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A THEORETICAL STUDY OF THE EFFECTS OF VEGETATION ON TERRAIN SCATTERING

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This study develops an analytical model for the scattering by a covering of vegetation on a randomly rough conducting surface. The foliage is analyzed using the Foldy-Twersky formalism for the coherent or average field and a new distorted wave born approximation (DWBA) is developed for the fluctuating field. The rough surface scattering is modeled using an interactive solution of the Magnetic Field Integral Equation (MFIE) and this approach is related to classical random surface methods. In addition, an improvement to this interactive approach is developed based on a Luneburg-Kline series for the current induced on the rough surface. The layer-on-the-surface problem is modeled by a single down and up passage of the appropriate coherent and incoherent fields as opposed to the exact solution which comprises an infinite number of passages. The key to the analysis is the conversion of all fields in the foliage to either coherent or incoherent quantities. All the important primary and secondary interactions are preserved in this model.

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1.0 The Effects of Vegetation on Terrain Scattering

1.1 Introduction

An important element in any effort to model the electromagnetic scattering from natural terrain is the inclusion of the effects of the various types of vegetative or foliage cover. That such coverage is important is obvious from just looking at terrain and being able to readily see which surface areas are covered by the different types of vegetation, e.g. crops, forests, scrub brush, and grass. In the language of scattering physics the phrase "just looking at terrain and . . ." is equivalent to saying that the bistatic scattering cross section of the vegetation covered surface in the visible frequency band is sufficiently sensitive to the types of cover to provide a quantitative means for discriminating between the various kinds of vegetation. With this more technical definition, it is important to take note of the distinguishing characteristics of the "measurement". That is, the transmitter/receiver geometry is bistatic and the transmitter and receiver are both operating over a very wide band of frequencies. This latter point is particularly important because such wideband measurements have no analog in the lower frequency ranges common to radar or communication, and so care should be exercised in comparing what is "seen" with what much lower frequency measurements or theory yield. Certainly, one would expect that as the frequency is continually decreased, there would come a point where the scattering is due predominately to the underlaying surface and there is very little scattering from the vegetation [1].

Unfortunately, most radars and communications systems operate in a frequency range that is somewhere *between* these two extremes. Consequently, there is a need for models involving scattering from (1) the vegetation layer alone, (2) the underlying surface alone, and (3) various degrees of interaction between the layer and the underlaying surface. Since ultra high spatial resolution scattering is not a goal of this study, the use of statistical means to describe the scattering should be appropriate. Ideally, this means that what is needed is the probability density function (pdf) for the scattered field because this will then permit a calculation of any statistical moment of the scattered field. However, the pdf of the scattered field is a very difficult quantity to obtain, in general, and so attention will be directed toward a less ambitious goal, namely, the modeling of the first and second moments of the scattered field.

Having established the goal of this study, the next step is to develop a rationale for achieving it. The rationale for this problem comes rather naturally from the importance of the two major contributors to the scattering. That is, the first thing that needs to be done is to insure that adequate models exist for both the foliage layer and the underlaying surface alone. Where deficiencies are noted, they need to be corrected. The final phase is to model the foliage layer on the surface by giving particular attention to the *interaction* of the two media. This is the real crux of the problem because it is this phase that permits a smooth transition between the complete high frequency shadowing of the surface by the vegetation to the low frequency transparency of the foliage to the incident radiation.

1.2 Vegetation Only

Before becoming too involved in the electromagnetics associated with wave propagation through and scattering by vegetation, it is advantageous to organize vegetation into four very broad categories. In particular, these categories are trees, brush, crops, and grasses. This is done primarily for discussion purposes because there is, in general, little that is unique about each category. Trees are usually characterized by their bulk size, relatively large leaves, and a central trunk. Brush is usually smaller in overall size, has a smaller leaf, lacks a well-defined central trunk, and has much less woody structure. However, these differences become less distinct when the trees are relatively young. Similarly, there is not much difference between new, emerging grain crops and grass except that the former is usually planted in periodic rows while the grass tends to be more uniform in its coverage. Despite the biological nonuniqueness of these categories, they provide a physically intuitive means of identifying types of vegetation.

In estimating the electromagnetic scattering behavior of a *collection* of vegetation, it would be most helpful if the behavior of a *single composite structure* of an entire entity were known. However, even single trees, bushes, and plants are difficult to describe from an electromagnetic scattering point of view. Thus, it is necessary to go one step further in simplicity and consider the scattering properties of the *component parts* of the tree, bush, plant, etc. In the case of trees and bushes, these component parts comprise leaves, branches, and trunks. With crops and grasses, the parts comprise leaves and stems or stalks. Although these component parts are interconnected by branches, stems, and twigs, most subsequent scattering analysis will *ignore* these interconnects and treat the component parts as devoid of physical contact with any other parts of the foliage.

This latter approximation is clearly a drastic one and therefore deserves some justification. The important assumption that will be made to rationalize this approximation is that each component part of the foliage is a much better absorber of electromagnetic energy than it is a scatterer. More formally, the approximation is that the absorption cross section is significantly greater than the scattering cross section. For the frequency range of interest to this study, it turns out that this is indeed a reasonably good assumption [1]. Hence, given that each component part of the vegetation absorbs much more incident energy than it scatters, consider what happens when a wave strikes two leaves connected by a stem or branch. In addition to the energy scattered from one leaf to another, there is also the possibility of energy being propagated in a transmission line mode along the connecting branch or stem. However, energy being transported between leaves via this mechanism suffers a *threefold attenuation*. First, it suffers absorption in the first leaf and then in the connecting branch and finally in the second leaf. Thus, it would seem reasonable to ignore the interconnection of the various parts of the vegetation based on the highly lossy nature of the branches and stems, i.e. the interconnecting medium.

1.2.1 The Coherent Field

The total field inside a vegetative medium can be mathematically split into the sum of a mean, average, or coherent part and a zero mean fluctuating part [2], e.g.

$$\vec{E}_t = <\vec{E}_t> + \delta\vec{E}_t \tag{1.1}$$

where $<\cdot>$ denotes the averaging operation, and $\delta\vec{E}_t$ is a zero mean field quantity. There are specific physical situations when analytical models for the average field have been derived. The first of these is the classic Foldy-Twersky result [3,4,5] in which the concentration of scatterers is sufficiently small that multiple scattering between them may essentially be ignored. The Lax-Twersky [6,7,8,9] result holds for higher concentrations of scatterers but with the restriction that they cannot differ very much in their dielectric properties from free space. Finally, rather high concentrations of scatterers have been analyzed recently [10] using a numerically based T-matrix approach.

The Lax-Twersky theory is difficult to apply to the vegetation problem because the component scatterers have such a large relative dielectric constant. The numerical T-matrix approach would be difficult to apply to the foliage problem because of the diverse shapes of the component scatterers. Furthermore, neither of the above methods is really necessary because the density of vegetation is relatively low. That is, the highest volume fraction occupied by vegetation will be less than 5%, and even this percentage is relatively unusual [1]. More typical volume fractions are 1% or less. With this low concentration of scatterers combined with their relatively high loss, the Foldy-Twersky theory for the coherent or mean field inside the foliated medium should be adequate [11].

The Foldy-Twersky theory predicts that in an unbounded volume of scatterers, the average field will be a plane wave having the wavenumber k_p given by [12]

$$k_p^2 = k_o^2 + 4\pi\int_0^\infty <f_p(\hat{k}_i,\hat{k}_i)| \ n(\alpha)d\alpha \tag{1.2}$$

where k_o is the free space wavenumber, $n(\alpha)d\alpha$ is the number of scatterers per unit volume having the size parameter α between α and $\alpha + d\alpha$, and the total number of scatterers per unit volume is ρ where

$$\rho = \int_0^\infty n(\alpha)d\alpha \tag{1.3}$$

The quantity $f_p(\hat{k}_i, \hat{k}_i)$ is the p^{th} vector component of the scattering amplitude of a single scatterer having size α. It is computed for a plane wave traveling in the direction $\hat{k}_i$ incident on the body and the body scattering in the direction $\hat{k}_i$. The polarization of the incident field in this calculation is taken to be $\hat{e}_p$ while the symbol $<\cdot]$ denotes that the scattering amplitude may need to be averaged over *all possible orientations* of the scattering body. An explicit expression for $f_p(\hat{k}_i, \hat{k}_i)$ is

$$f_p(\hat{k}_i,\hat{k}_i) = \frac{k_o^2}{4\pi}(\varepsilon_r - 1) \int_{V_\alpha} [\vec{E}_i(\vec{r}) \cdot \hat{p} / E^i] \exp(jk_o \hat{k}_i \cdot \vec{r}) dv \tag{1.4}$$

where ε_r is the relative dielectric constant of the scattering body, $\vec{E}_i(\vec{r})$ is the field inside the body due to an incident plane wave having amplitude E^i, polarization $\hat{p}$, and traveling in the $\hat{k}_i$ direction. The integration in (1.4) is over the volume of the scatter, V_α, and, alth ugh not explicitly shown, the calculation is for one specific orientation of the body. The av rages in (1.2) over orientation and size are necessary when there is a polydisperse mixture of scatterers having random orientation. As indicated in (1.2), it is possible for the average medium to be anisotropic, i.e. the propagation constant is different in different directions.

Equally important to the result in (1.2) are the limitations on its validity. These limitations were not particularly well understood until recently [12] when it was shown that they could all be essentially lumped together in the following condition;

$$(k_p - k_o) \max(\alpha) << 1 \tag{1.5}$$

where max(α) is the maximum meaningful dimension of the scattering body. For a sphere, max(α) is the diameter while for a randomly oriented thin disk it should be set equal to the diameter of the disk. The condition in (1.5) is essentially equivalent to requiring that the scattering properties of a body in free space and in the random medium are not appreciably different. It should be noted that if the concentration of scatterers increases then either the relative dielectric constant must approach unity, see (1.4), or the scatterers must become smaller, see (1.5), in order for (1.2) to remain valid. It is the interplay of the scatterer concentration, dielectric constrast, and size that determine the validity of the Foldy-Twersky result for the average or coherent field.

If there are a number of different distinct types of scatterers comprising the medium then (1.2) should be augmented to reflect this fact. For example, if there are M different types of scatterers then (1.2) should be written as follows;

$$k_p^2 = k_o^2 + 4\pi \int_0^\infty \sum_{m=1}^{M} < f_p(\hat{k}_i,\hat{k}_i)]_m n_m(\alpha) d\alpha \tag{1.6}$$

where $< f_p(\hat{k}_i,\hat{k}_i)]_m$ is the forward scattering amplitude for the m^{th} class of scatterers comprising $n_m(\alpha)d\alpha$ number per unit volume having a size parameter between α and $\alpha + d\alpha$. This result is particularly relevant to the vegetation problem because there are indeed a number of distinctly different types of scatterers (even when the actual foliage components are replaced by canonical shapes). Table I shows the major vegetation categories that have been postulated along with the actual component parts comprising these and the canonical approximations to these parts. The canonical shapes are the ones that would be used in calculating scattering amplitudes such as required by (1.6).

Clearly, there is a great deal of approximation involved in replacing exact shapes by canonical forms. However, it must be remembered that vegetation is a very complicated physical environment and it is essential that it be simplified as much as possible without distorting the important details. This was the approach used in an earlier attempt to go beyond pure empirical modeling [1] and it appeared to work quite well for wave propagation through trees. Since this work first appeared, there has been a great deal of literature on computing the scattering properties of various canonical shapes and it seems that there are a number of approximations that can be used [13,14,15]. These results are most welcome because calculating the scattering amplitudes of even the limited number of essential canonical shapes is not a trivial matter.

1.2.2 The Fluctuating Field

Equation (1.1) indicates that the total field inside the random medium comprises a coherent or average field and a zero mean fluctuating part. The coherent field attenuates exponentially due to absorption and scattering by the objects in the medium; the attenuation rate is determined by the imaginary part of (1.4). The fluctuating field gives rise to the incoherent scattered power and is therefore of prime importance to this study. The fluctuating field is, in general, more difficult to determine than the coherent field; however, reasonable success has been obtained using the so-called distorted wave Born approximation (DWBA) [16] at least for the frequency range of interest to this study. However, as the DWBA is now used [17], there appears to be a form of double accounting for the effects of the average medium. To show this, the DWBA will first be reviewed, an exact integral equation for the fluctuating field will be derived, and this result will be compared to the DWBA that is presently in use. The differences will be noted and discussed.

1.2.2.1 The Distorted Wave Born Approximation (DWBA)

Taylor [16] gives an excellent discussion of the DWBA as it applies to particle scattering by potentials. However, the same reasoning can be applied to the vegetation scattering problem and the rationale for the approximation centers around the *form* of (1.1). What (1.1) does is to split the unknown field into the sum of a largely dominant term, $<\vec{E}_t>$, and a small perturbation $\delta\vec{E}_t$. The dominant term $<\vec{E}_t>$ is known to a reasonably good approximation;

BASIC VEGETATION CATEGORY	ACTUAL COMPONENT PARTS	CANONICAL COMPONENT SHAPES
Tree	leaves/needles	disks/cylinders
	branches	cylinders with large range of radii
	trunks	cylinders with small range of radii
Brush	leaves	disks
	branches	cylinders
Crops	leaves	disks on thin elongated sheets
	stalks, stems	cylinders with relatively small range of radii
	product	variable
Grasses	leaves	short dipoles
	stems	cylinders

TABLE I

A categorization of basic vegetation types by actual and canonical component parts.

hence, $\delta\vec{E}_t$ can be determined fairly accurately by a rather crude approximation. The key to the success of the DWBA is that most of $\vec{E}_t$ is already known ($<\vec{E}_t>$) and it is only a slight improvement ($\delta\vec{E}_t$) that is sought. Of course, this also shows one of the limitations of the DWBA as applied to the vegetation problem. If $<\vec{E}_t>$ is to be the dominant term in $\vec{E}_t$, the DWBA cannot be expected to give an accurate estimate for $\delta\vec{E}_t$ very deep into the medium. This is because $<\vec{E}_t>$ will be very small for deep penetration into the medium and it is then that $\delta\vec{E}_t$ will become the dominant field quantity. This situation clearly violates the assumptions in the DWBA.

In the next section, an exact integral equation of the second kind for the fluctuating field will be developed and attention will be directed toward the Born term in this equation. Using this term only comprises a DWBA because it assumes that $<\vec{E}_t>$ is known and dominant. In the subsequent section, this result will be compared to a heuristic based DWBA. It will be shown that the heuristic approach leads to what appears to be a form of double accounting for the average properties of the medium. In this regard, the heuristic approach is not in agreement with the Born term from the exact expression for the fluctuating field.

1.2.2.2 An Exact Integral Equation for the Fluctuating Field

The *total* field at the point $\vec{r}$, $\vec{E}_t(\vec{r})$, due to an *incident* field, $\vec{E}_i(\vec{r})$, and the fields *scattered from objects located about the point* $\vec{r}$, $\vec{E}_s(\vec{r})$, *is given by*

$$\vec{E}_t(\vec{r}) = \vec{E}_i(\vec{r}) + \vec{E}_s(\vec{r}) \tag{1.7}$$

The scattered field due to the presence of the N objects can be written as follows [18];

$$\vec{E}_s(\vec{r}) = L\overline{\overline{K}}_\Sigma \vec{E}_t(\vec{r}_n + \vec{r}_o) \tag{1.8}$$

where L is the three dimensional integral over all space, i.e., $L = \int\int\int d\vec{r}_o$, and the dyadic operator $\overline{\overline{K}}_\Sigma$ is given by

$$\overline{\overline{K}}_\Sigma = -k_o^2 \sum_{n=1}^{N} (\varepsilon_{r_n} - 1) S_n(\vec{r}_o;\ \Omega_n)\overline{\overline{\Gamma}}(r - \vec{r}_n - \vec{r}_o) \tag{1.9}$$

In (1.9) ε_{r_n} is the relative dielectric constant of the n^{th} scatterer, $S_n(\vec{r}_o;\ \Omega_n)$ is the support of the n^{th} scatterer whose centroid is located by the position vector $\vec{r}_n$ so that

$$S_n(\vec{r}_o;\ \Omega_n) = \begin{array}{ll} 1 & \vec{r}_o \ \textit{inside}\ V_n \\ 0 & \vec{r}_o \ \textit{not inside}\ V_n \end{array} \tag{1.10}$$

and V_n is the volume of the n^{th} scatterer. The dyadic $\overline{\overline{\Gamma}}$ is given by

$$\overline{\overline{\Gamma}} = -P.V.\left[\overline{\overline{I}} + k_o^{-2}\nabla_o\nabla_o\right] g(\vec{r} - \vec{r}_n - \vec{r}_o) + \overline{\overline{I}}\delta(\vec{r} - \vec{r}_n - \vec{r}_o)/3k_o^2 \tag{1.11}$$

where P.V. denotes the principle value*, $\overline{\overline{I}}$ is the unit dyad, $\delta(\cdot)$ is the three-dimensional delta distribution, and

$$g(\vec{r} - \vec{r}_n - \vec{r}_o) = \exp(-jk_o|\vec{r} - \vec{r}_n - \vec{r}_o|)/4\pi|\vec{r} - \vec{r}_n - \vec{r}_o| \tag{1.12}$$

The variable Ω_n in the argument of the support function symbolizes the dependence on the orientation of the n^{th} scatterer. Equation (1.9) is general in that it allows scatterer-to-scatterer variation in the dielectric constant (ε_{r_n}), the size and shape (V_n), the orientation or alignment (Ω_n), and the location $(\vec{r}_n)$. It should be noted that calculation of the scattered field requires knowledge of the total field inside each scatterer. Thus, substituting (1.8) in (1.7) and then taking the point $\vec{r}$ inside each scatterer yields N coupled integral equations for the total field inside each of the N objects. Once these equations are solved, the scattered field at any point in space can be determined by straightforward integrations. For the random problem, a somewhat different methodology will be developed.

