A PARAMETER ESTIMATION ALGORITHM AND EXTENSIVE NUMERICAL SIMULATIONS FOR THE CAP MODEL

J. Ju, et al.
University of California
Department of Civil Engineering
Berkeley, CA  94720

30 November 1985

Technical Report

CONTRACT No. DNA 001-84-C-0304

Approved for public release; distribution is unlimited.

THIS WORK WAS SPONSORED BY THE DEFENSE NUCLEAR AGENCY
UNDER RDT&E RMSS CODE B341085466 Y99QMXSC00029,H2590D.

Prepared for
Director
DEFENSE NUCLEAR AGENCY
Washington, DC  20305-1000
DISTRIBUTION LIST UPDATE

This mailer is provided to enable DNA to maintain current distribution lists for reports. We would appreciate your providing the requested information.

☐ Add the individual listed to your distribution list.
☐ Delete the cited organization/individual.
☐ Change of address.

NAME: __________________________________________

ORGANIZATION: __________________________________

OLD ADDRESS

________________________________________________________________________

________________________________________________________________________

CURRENT ADDRESS

________________________________________________________________________

________________________________________________________________________

TELEPHONE NUMBER: (____)

SUBJECT AREA(s) OF INTEREST:

________________________________________________________________________

________________________________________________________________________

DNA OR OTHER GOVERNMENT CONTRACT NUMBER: __________________________

CERTIFICATION OF NEED-TO-KNOW BY GOVERNMENT SPONSOR (if other than DNA):

SPONSORING ORGANIZATION: __________________________________

CONTRACTING OFFICER OR REPRESENTATIVE: ____________________________

SIGNATURE: ________________________________________________________
A PARAMETER ESTIMATION ALGORITHM AND EXTENSIVE NUMERICAL SIMULATIONS FOR THE CAP MODEL

I. PERSONAL AUTHOR(S)
Ju, Jian-Wen; Simo, Juan C.; Pister, Karl S.; Taylor, Robert L.

II. SUPPLEMENTARY NOTATION
This work was sponsored by the Defense Nuclear Agency under RDT&E RMSS Code B341085466 Y99C.DLC00029 H2590D.

COSA CODES
Parameter Estimation; Colorado Concrete Data; Numerical Simulations; Plasticity; Cap Model

The inviscid two-invariant cap model is considered for geological materials such as concrete. A systematic constrained optimization procedure based on the Marquardt-Levenberg algorithm and the Armijo step-size rule is developed to determine values of the model parameters from available experimental data. The predictive capabilities of the cap model and the efficiency of the parameter estimation procedure are assessed through extensive numerical simulations based on well-documented experimental concrete data from the University of California.
PREFACE

This work was sponsored by the Defense Nuclear Agency under Contract No. DNA001-84-C-0304 with the University of California, Berkeley. This support and the interest and comments of Dr. Eugene Sevin are gratefully acknowledged.
TABLE OF CONTENTS

Section                                                                                   Page

Preface                                                                                   iii
List of Illustrations                                                                    v
1  Introduction                                                                            1
2  Basic Formulation of the Inviscid Cap Model                                             2
3  Parameter Estimation and Numerical Simulations                                         5
   3.1  Parameter Estimation, Marquardt-Levenberg Algorithm                               5
   3.2  Predictive Capabilities, Colorado Concrete Data                                    8
4  Closure                                                                                21
5  List of References                                                                      22

Appendices

A  Listings of Parameter Estimation Program                                                23
B  Example Input and Output for Appendix A                                                41
LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The yield surface for cap model</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Comparison of the experimental and simulated data for concrete test 1-2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Comparison of the experimental and simulated data for concrete test 1-3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Comparison of the experimental and simulated data for concrete test 2-2</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Comparison of the experimental and simulated data for concrete test 2-4</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Comparison of the experimental and simulated data for concrete test 3-5</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>Comparison of the experimental and simulated data for concrete test 3-17</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>Comparison of the experimental and simulated data for concrete test 4-7</td>
<td>17</td>
</tr>
<tr>
<td>9</td>
<td>Comparison of the experimental and simulated data for concrete test 4-12</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>Comparison of the experimental and simulated data for concrete test 5-1</td>
<td>19</td>
</tr>
<tr>
<td>11</td>
<td>Comparison of the experimental and simulated data for concrete test 5-2</td>
<td>20</td>
</tr>
</tbody>
</table>
SECTION I

