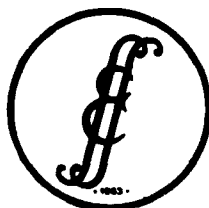ARO 18534.1-EG-CF

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ABSTRACTS

18th Annual Meeting
Society of Engineering Science, Inc.

September 2, 3 & 4, 1981

Brown University
Providence, Rhode Island 02912

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ABSTRACTS


18th ANNUAL MEETING

SOCIETY OF ENGINEERING SCIENCE

SEPTEMBER 2, 3, 4, 1981


BROWN UNIVERSITY

PROVIDENCE, RHODE ISLAND

## PREFACE

The technical program of the Eighteenth Annual Meeting of the Society of
Engineering Science consists of 36 sessions in which 196 papers are scheduled
for presentation. This book contains the abstracts of these paper  The pro-
gram covers recent developments in many areas of engineering science. Multi-
ple sessions have been organized on analytical dynamics, geomechanics, inelas-
tic structural dynamics, aerosols, material instability, fracture, plasticity,
and hydrodynamic stability, transition and turbulence.

The program includes two general lectures. Wednesday morning, Professor
M.D. Kruskal of Princeton University will speak on "Surrounded by Solitons".
Thursday morning, Professor J.B. Keller of Stanford University will speak on
"Interaction Between Mathematics and the Engineering Sciences". Professor
Keller is this year's recipient of the Society's Eringen Medal.

Much of the program was developed by members of the Technical Program
Committee. Their efforts in organizing interesting sessions are gratefully
acknowledged. Special thanks go to the two general lecturers, the authors,
and the session chairpersons for their contributions.

On behalf of the participants we would like to thank the U.S. Army Re-
search Office-Durham for financial support which defrayed part of the expenses
of the meeting.

We also express our thanks to Beth Googins, Sara Mancino, Debra Firth
and other secretaries of the Division of Engineering for their invaluable
assistance in handling correspondence and local arrangements.

Providence                                      R.J. Clifton
September, 1981                             J.R. Rice
                                               Co-Chairmen

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ORGANIZATION OF THE 18TH ANNUAL MEETING

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P.S. Symonds (Brown)
D.P. Updike (Lehigh)
J.H. Weiner (Brown)
U. Yuceoglu (Lehigh)

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Prediction of Thermophoretic Diffusion Effects in Flow Systems, With Applications to Materials Processing[a],[b]

Daniel E. Rosner[c]
High Temperature Chemical Reaction Engineering Laboratory
Yale University, New Haven, CT 06520 USA

## ABSTRACT

While rarely treated in engineering books on mass transport, it is shown that thermal (Soret or thermophoretic) diffusion significantly alters convective mass transport rates and important transition temperatures in nonisothermal flow systems[d] involving the transport of "heavy" species (vapors or particles) or "light" species. These conditions are commonly encountered in materials processing applications, e.g., chemical vapor deposition (CVD). Introduction of the Soret transport term is shown to produce mass transfer effects similar to those of "suction" and homogeneous chemical "sink" --indeed, this analogy provides a simple method of correlating and predicting thermophoretic effects in the abovementioned systems. In most cases the "suction" effect is dominant, with the important dimensionless group:

$$B_T = (\alpha_T Le)_w \cdot \frac{St_h}{St_m} \cdot \frac{\Delta T}{T_w}$$

playing the role of the thermophoretic suction parameter.[d] Here $St_h$ and $St_m$ are the ordinary heat and mass transfer coefficients, $\alpha_T Le$ is the ratio of the thermophoretic diffusivity to the mixture thermal diffusivity, and $\Delta T$ is the difference between the wall temperature and the neighboring fluid temperature. The effects of thermophoresis are illustrated in several specific cases, mainly involving vapor or particle transport through gases, with or without simultaneous chemical reactions. The methods outlined herein allow the rational inclusion of Soret transport effects in diverse applications, and, with the extensions indicated, should allow greatly improved engineering predictions of mass transfer phenomena in nonisothermal systems with large molecular weight disparities.

CZOCHRALSKI SILICON CRYSTAL GROWTH
SYSTEM MODELING

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## ABSTRACT

Heat and mass transfer processes are important in controlling the growth conditions and materials properties of large diameter Czochralski silicon crystals. An overall crystal growth system model based on these processes would greatly benefit the optimization of Czochralski silicon materials used in integrated circuit and solar cell manufacture. The status of presently available knowledge pertaining to such a model is assessed. Recent Czochralski hot-zone temperature measurements which would have impact on model development are presented.

# Transport Phenomena in Space Processing

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Commercial production of glasses for advanced applications often requires processing techniques substantially different from those in common use. In particular, containerless processing is desirable where melt temperatures are sufficiently high that the container wall reacts chemically with the melt and/or promotes crystallization. An ideal environment for containerless processing is provided by the NASA Space Shuttle program because the effective gravitational acceleration in earth orbit is especially low ($\sim 10^{-5}$g) and little levitation is necessary. In such an environment however, convective mixing and buoyant fining (bubble removal) are non-existent. Alternate mechanisms based upon thermocapillary forces have been proposed by Subramanian and Cole [1]. These forces are produced by surface tension gradients and are to be generated by spot heating the surface of a levitated molten glass sphere.

This talk is concerned with ongoing ground-based projects in support of our future shuttle experiments. They include the use of rotation as a means of collecting entrapped bubbles as an aid to subsequent thermal fining (the rotation experiments also have application to the production of glass microballoons for use as laser fusion targets), the rise of single gas bubbles in molten glass to provide information on the susceptibility of low melting glasses to interfacial contamination, thermocapillary convection in pendant drops (silicone oil) and in a cylindrical zone glass melt as a means of mixing, and an analytical investigation of the thermocapillary migration of a gas bubble inside a liquid drop as a means of fining.

1. Subramanian, R.S., and R. Cole, "Space Shuttle Experiments in Glass Processing," AIChE 71st Annual Meeting, Miami Beach, Florida, Nov. 1978.

# FINITE ELEMENT ANALYSIS OF CONVECTION AND
## MASS TRANSFER IN DIRECTIONAl SOLIDIFICATION*

Robert A. Brown and C. J. Chang

Department of Chemical Engineering
Massachusetts Institute of Technology
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The compositional uniformity of dopants and impurities in semi-
conductor crystals grown from the melt depends strongly on the shape of
the melt/solid interface and on the convective flow pattern in the
melt, especially near the interface. Finite-element analysis has been
developed as an analytical tool for studying the coupling between fluid
flow and heat and mass transfer in melt growth systems. In this paper
we outline an efficient method for calculating laminar buoyancy-driven
convection and interface shape in directional solidification. The
model system consists of melt and solid confined in a cylindrical am-
poule that is much longer than its radius. The thermal environment of
a typical furnace is mimicked by varying the boundary conditions on
temperature and heat flux along the length of the ampoule. The temper-
ature and flow fields are taken to be axisymmetric.

The finite-element method is a combination of Galerkin's technique
with a new Newton's method for calculating simultaneously the melt/-
solid interface shape and field variables, temperature, velocity, etc.
[1]. Besides giving quadratic convergence, the Newton's method offers
computer-aided methods for studying the uniqueness of the flow field
and interface morphology.

The structure and intensity of the flow depends on the Rayleigh
and Prandtl numbers. For melts with low Prandtl numbers ($P_r = 0.01$),
heat transfer in the melt and melt/solid interface shape vary only
slightly with increasing Rayleigh number, however the pattern of the
flow may change dramatically. Depending on the aspect ratio of the
melt and the Rayleigh number, the flow may be simply a single toroidal
cell or may separate into two cells stacked on top of each other. How
these effects impact dopant transfer in analyzed by solving the equa-
tion for dopant transport in the melt by finite element analysis. The
ranges of validity are discussed for previous models of mass transfer
that either neglect convection or include convection but neglect inter-
face curvature.

[1] Chang, C.J. and Brown, R.A., Finite element methods for buoyancy-
driven convection near a melt/solid phase boundary. J. Comp.
Physics, submitted (1981).

A Solution of the Mass Transport Equations

with Chemical Reactions by the Finite Element Method

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Thomas L. Theis
Aaron A. Jennings

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The equations governing the convective-dispersive transport of chemically reacting solutes in a saturated porous media are solved by the finite element method. Local chemical equilibrium is assumed throughout the domain.

The method presented keeps the mass transport calculations separate from the chemical equilibrium calculations. Thus, complicated non-linear chemical reactions and solid phase accumulations can be handled. In contradistinction to most other approaches to this problem, changes in the chemical system affect only a specific module in the program, thus leaving the mass transport model unchanged. This feature also makes the programming for multi component systems a relatively simple matter. This modular treatment of the chemistry facilitates the use of existing chemical speciation programs.

Several examples are presented demonstrating the effects of competitive adsorption and soluble complexation on the mass transport of a simple chemical system.

Session WM-2:  NEW DIRECTIONS IN ANALYTICAL DYNAMICS - I

Organizer:    L.Y. BAHAR (Drexel)
Chairpersons: L.Y. BAHAR (Drexel) and H.H.E.LEIPHOLZ (U.Waterloo)

* 9:40 - 10:40   R.W. BROCKETT (Harvard)
                 "Classical Mechanics, Geometry, and Control Theory"

 10:40 - 11:10   COFFEE BREAK

* 11:10 - 11:40  H.H.E. LEIPHOLZ (Waterloo)
                 "Active Structural Control"

* 11:40 - 12:10  L. MEIROVITCH (VPI)
                 "Feedback Control of Distributed-Parameter Systems"

 12:10 - 12:40   P.D. SPANOS (U. Texas, Austin)
                 "First Passage Problem for the Response Amplitude of a
                 Randomly Excited Structure"

# CLASSICAL MECHANICS, GEOMETRY, AND CONTROL THEORY

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## ABSTRACT

Recent development in the area of dynamical systems have given us both new insights into the properties of nonlinear differential equation models and new ways to organize what we know about solving nonlinear differential equations.  A central theme in this program has been the use of geometry, both as a source of new ideas and as a language for unifying and extending results.  Of course, most of the engineering of dynamical systems concerns the effect of exogenous inputs on dynamical systems; a subject which has been developed most vigorously by control theorists. In this talk I want to amalgamate these points of view giving prominence to those ideas which seem to have the most significance for modeling and control.  More specifically, the talk will describe recent developments in completely integrable Hamiltonian systems, bifurcation theory, chaos and strange attractors, the global formulation of nonlinear input-output models; and a discussion of new methods for investigating Liapunov stability.  Ideas from differential geometry and, more specifically, Lie theory will be introduced as needed.  Background information can be found in the book by R. Abraham and I. Marsden, Foundations of Mechanics (2nd ed.), Addison-Wesley, 1979; and in my paper "Control Theory and Analytical Mechanics" which appears in Geometric Control Theory (C. Martin and R. Hermann, Eds.), Math Sci Press, 1976.

# ACTIVE STRUCTURAL CONTROL

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Technological developments in the context of design and building of modern civil engineering structures like bridges, multi-storey buildings, TV-towers, antennas, etc., have led to situations in which these buildings are more and more exposed and sensitive to dynamic loading. The complexity of such situations and the relatively high flexibility of the structures involved force the designer frequently to abandon the field of a statical analysis and to take up a dynamical approach to the problems posed. Yet, dealing with the structure as a dynamical system suggests certainly the inclusion of structural control in one's investigations. Structural control against earthquake motions, vibrations due to wind gusts, explosions, and moving traffic loads has indeed been considered in the past in the literature and in actual design. This paper will give a certain survey of these attempts.

If there is a justification for talking specifically of structural control, then, it is due to the fact that the nature of the loading and of the structure subjected to this loading leads to specific, typical mathematical formulations and to pertinent problems. For example, one may have discrete systems with large degrees of freedom. How to optimize a control for such a system? How to place the observers and how to choose their order? How to reduce possibly the number of degrees of freedom of the system without affecting adversely the mathematical modelling of the system by omitting important features of its behaviour?

Also, one may have to deal with continuous systems. How could one effectively discretize such systems? This may be necessary in order to apply classical control theory to these systems. However, in this paper, the fact will be emphasized that actually the abstract mathematical formulation of the control problem for continuous elastic bodies leads to functional equations, variational problems with inequality constraints, and to a set of theoretical results which do exist already in the theory of partial differential operators and in functional analysis in the broadest sense. These results should be brought to the attention of the civil engineer dealing with control aspects of structural design. This paper will therefore be addressed specifically to the dissemination of such existing knowledge and techniques.

# Feedback Control of Distributed-Parameter Systems

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## Abstract

Feedback control of dynamical systems implies the generation of forces depending on the displacement, or on the velocity, or both on the displacement and velocity, where the forces are designed so as to drive the system to a given state, ordinarily an equilibrium state. This amounts to modifying the system so that, in the presence of feedback controls, the system eigenvalues possess negative real parts. The various methods of control differ in the manner in which this is accomplished.

Control of distributed systems represents one form or another of modal control, which implies that the system is controlled by controlling the individual modes. But, a distributed system possesses an infinity of modes, and it is not practical to control the entire infinity of modes. Hence, truncation is a virtual necessity.

Modal control is carried out in the time domain, which implies that the spatial dependence must somehow be eliminated. This is done by representing the dependent variable by a series of the space-dependent eigenfunctions multiplied by time-dependent generalized coordinates, known as modal coordinates. Truncation of the series implies that the distributed system is controlled through a surrogate discrete system. For linear control, the forces are linear combinations of the state variables, where the transformation matrix is the control gain matrix. For optimal control, the gain matrix is obtained by solving a matrix Riccati equation (Ref. 1). But, for the discrete system to be a good representation of the distributed system, the order of the discrete system must be relatively large. This is likely to cause computational difficulties, as solutions for matrix Riccati equations can be carried out successfully only for relatively low-order systems and they tend to blow up for high-order systems. Moreover, the computational time required tends to increase dramatically as the system order increases. A method circumventing these difficulties is the independent modal-space control (IMSC), whereby the controls are designed so that each mode can be controlled independently. The IMSC method leads to a set of independent second-order Riccati equations that are solved in parallel (Ref. 2). The solution of second-order Riccati equations presents no computational difficulties and the computational time for implementation of the scheme based on the IMSC is smaller than that based on coupled controls by several orders of magnitude. A numerical example corroborating these statements is presented.

References.
1. Anderson, B. D. O., and Moore, J. B., Linear Optimal Control, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971.
2. Meirovitch, L., and Baruh, H., "Control of Self-Adjoint Distributed Systems", Proceedings of the AIAA Guidance and Control Conference, Danvers, MA, August 11-13, 1980, pp. 1-10.

FIRST PASSAGE PROBLEM FOR THE RESPONSE AMPLITUDE

OF A RANDOMLY EXCITED STRUCTURE

by

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Numerical aspects of the solution of the Kolmogorov backward partial differential equation governing the first-passage problem associated with a Markovian approximation of the amplitude of the response of a lightly damped linear structure subjected to stationary broad-band random excitation are examined. Application of the technique of separation of variables on this equation leads to a boundary value problem.

A theorem is proved which provides analytical estimates of the eigenvalues of the boundary value problems as functions of the barrier of the first-passage problem. The analytical estimates and a recursive numerical scheme are used to compute the first twenty (20) eigenvalues for a range of barriers of practical importance.

An algorithm is presented for the computation of the eigenfunctions of the boundary value problem. This algorithm is found particularly efficient in computing the eigenfunctions at a large number of densely spaced points. The first five (5) eigenfunctions corresponding to the assumed barrier values are computed and plotted versus the initial value of the response amplitude.

Finally, the constant coefficients associated with the first twenty (20) eigenfunctions of a series solution of the Kolmogorov equation are computed and tabulated for barrier values identical to those assumed in determining the eigenvalues and the eigenfunctions of the problem.

The trends of the eigenvalues, eigenfunctions, and constant coefficients are examined qualitatively and quantitatively from a physical point of view, and correlation is made, when possible, with existing experimental data.

It is suggested that the analytical estimates and the numerical data obtained by the present study can be the basis of parameter studies investigating the reliability of randomly excited structures as a function of the barrier size, the damping and the stiffness of the structure, and the duration of the random excitation.

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Session WM-3:  ENVIRONMENTALLY-INFLUENCED FRACTURE/FATIGUE

Organizer:    R.O. RITCHIE (U. Cal., Berkeley)
Chairperson:  R.O. RITCHIE (U. Cal., Berkeley)

* 9:40 - 10:10   A.W. THOMPSON and I.M. BERNSTEIN (Carnegie-Mellon)
                 "The Role of Microstructure in Environmentally-Influenced Fracture"

* 10:10 - 10:40  R.P. WEI (Lehigh)
                 Environmentally-Influenced Fracture: Role of Hydrogen Embrittlement"

  10:40 - 11:10  COFFEE BREAK

* 11:10 - 11:40  F.P. FORD (GE, Schenectady)
                 "Environmental Influence on Crack Propagation: Role of the
                 Slip-Dissolution Mechanism"

* 11:40 - 12:10  R.O. RITCHIE (U. Cal., Berkeley)
                 "Environmentally-Influenced Fracture: Role of Crack Closure"

  12:10 - 12:25  Y. MURAKAMI (Northwestern and Kyushu Inst. Tech., Japan)
                 "A Consistent Method for Estimating the Effects of Small
                  Three-Dimensional Defects on Fatigue Strength"

  12:25 - 12:40  M.S. EL SAYED (Cairo, Egypt)
                 "Quantitative Description of Delay in Fatigue Crack Propagation"

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THE ROLE OF MICROSTRUCTURE IN
ENVIRONMENTALLY-INFLUENCED FRACTURE

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ABSTRACT

There have been only a few studies of corrosion fatigue which have
explicitly considered microstructure as an experimental variable. How
ever, emerging understanding of the role of microstructure in stress
corrosion cracking and hydrogen embrittlement, together with informa-
tion on microstructural effects on fatigue, can be combined to give a
broad perspective on the problem. Such a perspective will be presented,
and illustrated with recent results on several types of steels.

ENVIRONMENTALLY-INFLUENCED FRACTURE: ROLE OF HYDROGEN
EMBRITTLEMENT, R. P. Wei, Department of Mechanical
Engineering and Mechanics, Lehigh University,
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## ABSTRACT

Environmentally influenced fracture, particularly in
terms of chemically aggressive external environments, is a
long standing problem of technological importance and
scientific interest.  Two basic mechanisms have been invoked
to rationalize experimental observations and provide guid-
ance in engineering decisions -- namely, hydrogen embrittle-
ment and active-path dissolution (particularly for aqueous
environments).  Parallel fracture mechanics and surface
chemistry studies in recent years on the kinetics of
crack growth in gaseous environments and on the kinetics
of the associated chemical reactions have provided sup-
port for the hydrogen embrittlement mechanism(s).  Models,
based on the results of these studies, have provided con-
sistent interpretations of the data.  Experimental data
that are emerging from parallel fracture mechanics and
electrochemistry studies again tend to support hydrogen
embrittlement (vis-á-vis, active path dissolution) as the
responsible mechanism for environmentally assisted crack
growth in the aqueous environments.  A brief review of
the pertinent data will be given, and the role of hydro-
gen embrittlement in environmentally influenced fracture
will be discussed.

ENVIRONMENTAL INFLUENCE ON CRACK PROPAGATION ;
ROLE OF THE SLIP-DISSOLUTION MECHANISM

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The controlling parameters in the slip dissolution
model for the enhancement of crack-propagation in metal/
aqueous environment systems are reviewed, in order to
set limits to the model's validity under various envi-
ronmental and stressing conditions.  These experimental
conditions are bounded by situations where other models
of crack advance may be more applicable eg. hydrogen
embrittlement.

Examples of situations are given where the slip-disso-
lution model is able to predict quantitatively the crack-
propagation rates as a function of variables such as
temperature, stress-intensity, pH, stressing-mode and
electrode potential.  These examples are taken from data
on aluminum and iron-based alloys (eg. stainless, low-
alloy and carbon-steels) in various environments. Equal
attention is given, however, to uncertainties in the
model, which are normally associated with accurate defi-
nitions of, for instance, crack-tip strain rates under
constant-load or -displacement conditions.  It is under
these conditions of uncertainty that most debates arise
as to the precise mechanism of crack-advancement eg.
slip-dissolution, hydrogen embrittlement, or a combina-
tion of the two mechanisms.

ENVIRONMENTALLY-INFLUENCED FRACTURE:  ROLE OF CRACK CLOSURE

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ABSTRACT

In general, mechanisms for the environmental contribution to
cracking during corrosion fatigue crack propagation in engineering
materials involve such processes as hydrogen embrittlement and active
path corrosion (metal dissolution).  Recently, however, aberrations
of corrosion fatigue at very low (near-threshold) growth rates, approach-
ing the threshold $\Delta K_0$ below which cracks appear dormant, have shown
behavioral patterns totally inconsistent with such mechanisms.  For
example, in steels, under certain conditions, near-threshold crack
growth rates can be accelerated in helium gas and decelerated in hydro-
gen compared to behavior in room air.  Accordingly, a new mechanism for
the environmental influences on corrosion fatigue at near-threshold
levels is presented, based on the role of excess corrosion debris, gen-
erated within the crack, in promoting increased crack closure loads in
moist and oxidizing environments (termed "oxide-induced crack closure").
The mechanism, which is specific to conditions where crack tip dis-
placements are of the order of the size-scales of the corrosion debris,
is shown to be consistent with effects of load ratio, strength level,
environment and variable amplitude cycling, which have been reported
for near-threshold corrosion fatigue crack growth behavior.

A CONSISTENT METHOD FOR ESTIMATING THE EFFECTS OF

SMALL THREE-DIMENSIONAL DEFECTS ON FATIGUE STRENGTH

Y. Murakami

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on leave from Department of Mechanical Engineering,
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## ABSTRACT

The fatigue strength of specimens containing cracks, defects or
inclusions has been investigated separately, mostly by two-dimensional
rather than the required three-dimensional consideration. Considerable
progress in understanding fatigue cracks has been made in recent years,
while there has been almost no consistent approach for estimating the
effect of defects and inclusions on fatigue strength.

The fatigue limit of carbon steel is not the critical stress under
which no crack appears, but the threshold stress where the fatigue
crack developed under the stress level, stops propagating. Even the
fatigue limit of the specimen including defects or inclusion is deter-
mined by the nonpropagation condition of cracks emanating from them.
Then, on the basis of the analysis of stress intensity factors for
three-dimensional cracks with various shapes, cracks, defects and
inclusions can be consistently treated.

In our investigation, it is found that the maximum stress intensi-
ty factor along the front of a three-dimensional crack is approximately
proportional to the square root of cracked area. Therefore, the
approximate maximum stress intensity factors for the partially cracked
defects or inclusions can be estimated from the square root of the area
projected on the direction of the maximum tensile stress. Thus, the
square root of the projected area of defects can be regarded as the
controlling dimension that enables one to interpret all fatigue data
consistently. It is confirmed that the predictions are in good
agreement with the experimental data [1].

References

[1] Y. Murakami and M. Endo, Proc. Annual Meeting (1981-4) of JSME.

Quantitative Description of Delay in Fatigue Crack Propagation

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6, Baha El Din Ebn Hanna St.
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Sudden changes of load conditions leads to transient phenomena in propagation of cracks. It has been proved, that the drop of amplitude of stress, drop of mean value of stress and the overload lead to a distinctive transient deceleration. The value of transient deceleration and its durations depend on the parameters of loading before and after the sudden change conditions.

A quantitative description of transient phenomena is presented, that is based on the idea of the dominant role of residual stresses at the crack tip, the steady state fatigue crack propagation, and the threshold values. Experimental results measured on Cr-Ni steel specimens, of the course of transient propagation, are within an acceptable accordance with this quantitative description.

Session WM-4:  INELASTIC STRUCTURAL DYNAMICS - I

Organizer:    P.S. SYMONDS (Brown)
Chairperson:  G.R. ABRAHAMSON (SRI International, California)

* 9:40 - 10:10    S. KALISZKY (U. Wisc., Milwaukee and Tech. University, Budapest)
                  "Optimal Design of Dynamically Loaded Rigid-Plastic Structures"

* 10:10 - 10:40   M.J. MIKKOLA (VPI & SU) and J. MYLLYMAKI (Helsinki U.)
                  "Large Deflections of Impulsively Loaded Plane Frames"

  10:40 - 11:10   COFFEE BREAK

* 11:10 - 11:40   E.B. BECKER and G. FINE (U. of Texas, Austin) and
                  H.D. HIBBITT (Hibbitt & Karlsson, Providence)
                  "Some Simple Approximate Methods in Inelastic Structural
                  Dynamics"

* 11:40 - 12:10   P.S. SYMONDS (Brown)
                  "Applications of a Simple Approach to Peak and Final Deformations
                  due to Pulse Loading"

  12:10 - 12:25   H.L. WISNIEWSKI and A.D. GUPTA (Ballistic Res. Lab., Aberdeen  MD)
                  "Response Analysis of a Hemispherical Containment Structure
                  Subjected to Transient Loads"

  12:25 - 12:40   A.D. GUPTA and H.L. WISNIEWSKI (Ballistic Res. Lab., Aberdeen  MD)
                  "A Comparison of Structural Response Using a Sublayer Model
                  in a Finite Difference and a Finite Element Computer Code"

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# OPTIMAL DESIGN OF DYNAMICALLY LOADED RIGID - PLASTIC STRUCTURES

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An approximate method is presented for the optimal design of rigid-perfectly plastic inhomogeneous structures subjected to short-time dynamic pressures.  The structure is subdivided into homogeneous parts $V_k$ (k = 1, 2, ...n), in which the specific cost $C_k$ and the yield stress of material are given.  The location, size and shape of the parts are to be determined such, that the parts do not go out of prescribed domains $H_k$ and $G_k$, the permanent displacements $W_{ph}$ caused by the pressure do not exceed the permissible displacements $W_{oh}$ and the cost of the structure be a minimum:

$$C = \sum C_k V_k = \min!$$

subject to

$$H_k \subseteq V_k \subseteq G_k, (k = 1,2,...n); \quad W_{ph} \leq W_{oh}, (h = 1,2,...m)$$

For the estimation of permanent displacements kinematically [1] admissible stationary displacement fields are imposed on the structure[1]. Solutions can be developed by mathematicall programming.  Application is illustrated through the optimal design of thick-walled concrete cylinders reinforced by steel wires on their exterior surface and subjected to internal dynamic pressures[2].

[1]  Kaliszky, S.:  Approximate Solutions for Implusively Loaded Inelastic Structures and Continua.  Int. Journ. Non-linear Mech. Vol. 5 pp. 143-158 (1970).

[2]  Heinloo, M.; Kaliszky, S.:  Optimal Design of Dynamically Loaded Rigid-Plastic Structures. Journ. Struct. Mech. (to be published).

Large Deflections of Impulsively Loaded Plane Frames

by

Martti J. Mikkola* and Jukka Myllymäki

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Helsinki University of Technology, Finland

## ABSTRACT

The paper deals with a numerical study on the response of plane
frames subjected to impulsive loads, which are sufficiently severe
to cause plastic flow and large permanent displacements.

The equations of equilibrium are formulated using the total
Lagrangian and updated Lagrangian descriptions. The finite
element method is employed for spatial discretization. Numerical
time integration is carried out using the central difference
scheme and the trapezoidal rule. Material behavior is modeled by
elastic-plastic and elastic-viscoplastic relationships.

The computer method developed was applied to several test
cases on steel, titanium, and aluminum beams and frames. In the
analysis of the recent tests on steel and titanium frames by
Bodner and Symonds, satisfactory agreement with observed final
permanent deflections was obtained. Aluminum beams, frames, and
arches tested and analyzed by Hashmi, Al-Hassani, and Johnson and
by Witmer and Balmer were also investigated. In the case of a
cantilever loaded at the top it was observed that the computed
deflections were closer to experimental ones when the impulse
was applied in the form of a distributed time-dependent load over a
certain interval rather than as an initial velocity attached to
the tip mass. Although the agreement of finite element prediction
with the experimental results was satisfactory in general, deviations
due to element types and types of mass matrices were observed.

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SOME SIMPLE APPROXIMATE METHODS IN INELASTIC
STRUCTURAL DYNAMICS

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[2]Hibbitt and Karlsson, Providence RI

### ABSTRACT

For some problems involving large inelastic deformations of dynamically loaded
structures there is a significant discrepancy between the amount of detail used
in finite element modelling of the structure and that required from the analy-
sis. Stability considerations associated with detailed spatial modelling lead
to very expensive temporal integration--either explicit methods with small time
steps or implicit methods with attendant iteration and equation solving. Moti-
vated, in part, by the remarkably effective approximate-analytical method of
Symonds, see e.g. [1], we seek computational techniques for reducing the cost
of integrating the equations of motion of dynamically loaded structures.

Specifically, our search is guided by the following precepts:
    1. The method should be applicable to conventional finite element modelling
of structures and be capable of being easily incorporated in existing general
purpose codes.
    2. The desired results are gross measures of response, such as the value of
and time to reach maximum deflection, rather than the detailed time history of
stress distribution. By asking for less we expect to pay less for the computation.
    3. The method should not demand decisions of the user which require a priori
insight into the behavior of the system.

We employ alternate intervals of explicit (central difference) and implicit
(Newmark-Hilber Hughes) integration. The explicit integration is applied to
an approximate set of equations defined on a subspace of small dimension. The
basis of this subspace is a set of normal modes calculated by linearization at
the beginning of the explicit interval. During the implicit interval the full
set of equations is treated using Newton iteration to assure the satisfaction of
dynamic equilibrium. Economy dictates long explicit intervals and brief implicit
ones.

Numerical studies have been directed toward evaluating the effectiveness of the
method as a whole and of self adaptive procedures for determining the appropriate
dimension of the subspace, the time step used in and allowable extent of the
explicit intervals. For a certain class of problems the method appears highly
effective, while for another class of problems the efficacy is less pronounced.

1. Symonds, P. S., Finite Elastic and Plastic Deformations of Pulse Loaded
   Structures by an Extended Mode Technique, Int. Jour. Mech. Sci. 22,
   pp. 597-605, (1980).

- 22 -

Applications of a Simple Approach to
Peak and Final Deformations due to Pulse Loading

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Brown University
Providence  RI   02912

An impact or high intensity pressure pulse produces first elastic re-
sponse, then a complex intermingling of elastic and plastic response,
and finally an elastic response that is eventually damped out, leaving
permanent deformations.  A simple approach [1, 2, 3] to obtaining the
main deflections treats the response either as all elastic (usually
obeying small deflection linear field equations) or all plastic.  In
the latter case, the behavior is idealized as rigid-perfectly plastic
with appropriate dynamic yield stress, and the response consists of a
one-degree of freedom (mode form) motion, which is continued to large
deflections.  The initial amplitude is chosen so as to provide the
closest fit to the preceding velocity field in the sense of a least
square difference.  Recent applications are described to structures
subjected to finite pulses where elastic effects may play an essential
role, including rectangular frames under side loading on one column,
and non-uniform beams.  Illustrations are discussed of estimates of
peak and final deflections in the final general motion and of plastic
deformations in the initial phase of mainly local response, including
comparisons with complete numerical finite element solutions, and
with experiments.

1.  P.S. Symonds and J.L. Raphanel, Proc. Conf. on Mech. Props of
    Materials at High Rates of Strain Institute at Physics Conf.,
    No. 47 (J. Harding, Ed.), London, pp. 227- 287, 1979.

2.  P.S. Symonds, Int. J. Mech. Sci., Vol. 22, pp. 597- 605, 1980.

3.  P.S. Symonds, J. Mech. Eng. Sci., Vol. 22, No. 4, pp. 189- 197,
    1980.

RESPONSE ANALYSIS OF A HEMISPHERICAL CONTAINMENT STRUCTURE SUBJECTED TO
TRANSIENT LOADS.

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Mathematician
Aaron D. Gupta
Mechanical Engineer
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Aberdeen Proving Ground, MD.21005

Large deflection elastic-plastic response of a hemispherical containment shell
configuration clamped to a horizontal rigid foundation subjected to an internal
blast was analysed using a finite-difference structural response code, P-Trde 3.5.
The reflected overpressure loading was estimated from a scaled distance of the wall
from the point of detonation based on a conservative cube-root scaling law and
an exponential decay. The residual quasi-static overpressure was obtained from
an equation developed by Kinney and Sewell based on the ratio of the available
vent area and the internal volume.
          Only a quarter segment of the structure was modelled using 17 equal
width meshes in one layer and four Gaussian integration points in each mesh
( through the thickness ). The mild steel was represented by three sublayers
followed by a perfectly-plastic behavior and elastic-plastic unloading resulting
in a polygonal approximation.
          The results indicated the initiation of flexural waves at the boundary
propagating towards the pole, thereby altering the spherically symmetric breathing
mode of response of the centrally loaded hemisphere. The peak deflection predicted
by the code occured at the pole and permanent deformation was found to be quite
small after elastic oscillations were damped out. Transient strain components
at the inner and outer surfaces near the clamped edge showed the effect of bending
deformation superposed on the membrane components. In conclusion the protective
structure was found to be efficient and cost effective and capable of blast
containment with an adequate margin of safety.

# A COMPARISON OF STRUCTURAL RESPONSE USING A SUBLAYER MODEL IN A FINITE DIFFERENCE AND A FINITE ELEMENT COMPUTER CODE

**Aaron D. Gupta**
**Henry L. Wisniewski**

**Ballistic Research Laboratory**
**US Army** Armament Research and Development Command
**Aberdeen Proving Ground, MD 21005**

## ABSTRACT

An elastoplastic three-dimensional mechanical sublayer hardening model has been recently incorporated in a nonlinear finite element structural response code i.e., ADINA developed at M.I.T. by Professor K. J. Bathe. The model complements nonlinear geometric capability in the code with an improved nonlinear material representation in the form of a polygonal approximation of the constitutive relation.

The results from the modified code were compared with those from the existing isotropic and kinematic hardening models and the sublayer model from a finite-difference shell response code i.e., PETROS for an impulsively loaded clamped aluminum plate. Satisfactory correlation was obtained in both elastic and elastic-perfectly plastic material models but some discrepancy was observed during the plastic phase of the deformation possibly due to the difference in response behavior between the three-dimensional brick element and the two-dimensional shell element discretization. A plane stress sublayer hardening model is currently under development to allow a better evaluation of nonlinear material response.

**Speaker:** Aaron D. Gupta
**Telephone** Number: (301) 278-5574

<u>Session WM-5</u>:  MECHANICS OF HYDRAULIC FRACTURING

Organizer:    K.G. NOLTE (Amoco, Tulsa)
Chairperson:  K.G. NOLTE (Amoco, Tulsa)

* 9:40 - 10:40    K.G. NOLTE (Amoco, Tulsa)
                  "Industrial Hydraulic-Fracturing and Applied Mechanics"

  10:40 - 11:10   *COFFEE BREAK*

* 11:10 - 11:40   R.J. CLIFTON (Brown)
                  "Prediction of the Three-Dimensional Geometry of Hydraulic
                  Fractures"

* 11:40 - 12:10   P. BRUNN (Columbia)
                  "Particle Migration Phenomena for Certain Flows of a
                  Viscoelastic Fluid"

  12:10 - 12:40   R.B. STOUT, L. THIGPEN and J. PETERSON (Lawrence Livermore Labs.)
                  "Modelling Stochastic Microcrack Dependent Deformations in
                  Rock Materials"

## INDUSTRIAL HYDRAULIC-FRACTURING AND APPLIED MECHANICS

K. G. Nolte, Amoco Production Co., Tulsa, OK

INTRODUCTION: The purpose of this paper is to review hydraulic fracturing in the petroleum industry, and to highlight technology gaps which relate to applied mechanics. Fracturing has been extensively used since 1950 ($5 \times 10^5$ by 1968) to increase petroleum production. This results by creating in the rock a hydraulic wedge carrying a propping agent (e.g., sand) to form a high (relative to formation) permeability corridor to the wellbore. Recently, treatments an order of magnitude larger (e.g., 1000m penetration, 3000m$^3$ fluid, $10^6$ kgm sand, and $5 \times 10^5$) than the historical average have been used for the economic development of very low permeability formations at depths ~2500m and ~150°C A comparable increase has resulted in the need to understand the mechanics of the complex fluid/rock problem. However, a major portion of the literature is not applicable to the current problem, and a refocusing is required.

FLUID MECHANICS: The slurry generally consists of up to a 0.5 vol. fraction of sand in a water-based organic-polymer solution crosslinked with transitional metals, and at ambient conditions is more solid than fluid. These systems best meet the economic and functional requirements of crush resistant proppant, particle transport, minimal fluid loss and eventual degradation for the required residue-free backflow. Thus the usual assumptions are not applicable, i.e., Newtonian (very viscoelastic), laminar (some slip on degraded boundary), Stokes fall (high vol. fraction and major effect from cross-link network) and D'Arcy fluid loss (polymer filter cake built on formation wall). The more critical areas are the pressure gradients from flow, and the velocity field of the suspended particles which are subjected to time varying concentrations, gravity currents and gradients of normal stress differences.

SOLID MECHANICS: Recent measurements[1] during large treatments indicate that viscous stresses (pressure differences ~$10^4$kPa) dominate the fracture geometry (boundary and width changes) instead of the generally assumed material properties effects (~$10^2$kPa). Also, for the vertically planar fracture, the horizontal extension exceeds the vertical (>10X). Thus a vertical plane-strain assumption (vs. generally assumed horizontal) is more valid. The large viscous pressures and limited vertical growth (an economical requirement) indicate significant in situ stress difference in the various rock layers; i.e., larger horizontal stresses in bounding layers (generally shales) than target layer (e.g., sandstone). As a result, future work should be directed at oblique fracture growth in layered media with significantly different in situ stresses[2] and modulii. Also desirable is an understanding of the higher stresses in shales (i.e., smaller particles, bound water, etc.) than in the frictional dominated sandstones (e.g., due to relaxation and/or pressure/temp. histories).

[1]Nolte and Smith, "Interpret. of Fracturing Pressures," SPE 8297, 1979.
[2]Clifton and Abou-Sayed, "On the Computation...," SPE 7943, 1979.
 Cleary, "Analysis of Mechanisms and Procedures..., SPE 9260, 1980.

Prediction of the Three-Dimensional Geometry of Hydraulic Fractures

by

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Brown University
Providence RI 02912

A computational method is outlined for modeling the three-dimensional development of hydraulic fractures due to the injection of a non-Newtonian fluid at the well bore. The rock formation is modeled as an infinite, homogeneous, isotropic, elastic solid with *in situ* stresses that vary with depth. Vertical migration of the fracture is controlled by the presence of *in situ* compressive stresses that are larger in the layers above and below the gas or oil bearing layer than in the layer that is being fractured. This three-dimensional problem is made two-dimensional by formulating the elasticity problem as an integral equation that relates pressure on the crack faces to crack openings and by neglecting the component of the fluid velocity in the direction perpendicular to the fracture plane. Variational principles are used to derive the discrete form of the governing equations that are used for the computations. These equations are nonlinear and involve coupling between the elasticity of the formation and the flow of the fluid in the fracture. In addition the locations of the crack front and the fluid front are undetermined *a priori* and the gradient of the crack-opening displacement is unbounded at the crack front. Computational approaches for dealing with these complications are presented.

References:

1. Clifton, R.J. and Abou-Sayed, A.S., "A Variational Approach to the Prediction of the Three-Dimensional Geometry of Hydraulic Fractures", Proceedings of the SPE-DOE Symposium on Low-Permeability Gas Reservoirs, May 27-29, 1981, Denver, Colorado.

2. Clifton, R.J. and Abou-Sayed, A.S., "On the Computation of the Three-Dimensional Geometry of Hydraulic Fractures", Proceedings of the SPE-AIME Symposium on Low-Permeability Gas Reservoirs, May 20-22, 1979, Denver, Colordao.

3. Simonson, E.R., Abou-Sayed, A.S. and Clifton, R.J., "Containment of Massive Hydraulic Fractures", Soc. of Petroleum Engineers Journal, Vol. 18, No. 1, 1978, pp. 27-32.

Particle Migration Phenomena for Certain Flows
of a Viscoelastic Fluid

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The flow of a viscoelastic fluid over a spherical cap and between
hyperbolic cylinders is studied. Special attention is paid to the
resulting behavior of a rigid, non-neutrally buoyant sphere sus-
pended in the fluid. Anticipating that the flow fields studied can
be used to locally model the flow in a porous material, the results
obtained admit conclusions with regard to the detailed effect of sus-
pended matter.

Modelling Stochastic Microcrack Dependent Deformations in Rock Materials*

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A stochastic model for microcrack kinetics in rock material has been formulated by using some concepts from statistical mechanics. The model defines a stochastic or probablistic density function for the number of microcracks per unit volume per unit species type. For the present model, the species type is a hyperspace over measures for the microcrack area A, the microcrack orientation $\Omega$, the microcrack displacement opening U], and the microcrack displacement opening rate Ú]. A balance or accounting statement for each species of microcrack results in an integral-differential equation for the stochastic microcrack density function. This evolution equation is a microcrack transport equation similar to the Boltzmann equation in classical statistical mechanics.

Based on the above formalism, a simple failure model for rock material has been completed to illustrate concepts of the theoretical development. Through recent extensions, a deformation functional, which depends on the microcrack function space, has been obtained. This leads to approximations for the kinematic measures of strain and strain rate that also depend on the microcrack density function.

