CONSTITUTIVE BEHAVIOR OF FIBER REINFORCED SANDS

by

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Abstract:
The response of sands reinforced with discrete, randomly distributed fibers was determined under both static and dynamic loading conditions. Laboratory triaxial compression, resonant column, and torsional shear tests were used to measure the stress-deformation response and to observe the influence of various fiber properties, soil properties, and other test variables on constitutive behavior. In addition to the experimental program a theoretical model was developed, based on statistical theories of composites, to predict the fiber contribution to strength under static loads.

Randomly distributed fiber inclusions significantly increased the ultimate strength and stiffness of sands under the action of static loads in triaxial compression tests. The increase in strength and stiffness was a function of sand (cont. on reverse)
granulometry (i.e., gradation, particle size, and shape) and fiber properties (e.g., weight fraction, aspect ratio, and modulus). An increase in gradation and particle angularity of sands, and aspect ratio and modulus of fibers resulted in a greater contribution of fibers to strength.

The sand-fiber composites exhibited either a curved-linear or a bilinear failure envelope with the break or transition to a linear envelope occurring at a threshold confining stress termed the "critical confining stress". At confining stresses below critical the fibers slipped during deformation, and at confining stresses above critical the failure envelope of the composite paralleled that of sand alone.

The failure surface in triaxial compression tests of sands reinforced with discrete, randomly distributed fibers is planar, and is oriented in the same manner as predicted by the Coulomb theory, viz., (45° + 90°/2). This finding suggests an isotropic reinforcing action with no development of preferred planes of weakness nor of strength.

Randomly distributed fiber inclusions influenced the dynamic behavior of sand with respect to shear modulus and damping. The effect of fiber inclusion was evaluated as a function of shearing strain amplitude, confining stress, prestrain, number of cycles, fiber content, aspect ratio, and modulus. Fiber inclusions affected the dynamic response of sands in a manner analogous to the static response. For example, the dynamic modulus increased with increasing fiber aspect ratio, modulus, and with increasing fiber content up to a limiting weight fraction. Fiber inclusions affected damping as well; in general any fiber contribution to increased stiffness resulted in commensurately less damping.
CONSTITUTIVE BEHAVIOR OF FIBER REINFORCED SANDS

RESEARCH SUMMARY

The response of sands reinforced with discrete, randomly distributed fibers was determined under both static and dynamic loading conditions. Laboratory triaxial compression, resonant column, and torsional shear tests were used to measure the stress-deformation response and to observe the influence of various fiber properties, soil properties, and other test variables on constitutive behavior. In addition to the experimental program a theoretical model was developed, based on statistical theories of composites, to predict the fiber contribution to strength under static loads.

Randomly distributed fiber inclusions significantly increased the ultimate strength and stiffness of sands under the action of static loads in triaxial compression tests. The increase in strength and stiffness was a function of sand granulometry (i.e., gradation, particle size, and shape) and fiber properties (e.g., weight fraction, aspect ratio, and modulus). An increase in gradation and particle angularity of sands, and aspect ratio and modulus of fibers resulted in a greater contribution of fibers to strength. At low confining stresses strength was also proportional to the amount or weight fraction of fibers, up to some limiting content. Thereafter, the strength increase approached an asymptotic upper limit. At sufficiently high confining stresses, on the other hand, the strength continued to increase proportionately with fiber content, at least up to a weight fraction of 6%. The latter represents the maximum amount of fiber that can be mixed with the sand under practical conditions and still achieve a reasonable preselected density.

The sand-fiber composites exhibited either a curved-linear or a bilinear failure envelope with the break or transition to a linear envelope occurring at a threshold confining stress termed the "critical confining stress". At confining stresses below critical the fibers slipped during deformation, and at confining stresses above critical the failure envelope of the composite paralleled that of sand alone. The magnitude of the critical confining stress was insensitive to changes in sand particle size and fiber content.

The failure surface in triaxial compression tests of sands reinforced with discrete, randomly distributed fibers is planar, and is oriented in the same manner as predicted by the Coulomb theory, viz., \((45 + \phi/2)\). This finding suggests an isotropic reinforcing action with no development of preferred planes of weakness nor of strength. A statistical analysis was used to predict the expected orientation of randomly distributed...
fibers with respect to the shear failure surface. This expected or statistically most likely orientation was calculated to be ninety degrees, i.e., at right angles to the failure plane. The prediction of fiber contribution to strength (under static loads) was obtained by using a theoretical model which was based on statistical theories of composites and on limiting equilibrium. A comparison of predicted and observed values showed good agreement.