INTRODUCTION

The inviscid, two-invariant associative cap model was originally proposed by DiMaggio and Sandler [1,2] and an algorithm to implement the model in stress analysis programs was proposed by Sandler and Rubin [3]. To assess the predictive capabilities of the inviscid cap model, the extensive and well-documented data obtained in the experimental program at the University of Colorado [4] has been selected. A characteristic of this experimental work is the exercise of truly three dimensional non-conventional stress paths. Due to the non-conventional nature of the experimental data, standard fitting procedures based on the use of conventional tests to independently fit the cap surface, failure envelope and hardening law (see e.g. [5,6]) cannot be used. Hence, to obtain values for the cap parameters, an alternative constrained optimization procedure which employs a modified Marquardt-Levenberg algorithm and Armijo step-size rule is developed. This approach makes the fitting process completely systematic and renders the optimal values of the parameters in a least square sense.

In the simulations reported herein, six (6) tests are used to fit the seven parameters of the cap model, and the resulting model is exercised to predict the remaining sixty-one (61) tests. The resulting numerical predictions agree remarkably well, both qualitatively and quantitatively, with the experimental results.
SECTION 2

BASIC FORMULATION OF THE INVISCID CAP MODEL

The two-invariant, rate-independent elastoplastic associative cap model is characterized by the following constitutive equations:

\[ \varepsilon = \varepsilon^e + \varepsilon^p \]
\[ \sigma = \hat{\sigma}(\varepsilon^e) \quad \text{(elastic response)} \]
\[ \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial \phi(\sigma, \kappa)}{\partial \sigma} \quad \text{(associative flow rule)} \]
\[ \phi(\sigma, \kappa) \leq 0 \quad \text{(yield condition)} \]

where \( \varepsilon, \varepsilon^e, \) and \( \varepsilon^p \) denote the total, elastic and plastic strain tensors; \( \sigma \) denotes the stress tensor and \( \phi(\sigma, \kappa) = 0 \) is the yield surface in stress space. In addition, \( \kappa \) is the hardening parameter which for the cap model is related to the plastic volume change by a hardening law as described below. Loading/unloading conditions may be expressed in a compact manner by requiring that

\[ \phi(\sigma, \kappa) \leq 0, \quad \dot{\lambda} \geq 0, \quad \dot{\lambda} \phi(\sigma, \kappa) \equiv 0 \]

This is the so-called Kuhn-Tucker form of unilateral constraint conditions. Note that if \( \phi < 0 \) then \( \dot{\lambda} = 0 \) and the process is elastic. On the other hand, for loading, \( \dot{\lambda} > 0 \) and \( \phi = 0 \). In this latter case, \( \dot{\lambda} \) is determined by requiring that \( \dot{\phi} = 0 \); the so-called consistency condition leads to the classical elastoplastic tangent modulus.

The basic characteristic of the cap model is the form of the yield function \( \phi(\sigma, \kappa) \) which is specified in terms of two functions \( F_1 \) and \( F_2 \). The function \( F_2 \) denotes the so-called failure envelope surface whereas the function \( F_1 \) is referred to as the hardening cap. Functional forms for \( F_1 \) and \( F_2 \) are (see Fig. 1)

\[ \phi(\sigma, \kappa) \equiv \begin{cases} \sqrt{J_{2n}} - F_1(J_1) \leq 0 & \text{(failure envelope)} \\ \sqrt{J_{2n}} - F_2(J_1, \kappa) \leq 0 & \text{(cap surface)} \end{cases} \]

where \( J_1 \equiv \text{tr} \sigma \cdot J_{2n} \equiv \frac{1}{2} s \cdot s \) (\( s \) : stress deviator) and

