,
2. if two globes have the same size, only the globe held by the larger monster may be changed, and
3. a globe may not be changed to the same size as the globe of a larger monster.

By what sequence of changes could the monsters have solved this problem?

(b) Monster Move Problem

Three five-handed extra-terrestrial monsters were holding three crystal globes. Because of the quantum-mechanical peculiarities of their neighborhood, both monsters and globes come in exactly three sizes with no others permitted: small, medium, and large. The small monster was holding the large globe; the medium-sized monster was holding the small globe; and the large monster was holding the medium-sized globe. Since this situation offended their keenly developed sense of symmetry, they proceeded to transfer globes from one monster to another so that each monster would have a globe proportionate to its own size.

Monster etiquette complicated the solution of the problem since it requires that:

1. only one globe may be transferred at a time,
2. if a monster is holding two globes, only the larger of the two may be transferred, and
3. a globe may not be transferred to a monster who is holding a larger globe.

By what sequence of transfers could the monsters have solved this problem?
and then test the imaged size against the size of another globe that was held by another monster. In that case, the imposed load was higher because of the need to imagine the size change together with the comparison at a distance. Hence there is evidence for a positive correlation between the processing load imposed by the move operators, operator application time, and problem difficulty.

Another major finding was that subjects' move making exhibited a surprisingly regular pattern. Their moves could be dichotomized into an initial, exploratory phase, and a subsequent "final path" phase. The exploratory moves were made slowly, they occupied the major phase of the problem solving time, and they were more difficult (took much longer) in the harder isomorphs. Furthermore, subjects were as far from the goal after making these moves as they were at the beginning of the problem. In contrast, the final path moves were relatively error free, were made very rapidly, were executed at a similar speed across all problem isomorphs, and led almost immediately to a problem solution. This dichotomous pattern of slow or difficult move making that made no net progress, and whose length reflected the relative difficulty of the problems, followed by a rapid dash to a solution in the last minute or so of the solution process, regardless of isomorph, was characteristic of a sizeable majority of the subjects. Thus the exploratory moves seemed to bring the subjects to the point where they could move quickly and efficaciously towards a solution to the problem; that is move along the "final path".

This interpretation of the exploratory and final path phases provided a plausible link between move operator difficulty and problem difficulty. The issue was that although the processing load imposed by the move operators predicted the ordering of isomorph difficulty, the differences in move time were not great enough to account for the very large differences in problem solution time. For example, a relatively difficult problem that subjects solved in an average of 15 moves might have a time-per-move that was 3 seconds longer than the time-per-move of an easier problem. However, instead of taking 45 seconds longer (the product of the number of moves and the time differential), the harder problem actually took 10 or 15 minutes longer. To account for this discrepancy, the hypothesis was developed that move operator difficulty could prevent the planning of move sequences such as goal-subgoal pairs of moves, and that such planning is necessary for people to start the final path phase and solve the problem. To show that people do not solve the problem by randomly making moves, a random walk simulation was constructed and it used many more moves than the subjects in solving the problem. This was true even when the model was
parametrized with the same bias against backtracking evidenced by human subjects. Furthermore, an information processing analysis of the load imposed in making goal-subgoal pairs of moves showed that the harder isomorphs, the Monster Change problems imposed much higher memory loads than the Monster Move problems. The results of this analysis are presented in Table 2, in terms of the number of entities (globs, monster loci) that had to be simultaneously held in mind or imaged in order to make moves, or plan pairs of moves in the monster isomorphs. It can be seen that a move in the Monster Change problem always requires one more entity to be imaged than an equivalent move in the Monster Move problem. The direction of this difference is what would be expected from the pattern of move operator difficulties found for individual moves in the various isomorphs. Planning pairs of moves simply magnified the effects.

Insert Table 2 About Here

To test the hypothesis that subjects were planning their moves during the final path phase, the move latencies of the final path moves were analyzed for evidence of subgoal-goal pairs of moves. The analysis indicated that the subjects solved these five move minimum path problems in two rapid sequences of moves. One sequence advanced them to within three moves of the goal, and a second sequence advanced them to the goal. The evidence for these distinct sequences were the patterns of move latencies, which were long-short and long-short-short. This is what we would expect if the subject attained the ability to plan and execute a subgoal-goal move pair, as contrasted with the pattern if they made individual moves, or planned and executed all five final path moves as a compiled whole. The long-short pattern of move latencies is presumably due to a planning-plus-move step, followed by a move step. The planning time for the subgoal-goal pair of moves occurs prior to the first move, and contributes to the "long" of the long-short pair of latencies. According to this analysis, the last move would be a fast one, either because it was part of the second planned chunk of moves, or because it is simply executed rapidly as a single move that

\[2\] An example of a subgoal-goal move pair is that encountered in the Move Problem when trying to move the medium globe to the medium monster when the medium monster is already holding the large globe. The completion of the move requires the subgoal of "clearing" the medium monster by moving the large globe elsewhere, followed by the goal move of moving the medium globe to the medium monster.
Table 2: Processing Load. Each X represents an envisioned or imaged entity that must be kept in mind when planning moves.

<table>
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<th></th>
<th>Test</th>
<th>Image</th>
<th>Image</th>
<th>Image</th>
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<tbody>
<tr>
<td><strong>Move Rules</strong></td>
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<tr>
<td>Rule 2</td>
<td>X</td>
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<tr>
<td>Rule 3</td>
<td>X</td>
<td>X</td>
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<tr>
<td><strong>Move Planning</strong></td>
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<tr>
<td>Rule 2-Subgoal</td>
<td>X</td>
<td>X</td>
<td></td>
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</tr>
<tr>
<td>Rule 3-Subgoal</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
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<tr>
<td><strong>Change Rules</strong></td>
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<tr>
<td>Rule 2</td>
<td>X</td>
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<td>Rule 3</td>
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<tr>
<td>Rule 3-Subgoal</td>
<td>X</td>
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requires no subgoal planning. Approximately two-thirds of the subjects for whom latencies were recorded exhibited the long-short and long-short-short temporal patterns.

The major conclusion drawn from these findings was that subjects were able to rapidly solve the problem only after they had automated move making enough to be able to plan a subgoal-goal pair of moves. Furthermore, differences in isomorph difficulty were due to differences in the demands imposed by the various isomorphs' move operators: differences that required varying amounts of time before the move making could be automated (or "compiled" or "proceduralized"). Another demonstration of this was that subjects who were given training on the move operators prior to the presentation of a problem were able to solve the problems rapidly, and without significant differences between the Monster isomorphs. According to this account of the results, once the processing load had been reduced through the automation of move-making, the Monster problems should have been about as difficult as isomorphs whose move making imposed minimal processing loads. The Tower of Hanoi is such an isomorph because the move operator restrictions are inherent to its external representation; the disks block each others' removal from the pegs, so that it is impossible to remove a larger disk first. Figure 3 shows the solution times for Monster isomorphs under a number of conditions, the final path times for those isomorphs, and the solution time for the Tower of Hanoi problem. The Figure shows that the final path times, that is, the solution times once move making was automated, are about equal to the Tower of Hanoi solution time and are considerably less than the overall solution time for those isomorphs. These results provide further support for the conclusion that the processing load imposed by the move operators is minimal during the final path phase.