Randomly distributed fiber inclusions influenced the dynamic behavior of sand with respect to shear modulus and damping. The effect of fiber inclusions was evaluated as a function of shearing strain amplitude, confining stress, prestrain, number of cycles, fiber content, aspect ratio, and modulus. Fiber inclusions affected the dynamic response of sands in a manner analogous to the static response. For example, the dynamic modulus increased with increasing fiber aspect ratio, modulus, and with increasing fiber content up to a limiting weight fraction. The presence of fibers reduced prestrain effects often observed in unreinforced sands. The increase in dynamic modulus of fiber reinforced sand was decidedly more pronounced at high shearing strain amplitudes. The maximum percent increase in shear modulus as a result of fiber inclusions occurred over a narrow range of confining stresses from 3 to 10 psi (0.2 to 0.7 kg/sq.cm.). Fibers affected damping as well; in general any fiber contribution to increased stiffness resulted in commensurately less damping.

The dynamic response of sands reinforced with vertically oriented fibers was very similar to that of randomly distributed fibers (at the same weight fraction, modulus, aspect ratio, etc). This finding together with the observed mode of failure in the static compression tests supports the statistically derived prediction that the "expected" orientation of random fibers with respect to the shear plane is ninety degrees.

OBJECTIVES AND SCOPE OF RESEARCH

Background

Research on the constitutive behavior of soil-inclusion systems was initiated as part of Research Grant No. AFOSR 84-0189 which terminated in mid November 1986. In this earlier research the constitutive behavior of a sand reinforced with discrete, randomly distributed fibers was contrasted with that of the same sand reinforced with continuous, oriented fabric inclusions. This earlier research revealed the broad behavior and characteristic response of fiber/fabric reinforced granular soil.

Continuation funds were provided under Grant No. AFOSR 84-0189 to conduct additional tests to delineate more completely the influence of parameters already known to affect the stress-
deformation response of fiber reinforced sand, e.g., the amount of fiber, fiber aspect (length/diameter) ratio, fiber modulus and surface friction. New experimental tests were also undertaken to examine the influence of other soil/fiber/fabric parameters which were not investigated earlier, because of time and funding constraints, but which were likely to be important as well. These included the influence of soil granulometry (i.e., average particle size, gradation, and shape) relative to the fiber aspect ratio and fabric mesh size respectively. Past tests had been conducted only on a medium, uniform dune sand.

Objectives

Specific objectives of the continuation research were as follows:

1. To continue measuring experimentally the influence of such fiber properties as fiber aspect ratio, fiber modulus and surface friction characteristics on the stress-strain response and failure envelope of sands reinforced with these inclusions.

2. To initiate additional testing on the influence of grain size distribution relative to the fiber diameter/fiber aspect ratio and fabric mesh size on the stress-deformation behavior.

3. To examine in detail the mode of failure in fiber reinforced sand in relation to constitutive behavior.

4. To develop a statistically based model to describe and predict the shear strength increase of sands reinforced with discrete, randomly distributed fibers.

STATUS OF RESEARCH

The four objectives cited in the previous section have essentially been achieved. The research program was modified to concentrate only on fiber inclusions, and to examine the response of fiber reinforced sands to both static and dynamic loads respectively. Accordingly, the following research objectives were also included as part of the continuation research:

1. To determine experimentally the dynamic response of sand reinforced with discrete, randomly distributed vs. parallel oriented fiber arrays respectively.

2. To investigate the influence of critical fiber properties (concentration, aspect ratio, modulus, etc) and test parameters (mean confining stress, strain amplitude, number of cycles, etc) on the dynamic shear modulus and damping behavior.
A new laboratory testing apparatus and procedure was employed to investigate the stress-deformation and damping response of sand reinforced with oriented fibers. This technique consisted of subjecting cylindrical samples of sand with oriented fiber inclusions parallel to the long axis of the cylinder to torsional shear strain. The same apparatus was also used to test cylindrical samples of dry sand reinforced with discrete, randomly distributed fibers. The apparatus could subject such reinforced samples to very slow, quasi-static (5 secs/cycle) torsional rotations at relatively high strain amplitudes. The entire process was controlled by a microcomputer, and the test provided a stress-strain record over both loading and unloading cycles.