\[ F_1(J_1) \equiv \alpha - \gamma \exp(-\beta J_1) + \Theta J_1 \]
\[ F_2(J_1, \kappa) \equiv \frac{1}{R} \left[ \lambda \kappa - L(\kappa) \right]^2 - [J_1 - L(\kappa)]^2 \]
\[ L(\kappa) \equiv \langle \kappa \rangle = \begin{cases} \kappa & \text{if } \kappa > 0 \\ 0 & \text{if } \kappa \leq 0 \end{cases} \quad \text{(McInley bracket)} \]
Figure 1. The yield surface for cap model. $F_e$ and $F_c$ denote the failure envelope and the hardening cap surface, respectively. The shaded area is the "singular corner region".
Finally, the hardening parameter $\kappa$ is related to the plastic volume change $\varepsilon'_p \equiv tr \varepsilon'$ by the hardening law

$$
\varepsilon'_p(X) \equiv W \left( 1 - \exp \left[ -D X(\kappa) \right] \right)
$$

(5)

where $X(\kappa)$ is defined by

$$
X(\kappa) \equiv \kappa + R F_p(\kappa)
$$

(6)

In the above expressions, $\alpha, \beta, \gamma, \theta, W, D$, and $R$ are material parameters which characterize the two-invariant cap model considered here.
In order to assess the capability of the two-invariant cap model in predicting response behavior for actual materials such as concrete and geomaterials, model parameters need to be estimated from available experimental data. In this section, a parameter estimation procedure and an assessment of the predictive capability of the cap model are presented. This is followed by extensive numerical simulations for the Colorado concrete data.

3.1. PARAMETER ESTIMATION. MARQUARDT-LEVENBERG ALGORITHM.

It is characteristic of currently employed parameter estimation procedures for the cap model (see e.g. [5,6]) to fit separately the failure envelope, cap surface, and hardening law parameters. Typically, asymptotic failure points from TE, TC, SS, CTC, CTE, RTE, RTC and PL are used with a least-square fit procedure to estimate the failure parameters; whereas iso-plastic volumetric strain contours are employed to estimate the cap shape parameter $R$. The hardening law parameters $D$ and $H$ are determined from HC tests. Although this procedure provides a parameter fitting directly associated with the physical construction of the cap model, it has the following two major drawbacks: (a) a large amount (more than 20 tests) of conventional experimental data are required (e.g. CTC, CTE etc.), and (b) it is not possible to utilize some existing nonconventional experimental work: e.g., the results from the "Colorado" experimental program [4]. Hence, a more flexible and systematic parameter estimation procedure is needed. This is the objective of the following section.

**Optimization algorithm** The basic idea of the procedure advocated here is to regard the optimal fitting process for given experimental data as a least-square constrained optimization problem. In this context, the objective function $\Pi: \mathbb{R}^n \rightarrow \mathbb{R}$ is simply the sum-of-squares error function defined as

$$\Pi(\Psi) = \sum_{i=1}^{N} | |\sigma_f(\Psi, e_i) - \sigma_i^* | |^2$$

where

$N$: number of observations

---

$^+$ HL stands for triaxial extension, TC triaxial compression, SS simple shear, CTC conventional triaxial compression, CTE conventional triaxial extension, RTE reduced triaxial extension, RTC reduced triaxial compression, and PL proportional loading.

$^\dagger$ HC represents hydrostatic compression test.
**σ**: stress response from constitutive model considered

**σ^∗**: observed stress response

**Ψ**: parameter vector (in \( \mathbb{R}^7 \) for cap model)

**I**: \( I^{th} \) data point

In the following, this procedure will be illustrated using the cap model. It is, however, generally applicable to any constitutive model. The constraints imposed on the optimization problem emanate from physical restrictions placed on the cap parameters. For example, for a physically meaningful model one should have \( \alpha > 0 , \gamma > 0 , \alpha > \gamma , \theta > 0 , \beta > 0 , R > 0 , D > 0 , W > 0 \). These constraints define a feasible domain \( \Xi \subseteq \mathbb{R}^7 \), which is a convex polygon. The resulting constrained optimization problem is then expressed as

\[
\text{Find: } \min \Pi(\Psi) \text{ subject to } \Psi \epsilon \Xi
\]  

(8)