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Experimental Background: Transfer of Training and Problem Move Operators

If problem move operators are a major source of problem difficulty differences, the compatibility of the move operators of two problems should predict the amount of transfer that will be obtained between them. This idea was tested in an experiment reported in Kotowsky, Hayes, and Simon (1985) involving acrobats jumping between each others' shoulders on the tops of flagpoles, the acrobats corresponding to the disks of the Tower of Hanoi problem, or the globes of the Monster problems. As expected, the compatibility of
Figure 3: Solution times and final path times from Kolovsky, Hayes, & Simon 1985. The final path times for the Monster problems are close to the Tower problem solution time. Reprinted with permission from Academic Press.

![Graph showing solution times and final path times for different scenarios.](image-url)
the move operators was a major determinant of transfer. In one version of the experiment, the two problems were Monster Move problems in which the large spheres were free to move and blocked the small spheres from moving, and an Acrobat problem in which the move restriction was reversed: small acrobats were free to move and blocked large acrobats from moving. This move operator incompatibility resulted in relatively little transfer. In contrast, when the move operators were similar as in the Reverse Acrobat and Monster Move problems, where in both problems, large block small, there was positive transfer.

There were also interesting asymmetries between the problems in the amount of transfer that was obtained within pairs of problems. The amount of transfer from problem A to problem B was often different than the amount of transfer from B to A. Problem difficulty differences seemed to account for this asymmetry. If the source and target problems had different move operators, such as in the Monster Move and Acrobat problem pair, then any information acquired from the source problem required transformation of the move operator information in order to be useful in solving the target problem. Such transformations should be more resource competing on more difficult target problems, and result in less transfer. This is consistent with the direction of the results obtained with the Acrobat problems. In the Monster Move--Acrobat problem pair, although the individual transfer effects did not reach significance, the direction of transfer from the hard to the easy problem tended to be positive (+29.5%) whereas the transfer tendency from the easy to the hard problem was in the opposite direction (-28.9%). On the other hand, in the Monster Move--Reverse Acrobat problem pair, where the move operators were compatible, this was not found. There was positive transfer in both directions. The amount of transfer from the easy to the hard problem was +57.4% and the amount of transfer from the hard to the easy was +38.5%, with the former reaching significance as an individual effect. A similar tendency toward the occurrence of hard-easy asymmetry with more transfer to the easy target problem when move operators are not compatible has been found in a number of experiments. (See for example the transfer results reported for the Move--Change pair of problems in Hayes & Simon, 1977.)

In a similar vein, in an experiment described in Kotovsky, Hayes, and Simon, (1985), Hayes measured the amount of transfer between three isomorphs of the Tower of Hanoi that embodied different amounts of move operator information. These problems consisted of a standard Tower of Hanoi problem with the substitution of styrofoam balls of various diameters for the more usual disks. In one isomorph, called the "Peg Move" problem, the
balls had holes drilled in them so that they could be stacked on pegs that were inserted into holders. The move rules were the standard ones: (a) "only one ball may be moved at a time," (b) "if two balls are on a post, only the smaller is free to move," and (c) "a large ball may not be placed on top of a smaller one." The ability to stack the balls made the problem a standard Tower of Hanoi problem, and consequently easy to solve because the second rule is built in to the external representation. Because the balls are stacked, only the top one is free to move, and the subject does not have to think about the second rule in making moves. A second isomorph, called the "Dish Move" problem, consisted of the same external representation, but without the pegs. The subjects in this version faced a more difficult task because they had to remember the second rule (as well as the others) because the balls could not be stacked. The third isomorph was called the "Dish Change" problem. Its external representation was similar to the Dish Move problem, with the modification that each dish had a reserve dish placed behind it that contained two balls in addition to the ball in the main dish. The balls in the reserve dish were of the two sizes not included in the main dish. These additional balls could be "traded" for the ball in the main dish if the subject wished to change its size. For example, if the main dish had a medium ball, and the subject wanted to make it large, he or she did so by trading the medium ball from the main dish with the large ball from the reserve dish. The Dish Change problem shared an important feature of the Move Problems in that physically existant balls were moved back and forth between the main and reserve dishes. It was also similar to other Change Problems in that objects had to be compared at a distance. Thus it was possible to predict the ranking of difficulty of the problems from easiest to hardest: Peg Move, Dish Move, and Dish Change, based on the amount of informational load each problem imposed during move making.

The ranking of problem difficulty obtained from the experiment was as predicted. The median solution times of the Peg Move, the Dish Move and the Dish Change problems were 160 seconds, 241 seconds, and 342 seconds respectively. The increasing solution times across the three problems reflects their increasing difficulty and the increasing information load of their move operators. The transfer results from this experiment, which have not been reported elsewhere, are presented in Table 3 in terms of percentage reduction in solution time (transfer scores). The transfer scores indicate a great deal of transfer between problems that have similar move operators, i.e. the Move problems. The only other case yielding a sizeable amount of transfer involved two problems that appear to be similar but have different move operators, namely the Dish Move and Dish Change problems, where the
transfer is negative. The Dish Change problem took 59% more time when it followed a Dish Move problem than it did when it preceded the Dish Move problem. As in the Acrobat problems previously discussed, the more difficult target problem was not a good recipient of transferred skill if that skill had to be modified to be used. The Peg Move--Dish Change condition did not yield a similar result probably because of the large differences in the external representations (appearances) as well as the rule sets of those problems. It is likely that these differences were large enough so that attempts to transfer skill were not made.

An additional finding was that the Dish Change problem, in addition to being a poor recipient of positive transfer, was a poor source of transfer as well. Although part of this effect is no doubt due to the aforementioned difference in the move operators between it and the other problems, it did not even yield much self-transfer (reduction in solution time when the identical problem is administered twice)! This is quite opposite to the effect found with the other two problems which both yielded large amounts of self-transfer. One possible interpretation of this finding is that less is learned from a single exposure to solving more difficult source problems, (which are presumably more resource demanding), and therefore less is available for transfer, even to identical problems. The results of this experiment together with those of the Acrobat and Reverse Acrobat problems already discussed, support the conclusion that both move operator compatibility and problem difficulty are important and interacting determinants of transfer.

In summary, the results obtained by Kotovsky, Hayes, and Simon argue for two major conclusions. The first is that problem solving, at least across the range of problems tested, is a two phase process: (a) an initial phase that includes “problem exploration” during which people become expert enough at making moves to be able to plan, and (a) a subsequent “final path” phase during which people rapidly achieve a solution because they can plan move sequences that are within their processing limitations. The second conclusion is that move operator compatibility is a major determinant of transfer, with problem difficulty also having an effect on the ability to transform and apply a learned skill to a new problem. There was also an indication that problem similarity played a role in eliciting transfer, as in the Dish-Move to Dish-Change (but not the Peg-Move to Dish-Change) case. Given the demonstrated impact of problem representational issues on problem difficulty and transfer, we conducted an experiment to more precisely identify the effects of problem representation.
Table 3: Transfer scores between three isomorphs of the Tower of Hanoi that vary in difficulty and similarity

<table>
<thead>
<tr>
<th>Source Problem</th>
<th>Peg Move</th>
<th>Dish Move</th>
<th>Dish Change</th>
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<tbody>
<tr>
<td>Peg Move</td>
<td>78</td>
<td>55</td>
<td>7</td>
</tr>
<tr>
<td>Dish Move</td>
<td>34</td>
<td>78</td>
<td>-59</td>
</tr>
<tr>
<td>Dish Change</td>
<td>-12</td>
<td>-6</td>
<td>19</td>
</tr>
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</table>
on transfer and difficulty, and to localize the effect of transfer within the problem solving process. Having identified planning move pairs as the skill acquired in the problem exploration phase of problem solving in the aforementioned work, we proceeded to test the hypothesis that transfer might perform a similar function, and thus substitute for problem exploration in problems having similar representations.