Session <u>WM-6</u>:  CONTACT/VISCOELASTICITY/FLUID-SOLID INTERACTION

Chairperson:  T.C.T.TING (U. of Illinois)

9:40 - 10:10   T.C. SOONG (Xerox, Rochester)
"The *Static and Rolling Contact Study of Two Elastic-Layer Covered Cylinders*"

10:10 - 10:25   J. PADOVAN (University of Akron)
"Moving Updated Lagrangian Finite Elements"

10:25 - 10:40   G.A.C. GRAHAM (Simon Fraser Univ., Canada) and M. COMNINOU (Univ. Michigan)
"Periodic Solutions of Viscoelastic Boundary Value Problems"

10:40 - 11:10   COFFEE BREAK

11:10 - 11:40   Y. WEITSMAN and B.D. HARPER (TEXAS A&M)
"Optimal Cool-Down Paths and Residual Thermal Stresses in Cross-Ply Composite Material Laminates and Adhesive Joints"

11:40 - 11:55   T.M. WICKS (Univ. Missouri - Rolla)
"Approximation of a Class of Problems Concerning Fluid-Solid Interaction"

11:55 - 12:10   G.S.A. SHAWKI (University Qatar, Arabian Gulf)
"Hydrodynamic Influences of Surface Undulations in Journal Bearings"

12:10 - 2:15   LUNCH BREAK

"THE STATIC AND ROLLING CONTACT STUDY OF
TWO ELASTIC-LAYER COVERED CYLINDERS"

Tsai-C. Soong, Xerox Corporation, Rochester, N. Y.

At the interface of the contact arc between two elastic-
layer covered cylinders in steady rolling with a thin
sheet in the nip, there are four conditions to be satis-
fied at each point; namely, the normal pressure, the
interference, slip or nonslip between each cylinder and
the sheet. Since each point has to be examined and slip
or nonslip is determined independently from shear con-
sideration, a continuous, analytic solution is more
difficult than the collocation method where the number
of condition equations can be matched by an equal number
of parametric unknowns and solved iteratively.

In the beginning. one may assume all contact points are
nonslipping, that is, the creep velocity is the same,
and then solve for the interface shear stress and normal
pressure. If the shear stress at some points are found
to be more than the Coulomb friction can support, then
the slipping points are unlocked one by one with the
new condition that the shear stress is the Coulomb
friction.

Circular stress functions are used to represent the
stresses in the two layers of the cylinders, and con-
tact conditions in the nip and outside are rigorously
enforced. The attached figure shows an example of the
local speed variation in the contact arc with and
without a sheet in the nip.
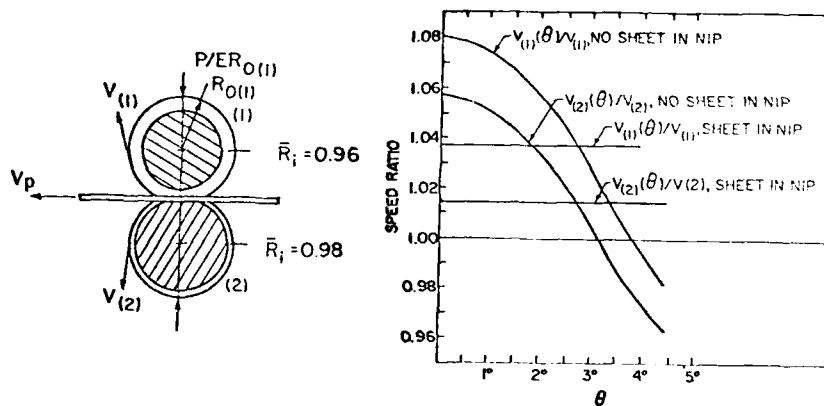
MOVING UPDATED LAGRANGIAN FINITE ELEMENTS
JOSEPH PADOVAN
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A natural outgrowth of bodies being in a state of moving contact
is the fact that each of the interacting structure are subject to
traveling load fields which can accelerate, decelerate as well as have
variable magnitude. The problem of rolling contact is a good example
of such structural interactions. While numerous analytical type linear
elasticity solutions have been developed for such problems, these are
generally limited to simple configurations. Static and dynamic con-
tact type problems have also been tackled by tne finite elememt (FE)
method. These FE solutions have mainly been employed for contact-
impact type problems wherein the time duration is small and where
only a small portion of the structural surfaces are involved. For
longer duration moving contact processes wherein large portions of the
interacting structural surfaces may have been subject to moving load
fields, the classical FE approaches are somewhat awkward to apply. In
this context, the presentation will overview the development of a
moving updated Lagrangian observer FE formulation which combines the
various benefits of the Eulerian and the classical total/updated Lag-
rangian approaches. This is achieved by locking the initial base of
the updated Lagrangian observer to the steady or accelerating/dece-
lerating motions of the interacting structure. As a consequence,
while the FE mesh is monitoring global structural deformations, it
appears to remain stationary during the rolling process. To generalize
the results, specialized isoparametric steady/transient large defor-
mation elements will be developed. In this context, the presentation
will include the formulation of the moving updated Lagrangian metho-
dology as well as give an overview of several numerical applications
to large deformation rolling contact problems.

# Periodic Solutions of Viscoelastic Boundary Value Problems

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## ABSTRACT

A primary source of mathematical interest in the solution of viscoelastic boundary value problems for cracked bodies derives from the unilateral nature of the boundary conditions that should be satisfied over the crack faces (e.g. see [1], [2]). If a point of the crack is in contact with the opposite crack face the normal displacement should be continuous and the normal tractions should not be tensile across the crack at that point. If the crack is open at a point the displacement field should be consistent with that and the traction across the crack surface should be as prescribed.

In this paper we have studied periodic solutions for a particular class of viscoelastic materials; those whose relaxation functions vanish at infinity. Particular problems considered are (i) a Griffith crack in an infinite viscoelastic body, (ii) a crack in a field of pure bending and (briefly) (iii) the interface crack. In all cases the applied loading is presumed to be periodic in time and the corresponding quasi-static elastic solutions would involve crack closure for extensive intervals of time. In contrast, for the viscoelastic solutions of this paper the cracks are entirely open essentially always. A feature of these solutions is that they are obtained from elastic solutions that involve material overlap: the right viscoelastic solution is obtained from the wrong elastic solution.

[1] G.A.C. Graham, Stresses and displacements in cracked linear viscoelastic bodies that are acted upon by alternating tensile and compressive loads, Int. J. Engng. Sc. $\underline{14}$ (1976), 1135-1142.

[2] G.A.C. Graham and G.C.W. Sabin, The opening and closing of a growing crack in a linear viscoelastic body that is subject to alternating tensile and compressive loads, Int. Journ. of Fracture $\underline{14}$ (1978), 639-649.

OPTIMAL COOL-DOWN PATHS AND RESIDUAL THERMAL STRESSES
IN CROSS-PLY COMPOSITE MATERIAL LAMINATES AND ADHESIVE JOINTS

BY

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## Abstract

When composite material laminates and adhesive joints are cooled
from the elevated cure temperature down to room temperature the ever
present geometric constraints introduce residual thermal stresses into
the structure. These stresses sometime approach the ultimate value.

By taking account of the viscoelastic response of the above ma-
terials, which is especially pronounced at elevated temperatures, it
is possible to reduce the magnitudes of the residual stresses. This
reduction can be attained by extending the cool-down time as well as
by following a special, optimal time-temperature cooling path.

The form of this optimal path was obtained previously for isotropic,
viscoelastic, thermoheologically simple materials.[1],[2] A significant
feature of this path is that it contains jump discontinuities at the
initial and final times.

In this paper the previous analysis is extended to some special
lay-ups of composite laminates and modified to include the case of
temperature-dependent coefficients of thermal expansion.

Computations, based upon recent data for graphite/epoxy materials
[3], were performed for symmetric, balanced, cross-ply laminates. It
was found that by taking an appropriate advantage of viscoelasticity
it may be possible to reduce the residual stresses by about 3%. Anal-
ogous computations were also performed for aluminum/epoxy joints.

References:

[1] Weitsman, Y. "Optimal Cool-Down in Linear Viscoelasticity". J. of
    Appl. Mech., ASME, Vol. 47, No. 1, pp. 35-39, (March 1980).
[2] Curtin, M. E. and Murphy L. F. "Optimal Temperature Paths for
    Thermorheologically Simple Viscoelastic Materials". Quart. App.
    Math., Vol. 38, No. 2, pp. 179-190, (July 1980).
[3] Kibler, K. G. "Time-Dependent Environmental Behavior of Graphite/
    Epoxy Composites". Tech. Report AFWAL-TR-80-4052, General Dynamics
    Corp., Forth Worth, Texas, (May 1980).

# APPROXIMATION OF A CLASS OF PROBLEMS
# CONCERNING FLUID-SOLID INTERACTION

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Abstract:

Fluid-solid interaction problems present a variety of
difficulties due to the different description  of motion which
is used for the fluid and solid.  This investigation is con-
cerned with such problems where a viscous fluid is partially
bounded by an elastic membrane.  Some channel flows, where the
wall consist of the membrane, are used as models.  Properties
of solutions representing stationary flows are discussed and an
analysis of finite element approximations of this problem is
given.  In particular we consider iterative schemes which un-
couple the fluid-solid problem and compare them with fully
coupled methods which provide both first and second order
convergence.

# HYDRODYNAMIC INFLUENCES OF SURFACE
# UNDULATIONS IN JOURNAL BEARINGS

By

G. S. A. SHAWKI

Professor and Dean, College of Engineering, Qatar
University, Doha, Qatar, Arabian Gulf.

This paper presents the results of both experimental and
computer-aided study of the performance of a cylindrical
bearing with finite length as influenced by surface undula-
tions intentionally produced on the bearing surface.
Experiments were conducted on an ? mm diameter x ? mm
length journal bearing.

It is herein established that with the load carrying
parameter, high values of wave amplitude $a_w$ and number of
waves along the bearing circumference $n_w$ are shown to lead
to lower eccentricity, hence to enhanced safety of operation.

Compared with a corresponding smooth bearing, $a_w$ ... an in-
crease in $a_w$ and in $n_w$ is shown to ... a slight ...
reduction in the coefficient of friction down to a minimum
value ... at $a_w = ? \mu m$, also ... in the ... minimum
before it regains its value for a smooth bearing at ...
values of $a_w$ and $n_w$.

When running at same eccentricity ratio $\varepsilon$, analytical as well
as experimental results show that higher
values of $a_w$ and $n_w$ force the journal

center to come closer to the ... which
means higher stability. Implications ...
are ... in the ... fig.

Analytical results indicate that, for
wavy bearings, finite hydrodynamic
pressure can still be developed even
under metal-to-metal contact. They also
show that the journal center ... that ...
a whirling motion of minute order.

From the ... viewpoint, wavy bearings
may well prove to be more advantageous
than perfectly smooth bearings as the
former can run at lower eccentricity
and friction values.

## References :

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Thesis, Cairo University, Cairo, Egypt.
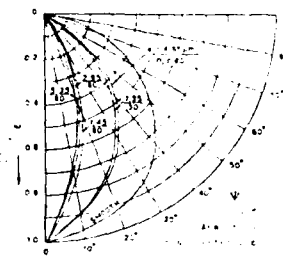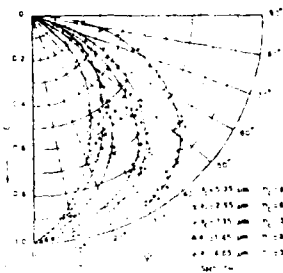
Session WA-1:  CONDENSATION:  SCIENCE AND APPLICATIONS

Organizer:     R.A. DOBBINS (Brown)
Chairperson:   R.A. DOBBINS (Brown)

* 2:15 - 3:15   P.P. WEGENER (Yale)
                "Vapor Condensation in the Supersaturated State:  A Survey of its
                 History, Experimental Methods, Current Applications, and Results"

* 3:15 - 3:45   R.P. ANDRES (Purdue)
                "A Continuum Model for the Microscopic Clusters Important
                 in Homogeneous Nucleation"

  3:45 - 4:15   REFRESHMENT BREAK

* 4:15 - 4:45   J.P. FRIEND (Drexel)
                "A Chemical Kinetic Mechanism for the Formation of Condensa-
                 tion Nuclei"

* 4:45 - 5:15   J.H. CLARKE and C.F. DELALE (Brown)
                "Nonequilibrium Condensation in One-Dimensional Nozzle Flows"

VAPOR CONDENSATION IN THE SUPERSATURATED STATE: A SURVEY OF ITS
HISTORY, EXPERIMENTAL METHODS, CURRENT APPLICATIONS, AND RESULTS.

Peter P. Wegener

Mason Laboratory, Yale University, P.O. Box 2159 Yale Station
New Haven, CT 06520

Phenomena and nomenclature of the great variety of processes of
condensation of vapors in the supersaturated state — in the absence of
foreign aerosols — will be recounted. The history of this field started
in the last century with rapid expansion experiments (1866), Laval steam-
nozzles (1883), and the classical Wilson cloud chamber (1897). In this
century Stodola's work on qualitative clarification of the steam noz-
zle process (1913) was followed by the first observations of condensa-
tion in supersonic wind tunnels by Wieselsberger and by Prandtl (1934).
In 1941 Oswatitsch finally saw the identity of the physics of condensa-
tion in steam and wind tunnel nozzles. He linked the delayed nucle-
ation process to the theoretical work by Volmer (1926), and Becker and
Döring (1935). After this the field began to evolve rapidly. The
invention of the diffusion cloud chamber by Langsdorff (1939), the
development of modern nozzle methods, advanced Wilson cloud chambers,
molecular beam experiments and other schemes took hold in different
branches of science and engineering and their relative merits will be
compared. In recent years — and often to the surprise of those in the
field — condensation problems of this nature have attained technological
importance. They are now encountered in combustion, rocketry, aero-
nautics (hypersonic and transonic wind tunnels) and astronautics, energy
(isotope separation and magnetohydrodynamic energy conversion), etc.,
and a number of examples in these fields will be given in detail. In
parallel to these developments the application of nucleation experiments
in basic physics or chemistry to improve our understanding of small
clusters and the liquid state, and new problems arising in stratospheric
chemistry are of interest.

Finally, the basic physical ideas — rather than the details of
nucleation theory - are presented in conjunction with a listing of the
contemporary attempts to improve expressions for the energy of formation
of critical clusters. Experimental results in conjunction with such
theoretical schemes will be demonstrated for some vapors and vapor mix-
tures as obtained with different experimental methods. It is hoped that
this overview of the field will aid our further discussions of the
fascinating field of nucleation. This is particularly important in
nucleation since the nature of the field requires the participation of
workers in many fields in order to progress. In turn a common view of
our different approaches may be helpful.

# A CONTINUUM MODEL FOR THE MICROSCOPIC CLUSTERS

## IMPORTANT IN HOMOGENEOUS NUCLEATION

Professor R. P. Andres
School of Chemical Engineering
Purdue University
West Lafayette, Indiana 47907

Estimation of the properties of small condensed aggregates or clusters, which contain fewer than 100 molecules, is often critical for prediction of the rate of homogeneous nucleation in a supersaturated vapor. Since the work of Gibbs over a hundred years ago, the free energy of small clusters has been estimated by means of a macroscopic model based on the bulk surface free energy and chemical potential of the stable condensed phase. Despite the fact that the surface free energy of small clusters cannot be equal to that of the bulk phase and that Gibb's expression does not account for free translation and rotation of these clusters, this macroscopic model has been remarkably successful. The reasons for this success and for the success of the steady-state nucleation theory of Becker and Döring, which incorporates Gibb's model, are explored in the present paper.

An empirical expression that is valid both in the limit of infinite size and in the limit of a three molecule cluster is presented. This new continuum model is compared to the classic expression implicit in the nucleation theory of Becker and Döring. It is also compared to detailed statistical calculations which treat small clusters as weakly bound van der Waals' polymers. Next, experimental rates of homogeneous nucleation are compared with the predictions of our model. Finally, the problem of the melting temperature of small particles is addressed, and a size dependent melting point for small particles is derived.

# A Chemical Kinetic Mechanism for the Formation of Condensation Nuclei

by James P. Friend
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The creation of condensation nuclei (measured at 300% relative humidity) by the photooxidation of sulfur dioxide in air has been observed in the laboratory.[1] A chemical kinetic mechanism is proposed[2] which is consistent with the observed rates of nuclei formation as a function of experimental conditions of water vapor concentrations and rates of $SO_2$ oxidation. The proposed immediate precursors to the nuclei are free radicals of the type $HSO_3$ and $HSO_5$ and their hydrates. The chemically stable molecules which result from the radical-radical combinations amongst the hydrated and unhydrated species are thought to be the nuclei upon which droplets form in an automatic condensation nucleus counter operating at 300 relative humidity. The nucleation process by this mechanism is compared with the thermodynamic-based theory of binary homogeneous nucleation for $H_2O$ and $H_2SO_4$ vapors. Possibilities for application of the mechanism to the formation of atmospheric aerosols are discussed.

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## References

1. R. Leifer, J.P. Friend, and M.P. Trichon, "The Mechanism of Formation of Stratospheric Aerosols", Proceedings of the International Conference on Structure, Composition and General Circulation of the Upper and Lower Atmospheres and Possible Anthropozenic Pertubations, As part of the IAMAP/IAPSO combined first special assemblies, Melbourne, Australia, January 14-25, 1974.

2. J.P. Friend, R.A. Barnes and R.M. Vasta, J. Phys. Chem., 1980 84, 2423-36.

Nonequilibrium Condensation in
One-dimensional Nozzle Flows

By JOSEPH H. CLARKE
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## Abstract

We consider the compressible nozzle flow of a vapor and gas mixture
with nonequilibrium homogeneous condensation. We use an asymptotic meth-
od with respect to two disparate parameters appearing in the nonequilib-
rium integral condensation-rate equation coupled to the equations of flow
and of state. We use a high-activation limit and another limit consider-
ation that signify a large nucleation time followed by a small droplet-
growth time. Under the limits, the coupled rate equation is so controlled
by the activation function $B(x)$ over its five zones that there appears a
one-to-one correspondence between these zones and the five characteristic
condensation zones of the nozzle flow itself. The five zones are initial
growth of condensate, further growth, onset region, nucleation zone with
growth, and droplet growth zone. The last zone can be simplified if we
are willing to neglect terms that imply a reasonably sizable order of er-
ror. What results are the rectilinear, one-dimensional flow equations
of a normal shock wave structured or resisted by droplet growth. More-
over, it is not necessary to specify, for the entire general analysis,
the several functions (of the specified independent thermodynamic coor-
dinates) dictated by the complex physics associated with the various
theories of nucleation and droplet growth.

Session WA-2:  NEW DIRECTIONS IN ANALYTICAL DYNAMICS - II

Organizer:    L.Y. BAHAR (Drexel)
Chairperson:  L.Y. BAHAR (Drexel) & H.H.E. LEIPHOLZ (U. Waterloo)

* 2:15 - 2:45    R.B. WASHBURN, JR. (Scientific Systems, Inc., Cambridge)
                 "Application of Qualitative Dynamics to Power System Stability"

* 2:45 - 3:15    L.Y. BAHAR and H.G. KWATNY (Drexel)
                 "When is a Newtonian Dynamical System Derivable from Lagrangian?"

* 3:15 - 3:45    P. HOLMES (Cornell, on sabbatical at U.C.-Berkeley)
                 "Bifurcations and Chaotic Motions in Nonlinear Dynamical Systems"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    R. GILMORE (Inst. Defense Anal., Arlington  VA)
                 "Catastrophe Flags in Conservative and Dissipative Systems"

  4:45 - 5:15    S. DOST and P.G. GLOCKNER (Univ. of Calgary, Canada)
                 "On the Stability of Viscoelastic Perfect Columns"

  5:15 - 5:30    A.K. AZAD and M.H. BALUCH (Univ. Petrol. & Minerals, S. Arabia)
                 "Buckling of an Initially Compressed Column by Subtangential
                  Force"

  5:30 - 5:45    B.S. SHAKER (King Abdul Aziz Univ., Saudi Arabia)
                 "Effect of Nonlinearity on the Response of a Forced System"

# APPLICATION OF QUALITATIVE DYNAMICS
## TO POWER SYSTEM STABILITY

Robert B. Washburn, Jr.

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54 Rindge Avenue Extension
Cambridge, MA  02140

The dynamic behavior of a large interconnected electric power system is extremely complex.  Yet in order to make a rational assessment ot system security or to decide what preventive control action to take in an emergency, it is necessary to make some prediction about the future behavior of the system.  Numerical simulation is the preferred approach for making such predictions and has the advantage over other approaches of providing quantitative predictions "closer to reality" than qualitative predictions.  Such simulations, however, can be extremely time-consuming to obtain and the results can be "myopic" in the sense that it is only possible to simulate future behavior for a limited number of specific contingencies.

This paper investigates an alternative, qualitative approach to making such future predictions.  Specifically, it considers application to power system stability of the qualitative methods of Poincaré, Birkhoff and others for studying the n-body problem of analytical mechanics. To illustrate this approach we present two simple examples which demonstrate the complexity of qualitative behavior which is possible for nonlinear power systems models and the power of the qualitative methods to analyze this behavior.  The first example considers dynamic behavior of a single damped generator connected to an infinite bus with periodically time-varying voltage phase.  The second example considers dynamic behavior of two generators coupled to each other and to an infinite bus. The first example is a non-autonomous one degree-of-freedom system; the second is an autonomous two degree-of-freedom system.  The two examples are related to each other and bear close resemblance to the restricted three-body problem discussed by J. Moser (Stable and Random Motion in Dynamical Systems, Princeton University Press, 1973).  In both examples we find so-called "chaotic" motions similar to those described by Moser (op cit.) in the three-body problem.  Further mathematical and physical details of this problem may be found in a report by Prof. Nancy Kopell (Northeastern University) and Robert Washburn.  This report can be obtained by writing to the latter at the above address.

WHEN IS A NEWTONIAN DYNAMICAL SYSTEM
DERIVABLE FROM LAGRANGIAN?[*]

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## ABSTRACT

It is well known that while a Lagrangian always leads to Newtonian
equations of motion through the application of Lagrange's equations in a
unique manner, the converse is not necessarily true. That is, a given
Newtonian dynamical system may not be derivable from a Lagrangian unless
it obeys the Helmholtz conditions of formal self-adjointness (FSA). There
exist, however, systems that are not FSA as given, but become so upon multi-
plication of the governing system of equations of motion by an appropriate
"multiplier matrix." Such systems are known as pre-formally self-adjoint
(PFSA). In general, there exists a set of non-unique multiplier matrices,
leading to many different Lagrangians, that result in Newtonian equations of
motion that are the same up to a multiplier matrix. It is possible, for
instance, to find a unique multiplier matrix by imposing the requirement
that it only depend on time.

The present paper extends the work of the authors on linear dynamical
systems with arbitrary constant parameters, to systems with time-dependent
coefficients, and a class of non-linear problems.

In the case of a linear time-varying system, the Lagrangian is ob-
tained by first reducing the equation of motion to a standard form (one
not containing the first derivative) by a time-dependent linear transform-
ation. The multiplier matrix for this reduced equation is a constant, non-
singular, symmetric matrix. The Lagrangian for the latter system is simply
that of coupled harmonic oscillations. A reverse coordinate transformation
yields the Lagrangian in the original coordinates. It is shown that under
a certain commutativity condition an energy-like first integral can be
written. An illustration is also provided.

The non-linear example considered is the Contopoulos problem. By ex-
tending the notion of FSA to variational forms, it is shown that a first
integral independent of the Hamiltonian recently derived by a direct method
follows. It is further shown that the same result can be arrived at by
constructing a Morse and Feshbach or Leipholz type Lagrangian for that
problem.

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BIFURCATIONS AND CHAOTIC MOTIONS IN NONLINEAR DYNAMICAL
SYSTEMS

by Philip Holmes

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Cornell University, Ithaca NY 14853

We will review some recent developments in the qualitative theory
of dynamical systems and bifurcation theory for vector fields and maps,
and discuss their application to nonlinear problems in mechanics. We
will distinguish between local bifurcations, in which new equilibrium
solutions and periodic orbits are created individually, and global bi-
furcations, in which infinite families of new solutions are created.
Bifurcations of the latter type are connected with the appearance of
chaotic motions and strange attractors.

We will show that such bifurcations occur naturally in a wide range
of systems, a prototypical example being a nonlinear oscillator with weak
damping and forcing, which might be either externally applied, or due to
mutual interactions with neighbouring systems. In situations such as
this, one is often dealing with a small perturbations of an integrable
Hamiltonian system, and it is the feature which permits us to make a
global analysis.

We will describe examples in solid mechanics, including the forced
oscillations of a buckled beam and coupled pendula.

# CATASTROPHE FLAGS IN CONSERVATIVE AND
# DISSIPATIVE SYSTEMS

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Elementary Catastrophe Theory is the study of the critical points (where $\nabla_x f(x;c)=0$) of a smooth function, and how these critical points move about, coalesce, and bifurcate as the parameters which describe this function change. The number of critical points changes whenever the Hessian, or stability matrix ($f_{ij} = \partial^2 f/\partial x_i \partial x_j$) becomes singular. Such points are called catastrophes. Elementary Catastrophe Theory is therefore intimately related to the equilibrium and stability proper-ties of systems governed by a potential, a Lyapunov function, or more generally, by a variational principle.

The presence of a catastrophe is indicated by many physical phe-nomena. Catastrophes leave "fingerprints" at the scenes of their crimes and wave "flags" to gain our attention. The phenomena which accompany a catastrophe are surveyed for classical conservative ($\ddot{x}_i = -\partial V/\partial x_i$) and dissipative ($\dot{x}_i = -\partial V/\partial x_i$) systems. A close parallel among these phenomena, which are illustrated for the cusp catastrophe, is exhibited. This parallel extends also to quantum mechanical ($i\hbar\dot{\psi} = (T+V)\psi$) and statistical mechanical ($i\hbar\dot{\rho} = [T+V,\rho]$) systems.

# ON THE STABILITY OF VISCOELASTIC PERFECT COLUMNS[*]

S. Dost and P.G. Glockner[+]

The dynamic stability of perfect columns made of a linearly visco-elastic material and subjected to a load, P, smaller than the Euler load, $P_e$, is examined. The solution to the governing intergro-differential equation subjected to appropriate boundary and initial conditions, is obtained by means of Laplace transforms. In addition an approximate solution is also derived using an approximation technique introduced in [1]. The solution shows that the time-dependent amplitude of vibration will grow to infinity as long as P is larger than some critical value, referred to as the "safe load limit", $P_\infty$ in [2]. For $P \leq P_\infty$, the column is stable. The approximate solution, on the other hand, yields a continuous spectrum of eigenvalues between $P_\infty$ and $P_e$ to each of which corresponds a specific "finite" critical time (see Table 1). Clearly, agreement between the two sets of results is poor for all load levels except $P = P_\infty$.

## References

1. Schapery, R.A., "A Method of Viscoelastic Stress Analysis Using Elastic Solutions", Journal of the Franklin Institute, Vol. 279, No. 4, pp. 268-289, 1965.
2. Vinogradov, A.M. and Glockner, P.G., "Stability of Viscoelastic Imperfect Columns", ASME 3rd Engng. Mech. Division Specialty Conference, Austin, Texas, 1979.
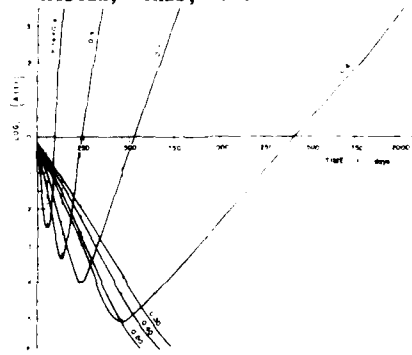
FIG 1 TIME -DEPENDENT BEHAVIOUR OF NONDIMENSIONAL AMPLITUDE OF VIBRATION, A(t), FOR SEVERAL LOAD LEVELS

TABLE 1. APPROXIMATE CRITICAL TIMES FOR SEVERAL LOAD LEVELS

| LOAD RATIOS $P/P_e$ | CRITICAL TIME $t_{cr}$ (days) |
|---|---|
| 0.90 | 4.7 |
| 0.80 | 11.2 |
| 0.70 | 22.4 |
| 0.60 | 44.0 |
| 0.51 | 129.0 |
| 0.50 | $\infty$ |

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# BUCKLING OF AN INITIALLY COMPRESSED
# COLUMN BY SUBTANGENTIAL FORCE

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According to Leipholz [1] and Kounadis [2], the stability of linear systems under the action of non-conservative forces is gaining importance in the area of hydromechanics and aeromechanics. In this paper, an attempt has been made to study the elastic buckling of a cantilever column as shown in figure and to determine the effect of the initial compressive force P on the critical value of the subtangential force Q. While such a problem has been referred to in [1], no final solution or numerical results seem to appear in literature.

The angle $\alpha$ for the subtangential force Q can be expressed as $\alpha = \beta y'(1)$, when $0 \le \beta \le 1.0$. Using kinetic criterion, the frequency equation for the free vibration of the column can be written from which the critical value of Q for given values of P and $\beta$ can be determined satisfying the stability criteria that the column is unstable when either the frequency is zero or complex. The values of Q corresponding to the latter case is obtained by coalescing two successive frequencies. The study shows that the static approach to determine Q is valid only for the range of $(\frac{1}{2}$   $\pi^2/2, \beta)$ $\le \beta \le 1$. For the case of P = 0, the minimum value of $\beta$ is $\frac{1}{2}$ for which the static approach is applicable. Values of Q are provided for various values of $\beta$ and P for $P \le \pi^2 EI/4\ell^2$.
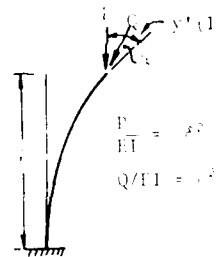


## References

(1)   Leipholz, H., "Stability Theory", Translated by Scientific Translation Service, Academic Press, 1970.

(2)   Panayotounakos, D.E. and Kounadis, A.N., "Elastic Stability of a Simple Frame Subjected to a Circulatory Load", J. of Sound and Vibration, Vol. 64 (2), pp.179-186, 1979.

# EFFECT OF NONLINEARITY ON THE RESPONSE OF A FORCED SYSTEM

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The equation of motion of a Forced Single degree of freedom system under the influence of slight viscous damping in the presence of the nonlinear terms $\sum_{m=2}^{N} \alpha_m u^m$ can be written as

$$\frac{d^2 u}{dt^2} + \omega^2 u = -2\epsilon \mu u + \epsilon \sum_{m=2}^{N} \alpha_m u^m - 2K \cos \Omega t$$

where $\omega$ is the natural frequency, $\mu$ is the viscous damping, $\epsilon$ is dimensionless parameter $\ll 1$, $\alpha_m$ are the coefficients of the nonlinear terms, it can be positive for the case of hard spring or negative for the case of soft spring and $K$ and $\Omega$ are the amplitude and frequency of excitation respectively.

The method of multiple scales is used to solve this system for both cases of soft excitation K=0 ($\epsilon$) as well as hard excitation K=0(1). Resonance cases including subharmonic, superharmonic and primary resonances may occur whenever $\omega = P\omega + Q\Omega$, where P and Q are integers such that $|P| + |Q| = m$, where $m = 2,3,\ldots.N$.

A uniformly valid solution, steady state response and frequency response equation for every resonant case are obtained using the method of multiple scales when N=5. An extensive study is made using the frequency response equation to show the effect of various parameters on the response amplitude. It is found that the presence of nonlin - earity parameters cause what is known as jump phenomina in the system. This phenomina is due to the multivaludness of the response amplitude. The uniformly valid solution is checked and found to compare well with the numerical solution of the system using runge-kutta integration scheme.

## References

1.      B.S.Shaker 1980 Nonlinear Oscillation of single-degree of freedom systems, Recent Advances in structural Dynamics (569-580).

2.      A.H.Nayfeh and D.T.Mook 1979 Nonlinear Oscilla - tions.New York, Wiley Interscience.

3.      A.H.Nayfeh 1973 Perturbation Methods. New York, Wiley Interscience.

MICROSTRUCTURAL ASPECTS IN METAL PLASTICITY

Organizer:     R.J. ASARO (Brown)
Chairperson:   R.J. ASARO (Brown)

* 2:15 - 2:45    J.D. EMBURY (McMaster Univ) and M.F. ASHBY (Univ. of Cambridge)
                 "The Representation of Fracture Processes in Fracture Maps"

* 2:45 - 3:15    R.G. DAVIES (Ford, Dearborn MI)
                 "Deformation and Fracture of High Strength Steels"

* 3:15 - 3:45    D.J. LLOYD (Aluminum Co. of Canada, Kingston, Ontario)
                 "The Deformation Morphology Developed in Some Aluminum
                 Alloys Deformed by Rolling"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    E.A. STARKE, JR. (Georgia Tech)
                 "Microstructural Aspects of Plastic Deformation in Alpha-Beta
                 Titanium Alloys"

  4:45 - 5:00    C.E.S. UENG (Georgia Tech)
                 "Superplastic Forming Process"

  5:00 - 5:30    L. ANAND and W.A. SPITZIG (U.S. Steel)
                 "Shear-Band Orientations in Plane-Strain"

## "The Representation of Fracture Processes in Fracture Maps"

J.D. Embury
Department of Metallurgy and Materials Science
McMaster University
Hamilton, Ontario
Canada

M.F. Ashby
Department of Engineering
University of Cambridge
Cambridge, England

In both the fabrication and application of engineering materials we must consider the competitive nature of various fracture and instability events. As these processes depend on the current stress state and the strain history it is important to devise methods of representing the various criteria in the form of maps which permit the comparison of the various fracture modes. The paper will present examples of failure maps to illustrate the occurrence of fracture modes such as cleavage, inter-granular fracture and void nucleation and growth for a variety of metallic and geological systems.

## References

The paper will utilize a variety of concepts which have been developed in the references given below.

(1) Patterson, M.S., Experimental Rock Deformation - The Brittle Field, Springer Verlag, 1978.

(2) Le Roy, G., Edward, G., Embury, J.D. and Ashby, M.F., A Model of Ductile fracture, Acta Met., 1981 (in press).

(3) Sowerby, R., Uko, D.K. and Tomita, Y., A Review of the Bauschinger Effect, Mat. Sci. & Eng., 1979.

DEFORMATION AND FRACTURE OF HIGH STRENGTH STEELS

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## ABSTRACT

The factors controlling both the strength and ductility of high strength conventional and dual-phase steels will be reviewed; special emphasis will be placed upon the change in work-hardening rate with strength.  In addition, a study has been made of the interaction of strength, inclusion shape, cold-work and "edge quality" on the fracture of sheared edges in high strength steels.  This has revealed that both inclusion shape control and localized cold work at the sheared edge are extremely important in controlling the fracture in these thin gage steels.  For dual-phase steels the strength of the martensite islands is one of the determining factors in both edge cracking and hydrogen embrittlement.

The Deformation Morphology
Developed In Some Aluminum
Alloys Deformed By Rolling

by

D. J. Lloyd

ABSTRACT

It has been appreciated for some time that the deformation morphology
in F.C.C. metals and alloys deformed to large strains can be extremely
inhomogeneous. Brown[1] examined the surface features developed in cold
rolled aluminum and showed the development of deformation and shear
banding. Similar features have been examined in copper and its alloys[2,3]
by surface replication and electron microscopy. In the present work
the deformation morphology developed during rolling in certain aluminum
alloys is revealed by a decoration technique. The development of the
various deformation features are discussed together with their relation-
ship to the underlying dislocation structures. The implications for
current work hardening theories at large strains are also considered.

[1] Brown, K., Journal of the Institute of Metals, 1972, Vol. 100, p. 341

[2] Hatherly, M. and Malin, A. S., Metals Technology, 1979, Vol. 6, p. 308

[3] Malin, A. S. and Hatherly, M., Metal Science, 1979, Vol. 13, p. 463

MICROSTRUCTURAL ASPECTS OF PLASTIC DEFORMATION
IN ALPHA-BETA TITANIUM ALLOYS

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ABSTRACT

The microstructures of $\alpha + \beta$ Ti alloys can be greatly varied by modifications in primary processing procedures and the deformation behavior is sensitive to microstructural variations.[1-3] By suitable selection of the working and annealing temperature with respect to the beta transus temperature, microstructure can be developed having equiaxed, platelet, or grain boundary alpha in a retained beta matrix; both phases can be fine, medium, or coarse, and continuous or non-continuous. In addition to deformation and property variations associated with microstructural modifications, mechanical properties can show substantial directionality, as a result of texture.[4] Since texture and microstructure can be controlled separately, to a large extent, their individual effects may be used in a beneficial manner. The relationships between processing, microstructure and deformation of an alpha-beta Ti alloy will be discussed in this talk.

1.  H. Margolin, P. N. Farrar, and M. A. Greenfield, in The Science and Application of Titanium, Pergamon Press, New York, 1970, p. 795.

2.  Howard G. Nelson, Dell P. Williams, and James E. Stein, Met. Trans., 3, (1972) p. 469.

3.  J. C. Chesnutt, C. G. Rhodes, and J. C. Williams, ASTM STM 600 (1976) p. 99.

4.  M. J. Harrington, A. W. Sommer, P. G. Reimers, and G. A. Alers in Titanium Science and Technology, Vol. 2, Plenum Press, New York, 1973, p. 1297.

# SUPERPLASTIC FORMING PROCESS

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Georgia Institute of Technology

Atlanta, Georgia 30332

Superplastic forming process with or without diffusion bonding has been recently proven to be a promising and innovative approach for manufacturing components with complicated shapes in many engineering problems. This new process can improve material utilization, reduce machining and assembling work, and cut down the cost of production. It is accomplished through the "one-piece" forming idea where the number of parts is greatly reduced. This new-forming process has been tried successfully in different branches of engineering, ranging from sophisticaed aircraft structures to turbine gear manufacturing, and to ground transportation vehicle designs.

Superplastic forming uses a special mechanical property of certain metallic materials which exhibit an exceptionally high ductility under a combination of suitable temperature and strain-rate conditions. Tensile elongations of several hundred percent are very common, while the range between one-and two-hundred percent is the most useful range for practical forming problems. This low load, slow strain-rate process takes place without appreciable strain hardening or necking, at temperatures of about half the absolute melting point or above. The essential microstructural condition is an extremely fine grain size, which permits deformation to occur largely by grain boundary sliding and not by the more usual slip and grain deformation process.

When in the superplastic condition, metal sheets can be deformed readily by ordinary sheet vacuum-forming methods, using atmospheric pressure or a superimposed gas pressure to gently force the heated metal sheet onto a female die which has the complicated geometrical shape desired. The metal assumes the shape of the die with great fidelity when deformed under the correct experimental conditions. Thus, once the die is made, then a large number of such components can be readily made with little trimming work necessary.

This new superplastic forming process has been employed successfully at Georgia Tech for making different configurations of cores used in a sandwich structure. Such "one-step" forming approach can provide many advantages over the conventional honeycomb shape. Detailed forming process, preparatory steps, experimental conditions, and certain mechanical properties of such products will be included and discussed in the presentation.

# Shear-Band Orientations In Plane-Strain

L. Anand and W. A. Spitzig

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## Abstract

Plane-strain experiments on a wide variety of
materials show that localized shear bands are generally
not inclined at 45° to the in-plane axes of principal
stresses.  In metals the shear bands are usually inclined
at angles less than 45° to the direction of the maximum
(most te..si'le) rrincipal stress, whereas in polymers and
geological materials they are usually inclined at angles
greater than 45°.

A bifurcation analysis that uses an incrementally
linear rate constitutive law exhibiting pressure-sensi-
tivity, dilatancy and non-coaxiality predicts that the
critical orientation of emergent shear bands from a
previously homogeneous plane deformation field is given by

$$\theta = \pm \arctan \left\{ \frac{(1+\mu)}{(1-\mu)} \frac{(1+\beta)}{(1-\beta)} \frac{(1-\tau/G)}{(1+\tau/G)} \right\}^{1/4} . \tag{1}$$

Here $\theta$ is the inclination of the shear-band to the maximum
principal stress direction; $\mu$ is a pressure-sensitivity
parameter; $\beta$ is a shear-induced-dilatancy factor; $\tau$ is a
shear stress defined in terms of the in-plane principal
stresses $(\sigma_1, \sigma_2; \sigma_1 \geq \sigma_2)$, by $\tau := (1/2)(\sigma_1 - \sigma_2) \geq 0$; and
$G$ is an instantaneous shear modulus for shearing parallel
to the principal directions of stress.

It is shown that the orientations of shear bands
in a metal, a polymer, and a sand are very well predicted
by various special cases of Equation (1).