There exists a wide variety of algorithms for solving the standard convex optimization problem (8) (e.g., see [9] for a review). The algorithm employed here is the well-known Marquardt-Levenberg algorithm together with the Armijo step-size rule [7-10]. This algorithm is essentially a hybrid of Newton and steepest descent (gradient) methods. It combines the ability of the steepest descent method to converge from an initial guess, which may be outside the region of convergence of other methods, with the asymptotic quadratic convergence characteristics of Newton’s method near the solution. The Marquardt-Levenberg algorithm can be summarized in the following form:

\[
\Psi_{i+1} = \Psi_i + \lambda_i h_i
\]  

(9)

\[
h_i = -[H_i + \eta_i D_i]^{-1} \nabla_i \Pi
\]  

(10)

\[
H_i = 2 Q_i^T Q_i \quad \text{(approx. Hessian)}
\]  

(11)

\[
Q_i = \frac{\partial \Psi}{\partial \Psi^T} \quad \text{(sensitivity matrix)}
\]  

(12)

\[
\eta_i = \text{Marquardt parameter}
\]

\[
D_i = \text{diagonal matrix of } \Pi_i \text{ or simply } I
\]

(13)

For problems where \( Q_i \) may not be easily constructed analytically the derivatives are typically computed by means of forward differences. However, central differences provide greater
accuracy in the vicinity of the solution (minimum); thus, central rather than forward differences are employed in computing \( Q \), when the solution is closely approached.

In addition, to minimize the number of function evaluations (stress responses), a \textit{rank one} update to the sensitivity matrix is used periodically (similar to the Quasi-Newton method)

\[
Q_{i+1} = Q_i + \frac{1}{||\Delta \psi_i||^2} [\sigma(\psi_{i+1}) - \sigma(\psi_i) - Q_i \Delta \psi_i] \Delta \psi_i^T
\]

(14)

where \( \Delta \psi_i = \psi_{i+1} - \psi_i \). In Eq. (10), for a given value of \( \eta \), Cholesky factorization of \( H_i - \eta D_i \) is employed to check for positive definiteness. If the factorization breaks down, i.e. \( H_i - \eta D_i \) is not positive definite, then \( \eta \) is increased. The algorithm summarized above can be systematically applied to any set of experimental data to obtain the optimal fit for the constitutive model under consideration in a least square sense.

\textbf{Error measurement} During the optimization process, a root-mean-square (RMS) type of error measurement is adopted. The optimization process is considered to reach its optimum when the RMS measure is minimized. The relevant measures are defined as follows:

\[
\Delta_N = \left[ \frac{\sum_{i=1}^{N} \left( \sigma_i^* - \sigma_i \right)^2}{N} \right]^{\frac{1}{2}} \quad \text{(RMS of error)}
\]

(15)

\[
\Gamma_N = \left[ \sum_{i=1}^{N} \frac{\left( \sigma_i^* - \sigma_i \right)^2}{N} \right]^{\frac{1}{2}} \quad \text{(RMS of observed responses)}
\]

(16)

\[
\delta_N = \frac{\Delta_N}{\Gamma_N} \quad \text{(normalized relative RMS error)}
\]

(17)

\textbf{Remark 3.1.} It is interesting to examine the sensitivity of the response under perturbations in cap model parameters. A finite difference sensitivity matrix \( Q \) is defined in dimensionless form:

\[
Q_{ij} = \frac{\Delta \sigma_i / \sigma_i}{\Delta \psi_j / \psi_j}
\]

(18)

where \( \sigma_i \) is a stress component (\( i = 1,...,6 \)) and \( \psi_j \) is a parameter component (\( j = 1,...,7 \)), respectively. A standard sensitivity analysis reveals that the response of the cap model is relatively insensitive to changes in the model parameters. By ordering the model parameters according to relative sensitivity in the response, one obtains in decreasing order of sensitivity:

\( W \rightarrow D \rightarrow R \rightarrow \alpha \rightarrow \Theta \rightarrow \gamma \rightarrow \beta \)

(19)

In summary, one obtains the following relative degree of sensitivity (from large to small):

- \textit{hardening parameters} \( \rightarrow \) \textit{cap parameters} \( \rightarrow \) \textit{failure parameters}
3.2. PREDICTIVE CAPABILITIES. "COLORADO" CONCRETE DATA.

In this section, we first examine the consistency of the "Colorado concrete" data [4], next we estimate the model parameters by exercising the procedure described above, finally we assess the predictive capability of the inviscid cap model.