Experiment One: Representation and Transfer

In this experiment, we attempted to measure the effect of representational overlap on transfer and identify the locus of transfer within the problem solving process. A set of problems that differed in one or more features of their problem spaces was used to determine the effect these differences would have on transfer. We predicted that the greater the representational overlap, the greater the amount of transfer obtained. This prediction is thus consistent with the "identical elements" conception of transfer of Thorndike and Woodworth (1901), and the shared production conception of Kieras and Bovair (1986). The basic procedure was to present pairs of problems, and measure the difficulty of the second or target problem, as a function of which first or source problem it followed.

Subjects

The subjects were 81 students at the Community College of Allegheny College who were given class credit for their participation.

The Problems

The problems used in this experiment were Monster isomorphs of the three disk Tower of Hanoi problem and were similar to the problems depicted in Table 1. The subjects' goal was to make moves in the problem space until each monster ended up with a globe whose size corresponded to his own. As is shown in that Table, the Monster Change problem moves involved the monsters changing the sizes of their globes in accordance with Change problem rules, and the Monster Move problem moves involved the monsters passing their globes back and forth according to that problem's rules.

Within each problem type, the problems could differ in the starting position in the problem space. The two starting places used in the current experiment are depicted as Start1 and Start2 in the problem space presented in Figure 1. Both starting positions are five moves from the goal. The problem space and the starting positions for the Change Problem were
chosen to be isomorphic to those for the corresponding Move problem. Two problems could also differ in the direction of the move operators (problem rules). There was a normal rule problem (depicted in Table 1) and a reverse rule problem. In a reverse rule problem, the smaller globes blocked the passing or changing of larger globes, in contrast to the standard problem in which larger globes blocked smaller globes. The rules for these reverse rule problems, while differing from the usual Monster Problem rules used by Hayes and Simon (1977), are more similar to the Tower of Hanoi problem in which smaller disks block larger disks from moving. To summarize, problems could be similar or different in representation (Move or Change), problem rules (normal or reverse), and solution path (Start1 or Start2).

Procedure

The problems were presented to the subjects on a MicroVAX computer, which displayed the stimuli, and recorded the responses and response latencies. The source and target problems were presented sequentially, with a short rest between. The subjects were run individually. Each subject was seated at the computer, given a brief introduction to the use of the mouse, and then given the first problem. Each problem was presented on the screen in three parts:

1. An introduction to the use of the mouse and instructions for "thinking out loud" while solving the problem.

2. Practice using the mouse with one "monster" on the screen. The procedure for making moves that S's practiced with one monster was identical to the procedure for making moves in the actual problem with three monsters.

3. A statement of the problem and the goal. These were very similar to the Change and Move problem instructions presented in Table 1, followed by the presentation of the problem itself (Figure 4). Response latencies were measured from the time that the problem was presented. Figure 4 presents the problem display.

Insert Figure 4 About Here

In the Move problem, a subject made moves by positioning the mouse pointer on the letter designating the size of the globe he wanted to move, in the box associated with the "destination" monster. On clicking the mouse, the desired globe moved to that destination box. In the Change problem, the subject made moves by positioning the pointer over the letter designating the size he wanted to change the globe into --the "destination" size-- in
Figure 4: Change Problem display
the box of the monster whose globe he wanted to change. On clicking the mouse, the globe in that box changed to the desired size. The computer checked move legality. When the subject violated a rule, the computer displayed a warning message citing the rule number that had been violated, and asked the subject if he wanted to review the rules. The only move allowed after an illegal move was the retraction of the illegal move. The only display difference between a Change problem and a Move problem was that in the Change problem each monster's box held only one globe which changed in size, while in the Move problem, each monster's box could hold from zero to three globes which could be moved from box to box.

**Representational overlap between problem isomorphs**

Of the total of six different problem isomorphs used in this experiment, three were isomorphs of the Monster Move problem, and three were isomorphs of the Monster Change problem. Within each major problem type (Move or Change), the source problems could differ from the target problem in a number of ways. These differences for the Change problem "Change 2 Regular" in target position are summarized in Table 4a, which shows the problems ranked in degree of overlap with the target problem. Change 2 Regular designates a Change problem with starting position 2, and regular (large blocks small) rules. The most overlap in problem representation occurred when the source and target problems were identical (Change 2 Regular followed by Change 2 Regular). Two problems that differed only in starting position were the next most related problems (Change 1 Regular followed by Change 2 Regular), problems that had similar rules that differed in their direction (normal rules vs reverse rules) were next (Change 2 Reverse followed by Change 2 Regular), and problems that differed across the Move-Change category (Move 2 Regular followed by Change 2 Regular), differed the most. The main prediction was that the greater the representational similarity or overlap, the more transfer would be obtained. The problems all shared the same move sequence except for the Change 1 and Move 1 problems which were not true isomorphs because of the different starting positions. Table 4a depicts the extent of the representational overlap of the four source problems with the target problem. Table 4b presents the same analysis for the Move target problem.

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Insert Table 4 About Here

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Table 4: Problem representational overlap (a) Change Problem  (b) Move Problem

(a) Target Problem is Change Regular

<table>
<thead>
<tr>
<th>Source Problem</th>
<th>General Represent</th>
<th>Rule</th>
<th>Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change 2 Reg.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Change 1 Reg.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Change 2 Rev.</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Move 2 Reg.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

(b) Target Problem is Move Regular

<table>
<thead>
<tr>
<th>Source Problem</th>
<th>General Represent</th>
<th>Rule</th>
<th>Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Move 2 Reg.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Move 1 Reg.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Move 2 Rev.</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Change 2 Reg.</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
The features that determine representational similarity have been discussed under methodology except for the first column labelled "general." This is a residual category that includes all of the similarities that exist between the problems that are not otherwise designated, such as the use of the mouse, the computer presentation, and the general style of problem (puzzle-like logic problems). This category represents an attempt to account for the similarities that exist even between the two most dissimilar problems.

Results: Representational Overlap

We predicted that increasing the representational overlap between source and target problem should lead to increasing amounts of transfer between the problems. The amount of transfer was determined by comparing the difficulty of the target problem with the difficulty of the same target problem when it was administered in the initial position (as a source problem for some other target problem). In the following Figures, we plot measures of target problem difficulty as a function of its representational overlap with the source problem it followed, and for purposes of comparison, indicate the same measure for the target problem when it was solved in initial position.3

The transfer results from the Change and Move problems are presented in Figure 5 which shows problem solving speed (the reciprocal of solution time) for the target problems, as a function of the amount of representational overlap. The abscissa depicts the amount of overlap as defined in Table 4, in the order of least to most. Figure 5 shows that the target problems were solved more quickly as the amount of overlap increased.

Insert Figure 5 About Here

The overall relation between representational overlap and problem solving speed is significant for the problems as a whole, \(F(1,79)=5.08, p < .05\), and marginally significant for the Change problems \(F(1,39)=2.97, p < .10\), and Move problems \(F(1,38)=2.78, p = .13\).