The findings described in this report formed the basis of a Doctoral Dissertation that was completed by Mohamad H. Maher and submitted to the University of Michigan Rackham School of Graduate Studies in August of 1987. A report describing in detail the static and dynamic response respectively of fiber reinforced sands has been submitted to the Air Force Office of Scientific Research under separate cover. This report is titled "Static and Dynamic Response of Sands Reinforced with Discrete, Randomly Distributed Fibers." The results of the static stress-deformation study formed the basis of a paper "Admixture Stabilization of Sands with Random Fibers" that was accepted for publication in the Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering. A copy of the paper is included as an appendix to this report.

PUBLICATIONS


PROJECT PERSONNEL

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1. T. Al-Refeai  
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   Ph.D. Degree awarded August 1985  
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2. Cho-sen Wu  
   GSRA (Sept 1984 – Oct 1986)  
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   Thesis Title: "Finite Element Analysis of Fabric 
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3. Mohamad H. Maher  
   GSRA (Sept 1985 – Aug 1988)  
   Ph.D. Degree to be awarded August 1988  
   Thesis Title: "Static and Dynamic Response of Sands 
   Reinforced with Discrete, Randomly 
   Distributed Fibers"  
   (Accepted appointment as Asst. Prof., Dept. Civil Engr.,  
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INTERACTIONS

A discussion and conference was held with Professor Iraj Noorany, Dept. of Civil Engineering, San Diego State University, in January 1988. Professor Noorany supervised a Masters thesis on the effect of soil reinforcement on the liquefaction potential of saturated sand that was completed in the Spring of 1987. This is the only other known study on the dynamic behavior of fiber/
fabric reinforced sands. The study consisted of performing cyclic triaxial tests on sand specimens reinforced with four commercially available products in nine different configurations within the reinforced specimens. These configurations ranged from four discrete layers of fabric disks, to fifteen layers of fabric strips, to randomly distributed polymeric fibers. One of the most significant conclusions of this study was that the more even the distribution of reinforcement throughout the specimen, the more effective it is in increasing the resistance to liquefaction. Specimens reinforced with only 0.38% by weight of randomly distributed polypropylene fibers exhibited a superior and marked resistance to liquefaction.

A discussion and conference was also held with Mr. Thomas Hoover, Research Engineer, CALTRANS Laboratory, Sacramento, California, in June 1988. CALTRANS is presently investigating the feasibility of using fiber reinforcement in compacted fill embankments. They are experimenting with fiberglass “roving” and various polyester/polypropylene monofilaments. Their proposed system appears to resemble closely the French “TEXSOL” method. Fiber strands are roller fed and blown into a sand simultaneously with its deposition. The sand-fiber mix is then compacted in the conventional manner. CALTRANS is presently conducting triaxial compression tests on large diameter (6-inch) triaxial samples prepared in this fashion.

NEW DISCOVERIES

No patents or inventions have resulted from the research described herein. It does appear, however, that findings from this research could provide the basis for developing practical applications for discrete, randomly distributed fiber reinforcement in soil masses. Unlike other types of reinforcement, that must be hand or machine placed in lifts or layers, random fiber reinforcement is essentially a variant of admixture stabilization. Considerable savings in labor might thus be achieved by batch mixing soil and fibers. A satisfactory method must still be developed, however, for mixing and placement in the field that avoids segregation and that yields a relatively uniform mixture of soil and fibers.

ADDITIONAL STATEMENTS

The research carried out to date with the support of the AFOSR Grant No. 84-0189 probably constitutes one of the most systematic and extensive studies on the properties and behavior of fiber reinforced sands. In addition to characterizing the behavior of fiber reinforced sands under static loading, the research has provided for the first time a large body of information on their dynamic behavior as well.
Admixture Stabilization of Sands

with

Discrete, Randomly Distributed Fibers

by:

D.H. Gray

&

M.H. Maher

THE UNIVERSITY OF MICHIGAN

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SYNOPSIS

Previous studies have shown that the inclusion of randomly distributed fibers significantly increase the shear strength of sands. Triaxial compression tests were used in this study to identify and evaluate sand/fiber parameters that affect strength and failure mechanics. Both sand granulometry, i.e., particle size, shape and gradation, and fiber properties, e.g., modulus, influenced constitutive behavior of sand-fiber composites. In addition to the experimental work, a theoretical model is presented to explain the sand/fiber interaction based on statistical theories of composites.
1.0 INTRODUCTION