Substituting (1.8) into (1.7) yields

$$\vec{E}_t = \vec{E}_i + L\overline{\overline{K}}_\Sigma\vec{E}_t \tag{1.13}$$

The total field at any point in space can be written as the sum of a conditional average, $<\vec{E}_t]$, and a zero-mean fluctuating part, $\delta\vec{E}_t$, so that (1.13) becomes

$$<\vec{E}_t] + \delta\vec{E}_t = \vec{E}_i + L\overline{\overline{K}}_\Sigma\vec{E}_t \tag{1.14}$$

This decomposition on the left side is a bit different from the one usually used and results from the need to accommodate the following situation unique to discrete object scattering. The conditional average $<\vec{E}\ ]$ is an average of the total field with the point of observation of the field held constant, i.e. the point of observation is the conditioned variable. Under most circumstances, the point of observation is a constant and so the conditional average equals the total average. This would be the case on the left hand side of (1.14), e.g. $<\vec{E}_t] = <\vec{E}_t>$. Furthermore, the conditionally averaged total field is the one that *is usually called just the "mean" or "average" or "coherent" field.* However, it is the *right side* of (1.14) that gives rise to a need for the conditionally averaged total field. That is, the random quantities in the expression for $\overline{\overline{K}}_\Sigma$, (1.9), are usually the scatterers' positions $(\vec{r}_n)$ sizes, (V_n), and orientations

*As it appears in (1.11), the P.V. excludes a small *spherical* volume centered at $r - r_n - r_o = 0$.

(Ω_n) . The field quantity on the right hand side of (1.14) has an *explicit* dependence on the coordinates $\vec{r}_n + \vec{r}_o$; $\vec{r}_n$ locates the centroid of the n^{th} scatterer while $\vec{r}_o$ ranges over its support V_n. Thus, the scatterer dependent terms on the right side of (1.14) may be written as follows:

$$L\overline{\overline{K}}_\Sigma \vec{E}_t = L\overline{\overline{K}}_\Sigma (\vec{r} - \vec{r}_n - \vec{r}_o)\, \vec{E}_t (\vec{r}_n + \vec{r}_o) \tag{1.15}$$

When $\vec{E}_t(\vec{r}_n + \vec{r}_o)$ is decomposed into a sum of a conditional average and a zero mean fluctuating term, there results

$$\vec{E}_t(\vec{r}_n + \vec{r}_o) = \,<\vec{E}_t(\vec{r}_o + \vec{r}_n)] + \delta\vec{E}_t(\vec{r}_n + \vec{r}_o) \tag{1.16}$$

If both sides of (1.16) are averaged, it is done by first performing a conditional average over all random variables except $\vec{r}_n$. Such a conditional average of $\delta\vec{E}_t$ yields zero because this is the way $\delta\vec{E}_t$ was formed, i.e. $<\delta E_t] = 0$. The total average of $\vec{E}_t$ thus amounts to averaging $<\vec{E}_t(\vec{r}_n + \vec{r}_o)]$ over the random variable $\vec{r}_n$.

The point of the above discussion is to illustrate the need for using a conditional average rather than a total average in decomposing $\vec{E}_t$. If a total average had been used, there would have been no way of determining what to do about the $\vec{r}_n$ dependence on the right side of (1.14). When using a conditional average it becomes clear what to do both when the point of field observation is random and nonrandom. In the latter case, the conditional average is equal to the full average.

Substituting (1.16) in the rhs of (1.14) yields

$$<\vec{E}_t] + \delta\vec{E}_t = \vec{E}_i + L\overline{\overline{K}}_\Sigma <\vec{E}_t] + L\overline{\overline{K}}_\Sigma \delta\vec{E}_t \tag{1.17}$$

Averaging this equation gives

$$<\vec{E}_t] = \vec{E}_i + L<\overline{\overline{K}}_\Sigma <\vec{E}_t]> + L<\overline{\overline{K}}_\Sigma \delta\vec{E}_t> \tag{1.18}$$

and subtracting (1.18) from (1.17) leads to the following integral equation for $\delta\vec{E}_t$;

$$\delta\vec{E}_t = L(1-P)\overline{\overline{K}}_\Sigma <\vec{E}_t] + L(1-P)\overline{\overline{K}}_\Sigma \delta\vec{E}_t \tag{1.19}$$

where P is the averaging operator, i.e.

$$P\left[\overline{\overline{K}}_\Sigma <\vec{E}_t]\right] = \,<\overline{\overline{K}}_\Sigma <\vec{E}_t]> \tag{1.20}$$

$$P\left[\overline{\overline{K}}_\Sigma \delta\vec{E}_t\right] = \,<\overline{\overline{K}}_\Sigma \delta\vec{E}_t> \tag{1.21}$$

At this stage, (1.19) is an exact integral equation for the fluctuating component of the total field. However, the fluctuating part of the total field is equal to the fluctuating part of the scattered field. This can be proved by writing (1.7) as

$$<\vec{E}_t] + \delta\vec{E}_t = \vec{E}_i + <\vec{E}_s] + \delta\vec{E}_s \tag{1.22}$$

averaging this equation and subtracting the result from (1.22) to give

$$\delta\vec{E}_t = \delta\vec{E}_s \tag{1.23}$$

Thus, (1.19) becomes

$$\delta\vec{E}_s = L(1-P)\overline{\overline{K}}_\Sigma <\vec{E}_t] + L(1-P)\overline{\overline{K}}_\Sigma\delta\vec{E}_s \tag{1.24}$$

which is now an integral equation for the fluctuating part of the *scattered field.* The Born term in (1.24) depends on the average of $\vec{E}_t$.

The average or mean part of the Born term in (1.24) is thus given by

$$LP\overline{\overline{K}}_\Sigma <\vec{E}_t] = \int\overline{\overline{K}}_\Sigma(\vec{r} - \vec{r}_n - \vec{r}_o)p(\vec{r}_n) <\vec{E}_t(\vec{r}_n + \vec{r}_o)]d\vec{r}_n \tag{1.25}$$

$$n = 1, 2, \ldots, N$$

which is a well defined quantity.

If all of the scatterers are confined* to the volume V, equation (1.24) relates the fluctuating scattered field in *this volume* to the fluctuating scattered field at *any point in space.* If the point of observation of $\delta\vec{E}_s$ on the left-hand side of (1.24) is taken inside V then (1.24) becomes an integral equation for $\delta\vec{E}_s$ (inside V). Solving this integral equation *inside V* subsequently allows the determination of $\delta\vec{E}_s$ *outside V* through the use of (1.24). The Born term in (1.24) depends on the fluctuating part of the product of the propagator and the averaged total field. It is interesting to note that there are no average or effective medium quantities appearing in (1.24) other than $<\vec{E}_t]$. The Born term should be dominant whenever $<\vec{E}_t]$ does not depart too much from free space propagation. When $<\vec{E}_t]$ violates this condition, it then becomes necessary to account for the second term in (1.24). For foliage and vegetation in the frequency range of interest to this study, the Born term in (1.24) should be adequate for most all cases, i.e.

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*This is equivalent to saying that the position vector $\vec{r}_n$, $n = 1, 2, \ldots, N$ locating the centroid of the n^{th} scatterer must lie within the volume $V - \Delta_n$ where Δ_n depends on where the centroid of the n^{th} scatterer is located (within the scatterer).

$$\delta\vec{E}_s \approx L(1-P)\overline{\overline{K}}_{\Sigma} < \vec{E}_t] \tag{1.26}$$

Even though this result comes from the Born term of an exact integral equation, it can be thought of as a distorted wave Born approximation (DWBA) because of the assumption of the dominant nature of the coherent or mean field [16]. Clearly, the approximation of (1.26) merits further study primarily because it comes from an *exact integral equation.*

1.2.2.3 Comparison With Other DWBA

There are other forms of the DWBA that have been applied to foliage. Lang [17,13] has developed a heuristic form of the DWBA in which the scatterers are immersed in a medium having the same effective dielectric constant as the mean or average field, e.g.

$$< \varepsilon_r >_p = k_p^2 / k_o^2, \tag{1.27}$$

where k_p^2 is given by (1.2). The scatterers are then assumed to be illuminated by $< \vec{E}_t]$ and to scatter independently of each other. Thus, any multiple scattering effects included in this model are contained entirely in the use of (1.27) as the background medium and $< \vec{E}_t]$ as the incident field. If the analysis technique derived in the previous section is applied to this scattering situation, the resulting Born term corresponding to (1.26) is

$$\delta\vec{E}_s = L(1-P) \sum_{n=1}^{N} \overline{\overline{K}}_A^n < \vec{E}_t] \tag{1.28}$$

where

$$\overline{\overline{K}}_A^n = -k_p^2(\bar{\varepsilon}_{r_n} - 1) S_n(\vec{r}_o; \Omega_n) \overline{\overline{\Gamma}}_{k_p}(\vec{r} - \vec{r}_n - \vec{r}_o) \tag{1.29}$$

is the propagator for a scatterer of relative dielectric constant $\bar{\varepsilon}_{r_n}$ buried in a medium having a wavenumber equal to k_p. Thus, $\overline{\overline{K}}_A^n$ is the average propagator for the n^{th} scatterer.

In comparing the heuristic result of (1.29) with the exact Born term in (1.26), there is only one essential difference and that is the use of an *effective background medium* rather than free space. It may be possible that the heuristic approach comprises a partial summation of some of the contributions from the second term on the rhs of (1.24); however, this could be difficult to prove. On the other hand, the early and pioneering work of Twersky [4,5,7] appears to argue *against* the use of an effective medium propagator and in favor of the free space propagator. In fact, the use of an effective medium propagator seems closely akin to *double* accounting for the mean field effects. There was not enough time to resolve this discrepancy between the exact and heuristic approaches. However, it is recommended that this point be investigated further since the two results lead to different answers and the difference may be appreciable for relatively thick foliage layers. A potentially fruitful approach to reconciling the two results is to try some form of selective partial summation of the second term on the rhs of (1.24). That is, solving (1.24) via iteration yields

$$\delta\vec{E}_s = \sum_{m=0} \{L(1-P)\overline{\overline{K}}_{\Sigma}\}^{m}\Big[L(1-P)\overline{\overline{K}}_{\Sigma} < \vec{E}_i]\Big] \tag{1.30}$$

or

$$\delta\vec{E}_s = L(1-P)\overline{\overline{K}}_{\Sigma}\sum_{m=0} \{L(1-P)\overline{\overline{K}}_{\Sigma}\}^{m} < \vec{E}_i] \tag{1.31}$$

and it may be possible to manipulate this latter series in such a fashion as to derive an approximate Born term which uses $\overline{\overline{K}}{}^{\eta}_{A}$ from (1.29) rather than $\overline{\overline{K}}_{\Sigma}$.

1.3 Rough Surface Only - A Perspective

For the problem of wave propagation through and scattering by discrete random media, there are a rather limited number of analytical techniques for dealing with this problem [19]. Conversely, when analyzing scattering by randomly rough surfaces, there seem to be a *great deal* of diverse methods capable of producing results [20]. The key word here is "seem" because, in fact, many of the methods lead to essentially the same results; the major differences are with the *starting point* and the *degree of approximation* necessary to achieve the result. In view of this observation, it would be highly desirable if a *fundamental approach* could be found which, when subjected to a hierarchy of approximations, could be shown to lead to a similar hierarchy of approximate results. The primary advantage resulting from finding such a technique is that, as the level of the approximation is reduced, there is a guaranteed *increase* in the accuracy of the resulting solution. Consequently, there is a *clear reason* for pursuing improvements in this technique because they will result in an improved solution.

The technique that appears to hold the greatest potential for leading to a rigorous solution to the rough surface scattering problem is the Magnetic Field Internal Equation (MFIE) for the current induced on a perfectly conducting surface. The infinite conductivity limitation, for the frequencies of interest to this study, is significant only near grazing incidence. Appendix A considers the case when the surface or interface is a dielectric. Essentially what happens is that there is both an electric *and* a magnetic current induced on the surface. These currents satisfy *coupled* integral equations of the second kind; the kernels of the equations are more complicated than with the MFIE, but the basic equations are the same as the MFIE. These complications are not unexpected as it is known that the dielectric surface may give rise to effects which are not observed with the conducting surface. For example, a Brewster angle phenomenon with the coherent scattered field should be possible along with a reentry effect in which the incident field enters one side of a bump on the surface and exits on the other side. As noted above, the effects are only significant near grazing incidence and they are very dependent on the imaginary part of the dielectric constant of the surface. The main point of this discussion is that if techniques can be developed for the MFIE which are not strongly dependent on the particular form of the kernel then it should be possible to translate the bulk of the methodology to the dielectric surface integral equations. As noted above, this topic is discussed more fully in Appendix A.

Our analysis of the MFIE as it applies to rough surface scattering is a continuing effort. For example, Section 2.1 presents a technique which extends the geometrical optics solution of the MFIE into the nonzero wavelength regime. However, it seems appropriate at this point to provide some perspective to previous analytical solutions of the MFIE; *that is, to show where they come from and how they fit in a well-ordered hierarchy of approximations.* As noted above, this is essential if we are to establish the fact that the MFIE leads to successively more accurate solutions as the input approximations are similarly improved.

The form of the MFIE that is most familiar is the following:

$$\vec{J}_s(\vec{r}) = 2\vec{n}(\vec{r}) \times \vec{H}^i(\vec{r}) + 2\vec{n}(\vec{r}) \times \int_{s_o} \vec{J}_s(\vec{r}_o) \times \nabla_o G(|\vec{r} - \vec{r}_o|) ds_o \tag{1.32}$$

where $\vec{J}_s$ is the surface current density, $\vec{n}$ is the unit normal to the surface, $\vec{H}^i$ is the incident magnetic field, and G is the free space Green's function

$$G(|\vec{r} - \vec{r}_o|) = \exp(-jk_o|\vec{r} - \vec{r}_o|)/4\pi|\vec{r} - \vec{r}_o| \tag{1.33}$$

Equation (1.32) can be rewritten in the following form

$$\vec{J}(\vec{r}) = 2\vec{N}(\vec{r}) \times \vec{H}^i(\vec{r}) + \int \left[-\vec{N}(\vec{r}) \times \nabla_o G \times \vec{J}(\vec{r}_o) \right] d\vec{r}_{t_o} \tag{1.34}$$

where

$$\vec{J} = J_s/(1 + \zeta_x^2 + \zeta_y^2)^{1/2}$$

$$\vec{N} = -\zeta_x \hat{x} - \zeta_y \hat{y} + \hat{z}$$

$$ds_o = d\vec{r}_{t_o}(1 + \zeta_z^2 + \zeta_y^2)^{1/2}$$

and $d\vec{r}_{t_o} = dx_o\, dy_o$. The $\zeta_x = \partial\zeta/\partial x$ and $\zeta_y = \partial\zeta/\partial y$ are the x and y components of the surface slope. The right hand side can be further manipulated by standard vector identities to yield

$$\vec{J}(\vec{r}) = 2\vec{N} \times \vec{H}^i + \int \{-2\nabla_o G[\vec{N}(\vec{r}) \cdot + 2[\vec{N}(\vec{r}) \cdot \nabla_o G]\} \vec{J}(\vec{r}_o) d\vec{r}_{t_o} \tag{1.35}$$

where

$$\nabla_o G[\vec{N}(\vec{r}) \cdot \}\vec{J} = \nabla_o G[\vec{N}(\vec{r}) \cdot \vec{J}(\vec{r}_o)] \tag{1.36}$$

In operator notation (1.35) can be written as follows:

$$\vec{J} = \vec{J}_i + L\overline{\overline{K}} \cdot \vec{J} \tag{1.37}$$

where

$$\vec{J}_i = 2\vec{N} \times \vec{H}^i \tag{1.38a}$$

$$L = \int (\quad) d\vec{r}_{t_o} \tag{1.38b}$$

$$\overline{\overline{K}} = \{-2\nabla_o G\, [\vec{N}(\vec{r}) \cdot + 2[\vec{N}(\vec{r}) \cdot \nabla_o G]\} \tag{1.38c}$$

Integral equations of the second kind such as (1.37) can be formally solved via iteration to yield

$$\vec{J} = \sum_{n=0} (L\overline{\overline{K}} \cdot)^n \vec{J}_i \tag{1.39}$$

This series can be thought of as a series of iterates, i.e.