3In the following Figure, the Change and Move source problems have similar solution speeds. This is contrary to the usual finding that Change problems are more difficult (as indeed these were in the target position). If solution times rather than reciprocals are plotted, the Change problem-Move problem difference is more apparent, the times yielding a ratio of about 3:2. In contrast to speed measures, solution time measures are more influenced by "slow" subjects, i.e., those who find the problems difficult.
Figure 5: The effect of representational overlap on problem solving speed.
taken as separate groups. Other measures of problem difficulty, (moves, errors, solution times, average move latencies) tend to show similar effects of representational overlap on target problem difficulty, although this is more the case for Change problems than the Move problems that were affected by some fairly extreme solution times (and numbers of moves) in one condition. By way of illustration, the time per move versus representational overlap data are presented in Figure 6 for the Change and Move problems. The relation between these measures and representational overlap was significant for the problems taken as a group, \( F(1.79) = 6.8, p = .01 \), and marginal for Change problems \( F(1.39) = 3.9, p < .06 \), and Move problems, \( F(1.38) = 3.3, p < .08 \), taken as separate groups. Our overall conclusion is that increasing representational overlap is related to decreased target problem difficulty; that is, representational similarity increases positive transfer.

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Insert Figure 6 About Here

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These results are in agreement with the previously discussed results of the Acrobat/Reverse Acrobat experiment which also showed significant transfer in the condition having high move operator compatibility. In addition, the results agree with those of the Dish Experiment where again, there was a large amount of transfer between problems having similar move operators, and little transfer between problems having different move operators.

The general conclusion that emerges from these experiments is that problem representation, in addition to exercising control over problem difficulty, is an important determinant of transfer. The results must be tempered by the realization that many factors affect target problem difficulty, and it therefore cannot be predicted as accurately as we would like solely on the basis of a subject's problem solving history, that is, on the basis of transfer. Nonetheless, these results suggest that a stronger prediction of target problem difficulty will become possible as we achieve the ability to further isolate the particular

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4A number of the measures in this experiment, particularly those that are responsive to extreme individual scores, exhibit a great deal of subject to subject variability both within as well as across conditions, resulting in some marginally significant results. This contrasts with the findings presented in the next section of the discussion of results where we report on a phase of the problem solving process where the variability tends to disappear.
Figure 6: The effect of representational overlap on move latencies.
problem features that provide the source of transfer, more accurately localize the effects of transfer within the problem solving process, and thus more precisely specify the degree of representational overlap between two problems. We turn now to those issues.

Results: Two Phase Solution Process

Although the transfer results presented above clearly demonstrate the effects of representational overlap, the locus of such effects has not yet been determined. In an attempt to address this issue, we analyzed our transfer data in accordance with our previous characterization of the solution process as consisting of two phases: a non-progress making exploratory phase, and a final path phase in which the subject rapidly advances to the goal.

The final path was defined as the set of moves made by each subject from their last occupation of a position five moves from the goal, to the attainment of a solution. A distance of five moves was chosen because it is the distance between the initial starting position and the goal, and thus consists of the subject traversing the entire start-to-goal distance. The move data was analyzed to determine the last occupancy of such a position, and the remaining moves and time were extracted from the move records. Kotovsky, Hayes, and Simon (1985) found that only a relatively small proportion of the problem solving time is spent on the final path, and its beginning is determined by the subject’s acquiring the ability to compile moves which allows them to plan ahead and thus quickly solve the problem. The current study constitutes the first test of that finding in a new problem situation. Furthermore, the entire set of move records is used, as opposed to using only those chosen by the additional criterion of temporal move patterns that evidence the attainment of move compilation ability. In this study, we simply take the end behavior of all the subjects to see if it exhibits the “mad dash to a solution” effect that was obtained in the earlier study.

The final path finding was replicated in this study. The subjects traversed the final path in a relatively short time and in relatively few moves. The median time to achieve a solution in the Change Problem from the last occupancy of a position five moves from the goal was 83 seconds, with a median of 7 moves required to traverse the distance. This contrasts with the total solution results (the amount of time and number of moves to traverse the distance to the goal from the beginning of the problem) of 434 seconds, and 21 moves. For the Move Problem, the results are similar: a final path median of 65 seconds and 7 moves, as compared to the total solution median of 350 seconds and 19 moves. (The mean number of moves for the final path and total solution respectively are 8.1 and 30.6 for the Change
problem, and 8.1 and 24.7 for the Move problem. The mean times for final path and total solution are 104 and 630 seconds for the Change problem and 108 and 441 seconds for the Move problem.) Thus the subjects do exhibit the predicted pattern of a large amount of exploratory behavior with no net progress toward the goal, followed by the very rapid attainment of a solution. The average time per move also reveals this dichotomous behavior: the average time per move being almost twice as fast during the final path phase of the solution process than it is during the exploratory phase.

The conclusion that the final path was achieved when subjects became able to plan and execute two move sequences was tested by searching for the temporal pattern of moves that suggested two goal-subgoal pairs of moves, followed by a quick final move. Because the planning of a goal-subgoal pair occurs prior to the first of the pair of moves, the expected pattern of move times is long-short, long-short. In addition, we expected a quick final move because minimal planning is required to move to the goal from a penultimate state. Table 5 presents the expected number of such temporal move patterns together with the number actually obtained from the final path records. In addition, the Table shows the same measures obtained from the most immediately prior subgoal situation in the exploratory phase of the move records. For the exploratory moves, single subgoal situations were identified and the Long-Short temporal pattern was searched for. For the final path, only the occurrence of the entire Long-Short-Long-Short-Short outcome was accepted as evidence of the ability to form and use subgoals, because on the final path we could be relatively certain what the subject was attempting. Even with this more stringent test, the evidence for subgoaling was extremely strong on the final path, and almost exactly at chance for the exploratory phase. Thus the conclusion is supported that the ability to plan and execute subgoal-goal pairs of moves is what differentiated the exploratory and final path phases.

Insert Table 5 About Here

Results: Locus of the Transfer Effect

Having established the existence of these two different phases of the problem solving process, we can proceed to examine which phase is likely to be facilitated by the prior solution of a similar problem. We predicted that the exploratory phase of the problem
Table 5: Comparison of subgoal planning in the final path and exploratory phases of problem solving. For each phase, the number of obtained move latency patterns consistent with subgoal planning is compared with the number expected by chance.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Final Path</th>
<th>Exploratory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Expected</td>
<td>2.4</td>
<td>6</td>
</tr>
<tr>
<td>Number Obtained</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>n</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>

p < 0.0001  p < 0.75
solving process would be shortened by the skill learned in solving the source problem. That is, the skill acquired on the earlier problem should reduce the exploratory phase of the process by helping the subject acquire the ability to plan and make moves. If the analysis of the solution process described in Kotovsky, Hayes, and Simon (1985) is correct, the locus of the transfer effect should be in the exploratory phase of the problem, because that is where the learning that is crucial for moving onto the final path occurs.