Traditional methods of earth reinforcement consists of introducing continuous inclusions, e.g., strips, fabrics, or grids, into an earth mass. These are normally oriented in a preferred direction and are introduced sequentially in alternating layers. An exception to this rule is the "Texol" system (Leflaive, 1986) in which a single monofilament is spun or injected in a random pattern simultaneously with the deposition of a sand. Soil strengthening using randomly distributed polymeric mesh elements has also been described by McGown et. al. (1986). These elements act to interlock particles and groups of particles in a unitary coherent matrix.

An earth mass stabilized with discrete, randomly distributed fibers resembles traditional earth reinforcement in many of its properties, (Gray and Al-Refeai, 1986) but mimics admixture stabilization in its preparation. Essentially, discrete fibers are simply added and mixed with the soil much the same as cement, lime, or other additives. One of the main advantages of randomly distributed fibers is the maintenance of strength isotropy and the absence of potential planes of weakness that can develop parallel to oriented reinforcements.

Unlike continuous oriented inclusions commonly used in reinforced earth structures, viz., strips, fabrics, and grids, only limited information has been recorded in the technical literature on fiber reinforcement (Gray and Ohashi, 1983; Andersland and Khattak, 1979; Hoare, 1977). These studies have tended to focus on the influence of fiber parameters such as fiber content, modulus, length/diameter ratio, etc. on strength/deformation response. A primary objective of this report is to describe the influence of soil granulometry as well, namely, the effect of gradation, average particle size, and shape. The other objective is to propose a theoretical model for predicting the influence of discrete, randomly distributed fiber inclusions on the constitutive behavior of sands.
2.0 THEORETICAL MODEL

In order to describe the soil-fiber interaction during triaxial compression tests and predict the contribution of fibers to the shear strength of soil, a model is presented herein. This model is based on the following: (a) a statistical analysis from the theory of fiber reinforced composites to predict fiber position and quantity with respect to the plane of shear failure; and (b) limiting equilibrium conditions (Coulomb Criterion) to predict the fiber contribution to soil shear strength.

2.1 Assumptions

The following assumptions were made with regard to our model:

a) Fiber length, L, and diameter, D, are constant.
b) Fibers have an equal probability of making all possible angles with an arbitrary axis.

c) The fibers in the soil mass and their equivalent points of intersection with the failure plane are randomly distributed following a Poisson process (Maher, 1988).
d) Fibers do not offer any bending resistance.
e) The sand-fiber composite has a bilinear failure envelope with the bilinearity break occurring at a threshold confining stress called $\sigma_{\text{crit}}$. At $\sigma < \sigma_{\text{crit}}$ the fibers lip during deformation, and at $\sigma > \sigma_{\text{crit}}$ the failure envelope of the composite parallels that of sand alone.

2.2 Number of Fibers Per Unit Volume and Corresponding Number of Fibers Intersecting a Unit Area.

Given a fiber volume fraction $\beta_f$, length, L, and diameter D; the average number of fibers per unit volume can be determined as follows:
\[ N_v = \frac{4\beta_f}{\pi D^2 L} \quad (1) \]

where the \( N_v \) = average number of fibers per unit volume. The number of fibers intersecting a unit area of a plane, say Coulomb Failure plane, is related to \( N_v \) and is given by:

\[ N_S = 2N_v \frac{L}{4} = \frac{2\beta_f}{\pi D^2} \quad (2) \]

This relationship is mathematically driven from our assumption "c".

2.3 - Characterization of Relevant Variables Associated With a Fiber Crossing the Plane of Shear Failure Plane in Soils.

Consider a random fiber, crossing the failure plane in a triaxial compression test (Fig.1), with variables \( i \) and \( \theta \) defining the position, and variable \( x \) as the smallest length of fiber on either side of the plane. Given a uniform distribution functions for each of these variables, we can obtain the following expected values:

\[ E(x) = \frac{L}{4} \quad (3) \]
\[ E(i) = \pi \quad (4) \]
\[ E(\theta) = \frac{\pi}{2} \quad (5) \]

Where \( E \) denotes an expected value.