Session WA-4:  INELASTIC STRUCTURAL DYNAMICS - II

Organizer:    P.S. SYMONDS (Brown)
Chairperson:  S. KALISZKY (Tech. University Budapest)

* 2:15 - 3:15    G.R. ABRAHAMSON (SRI International, California)
                 "New Methods for Dynamic Testing of Large Structures for
                 Computer Code Validation"

* 3:15 - 3:45    H. KOLSKY and J.M. MOSQUERA (Brown)
                 "The Dynamic Response of Fibre Reinforced Beams"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    M.J. FORRESTAL and D.B. LONGCOPE (Sandia Labs)
                 "Analvtical and Experimental Studies on Penetration into
                 Porous Rock Targets"

  4:45 - 5:15    E. HAUGENEDER and P. TORZICKY (Tech. Univ. Wien, Austria)
                 "Ultimate Load Analysis of Reinforced Concrete Panels"

  5:15 - 5:30    T.J. SUKANIT and J. PEDDIESON, JR. (Tenn. Tech. Univ)
                 "Axisymmetric Arbitrarily Large Deflections of Circular
                 Membranes"

Abstract

NEW METHODS FOR DYNAMIC TESTING OF LARGE STRUCTURES FOR
COMPUTER CODE VALIDATION

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        Data are lacking for validation of computer codes used
to calculate the response of large structures such as nuclear
power plants to dynamic loads from operational or accidental
disturbances. Over the past five years SRI has developed two
techniques for obtaining such data. A contained-explosion
technique has been developed that produces ground motion at
seismic frequencies. An array of vertical line sources is
placed in the ground some tens of feet from the structure to
be tested. Multiple firings in each line source produce
earthquake-like ground motion. The sources are reusable and
the ground motion is repeatable. An oscillating bubble
technique has been developed that produces local loads within
a structure and can be extended to produce ground motion at
seismic frequencies. An underwater void is suddenly pressurized
to produce an oscillating bubble. The amplitude and frequency
are controlled by choice of volumes and pressures. The technique
requires a volume of water of the order of 100 $m^3$, and has been
applied in two large structures. Accelerations in the structures
are repeatable within 10%. The main features of the two techniques
will be described and test results will be presented.

The Dynamic Response of Fiber-reinforced Beams

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Experiments are described in which beams of fiber-reinforced
materials are subjected to transverse dynamic loading.  The material
response when purely elastic as well as elastic and plastic deforma-
tions occur are discussed.  For elastic deformation it is shown that
the observed response agrees very closely with the theoretical pre-
dictions of Sayir [1].  Sayir has shown that when the ratio of the
extensional modulus of the beam is large compared with the shear
modulus of the matrix material the shear deformation of the matrix
material is the dominating physical parameter.  The specimens used
for these experiments were mainly fabricated in the laboratory and
consisted of steel wires embedded in matrices of lead and lead-tin
alloy as well as in natural rubber.  Commercially obtainable fiber
glass - Scotchply 1002, was also used for thes experiments.  The
work on dynamic plastic deformations was carried out on the specimens
with metal matrices and the transverse dynamic loading was achieved
either by the detonation of small explosive charges in contact with
the beams or by means of a "Hyge" impact testing machine.  In the
latter case high speed cine photographs were obtained by the use of
a "Fastax" high-speed cine camera.  The results of the experiments
showed that for large dynamic transverse deformations the plastic
yielding in shear of the matrix metal plays a predominant role.

[1]  Sayir, M., "Flexural Vibrations of Strong Anisotropic Beams",
     Ing. Archiv. 49 (1980), pp. 309-330.

Analytical and Experimental Studies on
Penetration into Porous Rock Targets *

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Mathematical models which predict forces on conical-nosed
penetrators for normal impact into porous rock targets are
developed and predictions from these models compared with
laboratory scale experiments. Constitutive description of the
target contains minimum detail; a linear hydrostat, a linear
shear failure-pressure relation and the material density.**
The analyses are further simplified by employing the cylin-
drical cavity expansion approximation which considers the
target as thin independent layers normal to the penetration
direction and allows only radial target motion. Governing
partial differential equations are reduced, via a similarity
transformation, to nonlinear ordinary differential equations,
and are solved both numerically and with an iterative method
which results in accurate closed-form solutions.

Laboratory scale experiments designed to verify the
mathematical models were also conducted. A light gas gun was
used to accelerate simulated geological targets to steady
velocity and impact the penetrators. Rigid body penetrator
motion was measured for the time corresponding to two nose
lengths of penetration with laser interferometry and accel-
erometers. Resultant forces were calculated from these data
and reasonably good correlation with predictions is observed.

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# ULTIMATE LOAD ANALYSIS OF REINFORCED CONCRETE PANELS

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Ultimate load analysis of reinforced concrete structures are commonly performed on the basis of a nonlinear stress-strain relationship and a stress criterion for crack advance. New cracks are assumed to develop or existing cracks are assumed to propagate if one of the principal stresses exceed the ultimate stress. Therefore we have to deal with cracked zones or so-called "smeared" cracks. This assumption allows a detection of the load-displacement path starting from an uncracked situation, but the results depend on the chosen finite element pattern.

Recently, Bazant and Cedolin have developed an objective method to evaluate displacements and stresses in plain and reinforced concrete structures /1,2/. This method is independent of the size of the elements; it is based on the calculation of the energy release rate and takes into account the bond slip between steel bars and concrete.

In the present work the energy criterion is adopted for the ultimate load analysis of reinforced concrete panels. Higher order elements are used and therefore the net reinforcement is "smeared" over parts of the panels. Special attention is paid to the development and propagation of cracks. The direction of propagating cracks is determined by the direction of the respective principal stresses. Results are compared to available test results from the literature.

Literature:

1. Z.P.Bazant and L.Cedolin, J. of the Engng. Mech. Div. 2 (1979) 297-315

2. Z.P.Bazant and L.Cedolin, J. of the Engng. Mech. Div. 6 (1980) 1287-1306

# AXISYMMETRIC ARBITRARILY LARGE DEFLECTIONS
## OF CIRCULAR MEMBRANES

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## ABSTRACT

The relative merits of various formulations of the equations govern-
ing arbitrarily large axisymmetric deflections of initially flat circu-
lar elastic membranes are discussed. Points singled out for attention
include the choice of stress resultants, the choice of strain measures,
and the choice of model for the thickness behavior. Numerical results
are obtained for the deformation due to an applied lateral pressure using
a specific set of idealized constitutive equations and employed to investi-
gate the accuracy of the Von Karman equations. Numerical results are
obtained for pure radial deformation using several sets of idealized
constitutive equations and employed to investigate the differences in
response exhibited by the associated prototype materials.

An interesting finding is that some formulations allow a reduction
to a single second-order differential equation governing a stress func-
tion. In particular, it was found that a prototype material for which
the Lagrangian stress resultants are related to the linear extensions
by the plane-stress form of Hooke's law leads to the equation

$$\psi''+\psi'/r-\psi/r^2+(Eh(1-\nu/(\psi^2+(rV)^2)^{1/2})-\nu((rV)^2)'/(2(\psi^2+(rV)^2)^{1/2}))/r = 0 \quad (1)$$

In (1) E, $\nu$, and h are, respectively, Young's modulus, Poisson's ratio,
and the undeformed thickness, r is the radial coordinate measured from
the symmetry axis, V is vertical stress resultant, and $\psi$ is the stress
function. The Lagrangian radial and circumfrential stress resultants
are found from the respective equations

$$N_r = (\psi^2 + (rV)^2)^{1/2}/r, \quad N_\theta = \psi' \quad (2)$$

Equations (1) and (2) are those associated with the Reissner membrane
theory. Thus, this formulation, usually interpreted as being valid
only for small strains, can also be interpreted as being exact when the
proper choices of stress resultants and strain measures are made.

Session WA-5:  MECHANICS OF GRANULAR MATERIALS

Organizer:     S. NEMAT-NASSER (Northwestern)
Chairperson:   S. NEMAT-NASSER (Northwestern)

* 2:15 - 3:15    J.T. JENKINS (Cornell)
                 "Continuum Models for Granular Materials"

* 3:15 - 3:45    J. GHABOUSSI (Univ. of Illinois, Urbana)
                 "Plasticity Models for Sands"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    M.M. MEHRABADI (Northwestern)
                 "On Micromechanical Description of Granular Material Behavior"

* 4:45 - 5:15    S.B. SAVAGE and M. SAYED (McGill)
                 "Experiments on Stresses Developed by Rapidly Sheared
                  Cohesionless Granular Materials"

# Continuum Models for Granular Materials

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We review recent attempts to formulate continuum theories for granular materials in three distinct flow regimes.

When the strain and strain rate are small, as in many situations in soil mechanics, modifications of the equations of classical plasticity have been proposed. Modifications must be made in order to include the internal friction and compressibility of these materials, and the development, with deformation, of anisotropy in the distribution of grain contacts.

For larger strains developed at modest strain rates, as encountered, for example, in slow geological flows and in some materials handling applications, detailed consideration of shearing on slip surfaces has led to various forms of rate independent flow rules. There is, as yet, little experiment and no agreement as to which form is appropriate.

Finally, for rapid shear flows, in which momentum is transferred in collisions between particles rather than by static contact forces, balance laws and constitutive equations have been introduced that exploit the analogy between these flows and those of a gas of hard, rough, imperfectly elastic spheres.

# PLASTICITY MODELS FOR SANDS

by

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## ABSTRACT

The basis for plasticity models for sand will be discussed along with
the examination of the supporting experimental evidence.  Modeling of yielding
and failure along with specific forms of yield surfaces and failure surfaces
will be discussed.  The use of normality rule (associated flow rule) in the
past has not been successful in reproducing the desired results in terms of
dilatancy volumetric strains.  The basis for a systematic use of non-
associated flow rule and related questions of uniqueness and stability will
be explored.  A specific method of modeling the dilatancy of sand based on
normalized plastic work will be presented along with extensive supporting
experimental data.

It will specifically be shown that the parameters involved in this
dilatancy model are independent of most of the bulk properties of sand,
such as relative density, and are only dependent on the intergranular
frictional properties  Finally, the requisites of modeling of cyclic
behavior of sands will be discussed and some specific forms will be
presented.

# ON MICROMECHANICAL DESCRIPTION OF
## GRANULAR MATERIAL BEHAVIOR*

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Some recent results [1-3] on the mechanics of cohesionless granular materials (with rigid granules) are summarized. The overall stresses are expressed in terms of the contact forces in two different but complementary ways: (1) by a statistical averaging over the sample volume of contact forces and "branches" which are vectors connecting the centroids of two contacting granules; and (2) by defining the overall tractions transmitted across an interior imagined plane as the sum of the contact forces which represent the mechanical effect of granules on one side of a unit area of this plane, upon those on the other side. Conditions under which the two representations of overall stresses are equivalent, are examined in detail. In addition, the corresponding kinematics is examined and the overall macroscopic deformation rate and spin tensors are developed in terms of the volume average of relevant microscopic kinematical variables. As an illustration of the application of the general expressions developed, two explicit macroscopic results are deduced: (1) a dilatancy equation which both qualitatively and quantitatively seems to be in accord with experimental observation; and (2) a non-coaxiality equation which seems to support the double-shearing models for granular materials and vertex plasticity models. Since the development is based on a microstructural consideration, all material coefficients entering the results have well-definied physical interpretations.

## References

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Experiments on Stresses Developed by Rapidly Sheared
Cohesionless Granular Materials

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H3A 2K6

Experimental results obtained for rapid shear flow of several
dry granular materials in an annular dynamic shear cell will be
described.  The purpose of the tests was to obtain information that
could be used to guide the theoretical development of constitutive
equations suitable for the rapid flow of cohesionless bulk solids
at low stress levels.

The shear cell consists of two concentric disk assemblies
mounted on a fixed shaft.  Granular material was contained in an
annular trough in the bottom disk and capped by a lipped annular
ring on the top disk.  The bottom disk can be rotated at specified
rates while the top disk is loaded vertically and is restrained from
rotating by a torque arm connected to a force transducer.  The
apparatus was thus designed to determine the shear and normal stresses
as functions of solids volume fraction and shear rate.

Tests were performed with spherical glass and polystyrene beads
of uniform diameters, spherical polystyrene beads having a bimodal
size distribution and with angular particles of crushed walnut shells.
At the lower concentrations at high shear rates the effects of inter-
particle collision are dominant and both shear and normal stresses
were found to be proportional to the square of the shear rate.  At
higher concentrations and lower shear rates dry friction between
particles becomes increasingly important and the stresses are
proportional to the shear rate raised to a power less than two.  All
tests showed a strong dependence upon solids volume fraction.

Session WA-6:  ELASTICITY/SHELL STRUCTURES

Chairperson:  J.L. SANDERS (Harvard)

2:15 - 2:45    F. ERDOGAN and F. DELALE (Lehigh)
               "Relatively Thin-Walled Cylindrical Vessels with a Part-
               Through Crack"

2:45 - 3:00    T.J. RUDOLPHI (Iowa State) and S-J. CHANG (Oak Ridge Nat. Lab)
               "The Edge Crack Problem Under Mixed Boundary Conditions"

3:00 - 3:15    A. MAEWAL and W. BOTTEGA (Yale)
               "Nonlinear Delamination Buckling and Growth in Laminates"

3:15 - 3:45    S-H. DAI (Univ. Rochester, and Nanjing Inst. Chem. Tech., PRC)
               "A Study of the Optimal Structural Parameters of Dished-Only
               Heads Operating Under Internal Pressures"

3:45 - 4:15    REFRESHMENT BREAK

4:15 - 4:30    N.C. HUANG (Notre Dame) and P.D. PATTILLO (Amoco, Tulsa)
               "Inelastic Buckling of Cylindrical Shells Subjected to
               Axial Tension and External Pressure"

4:30 - 4:45    Z. MOMEH and M. EL-NOMROSSY (Military Tech. College, Cairo)
               "Radial and Tangential Saddle-Cylinder Interface Pressure
               in Contact Problems of Shells"

4:45 - 5:00    M.H. BALUCH and A.K. AZAD (Univ. Petrol. & Min., Saudi Arabia)
               "One Variable Formulation of Cylindrical Shells with Trans-
               verse Shear Deformability"

5:00 - 5:15    V.I. FABRIKANT and T.S. SANKAR (Concordia Univ., Montreal)
               "Two-Dimensional Integral Equations in the Theory of Elasticity"

5:15 - 5:30    Y. TIAN-QUAN (Univ. British Columbia, Vancouver and Huazhong
               Inst. of Tech., Wuhan, PRC)
               "The Solution of Elastostatic Problems and the Principles of
               Minimum Potential Energy, Minimum Complementary Work, Under
               Fuzzy Boundary Conditions"

5:30 - 5:45    R.C. BATRA (Univ. Missouri-Rolla) and C. DAVINI (Univ. di
               Pisa, Italy)
               "An Existence Theorem in Linear Elasto-Statics Under Non-
               Classical Boundary Conditions"

6:00 - 7:30    COMPLIMENTARY RECEPTION
               Home of President and Mrs. Howard R. Swearer, 55 Power Street

# RELATIVELY THIN-WALLED CYLINDRICAL
# VESSELS WITH A PART-THROUGH CRACK

by

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The problem of a cylindrical vessel containing a part-through im-
bedded or surface crack is considered. The crack is assumed to occupy
a plane perpendicular to or containing the axis of the cylinder. The
general problem is analytically intractable and may be solved numeri-
cally, for example, by using the finite element technique. However,
for certain crack-cylinder geometries, one could use the line-spring
model developed by Rice and Levy to obtain an approximate solution and
one could obtain bounds for the stress intensity factors by solving the
related plane strain and axisymmetric elasticity problems. The line-
spring model is known to give reasonably good results if the thickness-
to-radius ratio of the cylinder is sufficiently small and if the crack
is relatively long. In this paper first the problem is solved by using
the line-spring model. It is assumed that the crack is either an im-
bedded flat elliptic crack or an internal or an external semi-elliptic
surface crack in an axial or a circumferential plane. The cylinder is
approximated by a shallow shell in which the transverse shear effects
are taken into account. Next, the plane strain and the axisymmetric
elasticity problems for the general thick-walled cylinder having an im-
bedded or a surface crack is considered. In each case the problem is
reduced to a singular integral equation which may be solved to any
desired degree of accuracy. The solution obtained from the plane strain
problem of a cylinder having a radial crack provides an upper bound for
the stress intensity factor in a cylinder which contains an axial part-
through crack of finite dimensions. Similarly, the elasticity solution
of the cylinder containing an axisymmetfic circumferential crack give
the bound for the corresponding finite part-through crack.

# THE EDGE CRACK PROBLEM UNDER MIXED BOUNDARY CONDITIONS

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An integral equation solution to the two-dimensional problem of an edge crack with prescribed stresses and/or prescribed displacements in a bounded region with in-plane deformation is presented.  By consideration of a bounded, unflawed region and a cracked, semi-infinite region, a coupled set of integral equations in the unprescribed quantities on the outer boundary and the line crack is obtained   The integrals on the outer boundary are the so-called boundary integral equation and the accompanying interior or field stress and displacement identities.  Similarly, for mixed type conditions on the crack surface, an integral relation between applied stresses and/or displacements is available from the semi-infinite solution along with the quadrature form field solution.  The field equations from both problems are used to couple the solutions by elimination of appropriate boundary variables.

A simple, boundary integral type of numerical solution is effected to approximate the coupled equations, resulting in a discretized approximation of the unprescribed variables on both the outer boundary and crack line.  With the crack stress then known, the stress intensity factor at the crack tip is readily obtained by simple quadrature.  The method is applied to several examples to demonstrate the validity and accuracy of the numerical solution.

# NONLINEAR DELAMINATION BUCKLING
## AND GROWTH IN LAMINATES[1]

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## Abstract

As a mechanism of degradation of compressive strength of laminates, buckling and subsequent growth of delaminations in layered plates has been the subject of a number of recent investigations [1,2]. In an attempt to assess the significance of geometrical nonlinearity in this failure mechanism, we consider the problems of (i) a partially debonded two-layered beam under axial compression and (ii) a penny-shaped delamination in a bilaminated circular plate subjected to uniform in-plane radial compression. With the condition at the delamination tip obtained in terms of surface energy as in [3], we solve the associated nonlinear free-boundary problems by using an asymptotic procedure for postbuckling analysis. The equilibrium paths obtained in this manner consist of (a) the prebuckling path along which the debonded region does not have any essential effect, (b) the postbuckling regime wherein the delamination buckles without growth and (c) the stage of simultaneous opening and growth of the debonded area. The stability of the equilibria is also examined and it is shown that some qualitative aspects of the overall structural behaviour are adequately predicted only if postbuckling stiffness of the delaminations is taken into account.

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3. Burridge, R. and Keller, J.B., "Peeling, Slipping and Cracking - Some One Dimensional Free-Boundary Problems in Mechanics", SIAM Review, 20, 1978, pp. 31-61.

# A STUDY OF THE OPTIMAL STRUCTURAL PARAMETERS OF DISHED-ONLY HEADS OPERATING UNDER INTERNAL PRESSURES

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## ASBTRACT

In the past, the use of cylindrical vessels with dished-only heads (without straight flange) was limited to low pressure conditions. However, they are being widely used now even in the process equipment of large size which operates under high pressure. This problem and related calculations are discussed in the present paper.

Factors influencing the edge loads and local stresses, produced at the junction of the head and shell, have been determined by using the shell bending theory. Based upon the relationship between these factors and with a view of controlling the magnitude of edge stresses, the optimal structural parameters of dished-only head are investigated.

To optimize the structure, the finite-elements method and the method of design of experiments are jointly used. To verify the evaluated results, the present paper reports a number of experimental tests on three different mild steel vessels in which the strains of the cylindrical shells and their dished-only head were measured under pressure by resistance strain gages. The nominal values of semi-central angle of spherical shell of dish-only head used were 60°, 75° and 80° respectively. The vessels, except for the 75° head vessel, were pressurized to destruction (Fig. 1, 2 and 3). Photoelastic stress analyses were also made on small plastic models of similar proportions. The stress distribution obtained from both experiments agreed with calculated results. As predicted, the vessels with their heads having the semi-central angle larger than the critical value (75°) had the best stress distribution. Furthermore, the optimal parameters obtained in the present work were in general agreement with those given in the ASME boiler and pressure vessel code, the Japanese and the French pressure vessel codes.

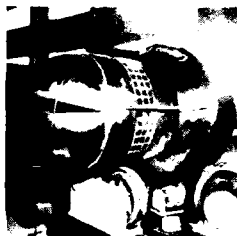Fig. 1.  = 75° vessel        Fig. 2.  = 80° vessel        Fig. 3.  = 60° vessel
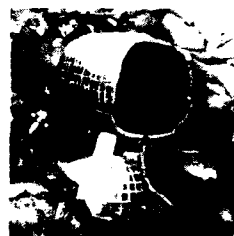
# Inelastic Buckling of Cylindrical Shells Subjected to Axial Tension and External Pressure

N. C. Huang* and P. D. Pattillo**

The frequency of occurance in oil well casing collapse in recent years has focused attention on the necessity to reexamine the effectiveness of the current design equation set by the American Petroleum Institute. In an effort to better understand the complexities involved in casing collapse, this paper will study the problem of the inelastic buckling of cylindrical shells subjected to axial tension and external pressure. The edges of the shell may either be fixed or free. Our analysis is based on Kirchhoff's assumptions associated with a finite deformation of the shell. Sanders' nonlinear theory of shells [1] is employed in our formulation. A numerical method based on the finite difference scheme is adopted for determining the critical condition for inelastic buckling. Incremental and deformation theories in plasticity are both included in the investigation.

The following conclusions can be drawn from our study: (1) For cylinders with fixed ends, as expected, the effect of bending in the prebuckling deformation of the shell is restricted within a layer near the edge of the shell. Within the practical range of shell thickness, if the length-diameter ratio of the shell exceeds 8, the effect of the prebuckling bending to buckling condition becomes insignificant. In this case, it is possible to use the infinite cylinder theory to predict the buckling condition for finite cylinders. However, for cylinders with free boundaries, we can always rely on the infinite cylinder theory for predicting the critical buckling condition of finite cylinders. (2) Our study finds that in determining the prebuckling deformation of the shell, we can employ either the deformation theory or the incremental theory. However, in evaluating the critical conditon for inelastic buckling, it is more suitable to use the deformation theory to compare with the experimental data. (3) It is found that the results of our study on casing collapse condition due to inelastic buckling within certain range of shell geometry agrees with the experimental observation data better than the existing equation for the design of oil well casings.

## References

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**
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RADIAL AND TANGENTIAL SADDLE - CYLINDER INTERFACE

PRESSURE IN CONTACT PROBLEMS OF SHELLS.

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## ABSTRACT

This work present, with experimental verifications, a simple analytical solution for the contact problem of saddle that supported cylindrical shell. The solution determine the radial and tangential components of the saddle-cylinder interface pressure. The solution is based on using the semibending theories of shells with both compressible and incompressible middle surface. The problem is solved numerically for different shell parameters and loading conditions at different saddle angles. A new distribution for the saddle-cylinder interface pressure are given whose tangential component has a great effect on the redistribution of the radial one. The analysis indicates that the semibending theory with compressible middle surface is the most suitable theory for studying the contact problem of saddle with cylindrical shell. Also the tangtial component of the saddle-cylinder interface pressure can not be simply neglected. All previous solutions considered the radial saddle-cylinder interface pressure and neglect the tangential one.

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# ONE VARIABLE FORMULATION OF CYLINDRICAL SHELLS
## WITH TRANSVERSE SHEAR DEFORMABILITY

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The equations for the linear theory of elastic shells, including the eff-
ects of transverse shear and transverse normal strain deformability, have
been formulated by Reissner [1] for the symmetric case and by Naghdi [2]
for the general loading case. These equations in general do not lend
themselves readily to closed form solutions, although solutions for some
particular problems do exist [3].

The objective of this work is to formulate the problem of the axisymmet-
ric cylindrical shell with transverse shear deformability in terms of one
variable, the transverse displacement w, much in the same fashion as the
cylindrical shell with infinite transverse shear stiffness. This is at-
tained by initially assuming a simplified relationship between the trans-
verse shear stress resultant V and the transverse displacement w, which
then leads to the one variable formulation. Such a formulation enables
one to obtain the influence of transverse shear on stress resultants and
displacements in a more simplistic fashion than the exact approach of [1]
and [2].

The one variable formulation has been used [4-7] for isotropic plates wi-
th transverse shear deformability and for sandwich plates.

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# TWO-DIMENSIONAL INTEGRAL EQUATIONS IN
## THE THEORY OF ELASTICITY

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Equations occurring in elasticity and electrostatics defined by

$$\int_a \int_0^{2\pi} \frac{F(\rho_0,\psi_0)\,\rho_0\,d\rho_0\,d\psi_0}{[\rho^2 + \rho_0^2 - 2\rho\rho_0 \cos(\psi-\psi_0)]^{\frac{1+\alpha}{2}}} = f(\rho,\psi)$$

$$\text{with } a \leq \rho \leq \infty, \quad 0 \leq \psi \leq 2\pi, \quad \alpha < 1 \tag{1}$$

is considered where $f$ is a given function, and the function $F$ is to be determined. The exact solution of this equation is obtained by a special integral representation of the kernel as

$$F(y,\psi) = - \frac{\cos(\pi\alpha/2)}{2^{\alpha}} \left[ \frac{\lambda(a,y,\psi)}{(y^2-a^2)^{\frac{1-\alpha}{2}}} + \int_a^y \frac{dr}{(y^2-r^2)^{\frac{1-\alpha}{2}}} \frac{d}{dr} \lambda(r,y,\psi) \right]$$

$$\lambda(r,y,\psi) = r^{1-\alpha} \int_r \frac{d\rho}{(\rho^2-r^2)^{\frac{1-\alpha}{2}}} \frac{d}{d\rho} \int_0^{2\pi} [2Re \frac{1}{1-r e^{i(\psi-\psi_0)/y}} - 1] f(\rho,\psi_0) d\psi_0 \tag{2}$$

Investigation of the properties of the solution leads to two theorems of general mathematical interest. (i) If a function $f(\rho,\psi)$ has integral representation (1) and the density $F(\rho_0,\psi_0)$ inside a circle $\rho_0 = a$, then the values of the function $f$ inside $\rho = a$ may be evaluated directly in terms of the function $f$ outside. (ii) If a function $f(\rho,\psi)$ has a density in an integral representation of the type (1) with zero in the lower limit of integration, and the function $f(\rho,\psi)$ outside, then the values of the $F$ outside $\rho = a$ may be evaluated directly in terms of the function $F$ inside.

As an example a non-homogeneous space whose elastic modulus is a power function of depth $E = E_0 z^\alpha$ ... with a penny-shaped crack ... An arbitrary normal pressure applied on both sides of the crack. In this case the normal ... is determined ... the normal displacement on the ...

# THE SOLUTION OF ELASTOSTATIC PROBLEMS AND THE PRINCIPLES OF MINIMUM POTENTIAL ENERGY, MINIMUM COMPLEMENTARY WORK, UNDER FUZZY BOUNDARY CONDITIONS

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Abstract. A solution of elastostatic problem is defined on the base of set theory and extended to the cases with fuzzy boundary conditions. Extension is also given for the principles of min potential energy and min complementary work with fuzzy boundary conditions,i.e.,for the former,

$$\left\{ \mathring{\underline{u}}_n \in \{u \mid \min \underline{v}; u = \underline{u}_n, s \in S_u\}\right\} \Longleftrightarrow \{\mathring{\underline{u}}_n \in S_{u1} = B_{u1} \cap B_\sigma \cap E\} \quad (27)$$

(Similar expression is obtained for the latter).

A dual relation $f_\varepsilon$ with reflexivity and symmetry in U (displacement function) space is used as the model of Fuzziness,i.e., $f_\varepsilon \triangleq \{(x,y) \mid d(x,y) \leqslant \varepsilon < \infty, \ x,y \in U\}$

A displacement function u is defined as fuzzy at boundary $s \in S_u$ by

$$\{u = \underline{u}, s \in S_u\} \Longleftrightarrow \left\{ \lim_{x_v \ni x \to s \in S_u} u(x) \in C(\underline{u},\varepsilon); \lim_{x_v \ni x \to s \in S_u} \sigma_{ij}(x)n_j = P_i(s)\right\} \quad (11)$$

Where $C(\underline{u},\varepsilon) \triangleq \{\underline{u} \mid d(\underline{u},\underline{u}) \leqslant \varepsilon < \infty\} = \underline{u} \circ f_\varepsilon\}$

Under assumptions:1.U,Y(stress function space) are sequentially compact;2.There exists a topological mapping $\Phi$, such that $\Phi:U \to Y$ (a hoeomorphism between U and Y);3.displacement function u(x),stress function $\sigma(x)$ are bounded and have cotinuous partial derivative in its domain (the elastic body and its sufficiently regular boundary S), we can prove:

$$\exists \mathring{\underline{u}}_n \in R_n = \{\underline{u}_n \mid d(\underline{u}_n,\underline{u}) = \varepsilon_n < \varepsilon, s \in S_u\}, \text{ such that}$$

$$\underline{v}(\mathring{\underline{u}}_n) = \inf_{\underline{u}_n \in R_n} \underline{v}(\underline{u}_n), \text{ and } \mathring{\underline{u}}_n \text{ is unique.} \quad (29)$$

(i.e., a solution of (27) exists and unique in $R_n$)

A quasisolution of an elastostatic problem is defined as an approximate solution with boundary conditions most closed to the origin's. And the existence of quasisolution of an elastostatic problem can be proved on the base of the above theorem (29).

# AN EXISTENCE THEOREM IN LINEAR ELASTO-STATICS UNDER
## NON-CLASSICAL BOUNDARY CONDITIONS

R. C. Batra, University of Missouri-Rolla, Rolla, MO U.S.A.
and
C. Davini, University di Pisa, Pisa, Italy

Abstract:

In an earlier paper [1], Batra derived boundary conditions
for a body which account somewhat for the deformations of the
surroundings of the body. This study suggests that in a mechan-
ical problem, the boundary conditions are of the type:

surface traction = functional of surface
displacements

He worked out explicitly this functional relationship for the
case when the loading device is a linear elastic half space.
However, if one considers an elastic membrane instead of a 3-
dimensional deformable continuum as an intimate contact loading
device and follow the procedure outlined in [1], one gets local
boundary conditions. These boundary conditions are non-classical
in the sense that the surface tractions depend upon the second
order surface gradients of the surface displacements. For a
linear elastic membrane, this functional relationship is a linear
one.

Here we consider a linear elastic body with its boundary
loaded by a linear elastic membrane. We show that for static
deformations of the system from a natural state, there exists
a unique weak solution of the problem provided the elasticities
satisfy certain conditions. These conditions imply that the
natural configuration of the system is stable with respect to
infinitesimal deformations. The differentiability properties
of the solution are also studied.

Reference

1.  R. C. Batra, On Non-Classical Boundary Conditions,
    Arch. Rational Mechs. Anal., 48, 163-191, 1972.

Session TM-1:   SOME FUNDAMENTAL ASPECTS OF AEROSOL BEHAVIOR

Organizer:    T.F. MORSE (Brown)
Chairperson:  J. CIPOLLA (Northeastern)

*  9:30 - 10:00    S. LOYALKA (Univ. Missouri)
                    "Kinetic Theory of Aerosol Motion"

* 10:00 - 10:30    R.K. CHANG, J.F. OWEN (Yale) and P.W. BARBER (Utah Univ)
                    "Resonances in the Elastic and Inelastic Light Scattering
                     from Dielectric Microparticles"

  10:30 - 11:00    COFFEE BREAK

* 11:00 - 11:30    S. HARRIS (SUNY, Stony Brook)
                    "A Microscopic Theory of Boundary Layer Phenomena in
                     Particulate Systems"

* 11:30 - 12:00    K.L. WALKER (Bell Labs., Murray Hill)
                    "Thermophoretic Deposition in the Manufacture of Optical Fibers"

* 12:00 - 12:30    A. ASHKIN (Bell Labs., Murray Hill)
                    "Applications of Radiation Pressure"

# KINETIC THEORY OF AEROSOL MOTION

by

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## ABSTRACT

Motion of aerosols in a non-uniform gas under the gradients of pressure, temperature, concentration and so forth is a problem of considerable interest in several branches of basic and applied engineering sciences. In recent years there has been progress in kinetic theory formulation and solution of initial boundary value problems of aerosol motion for all Knudsen numbers. We review this progress, particularly in conjunction with the frictional drag and thermophoresis problems for a single sphere. Implication of the recent work, and areas of future research that appear promising are also discussed.

Resonances in Elastic and Inelastic Light Scattering
from Dielectric Microparticles[*]

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New Haven, Connecticut 06520

and

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Salt Lake City, Utah 84112

Structurally defined natural modes of oscillation have recently
been shown to give rise to sharp peaks in elastic scattering, absorption,
emissivity, Raman, fluorescence, inelastic excitation, and optical levi-
tation force spectra from dielectric micro-objects. We have investigated
such resonances by studying angular distributions and spectral character-
istics of elastically scattered light from glass fibers (10 - 100 $\mu$m
diam), fluorescence excitation and emission spectra from glass fibers
coated with a thin layer of dye, and fluorescence emission spectra from
dye-embedded polystyrene spheres (10 $\mu$m diam).

Since resonance wavelengths and linewidths are strongly dependent
upon the size, shape, and refractive index of a microstructure, analyses
of resonance spectra provide a potentially useful technique for charac-
terizing the morphology of fibers and aerosols. Elastic scattering
spectra have been used to make highly accurate determinations of fiber
diameter. Furthermore, the angular distribution of elastically scattered
monochromatic radiation changes rapidly as a function of diameter (or
wavelength) near a resonance. Thus, the conventional technique for de-
termining fiber radius by monitoring angular distribution can be made
more sensitive by choosing incident wavelengths which satisfy resonance
conditions. Although inelastic emission spectra from bulk materials are
usually broad and featureless, sharp peaks in the inelastic spectra from
micro-objects occur at specific size-to-wavelength ratios for which reso-
nance conditions are satisfied. Thus, fluorescence and Raman scattering
are also potentially useful for structural characterization of micro-
particles.

Results of internal electric field calculations for dielectric cyl-
inders have provided insight into the physical nature of elastic scatter-
ing resonances and peaks in fluorescence emission spectra from dye-coated
or dye-embedded microstructures at resonance wavelengths. Under reso-
nance conditions, large fields are found to build up within the cylinder
near the perimeter and decay exponentially outside the boundary, consis-
tent with the notion that resonant modes correspond to internal waves
which travel around the cylinder, undergoing internal reflections at the
surface.

# A MICROSCOPIC THEORY OF BOUNDARY LAYER PHENOMENA IN PARTICULATE SYSTEMS

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We consider the steady absorption of Brownian particles in the
presence of an absorbing boundary (which can be plane, cylindrical,
or spherical). The classical macroscopic theory of this phenomena
is based on the use of the diffusion equation with the (physically)
incorrect boundary condition of vanishing particle density at the
absorbing surface. The use of a microscopic theory allows us to
use the correct boundary condition which applies to emergent particles
only. Our results indicate the presence of a boundary layer, the
extent of which depends on a number of factors, in which the predictions
of the macroscopic theory are not valid. In this region Fick's law
is only satisfied if the diffusion coefficient is spatially-dependent.
For the important case of a spherical absorber in the transition
regime our results are in substantial agreement with those found by
Fuchs on the basis of an ad hoc theory.

# THERMOPHORETIC DEPOSITION IN THE
# MANUFACTURE OF OPTICAL FIBERS

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The thermophoretic deposition of small particles where the migration velocity depends upon temperature gradients is studied theoretically for laminar tube flow. The prototype boundary value problem is that in which the wall temperature is suddenly decreased at a given axial position. Only some fraction of the particles initially present will deposit on the walls because of the ultimate relaxation of the temperature gradient. A Levéque solution for short distances is used to establish a scaling for the deposition efficiency. The effects of weak Brownian diffusion are treated rigorously, and limiting efficiencies for long tubes are determined numerically.

The theoretical results are applied to the Modified Chemical Vapor Deposition (MCVD) process for the fabrication of optical fibers. In the MCVD process, reactant gases consisting of primarily $SiCl_4$, $O_2$ and $GeCl_4$ flow through a rotating silica tube. A torch flame slowly traverses the exterior of the tube heating the tube to temperatures between 1500°C and 1900°C. The chlorides react rapidly at these temperatures forming submicron oxide particles which are deposited on interior walls of the tube downstream of the torch. The particulate layer is consolidated into a vitreous layer. The tube is collapsed into a solid rod or optical preform after the deposition of many layers. The preform is drawn into an optical fiber.

Thermophoresis is conclusively established as the particulate deposition mechanism in the MCVD process by comparing experimental measurements and quantitative theoretical predictions. The model predicts within four percent the experimentally measured deposition efficiencies, E, defined as the fraction of the silica in the gas stream (initially as $SiCl_4$) that is deposited. For normal MCVD operating conditions, the deposition efficiency is only a function of the equilibrium temperature, $T_e$, at which the gas and walls equilibrate downstream of the torch and the temperature, $T_r$, at which reaction occurs. The deposition efficiency is $\approx 0.8[1-(T_e/T_r)]$. It is determined that $T_e$ is a strong function of the torch traverse velocity, the traverse length, the temperature of the ambient environment, and the tube wall thickness but only a weak function of the gas flow rate.

# Applications of Laser Radiation Pressure

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Radiation pressure from lasers[1] exerts significant forces on small
particles of varying sizes, from individual atoms to small macroscopic
particles in the range from $\sim$ (.01-100) μm. These forces arising from
the momentum of the light are capable of strongly affecting the dynamics
of the particles. This capability permits one to stably trap small par-
ticles, levitate them against gravity, manipulate them singly, combine
them in pairs, and channel them selectively along laser beams. Small
particles can also be used as sensitive probes for measuring optical,
electric, radiometric, viscous, and other forces. This talk will em-
phasize the application of radiation pressure techniques to the measure-
ment of properties of macroscopic particles in aerosol size range such
as size, index of refraction, evaporation, condensation, and electric
charge.

1. A. Ashkin, Science 210, 1031 (1980).

Session TM-2: THERMOMECHANICS OF TWO-PHASE FLOWS

Organizer:    J. BATAILLE (Brown and Ecole Centrale de Lyon, France)
Chairperson:  P.F. MAEDER (Brown)

* 9:30 - 10:00    M. ISHII (Argonne Natl Lab) and G. KOCAMUSTAFAOGULLARI
                  (Univ. Wisconsin, Milwaukee)
                  "Two-phase Flow Formulation Based on Two-fluid Model"

* 10:00 - 10:30   R.T. LAHEY, JR. and M. PODOWSKI (RPI)
                  "The Analysis of Instability Modes in Two-Phase Systems"

  10:30 - 11:00   COFFEE BREAK

* 11:00 - 11:30   M. LANCE (Ecole Central de Lyon, Ecully, France)
                  "Turbulence Measurements in Two-Phase Bubbly Flows"

* 11:30 - 12:00   D.A. DREW (U.S.A.R.O. and RPI)
                  "One-Dimensional Burning Wave in a Particulate Bed"

  12:00 - 12:15   S.T.J. PENG (Cal. Inst. Tech., Jet Propulsion Lab)
                  "Constitutive Equations of Time-Dependent Shear Thickening
                  Solutions"

# Two-phase Flow Formulation Based on Two-fluid Model

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G. Kocamustafaogullari
Mechanical Engineering Department
University of Wisconsin-Milwaukee

A number of publications on two-phase flow formulation, constitutive relations and empirical correlations have appeared in the past. Recently, considerable efforts have been made in order to establish a very detailed two-phase flow model suitable for light water transient and accident analyses. Untill recently various mixture models have been used extensively because of its simplicity both in terms of the field equations and necessary constitutive relations. In view of the limited data base presently available and difficulties associated with detailed measurements in two-phase flow, the advanced mixture models such as the drift-flux model [1,2,3] are probably the most reliable and accurate analytical tool for ordinary two-phase flow problems.

However, more detailed treatment of two-phase motions through a two-fluid model [2,4,5] is also possible. The two-fluid model is formulated by considering each phase separately in terms of two sets of conservation equations governing the balance of mass, momentum and energy of each phase. The interaction terms which couple the transport of mass, momentum and energy across the interphase appear in the macroscopic field equations after using a proper averaging method.

Previous studies have indicated that unless the momentum interaction terms are accurately modeled, the advantage of the two-fluid model over the mixture model disappears. In addition to the standard interfacial drag force, at least two transient forces, i.e. the virtual mass and Basset forces, exist. However, these terms are not well established. It can be said that many experimental information necessary to develop an accurate two-fluid model are not available. Therefore, the present state of the arts in the two-phase flow instrumentation implies that considerable uncertainties exist in the constitutive relations for the two-fluid model. In spite of these shortcomings, however, there is no substitute available for modelling accurately two-phase phenomena where two-phase are weakly coupled. Examples of these are flow reversal, flooding and transient countercurrent flow. In the present paper, the two-fluid model formulation and the constitutive equations for the interfacial momentum transfer are presented.

[1] Zuber et al., GEAP-5417, Vol. 1 (1967).
[2] Ishii, "Thermo-fluid Dynamic Theory of Two-phase Flow," Eyrolle, Paris (1975).
[3] Ishii, ANL-77-47 (1977).
[4] Ishii and Mishima, ANL-80-111 (1980).
[5] Ishii and Zuber, AIChE J., 25, 843 (1979).

ABSTRACT

"THE ANALYSIS OF INSTABILITY

MODES IN TWO-PHASE SYSTEMS"

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The occurrence of instabilities in two-phase systems is normally unde-sirable. Indeed, instability modes can often lead to poor process control and/or physical damage to the equipment involved.

This paper will present an analytical approach which can be utilized to appraise the instability margin of boiling and condensing systems. The insta-bility modes of interest in this study are the so-called density-wave and Ledinegg (i.e., excursive) instability modes. It will be shown that these modes are uniquely related. In addition, the procedure to introduce neutronic feed-back, as may occur in a nuclear reactor, will also be discussed.