$$\vec{J} = \sum_{n=0} \vec{J}^{(n)} \tag{1.40}$$

where

$$\vec{J}^{(m)} = (L\overline{\overline{K}} \cdot)^m \vec{J}_i \tag{1.41}$$

and the zeroth order iterate $\vec{J}^{(0)} = \vec{J}_i$ is called the Born term. The scattered electric field resulting from the current $\vec{J}_s$ is given, in the far field approximation, as follows:

$$\vec{E}_s(\vec{R}_o) = jk_o\eta_o G(R_o)\hat{k}_s \times \hat{k}_s \times \int \vec{J}_s(\vec{r}) \exp(j\vec{k}_s \cdot \vec{r}) ds \tag{1.42}$$

where the vector $\vec{R}_o$ points from the origin of the coordinate system on the mean surface in the direction $\hat{k}_s$ to a distance R_o away. Also, the vector $\vec{k}_s$ is defined by $\vec{k}_s = k_o\hat{k}_s$. The integral in (1.42) can be written as

$$\vec{E}_s(\vec{R}_o) = jk_o\eta_o G(R_o)\hat{k}_s \times \hat{k}_s \times \int \vec{J}(r) \exp(jk_{s_z}\zeta) \exp(j\vec{k}_{s_t} \cdot \vec{r}_t)d\vec{r}_t \qquad (1.43)$$

where η_o is the impedance of free space and

$$\vec{k}_s = \vec{k}_{s_t} + k_{s_z}\hat{z}$$

The integral in (1.43) is recognized to be the 2-dimensional Fourier transform of the current $\vec{J}$ weighted by the exponential factor $\exp(jk_{s_z}\zeta)$, i.e.

$$\vec{E}_s(\vec{R}_o) = jk_o\eta_o G(R_o)\hat{k}_s \times \hat{k}_s \times F_2[\vec{J} \exp(jk_{s_z}\zeta)]$$

where

$$F_2[\,\cdot\,] = \int \cdot \exp(j\vec{k}_{s_t} \cdot \vec{r}_t)d\vec{r}_t.$$

Expanding the double curl operation leads to

$$\vec{E}_s(\vec{R}_o) = jk_o\eta_o G(R_o)F_2[\vec{J}_\perp \exp(jk_{s_z}\zeta)]$$

where

$$\vec{J}_\perp = \vec{J} - \hat{k}_s(\hat{k}_s \cdot \vec{J})$$

is the component of $\vec{J}$ which is perpendicular to $\hat{k}_s$. Assuming that the iterative series in (1.40) converges so that the series can be integrated term by term yields

$$\vec{E}_s(\vec{R}_o) = jk_o\eta_o G(R_o) \sum_{m=0} \vec{E}^{(m)} \qquad (1.44)$$

where

$$\vec{E}^{(m)} = F_2\left[\vec{J}_\perp^{(m)} \exp(jk_{s_z}\zeta)\right] \qquad (1.45)$$

The above development shows that there are *two* important elements to the determination of the scattered field. First, the current iterates must be computed from (1.41) and, second, the corresponding scattered field iterates must be calculated from (1.45). Of course, what *must* be done and what *is actually* done are two entirely different situations and this difference leads to various degrees of approximation. In the following material, an ordering of most of the classical approximations will be developed based upon their effect on the current and scattered field iterates. The purpose of this ordering is to put these classical approximations into a perspective which clearly indicates where we have been and were we need to proceed.

The first level of approximation is low frequency in nature. It starts with $\vec{J}^{(0)}\ \exp(jk_{s_z}\zeta)$ as an approximate form of the current and further simplifies this to

$$\vec{J}^{(0)}\ \exp(jk_{s_z}\zeta) \approx 2H_o\hat{z}\times\hat{h}_i[1 + j(k_{s_z} - k_{i_z})\zeta]\ \exp\,(-j\vec{k}_{i_t}\cdot\vec{r}_t) \tag{1.46}$$

where the slopes in $\vec{J}^{(0)}$ have been ignored and the surface roughness is assumed to be so small that

$$\exp[j(k_{s_z} - k_{i_z})\zeta] \approx 1 + j(k_{s_z} - k_{i_z})\zeta \tag{1.47}$$

The other factors in (1.46) come from the form of the incident magnetic field, i.e.

$$\vec{H}^i = H_o\hat{h}_i\ \exp\,(-jk_{i_z}\zeta - j\vec{k}_{i_t}\cdot\vec{r}_t)$$

Integrating (1.46) exactly to form $\vec{E}_s^{(0)}$ leads to a *scalar Bragg* result in the backscatter direction for the incoherent scattered power [21]. This result follows from the fact that $\vec{J}_{\perp}^{(0)}$ in the backscatter direction is polarized in the same direction as the incident electric field, and that it depends on ζ in a linear fashion.

The next level of approximation is to use an exact form for $\vec{J}^{(0)}$ but evaluate $\vec{E}_s^{(0)}$ by stationary phase techniques so that the result is a high frequency limiting form. This solution is frequently called *specular point* scattering [22]. It should be noted that whereas the scalar Bragg solution results from an exact integration of an approximate form for $\vec{J}^{(0)}$, the specular point result comes from an approximate integration of the exact $\vec{J}^{(0)}$. The specular point result assumes that $(k_{s_z} - k_{i_z})\zeta >> 1$, the surface slopes are relatively small, and the surface curvatures are also small (large radii of curvature). The first approximation permits the use of stationary phase; the second eliminates multiple scattering on the surface; the last avoids sharp edge diffraction effects.

A final approximation involving only $\vec{J}^{(0)}$ applies to composite surfaces. Such surfaces have a range of scales which satisfy both the large scale approximation common to specular point scattering *and* the small scale approximation to the scalar Bragg result [23]. If the surface can be approximated by a composite surface, then the integration to give $\vec{E}_s^{(0)}$ can be accomplished essentially exactly. This leads to the composite surface scattering result in which the specular point term dominates around the specular direction and the scalar Bragg term is important at other angles. In addition, the scalar Bragg result is tilted by the large scale surface slopes. It is important to remember however that the scalar Bragg result shows *no polarization sensitivity*.

A summary of what the zeroth order iteration of the MFIE leads to under certain approximations is given in Figure 1.1. It should be noted that there is no discussion of *when* $\vec{J}^{(0)}$ is a valid approximation for the total current. This question can be answered only by using improved estimates of the current to compute the scattered field. What is done above and summarized in Figure 1.1 is to show the consequences of (a) approximating $\vec{J}_s^{(0)}$ and doing the integration to get $\vec{E}_s^{(0)}$ exactly, (b) reversing this situation, and (c) using a combination of these manipulations.

The exact same procedure may be repeated when the next iteration of the current is included. That is, the current $\vec{J}$ is next approximated by

$$\vec{J} = \vec{J}^{(0)} + \vec{J}^{(1)} \tag{1.48}$$

where

$$\vec{J}^{(0)} = \vec{J}_i$$

and

$$\vec{J}^{(1)} = L\overline{\overline{K}} \cdot \vec{J}_i$$

It has recently been shown [24] that a low frequency approximation of $\vec{J}^{(0)} + \vec{J}^{(1)}$ along with the assumption of small surface slopes leads to a result which when integrated exactly to yield $\vec{E}_s^{(0)} + \vec{E}_s^{(1)}$ gives rise to the *vector Bragg* result. The vector Bragg result differs from the scalar solution obtained with $\vec{J}^{(0)}$ only in that for backscatter, the vector solution shows a polarization sensitivity whereas the scalar does not. This illustrates the inadequacy of the $\vec{J}^{(0)}$ iterate (or Born term) in so far as the scattered field in the off-specular direction is concerned.

The next level of approximation involves evaluating *all* integrations in the high frequency limit. Thus, when $L\overline{\overline{K}} \cdot \vec{J}_i$ is so evaluated it leads to $-\vec{J}_i$ in the shadowed parts of the surface, 0 on the illuminated parts of the surface, and the possibility of first order multiple scattering [25]. That is, first order multiple scattering means the reflection of an incidence ray from one point of the surface to another point. Generally, this first order ray optic multiple scattering is ignored because it makes the scattered field integration very hard to do. That current which remains is the shadowed current, i.e. $\vec{J}^{(0)} + \vec{J}^{(1)}$. If the integrations needed to compute $\vec{E}_s^{(0)} + \vec{E}_s^{(1)}$ are evaluated asymptotically (as $k_0 \to \infty$), what results is the shadowed specular point solution. Comparing this solution with the corresponding limiting behavior of $E_s^{(0)}$ alone shows that the inclusion of $\vec{J}^{(1)}$ in this limit gives rise to shadowing.

Finally, if the surfaces under study can be split into both large scale structure (ζ_ℓ) and small scale undulations (ζ_s) so that high frequency techniques can be used to predict the scattering from the large structure and low frequency methods can be used on the small scale structure then the surface can be treated as a composite structure, i.e. $\zeta = \zeta_\ell + \zeta_s$. One can subsequently show, though not easily [26], that if those parts of $\vec{J}^{(0)} + \vec{J}^{(1)}$ which depend upon ζ_ℓ are treated by high frequency asymptotics and those parts which depend on ζ_s are treated by low frequency approximations then the result is shadowed specular point scattering plus tilted vector Bragg diffraction. The tilting of the Bragg scattering is by the large scale surface slopes. It is interesting to note that the sum of *only* $\vec{J}^{(0)}$ *and* $\vec{J}^{(1)}$ lead to one of the best scattering models around, namely, the composite scattering model.

The results of including *both* $\vec{J}^{(0)}$ *and* $\vec{J}^{(1)}$ are summarized in Figure 1.2. In all categories there is a marked improvement over using $\vec{J}^{(0)}$ only. However, this is particularly so for the low frequency results where there is a polarization sensitivity not accounted for with $\vec{J}^{(0)}$ only.

The purpose of this section has been to put the various levels of rough surface scattering approximations into perspective. This has been done by relating the approximations to either the first two iterate solutions of the MFIE or the subsequent scattering integral calculations. It has been shown how this methodology can lead very naturally to a hierarchy of approximations. Most important of all, however, is the fact that the MFIE can generate improved solutions by simply increasing the order of the iteration. This establishes the MFIE as a useful means for studying rough surface scattering.

Finally, all of the above approximate solutions are available for the surface scattering part of the vegetation layer problem. Which one should be used is dictated by the frequency of interest and the surface roughness statistics.

1.4 Vegetation on a Rough Surface

Section 1.2 and 1.3 detail the techniques that can be used to estimate the scattering from an isolated patch of vegetation and bare ground, respectively. This section will develop and discuss a model for scattering from the combination of a layer of vegetation on a rough surface.

Some of the earliest work on this problem proposed using the simple addition of the scattering cross sections of the vegetation layer and the rough surface. However, it was quickly realized that some accounting for the influence of the vegetation layer on the rough surface scattering cross section was essential. Subsequent attempts to account for the vegetation simply attenuated the surface cross section by the loss suffered by the *coherent* power propagating down through the foliage and then back up to the foliage air interface. Use of this approximation led to some qualitative agreement with scattering measurements, but the model

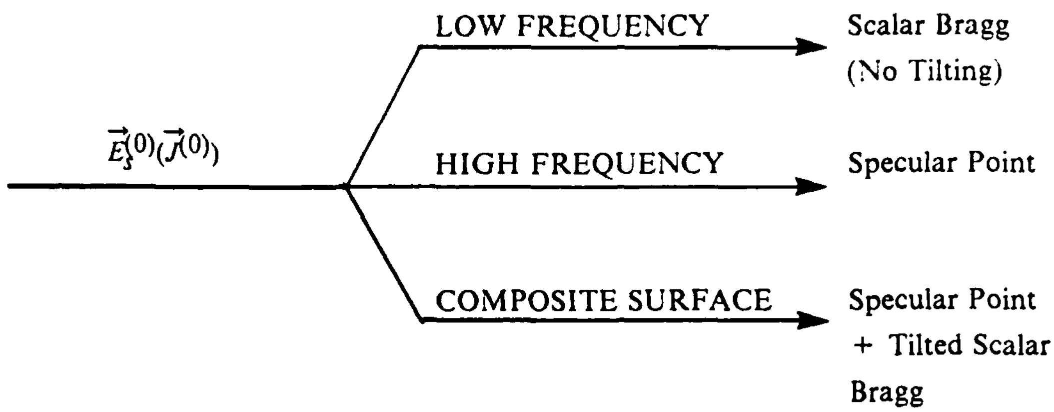

Figure 1.1 The description of the scattered field resulting from the zeroth order iteration of the MFIE current, $\vec{J}^{(0)}$, coupled with various approximations.

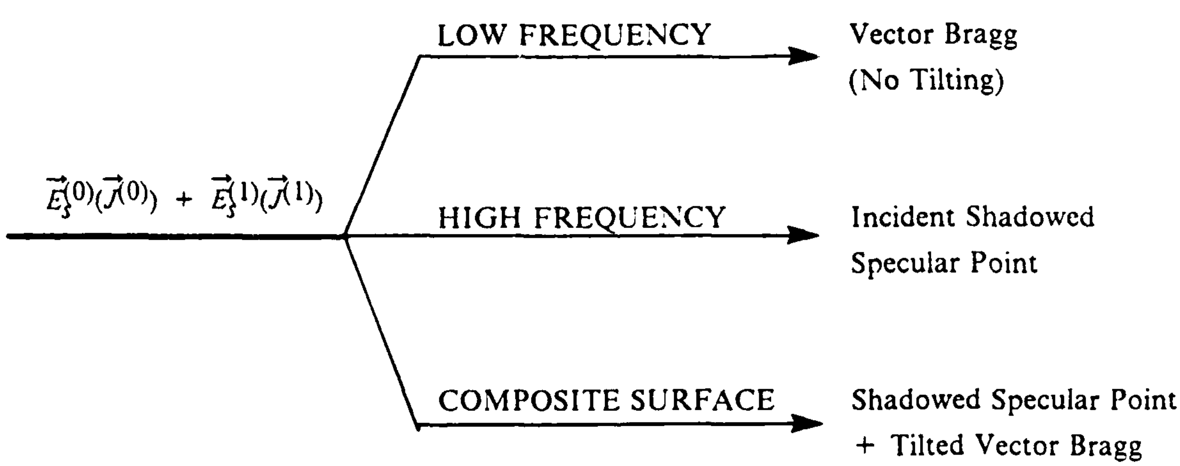

Figure 1.2. The description of the scattered field resulting from the zeroth and first order iteration of the MFIE current, $\vec{J}^{(0)} + \vec{J}^{(1)}$, coupled with various approximations.

also raised questions as to why it worked as well as it appeared to and when it could be expected to fail. Very little of this early work was ever published and most of this information was presented during informal meetings and discussions.

More recently, attempts were made to rigorously model certain subsets of this general problem (see [13] for a comprehensive list of relevant references). Unfortunately, most of these attempts chose to model the foliage as a continuous variation in dielectric constant and to thus use continuous random media theories as the analytical tools. While this approach did lead to some insight into the scattering problem, it also raised questions because of the need for "effective" or "equivalent" parameters in the continuous representation of the discrete random medium. Lang and Sidhu [13] overcame the limitations of these earlier models by using the Foldy-Twersky theory along with a version of the DWBA to model the scattering from a foliage layer on a flat earth. This was a significant advance because it clearly showed the interaction between the foliage components and the reflection from the flat surface. Unfortunately, it is not obvious how Lang and Sidhu's method of analysis could be extended to an arbitrarily roughened surface.

The model that will be developed here is very simple in concept but, as one might expect, complicated in detail. First, the field incident on the foliage is converted, inside the foliage, to a mean or average field and a zero mean fluctuating field. The mean field is determined by the Foldy-Twersky theory of Section 1.2.1 while the fluctuating field is based on the distorted wave Born approximation of Section 1.2.2.1. These two fields then act as incident fields on the rough surface which is approximated as perfectly conducting. The electric currents excited by these fields are computed using the Magnetic Field Integral Equation (MFIE) as discussed in Section 1.3. The next step is to let these currents radiate back up through the foliage. The fields radiated by the surface currents are then acted on by the foliage to give rise to another set of mean and fluctuating fields in the foliage [27]. These fields can be continued into free space via standard techniques such as volume and surface distributions of current or plane wave spectral methods. If the process is truncated at this point then what has been accounted for is one single downward and upward passage of the fields. If the process were to be continued, the interaction of the upward going fields with the foliage should be allowed to interact with the rough surface which, in turn, would scatter back up through the foliage. If the problem were to be analyzed exactly, this process should be continued an infinite number of times. However, there are very practical reasons for not continuing this process beyond the first downward and then upward pass. The most obvious of these is the computational complexity associated with each down-and-up field iteration. Furthermore, each iteration will also generate the need for *higher order* (multipoint) vegetation and surface statistics and these are simply not known. Finally, if there is an indication that more down-and-up field iterations are needed then perhaps there are simpler techniques available to deal with the entire problem. The single down-and-up field technique assumes that either the foliage *or* the surface is the dominant scatterer and that interactions between the two natural scatterers is small relative to these two dominant effects. To a certain extent, this approximation can be checked and this will be discussed later in the section.