The theoretical expected values of \( i \) and \( \theta \), indicate that "on average" a fiber will be oriented perpendicularly to the plane of failure. An experimental verification of this finding is given by Gray and Ohashi, (1983).
2.4 - Mechanics of Reinforcement.

Given the number and position of fibers with respect to the failure plane (Fig. 1), we can estimate their contribution to shear strength by using a limit equilibrium approach (Gray and Ohashi, 1983; Waldron, 1977). The shear strength increase due to fiber inclusion ($\Delta s$) can be shown to be:

$$\Delta s = N_s (t_f) (\sin \omega + \cos \omega \tan \phi)$$  \hspace{1cm} (6)

Where $t_f$ = maximum tensile stress developed in one root at any given shear displacement, $\omega$ = angle of shear distortion (see Fig. 2), and $\phi$ = angle of internal friction of sand.

The value for $t_f$ is a function of fiber response to the loading; and it varies for the case of stretching, slipping through soil, and breaking respectively:

$$t_f = \sqrt{4 \tau E Z/D} \sqrt{\sec \omega - 1}$$  \hspace{1cm} (Stretch.)  \hspace{1cm} (7)

$$t_f = 2 \tau L/D$$  \hspace{1cm} (Slip)  \hspace{1cm} (8)

$$t_f = \text{maximum tensile strength of fiber}$$  \hspace{1cm} (Break)  \hspace{1cm} (9)

Figure 1. Characterization of a Single Fiber Crossing the Plane of Shear Failure in Triaxial Compression Tests.
Where $\tau =$ shear stress at the fiber-soil interface, $E =$ fiber modulus, and $Z =$ thickness of shear zone (Fig. 2).

Figure 2. Characterization of a Single Fiber Crossing the Shear Zone, During Deformation in Triaxial Compression Tests: (a) Intact fiber (b) Deformed fiber.

Given the bilinearity of the failure envelope in fiber reinforced soil (Gray and Ohashi, 1983; Gray and Al-Refeai, 1986); and the existence of a critical confining stress $\sigma_{\text{crit}}$ (assumption d). We can show that:

$$\Delta_S = N_s \left( \frac{\pi D^2}{4} \right) (2\sigma \tan \delta \ L/D)(\sin \omega + \cos \omega \ \tan \phi)$$

$$0 < \sigma < \sigma_{\text{crit}}$$ \hspace{1cm} (10)

$$\Delta_S = N_s \left( \frac{\pi D^2}{4} \right) (2\sigma_{\text{crit}} \tan \delta \ L/D)(\sin \omega + \cos \omega \ \tan \phi)$$

$$\sigma > \sigma_{\text{crit}}$$ \hspace{1cm} (11)

Where $\Delta_S =$ shear strength increase due to fiber inclusion, $N_s =$ no. of fibers intersecting a unit area of failure plane, $\sigma =$ applied confining stress, $\sigma_{\text{crit}} =$ known or estimated value.
of critical confining stress, $\delta$ = skin friction angle of the fiber, and $Z$ = thickness of shear zone ($Z$ is estimated from Roscoe, 1970).

In order to further verify the model, it is important to experimentally define the critical confining stress and its relation to fiber-soil parameters. This was achieved by the following experimental program.

### 3.0 EXPERIMENTAL PROGRAM

#### 3.1 Test Parameters

We investigated the influence of a number of soil-fiber parameters on the stress-deformation response and constitutive behavior. Fiber parameters of interest included: tensile strength and modulus, surface roughness, length/diameter (aspect) ratio, and weight fraction. Fiberglass, natural reed, palmyra, and Buna-N rubber fibers were employed for this purpose (Table I). The experimental program included a detailed study on the effect of granulometry (grain size, shape, and gradation) on constitutive behavior. A variety of different sands ranging from well graded to uniform, and rounded to angular were tested (Tables II, III, and IV). A rapid, semi-automated triaxial testing set-up was developed and fiber-sand specimens were tested over a range of confining stresses up to 4 kg/cm$^2$ in order to fully define their stress-strain response and failure envelopes.
Table I. Fiber Properties