The final path performance measures for the target problems exhibit little variation regardless of the source problems. In contrast, our previous analyses of entire solutions exhibited significant variation in performance depending on the overlap of the source problems. Figure 7 shows that the target problems (and the source problems), have similar final path times. Although the total solution times might vary by 10 minutes or so, the final path times vary by less than 1 minute (much less, in the case of the Change problems). A similar result exists for the number of moves in the final path which exhibit relatively little variation from one transfer condition to another, and are similar for source and target problems. These results, showing the relative equality of final path moves across conditions, are depicted in Figure 8 for the Change and Move problems.

In contrast, the overall number of moves to a solution demonstrated the existence of a great deal of variability across problem conditions, with the means of the various overlap conditions ranging from 27.1 to 37.8 for Change problems, and 18.2 to 37.4 for Move problems. Of the measures used in this experiment, the one that is most descriptive of move operator application difficulty is time per move. In Figure 6 we showed that the time per move of the target problems varied with representational overlap. It is instructive to consider this move latency measure for the exploratory and final path phases separately. For exploratory moves, there were significant differences (one way anova) for all target problems. $F(1,79)=4.47, p < .05$, as well as for the Change problems separately. $F(1,39)=4.77, p < .05$, but not for the Move problems separately. $F(1,38)=.59, p < .45$. For the final path phase on the other hand, the time per move measure was not related to overlap. The corresponding Figures for all target problems, Change problems, and Move problems were $F(1,79)=1.13, p < .30$, $F(1,39)=0.026, p < .89$, and $F(1,38)=1.334$. 

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Insert Figures 7 & 8 About Here

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Figure 7: The effect of representational overlap on duration of final path

Representational Overlap

Median Final Path Time (minutes)

CHANGE TARGET
MOVE TARGET
CHANGE SOURCE
MOVE SOURCE
Figure 8: The effect of representational overlap on number of moves in final path phase.
p < 26, respectively; none approaching significance.

If the time per move for the first (source) problem is included, the result is even more striking, with large (and significant) differences in overall time per move and exploratory time per move, and no significant differences in final path time per move. For all problems (source and target) combined, the average time per move, exploratory time per move, and final path time per move yield F ratios of 4.06, p < 0.0001, 3.1, p < 0.0025, and 1.19, p = 0.3, respectively. The corresponding values for Change problems alone are, average time per move, F(4,56) = 6.61, p < 0.0025, exploratory time per move, F(4,56) = 4.72, p < 0.0025, and final path time per move F(4,56) = 1.29, p < .29. For Move problems, the average time per move result is F(4,55) = 3.84, p < .01, exploratory time per move, F(4,55) = 3.58, p < .025, and final path time per move, F(4,55) = 1.6, p < .2. Thus the considerable variation that exists across problems is present in early parts of the problem solving process, but it is absent in the final path phase. The exploratory phase move latencies are depicted in figure 9 and the final path move latencies in Figure 10. The average time per move results were presented earlier in Figure 6.

We conclude that the transfer that occurs between two problems acts by reducing the exploratory (move learning) phase of the problem solving process, while leaving the final path phase of the process essentially unchanged. Stated in more general terms, these problems are solved in the last minute or minute and a half whether or not a similar problem was solved immediately beforehand. Almost all of the variability in solution times occurs prior to this time. For example, the ratio of standard deviation to the mean for the final path times and moves is generally between 1/4 and 1/2 of the ratio for the exploratory times and moves within each condition. Thus the within-condition as well as between-condition variation in times is greatly reduced in the final path behavior. Positive transfer enables subjects to reach the final path phase sooner than when there is no transfer. However, transfer does not affect the final path phase itself because at that point subjects have already achieved the ability to plan two moves, (through either the transferred skill or through practice in the exploratory phase), and thus rapidly achieve a solution. The analysis of move operator application time supports the conclusion that it is move operator application that is learned.
Figure 9: The effect of representational overlap on exploratory phase move latencies
Figure 10: The effect of representational overlap on Final path phase move latencies.
during the exploratory phase of problem solving and that move operator application is also the target of transfer. The ability to easily make moves is, in this view, the key to being able to plan and execute goal-subgoal pairs of moves.

This first series of experiments has generated a number of conclusions about representation and transfer. These include:

1. Increased overlap in the representations of two problems increases the amount of skill that will transfer from the source problem to the target problem.

2. Replicating an earlier finding, the problem solving process can be divided into two phases. These are (a) an exploratory phase during which the subject learns to make moves and compile two-move sequences, and (b) a final path phase during which the subject rapidly and efficaciously closes on the goal by planning and executing two-move sequences.

3. The target of transfer is learning to make moves. This learning can substitute for some of the learning that would normally occur during the exploratory phase of the solution process. Once subjects are on the final path, there is very little variation in move application times, whether between target problems with quite different amounts of transfer from widely different source problems, or between the same problem in source and target position.

4. There is evidence of an interaction between target problem difficulty and transfer, such that increased target problem difficulty leads to less transfer from more distant problem isomorphs. Thus, information from a source problem that has to be transformed at the time of its application in a target problem can yield positive transfer, but is less likely to do so on harder problems that are more resource demanding.

**Experiment Two: The Effects of Stimulus and Representational Similarity on Transfer**

Experiment 1 demonstrated the influence of representation on transfer, and showed that the locus of transfer was in the exploratory phase rather than the final path phase of the search for a solution. The representational features that were included in that experiment exemplify the diversity of representational features that can affect transfer. The next series of experiments was conducted to attempt to more precisely predict the transfer effects of representation, by separating the effects of internal representation from those of the external task environment, or problem "adequate stimulus." The strategy here was to control subjects' internal representation of the problem (i.e. their problem space), independently of the external problem presentation, in order to determine the effects of the internal representation on transfer.
One root of this line of investigation can be traced back to an interesting idea advanced by Charles Osgood (1949) who posited a relationship between stimulus similarity, response compatibility, and the amount of transfer that would be obtained in verbal learning situations. He formulated the "Osgood Transfer Surface" to describe the relationship. This transfer surface summarized a large portion of the data on transfer. Osgood posited that response compatibility would produce positive transfer, and response incompatibility would produce negative transfer. More importantly for the issues we are considering, he advanced the idea that high stimulus compatibility would produce large amounts of transfer (whose sign depends on the response compatibility/incompatibility), and low stimulus resemblance would produce small amounts of transfer. The work of Gick and Holyoak (1983, 1987) and others suggests that although the "large" might have to be qualified a bit, Osgood's analysis applies as well to problem solving.

Osgood's emphasis on stimulus similarity and response compatibility managed to both summarize a significant portion of the data on transfer, and fit into a more behaviorist zeitgeist. However, the current cognitive science view of the centrality of representation in cognition led us to consider an extension of his elegant analysis. Newell and Simon (1972) clearly differentiated the problem space or internal representation of the problem from the external task environment. This differentiation is essential to their analysis of problem solving, and impels us to focus our attention on the internal representational aspects of problem solving to achieve an understanding of the problem solving process. The question that we attempt to answer here is whether the same thing holds true for understanding transfer; that is, whether it is internal representational rather than external stimulus properties that are the important determinants of transfer.