The foliage and the rough ground represent statistically different random processes. If there were some degree of correlation between the two, it would be necessary to use conditional averaging in forming the first two moments of the scattered field. That is, the foliage would be averaged holding the surface fixed and then the surface would be averaged. This process requires the use of conditional probability density functions. However, there is no reason to expect to a first order at least that there should be any correlation between the foliage and the surface. In fact we will go further and assume that the two processes are statistically independent; this simplifies the actual mathematical operations because the joint density function is just a product of marginal densities. Thus, the averaging over either the foliage or the surface can be done independent of the remaining random process. However, it should be remembered that if there is some correlation between the foliage and the surface then it can be accommodated in the model through the use of conditional probability density functions. This generality may be useful in dealing with very high resolution scattering because such a situation may emphasize the slight correlations between certain classes of foliage and terrain.

The terrain is assumed to be a perfectly conducting, randomly rough interface, having a zero mean about the $z=0$ plane and homogeneous but arbitrary statistics. The foliage or vegetation occupies the space immediately above the surface and up to an average height of $z=h$. It should be remembered, however, that this average height of the foliage is a description which must be somewhat carefully interpreted. For example, any coherent scattering which results from this average planar height is clearly open to question since the foliage interface, for coherent calculations, should be modeled as a *rough* interface. This roughness will certainly give rise to a very highly attenuated coherent field which can thus be ignored. This is not to say that there will never be a coherent field scattered from a discrete collection of scatterers; there clearly will be such a field when the volume fraction of objects approaches unity. However, for a *sparsely populated* medium, the existence of a coherent scattered field would appear to be a model anomaly rather than a physical reality.

The field incident on the foliage layer is assumed to have the following plane wave form:

$$\vec{E}_i = [E_v\hat{e}_v + E_h\hat{e}_h] \exp(-j\vec{k}_i \cdot \vec{r}) \tag{1.49}$$

where

$$\vec{k}_i = k_{i_x}\hat{x} + k_{i_y}\hat{y} + k_{i_z}\hat{z} \tag{1.50}$$

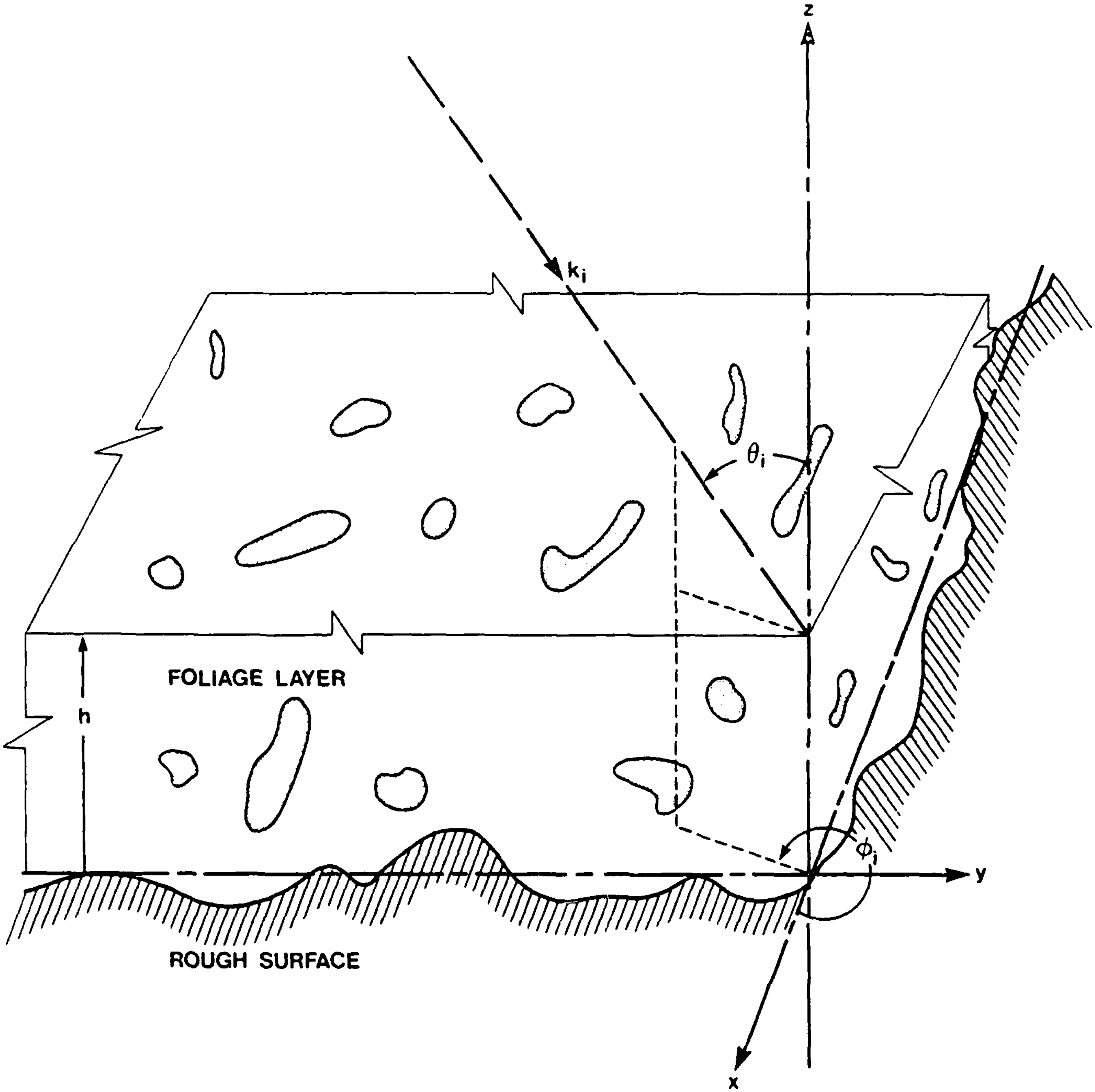

Figure 1.3. Geometry for the foliage layer on the rough surface.

and

$$k_{i_x} = -k_o \sin\theta_i \cos\phi_i \tag{1.51a}$$

$$k_{i_y} = -k_o \sin\theta_i \sin\phi_i \tag{1.51b}$$

$$k_{i_z} = -k_o \cos\theta_i \tag{1.51c}$$

The unit vectors $\hat{e}_v$ *and* $\hat{e}_h$ denote vertical and horizontal polarization directions;

$$\hat{e}_h = -\sin\phi_i\hat{x} + \cos\phi_i\hat{y} \tag{1.52a}$$

and

$$\hat{e}_v = -\cos\theta_i\cos\phi_i\hat{x} - \cos\theta_i\sin\phi_i\hat{y} + \sin\theta_i\hat{z} \tag{1.52b}$$

The angles are standard polar (θ_i) and azimuthal (ϕ_i) angles and they are explicitly defined in Figure 1.3. As discussed above, we ignore the possibility of any coherent reflection from the foliage layer. Thus, according to Section 1.2, the incident field is converted to the sum of a coherent field, $<\vec{E}_{fd}>$, and a fluctuating field, $\delta\vec{E}_{fd}$, inside the foliage. The average field in the foliage is given by

$$<\vec{E}_{fd}> = E_v\hat{e}_v \exp(-j\vec{k}_{i_v}\cdot\vec{r}) + E_h\hat{e}_h \exp(-j\vec{k}_{i_h}\cdot\vec{r}) \tag{1.53}$$

where the directions of $\vec{k}_{i_v}$ *and* $\vec{k}_{i_h}$ are the same as $\vec{k}_i$, i.e.

$$\hat{k}_{i_v} = \vec{k}_i/k_o \qquad \hat{k}_{i_h} = \vec{k}_i/k_o$$

but the complex amplitudes are solutions of (1.2) with p = v or h, e.g.

$$k_{\substack{v\\h}}^2 = k_o^2 + 4\pi\int_0^\infty <f_{\substack{v\\h}}(\hat{k}_i,\hat{k}_i)]_m n_m(\alpha)d\alpha \tag{1.54}$$

and $<f_{\substack{v\\h}}(\hat{k}_i,\hat{k}_i)]_m$ is the scattering amplitude of the m^{th} component of the foliage *averaged* over all possible orientation angles. As discussed in Section (1.2), the integration in (1.54) accounts for a size distribution of the m^{th} foliage components. The fact that the average field may have different wavenumbers or propagation constants for horizontal (h) and vertical (v) polarizations has been observed in some measurements [1]; however, the effect seems to disappear above about 800 MHz.

The fluctuating field $\delta\vec{E}_{fd}$ is a solution of the integral equation in (1.19); however, based on the discussion presented in Section 1.2.2 it should be possible to use only the Born term in (1.19) and this is equivalent to the distorted wave Born approximation (DWBA), i.e.

$$\delta\vec{E}_{fd} \approx L(1-P)\overline{\overline{K}}_{\Sigma} < \vec{E}_{fd}] \tag{1.55}$$

where L is the integral over all space, P is the averaging operator, $\overline{\overline{K}}_{\Sigma}$ is given by (1.9), and $< \vec{E}_{fd}]$ is essentially the average field evaluated at $\vec{r}_o$. The reader is reminded that a complete discussion of the terms in (1.55) and the approximations under which it is valid are given in Section 1.2.2. It should be emphasized that all of the quantities are known and so (1.55) is computable. There are two additional calculations that need to be done with the fluctuating field in (1.55). First, it is a primary source of incoherent power scattered back up into free space, i.e. $< |\delta\vec{E}_{fd}|^2 >$. In fact, in the absence of the terrain surface or for a very thick vegetation layer this is the *only* source of scattered power. The second calculation recognizes that $\delta\vec{E}_{fd}$ *and* $< \vec{E}_{fd}]$ are both incident on the terrain surface and they induce currents on the surface which reradiate back into the vegetative medium.

Within the limitation discussed in Section 1.3, the terrain will be approximated as a perfectly conducting interface. Consequently, the Magnetic Field Integral Equation (MFIE) describes how the induced current, $\vec{J}$, behaves on the surface, i.e.

$$\vec{J} = 2\vec{N} \times \vec{H}_{fd} + 2\vec{N} \times \int \vec{J} \times \nabla_o G d\vec{r}_{t_o} \tag{1.56}$$

The incident magnetic field $\vec{H}_f$ may be written as

$$\vec{H}_{fd} = < \vec{H}_{fd}] + \delta\vec{H}_{fd} \tag{1.57}$$

and

$$< \vec{H}_{fd}] = -\frac{1}{j\omega\mu_o}\nabla \times < \vec{E}_{fd}] \tag{1.58a}$$

$$\delta\vec{H}_{fd} = -\frac{1}{j\omega\mu_o}\nabla \times \delta\vec{E}_{fd} \tag{1.58b}$$

follow from Maxwell's equations. Since $< \vec{E}_{fd}]$ is a plane wave traveling in the $\hat{k}_i$-direction through a medium having an average relative dielectric constant $< \varepsilon_r >_p = k_p^2/k_o^2$ (p = v or h), (1.58a) becomes

$$< \vec{H}_{fd}] = \frac{(< \varepsilon_r >_p)^{1/2}}{\eta_o}\hat{k}_i \times < \vec{E}_{fd}] \tag{1.59}$$

where η_o is the impedance of free space $(\sqrt{\mu_o/\varepsilon_o})$. The choice of vertical or horizontal polarization in (1.59) determines both the polarization of $<\vec{E}_{fd}]$ *and* $<\varepsilon_r>_p$. If near zone and Fresnel zone fields are ignored in the calculation of $\delta\vec{E}_{fd}$ and, subsequently, $\delta\vec{H}_{fd}$ then (1.58b) can be written as follows;

$$\delta\vec{H}_{fd} = \frac{1}{\eta_o}\hat{k} \times \delta\vec{E}_{fd} \tag{1.60}$$

where $\hat{k}$ specifies the direction of propogation of $\delta\vec{E}_{fd}$. It is not necessary that $\hat{k}$ point in only one direction as, for example it would in the case of a plane wave. The average medium relative dielectric constant does not appear in (1.60) because according to the DWBA used in (1.55), the fluctuating field *propagates in free space*, this point was discussed in detail in Section 1.2.2.3. It should be noted that all the quantities in the Born term in (1.56) are known.

Substituting (1.57) into (1.56) yields

$$\vec{J} = 2\vec{N} \times <\vec{H}_{fd}] + 2\vec{N} \times \delta\vec{H}_{fd} + 2N \times \int \vec{J} \times \nabla_o G d\vec{r}_{t_o} \tag{1.61}$$

The total current $\vec{J}$ is split as follows;

$$\vec{J} = \vec{J}_{af} + \vec{J}_{\delta f} \tag{1.62}$$

where

$$\vec{J}_{af} = 2\vec{N} \times <\vec{H}_{fd}] + 2N \times \int \vec{J}_{af} \times \nabla_o G d\vec{r}_{t_o} \tag{1.63}$$

and

$$\vec{J}_{\delta f} = 2\vec{N} \times \delta\vec{H}_{fd} + 2N \times \int \vec{J}_{\delta f} \times \nabla_o G d\vec{r}_{t_o} \tag{1.64}$$

From (1.63), $\vec{J}_{af}$ is the current induced on the surface by the downward propagating average foliage field $<H_{fd}]$ while, from (1.64), $\vec{J}_{\delta f}$ is the current induced on the surface by the downward traveling fluctuating foliage field $\delta\vec{H}_{fd}$. These equations can be solved by iteration, such as discussed in Section 1.3, to yield

$$\vec{J}_{af} = \sum_{m=0} (L\overline{\overline{M}})^m (2\vec{N} \times <\vec{H}_{fd}]) \tag{1.65}$$

$$\vec{J}_{\delta f} = \sum_{m=0} (L\overline{\overline{M}})^m (2\vec{N} \times \delta\vec{H}_{fd}) \tag{1.66}$$

where L is two-dimensional integral over the z=0 plane and

$$\overline{\overline{M}} = -2\nabla_o G(\vec{r} - \vec{r}_o)\{\vec{N}(\vec{r}) \cdot + 2[\vec{N}(\vec{r}) \cdot \nabla_o G(\vec{r} - \vec{r}_o)]\} \quad (1.67)$$

The quantities in (1.67) are defined in Section 1.3.

The currents in (1.65) and (1.66) are now taken to reradiate upwards into free space. The calculation of the resulting fields can be accomplished via either the exact formula,

$$\vec{E}_{su} = -j\frac{\eta_o}{k_o}\nabla\times\nabla\times\int\vec{J}(\vec{r}_o)G(R-\vec{r}_o)d\vec{r}_{t_o}, \quad (1.68)$$

or the far field approximation,

$$\vec{E}_{su} \approx jk_o\eta_o G(R)\hat{k}_s\times\hat{k}_s\times\int\vec{J}(\vec{r}_o)\ \exp\ (jk_o\hat{k}_s\cdot\vec{r}_o)d\vec{r}_{t_o}, \quad (1.69)$$

or a plane wave spectral representation developed in Appendix B. The easiest of these to deal with is the far-field approximation of (1.69) but, of course, this does not make it correct. However, there do not appear to be any near-field effects which would be augmented or magnified by the foliage layer so the far-field approximation will be used. The total field scattered by the surface and up into free space may be written as

$$\vec{E}_{su} = \vec{E}_{asu} + \vec{E}_{\delta su} \quad (1.70)$$

where

$$\vec{E}_{asu} = jk_o\eta_o G(R)\hat{k}_s\times\hat{k}_s\times\int\vec{J}_{af}(\vec{r}_o)\ \exp\ (jk_o\hat{k}_s\cdot\vec{r}_o)d\vec{r}_{t_o} \quad (1.71)$$

and

$$\vec{E}_{\delta su} = jk_o\eta_o G(R)\hat{k}_s\times\hat{k}_s\times\int\vec{J}_{\delta f}(\vec{r}_o)\ \exp\ (jk_o\hat{k}_s\cdot\vec{r}_o)d\vec{r}_{t_o} \quad (1.72)$$

The quantity $\vec{E}_{asu}$ is the electric field scattered up into free space by the rough surface when illuminated by the *average foliage field* $<\vec{H}_{fd}]$. $\vec{E}_{\delta su}$ is the electric field scattered up into free space by the rough surface when illuminated by the *fluctuating foliage field* $\delta\vec{H}_{fd}$.