Colorado concrete data. This experimental program on concrete was performed at the University of Colorado (1983) and is well-documented [4]. The program consists of six major series of nonconventional multiaxial stress-strain curves. The total number of experiments is 67. The data are characterized by the following properties: (a) characteristic uniaxial compressive strength \( f_c \approx 4 \) ksi, (b) mean pressure \( \leq 8 \) ksi (c) truly triaxial states of stress for concrete, (d) nonconventional complicated stress paths, and (e) quasi-static loading.

The six major series of tests consist of the following:

1. A series of 12 cyclic triaxial tests, consisting of cyclic hydrostatic preloading to various stress levels, followed by proportional deviatoric stress cycles without reversal along triaxial compression, simple shear, and triaxial extension paths.

2. A series of 8 cyclic triaxial tests, consisting of cyclic hydrostatic preloading to various stress levels, followed by proportional deviatoric stress cycles with reversal along the same deviatoric paths as in Series 1.

3. A series of 17 tests consisting of hydrostatic loading, followed by proportional stress deviation, followed by a circular stress path within the deviatoric plane.

4. A series of 22 axisymmetric triaxial tests to explore load path effects. In addition to proportional and hydrostatic-deviatoric paths, this series contained staircase-type loadings to explore convergence to the proportional path, tests with hydrostatic stress increments with and without hydrostatic preloading, and tests under non-proportional loadings.

5. A series of 6 tests within the deviatoric plane, as well as a number of other tests specifically designed to check the meaning of loading and unloading.

6. A series of 2 tests of piecewise-uniaxial loadings.

Assessment of data consistency. Basically, the measures employed here are the same as those discussed in the previous section. For convenience, these measures are summarized as follows:

\[
\Delta_N = \left[ \frac{\sum_{i=1}^{N} ||\Delta e_i||^2}{N} \right]^{\frac{1}{2}} \quad \text{(see(15))} \tag{20}
\]

\[
\Gamma_N = \left[ \frac{\sum_{i=1}^{N} ||f_i||^2}{N} \right]^{\frac{1}{2}} \quad \text{(see(16))} \tag{21}
\]

\[
\delta_\lambda = \frac{\Delta_N}{\Gamma_N} \quad \text{(see(17))} \tag{22}
\]
Here $\varepsilon'$ refers to a strain measurement of test 'A'. An assessment of consistency for the "Colorado" concrete data may be obtained from the replicates of experiments available in the reported results [4]. The present analysis generally indicates reasonable consistency of the data. However, some serious discrepancies between replicates are also observed. See Table 1 below.

Table 1. Consistency of the Colorado concrete data [4].

<table>
<thead>
<tr>
<th>Tests</th>
<th>$\delta$ %</th>
<th>Major Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1 &amp; 1-10</td>
<td>13.5</td>
<td>CTC</td>
</tr>
<tr>
<td>1-4 &amp; 1-7</td>
<td>31.1</td>
<td>TC</td>
</tr>
<tr>
<td>1-6 &amp; 1-9</td>
<td>51.3</td>
<td>TE</td>
</tr>
<tr>
<td>2-3 &amp; 2-4</td>
<td>9.6</td>
<td>SS</td>
</tr>
<tr>
<td>2-7 &amp; 2-8</td>
<td>13.5</td>
<td>SS</td>
</tr>
<tr>
<td>3-1 &amp; 3-2</td>
<td>244.3</td>
<td>Circular</td>
</tr>
<tr>
<td>3-3 &amp; 3-4</td>
<td>47.2</td>
<td>Circular</td>
</tr>
<tr>
<td>3-10 &amp; 3-11</td>
<td>92.9</td>
<td>Circular</td>
</tr>
<tr>
<td>4-1 &amp; 4-2</td>
<td>10.9</td>
<td>Axisymmetric</td>
</tr>
<tr>
<td>4-6 &amp; 4-7</td>
<td>54.2</td>
<td>Axisymmetric</td>
</tr>
</tbody>
</table>