The results of Experiment 1 argued for the importance of representation for transfer, but did not clearly differentiate between the relevance of stimulus overlap and internal representational overlap. The dimensions of representation that were manipulated, such as rule reversal and Move-Change problem representations, would be hard to map onto a stimulus similarity dimension in any reasonable way, although the representational differences are not totally independent of differences in the stimulus properties for some of the isomorphs. Thus, although normal rule and reverse rule problems have identical stimulus properties, Move and Change problems are not identical on the display screen of the MicroVAX even though they were constructed to minimize stimulus property differences. The major difference between the stimuli of a Move problem and a Change problem is that a
monster in a Move problem can hold more than one globe, whereas a monster in the Change problem always holds only a single globe. This difference is one source of the move operator difficulty differences previously discussed, and thus is a potentially important difference between the two problems. In order to separate the effects of representation from those of stimulus similarity, the current series of experiments controlled for stimulus properties so that any differences in performance could be attributed to differences in representation.

Experiment 2a

The Problems

The basic experimental procedure used in this series of experiments was to sequentially present the subject with two problems to solve, both of which were isomorphs of the three-disk Tower of Hanoi Problem. Four problem isomorphs were used: two Size problems and two Depth problems. The Size Problems were very similar to the Change problems used in Experiment 1, with the exception of being presented in color. A black and white rendition of the display of the Size problem is given in Figure 11a. The displays consisted of three boxes (or so called "tunnels" in the Depth problem), each of which contained a sphere. The size of the sphere could be changed by positioning a cursor within the box, or tunnel, by means of a joystick, and pressing one of three keys on a keyboard that were labelled "small", "medium", or "large", in the Size problem, or "far", "middle", or "near" in the Depth Problem. The display of the Depth problem was identical to the display of the Size problem in some versions of the experiment. In other versions, an additional detail, perspective "windows", was added to the Depth problem display in order to facilitate the illusion that the spheres moved in depth. A rendition of a Size problem display modified in this way is given in Figure 11b. The inclusion or non-inclusion of the windows is noted in the description of the individual experiments.

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Insert Figure 11 About Here

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The move operator rules for making moves or changes in the Depth and Size isomorphs were made as similar as possible. They were always stated in terms of the darkness or lightness of the tunnel or box surrounding the sphere, instead of being stated in terms of
Figure 11: Experiment 2 displays shown with and without the windows that facilitated the depth illusion.
sphere size and monster size as they were in Experiment 1. The text of the rules for a Depth problem are given in Table 6a & b. The corresponding rule set for a size problem can be generated by substituting "change the size" for "change the position" and substituting "large", "medium", and "small" for "near", "middle" and "far" in that Table.

To determine the effect of representational similarity on transfer from one problem to the other, subjects were presented with two problems that had identical (or very similar) stimulus properties, with instructions that induced either the same or different internal representations. Two Size problems and two Depth problems were used the only difference between the two problems within each type was the starting position in the search space. The two types of problems were isomorphic, with Depth problem 1 (D1) having the same solution path as Size problem 1 (S1), and D2 the same path as S2. The difference between the Size and Depth problems was in the representation engendered in the subjects by the written problem statement. In contrast, the external representation (screen display) was either identical or very similar for all problems.

Procedure

Subjects saw the problem displays on an AED graphics terminal driven by a MicroVAX computer which also recorded their responses and response latencies. The subjects were seated at the terminal and the experimenter demonstrated the use of the joystick to control the cursor and the use of keyboard buttons to make moves. They were instructed to think aloud while solving the problems, and their verbal protocol was recorded. A general description of the problem was then given to the subject on a sheet of paper. When subjects were ready to begin, they turned over a second sheet that contained the specific problem statement. The description and problem statement given to subjects solving a Depth problem are given in Table 6a and b. Subjects' response latencies were measured from the time that they turned over the second sheet.

The Subjects

The subjects were 64 students at Carnegie-Mellon University and the Community College
(a)

This problem involves changing the positions of 3 identical spheres. Each sphere is in a separate 'tunnel' and the tunnels vary in how dark they are. You can move each sphere between three depths: Near, Middle, and Far.

To move a sphere, first position the black crosshair inside the sphere's tunnel using the silver joystick. Then hit one of the keys labelled Near, Middle, and Far to move the sphere to the desired depth.

(b)

Your task is to move the 3 spheres so that the sphere in the darkest tunnel is Near, the sphere in the next lightest tunnel is Middle, and the sphere in the lightest tunnel is Far. The 2 rules for moving the spheres are:

1) you may not change the position of a sphere if it is at the same depth as another sphere in a darker tunnel.

2) you may not change the position of a sphere if it will be at the same depth as another sphere in a darker tunnel.

If you break either rule, the computer will move the sphere but it will also ring a warning bell. If this happens, the only move you can make is the one that returns the sphere to its previous position.
of Allegheny County who were given course credit for their participation.

**Results**

We predicted there would be more transfer between problems whose representations were the same than between problems whose representations were different. In the first of the experiments, the displays of the Size and Depth problems were identical. The stimuli on the screen consisted of three squares with diagonals drawn in to facilitate the depth effect. There was a large transfer effect; the target problem was much more rapidly solved than the source problem, $F(1,59)=19.84$, $p < .001$. However, the effect of manipulating the representations of the source and target problems was not significant, although it was in the direction of our prediction that there should be more transfer between problems whose representations were the same: two Size problems or two Depth problems, than between problems whose representations were different: a Size problem followed by a Depth problem or Depth followed by Size $F(1,59)= 1.34$ $p < .26$.

In order for our hypothesis to be testable, it is necessary for subjects to form the intended internal representations of the problems. To check that subjects had formed the correct representations of the problems, we evaluated the verbal protocol statements that subjects made in announcing their moves. This analysis revealed that many subjects solving Depth problems did not perceive the spheres as moving in depth. The protocols showed that in these cases, the subjects often referred to depth moves as size changes. Thus, instead of saying “Now this one gets moved to near”, or “Now change this to far”, they would say “Now this one gets made large”, and “Now change this to small”. A subset (36 of the 64 subjects) gave representationally correct references in announcing their moves, and the results from their data was closer to the predicted effect. However, this subset included very few subjects who solved a Size source problem and a Depth target problem. It seemed that once subjects had solved a source problem with the size interpretation, they had great difficulty forming a depth interpretation of the target problem. The analysis of the protocols indicates that subjects had difficulty forming the intended internal representation of the Depth problem which may explain why we did not obtain the predicted effect.

In the next experiment we modified the display associated with the Depth problem to make it more perceptually viable, in the belief that this would increase its “availability” to the subject for further processing, or transfer. One useful result of this experiment is that it
raised an issue we explore later, that is, the availability of a representation. The basic hypothesis, based on these preliminary findings, is that if it is difficult to achieve or maintain a representation of a problem, then that problem will not be useful as a source of transfer to subsequent similar problems.

Experiment 2b

The second experiment used redesigned stimuli that were intended to be more easily perceivable as a depth display. To facilitate the achievement of a depth representation of the Depth problem, we introduced windows into the display associated with the Depth problem. These consisted of trapezoidally shaped windows in the walls of the tunnels of the display that helped produce the depth illusion and thus made it easier for the subjects to represent what they were doing as changing the position of spheres in the tunnels as opposed to changing their sizes. The experimental strategy was similar to that of Experiment 2a. To determine the effect of representational differences on transfer, problems were presented that had very similar stimulus properties, with rules that engendered either similar or different representations. We viewed the presence of the windows in the Depth condition, and their absence in the Size condition, as a minimal stimulus difference that would in itself not introduce a significant difference between the problems. The reason for this belief is that the windows were not a central feature of the displays, nor were they referred to in the statement of the problem or used as part of the problem move operators, (this issue is investigated further in Experiment 2c).