<table>
<thead>
<tr>
<th>Fiber Type</th>
<th>Diam. (mm)</th>
<th>Aspect ratio (L/D)</th>
<th>Spec. Gravity (g/cc)</th>
<th>Tensile Strength (Kg/cm²)</th>
<th>Tensile Modulus (Kg/cm²)</th>
<th>Skin Friction Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buna-N (Rubber)</td>
<td>1.1</td>
<td>20</td>
<td>1.26</td>
<td>0.42x10²</td>
<td>1.05x10²</td>
<td>30</td>
</tr>
<tr>
<td>Reed#0 (Natural)</td>
<td>1.0</td>
<td>20</td>
<td>0.47</td>
<td>0.47x10⁴</td>
<td>0.34x10³</td>
<td>30</td>
</tr>
<tr>
<td>Palmyra (Natural)</td>
<td>0.58</td>
<td>20</td>
<td>0.73</td>
<td>16.87x10⁴</td>
<td>1.81x10⁵</td>
<td>30</td>
</tr>
<tr>
<td>Glass#1 (Synth.)</td>
<td>0.3</td>
<td>60</td>
<td>2.7</td>
<td>7.14x10⁵</td>
<td>1.28x10⁴</td>
<td>21</td>
</tr>
<tr>
<td>Glass#2 (Synth.)</td>
<td>0.3</td>
<td>80</td>
<td>2.7</td>
<td>7.14x10⁵</td>
<td>1.28x10⁴</td>
<td>21</td>
</tr>
<tr>
<td>Glass #3 (Synth.)</td>
<td>0.3</td>
<td>125</td>
<td>2.7</td>
<td>7.14x10⁵</td>
<td>1.28x10⁴</td>
<td>21</td>
</tr>
</tbody>
</table>

Table II. Type of Soils Used for Gradation Effect.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Specific Gravity (g/cc)</th>
<th>D₅₀ (mm)</th>
<th>Coef. of Unif. (Cu)</th>
<th>Max. Void Ratio</th>
<th>Min. Void Ratio</th>
<th>Sphericity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Musk. Dune</td>
<td>2.65</td>
<td>0.41</td>
<td>1.5</td>
<td>0.78</td>
<td>0.50</td>
<td>0.90</td>
</tr>
<tr>
<td>50-50 Sand</td>
<td>2.59</td>
<td>0.47</td>
<td>3.3</td>
<td>0.86</td>
<td>0.58</td>
<td>0.55</td>
</tr>
<tr>
<td>Mortar Sand</td>
<td>2.70</td>
<td>0.60</td>
<td>4.13</td>
<td>0.62</td>
<td>0.32</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Table III. Type of Soils Used for Particle Size Effect.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Specific Gravity (g/cc)</th>
<th>(D_{50}) (mm)</th>
<th>Coef. of Unif. (Cu)</th>
<th>Max. Void Ratio</th>
<th>Min. Void Ratio</th>
<th>Sphericity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa 20-30</td>
<td>2.66</td>
<td>0.65</td>
<td>1.1</td>
<td>0.72</td>
<td>0.51</td>
<td>0.85</td>
</tr>
<tr>
<td>Ottawa 50-70</td>
<td>2.66</td>
<td>0.25</td>
<td>1.1</td>
<td>0.91</td>
<td>0.77</td>
<td>0.85</td>
</tr>
<tr>
<td>Glass Sphere#1</td>
<td>2.50</td>
<td>0.60</td>
<td>1.1</td>
<td>0.73</td>
<td>0.56</td>
<td>0.9</td>
</tr>
<tr>
<td>Glass Sphere#2</td>
<td>2.50</td>
<td>0.30</td>
<td>1.1</td>
<td>0.75</td>
<td>0.56</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table IV. Type of Soils Used for Particle Shape Effect.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Specific Gravity (g/cc)</th>
<th>(D_{50}) (mm)</th>
<th>Coef. of Unif. (Cu)</th>
<th>Max. Void Ratio</th>
<th>Min. Void Ratio</th>
<th>Sphericity</th>
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<td>2.66</td>
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</tr>
<tr>
<td>Musk. Dune</td>
<td>2.65</td>
<td>0.41</td>
<td>1.5</td>
<td>0.78</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>Sand #1</td>
<td>2.70</td>
<td>1.1</td>
<td>1.65</td>
<td>0.84</td>
<td>0.43</td>
<td>0.30</td>
</tr>
</tbody>
</table>

3.2 Test Results

Results of this testing program showed that sands reinforced with randomly distributed fibers exhibited either curved-linear or bilinear failure envelopes. Uniform, rounded sands exhibit the former behavior (Fig.3), whereas well graded and/or angular sands tend to exhibit the latter (Fig.4). The break in the bilinear curve or transition from a
curved to a linear envelope occurs at threshold confining stress which we refer to, in our theoretical model, as the "critical" confining stress. This critical stress is quite sensitive to certain soil-fiber parameters, e.g., fiber aspect ratio, grain shape and gradation (Figs. 5 and 6), but is relatively unaffected by others, e.g., amount of fiber and median grain size.