Model parameter estimation procedure. The actual data employed in the optimization process based on the Marquardt-Levenberg algorithm are obtained by arbitrarily selecting one test out of each of the six major series. Thus, a total number of 6 tests is used in the actual fit of the model. The quality of the fitting is satisfactory. Typical values of the RMS error found from back-prediction using the optimal material properties are: $\delta = 16\%$ for test 1-1 (CTC), $\delta = 8.5\%$ for test 2-3 (SS), $\delta = 26\%$ for test 3-11 (circular), $\delta = 11.5\%$ for test 4-11 (axisym.), etc. From this optimization procedure, we obtain the following set of parameters which best fits the observed experiments: $a = 3.86 ksi$, $\alpha = .11$, $\gamma = 1.16 ksi$, $\beta = .44 ksi^{-1}$, $R = 4.43$, $D = .0032 ksi^{-1}$, $W = .42$, $X_0 = 16 ksi$.

Predictive capability. After the optimal model parameters are obtained, the resulting cap model is used to predict the response of every other Colorado test which is not included in the optimization process (total number = 61). It is emphasized that the "prediction" here has nothing to do with optimal fitting, but is obtained by exercising the cap model using previously estimated parameters. In general, considering the experimental data scatter, the predicted response is in good agreement with the experimental results. It is noted that the overall qualitative behavior for the Colorado concrete data is captured. Values of the RMS error corresponding to a selected sample of simulations are summarized in Table 2 below.
The overall RMS and standard deviation of error for 61 tests are 26.6% and 14%, respectively. A comparison between experimental and predicted stress-strain curves is contained in Figures 2-11.

Table 2. Results of prediction. Inviscid case

<table>
<thead>
<tr>
<th>Tests</th>
<th>δ %</th>
<th>Major Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>12.4</td>
<td>SS</td>
</tr>
<tr>
<td>1-3</td>
<td>14.1</td>
<td>TE</td>
</tr>
<tr>
<td>2-2</td>
<td>17.</td>
<td>TE</td>
</tr>
<tr>
<td>2-4</td>
<td>11.7</td>
<td>SS</td>
</tr>
<tr>
<td>3-5</td>
<td>15.</td>
<td>Circular</td>
</tr>
<tr>
<td>3-17</td>
<td>11.6</td>
<td>Circular</td>
</tr>
<tr>
<td>4-7</td>
<td>14.</td>
<td>Axisymmetric</td>
</tr>
<tr>
<td>4-12</td>
<td>11.4</td>
<td>Axisymmetric</td>
</tr>
<tr>
<td>5-1</td>
<td>14.</td>
<td>Unsymmetric</td>
</tr>
<tr>
<td>5-2</td>
<td>17.</td>
<td>Unsymmetric</td>
</tr>
</tbody>
</table>

Assessment and evaluation. From the above fitting and prediction exercises, it may be concluded that the inviscid cap model generally exhibits good fitting and predictive capabilities for the Colorado concrete data. The simulations reported herein capture the overall qualitative behavior of the experimental response.
Figure 2. Comparison of the experimental and simulated data for concrete test 1-2. This is a cyclic simple shear test. The vertical axis is the major principal stress and the horizontal axis is one of the three principal strains. "EY" (solid-line) and "SY" (dash-line) represent the experimental and the simulated response in Y-direction, respectively. The diamond symbols signify the data points along "SY", in which "E" stands for the elastic mode, "C" for the cap mode and "T" for the tension cutoff mode. The r.m.s. error measure \( \delta \) = 12.4%.
Figure 3. Comparison of the experimental and simulated data for concrete test 1-3. This is a cyclic triaxial extension test. The vertical axis is the major principal stress and the horizontal axis is one of the three principal strains. "EX" and "SX" represent the experimental and the simulated response in X-direction, respectively. The r.m.s. error measure δ = 1.41%.
Figure 4. Comparison of the experimental and simulated data for concrete test 2-2. This is a cyclic triaxial extension test with stress reversal about the hydrostatic axis. "EZ" and "SZ" represent the experimental and the simulated response in Z-direction, respectively. The r.m.s. error measure $\delta = 17\%$. 
Figure 5. Comparison of the experimental and simulated data for concrete test 2-4. This is a cyclic simple shear test with stress reversal with respect to the hydrostatic axis. The r.m.s. error measure $\delta = 11.7\%$. 
Figure 6. Comparison of the experimental and simulated data for concrete test 3-5. This is a circular stress path on the 12 KSI octahedral plane. The r.m.s. error measure $\delta = 15\%$. 
Figure 7. Comparison of the experimental and simulated data for concrete test 3-17. This is a proportional loading path followed by cyclic circular stress path on two orthogonal planes, and finally followed by another proportional loading path. The r.m.s. error measure $\delta = 11.6\%$. 