Subjects

The subjects in this and the following two experiments were students at the Community College of Allegheny County who were given class credit for their participation. Forty four subjects were used in this experiment, 11 subjects in each of the 4 conditions: Size-Size, Size-Depth, Depth-Depth, Depth-Size.

Results

There was, as in Experiment 2a, massive transfer from the source to the target problem. The median solution times of the target problems are plotted for each condition in Figure 12. The connected points in the Figure show that target problems that were preceded by a source problem with a similar representation were solved more quickly than target problems that were preceded by a source problem with a different representation. The difference
between the amounts of transfer obtained from the same and different conditions was significant, \( p < 0.05 \) (Mann-Whitney U test). The two unconnected points are results from two control experiments discussed later.

In addition to the predicted effect, the transfer results also exhibited a small (and non-significant) interaction that we tentatively attribute to different availabilities of the Size and Depth problems. The solution times in Figure 12 suggest a greater difference in transfer between the Size-Size and the Size-Depth conditions than between the Depth-Depth and Depth-Size conditions. A possible explanation for this is that the Size representation is the more available of the two, and thus has the greater potential for transfer to similar problems. Depending on the measure, the interaction is at best marginally significant, but it does suggest further examination of the phenomenon, which we undertake in Experiment 2e.

We conclude that representation is a determinant of transfer, and that more transfer is obtained between problems sharing the same representation than between problems having different representations. One implication of this finding is that the Osgood transfer surface can be reinterpreted to refer to problem representation rather than stimulus similarity. That is, the similarity of the representations for two problems is the predictor of amount of transfer, rather than the similarity of the stimuli. This is not to argue that the similarity between the representations of two problems is totally independent of the similarity of their stimuli; we are not proposing a hallucinogenic model of problem representation. Rather, we argue that the representation the subject creates is the relevant variable, and it may be the product of several factors including the subjects' expectations (often induced by the problem instructions), and their problem solving history, as well as the stimulus qualities of the problem itself.

As was mentioned previously, the Size and Depth problem stimuli differed only slightly (via the presence or absence of windows), and by an amount that is unlikely to have produced the differential levels of transfer found between and within problem types. In order to rule out the possibility that these small stimulus differences somehow produced the different amounts of transfer that were obtained, we conducted two control experiments.
Figure 12: The effect of representational similarity of the source and target problems on solution times.
Experiment 2c: Different Representations, Same Stimulus

The purpose of the first control experiment was to determine the effect of the small stimulus differences introduced by having windows present in some problems' displays in this experiment, we measured the amount of transfer between two problems that had identical displays but different representations. Subjects solved a pair of problems with windows present in both problem displays, but with problem rules that defined the source problem as a Size problem and the target problem as a Depth problem. By making the stimuli identical, the only difference was the difference in representation engendered by the different rules. If the stimulus differences produced by the presence or absence of windows were responsible for the relatively small amount of transfer obtained in the Size-Depth and Depth-Size conditions of Experiment 2b, then the added stimulus similarity in the current experiment should result in more transfer than was previously obtained. Conversely, if the stimulus similarity (rather than representational similarity) was responsible for the relatively large amount of transfer obtained in the Size-Size and Depth-Depth conditions of Experiment 2b, then the identical displays used in the current experiment should yield a similarly large amount of transfer as in the equivalent conditions of the previous experiment.

The result was that the added stimulus similarity of having windows present in the displays of both problems had no effect on the amount of transfer. The eleven subjects in this control experiment exhibited relatively little transfer when compared to those in the Depth-Depth condition of Experiment 2b. They also exhibited no more transfer than subjects in the Size-Depth condition of Experiment 2b, as shown by the unconnected point in the different representation condition in Figure 12. The increased stimulus similarity occasioned by having windows present in both problems' displays did not produce an increase in transfer; the control of transfer was in the representations. Comparing target problem times in the stimulus different condition and the stimulus-similar condition (with size-depth representations in both cases), yielded an F(1,20)=0.3, p < .65. The nonsignificant tendency was in the opposite direction to that predicted by a stimulus based model; if anything, the target problem took slightly longer in the stimulus-similar condition than in the stimulus-different condition. The unconnected data point reflecting this result is slightly above the stimulus-different comparison point in that figure, rather than below it. In other words, making the stimuli the same did not increase transfer.

The results indicate that the stimulus difference introduced by the presence or absence of
windows in the Depth problem displays did not account for the transfer results we obtained. Rather, it was the difference in representations engendered by the problem rules that was the source of the different amounts of transfer obtained in the Size-Depth and the Depth-Depth conditions of Experiment 2b.

Experiment 2d: Same Representation, Different Stimuli

In the second control experiment, we measured the amount of transfer between two problems that had identical representations but different displays. This differs from Experiment 2c in which we demonstrated that stimulus similarity did not produce transfer when there were different representations for the two problems. Here we were interested in finding any effects that stimulus differences might produce when representations are the same. The subjects in this experiment solved two Size problems similar to the Size-Size condition of Experiment 2a. The only difference was that there were windows present in the display of the source problem but no windows in the display of the target problem. The purpose was to investigate the effect of introducing the stimulus difference, as a further control on Experiment 2b.

Results

There was no difference in the solution times for the target problem between the condition with the same representation and different stimuli, and the condition with the same representation and same stimuli. There was still massive transfer, with no significant difference between the two stimulus conditions. The results are depicted in Figure 12 by the unconnected point in the same-representation condition. The Figure shows that the introduction of the stimulus difference did not decrease the transfer. The results are almost identical to those obtained with the same representation and the same stimuli. The introduction of the slight stimulus difference between the two problems did not have the effect of reducing transfer. The fact that the representations of the two problems were the same (size-size), resulted in a great deal of transfer despite the difference in the problem stimuli introduced by the addition of the windows.

The overall conclusion about representation and transfer obtained from the second series of experiments is that the internal representation of a problem is the determining factor in transfer. The stimulus situation (in these experiments, the computer display of the problem) could make a difference in that it can influence the likelihood of forming and maintaining various representations, but it does not directly control the transfer of skill from problem to
problem. If representation is controlled, the stimulus differences are not of consequence.

Experiment 2e. An Independent Measurement of Representational Availability

At a number of points in this chapter we have argued that the representation of the Size Problem was somehow more attainable, and therefore more available, than the representation of the Depth Problem. Three results in particular have provided evidence for this argument. They are: (a) the contrast between the ambiguity of the results obtained from Experiment 2a and the stronger effect obtained in Experiment 2b which used windows in the Depth problem display (to make that representation more attainable), (b) the slight interaction obtained in Experiment 2b that suggested more transfer from the more available representation, and (c) the increase in the strength of the main effect found in Experiment 2a when the subjects not attaining the desired representation were removed from the analysis. Most of these subjects were in a depth condition and did not maintain the depth representation. These results all argue for the difficulty of evoking or maintaining the depth representation, and for the influence of that difficulty on transfer. In this last experiment, we independently measured the relative availability of the two representations, and thus determined if availability is skewed in a direction consistent with the empirical results.