Finally, it will be shown that the transfer functions resulting from linear analysis can be investigated using both classical servo techniques (e.g., Nyquist plots) or state variable techniques. The relationship between these two approaches will be quantified.

# TURBULENCE MEASUREMENTS IN TWO-PHASE BUBBLY FLOWS

## M. LANCE

Laboratoire de Mécanique des Fluides
de l'Ecole Centrale de Lyon (France)

※

Studies on two-phase flows are of great interest in many industrial situations. Recent developments of Energy Sciences and Nuclear Engineering cause an acute need for a better understanding of fundamental phenomena, in simple geometries. Of particular importance are turbulent mechanisms in gas-liquid flows, especially for the spatial distribution of each phase [1]. In order to complete the few investigations on this topic [2], [3], a research program on bubbly flows is being developped at the "Laboratoire de Mécanique des Fluides de l'Ecole Centrale de Lyon" by Pr Bataille and Drs Charnay, Lance, Marié. The aim is to obtain as detailed a description of the fluctuating motions in the liquid phase as one would expect nowadays for single phase turbulent flows.

An experimental facility has been built [4], which allows to study vertical co-current air water bubbly flows, in three basic patterns : homogeneous flow, shear flow and pure strained flow. In each case, the turbulence in the liquid is generated by a grid, through which air bubbles are injected. The results presented here concern the first one, which has been investigated by means of local optical probes, hot-film anemometry and by Laser Doppler Anemometry. Problems of local instrumentation and of signal processing are discussed. Fluctuating motions are characterized by the fluctuating intensity, the statistical moments, the time-correlations, the one-dimensional spectra, and the Reynolds stress tensor components. The evolution of these quantities following the non-dimensional parameters is determined and discussed. It is found that two flow regimes occur, the first one being dominated by bubble-effects, the second one by a strong interaction with the initial grid-generated turbulence. Bonds with a possible numerical modelling are considered.

References :

[1]     Drew-Lahey,
        RPI Report, 1978.

[2]     Serizawa, Katoaka, Mishigoshi,
        Int. J. of Multiphase Flow, 1975, vol. 2, n° 3, p. 221.

[3]     Sullivan, Houze, Buenger, Theofanous,
        OECD, CSWI Meeting, Paris, June 12-14, 1978.

[4]     Lance, Charnay, Bataille,
        La Houille Blanche, 1979, n° 5, p. 331.

[5]     Lance, Marié, Charnay, Bataille,
        ANS/ASME/NRC International Topical Meeting on Nuclear Reactor Thermal-Hydraulics.
        Saratoga, October 5-8, 1980.

ONE DIMENSIONAL BURNING WAVE IN A PARTICULATE BED

Dr. Donald A. Drew
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on leave at
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Abstract:

In order to evaluate the ability of two-phase flow models to give satisfactory
predictions in complicated flow situations, the models should be carefully
compared to simple flow situations.  While the ability of a model to predict a
simple flow does not guarantee its validity in a more complex situation, the
failure of a model in that simple situation practically precludes its use in any
more realistic context.

In some cases, the burning of a particulate monopropellant occurs as a wave of
burning, or a "flame."  In order to understand the mechanical aspects of this
process, a model has been developed which utilizes the dependence of burning
rate on pressure which prevails after the ignition temperature is reached.
Compressibility effects are also neglected.  Integrals of total momentum,
particle flow and total mass flow are derived.  The remaining equations are
analyzed in the phase plane in order to assess the possibility of existence of
plane wave solutions.  The equations of motion allow the propagation of a
one-dimensional burning wave provided the stresses transmitted by the solid
particles remain sufficiently low.  Results are shown for a specific solid
stress model.

<u>Constitutive Equations of Time-Dependent Shear Thickening Solutions:</u>

Steven T. J. Peng

Poly(methacrylic acid) (PMAA) in aqueous solution or FM-9 (the commercial name of an ICI product) in organic solvent exhibit strong time-dependent thickening behavior. The time needed to induce the thickening strongly depends on the shear rate. In this paper we derive the constitutive equation to describe this very unusual behavior in the framework of the Rivlin-Erickson formulation with consideration that the material constants will depend on the structure of the solution which will evolve with time as the imposed shearing continues. The obtained constitutive equation to describe the progressively shear thickening behavior is in the following form:

$$T = -pI + [\eta_0 + Kt^n ( \sqrt{II_{\dot{\gamma}} + hIII_{\dot{\gamma}}}\ )^{\frac{n}{b}}]\ A_1$$
$$+ Kt^m ( \sqrt{II_{\dot{\gamma}} + h\ III_{\dot{\gamma}}}\ )^{\frac{m}{b}}\ (A_1^2 - 1/2\ A_2)$$

where $T$ is the stress tensor; $\eta_0$ is the zero shear viscosity; $II_{\dot{\gamma}}$ and $III_{\dot{\gamma}}$ are second and third invariants of rate of strain tensor respectively; $K$, $n$, $m$ and $b$ are material constants which may depend on concentration and temperature. Finally, $A_1$ and $A_2$ are the Rivlin-Erickson tensors.

Session TM-3:  DYNAMIC PLASTICITY

Organizer:    J. DUFFY (Brown)
Chairperson:  H. KOLSKY (Brown)

*  9:30 - 10:00    R.B. SCHWARZ (Argonne)
                   "Dislocation Motion at High Strain Rates"

* 10:00 - 10:30    J. DUFFY (Brown)
                   "Dynamic Deformation of Mono and Polycrystalline Specimens
                   Using the Kolsky Bar"

  10:30 - 11:00    COFFEE BREAK

* 11:00 - 11:30    J.R. ASAY (Sandia)
                   "Shock Wave Studies of Elastic-Plastic Response at High
                   Pressure"

* 11:30 - 12:00    J. SHIOIRI and K. SATOH (Univ. Tokyo, Japan)
                   "An Ultrasonic Study of Dislocation Behavior at High Strain Rates"

* 12:00 - 12:30    L. SEAMAN, D.R. CURRAN and D.C. ERLICH (SRI International)
                   "An Anisotropic Plasticity Model for Shear Band Damage Under
                   High Rate Loading"

Dislocation Motion at High Strain Rates

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Dislocation motion in alloys for applied stresses above the yield
stress have been investigated analytically and through computer-
modelling methods. The dislocation was represented by a string in the
constant line-tension approximation and the solutes were modelled by
extended obstacles randomly distributed on the dislocation glide
plane. The presence of viscous-drag forces (phonon and electron
scattering) as well as thermal fluctuations were also considered.

In the absence of thermal fluctuations the motion has quite
different characteristics depending on the value of the viscous
drag.[1] For low viscous-drag forces a dislocation either glides at
high velocities ($v > v_s/50$, where $v_s$, where $v_s$ is the speed of sound in
the solid) or does not move at all. For high viscous forces, $v$ is a
continuous function of the applied stress, approaching zero as
$(\sigma - \sigma_c)^{1/2}$ where $\sigma_c$ is the yield stress at $T = 0$. For $\sigma \geq 2 \sigma_c$ the
motion is mostly drag-controlled, with characteristics similar to those
observed in the presence of low viscous forces.

For temperatures $T \geq 100$ K, the dislocation motion exibits the same
characteristics for all values of the viscous drag forces. For
$1/500 < \dfrac{v}{v_s} < 1/5$, the $\sigma - v$ relationships can be well approximated by

$$b\sigma = b\sigma_c(T,c,f_o) + Bv$$

where $c$ is the solute molar concentration, $f_o$ is the strength of the
dislocation-solute interaction force, and $B$ is the viscous drag
parameter. For higher dislocation velocities the extrinsic contribution
to the flow stress decreases and $b\sigma$ approaches asymthotically $Bv$.

(1) Schwarz, R. B., Phys. Rev. B 21. 5617 (1980).

Dynamic Deformation of Mono and Polycrystalline

Specimens Using the Kolsky Bar

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## Abstract

A description is presented of experiments in which  either single
crystal or polycrystalline specimens are deformed in shear.  These experi-
ments are performed in a torsional Kolsky (split Hopkinson) bar and the
imposed strain rates are either quasi-static  ($\sim 10^{-4}$ s$^{-1}$)  or dynamic
($\sim 10^3$ s$^{-1}$).  The instrumentation provides records of stress, strain and
strain rate as functions of time as well as stress-strain curves.  Poly-
crystalline specimens tested include aluminum, copper, zinc, magnesium
and mild steel [1,2]; single crystals are LiF [3] and aluminum [4].  The
experiments include jump tests in which an initial quasi-static strain
rate is changed abruptly to a dynamic rate, thus providing a measure of
strain rate sensitivity and of strain rate history effects in the material.
For LiF etch pit observations were made after testing; while for aluminum
thin samples of the crystals were examined by TEM to measure dislocation
density and observe dislocation distribution.

[1]   Senseny, P. E., Duffy, J., and Hawley, R. H., "The Effect of Strain Rate
      and Strain Rate History on the Flow Stress of Four Close-Packed Metals,"
      Journal of Applied Mechanics, March, 1978, pp. 60-66.

[2]   Wilson, M. L., Hawley R. H., and Duffy, J., "Strain Rate and Strain Rate
      History Effects in Two Mild Steels," Brown University Report NSF
      ENG 75-18532/8, March, 1979.

[3]   Chiem, C. Y. and Duffy, J., "Strain Rate History Effects in LiF Single
      Crystals During Dynamic Loading in Shear," Brown University Report,
      1979, NSF ENG 75-18532/11, to be published in Materials Science and
      Engineering, Vol. 48, No. 2, 1981, pp. 207-222.

[4]   Chiem, C. Y. and Duffy,J., report in preparation.

SHOCK WAVE STUDIES OF ELASTIC-PLASTIC RESPONSE AT HIGH PRESSURE*

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Abstract

Shock wave techniques provide a valuable tool for determining the dynamic response of materials at extreme loading rates. Early shock wave investigations focused on high pressure equation of state applications. More recently, the development of sophisticated driver and instrumentation techniques have provided a capability for detailed studies of fine structure in plane stress waves produced by several different loading processes. Present methods in use include steady shock compression, ramp loading, and unloading or reloading wave profiles from the shocked state which allow measurements of a variety of physical effects at high loading rates, such as dynamic yielding, viscoplastic behavior, and variation of shear strength with shock pressure. Recently, these methods have been extended to ultra high shock pressures, providing a capability for evaluating elastic-plastic properties of materials to shock pressures in the Mbar regime.

This discussion will include a brief review of shock wave methods for evaluating elastic-plastic effects and the use of plate impact techniques for studying the high pressure dynamic response of materials. Recent results obtained on an aluminum alloy shocked to maximum pressures of 600 kbar will also be discussed. These studies allow determination of shear strength variations with shock pressure, evaluation of viscoplastic properties in the shocked state, and have revealed a discrepancy between models used to describe dynamic elastic-plastic response and experimental results. The implications of this work to dynamic plasticity will be discussed.

# AN ULTRASONIC STUDY OF DISLOCATION BEHAVIOR
## AT HIGH STRAIN RATES

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The purpose of this work is to develop a new experimental means
of obtaining information on the behavior of dislocations under dynamic
plastic deformation. Presentation includes the theoretical background,
the experimental method and results, and discussions.

The effect of the dislocations under glide motion upon the atten-
uation and velocity of a superimposed ultrasonic wave are analyzed
theoretically. The analysis covers two starin-rate ranges in which
dominant rate controlling mechanisms of the moving dislocations are the
thermally assisted overcoming of point obstacles such as the forest
dislocations and the viscous damping, respectively. Time-resolved
measurements of the ultrasonic attenuation and velocity in specimens
undergoing dynamic plastic deformation were made utilizing a specially
devised ultrasonic apparatus, which has a resolution time of about 3
μsec and a frequency range of 5 to 50 MHz. Measurements were conducted
for polycrystalline aluminum of 99.99% purity and OFHC copper. The
dynamic plastic deformation was imposed by the split Hopkinson pressure
bar method. The strain rate range was about 50 to 3000 /sec. The
attenuation during the plastic deformation depends upon the strain rate
and the ultrasonic frequency, while the velocity is less sensitive to
the strain rate and frequency. The changes in the attenuation and ve-
locity which occur when the dynamic deformation is suddenly stopped can
be related to the contributions of the moving dislocations under the
deformation, which are a small fraction of all the dislocations exist.
Comparing the results of the experiments with the results of the theo-
retical analysis, the behavior of dislocations under dynamic plastic
deformation is discussed.

# AN ANISOTROPIC PLASTICITY MODEL FOR SHEAR BAND DAMAGE UNDER HIGH RATE LOADING[*]

L. Seaman, D. R. Curran and D. C. Erlich
SRI International, Menlo Park, CA 94025

A constitutive relation or model for shear banding has been developed to represent compressive loading at high shear strains in fragmenting cylinders and in penetrator impacts on targets. The model exhibits the inhomogeneous, anisotropic plastic flow at high strain rates observed in several steels.

For small plastic strains the model response is isotropic and homogeneous, following the von Mises yield criterion with nonlinear work-hardening. But above a critical threshold of plastic strain, shear bands begin to nucleate in the model. With continued plastic strains, the bands grow. Bands nucleate and grow in proportion to plastic shear strains on the planes of the bands. The shear bands represent an inhomogeneous deformation mechanism, and because the bands form on only a few planes, the deformation mechanism is also anisotropic.

The usual incremental plasticity flow law is used for the stress-strain relations until shear banding commences. With the presence of bands, the shear strength on damaged planes is reduced by the amount of shear banding and is proportional to the normal stress on the plane. Thus the bands are permitted to separate under tension but do not affect the strength under high compression.

Coalescence and fragmentation are provided in the model in a simple and very approximate way.

The model was developed from quantitative data from contained fragmenting cylinders of HF-1 high fragmentation steel and 4340 steel. Comparisons of fragmention in HF-1 and of shear band size distributions in 4340 with the model predictions are encouraging, but do not show exact correspondence. A clearer physical basis for the nucleation and growth processes is needed.

Session TM-4:  COMPOSITE MATERIALS AND LAMINATES

Organizer:     U. YUCEOGLU and D.P. UPDIKE (Lehigh)
Chairperson:  U. YUCEOGLU (Lehigh)

* 9:30 - 10:00     G.E.O. WIDERA (Univ. Illinois, Chicago) and N. MUNIR (Sargent
                   & Lundy, Chicago)
                   "The Edge Problem in Laminated Plates: An Asymptotic Analysis"

* 10:00 - 10:30    J.R. BARBER (Univ. New Castle-upon-Tyne, England) and
                   M. COMNINOU (Univ. Michigan)
                   "The Hot Axisymmetric External Crack Between Dissimilar
                    Materials"

  10:30 - 11:00    COFFEE BREAK

* 11:00 - 11:30    S.S. WANG (Univ. Illinois, Urbana)
                   "Interlaminar Stresses in Composite Laminated Plates Under
                    Bending"

* 11:30 - 12:00    T.W. CHOU (Univ. Delaware)
                   "Mechanical Behavior of Hybrid Composites"

* 12:00 - 12:30    U. YUCEOGLU and D.P. UPDIKE (Lehigh)
                   "Adhesive Layer Edge Stresses in Multi-Layer Composites"

# THE EDGE PROBLEM IN LAMINATED PLATES: AN ASYMPTOTIC ANALYSIS

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N. Munir
Sargent and Lundy, Chicago, IL 60603

In the first part of the present paper, the systematic derivation of a refined theory (i.e., first and second approximation equations) governing the elastic response of anisotropic plates is considered. The method employed to accomplish this is that of asymptotic integration which combines dimensional analysis with the expansion in powers of a small parameter of the solution of the three-dimensional theory (1-4). The advantages of employing the asymptotic method, in addition to its systematic nature, are that no a priori kinematic and/or static assumptions need be made, that transverse stresses naturally develop, and that thick plate corrections follow automatically. Even though general anisotropy is allowed for in the formulation only six elastic constants appear in the first approximation. The second approximation introduces six additional elastic compliances. These compliances relate first approximation stresses to the transverse strains, which are seen to be second approximation quantities. The paper concludes with an analysis of the stress distribution in the vicinity of the free edges of a two layer laminate. As indicated in (5), this problem is one of continuing interest and its solution will lead to a better understanding of laminate failure.

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(5) Pagano, N. J., "Stress Fields in Composite Laminates", AFML-TR-77-114, 1977.

# THE HOT AXISYMMETRIC EXTERNAL CRACK BETWEEN DISSIMILAR MATERIALS

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Maria Comninou
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Two half spaces of dissimilar material properties are
brought together and bonded over a circular region of radius $r = a$
to form an exterior axisymmetric interface crack. Due to loads
applied at infinity, the common boundary (i.e., the bond and
any contact region that develops) transmits tractions of speci-
fied resultants, P in tension and S in shear. This problem, as
well as the interior penny-shaped interface crack [1], involves
material interpenetration when the crack is assumed to be com-
pletely open [2,3]. Thus, for any value of the tensile force P,
and with S = 0 to retain axisymmetry, there is an annular contact
region adjacent to the bond, $a < r < b$, while separation occurs
for $r > b$. If the temperature is raised by an amount $\Delta T$, then
the contact region expands. The dependence of $P/\Delta T$ on $b/a$ is
examined, and the tractions over the bond and contact zones
are obtained. A closed form solution is given for full contact,
for which P vanishes.

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    Penny-Shaped Crack Reconsidered," Int. J. Engng Sci., Vol.
    16, pp. 765-772, 1978.

# INTERLAMINAR STRESSES IN COMPOSITE LAMINATED PLATES UNDER BENDING

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The problem of flexural bending of finite dimensional fiber-reinforced, laminated composite plates has been of significant interest in the analysis and design of laminated composite structures due to its technical importance. Approximate solutions given by Whitney [1] to incorporate transverse shear deformation and exact elasticity solutions by Pagano [2,3] for laminated plates in cylindrical bending have demonstrated clearly the limitations of the classical laminated plate theory. Due to the presence of geometric discontinuities, the respose near the boundaries may be more complex than in the interior of a finite dimensional laminate plate under bending, and current knowledge of the state of interlaminar stresses in this region remains very limited. In this paper, a study of interlaminar stresses near the geometric boundaries and in the interior of a composite laminate subjected to pure bending at its ends is presented. Formulation of the problem is based on the theory of anisotropic elasticity. By using Lekhnitskii's complex-variable stress potentials, a pair of coupled governing partial differential equations of the sixth order are obtained. The solution of the governing P.D.E.'s leads to the stress singularity at laminate edges and detailed stresses in the laminated plate. Effects of geometric and lamination variables on the interlaminar stresses are studied also.

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# MECHANICAL BEHAVIOR OF HYBRID COMPOSITES

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The term hybrid composite is used to describe composites containing more than one type of fiber material. Hybrid composites are attractive structural materials because they provide designers with the new freedom of tailoring composites and achieving properties that cannot be achieved in binary systems; it is a more cost-effective means of using expensive fibers such as graphite and boron; and hybrids may achieve a balance of stiffness and strength, and an increased elongation to failure.

The basic mechanical properties of hybrid composites have recently been reviewed by Chou and Kelly [1]. Following their terminology, hybrids can be categorized into three types, according to the arrangements of fibers and laminae. In the first type the different kinds of fibers are closely mixed together and infiltrated with a matrix material. The hybrid in this case is described as _intermingled_. The second type of hybrid is made by bonding together separate laminae, each containing just one type of fiber in a resin, and is known as _interlaminated_. The third type of hybrid consists of fabrics each of which contains more than one type of fiber, and it is called _interwoven_.

The purpose of this paper is to examine the stiffness, strength and "hybrid effect" of unidirectional composites. Both analytical and experimental results are presented for hybrids in interlaminated, intermingled and interwoven types.

## REFERENCE

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# ADHESIVE LAYER EDGE STRESSES IN MULTI-LAYER COMPOSITES

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Multi-layer composites as plate and shell structural elements has been used extensively in primary and secondary components of advanced structural systems.  In all these systems layers with different material characteristics and thicknesses are joined or bonded together by means of relatively thin and high-strength adhesive layers in order to meet the design requirements. Consequently stress distributions in adhesive layers and the analytical models which simulate them have received considerable attention in the scientific and engineering literature.  In current analytical models, the thin adhesive layer is assumed either to be of zero thickness or substituted by continously distributed mechanical compression-tension and shear springs [1,2].  Both of these models fail to predict some features of stress distributions near the edges of adhesive layers [1,2,3].  Adhesive stresses near the edges can influence the failure modes of the multi-layer composites.

Therefore, a simplified continuum model for the adhesive layer might be more appropriate in some cases.  The main objective of this study is to investigate the adhesive stresses in the adhesive layer of bonded joints and layered composites.  For this purpose a simplified continuum model developed by the present authors is used to account for the interlaminar stresses. Thus, the adhesive stresses are expanded in power series in the thickness coordinate z and terms of O(z) or less are maintained.  The governing equations of the adhesive layer are reduced to

$$[D]\{\sigma(x), s(x)\} = 0, \quad d\tau(x)/dx = -s(x)$$

$$\sigma_z(x,z) = \sigma(x) + zs(x), \quad \tau_{xz}(x,z) = \tau(x)$$

where [D] is a (2 x 2) matrix of  differential operators.  The stresses are further analyzed in terms of a boundary layer concept.  Applications of the theory to multi-layer plates and shells are discussed.

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     Cylindrical Shells" Emerging Technologies in Aerospace Structures,
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     ASME Publication, August 1980, pp 53-65.

Session TM-5:  MECHANICS OF EARTHQUAKE SOURCES

Organizer:     R. DMOWSKA (Brown)
Chairperson:  K. AKI (M.I.T.)

* 9:30 - 10:00    J.W. RUDNICKI (Univ. Illinois, Urbana)
                  "Energy Radiation from Seismic Sources"

* 10:00 - 10:30   K. AKI (M.I.T.)
                  "Strong Motion Prediction Based on the Barrier Model"

  10:30 - 11:00   COFFEE BREAK

* 11:00 - 11:30   S. DAS (Lamont-Doherty, Columbia Univ.)
                  "A Numerical Method for the Study of General Three-Dimensional
                   Rupture Propagation and Earthquake Source Mechanisms"

* 11:30 - 12:00   C.H. SCHOLZ (Lamont-Doherty, Columbia Univ.)
                  "Scaling Laws for Large Earthquakes, Consequences for
                   Physical Models"

* 12:00 - 12:30   W.F. LEITH and D.W. SIMPSON (Lamont-Doherty, Columbia Univ.)
                  "Seismological and Geological Studies of Induced Seismicity
                   at Nurek Reservoir"

Energy Radiation from Seismic Sources

by

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Energy changes associated with earthquakes are usually estimated
in one of two ways: from the transient dynamic displacements inferred
from seismograms or from measurements of the static deformation in the
epicentral region. The first method yields an estimate of the seismic
wave energy radiated to the farfield, the second an estimate of the
energy expended in overall deformation. Although the results of both
procedures are often referred to as "earthquake energy," they are not,
in principal, equal. Various expressions given in [1] for the energy
radiated by elastodynamic sources in infinite linear elastic solids are
reviewed. Application of the expressions to simple sources elucidates
the source of radiated energy in seismic faulting and clarifies the
relationship of radiated energy to other energies involved in earthquake
faulting.

For sources characterized by a moment density tensor, the radiated
energy is expressed in terms of the squares of far-field particle
velocities. If the source can be approximated as a point, the radiated
energy can be expressed in terms of the moment tensor. This approxi-
mation, however, typically overestimates the actual radiated energy.
An expression for the energy radiated by an extending crack-like source
is given in terms of fault surface traction and particle velocity [2].
This representation reveals the source of radiated energy as the dif-
ference during quasi-static and dynamic propagation of the energy flux
to the fault edge and the work-rate of the fault surface tractions.
If the fault surface tractions are time independent, the latter term
vanishes. If, in addition, the fault edge propagates near the limiting
speed (Rayleigh wave speed in plane strain, shear wave speed in anti-
plane strain), the dynamic energy flux to the fault edge is small, and
the remaining term in the expression for the radiated energy is the
excess of the static strain change over the quasi-static work done by
the fault surface tractions (called $W_O$ in [3]). In this special case,
the radiated energy can be determined from the static end-states.

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[2]  Kostrov, B. V., Izv. Earth Physics, 1, 23-40, 1974.

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# STRONG MOTION PREDICTION BASED ON THE BARRIER MODEL

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The concept of barrier model which has been developed by seismologists at M.I.T. since 1977 was successfully applied recently to the study of acceleration power spectra for several California earthquakes by Apostolos Papageorgiou, a student of M.I.T.'s Department of Civil Engineering. He used a specific barrier model which is essentially a rectangular Ben-Menahem-Haskell-type model filled with circular cracks of Brune-Sato-Hirasawa-Madariaga-type. In terms of the barrier model, the slip occurs only within the circular crack, and its fringe constitutes the barrier. The barrier is characterized by its length $d$ over which cohesive stress $\sigma_c$ exists. The barrier interval is twice the radius $a$ of the crack. The strength of the barrier is measured by Griffith's fracture energy $G$, which is proportional to $\sigma_c^2 d$. The most important result obtained so far is that the barrier interval $2a$ is roughly proportional to the amount of the slip. In other words, the stress drop within the barrier interval (circular crack) is roughly constant, about 300 bars, independent of earthquake magnitude. The barrier fracture energy, however, is greater for a larger slip. On a given fault, the barriers of different strengths are distributed, and the barrier intervals are greater for earthquakes with greater slip, because weaker barriers are broken by them. In terms of barrier strength, the 1966 Parkfield earthquake is similar to the 1933 Long Beach earthquake, and the 1952 Kern County earthquake is similar to the 1857 Fort Tejon earthquake.

Papageorgiou's result indicates that the high-frequency limit of acceleration spectrum is not due to attenuation effect as proposed by Hanks (1979) but due to source effect. In terms of barrier model, the barrier length $d$ is the crucial parameter determining the high-frequency limit. For a successful prediction of strong motion for a potential large earthquake, therefore, it is important to find how $d$ scales with earthquake magnitude.

A NUMERICAL METHOD FOR THE STUDY OF GENERAL THREE-DIMENSIONAL RUPTURE
PROPAGATION AND EARTHQUAKE SOURCE MECHANISMS

by S. DAS (Lamont-Doherty Geological Observatory  of Columbia University,
Palisades, New York 10964

We review a method to determine the displacement and the stress on a
crack for a planar shear crack of arbitrary shape propagating in an infin-
ite, homogeneous medium which is linearly elastic everywhere off the crack
plane.  The main idea of the method is to use a representation theorem in
which the displacement at any given point on the crack plane is written as
an integral of the traction over the whole crack plane.  The tractions are
weighted by the three-dimensional solution to Lamb's problem.  Such solu-
tions usually require one numerical integration, but fortunately the
necessary solutions are obtainable in closed form.  The weighting factor is
discretized over a space and time grid to solve the integral equation
numerically.  As a test of the accuracy of our numerical technique, we
compare the results with known solutions for two simple cases.  Applying
this method to spontaneous cracks on fault planes of constant strength, we
find that the terminal rupture velocity in the direction of purely anti-
plane rupture is $\beta$, the shear wave speed.  In the purely in-plane direc-
tion, it is $0.5\alpha$ or $\alpha$ depending on the yield strength, $\alpha$ being the com-
pressional wave speed.  We find the relationship between the average slip
and the average dynamic stress drop for long thin faults for our model,
which has physically acceptable stress singularities at the fault edges.
For such faults, which are much longer than wide, we find that the slip at
a point in the interior of the fault is controlled by the fault width.
This implies that two rectangular faults of the same width but varying
lengths have the same slip for the same average static stress drop, a fact
clearly contradicted by observations that fault slip increases with
earthquake size for earthquakes occurring along the same fault.  This
contradiction is resolved if the stress drops increase with fault length.

# SCALING LAWS FOR LARGE EARTHQUAKES
## CONSEQUENCES FOR PHYSICAL MODELS

Christopher H. Scholz
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The mean slip in large earthquakes is found to correlate linearly with fault length and is not related to fault width. Consequently, static stress drop increases with aspect ratio. Since aspect ratio tends to increase with seismic moment, there is a systematic increase of stress drop with moment for both strike-slip and thrust interplate earthquakes.

Dynamic models of rectangular faulting, however, clearly show that slip should be controlled by the fault width and scale with the dynamic stress drop. The only way that these models can be reconciled with the observations is if there is a correlation between dynamic stress drop and fault length, namely that the former determines the latter. There are several serious objections to that interpretation, however. An alternative is that these models are poor representations of large earthquakes probably because the boundary conditions that are assumed at the edges of the rupture are unrealistic. If it is assumed instead that the base of a large earthquake is in a plastic zone, it may be possible to relax the restriction that slip is zero at the base of the fault so that no healing wave propagates from there. In that case slip is determined by fault length and the observations may be interpreted as indicating that dynamic stress drop is nearly constant for these earthquakes. In this model, static stress drop is also a function of fault length, rather than width, and the conclusion quoted in the first paragraph that stress drop is a function of aspect ratio is incorrect. Instead, static stress drop is nearly constant. These two alternatives represent two extreme views of the mechanism of large earthquakes and predict very different scaling of the dynamic process.

Seismological and Geological Studies
of Induced Seismicity at Nurek Reservoir

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More than 1800 earthquakes greater than magnitude 1.4 have occured
during the first nine years of filling of the 300 m deep Nurek re-
servoir in Tadjikistan.  This is more than four times the average
rate of activity in the region prior to filling.  The increased
activity has occured in a series of bursts, the two most intense of
which were related to rapid increases in water level during reservoir
filling--to 105 m in 1972 and to 205 m in 1976.  All periods of high
seismicity occur when the water level is higher than it has been pre-
viously, however, all of the largest earthquakes and most of the bursts
of activity are triggered by decreases on the rate of filling of the
reservoir.  Extremely small changes in filling rate can trigger the
onset of activity; for example, the largest earthquakes all followed
decreases in filling rate of about 0.5 m/day, other bursts began soon
after the reservoir started to empty by rates as small as 0.2 m/day.
The increased activity which follows may start after delays as short
as just one to four days.

Not all of the reservoir area is seismic.  Seismicity, which is occur-
ing at depths of one to eight km, seems to be confined to the upper
plate of the Ionakhsh thrust, which dips southeast beneath the reser-
voir along an evaporite decollerment.  The folded sedimentary rocks
which comprise this thrust sheet can be inferred to have a highly
directional permeability, due to the alternation of permeable lime-
stones with impermeable shales and gypsums.  Seismicity seems to be
occurring in areas where fold structure allows reservoir water to
diffuse to depth, and not where structure inhibits this diffusion.

As the reservoir has approached maximum size, the area of induced
seismicity has increased as well.  From 1973 to 1978, activity in the
central reservoir area followed the growth upstream.  The first stage
of activity in 1971-1972 was characterized by low b-values, and included
the largest earthquakes of M=4.6 and M=4.3.  From 1973 to 1979, activity
was concentrated in the central reservoir area and b-values were higher.
No induced earthquakes larger than M=4.1 have occured since November 1972.
Considering reservoir loading and the lag-time seen in the water level
and seismicity data collected so far, we conclude that pore-pressure
is the dominant variable determining the time-distribution of induced
seismic activity, and permeability the dominant variable determining
the spatial distribution of induced activity.  The short delay times
and seismicity migration patterns imply very high permeabilities, and/or
that very low stress changes are necessary to induce an earthquake at
Nurek.

Session TM-6: RESIDUAL STRESSES

Organizer:    B.W. SCHMIEDESHOFF (U.S. Army Research, NC)
Chairperson:  B.W. SCHMIEDESHOFF (U.S. Army Research, NC)

* 9:30 - 10:00    E.H. LEE and R.L. MALLETT (STANFORD) and R.M. MCMEEKING
                  (Univ. Illinois, Urbana)
                  "Analysis of Residual Stresses Generated by Metal Forming
                   Processes"

* 10:00 - 10:30   S.L. PU (U.S. Army, Watervliet) and M.A. HUSSAIN (GE, Schenectady)
                  "Stress Intensity Factors for Radial Cracks in a
                   Thick-Wall Cylinder with Residual Stresses"

  10:30 - 11:00   COFFEE BREAK

* 11:00 - 11:30   E.F. RYBICKI (Univ. Tulsa)
                  "Studies of Residual Stresses in Girth Welded Pipes"

* 11:30 - 12:00   D.P.H. HASSELMAN, Y. TREE and A. VENKETESWARAN (VPI & SU)
                  "Residual Stress Relaxation in Brittle Structural Materials
                   Due to Creep by Crack Growth"

  12:00 - 12:30   C.P. BURGER and I. VOLOSHIN (Iowa State) and H.A. GOMIDE
                  (Federal Univ. of Uberlandia, Brazil)
                  "Three-Dimensional Strains During Hot Rolling of Billets
                   by Photoplastic Simulation"

12:30 - 1:45    LUNCH BREAK

1:45 - 2:15     S.E.S. ANNUAL BUSINESS MEETING

Analysis of Residual Stresses Generated by

Metal Forming Processes

by

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## Abstract

For metal fo,ming processes, in order to be able to predict the onset
of metal-forming defects, such as crack initiation. it is necessary to
determine the complete history of the stress distribution in the work-
piece from initial billet to final product.  This calls for elastic-
plastic analysis, since in plastic-rigid analysis stresses can only be
determined in the currently plastically deforming region which usually
constitutes only a small part of the workpiece.  The development  of
finite-deformation elastic-plastic continuum mechanics, finite-element
analysis and large digital computers is such that meaningful evaluations
of such stress-distribution histories can now be carried out.  These
include the residual stress distributions in the product, which if high
can be considered to be a type of forming defect.

Computed residual stress distributions are presented, with emphasis
on extrusion, and their dependence on process  conditions and parameters
is illustrated.  In particular, for a cylindrical billet, small reductions
of section, of the order of the yield point strain, can reverse the sign
of the longitudinal residual stress distribution compared with that
associated with large area reductions.  Such features are presented and
interpreted.

STRESS INTENSITY FACTORS FOR RADIAL CRACKS IN
A THICK WALL CYLINDER WITH RESIDUAL STRESSES

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Stress intensity factors for multiple, radial cracks in a thick-
wall cylinder without residual stress have been reported by several
investigators using different methods. It is a common practice, how-
ever, to introduce favorable residual stresses in such pressure vessels.
The method of thermal simulation has been found very effective in the
study of redistribution of residual stresses in a cylinder due to the
presence of notches and cracks in the cylinder. In this paper a com-
bination of thermal simulation, high order isoparametric elements and
the weight function method is used to determine stress intensity factors
for multiple, radial cracks in a pressurized and partially autofrettaged
cylinder.

Extensive numerical results are presented for multiple cracked
cylinders having an external diameter twice that of the internal diameter.
The addition of residence stress does not change the previous findings
that the cylinder with two diametrically opposed cracks is in general
the weakest configuration and that the stress intensity factor decreases
as the number of radial cracks increases.

STUDIES OF RESIDUAL STRESSES IN GIRTH WELDED PIPES

by

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ABSTRACT

    Welding inherently produces residual stresses.  Recent progress in
understanding residual stresses due to welding has been made through a
combination of experiments and computational modeling.  A finite element
computational model that represents residual stresses due to welding is
described.  Comparisons of residual stress data and computed values are
shown for surface stresses as well as through thickness stress distributions.
The finite element analysis has proven to be very useful in developing and
evaluating ways of controlling residual stress due to welding.  Examples
are presented for girth welded pipes.

RESIDUAL STRESS RELAXATION IN BRITTLE STRUCTURAL

MATERIALS DUE TO CREEP BY CRACK GROWTH

D. P. H. Hasselman, Y. Tree, and A. Venketeswaran

Department of Materials Engineering
Virginia Polytechnic Institute and State University

## ABSTRACT

Residual stress relaxation in brittle structural materials cannot
occur by dislocation-glide or climb. Creep by Nabarro-Herring or Coble
diffusional creep will occur only at elevated temperatures near the
fabrication temperature. Instead, at lower temperature levels residual
stress relief in brittle materials can occur only by the mechanisms of
creep by crack growth. In a number of cases this has lead to usually
highly catastrophic, spontaneous fracture.

A study was conducted of the mechanisms of residual stress relief
in polycrystalline aluminum oxides with residual stresses introduced by
thermoviscoelastic effects during rapid quenching. Residual stress
relief was observed to occur by elastic creep due to viscous grain boundary
cracking and cavitation at temperatures as low as 800°C. Such residual
stress relief was accompanied by irreversible increases in the dimensions
of the specimens. In a fine-grained alumina, residual stress relaxation
was accompanied by spontaneous disintegration of the specimen at
temperatures as low as 750°C.

Modified creep deformation maps were devised which include elastic
creep by crack growth for purposes of illustration of the experimental
observations.

THREE-DIMENSIONAL STRAINS DURING HOT ROLLING

OF BILLETS BY PHOTOPLASTIC SIMULATION

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An experimental method was developed that provides information
about three-dimensional strains through photoplastic simulation. Several
polymeric materials have been evaluated as model materials. Their de-
formation behavior and photo-optical properties can be adjusted to simu-
late the behavior of hot metal under large deformations by varying the
mixture ratios of the resins and test temperatures.

The paper reviews these developments and presents tantalizing
examples of the potential of the technique in extrusion, forging, rolling,
and upsetting. It presents detailed results for the strain fields that
occur during the rolling of thick billets. The photoplastic data is
handled through an interactive computer graphics procedure to produce
three-dimensional plots of the strain field. This form of presenting
the results highlights the extreme non-uniformity of the strains across
a section of the rolled billet. Areas of peak strain are the most likely
sources of defects in the finished product and should be avoided. The
technique presented in this paper can lead to the formulation of good
numerical models of the rolling process such that the parameters that
cause uneven flow can be identified and eliminated.

Session TA-1:  HYDRODYNAMIC STABILITY, TRANSITION AND TURBULENCE - I

Organizer:    J.T.C. LIU (Brown)
Chairperson:  J.T.C. LIU (Brown)

* 2:15 - 3:15    J.T. STUART (Imperial College, England)
                 "Hydrodynamic Stability, Transition and Turbulence"

* 3:15 - 3:45    C-M. HO (USC)
                 "Local and Global Dynamics of Coherent Structures"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    H. AREF (Brown)
                 "Dynamics of Vortex Structures in Two-Dimensional Flows"

* 4:45 - 5:15    R.E. KELLY and D. GOUSSIS (UCLA)
                 "Stability of a Liquid Film Flowing Down a Heated Inclined Plane"

  5:15 - 5:45    M.L. ADAMS (Univ. Akron) and A.Z. SZERI (Univ. Pittsburgh)
                 "Finite Rotating Disk Flow with Multiple Recirculation Cells"

Hydrodynamic stability, transition and turbulence

Abstract by J. T. Stuart

The topic of turbulence in fluid systems is, of course, an old
one and has been studied in a scientific manner at least since the time
of Osborne Reynolds in the 1880's. The problem of describing tur-
bulent, that is random or chaotic, motions in a fluid is still with us
although substantial advances have been made over the years.

One line of approach is to take the laminar, or streamline, flow
field, which is always a possible solution of the relevant equations,
and to subject this to a small oscillatory or other perturbation. Com-
plicated flow fields can be obtained by such means and in particular it
is possible to calculate critical Reynolds numbers for many flow fields
above which laminar flow is unstable. Such critical Reynolds numbers,
however, do not usually agree with experimentally-observed values
above which turbulence occurs. Moreover, the amplitudes and stresses
cannot be calculated by a linearized theory.

These deficiencies have stimulated much work in the last 30 years
or so on the effects of nonlinearities on the development of oscillations
and on the calculation of the critical Reynolds number. In part of this
lecture attention will be given to classical and prototype flow fields and
to some advances which have followed from linear and nonlinear theories
of instability of laminar flows, and from significant experiments.

In the 1950's an alternative approach to turbulence was proposed
by W. V. R. Malkus by his suggestion that the observed flow field in
thermal convection between horizontal planes should, subject to certain
constraints, maximise the heat transport. This simplifying suggestion
was evaluated mathematically by L. N. Howard some 20 years ago. He
showed that Malkus' idea, although not literally true, does have im-
mensely important consequences, in that it is possible to calculate an
upper bound to the heat transport in thermal convection. Moreover,
one to the excellent detailed work of Howard and those who followed,
particularly F. Busse, the difference between the observed heat trans-
port in experiment and the calculated maximising heat transport is
only a relatively small numerical factor. This is an enormous scien-
tific achievement for a problem as complicated as that of turbulent
motion. In the second part of the lecture the achievements of this par-
ticular line of research activity will be discussed for several flow fields.

# LOCAL AND GLOBAL DYNAMICS OF COHERENT STRUCTURES

**Ho,** Chih-Ming
Department of Aerospace Engineering
University of Southern California

A phenomenal model, the subharmonic evolution, is proposed to model
the free shear layers. This concept includes two physical mechanisms:
the local instability process and the global feedback, and can well des-
cribe the evolution of coherent structures. Near the origin of a shear
layer, the vortex develops from the initial instability waves. Its sub-
harmonic wave is selected by the subharmonic resonance mechanism and
amplifies. The vortex merging occurs at where the subharmonic saturates.
The merging location is determined by the global feedback mechanism. At
the merging location, the thickness of the shear layer doubles. Due to
the change of the length scale, the original subharmonic evolves from
amplifying local subharmonic to decaying local fundamental and the new
subharmonic starts another new cycle.