The next step in the process is to let the fields in (1.71) and (1.72) be incident upon the foliage layer *from* below. The reason for doing this is that this problem can be treated just like the first part, i.e. the downward passage of the free space incident field through the foliage. That is, the total field in the foliage due to $\vec{E}_{asu}$ *and* $\vec{E}_{\delta su}$ *incident from below* can be written as follows:

$$\vec{E}_{sf} = (\vec{E}_{asu} + \vec{E}_{\delta su}) + L\bar{\bar{K}}_{\Sigma}\vec{E}_{sf} \tag{1.73}$$

This equation is the same as (1.13) but with the incident field replaced by $(\vec{E}_{asu} + \vec{E}_{\delta su})$. We now develop the standard method of smoothing approach as a means to find the average and fluctuating upward going fields in the foliage. All averages are over the random quantities in the foliage and *not* the surface; averaging over an ensemble of surfaces will be delayed until later in the analysis. First, $\vec{E}_{sf}$ in (1.73) is rewritten in terms of its average and fluctuating parts, i.e.

$$< \vec{E}_{sf}] + \delta\vec{E}_{sf} = \vec{E}_{asu} + \vec{E}_{\delta su} + L\bar{\bar{K}}_{\Sigma} < \vec{E}_{sf}] + L\bar{\bar{K}}_{\Sigma}\delta\vec{E}_{sf} \tag{1.74}$$

Averaging this equation yields

$$< \vec{E}_{sf}] = \vec{E}_{asu} + LP\bar{\bar{K}}_{\Sigma} < \vec{E}_{sf}] + LP\bar{\bar{K}}_{\Sigma}\delta\vec{E}_{sf} \tag{1.75}$$

and subtracting (1.75) from (1.74) gives

$$\delta\vec{E}_{sf} = \vec{E}_{\delta su} + L(1-P)\bar{\bar{K}}_{\Sigma} < \vec{E}_{sf}] + L(1-P)\bar{\bar{K}}_{\Sigma}\delta\vec{E}_{sf} \tag{1.76}$$

Assuming for the moment that $< \vec{E}_{sf}]$ is known allows the integral equation in (1.76) to be solved by iteration to yield

$$\delta\vec{E}_{sf} = \vec{E}_{\delta su} + \sum_{n=1} \{L(1-P)\bar{\bar{K}}_{\Sigma}\}^{n}[< \vec{E}_{sf}] + \vec{E}_{\delta su}] \tag{1.77}$$

Comparing the integral equation in (1.76) with the corresponding equation for the first downward pass through the foliage, i.e. eqn. (1.24), it is noted that (1.76) has the additional term $\vec{E}_{\delta su}$. This, of course, is the field scattered upward by the surface when illuminated by the downward propagating fluctuating field. Thus, to the first pass scattered fluctuating field given by (1.55) must be added the scattering of $\delta\vec{E}_{fd}$ from the rough surface, i.e. $\vec{E}_{\delta su}$. It should be noted that both $\delta\vec{E}_{fd}$ *and* $\vec{E}_{\delta su}$ propagate in free space and not in the average medium.

In order to find the *total* fluctuating field due to the surface scattered fields, $< \vec{E}_{sf}]$ must be determined. To do this, (1.77) is substituted in (1.75) for $\delta\vec{E}_{sf}$ and, after regrouping terms, gives

$$< \vec{E}_{sf}] = \vec{E}_{asu} + LP\bar{\bar{K}}_{\Sigma}\sum_{n=0}\{L(1-P)\bar{\bar{K}}_{\Sigma}\}^{n}\vec{E}_{\delta su} + LP\bar{\bar{K}}_{\Sigma}\sum_{n=0}\{L(1-P)\bar{\bar{K}}_{\Sigma}\}^{n} < \vec{E}_{sf}] \tag{1.78}$$

which is the desired integral equation for $< \vec{E}_{sf}]$. In order to understand the meaning and implications of this equation, it is beneficial to write the corresponding equation for the mean field propagating *downward* through the foliage. This topic was covered in Section 1.2.1, but the integral equation for the mean field was not given. It is as follows:

$$< \vec{E_f}] = \vec{E_i} + LP\overline{\overline{K}}_\Sigma \sum_{n=0} \{L(1-P)\overline{\overline{K}}_\Sigma\}^n < \vec{E_f}] \tag{1.79}$$

where $\vec{E_i}$ is the field incident in free space. Comparing (1.78) and (1.79) shows that the only difference is the Born term and this is as it should be because the eigenvalues (k_p) for the mean field should be independent of downward or upward wave travel. What this comparison means is that the average field propagating upward from the surface and through the foliage is given, within the framework of the Foldy-Twersky approximation, by the Born term in (1.78) with k_o in (1.71) and (1.72) replaced by k_p (the wavenumber for the average field). That is,

$$< \vec{E_{sf}}] = \vec{E}^k_{asu} + LP\overline{\overline{K}}_{\Sigma_k} \sum_{n=0} \{L(1-P)\overline{\overline{K}}_{\Sigma_k}\}^n \vec{E}^k_{\delta su} \tag{1.80}$$

where

$$\vec{E}^k_{asu} = jk_p\eta_p G_k(R)\hat{k}_s \times \hat{k}_s \times \int \vec{J}_{af}(\vec{r_o}) \exp\,(jk_p\hat{k}_s \cdot \vec{r_o})d\vec{r}_{t_o} \tag{1.81}$$

$$\vec{E}^k_{\delta su} = jk_p\eta_p G_k(R)\hat{k}_s \times \hat{k}_s \times \int \vec{J}_{\delta f}(\vec{r_o}) \exp\,(jk_p\hat{k}_s \cdot \vec{r_o})d\vec{r}_{t_o} \tag{1.82}$$

As a reminder, $< \vec{E_{sf}}]$ is the upward propagating field in the foliage after having been scattered by the surface and averaged over all possible foliage configurations. A further simplification of (1.80) comprises taking only the n = 0 term in the series, i.e.

$$< \vec{E_{sf}}] \approx \vec{E}^k_{asu} + LP\overline{\overline{K}}_{\Sigma_k}\vec{E}^k_{\delta su} \tag{1.83}$$

This result has a number of interesting features which deserve comment. First, the field $\vec{E}^k_{asu}$ is the obvious contribution. It is the field produced by the surface scattering of the downward propagating field back up into the average medium characterized by the wavenumbers k_p, p = v or h. This is why all of the quantities, except $\vec{J}_{af}$, in (1.81) depend on k_p rather than k_o, i.e. $\vec{E}^k_{asu}$ is scattered into the *average medium*. The current $\vec{J}_{af}$, as it appears in (1.81), should also be computed using k_p rather than k_o, however, such a replacement is necessary only when there is significant multiple scattering on the surface because it *attenuates* the multiple scattering. In view of the fact that we have no surface scattering theories to deal with strong multiple scattering, there is no point to replacing k_o by k_p in computing the surface current.

The last terms on the rhs of (1.83) is deceiving because it appears that we are obtaining a coherent result from an incoherent field. The field $\vec{E}^k_{\delta su}$ is indeed a zero mean field as can be seen from (1.82) and (1.66); however, the average of the product of $\overline{\overline{K}}_{\Sigma_k}$ and $\vec{E}^k_{\delta su}$ is *not* zero. The cause of this is a highly cooperative effect in which the fluctuating field scattered by a given object is rescattered by the surface back up to the *same object* which, in turn, scatters the field a third time and back into free space. The very unique aspect of this scattering arrangement is that the total length of the scattering path is a *constant* regardless of where, in the random foliage layer, the scatterer is located. This is *not true* if there is only a *single scattering* from an object, such as causes $\delta\vec{E}_{fd}$, or if there is scattering between *different* objects. This particular source of coherent scattering is not expected to be large compared to $\vec{E}_{asu}$ or when the randomness of the surface is averaged over; however, there are instances where it might be important. To the author's knowledge, this effect has not been previously noted.

Before leaving the upward propagating average field in the foliage, as given by (1.83), we must get this field into the free space above the foliage layer. If the frequency is below about 400 MHz then there is a potential for a lateral wave existing at the interface because $<\vec{E}_{sf}]$ is propagating *into* a less dense medium and, in this case, the techniques developed by Tamir [28] should be used to complete the analysis. However, since the frequency of interest to this study is considerably above 400 MHz, the lateral wave can probably be ignored and the fields can be continued up into free space through the use of a simple Huygen's source on the z = h plane or the application of the plane wave spectral approach developed in Appendix B.

Finally, we turn our attention to the upward traveling fluctuating field as given by (1.77). Truncating the series in (1.77) with the n = 1 term yields

$$\delta\vec{E}_{sf} \approx \vec{E}_{\delta su} + L(1-P)\overline{\overline{K}}_{\Sigma}\{<\vec{E}_{sf}] + \vec{E}_{\delta su}\} \tag{1.84}$$

It is interesting to compare (1.84) with (1.55) which is the downward propagating fluctuating field resulting form the first pass through the foliage. The field $\vec{E}_{\delta su}$ is the result of the surface scattering $\delta\vec{E}_{fd}$ into the upward direction. The reaction of the foliage components to this field is given by the term

$$L(1-P)\overline{\overline{K}}_{\Sigma}\vec{E}_{\delta su}$$

and the remaining term in (1.84) is due to the usual interaction of the average field with the scatterers such as in (1.55). The important point to note is that the effect of an incident fluctuating field is twofold; it contributes by itself and by interacting with the scatterers. Substituting (1.83) into (1.84) yields

$$\delta\vec{E}_{sf} \approx \vec{E}_{\delta su} + L(1-P)\overline{\overline{K}}_{\Sigma}\{\vec{E}^k_{asu} + LP\overline{\overline{K}}_{\Sigma_k}\vec{E}^k_{\delta su} + E_{\delta su}\} \tag{1.85}$$

The interesting point about this result is that $\delta\vec{E}_{sf}$ includes fields which propagate in free space and fields which propagate in the average medium *up to a particle* and then they are scattered into free space.

1.4.1 Summary

This section has been concerned with the development of a model for the field scattered by a foliage or vegetation layer on a rough surface. Emphasis has been placed on the foliage aspects of the problem because they are the most difficult to properly account for and the assumed statistical independence of the foliage and the surface permits dealing with the surface averaging at any point in the problem. The analysis is based on the way in which a foliage layer converts an incident field into a coherent or average field and a zero mean fluctuating field. We use this particular decomposition to track the field down through the foliage and onto the rough surface. In the processes of doing this, we determine the field scattered by the foliage components back up into free space. This is the field that would exist if the foliage depth were infinite.

The integral equation for the current induced on the rough surface is solved by iteration and the fields rescattered back up into the foliage layer is expressed in terms of a standard diffraction integral. The source fields for the up-going problem included both coherent and incoherent terms in contrast to the down-going problem which had only a deterministic field as the source. These fields led to some additional interactions not present in the down-going problem. The net results were double pass average and fluctuating fields. The fluctuating fields could be continued directly into the space above the foliage layer because in the DWBA they propagate in free space even in the foliage layer. The average or coherent up-going fields require a Huygen's source or a plane wave spectral decomposition at the foliage-air interface because they propagate in an average medium while inside the foliage layer.

In view of the volume of equations developed in this section, it is useful to summarize the important ones. The results have been left in terms of field quantities and no surface averaging has been undertaken.

The fluctuating field scattered by the foliage components on the downward passage of the incident field is given by

$$\delta\vec{E}_{fd} = L(1-P)\overline{\overline{K}}_{\Sigma} < \vec{E}_{fd}] \tag{1.86}$$

where L is the integral over all space, P is the averaging operator, $\overline{\overline{K}}_{\Sigma}$ is given by (1.9), and $< \vec{E}_{fd}]$ is given by (1.53) with $\vec{r}$ replaced by $\vec{r}_o$. This field, $\delta\vec{E}_{fd}$, is valid in all space above the rough surface.

The fluctuating field scattered by the through-the-foliage passage of the surface scattered field is given by

$$\delta\vec{E}_{sf} = \vec{E}_{\delta su} + L(1-P)\overline{\overline{K}}_{\Sigma}\{\vec{E}^{k}_{asu} + LP\overline{\overline{K}}_{\Sigma_k}\vec{E}^{k}_{\delta su} + \vec{E}_{\delta su}\} \tag{1.87}$$

where $\vec{E}_{\delta su}$ is given by (1.72), $\vec{E}^{k}_{asu}$ is given by (1.81), $\overline{\overline{K}}_{\Sigma_k}$ is given by (1.9) with k_o replaced by k_p as given in (1.54), and $\vec{E}^{k}_{\delta su}$ is given by (1.82). This field, $\delta\vec{E}_{sf}$, is also valid in all space above the rough surface.

Finally, the average or coherent field propagating up through the foliage from the rough surface is given by

$$<\vec{E}_{sf}] = \vec{E}^{k}_{asu} + LP\overline{\overline{K}}_{\Sigma_k}\vec{E}^{k}_{\delta su} \tag{1.88}$$

This field is valid only in the foliage layer and a Huygen's source or a spectral decomposition is needed to continue these fields into the space above the foliage (see Appendix B).

The above relationships are the primary results of this study. It is recommended that they be used to develop numerical scattering results which can be checked against measured radar data.

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Appendix A

Integral Equations For Scattering From Dielectric Surfaces

When trying to determine the electric current induced on a perfectly conducting body or surface, there are three integral equation formulations that are familiar. The first is an integral equation of the second kind called the Magnetic Field Integral Equation (MFIE). The second is an integral equation of the first kind called the Electric Field Integral Equation (EFIE). These equations are based on the discontinuity or continuity of the tangential components of the stated fields on the conducting boundary. A third equation is of the first kind and it results from the requirement of zero total field inside the conducting medium. Of these equations or formulations, we have chosen the MFIE because a formal solution by iteration can always be developed. In fact this was done in Section 1.3 and the significance of the first two iterates was discussed. Convergence of the iterative series is always a potential problem but there is also the possible solution by partial summation methods. This is why we favor equations of the second kind over integral equations of the first kind.

When the surface becomes an imperfect dielectric or a lossy conductor, there are *many* possible ways to describe the field scattering problem. A good discussion of the possible formulations is given by Jones.* In addition to volume and surface integration formulations, he discusses the use of integral equations developed from the Leontovich impedance boundary condition. This latter formulation is approximate but very attractive because one can deal with either an electric or a magnetic current in a single integral equation. There are two difficulties with the technique. First, the technique is approximate** and its ability to describe scattering from *arbitrarily* roughened surfaces is as yet unknown. Second, the actual resulting equation is an integro-differential equation in that it involves the unknown current and its derivative. For these reasons, integral equations based upon the impedance boundary condition do not seem to be suitable to the random rough surface problem.

In searching for integral equations suitable for describing the induction of currents on the dielectric interface, we were guided by one primary condition. This condition was that the equations be sufficiently close to the MFIE that we could use MFIE-based methods to deal with the dielectric interface. That is, we did *not* want to deal with the dielectric interface as a fundamentally new and different problem. A number of formulations were investigated and the

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* Jones, D.S., *Methods In Electromagnetic Wave Propagation*, Oxford Press, pp. 887, 1979.

** Wang, D.S., "Limits and Validity of the Impedance Boundary Condition on Penetrable Surfaces", *IEEE Trans. Antennas & Propag.*, *AP-35*(4), pp. 453-457, 1987.

one developed by Muller*** was selected as the most suitable. In this approach Muller derives coupled integral equations of the second kind for the electric current $\vec{J_s}$ and the magnetic current $\vec{K_s}$. The resulting equations are as follows;

$$\vec{J_s}(\vec{r}) = (\frac{2}{\mu_r+1})\vec{J_s^i}(\vec{r}) + \frac{2}{(\mu_r+1)}\hat{n}(\vec{r})\times\int\vec{J_s}(\vec{r_o})\times[\nabla_o G_o - \mu_r\nabla_o G]ds_o$$

$$-j\frac{k_o}{\eta_o(1+\mu_r)}\hat{n}(\vec{r})\times\int\{\vec{K_s}(\vec{r_o})[G_o - \mu_r\varepsilon_r G] + k_o^{-2}[\vec{K_s}(\vec{r_o})\cdot\nabla_o][\nabla_o G_o - \nabla_o G]\}ds_o \qquad (A1)$$

$$\vec{K_s}(\vec{r}) = (\frac{2}{\varepsilon_r+1})\vec{K_s^i}(\vec{r}) + \frac{2}{(\varepsilon_r+1)}\hat{n}(\vec{r})\times\int\vec{K_s}(\vec{r_o})\times[\nabla_o G_o - \varepsilon_r\nabla_o G]ds_o$$

$$+j\frac{k_o\eta_o}{(1+\varepsilon_r)}\hat{n}(\vec{r})\times\int\{\vec{J_s}(\vec{r_o})[G_o - \mu_r\varepsilon_r G] + k_o^{-2}[\vec{J_s}(\vec{r_o})\cdot\nabla_o][\nabla_o G_o - \nabla_o G]\}ds_o \qquad (A2)$$

where ε_r *and* μ_r are the relative permittivity and permeability of the lower medium, $\eta_o = \sqrt{\mu_o/\varepsilon_o}$ is the characteristic impedance of free space, $k_o = 2\pi/\lambda_o$, $k = k_o\sqrt{\mu_r\varepsilon_r}$, and

$$\vec{J_s^i} = \hat{n}\times\vec{H^i} \qquad \vec{K_s^i} = \vec{E^i}\times\hat{n} \qquad (A3)$$

$$G_o = \exp(-jk_o|\vec{r} - \vec{r_o}|)/4\pi|\vec{r} - \vec{r_o}| \qquad (A4)$$

$$G = \exp(-jk|\vec{r} - \vec{r_o}|)/4\pi|\vec{r} - \vec{r_o}| \qquad (A5)$$

The operator ∇_o is the conventional three dimensional gradient operator evaluated on the surface.

Of particular note with these equations is that they are second kind integral equations and they involve only the unknown currents and not their derivatives. It is only necessary to solve (A1) for J_{s_x} *and* J_{s_y} and (A2) for K_{s_x} *and* K_{s_y} because the currents must be tangential to the surface. That is, $\hat{n}\cdot\vec{J_s} = 0$ and $\hat{n}\cdot\vec{K_s} = 0$ so that

$$J_{s_z} = \zeta_x J_{s_x} + \zeta_y J_{s_y} \qquad (A6)$$

and

$$K_{s_z} = \zeta_x K_{s_x} + \zeta_y K_{s_y} \qquad (A7)$$

*** Muller, D., *Foundations of Mathematical Theory of Electromagnetic W...s*, Springer-Verlag, pp. 1969.

where ζ_x *and* ζ_y are the x and y components of the surface slope. Thus, (A1) and (A2) can be rewritten as a matrix integral equation of the second kind for J_{s_x}, J_{s_y}, K_{s_x} and K_{s_y}. While the increase in dimensionality relative to the conducting interface case is not insignificant, it does not cause any fundamentally new problems. The major increase in difficulty comes from the need to deal with the second derivative of the Green's functions at the source point. However, this is also is not a fundamentally new problem.