Methodology

Subjects in this experiment were asked to step into a room and make a judgement about something they would be shown on a computer screen. The displays on the screen were identical to those used in Experiment 2b. The subject was shown either a Size problem display without windows, or a Depth problem display with windows. The experimenter then asked subjects to describe what they saw as he or she made the sphere change size/position. The experimenter noted whether their response referred to a size change or a position (depth) change. They were asked to rate, on a five point scale ranging from "not at all" to "totally", "how much it looks like the sphere is getting larger and smaller", if they had first described it as changing size, or to rate "how much it looks like the sphere is getting closer and further away", if they had first described it as moving in depth. After making the rating on a written response form, subjects were prompted for the alternative representation by being told that "some people see the display as a sphere changing size, growing larger and smaller", if they had responded with a depth representation, or that "some people see the sphere as changing position by moving in and out of a tunnel", if they had reported the size representation. They were asked if they could see it that way.
and to rate the alternative perception on the same five point scale.

Subjects

The subjects were 40 students at the Community College of Allegheny County who were given a donut for their participation.

Results

The results were strongly in the predicted direction; the depth representation was much harder to obtain than the size representation. All 20 subjects who were asked to describe the Size (no window) problem display described the sphere as changing in size. In contrast, only 9 of the 20 subjects who saw the Depth (window) problem display described the sphere as moving in depth. \( \chi^2 = 11.61, (df=1), p < 0.001 \). When asked if they could see it the other way, "Some people see it as moving in depth/changing in size, can you see it that way?" the subjects reported that they were able to. However, their ratings of the goodness of the display on the five point scale favored their first reported answer. Subjects who first perceived the sphere as changing position, when asked, gave that interpretation a higher rating than the size interpretation, whereas those who first perceived it changing in size, gave that interpretation a higher rating, even though all subjects reported being able to see it both ways. The overall preponderance of Size interpretations over depth interpretations indicate that the size representation is more available from the Size problem display than the depth representation is from the Depth problem display. Furthermore, the consistency of the goodness ratings with the initial interpretations shows that once subjects have formed a representation of a display, they tend to stick with it, preferring it to the alternative representation of the same display.

The relative availability of the size and depth representations as measured in this experiment agrees with the predictions derived from the empirical transfer results. Hence, there is further evidence to support the tentative conclusion reached previously that the availability of a representation is a determinant of how readily it will transfer to other problems. The relative unavailability of the depth representation can explain the interaction between the size and depth conditions in Experiment 2b, in which the Size problems seemed to offer more transferable skill or knowledge than the Depth problems. Furthermore, it can explain why, without windows in the display, it was very difficult to obtain the depth representation after the size representation, and difficult to obtain cross representation transfer from depth to size, or even obtain within representation transfer from depth to
depth. The ability to measure the availability of alternative representations, and use them to explain differences in transfer is a potentially useful addition to our findings about the role of representation in transfer.

Discussion

In total, these experiments demonstrate that transfer depends on the internal representation of problems. The first series of experiments, building on previous work, argued for the importance of representational similarity as a predictor of transfer. The amount of transfer obtained was shown to be related to the degree of representational overlap that existed between a pair of problems. Greater representational overlap resulted in increased transfer. The dimensions of overlap included features of the problem representation such as solution path, move operator rules, and Move-Change representational differences. The first series of experiments also showed that the locus of transfer was largely confined to the exploratory phase of the problem solution process. By the time subjects were able to move efficaciously in the problem search space, they required very little time to solve the problem, and this was almost independent of the particular transfer condition or problem order. The final path time simply did not vary by much, taking a minute or so in most target problem conditions. This finding about transfer is significant not only because it identifies the locus of the transfer effect, but also because of its relation to our understanding of the solution process. Kotovsky, Hayes, and Simon (1985) showed that subjects had to practice move making in order to be able to automate or compile the process of planning short (two move) sequences of moves before they could plan ahead. When the ability to plan two move sequences was attained, a solution was achieved in a very short period of time. In fact, all of the isomorphs considered there were solved in essentially the same time. A similar analysis was performed in the current experiment, and yielded the same conclusion; attaining the ability to plan and execute two move sequences was a precursor of the final path behavior.

Our findings about transfer are similar. The analysis of move operator application times strongly supports the view that the locus of transfer is in the exploratory phase and not the final path phase of problem solving. This analysis showed that the exploratory move application times exhibited significant variation with problem condition, whereas the final path move operator application times exhibited very little variation regardless of transfer history or problem order. The interesting implication of this result is that transfer seems to do what
practice does. By automating move making, transfer allows enough move compilation to occur so that planning is possible. If this interpretation is accurate, then it is likely that the design of training materials can be targeted at those processes that allow such move automation to occur. By knowing how transfer occurs and what phase of the problem solving process it affects, it is possible that better practice regimens can be designed, and more effectively monitored to assess the progress of training.

The second series of experiments demonstrates that it is the internal representation of a problem that determines transfer, and this representation can operate independently of the stimulus features of problems. This finding indicates that models of transfer that try to predict the direction and magnitude of transfer on the basis of stimulus overlap and response compatibility should be translated into representational terms. Such a change might make them more effective, especially in situations where the stimulus properties are not predictive of the representations that will be evoked. In addition, by demonstrating that the availability of a representation can be directly and independently assessed, and that availability has a sizable effect on transfer, we have suggested a methodology for assessing the likelihood of transfer, as well as a method for increasing the amount of transfer that will be attained. By modifying or initially designing problems to evoke particular representations, we can increase the likelihood of those representations that will produce positive transfer being available, evokable, and therefore useable in subsequent problem solving experiences. A question that arises from these considerations is to what extent these findings generalize to other domains both within problem solving, and in other areas as well. We very much hope that our findings about transfer transfer. Some work that has been reported by others suggests that this might be so. The work of Singley and Anderson (1985) on transfer of text editing skills is a particularly promising example of similar findings in a quite different domain.

The analysis we have presented is lodged within the view of problem solving that Newell and Simon have developed over the last quarter century. It is a view predicated on the ability to understand problem solving not only as an externally observable process, but also one that is internally driven. It conceives that process as being dependent on an internal representation of the problem and its various components: a representation that is empirically measurable, computer modelable, and scientifically understandable. The key elements of that conception are predicated on a differentiation between the external task environment that problem solving takes place in, and an internal representation of that environment that
constitutes the problem space the subject works in as he or she moves from start to goal. The methodology of means-ends-analysis operates in that problem space, relying on move operators that make progress through it. By applying that type of analysis to transfer, we have tried to show that an understanding of transfer is not only possible within this kind of conception, but that this conception is necessary for the attainment of any complete understanding of transfer. This is particularly the case for the centrality of the representation of the move operator as a determinant of transfer across problems. According to this analysis, the manner in which people represent a problem determines the generalizable and transferable knowledge that they will attain. Furthermore, the availability of that representation is a determinant of the likely success of transfer. We have demonstrated that it is possible to measure which of a set of alternate representations particular problems elicit, that those representations predict the direction of transfer, and that those representations can be empirically manipulated, so as to control transfer within a set of problems that are not only isomorphic but also virtually identical in their stimulus properties. In doing so, we have extended the power of that analysis to a new area that is fully consistent with the work that has come before.

We have perused some of the vineyards sown by Herbert Simon and tasted a few of the grapes. Although the sign over the entrance proclaims a limited capacity, they come in many full-bodied varieties, and there are large bunches of them. We have gotten much sweetness out of them, and the seeds hold out the promise of much more to come.
References