This model can provide a rationale to explain the paradox of the
linear stability analyses being able to interpret a non-linear shear flow.
A very simple formula of the spreading rate is derived according to the
subharmonic evolution model. The physical insights of many experimental
results are revealed.

Dynamics of Vortex Structures in Two-Dimensional Flows

by

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The discovery of coherent vortex structures in many predomin-
antly two - dimensional laboratory and geophysical shear flows,
even in the turbulent regime, has led to a revival of interest in
vortex dynamics. The ability to perform well controlled, compre-
hensive, numerical simulations has been of considerable importance.

A review of some recent developments is presented. The em-
phasis is on new qualitative features such as the emergence of
chaotic solutions in few vortex systems, the persistence of well-
defined vortices in "strongly turbulent" flows, and the relation-
ship between statistical self-similarity and dominant modes of
interaction of vortex structures. Relationships and analogies
with topics in the theories of dynamical systems, turbulent shear
flows, solitons and plasma dynamics are pointed out. In conclusion
some brief remarks suggesting new areas of application for vortex
methods are advanced.

STABILITY OF A LIQUID FILM FLOWING

DOWN A HEATED INCLINED PLANE

by

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When an isothermal layer of viscous fluid with a free surface
flows down an inclined plane, it first becomes unstable to long wave-
length disturbances at a critical value of the Reynolds number which
decreases as the angle of inclination from the horizontal increases[1].
Roca[2] has investigated the case when the plane is heated, but he
neglected the streamwise perturbation density term.  When this term is
included, we have found that the hydrodynamic mode of instability is
actually stabilized by heating from below.  A thermal mode can also
exist, however, which is more unstable at low values of the Reynolds
number.  The regions of instability for the two modes are separated,
however, so that, for a fixed value of the Rayleigh number, stable flow
occurs for an intermediate range of Reynolds numbers.  This corridor of
stability increases as the Rayleigh number increases.

References:

1.    C. - S. Yih, Dynamics of Nonhomogeneous Fluids (Macmillan, 1965),
      pp. 180 - 194.

2.    R. Roca, "Gravitational instability of a liquid layer flowing
      down an inclined plane and heated from below", J. de Mechanique,
      v.5 (1966), pp. 117 - 147.

# FINITE ROTATING DISK FLOW WITH MULTIPLE RECIRCULATION CELLS

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## ABSTRACT

Solutions were developed for the primary laminar steady flow field that occurs in an incompressible, isoviscous, Newtonian fluid which is contained between two finite parallel disks. One disk rotates at constant angular velocity and the other is held stationary, and either a source or a sink is located concentric to the axis of rotation. The analysis is general, containing all terms of the Navier-Stokes equations for rotationally symmetric flows, and produces a four-parameter family of solutions. The high Reynolds number flow contains multiple cells, arranged along the radius, and the flow appears to be uniquely defined by the boundary conditions and the Reynolds number. In the figures below are equipotential stream function lines (streamlines) which show development of a three-cell flow structure at the highest rotational Reynolds number computed. The algorithm employed was not suitable for studies at significantly higher Reynolds number. However, these results suggest that for even higher Reynolds number, more cells would develop, i.e., the cells would "multiply" as Reynolds number is further increased. A numerical study of the time transient problem, coupled with an experimental program is now in progress and should answer the question regarding cell multiplication.
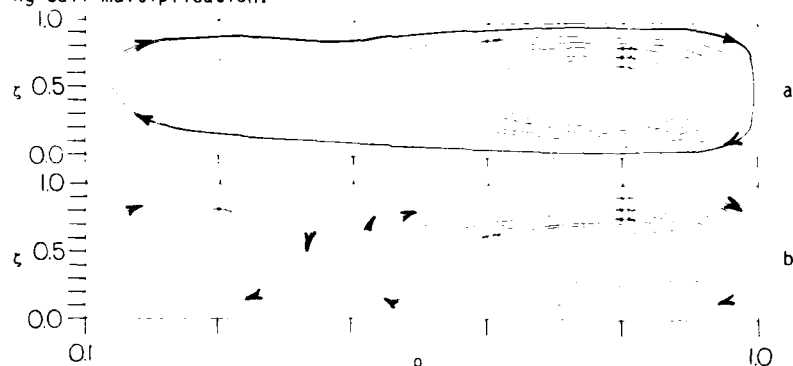
Equipotential stream function lines in the radial plane; $r_1/r_2 = 0.1$, $\sigma = 10$, $R_Q = 1$, (a) $R_E = 2000$, (b) $R_E = 14000$; top disk rotates, bottom disk is stationary

$\zeta = z/s$   $z =$ axial coordinate   $s =$ axial gap   $Q =$ throughflow
$\rho = r/r_2$   $r =$ radial coordinate   $r_1 =$ inner radius   $\omega =$ angular velocity
$\sigma = r_2/s$   $R_Q = Q/2\pi s \nu$   $r_2 =$ outer radius   $\nu =$ kinematic viscosity
$R_E = r_2^2 \omega/\nu$

Session TA-2:  MATERIAL INSTABILITY AND FAILURE - I

Organizer:    A. NEEDLEMAN and R.D. JAMES (Brown)
Chairperson:  A. NEEDLEMAN (Brown)

* 2:15 - 2:45    N. TRIANTAFYLLIDIS (Univ. Michigan)
                 "On the Large Deformation of Elastic Materials with Non-
                  Elliptic Strain Energy Density Functions"

* 2:45 - 3:15    S.S. ANTMAN (Univ. Maryland)
                 "Stability of Motion of Structures with a Nonlinear
                  Material Response"

* 3:15 - 3:45    K.W. NEALE (Univ. Sherbrooke)
                 "Inertial Effects in Neck Development"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    M. PITTERI (Univ. Padova, Italy)
                 "On the Kinematics of Twinning"

* 4:45 - 5:15    R.D. JAMES (Brown)
                 "The Jump in Temperature Across a Propagating Phase Boundary"

* 5:15 - 5:45    J.W. HUTCHINSON and P.S. STEIF (Harvard)  K.W. NEALE (Univ.
                 Sherbrooke)
                 "Neck Propagation in Polymer-Like Materials"

ON THE LARGE DEFORMATIONS OF ELASTIC MATERIALS WITH
NON-ELLIPTIC STRAIN ENERGY DENSITY FUNCTIONS

by N. Kikuchi* and N. Triantafyllidis**

ABSTRACT

In this paper we are concerned about certain general aspects of the
behavior of elastic materials which, at certain levels of deformation,
admit solutions with discontinuous strains (i.e. they possess non-elliptic
strain energy densities).

First, we show that all stable solutions to boundary value problems
involving the aforementioned materials do not admit any finite region where
the body is in the hyperbolic regime.

Then, we classify these materials into different categories, according
to their behavior near infinite strains. Some examples are provided and
the behavior of the discretized version of these problems is also briefly
discussed.

Finally a theorem of a general nature is outlined, which asserts that
if a boundary value problem for such a material has a stable solution,
this solution can be obtained by solving an equivalent problem with a
"convexified" strain energy density function.

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Stability of Motion of Structures with a Nonlinear Material Response

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This lecture is devoted to the analysis of classical dynamic stability problems for structures having a nonlinear material response. Special attention is devoted to problems in which the loading is nonconservative, to problems in which the loading is periodic in time, and to problems in which shocks form. If the structure is non-linearly elastic and if it is described by a geometrically exact theory, then it is governed by a quasilinear hyperbolic system of equations, which are notoriously difficult to analyze because their solutions are likely to develop shocks. Some of the analytic difficulties vanish if the material has a suitable viscoelastic dissipative mechanism. In this case the quasilinear hyperbolic system is replaced by a quasi-linear parabolic system. The study of stability and global behavior devolves on an intricate interplay between these systems and their linearizations. In a number of cases new phenomena, not predicted by classical engineering theories, occur.

INERTIAL EFFECTS IN NECK DEVELOPMENT

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Abstract

A special problem is analyzed to illustrate the effects of inertia on the development of necks in dynamically-loaded solids. An incompressible power-law creeping material characterized by $\dot{\varepsilon} = \alpha\sigma^n$ in simple tension is considered.

The problem, previously examined in [1] neglecting inertial effects, concerns the plane strain tensile loading of an infinite strip with a slight nonuniformity in the form of a sinusoidal thickness variation. A linearized solution for the influence of inertia on the amplification of the nonuniformity is obtained along the lines described in [1]. Among the parameters discussed are the applied loading rate and the ratio of the wavelength of the nonuniformity to the thickness of the strip. We emphasize the basic differences in behaviour when inertial effects are included.

[1]  J. W. Hutchinson, K. W. Neale and A. Needleman:  Mechanics of Sheet Metal Forming, edit. D. P. Koistinen and N.-M. Wang, Plenum (1978) 111.

On the Kinematics of Twinning

M. Pitteri

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Most theories of twinning are essentially kinematic and aim at
explaining some features of the phenomenon, like twinning planes
and twinning directions for various crystal species. We outline
a kinematic theory of twinning which is essentially a continuum
theory in the sense that it involves gross quantities. On the
other hand, the material symmetry of the constitutive equations
is such as to retain some features of the description of a crys-
tal lattice. We show that the definition we propose is consis-
tent with a number of twinning modes in the cubic, tetragonal and
hexagonal classes and show that, in an extremely simplified case,
we get results consistent with experience from the kinematics we
propose and an equilibrium theory.

# The Jump in Temperature Across
# A Propagating Phase Boundary

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Unlike the phases of ordinary fluids, solid phases are often found to occur in metastable equilibrium, especially at low temperatures. At constant temperature a stress-extension test on a bar made of a material which allows the co-existence of two solid phases will often produce a large hysterysis loop. It is then impossible, by static measurements alone, to determine the values of stress $\tau^*$ and temperature $\theta^*$ at which the two phases have the same specific free energy. I explore the idea that by the measurement of the jump in temperature across a steadily propagating phase boundary, the values of $\tau^*$ and $\theta^*$ can be determined.

An application to the design of shape memory engines will be presented.

NECK PROPAGATION IN POLYMER-LIKE MATERIALS

J. W. Hutchinson[*], K. W. Neale[+] and P. S. Steif[*]

## Abstract

Some polymeric materials are characterized by a nominal (engineering) stress-strain curve in simple tension which first increases to a local maximum, decreases to a local minimum, and then upturns strongly with additional straining. A thin cylinder or strip of such a material will propagate a neck along its length when pulled in tension. The steady-state problem has been studied in which, for an observer traveling with the necking front, the up-stream state and velocity are uniform as is the downstream velocity. Steep gradients occur in the necking region and three-dimensional effects must be accounted for to relate upstream and downstream conditions and to predict the velocity of the neck front. Both time-independent and time-dependent material models are considered.

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4/6/81

Session TA-3:  ATOMISTIC MODELLING OF DISLOCATION AND CRACKS

Organizer:    J.H. WEINER (Brown)
Chairperson:  J.H. WEINER (Brown)

* 2:15 - 2:45    V. CELLI and D. EVANS (Univ. Virginia)
                 "Lattice Dynamical Theory of Dislocation Mobility"

* 2:45 - 3:15    R.M. THOMSON and E.R. FULLER, JR. (NBS, Washington)
                 "Lattice Theory of Fracture"

* 3:15 - 3:45    A. PASKIN (Queens College, visiting Brookhaven), D.K. SOM
                 (Queens College) and G.J. DIENES (Brookhaven)
                 "Lattice Trapping and Crack Propagation"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    B. MORAN and W.G. HOOVER (UC/Davis-Livermore)
                 "Atomistic Fracture Studies:  Dynamic Stress Intensity and
                 J Integral"

  4:45 - 5:15    R.B. STOUT (Lawrence Livermore Natl Lab)
                 "Modelling Dislocation Kinetics, Deformations and Thermo-
                 dynamics for Crystalline Solid Materials"

Lattice Dynamical Theory of Dislocation Mobility

V. Celli and D. Evans

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Analytical solutions for the uniform motion of a dislocation in
a lattice were obtained long ago for a one-dimensional model of an
edge dislocation and for a straight screw dislocation (which gives a
two-dimensional strain field), un 'er the assumption of linear inter-
atomic bonds that "snap" at the passage of the dislocation core. The
applied stress (or the strain s) required to maintain the dislocation
in uniform motion at velocity v is found to be large, due to the gener-
ation of lattice waves, but drops to zero at one particular velocity
in one dimension. Replacement of the snapping bond model by more
realistic force laws in the dislocation core leads to a great decrease
of the phonon generation due to the rearrangement of bonds. Results
for straight screw dislocations show that the strain vs. velocity curve
still can show a deep minimum (where $s < 10^{-5}$) in the region $v \sim 0.5\ c$,
where c is the speed of transverse sound. Certain lattice vibrations
are found to favor the propagation of a dislocation, even against the
applied stress. Thus, the effect of the thermal vibrations is subtle
and model-dependent.

Efforts to extend the calculations to edge dislocations have met with
computational difficulties. A general analysis can be given, however,
of the propagation of a non-linear disturbance, such as the disloca-
tion core, in a discrete linear elastic medium; this leads to a
physical interpretation and to a generalization of the results found
with particular models.

# LATTICE THEORY OF FRACTURE

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## Abstract

Atomic effects in fracture relate to slow crack growth which often
precedes dynamic fracture in brittle materials either with or without
assistance from external chemical environments. In addition, atomic
effects are crucial in determining the stability of a crack against
shear breakdown, which is a necessary condition for brittle fracture
either with or without attendant plastic deformation in the surrounding
material.

Two primary techniques have been developed to explore the atomic
structure of cracks based on assumed ad hoc two-body force laws between
the atoms. We will describe the lattice static theory in its most
general form in terms of the lattice Greens functions of the material,
demonstrate the graphical solution it provides, and explain a theorem
which shows how under certain conditions the mechanics of the problem
splits into a part relating to the crack tip bond, a part relating to
the compliance of the remainder of the lattice, and a part relating to
the external macroscopic machine providing the force. When only one
atom pair in the system is nonlinear, a complete solution of the problem
can be obtained (often analytic) easily.

The broad outline of chemical effects in fracture can be displayed from
this theory. In particular, the pressure dependence of the crack
velocity is associated with the order of the reaction at the crack tip,
and the activation energy and activation volume are intimately associated
with the critical nucleus for double kink formation. Additional features
of possible chemical phenomenon relate to bridging reactions at the tip,
and the competition between parallel paths in configuration space which
lead to various chemical final states.

A current assessment of problems which may be accessible to this lattice
theory and other computer simulation is provided, together with the
author's opinions on the limitations of these theories.

Abstract

Lattice Trapping and Crack Propagation

A. Paskin and Dilip K. Som
Queens College of the City University of New York

and

G. J. Dienes
Brookhaven National Laboratory


Lattice trapping[1] is an unexpected result obtained in a number of
atomic treatments of crack propagation. It has been widely accepted[2]
as a fundamental consequence of the discrete periodic nature of
solids. Molecular dynamic simulation was used in the present inves-
tigation[3] to study crack behavior in an atomistic two-dimensional
solid. Using a relatively large two-dimensional sample of about
11000 atoms and a relatively long-range Lennard-Jones potential,
lattice trapping is found to be a negligible effect. Sample size
effects and the non-linearity of the modulus were studied and found
not to affect the lattice trapping result. Negligible lattice
trapping seems mainly to be a consequence of the range of the potential;[4]
long-range continuous potentials produce negligible lattice trapping.
The role of the range of the potential and the importance of the bond
snapping approximation[1] in lattice trapping will also be demonstrated
using some of the earlier lattice static calculations.[1]

[1]Thomson, R. M., Hsieh, C. and Rana, V. 1971, J. Appl. Phys. 42, 3154-60.
[2]Lawn, B. R. and Wilshaw, T. R. "Fracture of Brittle Solids" (Cambridge
    University Press, 1975), Chap. 7.
[3]Paskin, A., Som, D. K. and Dienes, G. J. 1981, J. Phys. C 14, L171-76.
[4]Sinclair, J. E. 1975, Phil. Mag. 31, 647-71.

Atomistic Fracture Studies. Dynamic Stress Intensity and J. Integral.* B. MORAN and W. G. HOOVER, Department of Applied Science, University of California at Davis-Livermore, Livermore, California 94550.

The close-packed two-dimensional triangular lattice with nearest-neighbor springs is a convenient standard material for fracture studies. We have carried out a series of simulations using this material to evaluate continuum fracture theories and to understand macroscopic stress fields from a microscopic viewpoint. Accurate critical stress intensity factors, $K_I^C$, are obtained from atomic strips up to 20 atoms wide. Propagation of cracks along a prestressed strip, with displacement boundary conditions giving a linear decrease in strain with distance, led to crack arrest in regions with static $K_I$ values about 15% less than $K_I^C$. The precise amount of overshoot depends upon the details of the forces. A typical arrested configuration is shown below. The arrow indicates the location at which a static crack would be stable.
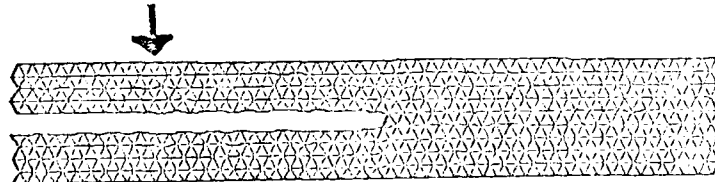


Dynamic cracks have been studied in order to compare the dynamic stress intensity factors with the shadow-pattern measurements made by Kalthoff, Beinert, Klemm, and Winkler at the Fraunhofer Gesellschaft Institut fur Festkorpermechanik, in Freiburg. We have used several different methods based on the J-Integral approach to estimate dynamic stress intensity.

Modelling Dislocation Kinetics, Deformations, and
Thermodynamics for Crystalline Solid Materials*

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Using concepts from statistical mechanics, a stochastic density
function for dislocations is defined as a scalar function for the number
of dislocations per unit volume per unit species. A balance statement
for the dislocation density results in a Boltzmann-type equation. This
equation contains functional degrees of freedom to model the evolution of
the stochastic dislocation density.

The deformation response of an idealized crystalline solid material
is modelled as a continuum deformation of a crystalline lattice structure
plus the discontinuous deformation due to dislocation kinetics. The
lattice deformation is defined by a function as in continuum mechanics.
The discontinuous deformation is defined by a functional that depends on
the stochastic dislocation density function space. It is shown that a
differential strain measure does not exist when dislocation kinetics
occurs. However, for the thermodynamic modelling, a tensor function can
be defined for a measure of only the recoverable lattice structural
strain.

In the thermodynamic modelling, an internal energy functional is
defined that depends on the entropy, the recoverable lattice strain, the
scalar dislocation density, and the mass density. The conjugate variable
to the recoverable strain is the thermodynamic stress; and the conjugate
variable to a dislocation species is the thermodynamic chemical potential
of the dislocation species. From this, the phenomenological plastic
yield concept becomes equivalent to a thermodynamic criteria of a
stress-lattice work term becoming equal to (or greater than for
irreversible thermodynamics) the dislocation chemical potential. The
approach suggests, in terms of Onsager's formalism, forms for models of
irreversible thermodynamic processes during material deformation.

Session TA-4: NONLINEAR ANALYSIS OF COMPOSITE PLATES AND SHELLS

Organizer: J.N. REDDY (VPI & SU)
Chairperson: V.B. VENKAYYA (Wright-Patterson, Ohio)

* 2:15 - 2:45   C-Y. CHIA (Univ. Calgary, Canada)
"Nonlinear Vibration of Rigidly Clamped Antisymmetric
Cross-Ply Plates"

* 2:45 - 3:15   J.N. REDDY and W.C. CHAO (VPI & SU)
"Large Deflection Vibration of Cross-Ply Laminated Rectangular
Plates with Certain Edge Conditions"

* 3:15 - 3:45   D.W. SCHMUESER (GM Res. Lab., Warren)
"Effects of Bimodular Material Behavior on Nonlinear Stress-
Strain and Strength Response of Laminated Composite Shells"

  3:45 - 4:15   REFRESHMENT BREAK

* 4:15 - 4:45   C.C. CHAMMIS and R.A. AIELLO (NASA Lewis Res., Cleveland)
"A Unified Nonlinear Laminate Theory"

* 4:45 - 5:15   M.W. HYER (VPI & SU)
"Thermal Deformations of Unsymmetrically Laminated Composites
Plates"

# NONLINEAR VIBRATION OF RIGIDLY CLAMPED
## ANTISYMMETRIC CROSS-PLY PLATES

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In recent years the use of laminated composites as structural members
has increased considerably.  A considerable amount of research work, there-
fore, is under progress regarding the elastic behavior of laminated aniso-
tropic plates.  In vibration analysis the system of coupled nonlinear
differential equations in the sense of von Karman obtained by Herrmann [1]
for isotropic plates was extended to generally laminated plates by Whitney
and Leissa [2].  Since then, approximate solutions for large amplitude
flexural vibrations of some laminated plates have been discussed by several
investigators.  A review of the literature can be found in Ref. [3] or else-
where.

This paper is analytically concerned with nonlinear free flexural
vibration of an antisymmetric cross-ply rectangular plate.  The four edges
are assumed to be rigidly clamped.  The dynamic von Karman-type equations
of the plate [3] are expressed in terms of three displacements.  An
approximate solution is formulated by use of a modified Galerkin's method.
Satisfying the out-of-plane boundary conditions for clamped edges a single
term in the separable form is used for the transverse displacement.  The
two inplane displacements are expressed in the form of polynomeals of the
sixth degree with coefficients chosen so that the equations of inplane
equilibrium and inplane boundary conditions for the vanishing of inplane
displacements are satisfied.  The Galerkin procedure is now applied to
the equation of transverse motion and furnishes a differential equation
for the time function.  This equation is of the Duffing type whose solution
is well known.  The amplitude-period response curves are presented for
different values of material properties, aspect ratio and number of layers.

REFERENCES:

1.  G. Herrmann, Influence of Large Amplitude on Flexural Motions of
    Elastic Plates, NACA TN 3578, 1955.

2.  J. M. Whitney and W. Leissa, Analysis of Heterogeneous Anisotropic
    Plates, Journal of Applied Mechanics, Vol. 36, 1969, pp. 261-266.

3.  C. Y. Chia, Nonlinear Analysis of Plates, McGraw-Hill International
    Book Co., 1980.

# LARGE DEFLECTION VIBRATION OF CROSS-PLY LAMINATED RECTANGULAR PLATES WITH CERTAIN EDGE CONDITIONS

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## ABSTRACT

Over the past few years fibre-reinforced laminates have found increasing application in many engineering structures. Recent developments in the analysis of plates laminated of fibre-reinforced materials is largely dominated by static (bending) analysis, and the large amplitude oscillations of layered composite plates have received relatively less attention. All of the previous works in the large amplitude free vibrations of composite plates have employed either the Galerkin method or the perturbation method to reduce the governing equations to the Duffing type equation. The finite-element analyses of the large deflection vibration of laminated plates are scarce in the open literature.

In the present paper the large-amplitude free vibrations of layered composite (cross-ply) plates is studied using a shear-flexible finite element [1]. The finite element is based on a combination of the shear deformable plate theory of Whitney and Pagano [2] and the nonlinear plate theory of Von Karman. The coupling among individual vibration modes of the generalized displacements is taken into account. Numerical results showing the effect of the boundary conditions, aspect ratio, side-to-thickness ratio, and material orthotropy on the fundamental models of vibration are presented. Present results agree very closely with the approximate analytical solutions of Chia and Prabhakara [3], and Chandra [4]. Apparently, the present paper is the first one which considers the finite-element analysis of the nonlinear oscillations of layered composite plates.

1. Reddy, J. N., "A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates," Int. J. Numer. Math. Engng., Vol. 15, pp. 1187-1206, 1980.

2. Whitney, J. M. and Pagano, N. J., "Shear Deformation in Heterogeneous Anisotropic Plates," J. Appl. Mech., Vol. 37, pp. 1031-1036, 1970.

3. Chia, C. Y. and Prabhakara, M. K., "A General Mode Approach to Nonlinear Flexural Vibrations of Laminated Rectangular Plates," J. Appl. Mech., Vol. 45, pp. 623-628, 1978.

4. Chandra, R., "Large Deflection Vibration of Cross-Ply Laminated Plates with Certain Edge Conditions," J. Sound and Vibration, Vol. 47, No. 4, pp. 509-514, 1976.

EFFECTS OF BIMODULAR MATERIAL BEHAVIOR
ON NONLINEAR STRESS-STRAIN AND STRENGTH RESPONSE
OF LAMINATED COMPOSITE SHELLS

by

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## ABSTRACT

Bimodular materials are those which exhibit different material response when stressed in compression rather than tension. Strength predictions of laminated composites can be affected by bimodular material response and nonlinear stress-strain behavior in individual laminae. Most applications of phenomenological failure models to laminated composites assume that lamina yield and ultimate lamina strength are the same. In addition, most failure models do not account for bimodular material response in a manner consistent with the basic postulates of continuum mechanics.

The present paper determines the effects of two bimodular material models on strength predictions for graphite/epoxy and glass/epoxy composites. Nonlinearities of composite laminae in transverse compression and shear are treated in an approximate manner by employing piece-wise linear and spline representations for the stress-strain curves. The Tsai-Wu stress and Sandu strain-energy failure criteria are applied to predict failure of thin, laminated shells. The strength predictions are based on numerical solutions to Sander's nonlinear axisymmetric shell equations. Numerical solutions for internally pressurized rings and compressed cylinders are compared to experimental data cited from the literature.

A UNIFIED NONLINEAR LAMINATE THEORY

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## ABSTRACT

A coupled thermomechanical, time dependent (unified) nonlinear laminate
theory is described.  This unified theory is based on approximate composite
micromechanics where the coupled thermomechanical and time dependent fiber and
matrix characteristics are properly accounted for.  The approximate composite
micromechanics are used to predict the corresponding coupled thermomechanical,
time dependent characteristics (constitutive relationships) of the ply.  These
ply constitutive relationships are then integrated over the various plies to
obtain the corresponding commposite (laminate) constitutive relationships.  The
theoretical development is streamlined for application to fiber/metal matrix
composites used at high temperatures.  Suitable functional relationships are
assumed for thermomechanical and time dependent behavior in order to aid the
integration and enhanace the numerical computations.  Unique features of this
unified theory include generally orthotropic thermomechanical and time dependent
behavior of the in situ matrix, no flow rule, and in situ ply constitutive
relationships.  Application of the theory to limited cases will be described to
illustrate the effectiveness, advantages and limitations of this unified theory.

Thermal Deformations of Unsymmetrically
Laminated Composites Plates

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The deformations associated with the cooling of unsymmetrically
laminated plates from their cure temperature is a geometrically non-
linear phenomenon. Failure to include the geometric nonlinearities will
result in predicted deformations which are quite different than those
observed experimentally. This paper discusses the analysis which must
be conducted to correctly predict the room-temperature shapes of unsym-
metrically laminated composite plates. The predictions are compared
with experimental results.

Experimental observations indicate the cured, room-temperature
shapes of unsymmetrically laminated composites are closely approximated
by right circular cylinders. In addition, these laminates appear t
*have two room-temperature shapes,* both cylindrical. A second cylinder
can be obtained from the first by a simple snap-through action. Clas-
sical lamination theory, in the sense of linear elastic behavior, linear
strain-displacement relations, and assumed validity of Kirchhoff's
hypothesis, predicts the room-temperature shapes of all unsymmetric
laminates to be a unique saddle shape. The discrepancy between classi-
sical theory and experimental observations can be resolved by extending
the classical theory to include a nonlinear strain-displacement rela-
tion. This paper uses this extension, together with a Ritz-minimization
of the total energy, to obtain predicted shapes. Seeking the first
variation of the total potential energy, using assumed deformations,
leads to a set of algebraically nonlinear equations. These equations
are solved for the shapes of the laminates. For all laminates there are
predicted to be either one equilibrium shape or three equilibrium
shapes, depending on the thickness, length, and width of the laminate.
For laminates with one predicted shape this shape is a saddle. A sta-
bility analysis, based on positive definiteness of the total potential
energy, indicates this shape is stable. The saddle, however, is shal-
lower than that predicted by the classical theory. For laminates with
three predicted shapes, two shapes are stable and one is not. The two
stable shapes are cylindrical, of opposite curvature, and with cylinder
axes at right angles to each other. The one unstable shape is a saddle.
Whether the stable saddle or stable cylinders exist is strictly a
function of the size of the laminate. For a $[0_2/90_2]_T$ laminate, for

example, any laminate larger than 50 x 50 mm will be cylindrical.

Experimental data confirms the predicted curvature of the cylinders
and shows the transition from stable saddle to stable cylindrical shape
as the laminate size increases.

Session TA-5:  CRUSTAL STRESS AND STRESS REDISTRIBUTION PROCESSES

Organizer:    R. DMOWSKA (Brown)
Chairperson:  P. MOLNAR (M.I.T.)

* 2:15 - 2:45    F.K. LEHNER, V.C. LI and J.R. RICE (Brown)
                 "Diffusion of Stress and Deformation Along Rupturing
                  Tectonic Plate Boundaries"

* 2:45 - 3:15    R.S. STEIN (Lamont-Doherty)
                 "Time-Dependent Rupture and Possible Viscoelastic Rebound
                  Associated with a Small Crustal Earthquake Sequence"

* 3:15 - 3:45    V.C. LI and R. DMOWSKA (Brown)
                 "Crustal Scale Instability and Precursory Seismicity"

  3:45 - 4:15    REFRESHMENT BREAK

* 4:15 - 4:45    M.D. ZOBACK (USGS, Menlo Park)
                 "In-Situ Stress Measurements and Earthquake Mechanics"

* 4:45 - 5:15    L. JONES (M.I.T.), W. BIQUAN and X. SHAOXIE (Inst. of Geophysics,
                 Beijing, China) and T.J. FITCH (Lincoln Labs)
                 "The Foreshock Sequence of the 4 February 1975 Haicheng
                  Earthquake (M ≈ 7.3)"

# Diffusion of Stress and Deformation Along
# Rupturing Tectonic Plate Boundaries

F.K. Lehner, V.C. Li and J.R. Rice
Division of Engineering, Brown University, Providence, R.I. 02912

The episodic occurrence of large earthquake ruptures along active
tectonic plate boundaries is accompanied by time-dependent stress alter-
ations and deformations which depend strongly on the mechanical layering
of the Earth's upper mantle and crust. Knowledge about one can enhance
one's understanding of the other and this seems to hold, in particular,
for certain aspects of the temporal and spatial distribution of large
earthquake ruptures. We present an analysis [Lehner, Li and Rice, 1981,
to appear in J. Geophys. Res.] of the role played by viscoelastic litho-
sphere/asthenosphere coupling in the time-dependent redistribution of stress
along plate boundaries or other seismic lineaments following great earth-
quakes. The study is based on a generalization by Rice of Elsasser's
model of stress-diffusion [Rice, 1980, in "Physics of the Earth's Interior",
A.M. Dziewonski and E. Boschi (eds.), North Holland], in which general
elastic plane stress deformations are allowed in lithospheric plates
which are coupled in an elementary way to a (Maxwellian) viscoelastic
asthenosphere. Solutions are developed which describe the large-scale
quasistatic distribution of thickness-averaged stresses in the lithosphere
at or near stationary or travelling rupture zones, modeled here by either
crack-like zones of fixed stress drop or dislocation-type slip zones.
Sudden ruptures shed load onto the asthenosphere which is gradually trans-
ferred back to the lithosphere by a slow relaxation process. The spatial
and temporal characteristics of the predicted stress alterations suggest a
significant role of lithosphere/ asthenosphere coupling effects in trigger-
ing interactions of great earthquakes, patterns of prolonged aftershock
activity, and the breaking of barriers or gaps by time-dependent stressing.
We also study the large scale response of the lithosphere to periodic slip
at a transform or subduction-type plate boundary, as described by an
appropriate limit cycle solution [Lehner and Li, EOS, Vol. 61 (46), 1051,
1980]. The periodic behavior of displacements and stress, their decay
away from the plate boundary, and a resolution into co-seismic and post-
seismic stress alterations are obtained and their dependence on plate
velocity, recurrence time, and a characteristic relaxation time investi-
gated. Post-seismic stress alterations of equal sign as co-seismic stress
jumps appear gradually at distances beyond one lithosphere thickness and
become progressively more important than the latter. They are followed
by a much slower post-seismic stress recovery, extending typically over
80% of the cycle length. Finally, we propose a new interpretation, based
on our results, of post-seismic strain rate data to be taken along a survey
line perpendicular to a transform fault, which would provide independent
estimates of lithosphere and asthenosphere parameters.

# TIME-DEPENDENT RUPTURE AND POSSIBLE VISCOELASTIC REBOUND ASSOCIATED WITH A SMALL CRUSTAL EARTHQUAKE SEQUENCE

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An unusually dense network of coordinated precise geodetic measurements, including leveling, triangulation, trilateration, and gravity, were undertaken before and following the $M_L$ 5.2 Homestead Valley, California earthquake sequence of 15 March 1979. The observations are modeled as dislocations in an elastic half-space; they provide evidence for time-dependent rupture with no more than one quarter of the total moment released seismically. During the first 1 1/2 years following a 3-hr, 4-shock sequence ($M_L$ 4.5-5.2), the initial 5 x 6 km (depth x length) slip surface yielded bilaterally along strike to 5 x 30 km, accompanied by the propagation of aftershocks. The seismic slip averaged 60 cm in the initial 5 x 6 km region, with less than 2 cm of additional post-seismic slip. Rupture barriers at the ends of the seismic patch did not slip appreciably at any time and remained continuously aseismic.

During the first 15-600 days after the seismic sequence, a 15% relaxation of surface deformation took place at an exponentially decaying rate in the epicentral region; this deformation is opposite in sense to the coseismic changes. Postseismic relevels oriented parallel to the extended slip surface are modeled by 10-15 cm of buried aseismic slip. During approximately the same period, 10-15 cm of horizontal displacement developed post-seismically at large distances from the fault trace, at 15-20 km. The vertical and horizontal post-seismic observations may be explained by aseismic crustal slip being induced in regions of increased shear stress and propagating to regions of lower stress, beyond the barriers that terminated the seismic slip. Subsequent epicentral relaxation and broad-scale displacements may result either from viscoelastic rebound if a channel or layer with effective viscosity of $10^{18}$-$10^{19}$ Poise underlies the fault at depths of about 10 km, or from deep post-seismic fault slip of about 50 cm. The geodetic observations suggest that ductile or viscous behavior in the upper lithosphere can be triggered by small earthquakes.

# CRUSTAL SCALE INSTABILITY AND PRECURSORY SEISMICITY

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We present a simple asperity model to explain features of seismicity patterns, including quiescence periods, observed before some large earthquakes. The patterns are interpreted in the context of the upward progression of a zone of slip, towards the earth's surface, from some depth of the lithosphere at a plate boundary. The slip zone may be blocked by local strength asperities along strike, these asperities may be zones of real alteration of inherent strength, or instead may be zones which are currently stronger due to a local slowdown of a basically rate-dependent frictional response. For any segment (between two asperities) that is relatively shorter than adjacent ones, a higher stiffness would result in diminishing of slippage in the blocked region, producing a spatial contrast of reduced seismicity. Quiescence over space and time would be maintained until stress concentration forces slip through one of the asperities, initiating a rupture from one end of the gap zone. This model is supported by seismicity evidence such as doughnut-shaped patterns of seismicity (Mogi, 1969), precursory clustering of seismic activity at the two ends of a gap zone, and propagation of rupture into the quiet region (Kelleher and Savino, 1975). This model suggests the possibility of lower b-values, perhaps associated with higher stress drops, as well as different waveforms and frequency content of seismic events that cluster around the future hypocenter, in comparison with the same characteristics of background seismic activity. For earthquake prediction purposes, the model emphasizes both the spatial and temporal consideration of seismicity changes preceding the main event.

In-Situ Stress Measurements and Earthquake Mechanics

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Knowledge about the state of stress in the crust is essential to progress in both earthquake prediction and earthquake hazards assessment research. In trying to assess earthquake hazards in intra-plate areas (such as the Central and Eastern U.S.) it is critical to determine if the sites of large historic earthquakes are unique. That is, do places like New Madrid, Missouri, and Charleston, South Carolinia represent anomalously weak zones in the crust responding to a relatively uniform stress field, or do they represent areas of concentrated stress? Data compiled to date suggests that the intraplate stress field is fairly uniform over broad regions and sites of large historic earthquakes can often be associated with reactivated zones of weakness in the crust responding to a regional stress field. However, only sparse in-situ stress data is presently available to test this hypothesis and reasonable physical mechanisms to explain why some zones have unusually low strength has yet to be found

In the field of earthquake prediction, understanding the physical processes leading-up to a major earthquakes on the San Andreas Fault for example, is impeded by our uncertainity about whether the seismic fault slip is a high stress ( 100 MPa) or low stress (~10MPa) phenomena. Laboratory studies of the frictional strength of rock at representative pressures suggest that the higher estimate is correct but the lack of observable frictional heat dissipation at the fault suggests a mean shear stress value similar to the lower value. To address this fundamental question, measurements have been made both of the variation of stress with distance away from the San Andreas fault zone, and the variations of stress with depth near the fault zone. Unfortunately, the measurements made to date can not be uniquely interpreted to support or reject either alternative. Increasing shear stress with distance away from the San Andreas was observed on two profiles of stress measurements made perpendicular to it. While the profile measurements are generally consistent with weak fault models, other interpretations of the data are possible. Data from a ~1-km deep well near the San Andreas in Southern California, however, shows that shear stress increases at an apparent rate of about 10 MPa/km; a value consistent with high stress at depth if extrapolation of the data to seismogenic depths is reasonable. As more data becomes available, especially from greater depth, stress measurements should be able to constrain postulated fault models and thus contribute directly to establishing a physical basis for earthquake prediction.

The Foreshock Sequence of the 4 February 1975
Haicheng Earthquake (M=7.3)

Lucile Jones (Massachusetts Institute of Technology), Wang Biquan,
Xu Shaoxie (Institute of Geophysics, Beijing, China), T.J. Fitch
(Lincoln Laboratory, Cambridge, Mass.)

We have examined the locations and radiation patterns of the fore-
shocks of the 4 February 1975 Haicheng earthquake (M=7.3).  Using
arrival times from six local seismic stations, the foreshocks and
mainshock were located relative to a master event.  The foreshocks
occurred in a tight cluster that elongated with time.  Before the
largest foreshock, the activity was located within a small, approxi-
mately equidimensional volume with a diameter of about two kilometers.
After the largest foreshock, the activity spread northwest and south-
east forming a six kilometer long, northwest trending zone.  First
motions and ratios of P to S amplitudes indicate that two different
faulting mechanisms occurred during the foreshock sequence.  The two
radiation patterns can tentatively be correlated with different parts
of the zone.  The hypocenter of the mainshock was not located on the
same fault as that defined by the foreshock's hypocenters but rather
was located six kilometers southsouthwest of and several kilometers
shallower than the foreshock cluster.  We think this large separa-
tion between foreshocks and mainshock in a direction perpendicular
both to the plane of rupture of the mainshock and to the trend of
the foreshocks might be the result of an en enchelon step in the
fault that slipped during the mainshock.  An analysis of the change
in stress due to slip on the possible foreshock faults shows that
the increase in shear stress on the mainshock fault caused by the
foreshocks can be no more than 4% of the stress drop associated with
the foreshocks.

Chairperson:  C.E. TAYLOR (U. of Florida)

2:15 - 2:45    C.W. SMITH (VPI & SU)
               "Experimental Observations on Sub-Critical Crack Growth"

2:45 - 3:00    J.T. PINDERA (Univ. Waterloo)
               "Analytical Foundations of the Isodyne Photoelasticity"

3:00 - 3:15    J.T. PINDERA and B.R. KRASNOWSKI (Univ. Waterloo)
               "New Experimental Method for Determination of the Stress
                Intensity Factors"

3:15 - 3:30    T. KOBAYASHI and R. CHONA (Univ. Maryland)
               "Examination of a Strip-Zone Model in Fracture Mechanics"

3:30 - 3:45    A. SHUKLA, R. CHONA and W.L. FOURNEY (Univ. Maryland)
               "The Variation of Non-Singular Stress Function Coefficients
                in Different Fracture Test Specimens"

3:45 - 4:15    REFRESHMENT BREAK

4:15 - 4:45    T. KOBAYASHI and N. SAAD (Univ. Maryland)
               "Studies of Micro-Crack Formation and its Effects on Crack
                Behavior Part I:  Development of Mica-Epoxy Composite
                Material for Simulation of Micro-Cracks"

4:45 - 5:15    H.P. ROSSMANITH (Tech. Univ. Vienna)
               "How 'Mixed' is Dynamic Mixed-Mode Crack Propagation? -
                A Dynamic Photoelastic Study"

5:15 - 5:30    D.D. RAFTOPOULOS (Univ. Toledo) and B. FARAHMAND (Tri-State
               Univ., Indiana)
               "Crack Subjected to Compressive Loading"

5:30 - 5:45    M. VAN OVERMEIRE and M.A.K. FAHMY (Vrije Universiteit, Brussels)
               "Finite Element Analysis of Double Circular Arc Tooth
                Profile Gears"

6:30 - 10:30   SES BANQUET (Andrews Hall)

# EXPERIMENTAL OBSERVATIONS ON
# SUB-CRITICAL CRACK GROWTH

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The majority of service fractures in structural elements are preceded by sub-critical flaw growth, usually under boundary conditions and problem geometries of extreme complexity. For over a decade the writer and his associates have been studying such problems through the frozen stress photoelastic method [1], [2], and have recently developed a procedure for simulating fatigue cracks under certain conditions [3]. This development allows the investigator to obtain experimental estimates of both flaw shape and stress intensity factors (SIF) where neither are known a-priori. In the present paper, results of applying the method to several three-dimensional finite cracked body problems, will be discussed, revealing

    i) varying SIF distributions along flaw borders
    ii) non-self similar flaw growth
   iii) non-planar flaw growth
   iv) absence of shear modes during flaw growth

with quantitative measurements of these phenomena.