The point of the above discussion is to show that the penetrable rough interface problem can be dealt with using a set of equations that are very similar to the MFIE describing the conducting interface. This means that if techniques can be developed to deal with the MFIE which are not tailored to the very special form of the kernel of the integral equation then they can also be applied to the penetrable interface also. Of course, special kernel dependent techniques can also be translated from one problem to the other as long as *both* kernels have the same property. In summary, it is the similarity of the MFIE with (A1) and (A2) which justify our emphasis on developing analytical techniques for solving the MFIE.

Appendix B

Continuation of the Coherent Field Into Free Space

Section 1.4.1 summarizes the various contributions to the scattered field in the region of space above the foliage layer. These contributions comprise coherent and fluctuating fields and the latter present no difficulty because they are easily continued into free space (see the discussion in Section 1.4.1). However, the coherent or average fields propagate, when in the foliage, as in an average or effective medium. This medium is characterized by the relative dielectric constant ε_{r_p} where

$$\varepsilon_{r_p} = k_p^2/k_o^2 \qquad p = v \, or \, h \tag{B1}$$

When these coherent fields strike the foliage-air interface, there will be reflected and transmitted fields. The reflected field is ignored because the dielectric contrast between free space and the average foliated medium is not very significant. However, a proper accounting for the change in medium must be undertaken for the transmitted field.

The average incident field at z = h, coming up through the foliage, is given by (1.88), i.e.,

$$< \vec{E}_{sf}] = \vec{E}^k_{asu} + LP\bar{\bar{K}}_{\Sigma_k}\vec{E}^k_{\hat{o}su} \tag{B2}$$

The k superscript on the fields and the k subscript on the operator $\bar{\bar{K}}_{\Sigma}$ indicate that the medium is characterized by the wavenumber associated with the average field in the foliage medium. We now have to continue this field into free space above the foliage. It is well known* that the fields above the z = h plane can be determined from the spectral representation

$$< \vec{E}(\vec{r}_t, z)] = (2\pi)^{-2}\int \vec{F}(\vec{k}_t) \exp\left(-j\kappa[z-h] - j\vec{k}_t \cdot \vec{r}_t\right) d\vec{k}_t \tag{B3}$$

where

$$\kappa = \begin{cases} (k_o^2 - \vec{k}_t \cdot \vec{k}_t)^{1/2} & k_o^2 > \vec{k}_t \cdot \vec{k}_t \\ -j(\vec{k}_t \cdot \vec{k}_t - k_o^2)^{1/2} & \vec{k}_t \cdot \vec{k}_t > k_o^2 \end{cases} \tag{B4}$$

and $d\vec{k}_t = dk_x dk_y$, $\vec{k}_t = k_x\hat{x} + k_y\hat{y}$. Equation (B3) represents the average field above the foliage as a superposition of plane waves whose complex vector amplitudes are given by $\vec{F}(\vec{k}_t)$.

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* Collin, R.E., *Antennas and Radiowave Propagation*, McGraw-Hill, New York, 1985.

Since $<\vec{E}(\vec{r}_t, z)]$ satisfies Maxwell's source free equations, we must have $\nabla \cdot <\vec{E}(\vec{r}_t, z)] = 0$. This implies that all the vector components of $\vec{F}$ are not independent and, in fact,

$$(\vec{k}_t + \kappa\hat{z}) \cdot \vec{F} = 0$$

or

$$F_z = -\frac{(k_x F_x + k_y F_y)}{\kappa} \tag{B5}$$

The x and y-components of $\vec{F}$, denoted as

$$\vec{F}_t = F_x\hat{x} + F_y\hat{y}, \tag{B6}$$

may be determined from (B3) by setting z = h and taking the inverse two-dimensional Fourier transform of both sides to yield

$$\vec{F}_t(\vec{k}_t) = \int <\vec{E}_t(\vec{r}_t, z=h)] \exp(j\vec{k}_t \cdot \vec{r}_t) d\vec{r}_t \tag{B7}$$

In (B7)

$$<\vec{E}_t(\vec{r}_t, z=h)] = <E_x(\vec{r}_t, z=h)]\hat{x} + <E_y(\vec{r}_t, z=h)]\hat{y} \tag{B8}$$

is taken to be the x and y components of the field in (B2); that is, the upward traveling field in the foliage layer. Thus, (B7) can be written as follows;

$$\vec{F}_t(\vec{k}_t) = \int <\vec{E}_{sf}(\vec{r}_t, z=h)] \exp(j\vec{k}_t \cdot \vec{r}_t) d\vec{r}_t \tag{B9}$$

When the point of observation is sufficiently far from the illuminated area on the interface at z = h, the integral in (B3) can be asymptotically evaluated to yield

$$<\vec{E}(\vec{r}_t, z >> h)] \sim j2k_o \cos\theta_s G(R)\vec{F}(-\vec{k}_{s_t}) \tag{B10}$$

where

$$G(R) = \exp(-jk_o R)/4\pi R \tag{B11}$$

and

$$\vec{k}_{s_t} = -k_o \sin\theta_s \cos\phi_s\hat{x} - k_o \sin\theta_s \sin\phi_s\hat{y} \tag{B12}$$

The angles θ_s *and* ϕ_s are the polar and azimuthal scattering angles while R is the distance from the top of the foliage layer to the point of observation. In this limit, all we need is the two-dimensional Fourier transform of the total average field at $z = h$. In terms of the parts of $< \overrightarrow{E}_{sf}]$, as given by (B2), the overall process of getting from the current on the rough surface up through the foliage and into free space is a bit more involved.

Appendix C
The Conversion of Fields Into Scattering Cross Sections

While the fields obtained in the main body of this section are fundamental quantities they are not the most useful to a radar engineer. For incoherent radars and even most coherent radars, the characteristic of primary interest is the *scattering cross section* of the terrain. This quantity is related to the second moment of the scattered field so it is determined by the fluctuating component of the scattered field in all directions except the specular direction. In the specular direction, the scattered power comprises both coherent and incoherent parts although the coherent part is frequently negligable. If the coherent power is not negligable, it gives rise to a range-dependent scattering cross section.

With scattering from bare ground, the cross section of interest is the scattering cross section of the surface per unit scattering area or $\sigma°$. This normalized monostatic cross section is determined from the fluctuating part of the scattered field as follows;

$$\sigma°_{pq} = \lim_{\substack{R\to\infty \\ A\to\infty}} \{4\pi R^2 < |\delta\vec{E}_s \cdot \hat{q}|^2 > / |\vec{E}_i|^2 A\} \qquad (C1)$$

where R is the distance from the radar to the midpoint of the illuminated area, A is the illuminated area on the surface, and E_i is the incident field. The subscripts p and q denote that the polarization p is transmitted while the q-component of the backscattered field is sampled. This normalized cross section depends on only two radar parameters, namely, the frequency and the polarization.

The normalized cross section $\sigma°$ is adequate for bare surfaces and surfaces having vegetation layers which are small in depth compared to the range resolution of the radar. However, when the foliage depth exceeds the radar range resolution, the surface scattering cross section should be replaced by a volume scattering cross section. That is, instead of (C1), the following *volume* cross section should be used;

$$\sigma^v_{pq} = \lim_{\substack{R\to\infty \\ V\to\infty}} \{4\pi R^2 < |\delta\vec{E}_s \cdot \hat{q}|^2 > / |\vec{E}_i|^2 V\} \qquad (C2)$$

where V is the illuminated volume and R is the distance from the radar to the midpoint of the volume. It should be noted that just as (C1) assumes completely incoherent surface scatter so does (C2) assume completely incoherent volume scatter. That is, under the complete incoherence condition, either $\sigma°_{pq} A$ or $\sigma^v_{pq} V$ (whichever is appropriate) can be used in the conventional radar equation for the target scattering cross section to give the power backscattered by the terrain.

As summarized in Section 1.4.1 there are essentially three fields to consider when dealing with the scattering from vegetation covered terrain. The first field is due to scattering by the foliage of a field incident from free space. This field is a fluctuating field in that the foliage layer is assumed to be so large in terms of a wavelength that there is essentially no coherent backscattering. The other two fields are the coherent and incoherent fields which have propagated down through the foliage and have been scattered by the surface back up through the foliage. Of these three fields, the two of most interest are the fluctuating field scattered by the foliage on its way to the surface and the fluctuating field scattered by the surface back up through the foliage.

Scattering of the free-space incidence field (propagating down to the surface) by the foliage is described within the distorted Born approximation by (1.86). The mean square value of this quantity may be written as follows;

$$< |\delta \vec{E}_{fd}|^2 > \; = \; < L\overline{\overline{K}}_\Sigma < \vec{E}_{fd}] \cdot L\overline{\overline{K}}_\Sigma^* < \vec{E}_{fd}]^* > \; - \; < L\overline{\overline{K}}_\Sigma < \vec{E}_{fd}] >< L\overline{\overline{K}}_\Sigma < \vec{E}_{fd}] >^* \qquad (C3)$$

If the point of observation is sufficiently far removed from the scattering region, the far field approximation for the Green's function in (1.11) may be used to yield

$$\overline{\overline{K}}_\Sigma \approx k_o^2 g(R) \sum_{n=1}^{N} (\varepsilon r_n - 1) S_n \exp[jk_o \hat{k}_s \cdot (\vec{r}_o + \vec{r}_n)]\{1 - \hat{k}_s(\hat{k}_s \cdot\} \qquad (C4)$$

where $\hat{k}_s$ denotes the direction of the scattering observation. Rewriting (C4) as

$$\overline{\overline{K}}_\Sigma = k_o^2 g(R) \sum_{n=1}^{N} Q_n(\vec{r}_o + \vec{r}_n)$$

permits the following expression for the first term on the right side of (C3);

$$L\overline{\overline{K}}_\Sigma < \vec{E}_{fd}] \cdot L\overline{\overline{K}}_\Sigma^* < \vec{E}_{fd}]^* = \left(\frac{k_o^2}{4\pi R}\right)^2 \sum_{n=1}^{N} |LQ_n(\vec{r}_o + \vec{r}_n) < \vec{E}_{fd}(\vec{r}_o + \vec{r}_n)]|^2$$

$$+ \left(\frac{k_o^2}{4\pi R}\right)^2 \sum_{n=1}^{N} LQ_n(\vec{r}_o + \vec{r}_n) < E_{fd}(\vec{r}_o + \vec{r}_n)] \cdot \sum_{\substack{m=1 \\ m \neq n}}^{N} LQ_m^*(\vec{r}_o + \vec{r}_m) < E_{fd}^*(\vec{r}_o + r_m)] \qquad (C5)$$

What (C5) does is to separate out the N product terms which contain like elements. Assuming that the scatterers are independent of each other, an assumption which is consistent with the distorted Born approximation, an average of (C5) yields

$$< L\overline{\overline{K}}_\Sigma < E_{fd}] \cdot L\overline{\overline{K}}_\Sigma^* < E_{fd}]^* > \approx \left(\frac{k_o^2}{4\pi R}\right)^2 N \int < |LQ_n(\vec{r}_o + \vec{r}_n) < \vec{E}_{fd}(\vec{r}_o + \vec{r}_n)]|^2] p(\vec{r}_n) d\vec{r}_n$$

$$+ (\frac{k_o^2}{4\pi R})^2 N^2 (1 - 1/N) \,|\int L < Q_n(\vec{r}_o + \vec{r}_n)] < \vec{E}_{fd}(\vec{r}_o + \vec{r}_n)] p(\vec{r}_n) d\vec{r}_n|^2 \qquad (C6)$$

where $p(\vec{r}_n)$ is the probability density function for the location of the scatterers, symbolized by the vectors r_n $n = 1,2,...,N$. As $N \rightarrow \infty$, the second term on the right side of (C6) goes to the negative of the second term on the right side of (C3) so they cancel and this leaves

$$< |\delta \vec{E}_{fd}|^2 > \; = (\frac{k_o^2}{4\pi R})^2 N \int \; < |LQ_n(\vec{\lambda}_o + \vec{r}_n) < \vec{E}_{fd}(\vec{r}_o + \vec{r}_n)]|^2] p(\vec{r}_n) d\vec{r}_n \qquad (C7)$$

The $< \cdot]$ brackets inside the integrand denote a conditional average over scatterer size, shape, orientation, and dielectic constant.

Substituting from (C4) and simplifying, yields the following result;

$$< |\delta \vec{E}_{fd}|^2 > \; = (\frac{k_o^2}{4\pi R})^2 N < (\varepsilon r_n - 1)^2 > \iint \int_{<V_s>} \left[1 - (\hat{k}_s \cdot \hat{e}_{fd})^2\right] < E_{fd}(\vec{r}_o + \vec{r}_n)] < E^*_{fd}(r'_o + \vec{r}_n)]$$

$$\cdot \, p(\vec{r}_n) \exp[jk_o \hat{k}_s \cdot (\vec{r}_o - \vec{r}\,'_o)] d\vec{r}_o d\vec{r}\,'_o \, d\vec{r}_n \qquad (C8)$$

where

$$< \vec{E}_{fd}] \; = \; < E_{fd}] \, \hat{e}_{fd}$$

and $< V_s >$ is the average volume of a scatterer and shape of a scatterer. For plane wave illumination of the foliage layer, the average field in the layer is given by

$$< E_{fd}(\vec{r}_o + \vec{r}_n)] = E_i \exp[-j\vec{k}_{i_t} \cdot (\vec{r}_{o_t} + \vec{r}_{n_t}) - jk_p(z_o + z_n)] \qquad (C9)$$

where

$$\vec{k}_{i_t} = k_{i_x} \hat{x} + k_{i_y} \hat{y} \qquad (C10)$$

with k_{i_x} and k_{i_y} given by (1.51a) and (1.51b), and k_p is the propagation constant of the average medium given by (1.54). The average field has its transverse (to z) wavenumber equal to $\vec{k}_{i_t}$ in order to be in phase synchronism with the free space incidence field across the upper average boundary (z = h). Substituting (C9) in (C8) yields

$$< |\delta E_{fd}|^2 > = (\frac{k_o^2}{4\pi R})^2 N < (\varepsilon r_n - 1)^2 > \iint \int_{<V_s>} [1 - (\hat{k}_s \cdot \hat{e}_{fd})^2] |E_i|^2 \exp[j(\vec{k}_{s_t} - \vec{k}_{i_t}) \cdot (\vec{r}_{o_t} - \vec{r}\,'_{o_t})$$

$$+ jk_{s_z}(z_o - z_o') - j(k_o z_o - k_p^* z_o') - 2k_p^i z_n]p(\vec{r}_n)d\vec{r}_o d\vec{r}_o' \, d\vec{r}_n \qquad (C11)$$

where the conjugate on $k_p = k_p^n - jk_p^i$ is necessary because the propogation constant for the average field may be complex. (C11) may be rewritten as follows,

$$< |\delta E_{fd}|^2 > = \left(\frac{k_o^2}{4\pi R}\right)^2 N < (\varepsilon_{r_n} - 1)^2 > |E_i|^2 \int | \int_{<V_s>} [1 - (\hat{k}_s \cdot \hat{e}_{fd})^2]$$

$$\cdot \exp[j(\vec{k}_{s_t} - \vec{k}_{i_t}) \cdot \vec{r}_{o_t} + j(k_{s_z} - k_p)z_o]d\vec{r}_o |^2 \exp(-2k_p^i z_n)p(\vec{r}_n)d\vec{r}_n \qquad (C12)$$

The integral over $< V_s >$ has the same form as the scattering pattern of a scatterer of volume $< V_s >$ and supporting an internal field of the form in (C9). Thus, with

$$P(\vec{k}_{i_t} + k_p\hat{z};\ \vec{k}_s) = \int_{<V_s>} [1 - (\hat{k}_s \cdot \hat{e}_{fd})^2] \exp[j(\vec{k}_{s_t} - \vec{k}_{i_t}) \cdot \vec{r}_{o_t} + j(k_{s_z} - k_p)z_o]d\vec{r}_o \qquad (C13)$$

(C12) becomes

$$< |\delta E_{fd}|^2 > = \left(\frac{k_o^2}{4\pi R}\right)^2 N < (\varepsilon_{r_n} - 1)^2 > |E_i|^2 |P(\vec{k}_{i_t} + k_p\hat{z}; \vec{k}_s)|^2 \qquad (C14)$$

$$\cdot \int \exp(-2k_p^i z_n)p(\vec{r}_n)d\vec{r}_n$$

If the scatterers are assumed to be uniformly distributed about the volume $V = A \cdot h$ where h is the depth of the foliage layer and A is the area in the z = 0 plane (see Figure 1.3) then

$$P(\vec{r}_n) = \begin{cases} 1/Ah & \vec{r}_n \text{ in } V \\ 0 & \vec{r}_n \text{ not in } V \end{cases} \qquad (C15)$$

and (C14) becomes

$$< |\delta E_{fd}|^2 > = \left(\frac{k_o^2}{4\pi R}\right)^2 N < (\varepsilon_{r_n} - 1)^2 > |E_i|^2 |P(\vec{k}_{i_t} + k_p\hat{z};\ \vec{k}_s)|^2$$

$$\cdot \frac{1}{2k_p^i h}[1 - \exp(-2k_p^i h)]$$

Substituting this into (C2) yields the following result for the scattering cross section per unit scattering volume:

$$\sigma^{v} = \frac{N}{V}\frac{k_o^4}{4\pi} < (\varepsilon r_n - 1)^2 > | P(\vec{k}_{i_t} + k_p\hat{z};\vec{k}_s)|^2[1 - \exp(-2k_p^i h)]/2k_p^i h \tag{C16}$$

If the scattering amplitude is normalized to its maximum value, i.e. $< V_s >$, (C16) becomes

$$\sigma^{v} = \frac{k_o^4}{4\pi} < V_s >< (\varepsilon r_n - 1)^2 > | P(\vec{k}_{i_t} + k_p\hat{z};\vec{k}_s)/ < V_s > |^2$$

$$\cdot v_c[1 - \exp(-2k_p^i h)]/(2k_p^i h) \tag{C17}$$

where v_c is the average fractional volume occupied by the scatterers or

$$v_c = N < V_s > / V \tag{C18}$$

This result differs from what would be obtained with previous results in the appearance of the factor $2k_p^i h$. That is, previous results would erroneously replace this by $4k_p^i h$.