Figure 3. Failure Envelopes for a UNIFORM, SUBROUNDED Sand Reinforced with Random Glass Fibers @ 3% by Wt. Fiber Length/Diameter Ratio = 60, 80, 125.
Figure 4. Failure Envelopes for a WELL GRADED, SUBANGULAR Sand Reinforced with Random Glass Fibers @ 3% by Wt. Fiber Length /Diameter Ratio = 60, 80, 125.

Figure 5. Influence of Soil Gradation and Aspect Ratio on the Critical Confining Stress. Glass Fibers @ 3% by Wt.
Figure 6. Influence of Particle Shape and Aspect Ratio on the Critical Confining Stress. Glass Fibers @ 3% by Wt.

The amount of strength increase induced by reinforcement with randomly distributed, short fibers also depends strongly on certain soil-fiber parameters, e.g., fiber aspect ratio and modulus; grain size, gradation, and shape (Figs. 7, 8, and 9); and amount of fiber present. The increase in strength with increasing amounts of fiber is almost linear at high confining stresses and/or fiber aspect ratios but approaches an asymptotic upper limit at lower values of these two parameters (see Fig. 9). The maximum increase in strength on a relative basis occurs at confining stresses less than the critical value (Fig. 8). These findings make it possible to select fibers (by aspect ratio and type of fiber) to achieve optimal reinforcement for a particular stress environment and soil granulometry.
Figure 7. Influence of Fiber Modulus on Fiber Contribution to the Strength of Reinforced Sand (same fiber volume fraction and aspect ratio).

Figure 8. Influence of Average Particle Size on Fiber Contribution to Strength Increase in Sands with Same Gradation and Particle Shape. Glass Fibers @ 3% by Wt., Aspect Ratio = 125.
4.0 CONCLUSIONS

1. The failure surface in a triaxial compression test of randomly distributed, fiber-reinforced sand is planar and oriented in the same manner as predicted by the Coulomb theory, viz., $(45 + \phi/2)$. This finding suggests an isotropic reinforcing action with no development of preferred planes of weakness or strength.

2. The failure envelopes in our tests were either curved-linear or bilinear with the transition or break occurring at a confining stress denoted as the "critical confining stress", $\sigma_{\text{crit}}$.

3. An increase in fiber aspect ratio, L/D, resulted in a lower $\sigma_{\text{crit}}$ and more effective fiber contribution to increased shear strength.

4. An increase in fiber amount or weight fraction, $W_f$, had no effect on $\sigma_{\text{crit}}$, but it did influence strength significantly.

Figure 9. Influence of Fiber Content and Aspect Ratio on Strength Increase in Muskegon Dune Sand at Low and High Confining Stresses, Respectively.
5. Shear strength increases approximately linearly with increasing amounts of fiber and then approaches an asymptotic upper limit that is governed mainly by confining stress and fiber aspect ratio.

6. Very low modulus fibers (e.g., rubber) contribute little to increased strength in spite of superior pullout resistance (low $\sigma_{\text{crit}}$).

7. An increase in soil gradation, $C_u$, resulted in lower $\sigma_{\text{crit}}$, and higher fiber contribution to strength (all other factors constant).

8. An increase in particle sphericity resulted in a higher $\sigma_{\text{crit}}$, and lower fiber contribution to strength (all other factors constant).

9. An increase in soil grain size, $D_{50}$, had no effect on $\sigma_{\text{crit}}$; however, it reduced the fiber contribution to strength (all other factors constant).

10. Experimental lower and upper bound values of fiber-soil parameters, in comparison with the theoretical model presented herein permit an estimate of the likely contribution to shear strength of sands as a result of reinforcement with short, randomly distributed fibers.

5.0 REFERENCES


6.0 ACKNOWLEDGEMENT

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