## REFERENCES

[1]  Smith, C. W., "Use of Photoelasticity and Progress in Related Areas" Chapter 1 Experimental Techniques in Fracture Mechanics 2 - Society for Experimental Stress Analysis Monograph. A. S. Kobayashi, Ed., pp. 3-58 (1975)

[2]  Smith, C. W., "Photoelasticity in Fracture Mechanics," Experimental Mechanics, V20, N. 11, pp. 390-396. Nov. (1980)

[3]  Smith, C. W. and Peters, W. H., "Prediction of Flaw Shapes and Stress Intensity Distributions in 3-D Problems by the frozen Stress Method," Preprints of the Sixth International Conference on Experimental Stress Analysis - Munich, pp. 821-824. Sept. (1978).

# ANALYTICAL FOUNDATIONS OF THE ISODYNE PHOTOELASTICITY

by

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It has been shown by Pindera and Mazurkiewicz [1] , that the photoelastic isodynes represent geometric loci of points at which the total normal stress intensity acting on the corresponding cross-section along characteristic direction is constant.

$$I^x = I^x (x,y) = S_s m_s (x,y) = const. = P_y'(x,y), \; etc.$$

For a plane stress state: $P_y(x,y_o) = bP_y' = {}_o^{.x} (b\sigma_y) \; dx,$

Consequently, the isodynes represent functions $I^x$ and $I^{\ddot{}}$ partial derivatives of which are proportional to the corresponding normal stress components:

$$\sigma_y = \frac{\partial}{\partial x} (I^x); \quad \sigma_x = \frac{\partial}{\partial y} (I^y)$$

Obviously, the functions $I^x$ and $I^y$ represent partial derivatives of Airy's stress function for plane stress states:

$$I^x = I^x (x,y) = \frac{\partial}{\partial x} \phi(x,y), \quad I^y = I^y (x,y) = \frac{\partial}{\partial y} \phi(x,y)$$

Followingly, the shear stress components can be obtained by differentiation of the isodyne in the direction normal to the characteristic direction, taking into account the boundary conditions:

$$\tau_{xy} = - \frac{\partial^2 \phi}{\partial x \partial y} = - \frac{\partial}{\partial y} (I^x) + F_o^y = - \frac{\partial}{\partial x} (I^y) + F_o^x$$

where the functions $F_o$ can be readily evaluated.

Figures illustrate isodyne measurements using two basic techniques: the amplitude modulation and the frequency modulation.

Sample of References

[1] Pindera, J.T., Mazurkiewicz, S.B., "Photoelastic Isodynes: A New Type of Stress Modulated Light Intensity Distribution", MECHANICS RESEARCH COMMUNICATIONS, Vol. 4, 1977, 247-252.
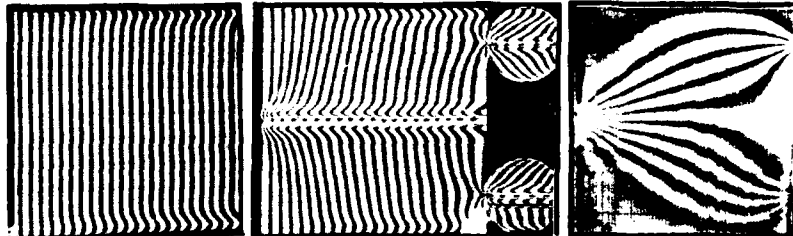
NEW EXPERIMENTAL METHOD

FOR DETERMINATION OF THE STRESS INTENSITY FACTORS

by

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Applicability of analytical, numerical and experimental methods in actual engineering determinations of the stress intensity factors, K, is limited, because of known reasons [1].

The major experimental methods applied in fracture mechanics to determine K-values such as photoelasticity, interferometry, shadow method, acoustic emission, moiré, do not supply information on the value of the normal stress component which is necessary to evaluate K-factors. The majority of these methods is based on analytical solutions for stress states in the region of crack tip in an infinite plate, and on other simplifying assumptions. As a result, the reliability of data evaluated from the experimentally obtained parameters is often difficult to assess.

It has been shown by J. T. Pindera, et al., [2], that the recently developed isodyne photoelasticity allows to directly determine all three components of a plane stress field along arbitrarily chosen cross-section:

$$\sigma_x = \sigma_x(x_0, y); \quad \sigma_y = \sigma_y(x, y_0); \quad \tau_{xy} = \tau_{xy}(x, y)$$

No auxilliary relationships are needed and no additional assumptions must be made.

Typical isodyne fringes in a bar with a crack, loaded by four forces are presented in the figures below. For comparison, the corresponding isochromatics field is included.

Samples of References

[1]  Kobayashi, A.S., ed., "Experimental Techniques in Fracture Mechanics", The Iowa State University Press and the SESA, Vol. 1 and 2, 1973, 1975.
[2]  Pindera, J.T., Mazurkiewicz, S.B., "Photoelastic Isodynes:  A New Type of Stress Modulated Light Intensity Distribution", MECHANICS RESEARCH COMMUNICATIONS, Vol. 4, 1977, 247-252.
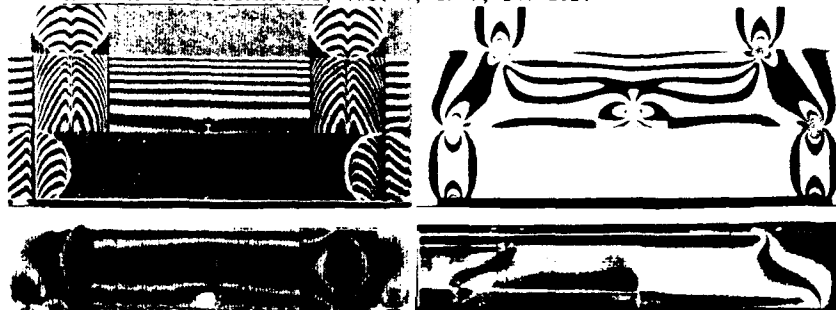
# EXAMANATION OF A STRIP-ZONE MODEL IN FRACTURE MECHANICS

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## ABSTRACT

Dynamic photoelastic studies of crack propagation in steels revealed the presence of a small-scale yield zone at the crack tip. Analysis of isochromatic fringe patterns around a crack tip with a small-scale yield zone and determination of fracture parameters such as stress intensity factor and yield zone size require a stress field model which accounts for yielding in the vicinity of the crack tip. This paper presents a computer-aided analysis and experimental verification of isochromatic fringe patterns around a crack tip in a strip-zone model.

An experimental strip-zone model was produced by bonding two halves of crack-line-wedge-loading (CLWL) specimen of Homalite 100 with a tough structural adhesive. The adhesive exhibited stress-whitening at the yield strength and this phenomenon was utilized to assess the yield zone region ahead of a crack tip. Isochromatic fringe patterns were photographed together with a stress-whitened zone. Crack opening displacements at two locations specified by the ASTM E561-73T (R-curve determination) method were recorded to assess the effective crack tip position.

Theoretical isochromatic patterns associated with a strip-zone model were generated by combining the generalized Westergaard equations:

$$Z(z) = \sum_{n=0}^{n=N} A_n \, z^{n-1/2} \quad \text{and} \quad Y(z) = \sum_{m=0}^{m=M} B_m \, z^m$$

where $Z(z) = \dfrac{K}{\sqrt{2\pi b_0}} \tan^{-1}\sqrt{\dfrac{b_0}{z}}$ is used to model plastic strip-zone. In the equation, $b_0$ represents the length of the strip zone. In addition, a factor, $\lambda$ (varying from 0 to 1), is introduced to change the degree of removal of the singularity by the yielded zone. The experimental isochromatic patterns were used to provide input data to a multiple-point least square algorithm for evaluation of the parameters of interest such as $K$, $b_0$, and coefficients $A_n$ and $B_n$. Adequacy of the above theoretical model in describing the experimental results with a small-scale yielding is discussed. Analysis methods such as R-curve technique are also examined.

# THE VARIATION OF NON-SINGULAR STRESS FUNCTION COEFFICIENTS
# IN DIFFERENT FRACTURE TEST SPECIMENS

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ABSTRACT

It has been demonstrated [1,2,3] that stress field representations
incorporating non-singular terms are frequently necessary if the stress
state surrounding the tip of a running or a stationary crack in a finite
plate specimen is to be adequately modelled. A failure to include at least
the two lowest-order non-singular terms ($z^0$ and $z^{1/2}$) can seriously affect
the ability to obtain accurate and reliable values of the stress intensity
factor, K, from experimental isochromatic, isopachic, and caustic data.

The isochromatic fringe pattern surrounding a single-ended opening
mode static crack can be represented by [2,3,4]

$$\tau_m = 2|Y + \tfrac{1}{2}i(z - \bar{z})(Z' + Y')| \tag{1}$$

where $Z(z) = \sum_{n=0}^{n=N} A_n z^{n-\frac{1}{2}}$, $Y(z) = \sum_{m=0}^{m=M} B_m z^m$, and the origin of coordinates

is located at the crack tip, with the negative x-axis coinciding with the
crack faces. The increase in complexity of eqn. (1) for the case of a
constant speed running crack is only moderate, and the same functions $Z(z)$
and $Y(z)$ can be used with a crack speed dependent modification of the vector
$z = x + iy$ [5].

This paper examines variations with crack tip location in the first two
non-singular term coefficients, $B_0$ and $A_1$, as the crack is extended across
MCT, RDCB, and Ring-type specimens in both static and dynamic situations.
Particular emphasis is placed on comparing the results from the Ring specimen
with the systematic variation that has been observed in MCT and RDCB specimens
for the static case [6]. Finally, static and dynamic behaviours are compared,
and the results are discussed with particular attention to the consequences
for experimental determinations of K.

References

1. Rossmanith, H.P. and G.R. Irwin, University of Maryland Report, June 1979.
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STUDIES OF MICRO-CRACK FORMATION AND ITS EFFECTS ON CRACK
BEHAVIOR

PART I: DEVELOPMENT OF MICA-EPOXY COMPOSITE MATERIAL FOR
SIMULATION OF MICRO-CRACKS

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ABSTRACT

Micro-crack formation zones developed near a crack tip in
rocks and ceramics play a significant role in fracture
behavior of these materials. In order to study micro-crack
formation near a crack tip and its effects on crack behavior,
a mica-epoxy composite material was developed.

Multi-layer mica flakes were embedded in an epoxy matrix.
Under high stress in the vicinity of a crack tip the mica
delaminated, simulating the micro-cracking along grain
boundaries or crystal planes in rocks and ceramics. Fig. 1
shows the development of a micro-cracking zone in a compact
tension specimen.

During the test the load and crack opening displacement
were measured following the ASTM E561-73T (R-curve
determination) procedure. Through the compliance calibration
method the effective crack length and the $K_R$ were computed.
The R-curve is shown in Fig. 2. This experiment clearly
demonstrates the formation of micro-cracks and their effects
on the overall behavior of the specimen. The paper discusses
the significance of micro-crack formation on fracture
toughness characterizations, and the advantages of the use of
mica-epoxy composite for detailed studies of interaction
between the micro-crack formation zone and the specimen
geometry and size.
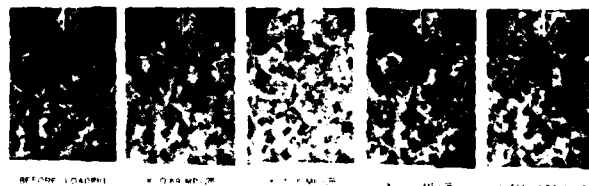
Fig. 1 Development of Micro-Cracks
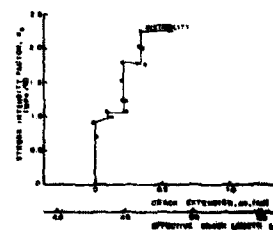


Fig. 2  R-curve for Mica-
Epoxy Composite

HOW 'MIXED' IS DYNAMIC MIXED-MODE CRACK PROPAGATION ?
A DYNAMIC PHOTOELASTIC STUDY .

by

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## Abstract

Rapid crack propagation, crack division and the interaction
between stationary or moving cracks with elastic waves is of
importance in many fields of applied fracture mechanics (oil
shale exploitation, mining engineering, etc.). The processes of
crack branching and dynamic crack propagation along curved crack
paths form most challenging fundamental problems. Dynamic photo-
elasticity provides a useful technique to record transient stress
wave patterns and isochromatic fringe loops at the tips of stat-
ionary or running cracks. The isochromatic fringe pattern associ-
ated with a crack moving rapidly along a curved path is shown in
Fig 1. The asymmetric fringe pattern - when evaluated - yields a

mixed-mode stress-intensity factor $\bar{K}=K_1-iK_2$. Numerical fringe data
evaluation shows the mode-2 contribution to be a function of the
particular selection of data points for measurement purposes.
Similar considerations apply to crack branching, where the fringe
systems associated with individual branch tips show decreasing
'mixed-mode' distortion with decreasing distance from the branch
crack tips ( as shown in Fig 2 for one branch tip).
The paper compares experimentally recorded fringe data with
numerical results in an attempt to resolve the dynamic photoelastic
mode-2 problem.

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CRACK SUBJECTED TO COMPRESSIVE LOADING

by

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Research on brittle fracture was concentrated mostly around cracks that are in a state of opening, sliding, or tearing mode. To this end, little attention was given to the understanding of the behavior of open cracks subjected to compressive loading. An interferometric method is presented herein, to investigate the behavior of the stress field at the locality of the tip of an open normal edge crack subjected to pure compression.

A convergent monochromatic collimated light beam of He-Ne laser was directed to the tip of the loaded crack under investigation. Due to the intensification of the stresses, the beam deviated and, consequently, a caustic was received on the screen which was located at some distance away from the specimen. For a compressed crack, the focal point of the convergent beam must fall between the specimen and the screen, otherwise, the caustic cannot form. For this optical arrangement, the shape of the caustic of the compressed crack resembled the caustic of mode one (1). Consequently, the transverse diameter of the caustic was measured and it was used to calculate the stress intensity factor Experimental observations show that; for a given applied load and a given crack length, the behavior of the compressed crack is the same as that of tension (2) as long as overlapping* and buckling are prevented.

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*Overlapping refers to the case where the two edges of the crack contact each other when the applied load is compressive.

(1) P. Manogg, Dissertation, Ph.D., 1964, Albert-Ludwings Universitaet Zu Freiburg i Br.

(2) Koiter and Benthem, Mechanics of Fracture 1, Chap (3). Noordhoff, Leyden, The Netherlands (1973), 131-178.

# FINITE ELEMENT ANALYSIS OF DOUBLE CIRCULAR ARC TOOTH PROFILE GEARS.

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Although the finite element method has been used successfully in the stress analysis of mechanical structures, it has not found a wide application in gear problems.

This paper presents a finite element study of double circular arc tooth profile (wildhaber - Novikov) gears, which have proved to be at least 3 to 5 times stronger than involute gears and replace the latter in some applications, demanding a high load-carrying capacity.

The complexity of the bending stress distribution, the effect of the shear deformation, the two semi-ellipsoidal contact load distribution on addendum and dedendum, the varying stiffness of the teeth due to the change of helix angle, ask for a three-dimensional analysis of this gear problem.

No theoretical analysis is able to take all these factors into consideration.

The analysis was carried out for static linear elastic conditions and the effect of tangential frictional forces at the contact surface was neglected.

The gear tooth was discretised by 80 three-dimensional PR-20 elements, and for solution the SAP-5 program was used.

From these results, a simple analytical expression for the deflection was deducted, namely :

$$w = S \; \eta \; P \; m_v^{-1} \qquad \text{(in } \mu m\text{)}$$

with S material factor, $\eta$ deflection coefficient (in $\mu m$ N), $m_v$ normal module (in mm) P the combined load on addendum and dedendum points of contact and w the deflection (in $\mu m$). Also a form factor Y for the double circular arc tooth profile gears is experimentally and numerically derived and introduced into the load capacity formula :

$$P_{t_{max}} = 0.465 \; \sigma_{b_{max}} \; Y \; \frac{F^2}{N} \frac{D_p}{m_v} \cos^2 \beta \qquad \text{(in N)}$$

where $\sigma_{b_{max}}$ maximum bending strength of material, F face width, N number of teeth, $D_p$ pitch circle diameter and $\beta$ helix angle.

The value of $\eta$, the deflection coefficient, was compared with previous experimental values, obtained with three-dimensional holographic interferometry and a difference of 10 % was found between these experimental values and numerical results. Comparison was also made with photoelastic and strain gage measurements and here also a reasonable agreement was found.

Session FM-1:  HYDRODYNAMIC STABILITY, TRANSITION AND TURBULENCE - II

Organizer:    J.T.C. LIU (Brown)
Chairperson:  J.T. STUART (Imperial College)

* 8:30 - 9:30    R.S. BRODKEY (Ohio State) and J.M. WALLACE (Univ. Maryland)
                 "Coherent Structures:  The Delta Conferences"

* 9:30 - 10:00   J.T.C. LIU (Brown)
                 "Energy Transfer Between Coherent Structures and Fine-Grained
                 Turbulence"

  10:00 - 10:30  COFFEE BREAK

  10:30 - 11:00  C.G. SPEZIALE (Stevens Inst., Hoboken)
                 "A Nonlocal Field Theory Approach to Wall Turbulence"

  11:00 - 11:30  M.B. LONG, D. FOURGUETTE, H.M. TZENG and R.K. CHANG (Yale)
                 "A Light Scattering Technique for Two-Dimensional Gas
                 Concentration Measurements in a Forced Flow"

Coherent Structures:   The Delta Conferences

by

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The Ohio State University
and
James M. Wallace
Department of Mechanical Engineering
University of Maryland

The study of coherent structures in turbulent shear flows has
become widespread in the last 15 years.  The proliferation of ideas,
experiments, theories, ...., has become so great that close contact
among researchers striving for a solution to the problem seemed to
many to be disappearing.  Thus, a small group of closely associated
people has been brought together each year for the last three years
to discuss the present status of the field; i.e., what can we agree
upon that we do know, what is still clearly in question, and what
needs to be done?  The logic behind the small, closely associated
group was to avoid presentations of personal views in favor of a
broader discussion of the objectives of the conferences.  The ses-
sions were designed to push the participants beyond their previously
held views and to provide new research ideas.  The present presen-
tation is an annotated outline of the results of this effort.  No
attempt is made to set forth any particular individual or groups of
individuals' views.  Rather, an attempt is made to present the
cooperative discussion of what we know, determine what we do not
know, and suggest how we might expand our knowledge.  We have
attempted to refocus on our goals and to identify what means we
have to achieve them.

ENERGY TRANSFER BETWEEN COHERENT STRUCTURES AND

FINE-GRAINED TURBULENCE

by

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Some of our recent works (e.g. Gatski & Liu 1980; Mankbadi & Liu 1981) are
reviewed and new results presented on the development of large-scale coherent
structures in turbulent free shear flows.  While there is now no doubt that such
coherent structures are the result of mechanisms well known to hydrodynamical
instabilities, which are of a dynamical nature for free flows, their evolution
downstream in a real turbulent flow faces a number of modifications not familiar
to their existence in laminar free flows.  In addition to the modifications of
the energy transfer from the steady mean flow to the coherent structures through
the distortion of the mean motion by the disparate fine-grained turbulence and
the effective diffusion of coherent energy, direct energy exchanges between the
large-scale coherent and fine-grained oscillations take place as the flow evolve
downstream.  The efficiency of such exchanges, which is dissipative on the
average with respect to the coherent structure, and the ability or inability of
the coherent structure to extract energy from the mean motion to overcome the
dissipative mechanisms are strong spectral properties of the coherent oscillations.
Computational efforts applied to a simplified problem and approximate methods
applied to physical situations are used to explain observations both qualitatively
and quantitatively.

Gatski, T. B. & Liu, J. T. C. 1980 Phil. Trans. Roy. Soc. Lond.
A 293, 473-509.

Mankbadi, R & Liu, J. T. C. 1981 Phil. Trans. Roy. Soc. Lond. A 298,
541-602.

# A NONLOCAL FIELD THEORY APPROACH TO WALL TURBULENCE

## Charles G. Speziale

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ABSTRACT

The problem of developing a Reynolds stress closure in turbulence has been the subject of a considerable amount of research during the past fifty years. The earliest work consisted of the mixing length theories of Prandtl while most of the current research has been directed toward the development and refinement of second-order closure models (c.f., Launder, Reece, and Rodi [1]). However, mixing length theories are geometry-dependent and second-order closure models, which are also not properly invariant, have other associated problems which make it doubtful that they can be rigorously applied to wall turbulence that is strongly anisotropic (see Speziale [2]).

In this paper, it will be shown that the nonlocal Stokesian fluids of Eringen [3], which employ integral representations for the stresses, can serve as a useful tool in the description of wall turbulence (see Speziale and Eringen [4]). More specifically, it will be shown that there exists a subclass of nonlocal Stokesian fluids that are frame indifferent (as well as geometry-independent) and in the limiting case of certain special geometries reduce to the usual mixing length theories. However, unlike in the case of the mixing length theories, within the framework of the nonlocal theory the conditions of realizability (i.e., positiveness of the turbulent kinetic energy and vanishing Reynolds stresses at moving or stationary solid boundaries) can be satisfied identically independent of geometrical constraints.

Comparisons will be made between the nonlocal theory and several other popular models of turbulence (e.g., the $k$-$\epsilon$ model and the Kolmogorov model) for fully-developed turbulent internal flows. It will be shown that for this important class of turbulent flows the nonlocal theory is simpler and includes physical features that are absent in many of the more popular models of turbulence.

REFERENCES

[1] B. Launder, G. Reece, and W. Rodi, J. Fluid Mech. 68, 537 (1975).

[2] C. Speziale, Phys. Fluids 22, 1033 (1979).

[3] A.C. Eringen, Int. J. Eng. Sci. 10, 561 (1972).

[4] C. Speziale and A.C. Eringen, Comp. & Math. Applic. 7, 27 (1981).

A Light Scattering Technique for Two-Dimensional Gas Concentration
Measurements in a Forced Flow[*]

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A nonintrusive optical technique which can yield quantitative in-
formation on the spatial characteristics of the gas concentration within
"forced" turbulent flows has been developed. The technique makes it
possible to infer the nozzle gas concentration at 10,000 points within
a plane inte-secting the flow, thus making it ideal for studying the
spatial characteristics of large-scale organized structures. In the
technique, naturally occurring instabilities of the turbulent flow are
triggered with an acoustic excitation. By making time-resolved measure-
ments in phase with this excitation, the large-scale structures are made
to appear statistically stationary. To obtain these time-resolved
measurements, the light from an argon-ion laser is focused into a thin
sheet which is made to intersect the center of an axisymmetric jet flow.
The laser is gated on for a time short compared to the characteristic
time for change in the flow and the gating is done in phase with the
acoustic excitation. Measurements are made by recording the intensity
of the radiation scattered by the nozzle gas with a computer-controlled
low-light-level TV camera. Various mechanisms for scattering the laser
light have been used. The largest signal is obtained when the nozzle
gas is seeded with small aerosol particles and the elastically scattered
light from these aerosols is recorded. However, because the measure-
ments are made in phase with the excitation, only events with a similar
origin will be recorded and, therefore, weak scattering mechanisms such
as Rayleigh scattering or Raman scattering can be used. The advantage
of these weak scattering mechanisms is that they are sensitive to
particular gas species and, therefore, have potential for measuring the
characteristics of flows in which chemical reactions occur (e.g., turbu-
lent diffusion flames). Regardless of the scattering mechanism used,
the scattered intensities are digitized under computer control and the
resulting quantitative concentration maps are stored on magnetic tape
for later analysis.

By triggering the laser to flash at successively later times with
respect to the phase of the excitation, the equivalent of a time evolu-
tion for the forced flow can be obtained. In addition, various acoustic
forcing techniques have been utilized and the results give an indication
of the mechanism by which the acoustic wave interacts with the shear
layer in the jet.

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Session FM-2: NUMERICAL ANALYSIS AND STABILITY OF SHOCK WAVES

Organizer:    C.M. DAFERMOS (Brown)
Chairperson:  C.M. DAFERMOS (Brown)

* 8:30 - 9:00     A. MAJDA (UC, Berkeley)
                  "Stability and Instability for Shock Waves"

* 9:00 - 9:30     R.J. DIPERNA (Univ. Wisconsin)
                  "Convergence of Approximate Solutions to Conservation Laws"

* 9:30 - 10:00    J. GLIMM (Rockefeller Univ., NY)
                  "Front Tracking Methods for Hyperbolic Conservation Laws"

  10:00 - 10:30   COFFEE BREAK

* 10:30 - 11:00   R. KOHN (Courant, NYU)
                  "The Mathematics of Nonlinear Strain"

  11:00 - 11:30   S.K. AGGARWAL and J.I. RAMOS (Carnegie-Mellon)
                  "A Numerical Study of One-Dimensional Chemically Reacting Flows"

STABILITY AND INSTABILITY FOR SHOCK WAVES

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## Abstract

Rigorous results regarding the nonlinear stability of multi-
dimensional shock fronts in gas dynamics will be presented. Also, a
theory which predicts the experimentally observed formation of Mach
Stems in reacting shock fronts will be developed. The theoretical
mechanisms in these two cases responsible for the differing observed
behavior will be compared and contrasted.

CONVERGENCE OF APPROXIMATE SOLUTIONS TO CONSERVATION LAWS

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## Abstract

We shall consider a class of hyperbolic systems of conservation
laws in one space dimension, modelled on the equations of inviscid com-
pressible fluid dynamics, together with a class of parabolic conservation
laws with viscosity, modelled on the compressible Navier-Stokes equations.
We shall discuss convergence of the parabolic solutions to the hyperbolic
solutions as the viscosity vanishes.

FRONT TRACKING METHODS FOR HYPERBOLIC CONSERVATION LAWS

James Glimm
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## Abstract

A sharply resolved interface is propagated with zero numerical diffusion in a hydrodynamics code for nonlinear hyperbolic or coupled hyperbolic and elliptic equations in two space dimensions. The interface is traced as an extra computational degree of freedom, and advanced dynamically by use of characteristic wave speeds obtained from the solution of a Riemann problem, locally at each mesh point along the interface.

THE MATHEMATICS OF NONLINEAR STRAIN

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## Abstract

The fundamental "Korn's inequality" in linear elasticity states
that the linear elastic energy of the deformation controls its distance
to an (infinitesimal) rigid motion. I shall describe analogous estimates
for (nonlinear) hyperelasticity and discuss the mathematical and
physical relevance of such estimates.

# A NUMERICAL STUDY OF ONE-DIMENSIONAL CHEMICALLY REACTING FLOWS

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## ABSTRACT

Numerical experiments have been conducted to analyze the laminar flame propagation in one-dimensional enclosed combustors. The aim of these experiments was to state the relative efficiency and accuracy of different numerical schemes in the analysis of a physical phenomena where the space and time step restrictions imposed by the finite-difference form of the conservation equations of mass, momentum and energy were not as important as those imposed by the chemical processes occurring in a premixed combustor. For this purpose the governing equations in the conservation form were finite-differenced for a very simplified combustor. Three different numerical methods were used: explicit, fully implicit and quasilinear (1). Some sample results are shown in Fig. 1, which gives the temperature profile as a function of space at different instances. The results of the 3 methods are plotted in the same figure and are indistinguishable from each other. (The maximum temperature difference between the methods was about 2%.) This figure indicates a very steep temperature and species gradient across the flame which behaves like a discontinuity. The flame decelerates when approaching the right end of the combustor and is very sensitive to the initial temperature and species profiles. In terms of computational efficiency the explicit method was by far superior to the implicit and quasilinear methods.
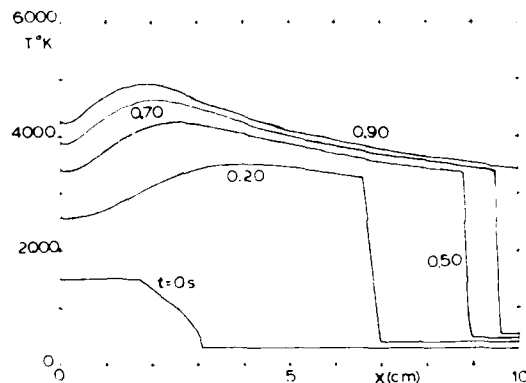
Figure 1:
Temperature Profiles.

(1)  R.E. Bellman and R.E. Kalaba: Quasilinearization and Nonlinear Boundary Value Problems, American Elsevier, New York (1965).

Session FM-3:  MECHANICS OF DYNAMIC FRACTURE

Organizer:    L.B. FREUND (Brown)
Chairperson:  J.W. DALLY (URI)

* 8:30 - 9:00     J.D. ACHENBACH (Northwestern)
                  "Plasticity Effects in Dynamic Crack Propagation"

* 9:00 - 9:30     A.J. ROSAKIS and L.B. FREUND (Brown)
                  "The Method of Caustics in the Presence of Crack Tip Plasticity"

* 9:30 - 10:00    D.A. SHOCKEY (SRI Intl), J.F. KALTHOFF, W. KLEMM and S. WINKLER
                  (Fraunhofer Inst. fur Werkstoffmechanik, Germany)
                  "Simultaneous Measurements of Stress Intensity and Toughness
                   for Fast Running Cracks in Steel"

  10:00 - 10:30   COFFEE BREAK

* 10:30 - 11:00   K. RAVI CHANDAR and W.G. KNAUSS (Caltech)
                  "Experiments in Dynamic Crack Propagation"

  11:00 - 11:30   K.S. KIM and J.W. PHILLIPS (Univ. Illinois, Urbana)
                  "Focal-Plane Analysis of the Optical Pattern Produced by a
                   Mode-I Crack"

  11:30 - 11:45   J.P. DEMPSEY (Clarkson) and P. BURGERS (U.Penn)
                  "Dynamic Bifurcation of Stationary Cracks in Antiplane Strain"

Plasticity Effects in Dynamic Crack Propagation

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In fast fracture the mass density of the material affects the fields
of stress and deformation.  In this presentation the influence of plastic
deformations on the dynamic fields in the immediate vicinity of a rapidly
propagating crack tip will be discussed.  It has been found that various
models of material behavior predict essentially different near-tip fields.
For the elastic perfectly-plastic model a boundary-layer effect occurs
which indicates very strong dynamic effects in a very small zone near the
crack tip, see Refs.[1] and [2].  For the Mode III case, a complete
solution of the relevant shear strain has been obtained in the plane of
the crack, ahead of the propagating crack tip.  For $J_2$-flow theory with
linear strain hardening and a crack-tip speed smaller than a certain
critical velocity, the stresses are of a singular form which is very
similar to the one for the corresponding quasi-static problem, as dis-
cussed in some detail in Refs.[3] and [4].  Finally, we will also
mention a material model for a work-hardening material that does not
require the statement of a separate yield condition, nor does it require
that loading and unloading be considered separately, [5].  Plastic
deformations always exist, but they are negligibly small when the mater-
ial should be essentially elastic.  This model has advantages for numeri-
cal analysis.

1.    J. D. Achenbach and V. Dunayevsky, "Fields Near a Rapidly Propagating
Crack Tip in an Elastic Perfectly-Plastic Material," J. Mech. Phys. of
Solids, in press.

2.    V. Dunayevsky and J. D. Achenbach, "Boundary Layer Phenomenon" in the
Plastic Zone near a Rapidly Propagating Crack Tip," Int. J. Solids and
Structures, in press.

3.    J. D. Achenbach and M. F. Kanninen, "Crack Tip Plasticity in Dynamic
Fracture Mechanics," Fracture Mechanics, N. Perrone et al (eds.), The
University of West Virginia Press, 649 (1978).

4.    J. D. Achenbach, M. F. Kanninen and C. H. Popelar, "Crack Tip Fields for
Fast Fracture in an Elastic-Plastic Material," J. Mech. Phys. of Solids,
in press.

5.    J. Aboudi and J. D. Achenbach, "Rapid Mode-III Crack Propagation in a
Strip of Viscoplastic Work-Hardening Material," Int. J. Solids and Struc-
tures, in press.

# THE METHOD OF CAUSTICS IN THE PRESENCE OF

## CRACK TIP PLASTICITY

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The shadow spots which are obtained in using the optical method of caustics to experimentally determine dynamic stress intensity factors are usually interpreted on the basis of a static elastic crack model. Here, both crack tip plasticity and inertial effects are included in the analysis underlying the use of the method in reflection. For dynamic crack propagation in the two-dimensional tensile mode which is accompanied by a Dugdale-Barenblatt line plastic zone, the predicted caustic curves and corresponding initial curves are studied within the framework of plane stress and small scale yielding conditions. These curves are found to have geometrical features which are quite different from those for purely elastic crack growth. Estimates are made of the range of system parameters for which plasticity and inertia effects should be included in data analysis when using the method of caustics. For example, it is found that the error introduced through the neglect of plasticity effects in the analysis of data will be small as long as the distance from the crack tip to the initial curve ahead of the tip is more than about twice the plastic zone size. Also, it is found that the error introduced through the neglect of inertial effects will be small as long as the crack speed is less than about 20% of the longitudinal wave speed. Finally, some recent observations are reported on the use of the method of caustics when the initial curve is within the near tip plastic zone of a stationary crack in a ductile material.

# SIMULTANEOUS MEASUREMENTS OF STRESS INTENSITY AND TOUGHNESS FOR FAST RUNNING CRACKS IN STEEL

D. A. Shockey,[*] J. F. Kalthoff,[+] W. Klemm[+] and S. Winkler[+]

## Abstract

Simultaneous measurements of the dynamic stress intensity factor $K_I^{dyn}$ and the dynamic fracture toughness $K_{ID}$ were made in a high-strength steel to investigate the relation between energy delivered to and energy absorbed by rapidly propagating cracks. The heat generated by the propagating crack, and hence $K_{ID}$, was calculated from temperature profiles measured by sensitive thermocouples near the crack path. The values of $K_I^{dyn}$ at the same locations were obtained by the shadow optical method of caustics from high speed photographs of the moving crack tips. The results indicate that the energy absorbed is less than the energy delivered to the running crack tips and therefore suggest that, in its present form, the energy balance method for determining dynamic fracture toughnesses may lead to erroneous results. Additional work is necessary to verify these inferences.

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+Institut für Werkstoffmechanik, 7800 Freiburg, Germany

# EXPERIMENTS IN DYNAMIC CRACK PROPAGATION

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Crack branching is a recurring phenomenon in high speed crack propagation. In the past much discussion has centered on the role of the crack propagation velocity on precipitating the event. There exists evidence that branching is initiated by small flaws away from the main crack axis which grow under the stress pulse associated with the tip of the dynamically growing crack. Successful linkage of these cracks with the main crack can lead to branching.

The experimental method to achieve reproducible stress wave loading is described along with analysis of caustics which verify the reliability of the measurements.

# Focal-plane Analysis of the Optical Pattern
# Produced by a Mode-I Crack

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In recent years, the optical method of caustics [1-4], or shadow-spots, has been used to investigate both stationary and running cracks in both transparent and opaque materials. One of the appealing features of the method is its conceptual and practical simplicity.

However, for dynamic crack-growth studies, cameras with high framing rates have usually been required to record the caustics at specific instants in time; the caustics are then analyzed one-by-one to determine the stress-intensity factor $K_I(t)$ at the discrete values of time $t$ for which exposures are made.

In this paper, a simple experimental method is outlined for determining $K_I(t)$ as a *continuous* function of $t$ for a single Mode-I crack whose tip may or may not be moving. The method eliminates the need for a camera altogether.

Consider the classical setup for transmitted-light caustics, in which parallel incident light strikes a transparent model at normal incidence. If the model is unloaded, the light leaving the model is also parallel; and if this emitted light is collected by a converging lens of focal length $c$, all the emitted light will be focussed to a single point P in the focal plane of the lens. When the model contains a loaded crack, light rays leaving a generic point r (with respect to the crack tip in the specimen midplane) will pass through the focal plane at a point r' (relative to P) given by a mapping similar to that used in the theory of caustics. It can be shown that, if r and r' have polar coordinates $(r, \theta)$ and $(r', \theta')$, respectively, then, in the *quasistatic* case $r' = Ar^{-3/2}$ and $\theta' = 3\theta/2$. Here,

$$ A = c d K_I t^{1/2} / \sqrt{2\pi}, $$

where $c$ is the same elasto-optic constant as that used in the theory of caustics, and $d$ is the specimen thickness. It is subsequently shown that, in the focal plane, the intensity $I$ of the light striking any fixed finite-size area $L$ that does not include the focal point P is given by

$$ I = B / K_I^{4/3}, $$

where the constant $B$ is the product of several experimental quantities, including a "shape factor" for the subframe area $L$. In an experiment, $I(t)$ can be measured with a photodiode. An important feature of the method is that $I(t)$ is insensitive to the location of the crack in the specimen plane, and if $L$ is chosen properly, $I(t)$ is also insensitive to the orientation of the crack.

The analysis of a running crack shows that, although the mapping $r \to r'$ becomes more complex, the formula given above for $I$ is still correct; the factor $B$ becomes a known function of crack velocity.

It also appears possible to determine independently the values of $K_I$ at each of two bifurcating cracks if two photodiodes are employed in the focal plane. Also, it appears that the velocity of the crack can be measured simultaneously by a variation of the proposed scheme.

1. Manogg, P. ...

2. ...

3. ...

4. ...

DYNAMIC BIFURCATION OF STATIONARY CRACKS IN

ANTIPLANE STRAIN

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and

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A semi-infinite crack in an infinite linear elastic body is sub-
jected to constant magnitude antiplane loading at time $t = 0$. At the
same instant the crack is assumed to bifurcate and propagate at constant
velocity, v, into the body with each branch making an angle $\kappa\pi$ with the
original crack line. Using the self-similar properties of the problem
and a method proposed in [1], analytic solutions for the stress are
obtained for $\kappa = 0, \frac{1}{2}$ [2]. For $0 < \kappa < \frac{1}{2}$, screw dislocations, which
are emitted at constant velocity from the original crack tip and are
growing linearly with time, are superimposed to derive a Cauchy
singular integral equation which is solved numerically [3]. The
dynamic stress intensity factor at the tip of the bifurcated cracks
is obtained as a function of v and $\kappa$, from which some conclusions about
crack initiation are drawn.

1.    Achenbach, J. D., Bifurcation of a Running Crack in Antiplane
      Strain, Int. J. Solids Structures, Vol. 11, pp. 1301-1314, 1975.

2.    Burgers, P., and Dempsey, J. P., Two Analytical Solutions for
      Dynamic Crack Bifurcation in Antiplane Strain, submitted to
      J. Appl. Mech..

3.    Burgers, P., Dynamic Propagation of a Kinked or Bifurcated
      Crack in Antiplane Strain, to be submitted to J. Appl. Mech..

Session FM-4:  CARDIC-PULMONARY MECHANICS/BIOMECHANICS

Organizer:    D.K. BOGEN (Harvard)
Chairperson:  T.A. MCMAHON (Harvard)

* 8:30 - 9:00    D.K. BOGEN, T.A. MCMAHON (Harvard) and A. NEEDLEMAN (Brown)
                 "Influence of Regional Myocardial Inhomogeneities on Cardiac
                 Function"

* 9:00 - 9:30    J.J. FREDBERG (Cambridge Collaborative, Inc. Cambridge  MA)
                 "Tracheal Mechanical Properties Inferred from Acoustic
                 Reflections Measured at the Mouth"

* 9:30 - 10:00   R.D. KAMM, C.J. JOSHI, J.M. COLLINS, A.H. SHAPIRO, A.S. SLUTSKY
                 and J.M. DRAZEN (M.I.T.)
                 "Experimental Studies of Gas Exchange in High Frequency
                 Pulmonary Ventilation"

  10:00 - 10:30  COFFEE BREAK

* 10:30 - 11:00  C.S. PESKIN and D.M. MCQUEEN (Courant, NYU)
                 "Computer Design and Evaluation of Artificial Heart Valves"

  11:00 - 11:15  R.N. VAISHNAV and J. VOSSOUGHI (Catholic Univ., Washington)
                 "A Note on the Use of Bergel's Modulus in Arterial Mechanics"

  11:15 - 11:30  A. VOLOSHIN (Iowa State) and J. WOSK (Hillel Jaffe Hospital, Israel)
                 "Attenuational Capacity of the Low Back Pain Patients' Musculo-
                 skeletal System"

  11:30 - 11:45  N.A. RUBAYI (Southern Illinois Univ.) and A.A. RUBAY1 (Univ.
                 Baghdad, Iraq)
                 "Optimization of Tooth Implant Design"

Influence of Regional Myocardial Inhomogeneities on Cardiac Function

by

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and

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Regional variations in heart muscle properties commonly occur in the setting of ischemic heart disease, in which portions of the heart wall receive an inadequate blood supply.  Not only may blood-deprived regions contract subnormally, but noninvolved regions may attain compensating increases in contractility.  Here, such regional inhomogeneities are investigated by means of a mathematical model of the injured heart.