A consequence of the sparse scatterer assumption is that (C17) very nearly splits into the product of two distinctly different factors. The first factor, with the exception of its dependence on the propagation constant of the average medium, is a function of the properties of a *single average scatterer* only. The other factor depends on *average medium parameters* only. Since P or the scattering pattern is the only function in (C17) which depends on the scattering direction, this shows that the "scattering pattern" of the ensemble is determined largely by the scattering pattern of an average scatterer.

The computation of the normalized scattering cross section due to the fields which propagate down through the foliage strike the underlying surface, and then are rescattered back up through the foliage is considerably more difficult. This is due primarily to the fact that there are essentially three random scattering processes occuring in this situation. Equation (1.87) gives the weak volume scattering approximation for the fluctuating field described above, e.g.

$$\delta\vec{E}_{sf} = \vec{E}_{\delta su} + L(1-P)\overline{\overline{K}}_{\Sigma}\{\vec{E}^{k}_{asu} + LP\overline{\overline{K}}_{\Sigma}\vec{E}^{k}_{\delta su} + \vec{E}_{\delta su}\} \tag{C19}$$

$\vec{E}_{\delta su}$ is the field scattered into free space by the rough surface when illuminated by $\delta\vec{E}_{fd}$. The term $L(1-P)\overline{\overline{K}}_{\Sigma}\vec{E}_{\delta su}$ represents the fluctuating field scattered by the particles when supporting the field $\vec{E}_{\delta su}$. $\vec{E}^{k}_{asu}$ is the field scattered into the *average* medium by the surface when illuminated by $< \vec{E}_{fd} >$. The term $L(1-P)\overline{\overline{K}}_{\Sigma}\vec{E}^{k}_{asu}$ is the fluctuating field scattered by the particles when supporting the field $\vec{E}^{k}_{asu}$. Finally, the term

$$L(1-P)\overline{\overline{K}}_{\Sigma}LP\overline{\overline{K}}_{\Sigma_k}\vec{E}^{k}_{\delta su}$$

represents a double scattering of the fluctuating field $\vec{E}_{\delta su}$ and it is not significant compared to the other terms in (C19) when there is a sparse population of scatterers. Thus, (C19) can be reduced to the following;

$$\delta\vec{E}_{sf} \approx \vec{E}_{\delta su} + L(1-P)\bar{\bar{K}}_{\Sigma}\{\vec{E}^{k}_{asu} + \vec{E}_{\delta su}\} \qquad (C20)$$

Assuming that there is no correlation between these various fields and recognizing that each are zero mean quantities results in the following expression for the mean square value of $\delta\vec{E}_{sf}$;

$$< |\delta E_{sf}|^2 > = < |\vec{E}_{\delta su}|^2 > + < |L(1-P)\bar{\bar{K}}_{\Sigma}\vec{E}^{k}_{asu}|^2 >$$

$$+ < |L(1-P)\bar{\bar{K}}_{\Sigma}\vec{E}_{\delta su}|^2 > \qquad (C21)$$

The first term on the right of (C21) is due to surface scattering. After considerable algebra, it may be reduced to the following form;

$$< |\vec{E}_{\delta su}|^2 > = (\frac{k_o\eta_o}{4\pi R})^2 \int\int < [1-(\hat{k}_s \cdot \hat{j}_{\delta f})^2] J_{\delta f}(\vec{r}_s) J^*_{\delta f}(\vec{r}_s{}')$$

$$\cdot \exp[jk_o\hat{k}_s \cdot (\vec{r}_s \cdot \vec{r}_s{}')] > d\vec{r}_{t_s}\, d\vec{r}\,'_{t_s} \qquad (C22)$$

where η_o is the impedance of free space, $\vec{J}_{\delta f}$ is the current induced on the rough surface by the downward propagating foliage field $\delta\vec{E}_{fd}$ (and satisfies equation (1.64),

$$d\vec{r}_{t_s} = dx_s dy_s \qquad d\vec{r}\,'_{t_s} = dx_s'\, dy_s' \,,$$

and $\hat{j}_{\delta f}$ denotes the direction of $\vec{J}_{\delta f}$ This result can be reduced to a surface scattering cross section through the use of (C1), e.g.

$$\sigma^{\circ} = (\frac{k_o^2\eta_o^2}{4\pi|E_i|^2}\frac{1}{A}) \int\int < [1-(\hat{k}_s \cdot \hat{j}_{\delta f})^2] J_{\delta f}(\vec{r}_s) J^*_{\delta f}(\vec{r}_s{}')$$

$$\cdot \exp[jk_o\hat{k}_s \cdot (\hat{r}_s \cdot \vec{r}_s{}')] > d\vec{r}_{t_s} d\vec{r}\,'_{t_s} \qquad (C23)$$

This is about as far as this formulation can be taken without a specific form for the current. However, it must be remembered that the average in (C23) involves an average over the random properties of the surface *and* the random properties of the discrete scatterers including their positions. The A^{-1} factor in (C23) will be cancelled by one of the integrations over x_s and y_s.

For the second and third terms in (C20), the following relationship developed in an earlier part of this appendix is useful;

$$< |L(1-P)\bar{\bar{K}}_{\Sigma}\vec{E}|^2> \; = (\frac{k_o^2}{4\pi R})^2 < (\varepsilon_{r_n}-1)^2 > N \int \int_{<V_s>} < [1-(\hat{k}_s \cdot \hat{e})^2]$$

$$\cdot E(\vec{r}_o+\vec{r}_n)E^*(\vec{r}_o' + \vec{r}_n) > \exp[jk_o\hat{k}_s \cdot (\vec{r}_o - \vec{r}_o')]d\vec{r}_o d\vec{r}_o' \tag{C23}$$

The averaging in (C23) inside the integral is over the random positioning of the scatterers (for the upward traveling field), the random surface, and the random locations of the scatterers for the downward traveling field. The form of (C23) suggests that to a good first approximation the second and third terms in (C20) give use to a scattering cross section per unit scattering volume similar to the one in (C16) but modified by the random surface effects. For example, the second term on the right of (C20) gives

$$\sigma_{asu}^{v} \approx \frac{k_o^4}{4\pi} < V_s > < (\varepsilon_{r_n}-1)^2 > |P(\vec{k}_{s_t}+k_p\hat{z};\ \vec{k}_s)/<V_s>|^2$$

$$\cdot \{v_c[1-\exp(-2k_p^i h)]/(2k_p^i h)\}S_{asu} \tag{C24}$$

while the third term yields

$$\sigma_{\delta su}^{v} \approx \frac{k_o^4}{4\pi} < V_s > < (\varepsilon_{r_n}-1)^2 > |P(\vec{k}_{s_t}+k_p\hat{z};\ \vec{k}_s)/<V_s>|^2$$

$$\cdot \{v_c\left[1 - \exp(-2k_p^i h)\right]/(2k_p^i h)\}S_{\delta su} \tag{C25}$$

where

$$S_{asu} = \frac{k_o^2 n_o^2}{A} \int\int < [1-(\hat{k}_s \cdot \hat{j}_{af})^2]J_{af}(\vec{r}_s)J^*{}_{af}(\vec{r}_s')$$

$$\cdot \exp[jk_o\hat{k}_{s_t} \cdot (\vec{r}_{s_t} - \vec{r}_{s_t}') + jk_p(\zeta - \zeta')] > d\vec{r}_{t_s} d\vec{r}_{t_s}' \tag{C26}$$

$$S_{\delta su} = \frac{k_o^2 n_o^2}{A} \int\int < [1-(\hat{k}_s \cdot \hat{j}_{\delta f})^2]J_{\delta f}(\vec{r}_s)J^*{}_{\delta f}(\vec{r}_s')$$

$$\cdot \exp[jk_o\hat{k}_s \cdot (\vec{r}_s - \vec{r}_s')] > d\vec{r}_{s_t} d\vec{r}_{s_t}' \tag{C27}$$

Of course, the averaging in (C26) and (C27) must include not only the random surface but the discrete scatterers as the field is scattered down to the surface.

To the level that has been developed above, the effects of a random foliage layer on a random surface may be described by a scattering cross section and two volume cross sections. It should be remembered that no coherent fields reflected from the surface have been included. Clearly, there is much work to be done on this problem. For example, one of the things that should be done next is to use some classical approximations for the surface current (such as physical optics and perturbation) and carry the above calculations through to completion. This would certainly give a better feel for the relative importance of the various effects above.

2.0 Asymptotic Surface Scattering

In Section 1.3 we reviewed a number of approximate methods for dealing with surface scattering and their relationship to iterations of the MFIE. Using these iterates along with asymptotic techniques to determine the scattered field provided further insight into the limitations and capabilities of the iterative approach. However, the iterative approach is not necessarily the one to use to improve on known asymptotic results because such improvements may possibly comprise the partial summation of an *infinite* number of iterates. There are other methods for extending asymptotic results and the purpose of this section is to apply and develop one such technique. These *asymptotic extention methods* generally augment iterative solutions.

2.1 Luneburg - Kline Expansion for the Surface Current

2.1.1 Introduction

One of the few scattering properties that are known about certain classes of rough surfaces is how they scatter electromagnetic energy in the high and low frequency limits. The caveat "certain classes" is necessary because even in these asymptotic frequency ranges, there are some surfaces which are not amenable to classical analytical techniques. For these surfaces or more general ones, there are problems with existing theories or models when the frequency of interest is *between* the low or high asymptotic limits [1]. The problems are not with a lack of models but with a good understanding of the limitations of the models. For example, there has recently been some very good work appear on a phase perturbation approach to rough surface scattering [2, 3]. This Rytov-like method has shown promise of extending the range of conventional power series-like field perturbation theory to the point where the rms surface roughness is the order of an electromagnetic wavelength [4]. Of equal importance, however, is the fact that some of the advocates of this approach have very carefully studied the validity of this method by applying it to deterministic surfaces. This type of validation analysis must be done if the method is to be useful as an engineering tool.

Phase perturbation is essentially a low frequency method whose upper frequency range has been extended by partial summation [3]. This approach of extending a low frequency method into higher frequency ranges is fairly common. It is based on the fact that most low frequency techniques do have higher order "terms" which can formally be derived; some of these terms can also be computed. The problem with such extensions is that it is never quite clear when the higher "terms" converge and when they give meaningless results. Little, if any, work has been done in the rough surface literature on extending high frequency solutions into lower frequency regions. However, a basic technique for doing so has existed for a number of years; that is, the Luneburg-Kline (LK) representation for the field scattered by a large object [5]. The classical Luneburg-Kline (LK) representation is a series in powers of the electromagnetic wavelength (or inverse powers of the electromagnetic wavenumber) for the field scattered from a body. The zeroth order or wavelength independent term corresponds to the geometrical or ray optics prediction for the reflected field. This term depends on the curvature and the reflection coefficient of the surface at the point of reflection (the so-called specular point). The term that varies as k_o^{-1} also depends on these same quantities but, in addition, appears to be sensitive to the distance from the specular point to the shadow boundary [6]. This distance dependence indicates the nonlocal nature of the first wavelength dependent term.

Lee [7] has recently obtained the k_o^{-1} coefficient for the scattered field and the current on a perfectly conducting convex body. This was a significant result but it was a bit too restrictive for application to the rough surface problem. For example, Lee's solution must be augmented by possible multiple reflections on the surface, the presence of a creeping wave where the incident field just grazes the surface, and any possible edge diffraction where the surface has a relatively sharp edge-like behavior. In addition, the solution obtained by Lee was very complicated from an algebraic point of view and involved surface characteristics whose statistics are not necessarily well know.

Thus, with an eye toward possibly generalizing Lee's results and also obtaining more insight into the L-K approach, an alternate methodology was developed [8]. First, rather than expanding the scattered and incident fields in L-K series and then applying the boundary conditions to obtain the expansion coefficients, the surface current induced on the surface by the incident field was expanded in an L-K series. This expansion was used in the Magnetic Field Integral Equation (MFIE) to generate a hierarchy of integral equations for the expansion coefficients. This sequence of integral equations exhibited some rather interesting properties. First, it was recurrsive in that knowledge of the *nth* integral equation solution determined the n + 1 integral equation solution. Second, the "source" or Born term in these integral equations depended entirely on the asymptotic evaluation of a *known integral* in inverse powers of k_o. Finally, and most interesting of all was the fact that each integral equation could be solved as they were all just like the equation obtained by iteratively solving the MFIE in the high frequency limit.

There were two immediate consequences of this work. First, it was found that the L-K representation for the current, even though forced to satisfy an *exact* integral equation, produced a zero current on the part of the surface shadowed from the incident field. That is, even the higher order terms in $1/k_o$ were identically zero in the shadow zones of the surface. Although it is not directly obvious why this is the case, it is apparent that since the L-K representation is asymptotic, it fails to converge to the value of current which is *correct for all frequencies*. Thus, in the shadow zones of the surface, the L-K representation is valid in the optical limit only. This result, in itself, is significant because it is the first time that a limitation of the L-K representation has been obtained via an exact analysis. A second major result of this work was to show that all the L-K expansion coefficients could be determined from the behavior (as a function of k_o) of a two-dimensional integral of known functions. Although time did not permit the evaluation of this integral, there are techniques for doing so [9] and it is suggested that this is a useful avenue for future work.

2.1.2. Analysis

The problem to be addressed here is the determination of the electric current density $\vec{J}_s$ induced on the surface $z=\zeta(\vec{r_t})$ by an incident magnetic field $\vec{H}^i$. The surface $z=\zeta(\vec{r_t})$ separates free space $(z>\zeta)$ from a perfectly conducting medium $(z\leq\zeta)$. The unit vector $\hat{k}_i$ specifies the direction of travel of the incident field. The electric surface current density $\vec{J}_s$ must satisfy the Magnetic Field Integral Equation (MFIE) as follows;

$$\vec{J}_s(\vec{r}\,) = 2\hat{n}(\vec{r}\,)\times\vec{H}^i(\vec{r}\,) + 2\hat{n}(\vec{r}\,)\times\int\vec{J}_s(\vec{r}_o)\times\nabla_o G(\vec{r} - \vec{r}_o)ds_o \tag{2.1}$$

In (2.1), $\hat{n}(\vec{r}\,)$ is the unit normal to the surface at the point $\vec{r} = \vec{r}_t + \zeta(\vec{r}_t)\hat{z}$ and is given by

$$\hat{n}(\vec{r}\,) = [-\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z}]/(1 + \zeta_x^2 + \zeta_y^2)^{1/2} \tag{2.2}$$

where $\zeta_x = \partial\zeta/\partial x$ and $\zeta_y = \partial\zeta/\partial y$ are the x and y surface slopes at the point $\vec{r}$ on the surface. $G(\vec{r} - \vec{r}_o)$ is the free space Green's function, i.e.

$$G(\vec{r} - \vec{r}_o) = \exp(-jk_o|\vec{r} - \vec{r}_o|)/4\pi|\vec{r} - \vec{r}_o|, \tag{2.3}$$

and ∇_o is the conventional three-dimensional gradient evaluated on the surface ($z = \zeta$ and $z_o = \zeta_o$). Noting that the area integration over the surface can be converted to one over the $z=0$ plane through

$$ds_o = (1 + \zeta_x^2 + \zeta_y^2)^{1/2}\; d\vec{r}_{t_o}$$

where $d\vec{r}_{t_o} = dx_o dy_o$, (2.1) can be rewritten as follows;

$$\vec{J}(\vec{r}\,) = 2\vec{N}\times\vec{H}^i + 2\vec{N}\times\int\vec{J}(\vec{r}_o)\times\nabla_o G(|\vec{r} - \vec{r}_o|)d\vec{r}_{t_o} \tag{2.4}$$

where

$$\vec{J}(\vec{r}\,) = \vec{J}_s(\vec{r}\,)(1 + \zeta_x^2 + \zeta_y^2)^{1/2} \tag{2.5}$$

and

$$\vec{N} = -\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z} \tag{2.6}$$

If $\vec{J}(\vec{r})$ can be determined from (2.4), the scattered field can be found from the following integral expression;

$$\vec{H}^s(\vec{R}) = \nabla\times\int\vec{J}(\vec{r})G(|\vec{R}-\vec{r}|)d\vec{r}_t \tag{2.7}$$

As a preparatory step to introducing the Luneburg-Kline expansion, the k_o dependence introduced by the incident field is removed. That is, with

$$\vec{J}(\vec{r}) = \vec{L}(\vec{r})\exp(-j\vec{k}_i\cdot\vec{r}), \tag{2.8}$$

$$\vec{H}^i(\vec{r}) = \vec{h}_i\exp(-j\vec{k}_i\cdot\vec{r}), \tag{2.9}$$

and $\vec{h}_i$ a constant, (2.4) may be rewritten as follows;

$$\vec{L}(\vec{r}) = 2\vec{N}\times\vec{h}_i + 2\vec{N}\times\int\vec{L}(\vec{r}_o)\times\nabla_oG(|\vec{r}-\vec{r}_o|)\exp(j\vec{k}_i\cdot\Delta\vec{r})d\vec{r}_{t_o} \tag{2.10}$$

where $\Delta\vec{r} = \vec{r} - \vec{r}_o$. The purpose of (2.8) is to remove the known high frequency behavior or dependence on k_o from the current. That is, as $k_o\to\infty$ it is known that $\vec{L}(\vec{r})$ is independent of k_o. The modified current $\vec{L}(\vec{r})$ is expanded in a Luneburg-Kline series, i.e.

$$\vec{L}(\vec{r}) = \sum_{n=o}\frac{\vec{j}_n(\vec{r})}{k_o^n}, \tag{2.11}$$

where the vector expansion coefficients $\vec{j}_n(\vec{r})$, n=0,1,..., are independent of the electromagnetic wavenumber k_o. (2.11) is next substituted into (2.10) and it is assumed that term by term integration is permissible* so that the following results;

$$\sum_{n=0}\vec{j}_n(\vec{r})k_o^{-n} = 2\vec{N}\times\vec{h}_i + 2\vec{N}\times\sum_{n=0}k_o^{-n}\int\vec{j}_n(\vec{r}_o)\times\nabla_oG\exp(j\vec{k}_i\cdot\Delta\vec{r})d\vec{r}_{t_o} \tag{2.12}$$

In order for (2.12) to be satisfied, the integral term must also have a Luneburg-Kline expansion. That is, if

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* This assumption is tantamount to requiring that the L-K series be uniformly convergent over $\vec{r}_{t_o}\in(-\infty,\infty)$; a requirement that is probably not satisfied.

$$\vec{T}_n(\vec{r}, k_o) = \int \vec{j}_n(\vec{r}_o) \times \nabla_o G \ \exp\ (j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.13}$$

then it may be written as follows;

$$\vec{T}_n(\vec{r}, k_o) = \sum_{m=0} \vec{T}_{nm}(\vec{r})/k_o^m \tag{2.14}$$

where the $\vec{T}_{nm}$ expansion coefficients are independent of k_o. Combining (2.13) and (2.14) yields

$$\vec{T}_{no} + \frac{\vec{T}_{n1}}{k_o} + \frac{\vec{T}_{n2}}{k_o^2} + \ldots = \int \vec{j}_n(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\ exp\ (j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.15}$$

so that the vector expansion coefficients can be determined as follows;

$$\vec{T}_{no} = \lim_{k_o \to \infty} \int \vec{j}_n(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\ exp\ (j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o}. \tag{2.16a}$$

$$\vec{T}_{n1} = \lim_{k_o \to \infty} k_o \left[\int \vec{j}_n \times \nabla_o G\ exp\ (j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} - \vec{T}_{no} \right] \tag{2.16b}$$

$$\vec{T}_{nm} = \lim_{k_o \to \infty} k_o^m \left[\int \vec{j}_n \times \nabla_o G\ exp\ (j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} - \sum_{p=0}^{m-1} \frac{\vec{T}_{np}}{k_o^p} \right] \tag{2.16c}$$

Substituting (2.15) into (2.12) yields

$$\sum_{n=0} \vec{j}_n(\vec{r})k_o^{-n} = 2\vec{N} \times \left[\vec{h}_i + \sum_{n=0} \sum_{m=0} \vec{T}_{nm}(\vec{r})k_o^{-n-m} \right] \tag{2.17}$$

so that equating like powers of k_o yields

$$\vec{j}_o(\vec{r}) = 2\vec{N} \times \{\vec{h}_i + \vec{T}_{oo}\} \tag{2.18a}$$

$$\vec{j}_1(\vec{r}) = 2\vec{N} \times \{\vec{T}_{10}(\vec{r}) + \vec{T}_{01}(\vec{r})\} \tag{2.18b}$$

$$\vec{j}_2(\vec{r}) = 2\vec{N} \times \{\vec{T}_{11}(\vec{r}) + \vec{T}_{20}(\vec{r}) + \vec{T}_{02}(\vec{r})\} \tag{2.18c}$$

etc.

Substituting from (2.16) into (2.18) yields the following sequence of integral equations for the vector expansion coefficients;

$$\vec{j}_o(\vec{r}) = 2\vec{N} \times \vec{h}_i + 2\vec{N} \times \lim_{k_o \to \infty} \int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o} \tag{2.19a}$$

$$\vec{j}_1(\vec{r}) = 2\vec{N} \times \lim_{k_o \to \infty} \{k_o \left[\int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o} - \vec{T}_{oo} \right]\}$$

$$+ 2\vec{N} \times \lim_{k_o \to \infty} \int \vec{j}_1(\vec{r}) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o} \tag{2.19b}$$

$$\vdots \qquad \vdots$$

This sequence of integral equations has a number of interesting properties. First, except for the source or Born term, all the integral equations *are identical* in form in that they appear as

$$\vec{j}_p(\vec{r}) = 2\vec{N} \times \vec{s}_p(\vec{r}) + 2\vec{N} \times \lim_{k_o \to \infty} \int \vec{j}_p(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o} \tag{2.20}$$

$$p = 0,1,2,...$$

Also, since the source term, $2\vec{N} \times \vec{s}_p(\vec{r})$, in (2.20) depends on $\vec{j}_o$, $\vec{j}_1$,..., and $\vec{j}_{p-1}$, the integral equations are *recurrsive*. Thus, if $\vec{j}_o$ can be determined then it should be possible to determine *all* higher order vector coefficients. Because the vector expansion coefficients, $\vec{j}_n(\vec{r})$, are independent of k_o, it is further noted that the source term $\vec{s}_p(\vec{r})$ is determined almost completely by a Luneburg-Kline series representation for the following integral

$$\int \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o}.$$

For example, from (2.19a) it follows that

$$\vec{s}_o(\vec{r}) = \vec{h}_i \tag{2.21a}$$

while from (2.19b)

$$\vec{s}_1(\vec{r}) = \lim_{k_o \to \infty} k_o \Big[\int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o}$$

$$- \lim_{k_o \to \infty} \int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r})\, d\vec{r}_{t_o} \Big] \qquad (2.21b)$$

It should be noted that the presence of $\vec{j}_o(\vec{r}_o)$ under the integral signs in (2.21b) has a very minimal effect because the dominant terms are the ones which depend upon k_o.

One final but very important point about the sequence of integral equations in (2.19) is that *their solutions are known*. In fact, (2.19a) is a slightly altered form of the Magnetic Field Integral Equation in the high frequency ($k_o \to \infty$) limit, so its solution is given by [10]

$$\vec{j}_o(\vec{r}) = \begin{cases} 2\vec{N} \times \vec{h}_i & (\vec{r} \text{ not } \textit{shadowed}) \\ 0 & (\vec{r} \textit{ in shadow}) \end{cases}$$

There is also the possibility of a multiple scattering contribution to $\vec{j}_o(\vec{r})$ from other points on the surface [10]. Thus, for the n*th* vector expansion coefficient, the solution is (except for the contributions of multiple ray bounces on the surface)

$$\vec{j}_n(\vec{r}) = \begin{cases} 2\vec{N} \times \vec{s}_n(\vec{r}) & (\vec{r} \text{ not } \textit{shadowed}) \\ 0 & (\vec{r} \textit{ in shadow}) \end{cases} \qquad (2.22)$$

The complete surface current density is thus obtained by combining (2.5), (2.8), (2.11), and (2.22), i.e.

$$\vec{J}_s(\vec{r}) = (1 + \zeta_x^2 + \zeta_y^2)^{-1/2}\, exp\,(-j\vec{k}_i \cdot \vec{r}) \sum_{n=0} \frac{\vec{j}_n(\vec{r})}{k_o^n} \qquad (2.23)$$

One of the immediate consequences of (2.23) is that the current on any shadowed portion of the surface is identically zero. This is obviously a high frequency approximation, but the analysis presented above makes no *explicit* approximations and, in fact, appears to be exact. Clearly, an exact analysis cannot lead to an approximate result. What is happening in this case is that the L-K series is doing the best job that an *asymptotic* series can do in representing the current in the shadowed parts of the surface. The failure of the L-K asymptotic series is linked to the fact that the current in the shadow-zones of the surface *cannot be represented* by an asymptotic series of the L-K form. To prove this, recall that the definition of an asymptotic series such as (2.11) is that if S_m represents the partial sum of the first m+1 terms * then

$$\lim_{k_o \to \infty} k_o^m \left[\vec{L}(k_o, \vec{r}) - \vec{S}_m \right] = 0 \tag{2.24}$$

for all values of m. In the limit as $k_o \to \infty$, the current in the shadow-zones of the surface is zero. Thus, from (2.24) with m=0, $\vec{S}_o = 0$ or $\vec{j}_o(\vec{r}) = 0$. For m=1, (2.24) yields

$$\lim_{k_o \to \infty} (k_o \vec{L}) = \lim_{k_o \to \infty} (k_o \vec{S}_1)$$

but $\vec{S}_1 = \vec{j}_1 / k_o$ because $\vec{j}_o = 0$, so this leads to

$$\vec{j}_1 = \lim_{k_o \to \infty} (k_o \vec{L}) \tag{2.25}$$

It is well known that the current in the shadow zone has the form of a creeping wave [11]

$$\vec{L} \sim \sum_{\ell=1} \vec{C}_\ell \exp\{-j[k_o \delta + \exp(-j\pi/3)\beta_\ell (k_o/2)^{1/3} \int_o^\delta \kappa^{2/3} d\delta]\} \tag{2.26}$$

where C_ℓ are the launching amplitudes, δ is the distance measured along the surface, β_ℓ is a root of the Airy integral [11], and κ is the curvature of the surface at the distance δ measured along the surface. Thus, in the limit as $k_o \to \infty$, each term in the series of (2.26) exhibits an exponential dependence on k_o. Furthermore, since the real part of this dependence leads to an *exponential* decay with distance, it is clear that

$$\vec{j}_1 = \lim_{k_o \to \infty} (k_o \vec{L}) = 0$$

and, in fact, *all* of the higher $\vec{j}_n$'s will vanish. Thus, the only acceptable asymptotic series for the current in the shadow region is the *null* series. The reason for this is contained in the definition of the asymptotic series, e.g. (2.24). That is, the only series of the form

$$\vec{S}_m = \sum_{n=0}^{m} \vec{j}_n(\vec{r}) / k_o^n$$

which can satisfy (2.24) for all m *in the shadowed* regions of the surface is the null series. Note that it is the constraint imposed by the *form* of the asymptotic series which dictates the end result. *Thus, other than the null series result, the current in the shadow region does not have an*

* $\vec{S}_m = \sum_{n=0}^{m} \vec{j}_n(\vec{r}) / k_o^n$

asymptotic series representation. This is an important result because it is the first time (to the author's knowledge) that the failure of an L-K series in the shadow region has been both demonstrated and explained.

While the L-K series does not lead to an *exact* result for the surface current density, it still holds the potential for providing a tractable improvement to a pure geometrical optics solution. This fact is demonstrated by Ansorge's [12] calculations for scattering by a dielectric sphere using the ray optics field approach. The attractiveness of the current approach as developed here is due in large part to the fact that the complete L-K representation can be developed *entirely* from an integral of known functions, i.e.

$$\int \nabla_o G(|\Delta\vec{r}|)\, exp\,(j\vec{k}_i \cdot \Delta\vec{r}\,)d\vec{r}_{t_o}.$$

While a complete L-K series development for this integral is prohibitive, it should be possible to obtain the terms up to and including k_o^{-2} [9]. One of the primary advantages of obtaining the k_o^{-1} and k_o^{-2} corrections to the k_o^0 asymptotic expansion of this integral is the recovery of some of the cross polarizing properties of a rough surface in the high frequency (but not optical) limit.

Having developed the Luneburg-Kline expansion for the current, it is a relatively straightforward matter to find the scattered field. The far-field approximation for the scattered magnetic field is

$$\vec{H}_s(\vec{R}\,) = -j\,\frac{\exp(-jk_oR)}{4\pi R}\,\hat{k}_s \times \int (k_o\vec{J}\,)\, exp\,(j\vec{k}_s \cdot \vec{r}_o)d\vec{r}_{t_o} \tag{2.27}$$

where R is the distance from the origin on the surface to the point of observation and $\hat{k}_s$ specificies the direction, e.g. $\vec{R} = R\hat{k}_s$. Substituting the Luneburg-Kline expansion for $\vec{J}$ in the above and assuming an interchange of the summation and integration yields

$$\vec{H}_s = -j\,\frac{\exp(-jk_oR)}{4\pi R}\,\hat{k}_s \times \sum_{n=0} \frac{1}{k_o^n} \int \frac{\vec{j}_n(\vec{r}_o)}{k_o}\,\exp[j(\vec{k}_s - \vec{k}_i)\cdot\vec{r}_o]\, d\vec{r}_{t_o} \tag{2.28}$$

The integral is expanded in a Luneburg-Kline series as follows;

$$\int \frac{\vec{j}_n(\vec{r}_o)}{k_o}\,\exp[j(\vec{k}_s - \vec{k}_i)\cdot\vec{r}_o]d\vec{r}_{t_o} = \sum_{m=0} \frac{\vec{h}_{nm}}{k_o^m} \tag{2.29}$$

where

$$\vec{h}_{no} = \lim_{k_o \to \infty} \int \frac{\vec{j}_n(\vec{r}_o)}{k_o} \exp[j(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_o] \, d\vec{r}_{t_o} \tag{2.30a}$$

$$\vec{h}_{n1} = \lim_{k_o \to \infty} k_o \left[\int \frac{\vec{j}_n(\vec{r}_o)}{k_o} \exp[j(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_o] \, d\vec{r}_{t_o} - \vec{h}_{no} \right] \tag{2.30b}$$

and

$$\vec{h}_{np} = \lim_{k_o \to \infty} k_o^p \left[\int \frac{\vec{j}_n(\vec{r}_o)}{k_o} \exp[j(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_o] \, d\vec{r}_{t_o} - \sum_{m=0}^{p-1} \frac{\vec{h}_{nm}}{k_o^m} \right] \tag{2.30c}$$

so that $\vec{H}_s$ is given by

$$\vec{H}_s = -j \frac{e^{-jk_oR}}{4\pi R} \hat{k}_s \times \sum_{n=0} \sum_{m=0} \frac{\vec{h}_{nm}}{k_o^{n+m}} \tag{2.31}$$

The geometrical optics field is proportional to $\vec{h}_{oo}$ which is determined by $\vec{j}_o$ only. The part of $\vec{H}_s$ which varies as k_o^{-1} depends on both $\vec{j}_o$ *and* $\vec{j}_1$, the part that varies like k_o^{-2} requires $\vec{j}_o$, $\vec{j}_1$, *and* $\vec{j}_2$, and so forth for higher order terms. It should be noted that with the physical optics approximation, one obtains only those contributions from $\vec{j}_o$. However, it is also known that only the *frequency independent* term is always accurate and the above series shows why this is true, i.e. the $\vec{j}_1$ contribution to the k_o^{-1} term is ignored, the $\vec{j}_1$ *and* $\vec{j}_2$ contributions to k_o^{-2} term are ignored, etc. Inclusion of these $\vec{j}_1$, $\vec{j}_2$, etc., terms will definitely improve the frequency dependence of the result relative to the physical optics approximation. There is another advantage to including these terms. Because of the asymptotic nature of the series being dealt with here, each term by itself will be accurate only over a limited range of observation directions. For example, the part of $\vec{H}_s$ contributed by $\vec{j}_o$ is truly only accurate about the specular direction, i.e., $\vec{k}_{s_t} = \vec{k}_{i_t}$ and $k_{s_z} = -k_{i_z}$. Thus, if more $\vec{j}_n$ terms are included it should make the resulting expression for $\vec{H}_s$ more accurate in directions *away* from specular. Clearly, the calculation of these $\vec{j}_n$'s can lead to definite improvements in existing models and is therefore strongly recommended.

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