In this model the left ventricle is represented as an initially spherical, nonlinear membrane, divided into two axially symmetric regions of differing material properties (Bogen et al, 1980).  Regional muscle properties are further described by a set of nonlinear elastic constants. Diastolic and systolic pressure-volume curves are subsequently calculated using a finite element method.

Results are given for three types of inhomogeneities:  (1) those resulting from localized decreases in contractility, (2) those resulting from myocardial infarction, or total loss of local heart muscle function, and (3) those resulting from compensating increases in contractility in noninvolved regions.

References:   Bogen, D.K., Rabinowitz, S.A., Needleman, A., McMahon, T.A. An Analysis of the Mechanical Disadvantage of Myocardial Infarction in the Canine Left Ventricle.  Circ. Res. 47: 728-741, 1980.

# Tracheal Mechanical Properties Inferred From Acoustic Reflections Measured at the Mouth

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Insofar as mechanical waves can propagate through airways and lung parenchyma, non-invasive measures of these waves may reveal the geometrical and mechanical properties of these tissues.  In a recent study of normal subjects at FRC we found that detailed geometrical features of upper and central airways (expressed as airway cross-sectional areas vs. distance along the airway) could be inferred from acoustic reflections measured at the mouth (Fredberg et al., 1980).  In this report we test the more general hypothesis that the area-pressure characteristics and hysteresis of intrathoracic central airway segments can be inferred from acoustic reflections measured at the mouth.  Using acoustic reflections we have studied double vital capacity maneuvers (TLC-RV-TLC) in 7 subjects, and observed variations in tracheal geometry, absolute and relative distensions of airways and lung tissue, and absolute and relative hysteresis of airways and lung tissue.  With simultaneous pleural and mouth pressure data the distensibility of airway walls were computable also.  In the absence of other non-invasive procedures for measurement of airway distensions and distensibility, we have compared our results with data gathered from human autopsy material and found adequate agreement.

Fredberg, Wohl, Glass, Dorkin, "Airway area by acoustic reflections measured at the mouth," J. Appl. Physiol.: Resp., Environ., Exercise Physiol., 48, 4, 749-758, 1980.

EXPERIMENTAL STUDIES OF GAS EXCHANGE IN HIGH FREQUENCY PULMONARY
VENTILATION

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Of the several possible mechanisms of gas exchange in high frequency
ventilation (HFV), we have examined those which, as a group, can be
termed "augmented transport".  This group includes Taylor-like augmented
diffusion (Taylor, Proc. R. Soc. A, 1953), the combined effects of axial
convection and radial mixing in turbulent or "mixed" flows (flows con-
taining secondary currents)(Taylor, Proc. R. Soc. A, 1954; Scherer et al.,
J. Appl. Physiol., 1975), and the net convective transport associated
with differences in shape between the inspiratory and expiratory velo-
city profiles (Haselton et al., Science, 1980).  Our studies encompass
both experimental measurements to elucidate these transport phenomena,
and the application of these experimental results to models of pulmonary
gas exchange in HFV.

By analogy to the studies of Taylor for steady flow, and Watson (personal
communication, 1980) for oscillatory flow, we postulate that gas exchange
in the lung can be likened to a diffusion process in which the effective
diffusion coefficient ($D_{eff}$) is determined by flow characteristics, air-
way geometry and gas properties.  Accordingly, the time-averaged mass
flux can be written:

$$j = D_{eff} \frac{dc}{dx} \tag{1}$$

where c is the time-mean, cross-sectional average concentration.  Note
that in this approximation $D_{eff}$ contains not only the combined effects
of axial convection and radial molecular diffusion, but also transport
processes that are entirely convective in nature.

Experiments have been completed for purely oscillatory laminar flow in
a long, rigid, uniform tube.  The results show good agreement with
Watson's theory, and can be expressed as

$$\frac{D_{eff}}{D_m} = 1 + f(\alpha, Sc) \cdot Pec^2 \tag{2}$$

where $f(\alpha, Sc)$ (a function of the Womersley number and Schmidt number)
is constant for $\alpha < 1$ and varies as $\alpha^{-3}$ for $\alpha >> 1$, $D_m$ is the molecular
diffusion coefficient, and Pec is the Peclet number.  Eq. (2) applies
only if, in addition to the previously stated conditions, the flow is
laminar.  Sergeev (Fluid Dynamics, 1(1), 1966) and Merkli et al (J. Fl.
Mech., 68, 1975) have shown that the transition to turbulent flow
occurs when $Re/\alpha \approx 350-700$, where Re is the Reynolds number.

Two additional sets of experiments are currently underway.  These will
examine gas transport first, in turbulent oscillatory flow through a
tube of uniform area, and second, in oscillatory flow through a symmetri-
cally branching, 5-generation model.  These results will be cast into
the form of Eq. (1) to determine appropriate expressions for $D_{eff}$.

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# COMPUTER DESIGN AND EVALUATION OF ARTIFICIAL HEART VALVES

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This talk is concerned with the practical application of a computational method that we have developed for determining the flow pattern of blood in the heart and predicting the performance of artificial heart valves. In this approach, we solve the coupled equations of motion of the blood, the muscular heart wall, and the heart valve. The results of such a calculation constitute a prediction of how the specified valve will perform under the specified physiological conditions. Such a prediction can be used to optimize the parameters of a given design without actually fabricating a prototype valve for each set of parameters. It can also be used to study the sensitivity of a given design to changes in the physiological situation in which the heart valve must operate.

To illustrate the use of our method, we shall present a computational study of the fluid dynamics of pivoting disc prosthetic mitral valves. Both flat and curved valves will be considered. As a design criterion, we use the velocity of flow through the smaller opening of the valve, since this is the place where flow tends to be stagnant and clots tend to form. For flat pivoting discs we determine the optimal position of the pivot with respect to this criterion. For curved discs, we determine the optimal combination of pivot point and curvature. We find that small amounts of curvature can make a dramatic improvement in valve performance.

A computer - generated movie will be shown.

References:
1. Peskin CS, "Numerical Analysis of Blood Flow in the Heart." J. Comput. Phys. 25: 220-252, 1977.
2. Peskin CS and McQueen DM, "Modeling Prosthetic Heart Valves for Numerical Analysis of Blood Flow in the Heart." J. Comput. Phys. 37: 113-132, 1980.
3. McQueen DM, Peskin CS, and Yellin EL, "Fluid Dynamics of the Mitral Valve: Physiological Aspects of a Mathematical Model." Preprint, 1980, available on request.

# A NOTE ON THE USE OF BERGEL'S MODULUS IN ARTERIAL MECHANICS

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Consider an arterial segment under a state of stress given by circumferential, longitudinal and radial true stresses $S_\theta$, $S_z$ and $S_r$, respectively, along with the associated extension ratios $\lambda_\theta$, $\lambda_z$ and $\lambda_r$. A popular way to study the incremental properties of the tissue is to use the differentials $P_\theta$, $P_z$ and $P_r$ of $S_\theta$, $S_z$ and $S_r$, respectively, as the incremental stresses, and the differentials $e_\theta$, $e_z$, and $e_r$, of $\ln\lambda_\theta$, $\ln\lambda_z$, and $\ln\lambda_r$ as the incremental strains, and express the incremental stresses as linear combinations of the three incremental strains. That is, $e_\theta = C_{\theta\theta}P_\theta + C_{\theta z}P_z + C_{\theta r}P_r$, etc. It can be shown [1] that only three coefficients $C_{\theta\theta} = 1/E_\theta$, $C_{zz} = 1/E_z$, and $C_{rr} = 1/E_r$ are needed to describe the incremental properties of the tissue in the neighborhood of a state of large deformation. If we now assume that under *in vivo* pulsatile deformations an arterial segment essentially preserves its length ($e_z = 0$), and that the incremental radial stress $P_r$ is small in comparison with $P_\theta$ and $P_z$, then we can show that $E_\theta = (1 - \nu_{\theta z}\nu_{z\theta}) P_\theta/e_\theta$, where $\nu_{\theta z} = -C_{\theta z}/C_{zz}$ and $\nu_{z\theta} = -C_{z\theta}/C_{\theta\theta}$, are two of the six Poisson's ratios. Thus, the circumferential incremental elastic modulus can be computed simply from the *in vivo* ratio of the incremental true stress $P_\theta$ and the incremental strain $e_\theta$ under the conditions of no change in segment length L. If we assume that the arterial segment is thin-walled, $S_\theta \approx pR/h$ and $e_\theta \approx dR/R$ where $p$ = intravascular pressure, $R$ = midwall radius and $h$ = wall thickness [1], we get the value of $E_\theta$ as

$$E_\theta = (1 - \nu_{\theta z}\nu_{z\theta}) \frac{d(pR/h)}{dR/R} \approx (1 - \nu_{\theta z}\nu_{z\theta})(\frac{dp}{dR} \frac{R^2}{h} + 2S_\theta)$$

In the above equation, we have used the facts that $d(2\pi RLh) = 0$ and $dL = 0$. If we now assume that $\nu_{\theta z} = \nu_{z\theta} = 1/2$, then

$$E_\theta = 0.75 \frac{dp}{dR} \frac{R^2}{h} + 1.5S_\theta = E_{inc} + 1.5S_\theta \tag{1}$$

The first term in Eq. (1) is recognizable as the "thin-wall" version of the incremental modulus $E_{inc}$ introduced by Bergel [1961]. $E_{inc}$ is a useful quantity and appears directly in the famous Moens-Korteweg equation for the velocity of pulse propagation. Eq. (1) shows that $E_{inc}$ is smaller than $E_\theta$ by $1.5S_\theta$. It is reasonable to use $E_{inc}$ for comparison of properties of a given blood vessel, but for comparison of different blood vessels, or for comparison of the results of a tube test with a uniaxial test, a measure such as $E_\theta$ is preferable.

(1) Patel, D.J. and Vaishnav, R.N. 1980. Basic Hemodynamics and Its Role in Disease Processes. Univ. Park Press, Baltimore, MD, 504pp.
(2) Bergel, D.H. 1961. J. Physiology, Vol. 156, pp. 445-457.

# ATTENUATIONAL CAPACITY OF THE LOW BACK PAIN PATIENTS' MUSCULOSKELETAL SYSTEM

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A new noninvasive in vivo method for quantitative evaluation of the human musculoskeletal system's shock absorbing capacity is presented. An accelerometric technique has been employed to obtain quantitative values necessary to evaluate the attenuational capacity of the different parts of the human musculoskeletal system. The accelerometers were connected securely to each subject's forehead and medial femoral condyle. Each subject walked barefoot along a walkway eight meters long using his normal step length. This noninvasive in vivo technique allows measurement of bone vibration resulting from propagated shock waves harvested during each heel strike.

The measurements were performed on four groups: healthy, painful knee, menisectomized, and low back pain (a total of 81 subjects). The obtained results support the idea that the repetitive loading caused by gait generates intermittent waves that propagate through the entire human musculoskeletal system. These waves are gradually attenuated and dissipated along their course toward the skull by natural shock absorbers, such as the foot, knee, intervertebral disk, etc. This is a rational form of absorption and dissipation of energy invading the human muscoloskeletal system during the heel strike.

The etiology of most low back pain (LBP) is unknown; but it is known that mechanical stress can aggravate it. In light of this, the obtained results seem significant. As can be seen from Table 1, the attenuational capacity of the subjects in the LBP group is 17% less than that of the others. The t-test confirms that the obtained difference is significant.

Contemporary methods for examination of the human musculoskeletal system may be complemented by using the described technique for quantitative evaluation of the subject's shock attenuation capacity. This could be extremely helpful in early diagnosis of deficiencies in the human musculoskeletal system.

Table 1. Attenuational Capacity of Each Group.

| | Low Back Pain | Healthy | Painful Knee | Menisec- tomized | P-value for LBP vs Others |
|---|---|---|---|---|---|
| Attenuational Capacity | 1.988 | 2.433 | 2.324 | 2.310 | 0.0010 |

# OPTIMIZATION OF TOOTH IMPLANT DESIGN

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In this study, the authors utilize the two-dimensional photoelastic analysis in order to optimize the shape of a tooth implant blade containing multiply connected regions. Lately, there has been success in use of prosthesis by dental implantologists, yet more reliable and safer dental implants are still sought.  So far, the design of the root of the dental implant has been treated imperically; however, the shape and discontinuities of the implant have great effect upon the stress distribution and concentration not only in the implant itself but also on the alveolar bone supporting the implant.

Eleven PSM-1 photoelastic models as shown in Figure 1 were analyzed.  All models were subjected to the same magnitude of the simulated uniform occlusal pressure and a lateral force and they all were supported as indicated by Model 11.  The models were analyzed in the transmission type polariscope and the stress distributions were obtained for various discontinuities.

The present analysis revealed that model 2 with a slot yielded the least amount of stress at the edge of the slot as compared with other models.  Therefore model 2 is considered an optimum design for an implant blade.
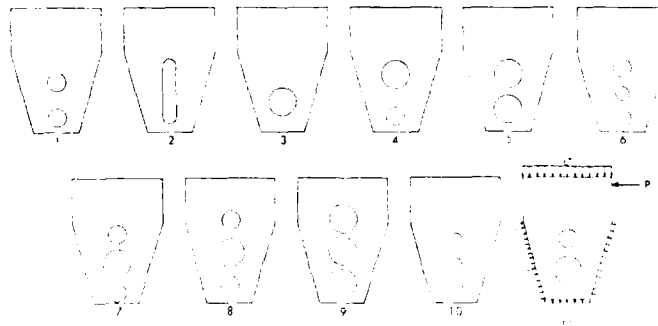


Figure 1

Session FM-5:  FRICTIONAL CONSTITUTIVE RELATIONS AND FAULT
SLIP INSTABILITIES

Organizer:    A.L. RUINA (Cornell)
Chairperson:  J.H. DIETERICH (USGS, Menlo Park)

* 8:30 - 9:00     D.L. TURCOTTE (Cornell)
                  "On the Relation Between Thermal Instabilities, Melting,
                  and Lithification and the Stick-Slip Mechanism"

* 9:00 - 9:30     L.W. TEUFEL (Sandia Labs)
                  "Frictional Instabilities in Rock:  Effect of Stiffness,
                  Normal Stress, Sliding Velocity, and Rock Type"

* 9:30 - 10:00    A.L. RUINA (Cornell)
                  "The Meaning of Static Friction in a State Variables Con-
                  stitutive Law"

  10:00 - 10:30   COFFEE BREAK

* 10:30 - 11:00   J.H. DIETERICH (USGS, Menlo Park)
                  "Extrapolation of Laboratory Experiments to Earthquake Faulting"

  11:00 - 11:30   J.R. RICE (Brown)
                  "Studies on the Stability of Frictional Slip"

  11:30 - 12:00   J. DUNDURS (Northwestern) and M. COMNINOU (Univ. Michigan)
                  "Illustrative Example for Friction in Elasticity"

# ON THE RELATION BETWEEN THERMAL INSTABILITIES,

## MELTING, AND LITHIFICATION AND THE STICK-SLIP MECHANISM

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Mantle and crustal rocks under a wide range of conditions are believed to exhibit a fluid behavior with a Newtonian viscosity which is exponentially temperature dependent. The dependence coupled with frictional heating (viscous dissipation) can lead to thermal runaway if a constant stress is applied. When elasticity is also included a thermal instability is obtained. Motion on a viscous zone occurs during short periods of time with long periods of quiescence in between. Elastic energy is stored during the quiescent periods and is released during the short periods of flow. The fluid zone heats up as flow occurs due to frictional heating and cools during the quiescent zones. However, the periods of flow are not sufficiently short to be associated with earthquakes. The decrease in viscosity is not sufficient to cause really rapid motion.

One way in which frictional heating can cause an earthquake is by melting on the fault. The viscosity of magma is sufficiently small that large velocities are obtained. However, partial melting is rarely observed on surface fault exposures. This mechanism is unlikely to be important on shallow faults but may be responsible for the deep seismicity associated with subduction zones.

Fault gauge (finely granulated rock) is usually associated with shallow fault zones. Fault zones are in general saturated with ground water. It is known that sediments can be lithified (turned to rock) in a relatively short time. Various mechanisms can contribute to lithification, an example is pressure solution. Solid material is precipitated due to high stress levels at points of contacts between grains and precipitate on free surfaces. After an earthquake the powdered fault gauge could become lithified in a matter of weeks. Stress would then have built up until the newly cemented contacts could be broken. This could be an important mechanism in the stick-slip behavior of faults.

Frictional Instabilities in Rock:  Effect of Stiffness,
Normal Stress, Sliding Velocity, and Rock Type*

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## Abstract

A systematic experimental study has been conducted to examine in
detail the parameters which may affect the transition from stable
sliding to stick-slip sliding for rock discontinuities in triaxial
compression.  Dry, room temperature, triaxial compression tests were
conducted on $35^\circ$ precuts of four different rock types, at confining
pressures to 50 MPa, and constant axial piston velocities of $2 \times 10^{-2}$
to $5 \times 10^{-7}$ cm/s.  The experimental results clearly indicate that the
transition from stable sliding to stick-slip is dependent on (1) the
stiffness of the specimen-machine system, (2) the normal stress across
the sliding surface, and (3) the frictional characteristics of the
sliding surface which vary with time of contact and sliding velocity.
As a result of time and velocity dependent friction, there is a critical
sliding velocity for the onset of stick-slip for each rock type.  For
sliding velocities greater than the critical velocity, stable sliding
occurs.  Once stick-slip is achieved a further decrease in the sliding
velocity results in an increase in the magnitude and seismic efficien-
cy of the events and a corresponding increase in the coefficient of
friction.  For a given rock type the critical velocity for the onset
of stick-slip decreases with an increase in the stiffness of the
loading machine and/or a decrease in the normal stress across the
sliding surface.  Measurements of the effective stiffness of the
specimen and machine (for a given machine stiffness) clearly show that
the effective stiffness of the system does not change significantly
for different rock types, suggesting that differences in the critical
sliding velocity are due primarily to differences in the frictional
properties of the sliding surfaces.  Differences in frictional proper-
ties of the different rock types are correlated to indentation
hardness.

THE MEANING OF STATIC FRICTION IN A
STATE VARIABLES CONSTITUTIVE LAW

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Stick-slip, the episodic occurence of sudden slip on surfaces
in frictional contact, is a possible model for earthquakes.  In this
picture earthquakes occur when the shear stress becomes equal to the
frictional strength of the fault.  The meaning of frictional strength
is the same as what is commonly called the static friction.  When
slip begins the friction force drops to a lower value, that, if
constant is called the kinetic friction.  This drop from static fric-
tion to kinetic friction occurs for both homogeneous slip and for the
propagation of a frictional crack.  Understanding the crack from the
point of view of fracture mechanics depends on understanding the drop
from the static to the kinetic friction.  Even if the slip is homo-
geneous, the nature of the instability (preseismic slip for example)
can only be understood if the drop from static to kinetic friction,
and thus static friction itself, is understood.

Traditional friction laws cannot account for static friction,
kinetic friction and the drop off from one to the other.  A new class
of friction laws developed from the work of Dietrich (1978, 1979) can
include all of these things.  In these friction laws the friction
force is taken to be a function of the surface state and of the slip
velocity.  The state of the surface is described by one or more state
variables each of which is defined by a function giving its rate of
change as a function of its current value and of the instantaneous
slip rate.

Depending on the exact form of the internal variable characteri-
zation static friction may or may not emerge as basically dependent
on time of stationary contact.  Time dependence of static friction
may, however, seem to appear as a consequence of particular loadings
even if it is not a basic property of the friction law.  Also the
static strength may appear as either an increasing or decreasing func-
tion of the loading rate as slip is initiated - depending on the
loading conditions.

This talk will introduce the state variable description of fric-
tion and show the interpretations of static friction that it allows.

# EXTRAPOLATION OF LABORATORY EXPERIMENTS TO EARTHQUAKE FAULTING

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Slow stable slip prior to rapid unstable slip appears to be
a ubiquitous feature of laboratory fault experiments. Analysis
of simple spring-slider models and two-dimensional elastic
models that employ fault constitutive relations derived from
laboratory experiments indicates that stable premonitory slip is
intrinsic to the process leading to unstable slip. Extra-
polations of laboratory observations to earthquake faulting are
subject to significant uncertainties but suggest the possibility
of significant amounts of premonitory slip as precursors to
earthquakes. One source of uncertainty in such extrapolations
arises from the possibility that precursory slip may occur on a
small patch of a fault and that unstable earthquake slip, once
initiated, may propagate well beyond the area of nucleation.

# Studies on the Stability of Frictional Slip

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Frictional slip experiments by Dieterich (1978,79,80) and Ruina
(1980) show that the shear strength $\tau$ of a sliding surface depends
not only on the normal stress $\sigma$, but also on the slip <u>rate</u> $\dot\delta$ and on
the <u>state</u> of the surface (as conditioned by prior slip). This state
itself evolves with ongoing slip. Assuming that a finite set of var-
iables $\phi_1, \phi_2, \ldots, \phi_n$ characterize adequately the state, Ruina (1980)
suggests the general constitutive form (for sliding at constant $\sigma$)

$$\tau = f(\dot\delta, \phi_1, \ldots, \phi_n) \; ; \; \dot\phi_k = g_k(\dot\delta, \phi_1, \ldots, \phi_n) \; , \; k=1,\ldots,n \quad . \quad (1)$$

The following features of this constitutive description are assumed,
consistently with the Dieterich-Ruina experiments: (i) For slip at
constant speed $\dot\delta$, the $\phi$'s evolve stably to unique values, dependent on
$\dot\delta$, satisfying $g_k=0$, $k=1,\ldots,n$ (i.e., a unique <u>steady</u> state exists for
each $\dot\delta$); (ii) $\partial\tau/\partial\dot\delta > 0$ when the derivative is computed at fixed $\phi$'s; but
(iii) $d\tau/d\dot\delta < 0$ (in the range of interest for stick slip instability) if
the steady state values of the $\phi$'s, in terms of $\dot\delta$, are used to evaluate
$\tau$.

A constitutive formulation in this class, adequate to match Ruina's
experiments on quartzite over the velocity range 0.01 to 1 $\mu$m/s is the
two-state-variable form (Ruina, 1980)

$$\tau = C\sigma + A\sigma \ln \dot\delta + \phi_1 + \phi_2 \; ; \; \dot\phi_{1,2} = -(\dot\delta/L_{1,2})(\phi_{1,2} + B_{1,2} \, A\sigma \ln \dot\delta) \quad (2)$$

where $C \approx 0.55$ (when $\dot\delta$ in $\mu$m/s), $A \approx 0.01$, $B_1 \approx B_2 \approx 1$, $L_1 \approx 0.3$ $\mu$m and
$L_2 \approx 5$ $\mu$m for the quartzite mentioned.

For the simpler, one-state-variable form of his constitutive des-
cription, Ruina (1980) derived conditions for stability of steady slid-
ing of a spring-block arrangement, and for the existence of quasi-
statically propagating, small-amplitude creep waves in a simple model of
<u>elastic</u> bodies in sliding contact. In the present work linearized sta-
bility conditions are established for the entire class of constitutive
relations in the form of (1), and specialized to the two-state-variable
form of (2).

It is thus shown that linearized instability of the spring-block
arrangement is always of the flutter type (oscillations in $\dot\delta$ of growing
amplitude), and that this occurs when the spring constant k is less than
a certain critical value $k_{cr}$. Further, a Hopf (1942) bifurcation occurs
at $k_{cr}$ such that finite amplitude periodic oscillations of $\dot\delta$, with con-
tributions from higher harmonics, occur in the neighborhood of $k_{cr}$.

Finally, conditions for existence of propagating, small-amplitude
creep waves are established for sliding elastic solids in contact. For
unbounded elastic half-spaces in contact, the conditions can be met by
considering wavelength $\lambda$ of a velocity perturbation such that $G/\lambda$ (G=
shear modulus) is of order of $k_{cr}$; waves grow in amplitude as they prop-
agate for longer $\lambda$, decay for shorter $\lambda$. For the two-state-variable law
of (2) with $B_1 = B_2 = 1$, the speed of propagation at critical conditions is

$$\text{speed} \approx (G/3A\sigma)[(L_1+L_2)/\sqrt{L_1 L_2}]\dot\delta \approx 100(G/\sigma)\dot\delta \text{ for quartzite.}$$

ILLUSTRATIVE EXAMPLE FOR FRICTION IN ELASTICITY

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Frictional contact is perhaps one of the less well understood topics in
elasticity, as it is beset by mathemati al and computational difficul-
ties. One major difficulty and the cause of continuing debate is the
question of existence and uniqueness of solutions associated with prac-
tical friction laws. The other difficulty is computational and centers
on the question of how numerical schemes must be tailored to pick up
the subtle but fundamental intricacies of interfacial slip. Moreover,
no analytical solutions suitable for testing general propositions or
numerical methods are known. An elasticity problem is constructed and
solved here f i this purpose. It involves the unbounded solid with a
semi-infinite cut. The body is first compressed in a direction normal
to the cut and subsequently subjected to a concentrated force at the
tip of the cut. The concentrated force causes propagating separation
and slip zones. The problem can be solved in closed form, and it is
possible to follow load paths involving a general time change in the
components of the force, which in a sense can be classified as loading,
unloading and reloading. The results illustrate several interesting
aspects of friction in elasticity, such as the difference between weak
and strong friction, the nature of residual stresses left upon a com-
plete removal of the force, and the possibility of a discontinuous
extension of slip zones. Moreover, some general statements can be made
about periodically fluctuating loads.

Session FM-6:   ELASTODYNAMICS AND VIBRATIONS

Chairperson:    L.H.N. LEE (Notre Dame)

8:30 - 9:00    A. MIODUCHOUSKY ...i J.b. HADDOW (Univ. of Alberta, Canada)
               "Solution of Some Two-Dimensional Elastodynamic Problems"

9:00 - 9:30    G. DASGUPTA (Columbia Univ.)
               "Exterior Problems by Finite Element Cloning"

9:30 - 9:45    D.J. INMAN (SUNY/Buffalo) and A.N. ANDRY, JR. (Michigan State)
               "The Nature of the Temporal Solutions of Damped Distributed
               Systems with Classical Normal Modes"

9:45 - 10:00   J. PADOVAN (Univ. of Akron)
               "Eigenvalue Properties of Nonconservative Structure Subject to
               Moving Disturbances"

10:00 - 10:30   COFFEE BREAK

10:30 - 10:45   Y. KAI-YUAN (Lanzhou Univ., Beijing, China)
                "Dynamic Theory of Trains Passing Through a Railway Bridge Con-
                sidering Effects of the Masses and Inertia Forces of Moving Loads"

10:45 - 11:00   M. EL-ESSAWI and S. UTKU (Duke) and M. SALAMA (Cal. Inst. Tech.)
                "Systematic Generation of Nonlinear Discretized Dynamic Equili-
                brium Equations of Spinning Cantilevers"

11:00 - 11:15   A. AKAY (Wayne State, Detroit)
                "Vibratory and Acoustic Response of Beams to Impact Loads"

11:15 - 11:30   M. VAN OVERMEIRE (Vrije Universiteit, Brussels)
                "Natural Vibration Problem of Viscoelastic Materials"

11:30 - 11:45   M. VAN OVERMEIRE (Vrije Universiteit, Brussels)
                "Frequency Domain Analysis for Linear Viscoelastic Dynamical
                Problems"

12:00 - 1:30   LUNCH BREAK

Solution of Some Two-Dimensional
Elastodynamic Problems

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The plane stress unloading waves emanating from a suddenly punched
circular hole in a thin plate subjected to a uniform uniaxial tension
field are considered, Fig. 1, and then the plain strain problem of a
sudden application of a spatially uniform pressure in an oval cavity
in an unbounded homogeneous isotropic solid is discussed, Fig. 2.  A
modification of the method of near characteristics [1], originally
developed for problems of gas dynamics with two or three spatial vari-
ables is used to obtain numerical solutions to the governing equations,
which have three independent variables, time and two plane polar co-
ordinates [2], [3].  Next, the generalized plane stress, or plane strain
elastodynamic problem of an unbounded body subjected to sudden applica-
tion of tractions at the surface of a circular hole is considered.
Although in general this problem involves two spatial variables and
time, it is shown that the method of characteristics for one spatial
variable and time is applicable and a numerical procedure is proposed.
The thin plate problem, Fig. 1, is   discussed again and additionally
the solution of wave  propagation due to suddenly applied normal stress
on part of circumference of a cylindrical hole, Fig. 3, is presented
[4].
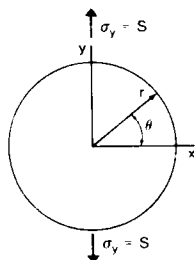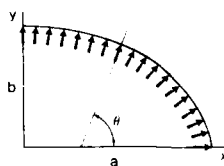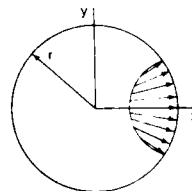


Figure 1          Figure 2          Figure 3

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# EXTERIOR PROBLEMS BY FINITE ELEMENT CLONING

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## ABSTRACT

A computational scheme is presented to evaluate the impedance matrices for time-harmonic excitations of linearly damped homogeneous unbounded media. The radiation damping losses are captured with the same order of accuracy as in a stipulated finite element model to be used in the (complimentary) interior domain of arbitrary shape. Some applications of the present technique could be cited in: modeling foundations for embedded structures; analyzing the effects of ocean waves on offshore structures; solving the electromagnetic field problems with open boundaries; studying the propagation of dislocations in crystalline media.

A quadratic matrix equation is developed. The solvent relates to the impedance matrix of the unbounded domain. The mass, damping and stiffness matrices of finite elements of similar geometry, whose infinite assembly constitutes the said unbounded domain, furnish the coefficents. The governing equation is then converted into the associated quadratic eigenvalue problem. It is shown that for each eigenvector the product of the pair of eigenvalues is unity. The discrete analog of the Sommerfeld's radiation condition, which demands that the inverse of the solvent should be a contraction map, is derived. Thus, those eigenvalues whose absolute values are greater than or equal to unity are admitted to formulate the solvent associated with the outgoing waves. The difficulties in constructing the complicated analytical decay expressions, which are essential to account for the radiation damping losses in all the currently available discrete schemes, are thereby alleviated.

A homogeneous elastic stratum undergoing antiplane harmonic motion is illustrated as a benchmark problem. An excellent comparison between the analytical results and the proposed finite element calculations is demonstrated. The cut-off frequency and the imaginary parts of the response indicating the radiation damping losses are very accurately reproduced. Techniques of modeling three-dimensional infinite and semi-infinite domains with arbitrary boundary geometry are also described.

The Nature of the Temporal Solutions of Damped
Distributed Systems With Classical Normal Modes

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This work considers the nature of the solutions of damped distributed parameter systems which may be described by the following partial differential equation:

(1)   $u_{tt}(x,t) + L_1 u_t(x,t) + L_2 u(x,t) = 0$ in $\Omega$

(2)   $Bu(x,t) = 0$ on $\partial\Omega$

where (i) $(\cdot)_t$ indicates partial differentiation of $(\cdot)$ with respect to the time $t$, (ii) $\Omega$ is a bounded, open region of $R^n$, $n = 1,2,3$. (iii) $L_1$ and $L_2$ are linear spatial (possibly differential) operators and (iv) B is a linear operator which reflects the boundary conditions. With additional assumptions, equations (1) and (2) can adequately describe vibration problems related to damped beams, plates, shells, etc. Caughey and O'Kelly [1] have shown that if $L_1$ and $L_2$ are self-adjoint, positive definite and commute (with respect to $Bu=0$) then the system (1) and (2) has a solution given by

(3)   $u(x,t) = \displaystyle\sum_{n=1}^{\infty} a_n(t)\phi_n(x)$

where $\phi_n(x)$ are a complete set of eigenfunctions (called classical modes) of $L_1$ and $L_2$. Under these assumptions, this paper derives necessary and sufficient conditions for the temporal functions $a_n(t)$ to be periodic (or to be aperiodic) for all n. These conditions are stated in terms of the definiteness of certain combinations of the coefficient operators $L_1$ and $L_2$ and are easy to check. Furthermore, they yield relationships between the stiffness, damping and mass parameters of a given system which may be used as design criteria to obtain a desired response.

The work is an extension to distributed systems of similar theorems for lumped parameter systems recently derived by the authors in [2]. Examples of the utility of the results are given, along with their derivation and comparison to problems in the current literature.

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EIGENVALUE PROPERTIES OF NONCONSERVATIVE
STRUCTURE SUBJECT TO MOVING DISTURBANCES
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Over the years, free vibration problems of structure with fixed
boundary restraints and stationary time dependent loads have received
much attention. One of the more interesting properties of such pro-
blems lies in the fact that when the spectral characteristics of the
disturbing forces and structural free modes coincide, potential re-
sonances can be achieved in the undamped case. This is a direct out-
growth of the fact that due to the stationariety of the time varying
load, fixed patterns of stress waves are set up in the structure.
When the phasing between such wave patterns and the external work
source is "tuned", resonances can occur. For the more general case of
moving externally applied loads/disturbances, such resonances and wave
patterns no longer coincide with the free spectral characteristics.
Although numerous aerospace and commercial structures are subject to
moving loads/disturbances, little is known of the spectral characte-
ristics of such problems. In the context of the foregoing, the pre-
sentation will consider the influence of moving loads/disturbances on
the spectral characteristics of structure modeled by 3-D nonpolar
elasticity theory wherein the fields are treated as small excursions
superposed on a potentially large initial state. To generalize the
results, the influence of nonconservatism and generalized inertial
fields will be admitted in the problem. The main emphasis will be
given to outlining the various properties of the eigenvalue/function
problem arising out of the foregoing continuum model. This will be
achieved by establishing a family of generalized Rayleigh quotients.
Employing the quotients, various types of spectral and critical speed
shifts and bifurcations induced by the moving disturbances will be
outlined. To add further generality to the development, the results
will be specialized to the eigenvalue/function problems arising from
nonconservative gyroscopic finite element/difference simulations of
such "moving" problems.

Dynamic Theory of Trains Passing Through a Railway Bridge

Considering Effects of the Masses and Inertia Forces of Moving Loads

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In this paper, we investigate the dynamic calculation of the whole
process of trains passing through a railway bridge, considering effects
of the mass and the damping effect of the bridge as well as the masses
of moving loads, small parameter method being used. In the first part,
we obtain the general solution of dynamic equation of the railway
bridge under the action of an arbitrary single moving load ($= PF(t)$,
where $P$ is a constant, $F(t)$ arbitrary function of time $t$). In
the second, we calculate the case of a single moving load with constant
weight, i.e. $PF(t) = Q$, where $Q$ is a constant. In the third, we
calculate the case of $PF(t) = P \sin ( t + )$, where is the re-
duced circular frequency, phase angle. In the fourth, we calculate
the mentioned results to establish the dynamic theory of trains passing
through a railway bridge. In the last, as an example, we preliminarily
calculate the well known Newark Dyke Bridge and discuss the results.

# SYSTEMATIC GENERATION OF NONLINEAR DISCRETIZED

## DYNAMIC EQUILIBRIUM EQUATIONS OF SPINNING CANTILEVERS

M. El-Essawi[+], S. Utku[§], and M. Salama[*]

The Rayleigh-Ritz procedure, in conjunction with any admissible trial solution in terms of undetermined functions of time and known yet unspecified coordinate functions of space, is systematized to obtain the coefficient matrices of the second order nonlinear ordinary differential equations of dynamic equilibrium of a spinning cantilever with initial geometric imperfections. In the formulation, second order nonlinear strain-displacement and velocity-displacement relationships are used, and the material is assumed linearly elastic. Systematic forms for the discretized energy density expressions are provided. The fact that the coordinate functions are represented parametrically serves to unify the discretization approaches. The choice of the coordinate functions includes both sets of continuous and piecewise continuous functions employed in the conventional Raleigh-Ritz method and the finite element method, respectively. A computer program for the systematic generation of the coefficient matrices involved in the governing equations is described. The degree of the nonlinearity, the type and number of the coordinate functions, the ordering of the vector representing the complete list of the undetermined functions of time, the imperfection functions, and the time independent forcing functions are taken as parameters of the program for the complete simulation.

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[*] Member of the Technical Staff, Division of Applied Mechanics, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91103.

# VIBRATORY AND ACOUSTIC RESPONSE OF BEAMS TO IMPACT LOADS

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A theoretical and experimental analysis of vibratory and acoustic
response of beams to time-dependent loads is developed.  Specific
cases of impact loading are examined.  The energy input into a beam is
calculated in both the time and frequency domains.  The damped vibra-
tory response of a beam is calculated by considering the structural
loss factor of the beam in the form of complex stiffness.  Results are
given in closed-form solutions and plots.  Damping due to acoustic
radiation is investigated using the "Radiation Efficiency" concept.

Vibrations of the beam and the resulting acoustic radiation are
measured for various impact conditions.  Acceleration and pressure-
time histories and their frequency characteristics are compared with
the theoretical results.

NATURAL VIBRATION PROBLEM OF VISCOELASTIC MATERIALS.

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The dynamic equilibrium equation of a structure (eq. 1) can

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \qquad (1)$$

be simplified by a modal superposition technique for linear structures. Here the displacements $\{u\}$ are expressed as a linear combination of eigenvectors $\{\phi_i\}$, which are obtained from the solution of the eigenproblem.  Only the lower eigenfrequencies and modes need to be taken into account, as the response to higher modes is critically damped and unimportant.

For viscoelastic materials, the stiffness matrix $[K]$ becomes frequency dependent, because it is a function of the complex elasticity modulus $E^{*}$ (equation 2) :

$$E^{*} = E(\omega) \left[ 1 + jn(\omega) \right] \qquad (2)$$

The solution of the modified eigenvalue problem (equation 3) is usually found in two steps [1]

$$[K(\omega_i)]\{\phi_i\} = \omega_i^2 [M]\{\phi_i\} \qquad (3)$$

However, using the Sturm Sequence Method [2], the solution is straightforward.  Only minor changes are necessary : for the calculation of the sign count in $\omega_i$, the current value of $E(\omega)$ namely $E(\omega_i)$ is used here for the stiffness matrix $[K]$.

The advantages of the normal Sturm algorithm remain, and make this method attractive for solving the natural vibration problem of viscoelastic materials.

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   "Sturm Sequence Method for Medium Band Matrices".
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# FREQUENCY DOMAIN ANALYSIS FOR LINEAR VISCOELASTIC DYNAMICAL PROBLEMS.

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On the dynamic equilibrium of a linear viscoelastic material (equation 1)

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + \int_0^t [K(t-\tau)]\{u(\tau)\}\, d\tau = \{F(t)\} \tag{1}$$

a Fourier transformation is applied, which gives as result

$$-\omega^2[M]\{u^*(\omega)\} + j\omega[C]\{u^*(\omega)\} + [K]\{u^*(\omega)\} = \{F^*(\omega)\} \tag{2}$$

supposing that the mass matrix M and the damping matrix C are time independent. The formulation of equation (2) is less complicated but unfortunately we have to solve now a system of 2n equations and 2n unknowns of the structure. This large numerical effort can be mini-mized by solving for the basic eigenvalues of the system and trans-forming the equations to a smaller system, expressed in modal coordi-nates (equation 3).
The necessary number of modes (m) to be taken into account for obtai-ning an accurate solution, can be found by comparing the eigenspec-trums of structure and applied forces.
For most vibration problems, m is much smaller than n, with as a con-sequence that the time required for solution of equation (3), is not the crucial phase of the solution process.

$$-\omega^2[m]\{x^*(\omega)\} + j\omega[C]\{x^*(\omega)\} + [k^*]\{x^*(\omega)\} = \{f^*(\omega)\} \tag{3}$$

Recovery of $\{u(t)\}$ is also done by FFT [1] :

$$\{u(t_\ell)\} = \sum_{i=1}^{m} \{\phi_i\} \sum_{k=0}^{N-1} x_i^*(\omega_k)\, e^{j\,\omega_k t_\ell} \tag{4}$$

For structures, which can be formulated in the frequency domain, such as linear viscoelastic structures, and whose behaviour can be repre-sented by a limited number of modes, this method is numerically the most efficient approach.

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Session FA-1:  HYDRODYNAMIC STABILITY, TRANSITION AND TURBULENCE - III

Organizer:    J.T.C. LIU (Brown)
Chairperson:  R.S. BRODKEY (Ohio State)

* 1:30 - 2:30    M.B. LONG, R.K. CHANG and B-T CHU (Yale)
                 "Experimental Study of the Structure of a Turbulent Jet by
                  Light Scattering"

* 2:30 - 3:00    A.K.M.F. HUSSAIN (Univ. Houston)
                 "Large-Scale Coherent Structures: What, Why and How?"

  3:00 - 3:30    REFRESHMENT BREAK

  3:30 - 4:00    J.M. DORREPAAL (Old Dominion Univ.)
                 "Burgers Approximation for Flow Past a Circular Cylinder"

  4:00 - 4:30    K. OKADA and M.N.L. NARASIMHAN (Oregon State)
                 "Synthetic Method in Thermal Boundary Layer Transition"

# Experimental Study of the Structure of a Turbulent Jet by Light Scattering[*]

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Recent experiments conducted at Yale University on the structure of a turbulent jet using a Lorenz/Mie light scattering technique are reviewed. Fluid from a 4 mm round nozzle is seeded with uniformly dispersed submicron size aerosol particles. Assuming that each unit mass of the nozzle fluid is always "tagged" by approximately the same number of aerosol particles with approximately the same size distribution, the degree of mixing of the marked nozzle fluid with the unmarked surrounding air will be reflected in a change in the aerosol concentration. If a sheet of radiation is allowed to pass through the mixing layer, the instantaneous distribution of the aerosol (and, therefore, the nozzle fluid) concentration in the sheet can be monitored and inferred from the distribution of the elastically scattered radiation. The scattered light from the sheet is digitalized at 10,000 points in a 100x100 array and stored in the computer. Subsequently, the record allows one to examine both quantitatively and qualitatively the degree of mixing of the nozzle fluid with the surrounding air in a plane.

From the large number of records stored in the computer, statistical information relevant to turbulent mixing can be deduced. In particular, the distribution of the mean concentration, the rms fluctuation and the longitudinal covariance in a meridian plane, as well as other statistical characteristics of the jet are presented. The physical processes responsible for the observed phenomena may be deduced by examining the large number of digitized instantaneous realizations stored in the computer. Thus, the characteristic "shoulders" in the mean concentration profile and the "depressions" in the rms profile are shown to be consequences of the vortical mixing mechanism. A second mixing mechanism which dominates farther downstream is responsible for the development of small scale concentration fluctuation and may be attributed to the instability and ultimate disintegration of vortex rings. The net effect of the instability is the production of bursts of nozzle fluid projected radially outward. Finally, some preliminary results on the structural change of a mixinglayer subjected to a forced acoustic excitation will be presented.

LARGE-SCALE COHERENT STRUCTURES:  WHAT, WHY AND HOW?

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The large-scale coherent structures in turbulent shear flows
have been the target of many experimental (and some theoretical) in-
vestigations[1-7] in recent years.  However, there is little agreement
on what is precisely meant by coherent structures, what features of
the structures are important, how these should be measured, and why
the knowledge of the details of the structures are of importance
in formulating a viable theory of shear flow turbulence.  The experi-
mental difficulties in educing distributions of properties over the
spatial extents of structures will be discussed.[8,10]  Detailed data
on the coherent structures, either naturally-occurring or induced,
in a number of free turbulent shear flows [11-16] will be reviewed
and some consequences will be examined.  Also to be discussed are
the relative contributions of the large-scale coherent structures and
fine-scale turbulence to the Reynolds stress and turbulence production
in turbulent shear flows.

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BURGERS APPROXIMATION FOR FLOW PAST A CIRCULAR CYLINDER

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The Navier-Stokes equation governs the steady two-dimensional flow of a viscous incompressible fluid and if expressed in terms of nondimensional polar coordinates $(r, \theta)$, has the form

$$[\nabla^2 + \frac{R}{r}(\frac{\partial \Psi}{\partial r} \frac{\partial}{\partial \theta} - \frac{\partial \Psi}{\partial \theta} \frac{\partial}{\partial r})]\omega = 0 \tag{1}$$

where $\nabla^2$ is the Laplacian, $\Psi(r, \theta)$ is the stream function, $\omega(r, \theta) = -\nabla^2 \Psi$ is the vorticity and R is the Reynolds number. Except for those problems in which the nonlinear terms vanish identically, the solution of (1) is formidable even in the simplest of geometries.

An analytic approach which has enjoyed some success in finding approximations to the solutions of (1) involves linearizing the equation and then solving the resulting problem exactly. Burgers approximation falls into this category but is more sophisticated than the better known Stokes and Oseen approximations. In Burgers flow the convective terms $\frac{\partial \Psi}{\partial r}$, $\frac{\partial \Psi}{\partial \theta}$ in (1) are replaced by $\frac{\partial \Psi_0}{\partial r}$, $\frac{\partial \Psi_0}{\partial \theta}$ where $\Psi_0(r, \theta)$ is the stream function for the corresponding irrotational inviscid flow. The convecting flow therefore "sees" obstacles around which the fluid must pass and in this regard is a definite improvement over the Oseen approximation.

In this paper it will be shown that the vorticity equation for Burgers flow past a circular cylinder is separable after an exponential quantity is factored out. The ordinary differential equations which result are different versions of Mathieu's equation. The vorticity is given by

$$\omega(z, \theta) = -\exp(R \cosh z \cos \theta) \sum_{n=1}^{\infty} C_n \text{Gek}_n(z, -\frac{1}{4}R^2) \text{se}_n(\theta, -\frac{1}{4}R^2) \tag{2}$$

where $z = \ln r$, $C_n$ are constants to be determined and the functions $\text{Gek}_n$ and $\text{se}_n$ are Mathieu functions, the former behaving asymptotically like $\exp(-R \cosh z)$ as $z \to +\infty$, the latter being odd in $\theta$ and having period $2\pi$. The stream function for the flow is recovered using a Green's function approach and the constants $C_n$ are determined by invoking the no-slip condition.

The solution has been analyzed numerically and predicts incipient separation at the rear of the cylinder when $R = 1.12$. This agrees with the work of Skinner [1] who predicts the same value of R by applying singular perturbation theory to the full nonlinear problem. (See [2]).

[1] L. A. Skinner (1975), Quart. J. Mech. Appl. Math. 28, 333-340.

[2] M. Van Dyke (1975) Perturbation Methods in Fluid Mechanics (Annotated Edition), The Parabolic Press, Stanford, California, p. 236.

SYNTHETIC METHOD IN THERMAL BOUNDARY LAYER TRANSITION

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## Abstract

The problem of a general incompressible viscous fluid flow past a flat plate with heat transfer due to forced convection is considered in this paper. The synthetic method developed by Seth is applied to the Navier-Stokes *equations* and the equation of energy governing the flow to obtain the dynamic and thermal boundary layer solutions as asymptotic limits of an extended field. As a result, new formulas are derived for both the dynamic and thermal boundary layer thicknesses. Also, algorithms for estimating all the parameters involved in the analysis are provided and boundary layer functions based on the new solutions are determined.

Session FA-2:  PLASTICITY THEORY

Chairperson: L. MALVERN (U. of Florida)

1:30 - 2:00    T. LEHMANN (Univ. Bochum, Germany)
               "Some Remarks on the Decomposition of Deformations and
               Mechanical Work"

2:00 - 2:30    S.R. BODNER (Technion, Israel) and J. ABOUDI (Univ. Tel-Aviv)
               "Stress Wave Propagation in Rods of Elastic-Viscoplastic
               Materials"

2:30 - 3:00    X. MARKENSCOFF (UC, Santa Barbara) and R.J. CLIFTON (Brown)
               "Radiation from Expanding Circular Dislocation Loops and
               Elastic Precursor Decay"

3:00 - 3:30    REFRESHMENT BREAK

3:30 - 4:00    E.T. ONAT (Yale)
               "Representation of Inelastic Behaviour"

4:00 - 4:30    H. GHONEIM and Y. CHEN (Rutgers)
               "A Viscoelastic-Viscoplastic Constitutive Equation and its
               Finite-Element Implementation"

Some remarks on the decomposition of deformations
and mechanical work

by
Th. Lehmann

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In arbitrary deformation processes (e.g. *non-isothermic elastic-plastic deformations* ) the applied mechanical work has to be decomposed in a first step into one part which is dissipated at once and into the remaining part. The remaining part has to be balanced with the changes of the free internal energy or free enthalpy, respectively.

The non-dissipated work can be split once more into one part connected with reversible processes in the strong sense like elastic deformations or solid phase transformations, respectively, and into another part which may be recoverable or not. The first part is governed by thermodynamical state equations. The second part represents the stored energy corresponding to certain changes of the internal structure of the material like isotropic and anisotropic hardening. It depends on the process history, whether this stored (mechanical) energy will be dissipated later on (totally or partially) or not. Energy stored in anisotropic hardening e.g. can at least partially recovered in reverse loading processes (Bauschinger effect).

The *definition and decomposition of strain measures* have to fulfill  certain physical requirements resulting from the decomposition of the mechanical work. This will be discussed in some details.

# STRESS WAVE PROPAGATION IN RODS OF ELASTIC-VISCOPLASTIC MATERIALS

by

S.R. Bodner          and          J. Aboudi

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Technion, Haifa, Israel               University of Tel Aviv, Israel

## Abstract

The unified elastic-viscoplastic constitutive equations of Bodner-Partom [1] and a newly developed numerical procedure based on finite differences are used to solve a variety of stress wave propagation problems in semi-infinite and finite rods. Those equations are characterized by the absence of a reference "static" stress-strain curve or yield criterion and of loading/unloading conditions [2], which make them suitable for problems of this kind. Stress and inertia effects are taken to be uniaxial, and transverse strains enter the formulation through the constitutive equations. Solutions are obtained for various cases of a constant stress or velocity imposed at one end of the rod for either an indefinite time or a prescribed time duration. Some exercises were performed with two steps in the applied velocity, the first being sufficient to induce appreciable plastic deformation at a slow rate and the second rapidly applied after a time interval.

In most of the examples, work hardening was assumed to be isotropic and the numerical results were obtained for the material constants of titanium as described in [1]. For the finite rod case, exercises were also run for anisotropic work hardening (Bauschinger effect) based on the anisotropic work hardening theory of Stouffer-Bodner [3] modified to give isochoric plastic deformations. For the uniaxial stress case, the modified hardening theory is essentially the same as that described in [4].

The numerical results show a number of interesting features some of which do not seem to have been predicted by other theories on viscoplastic wave propagation and which appear to be compatible with experimental observations. These include the absence of a sharp elastic front and the existence of low amplitude wave components traveling at the dilatational and higher velocities at very short times. Additional results are a pronounced stress peak and subsequent stress relaxation at the struck end, non-constant propagation velocities of equal strain levels, propagation at elastic velocities of superimposed waves, and the tendency of the plastic strains near the struck end to be almost constant (a strain "plateau") despite the strong rate dependence of the material. The results for the finite bar show the complicated effects of unloading and reflections.

## References:

1.  Bodner, S.R. and Partom, Y., J. Appl. Mech., 42, 1975, 385.
2.  Bodner, S.R., "Mechanical Behavior of Materials Under Dynamic Loads," (U.S. Lindholm, ed.), Springer-Verlag, 1968, 176-190.
3.  Stouffer, D.C. and Bodner, S.R., J. Engng. Sci., 17, 1979, 757.
4.  Bodner, S.R., Partom,I. and Partom, Y.,J. Appl.Mech., 46,1979, 805.

Radiation from Expanding Circular Dislocation Loops

and Elastic Precursor Decay

by

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and

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Brown University
Providence, Rhode Island 02912

### Abstract

The wave-front behavior is determined for circular dislocation loops which are at rest and suddenly start expanding with constant radial velocity $V$ in their slip-plane. The formulation is based on the three-dimensional Green's function as in Mura (1).

Superposition of solutions for many loops set in motion by an incident plane shear wave is used to relate the decay of the wave amplitude at the front of the plane wave to the density and velocity of dislocations at the wave-front. The resulting precursor decay relation is the same as the one derived from an elasto-visco-plastic model of the material, and also coincides with the analogous precursor decay analysis by Clifton and Markenscoff (2) for straight dislocations.

(1)  T. Mura, "Continuous Distributions of Moving Dislocations," Phil. Mag., Vol. 8, pp. 843 (1963).

(2)  R.J. Clifton and X. Markenscoff, "Elastic Precursor Decay and Radiation from Nonuniformly Moving Dislocations," J. Mech. Phys. Sol., (to appear).

# REPRESENTATION OF INELASTIC BEHAVIOUR

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SUMMARY

In the presence of elevated temperatures and/or stress most materials exhibit mechanical behavior that is <u>nonlinear</u> and <u>hereditary</u> (inelastic). When this kind of behavior is present the strain of a material element will depend (nonlinearly) not only on the current *stress* and temperature but also on the temperatures and stress previously applied to it. The present paper is concerned with the problem of mathematical representation of the relationship that exists between histories of stress and <u>isothermal</u> deformations. The thesis of the paper is that the study of <u>geometric</u> and <u>global</u> aspects of representations of mechanical behavior based on state variables and differential equations provides a unified point of view that enables one to compare existing representations and more importantly offers rational ways of improving a given representation or of constructing a new one.

It is known that the response of a material to various kinds of mechanical stimuli can be described conveniently and in geometrical terms by the corresponding evolution of the state point(that represents the internal state of the material in a suitable space). Moreover, as seen in Section 2, the vector field associated with test episodes where deformation is kept constant plays a central role in determining the motions in state space in any test. We emphasize in Section 3 that each classical mode of testing (e.g. creep experiments) can take the state point to only certain parts of the state space. It follows then that a judicious combination of various types of tests would be needed to explore the entire state space $\Sigma$. Section 4 is devoted to the study of intimate relation that exists between creep of metals and their seemingly time independent plastic behavior. In Section 5 the idea of using the current yield surface (or more exactly the parameters that define it) as the internal state is explained in the setting of <u>cyclic straining</u>. In the last section we consider general but still small and isothermal deformations of materials. Here the question of material symmetry both in virgin and deformed material becomes an important issue and the need arises for <u>tensorial</u> state variables. We illustrate this Section with a short essay on representation of internal damage in creep of polycrystalline materials.

# A VISCOELASTIC-VISCOPLASTIC CONSTITUTIVE EQUATION
## AND ITS FINITE-ELEMENT IMPLEMENTATION

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## Abstract

A viscoelastic-viscoplastic constitutive model for isotropic
materials undergoing isothermal infinitesimal deformation is proposed.
The model is based on the assumption that the total strain rate is
decomposable into a viscoelastic and a viscoplastic portion.
Consequently, the model consists of a linear viscoelastic model in
series with a modified plasticity model. This modified plasticity
model adopts the classical Drucker-Prager yield surface with isotropic
hardening and the associative flow rule of the invicid theory of
plasticity. However, hardening is assumed to be a function of both
the viscoplastic strain as well as the total strain rate. In this
manner, the proposed model acquires the advantage of having both the
initial and the subsequent yield surface to be a function of the strain
rate, a property which has its experimental supportive evidence for
viscoelastic materials such as polymers and some metals at highly
elevated temperatures.

A finite-element algorithm is developed to implement the constitu-
tive equation. This algorithm adopts a combination of the tangent
stiffness matrix and the initial load approach. Two numerical examples
are given to demonstrate the capability of the proposed model in depicting
the rate and pressure effect on the mechanical behavior of some classes of
viscoelastic-viscoplastic materials.

Session FA-3: DIRECT AND INVERSE SCATTERING PROBLEMS FOR QNDE
APPLICATIONS

Organizer:    J.D. ACHENBACH (Northwestern)
Chairperson:  J.D. ACHENBACH (Northwestern) & J.E. GUBERNATIS (Los Alamos)

* 1:30 - 2:00    J.E. GUBERNATIS (Los Alamos)
                 "Elastic Wave Scattering Theory with Application to Non-
                 destructive Evaluation"

* 2:00 - 2:30    L. ADLER (Ohio State)
                 "Experimental Work in Ultrasonic Scattering from Discontinuities"

* 2:30 - 3:00    J.H. ROSE (Iowa State)
                 "Ultrasonic Inverse Scattering from Volume Flaws"

  3:00 - 3:30    REFRESHMENT BREAK

* 3:30 - 4:00    J.D. ACHENBACH and A. NORRIS (Northwestern)
                 "Crack Characterization by the Combined Use of Time-Domain and
                 Frequency-Domain Scattering Data"

  4:00 - 4:15    L.S. FU (Ohio State) and T. MURA (Northwestern)
                 "Elastodynamic Fields of an Ellipsoidal Inhomogeneity"

  4:15 - 4:30    L.S. FU (Ohio State)
                 "Volume Integrals of Ellipsoids Associated with the Inhomo-
                 geneous Helmholtz Equation"

Elastic Wave Scattering Theory with
Application to Nondestructive Evaluation
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Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM  87545

Recent developments in elastic wave scattering theory will be
reviewed.  The focus will be on those developments which have been
useful for defect characterization studies in the nondestructive
evaluation of materials; accordingly, different approaches to
scattering calculations will be discussed with respect to their
success and limitations for different defect types and different
frequency ranges of the probing elastic waves.  For definiteness,
the integral equation approach to the scattering will be presented
to illustrate characteristic features of an elastic wave scattering
theory and important aspects of several useful computational
approaches.

Experimental Works in Ultrasonic Scattering from Discontinuities.*

Laszlo Adler
Ohio State University
Department of Welding Engineering

A general experimental system to obtain ultrasonic data scattered from discontinuities in solids will be described.  Measurement techniques to obtain angular and frequency distribution of scattered longitudinal shear and surface waves from bulk and surface discontinuities will be discussed.  The quantitative relationship between the measured ultrasonic parameters and parameters of the discontinuity (size, shape, orientation, surface roughness) will be emphasized.  Experimental results will be analyzed based on existing analytical works.

# ULTRASONIC INVERSE SCATTERING FROM VOLUME FLAWS

J. H. Rose
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The ultrasonic inverse problem for an isolated flaw in the bulk of a structural material consists in part of the following three determinations. First, is the flaw a crack, void or inclusion? Second, what are the geometric features of the flaw, such as its size shape and orientation? Finally, if the flaw is an inclusion we need to know its material composition. In this talk, we review [1] primarily from the point of view of the Inverse Born Approximation, the determination of the flaws geometry.

The primary inversion algorithm used is

$$\gamma(\vec{r}) \cong \text{const.} \int d^3\vec{k} \; e^{-2i\vec{k}\cdot\vec{r}} \; \frac{A(\vec{k})}{k^2} \qquad (1)$$

Here $\gamma(\vec{r})$ defines the flaw's shape and is defined to be ideally one for $\vec{r}$ inside the flaw and zero for $\vec{r}$ outside the flaw. $A(\vec{k})$ is the longitudinal to longitudinal, pulse-echo scattering amplitude and $\vec{k}$ is the wavevector. We discuss the derivation of Eq. 1 briefly. Consideration is given to establishing its utility for a wide class of flaws using numerically exact theoretically generated scattering data. Flaws discussed include a spherical void with a circumferential crack. Then we turn to establishing the experimental problems of implementing Eq. 1 experimentally. Noise, bandwidth requirements and material anisotropy are mentioned. The latter is of crucial importance for this algorithm and leads to the need for long wavelength data in the scattering amplitudes. Finally, if time permits, we will discuss the connection of the Inverse Born Approximation to other methods such as the Physical Optics Method and various algorithms incorporated in acoustic imaging systems.

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[1] J. H. Rose and J. A. Krumhansl, J. App. Phys. 50, 2951 (1979).

Crack Characterization by the Combined Use of Time-Domain and

Frequency-Domain Scattering Data

by

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The problem of locating cracks and determining their extent is one of fundamental concern in the non-destructive evaluation of materials.

First we consider the problem of finding the general location of the crack. We assume that the material is homogeneous. Either specularly reflected or edge-diffracted signals can be used. From Fermat's principle we obtain an inversion procedure which uses only the time delay of the first received signals as input. It can be shown that the inversion is generally unique, and that it results in determining a base-point on the crack.

For signals produced by edge diffraction, there will be two flashpoints on the crack edge. An inversion method based on first arrival times and on the resonance spacing of the high-frequency spectrum has been developed which yields planes containing the flashpoints relative to the base-point. A number of such planes defines the crack edge.

The method has been extended to anisotropic material behavior, and to the case that the relevant wave speeds are not known a-priori, but must be obtained as part of the inversion procedure. The method has yielded excellent results for synthetic crack-scattering data.

# ELASTODYNAMIC FIELDS OF AN ELLIPSOIDAL INHOMOGENEITY

L.S. Fu*, Ohio State University, U.S.A.
T. Mura, Northwestern University, U.S.A.

Elastic fields of a single ellipsoidal inhomogeneity embedded in
an infinite elastic matrix subjected to plane time-harmonic waves are
studied by employing the concept of dynamic eigenstrains, and the
extended method of inclusions [1]. Using the dynamic version of the
Betti-Rayleigh reciprocal theorem, an integral representation of the
displacement field is given in terms of the eigenstrains [2]. The
interaction energy due to the presence of the inhomogeneity is also
determined by the eigenstrains [3]. Expanding the eigenstrains and
applied strains in polynomial form in the position vector $\bar{r}$ [4,5] and
satisfying the equivalency conditions at every $\bar{r}$ inside the region
occupied by the inhomogeneity, the governing simultaneous algebraic
equations for the unknown coefficients in the eigenstrain expression are
derived. Reasonably convergent series of the volume integrals associat-
ed with the inhomogeneous Helmholtz equation are obtained by using
Taylor series expansion, multinomial theorem, and the work of Dyson [6].
Results for special cases such as spherical and disk-shaped inhomogenei-
ties will be reported.

References:

[1]  T. Mura, Applied Micromechanics, Monograph, Northwestern University,
     (May 1980)

[2]  P. Wheeler and T. Mura, J. Appl. Mech., 40 (1973) 498-502.

[3]  L.S. Fu, NASA Contractor Report, NSG 3269, (December 1980).

[4]  R.J. Asaro and D.M. Barnett, J. Mech. Phys. Solids, 23 (1975) 77-83.

[5]  Z.A. Moschovidis and T. Mura, J. Appl. Mech. 42 (1975) 847-852.

[6]  F.W. Dyson, Quart. J. Pure & Appl. Math. XXV (1891) 259-288.

# VOLUME INTEGRALS OF ELLIPSOIDS ASSOCIATED WITH THE INHOMOGENEOUS HELMHOLTZ EQUATION

L.S. Fu
Department of Engineering Mechanics
Ohio State University, U.S.A.

Results of volume integrals associated with the integration of the inhomogeneous Helmholtz equation [1] [2], carried out for an ellipsoidal region, are presented. The volume integrals take the form of

$$I(\bar{r}) = \iiint_{\Omega} \rho(\bar{r}') \, R^{-1} \exp i\alpha R \, dV' \qquad (1)$$

where $\rho(\bar{r}')$ is the density distribution of the form $x_i^{l_1} \, x_j^{l_2} \, x_k^{l_3}$, $\Omega$ is the ellipsoidal region where $\rho(\bar{r}')$ is distributed, and $R$ is the distance between the source and observation points, $R = |\bar{r}-\bar{r}'|$. Expanding $R^{-1} \exp i\alpha R$ in appropriate Taylor series expansions, and using multinomial theorem, the I-integrals can be obtained as relatively convergent series for regions outside and inside the ellipsoid. *Derivatives of these integrals are easily evaluated.* When $\alpha \to 0$, the results reduce directly to the potentials of ellipsoids of variable densities [3].

References:

[1] R.E. Collin, Field Theory of Guided Waves, McGraw-Hill (1960).

[2] P.M. Morse and H. Feshbach, Methods of Theoretical Physics (1953).

[3] F.W. Dyson, Quart. J. Pure & Appl. Math., XXV, (1891) 259-288.

Acknowledgement:

Session FA-4:  MATERIAL INSTABILITY AND FAILURE - II

Organizers:   A. NEEDLEMAN and R.D. JAMES (Brown)
Chairperson:  R.D. JAMES (Brown)

* 1:30 - 2:00    P.S. STEIF (Harvard)
                 "Strain Localization in Amorphous Metals"

* 2:00 - 2:30    D. PEIRCE (Brown)
                 "Finite Deformation and Localization in Ductile Single Crystals"

* 2:30 - 3:00    K.S. HAVNER (North Carolina State)
                 "A Minimum-Work Postulate for Dead Loading of FCC Crystals in
                 Multiple-Slip Orientations"

  3:00 - 3:30    REFRESHMENT BREAK

* 3:30 - 4:00    S. NEMAT-NASSER and T. IWAKUMA (Northwestern)
                 "A Microscopic Derivation of Constitutive Relations for Poly-
                 crystalline Solids at Finite Strains"

* 4:00 - 4:30    O. RICHMOND, E.J. APPLEBY and W.A. SPITZIG (US Steel)
                 "An Attempt to Model the Effect of Sulfide Stringers on the
                 Strength and Ductility of Steel Plates"

# STRAIN LOCALIZATION IN AMORPHOUS METALS

Paul S. Steif

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## Abstract

Inhomogeneous flow in metallic glasses is discussed in this talk within the context of continuum mechanics. Motivated by similar work for elastic-plastic solids, the possibility of strain localization into a shear band is examined for a metallic glass which is modeled as a nonlinear visco-elastic solid. In simple tension the constitutive law is expressed by a set of two coupled differential equations. The first equation is the stress-strain-rate relation which involves an internal parameter, the free volume, a measure of the ease with which atomic rearrangements leading to deformation can occur. The second equation gives the rate of change of free volume with stress. The essential features of the localization problem are brought out through an analysis of the constitutive law which reveals a catastrophic softening via free volume creation. Analytic expressions for the stress at catastrophic softening agree very closely with the stress at strain localization calculated from the numerical solution of the full set of shear band equations.

*Finite Deformation and* Localization

in

Ductile Single Crystals

Daniel Peirce
Brown University

Abstract

In ductile materials, bifurcation from an approximately homogeneous deformation state into a drastically nonuniform *mode is* widely observed. A frequent precursor to ductile failure, localization of strain is a significant phenomenon; conditions *for its* onset need to be carefully understood.

To this end, the present computational studies have concentrated on an examination of large deformations in face-centered cubic single crystals. For the three-dimensional crystal, homogeneous tensile deformations, featuring both single and double slip, have been modeled using Asaro's constitutive description [1]. With the two-dimensional idealization of symmetric double slip suggested by Asaro [1], finite-element calculations have been made. The resulting solutions for plane strain boundary value problems, based on the single crystal constitutive law, account for necking in the crystal, and permit one to follow the deformation after the shear band has formed. In agreement with the observations of Chang and Asaro [2], the present analysis shows that the lattice within a shear band rotates in such a way that the shearing material experiences substantial geometrical softening.

References

[1]  R. J. Asaro, Acta Met. 27, 445 (1979).

[2]  Y. W. Chang and R. J. Asaro, to appear in Acta. Met.

A Minimum-Work Postulate for Dead Loading of F.C.C. Crystals
in Multiple-Slip Orientations*

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A complete analysis of finite plastic straining of axially-loaded
f.c.c. crystals in double and 6-fold symmetry positions is presented
for each of three hardening theories. These are (i) classical Taylor
hardening[1], (ii) a two-parameter hardening rule suggested in the
metallurgical literature (cf.[2]), and (iii) the "simple theory" of
rotation-dependent crystal anisotropy [3]. We consider that only nom-
inal axial stress is specifiable (with respect to an embedded material
line in a tensile test or material plane in a compression test),whence
neither material spin nor lattice spin is known *a priori* relative to
the loading-axis reference frame. As a consequence, each system of
constitutive inequalities permits a multiplicity of strain-rates and
rotations, including those of (a) equal-slipping (and corresponding
rotation along the symmetry line) in double-symmetry orientations,
and (b) axis-stability, accompanied by axisymmetric deformation, in
the 6-fold symmetry position. From experimental evidence solutions
(a) and (b) appear physically most likely, at least in the tensile
test, for their respective initial positions. It is proved that these
solutions, among all those satisfying the constitutive equations for
each of the three hardening laws considered, correspond to the minimum
rate of plastic-working for a given (nominal) axial stress-rate and
initial axis orientation.
In addition, the theories are contrasted with empirical evidence
that defines the essential features of finite deformation and latent
hardening of f.c.c. crystals in various single and multiple-slip or-
ientations. It appears that only the simple theory, augmented by the
minimum-work postulate, is consistent with this diverse evidence.

[1] Taylor, G.I. and Elam, C.F., The distortion of an aluminium crystal
    during a tensile test. Proc.R.Soc. A102, 643-667 (1923).
[2] Jackson, P.J. and Basinski,Z.S., Latent hardening and the flow stress
    in copper single crystals. Can.J.Phys.45, 707-735 (1967).
[3] Havner, K.S. and Shalaby, A.H., A simple mathematical theory of
    finite distortional latent hardening in single crystals. Proc.R.
    Soc. A358, 47-70 (1977).

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- 221 -

A MICROSCOPIC DERIVATION OF CONSTITUTIVE RELATIONS
FOR POLYCRYSTALLINE SOLIDS AT FINITE STRAINS

by

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A self-consistent method is used to derive elastoplastic rate
constitutive relations of rate independent polycrystalline solids at
finite strains. Unlike the corresponding problem for small strains,
the calculation of the local quantities such as the objective stress
rate and the stress in terms of the corresponding overall quantities
presents a difficult problem. Various aspects of this problem are
discussed and illustrated. The results are then used to obtain the
overall instantaneous (elastoplastic) moduli by means of various
commonly discussed (for small strains) self-consistent methods. The
effects of changes in the grain shape and formation of texture, on
the overall behavior, are mentioned.

AN ATTEMPT TO MODEL THE EFFECT OF SULFIDE STRINGERS
ON THE STRENGTH AND DUCTILITY OF STEEL PLATES

By   O. Richmond, F. J. Appleby, and W. A. Spitzig*

## Abstract

The inclusion populations of three steel plates containing
different volume fractions of sulfide stringers are determined.
Scanning-electron-microscope-based automatic image analysis is used
to determine the size and shape distributions of the populations,
and the automatically constructed Dirichlet cell structures
corresponding to the populations are used to determine the dispersion.

Unit cells containing single inclusions are defined from
the mean values of size, shape, and spacing of the populations.
The plane-strain mechanical behavior of the unit cells is then
computed using a large-strain elastoplastic finite-element method,
assuming the noncoherent inclusions behave as voids.  Finally,
the calculated mechanical response is compared with the actual
through-thickness and transverse plane-strain behavior of the
prototype materials.

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  Pennsylvania 15146.

Session FA-5: MACROSCOPIC PROPERTIES OF DISORDERED MEDIA

Organizer: R. BURRIDGE (Courant, NYU)
Chairperson: R. BURRIDGE (Courant, NYU)

* 1:30 - 2:30   G. PAPANICOLAOU (Courant, NYU)
               "Upper and Lower Bounds for Effective Parameters of Heterogenous
                Media"

* 2:30 - 3:00   R. BURRIDGE (Courant, NYU) and J.B. KELLER (Stanford)
               "Poroelasticity Equations Derived from Microstructure"

  3:00 - 3:30   REFRESHMENT BREAK

  3:30 - 4:00   O.R. WALTON (Livermore)
               "Particle Dynamics: A Computational Method for Jointed Systems
                and Granular Materials"

  4:00 - 4:30   Y. BENVENISTE and J. ABOUDI (Tel-Aviv University)
               "Mixture Theories for Modeling the Dynamic Response of
                Composite Materials"

Upper and Lower Bounds for Effective Parameters
of Heterogenous Media

George Papanicolaou
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We consider first the definition and properties of effective parameters
such as effective dielectric constants, conductivity, etc.

We then derive bounds (including the Hashin-Schtrikman bounds) by varia-
tional methods as well as by analytical methods (introduced by D. Berg-
man). We investigate the mathematical basis of these methods and the
range of their applicability.

POROELASTICITY EQUATIONS DERIVED FROM MICROSTRUCTURE

Robert Burridge
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and

Joseph B. Keller
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Stanford, CA 94305

Abstract

Equations are derived which govern the linear macroscopic mechani-
cal behavior of a porous elastic solid saturated with a compressible
viscous fluid. The derivation is based on the equations of linear
elasticity in the solid, the linearized Navier-Stokes equations in the
fluid, and appropriate conditions at the solid-fluid boundary. The
scale of the pores is assumed to be small compared to the macroscopic
scale, so that the two-space method of homogenization can be used to
deduce the macroscopic equations. When the dimensionless viscosity of
the fluid is small, the resulting equations are those of Biot, who
obtained them by hypothesizing the form of the macroscopic constitutive
relations. The present derivation verifies those relations, and shows
how the coefficients in them can be calculated, in principle, from the
microstructure. When the dimensionless viscosity is of order one, a
different equation is obtained, which is that of a viscoelastic solid.

# Particle Dynamics — A Computational Method for Jointed Systems and Granular Materials[*]

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## ABSTRACT

The bulk response of granular or highly jointed materials to applied loads is often dominated by displacements of the particles involved with little deformation of the individual grains or blocks. Explicit numerical integration of the coupled equations of motion of the particles in such systems allows the overall motion to be studied while at the same time providing a unique opportunity to obtain an understanding of how interparticulate forces affect the bulk properties of granular materials. The effects of interparticulate friction, cohesion, arbitrary particle shapes and overall fabric of a system of particles as well as gravity flow of granular materials are being studied using a "particle-dynamics" model. The model calculates the motion of large numbers of (up to a few thousand) arbitrarily shaped two-dimensional polygonal blocks interacting with elastic and viscous normal forces, tangential surface friction, gravity and applied loads. Unlimited displacements and rotations of the particles are allowed and joints or contacts are automatically created or deleted as the motion progresses. The technique is similar to molecular dynamics models that have been used to determine macroscopic material properties from explicit calculation of the molecular motions.[1]

The construction of efficient data structures and searching algorithms to determine the near neighbors of each particle were a major part of the programming effort involved in developing the model.[2] Several simplifying assumptions are made in order to explicitly treat the complex motion of large systems of particles including:

- Quasi-rigid particles that retain their shape identity.
- Linear or nonlinear springs at points of contact.[3]
- Velocity dependent damping at contact points to simulate all plastic energy absorption phenomena
- Finite shear-displacement proportional to shear force before initiation of sliding.

Comparison with analytic solutions for simple systems have verified the model's ability to calculate collision dynamics, rotational motion, surface friction and gravity driven failure of multi-block configurations. Direct comparison with dynamic physical tests have verified the model's ability to predict the motion of macroscopic systems. Computer generated movies of rocking, sliding and toppling block systems closely follow the motion observed in physical tests of identical configurations. Calculation of gravitational flow of systems of irregular polygons (see Fig. 1) show formation of temporary arches followed by dynamic rupture and reformation of new arches. High speed movies of physical tests on similar configurations show qualitatively similar arching and dynamic breaking of arches.
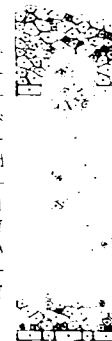
Figure 1

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1. B. J. Alder and T. E. Wainwright, *J. Chem. Phys.* Vol. 31, No. 2 (1959).
2. O. Walton, *Particle-Dynamics Modeling of Geological Materials.* LLNL Rept. UCRL-52915, 1980.
3. P. Cundall, *Rational Design of Tunnel Supports*, US Army Corps of Engrs., Tech. Rept. MRD-2-74, 1974.

MIXTURE THEORIES FOR MODELING THE DYNAMIC

RESPONSE OF COMPOSITE MATERIALS

by

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Abstract

Mixture theories developed in the last years by the authors are reviewed
and discussed. Some of the theories represent three-dimensional elasto-
dynamic phenomena by equivalent two-dimensional models and are applied to
investigate dynamic crack problems in laminated and fiber reinforced media.
Composites with constituents obeying a variety of constitutive laws are con-
sidered. These include laminated solids with linear isotropic elastic, linear
anisotropic elastic, nonlinear elastic, thermoelastic and elastic-viscoplastic
laminae. Debonding phenomena and large deformations are also treated. In all
of the formulated theories, the appearing interaction terms are determined and
related to the constituent material parameters. The proposed theories repre-
sent the dynamic response of the considered composites to a good degree of
accuracy and are governed by field equations which are suitable for solution
by proper numerical methods.

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Session FA-6:  AEROSOL DYNAMICS

Chairperson:  T.F. MORSE (Brown)

1:30 - 2:00    Y. TAMBOUR (Boston University)
               "Coagulation Breakup and Reflection Theory for A Population of
                Colliding Aerosol Particles - A Sectional Approach"

2:00 - 2:30    L. TRILLING and H.Y. WACHMAN (MIT)
               "Nucleation in a Rapidly Expanding Jet of Water Vapor"

2:30 - 3:00    T.F. MORSE (Brown) and J. CIPOLLA (Northeastern)
               "Some Fundamental Aspects of Aerosol Behavior"

3:00 - 3:30    REFRESHMENT BREAK

Coagulation Breakup and Reflection Theory for A Population of Colliding

Aerosol Particles - A Sectional Approach

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## Abstract

A new sectional representation for simulating the dynamics of a population
of colliding aerosol particles of arbitrary <u>velocity</u> and size distributions is
developed.  The particle population may be comprised of single molecules, dimers,
trimers...,pre-nucleation embryos and larger aerosol particles.  Binary particle
collisions are considered and processed as coagulation, breakup and reflection,
due to these collisions, are accounted for.  The approach, based on dividing the
particle velocity and size domains into arbitrary sections and dealing only with
one integral quantity in each section (e.g. number, surface area, mass or volume),
has the advantages that the sectional integral quantity is conserved and collisions
between particles of all sizes and velocities are properly accounted for.  Another
advantage of the present theory lies in the ability to simulate a whole particle
population by a relatively small number of conservation equations, which is simply
equal to the number of sections.

# NUCLEATION IN A RAPIDLY EXPANDING JET OF WATER VAPOR

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In a recent series of experiments,[1] we have examined the expansion of a jet of water vapor from stagnation pressures of 1 to 10 atm to chamber pressures of $10^{-11}$ atm at stagnation temperatures of 200°K to 500°K. One object was to make a simultaneous time of flight and mass spectrometry analysis of the velocity distribution of individual clusters of size 1 to 40 molecules (the range of our spectrometer).

We found that at the end of the (substantially adiabatic) expansion, the smaller clusters ($1 \leq i \leq 5$) were in translational equilibrium at a temperature of some 30°K to 60°K and the shape of their number distribution curve $n_i/n_1(i)$ was essentially independent of stagnation conditions.

On the other hand, the velocity distribution traces of the larger clusters ($5 \leq i \leq 30$) were all identical, indicating that their translational temperatures were proportional to cluster size. The number distribution curves $n_i/n_1(i)$ for these larger clusters were sensitive to stagnation conditions; in particular, as $p_{stag}$ increased, the number of any particular clusters in the size range ($10 \leq i \leq 30$) was nearly independent of cluster size.

We attempt to explain these observations by means of the following model. The smaller clusters are formed in the early part of the expansion as a result of triple collisions $A_i + A_1 + A_1 \rightarrow A_{i+1} + A_1$, the third particle being required to carry off excess energy of formation and to partly equilibrate $A_{i+1}$.

As these clusters grow, the energy of formation changed into kinetic energy by the capture of additional monomers can be distributed among a cloud of increasingly numerous loosely bound molecules, so that beyond a cluster size of (say) $i=4$, growth may also proceed by monomer capture in binary collision; by this time, the gas is very cold so that the monomers and clusters travel at substantially the same velocity in the same direction and the cluster $A_{i+1}$ simply "acquires" a monomer with negligible velocity change; its translational temperature is therefore proportional to its mass as indicated by experiments (Figs. 1,2). At the same time, its internal temperature increases as a result of the monomer capture; one would therefore not expect that this process can go on to make very large clusters; we have indications that it may change as the cluster size approaches 30 molecules.

The critical argument of the model is based on a comparison of the rate of formation of $(i+1)$ clusters from $(i)$ clusters and monomers by triple and binary collisions; beyond a size of order $5 < i < 10$ binary collisions leading to surviving clusters directed to the detector are more frequent than triple collisions and contribute the bulk of the signal.

1. Dreyfuss, D. D. (1980): Scattering of water cluster beams from surfaces. Ph.D. Thesis, M.I.T.

FIGURE 1. TRANSLATIONAL CLUSTER TEMPERATURE IN WATER VAPOR JET EXPANSION FROM $T_{stag} = 348°K$, $p_{stag} = 275$ torr



FIGURE 2: TIME-OF-FLIGHT TRACES OF TYPICAL WATER CLUSTERS
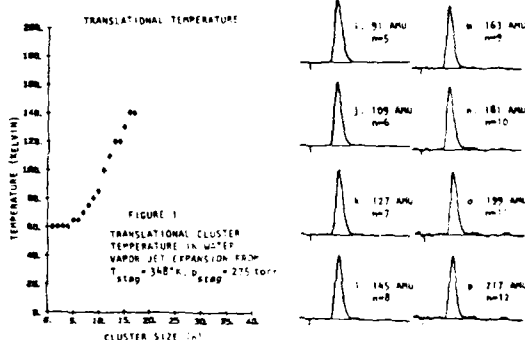
SOME FUNDAMENTAL ASPECTS OF AEROSOL BEHAVIOR

BY

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AND

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Abstract

The interaction of laser radiation with a strongly absorbing aerosol
in the Rayleigh limit is considered.  The gas itself is transparent to
the radiation, in this case 10 micron radiation from a carbon dioxide
laser, and the energy transferred to the gas is through the laser radia-
tion absorbed by the aerosol particles.  The particles are sub-micron in
size.  Consequently, for a sufficiently dense distribution of aerosol
particles, the gas becomes strongly heated.  Approximate calculations
indicate that this heating may be of the order of $10^{3}°K$.  Further, gradi-
ents in the laser intensity (through the absorptive heating of the aero-
sol) will manifest themselves as gradients in the gas itself.  This will
induce a subsequent thermophoretic motion in the aerosol particles, and
it is the study of the motion of these particles that forms the central
focus of our interest. We also note that this additional laser heating
may be of such an order as to enable one to investigate more conveniently
the properties of high temperature quartz and the interaction of a swarm
of liquid $SiO_2$ aerosol spheres.

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