Comparison of Experimental and Simulated Data for Concrete Test 3-17.
Figure 8. Comparison of the experimental and simulated data for concrete test 4-7. This is a cyclic axisymmetric triaxial compression test. The r.m.s. error measure $\delta = 14\%$. 
Figure 9. Comparison of the experimental and simulated data for concrete test 4-12. This is another cyclic axisymmetric triaxial test. The r.m.s. error measure $\delta = 11.4\%$. 
Figure 10. Comparison of the experimental and simulated data for concrete test 5-1. This is an unsymmetric triaxial test. The r.m.s. error measure $\delta = 14\%$. 
Figure 11. Comparison of the experimental and simulated data for concrete test 5-2. This is an unsymmetric triaxial test. The r.m.s. error measure $\delta = 17\%$. 
SECTION 4
CLOSURE

A systematic estimation procedure for the parameters involved in the cap model to given experimental data has been developed, based on a modified Marquardt-Levenberg optimization algorithm. This procedure has been applied to the extensive experimental program carried out at the University of Colorado and reported in [4]. It is emphasized that due to the nonconventional character of this experimental data, standard fitting procedures (e.g., Desai [5,6]) based on conventional tests can not be employed. The numerical simulations performed on the basis of these data support the good predictive capabilities of the cap model for concrete materials.
SECTION 5

LIST OF REFERENCES


APPENDIX A
LISTING OF PARAMETER ESTIMATION PROGRAM

```fortran
program fit

C.... Program for parameter fitting from experimental test data
for the cap model
implicit double precision(a-h,o-z)
common/fix/bulkm,shearm,zm
common/aa/n(6),mm
common/prop/Itype,tcut,fcut
common ystar(500,6),w(500,6)
dimension f(3000),q(21000),work(6063)

C.... Specify elastic material parameters and initial cap (z) parameter:
read(5,*) bulkm,shearm,zm
write(6,2004) bulkm,shearm,zm

C.... Input size of experimental steps
read(5,*) (n(j),j=1,6)
mm=n(1)+n(2)+n(3)+n(4)+n(5)+n(6)
do 10 j=1,6
if(n(j).gt.500) stop 5 -
10 continue

C.... Input Experimental Data for y (sig-33)
do 20 j=1,6
read(5,1000) (ystar(lj),l=1,n(j))
20 continue

C.... Call Optimization Algorithm
C Modified Levenberg-Marquardt Algorithm.
call opt(q,f,work)
stop

C Format Statement
format(8f10.0)
2004 format(//
  * t5,'BULK MODULUS'    = ',d14.6/
  * t5,'SHEAR MODULUS' = ',d14.6/
  * t5,'INITIAL CAP ( Z )' = ',d14.6/
end
```
C subroutine opt(f,q,work)

C Program to calc. the optimal values for cap model
C by the Modified Levenberg-Marquardt algorithm.
C The optimization criterion is with respect to the
C least square norm.
implicit double precision(a-h,o-z)
external func
common/fix/bulkm,shearm,zm
common/prop/ltype,tcut,fcut
common/aa/n(6),mm
common/bb/indi
common/ha/old(7)
common ystar(500,6),w(500,6),y(500,6)
dimension parm(4),para(7),f(1),q(mm,1),g(28),work(1)
dimension sosss(6),sosorr(6),rmsso(6),rmsyy(6),relI(6)
dimension eigval(7),eigvec(7,7),wk(7)
C Parameters for IMSL — ZXSSQ
ixjac-mm
read(5,*) nsig,eps,delta,maxfn,iopt
if(iopt.eq.2) read(5,*) (parm(l),l=1,4)
C Initial guess for parameters alpha w
read(5,*) (para(j),j=1,7)
write(6,2000) (para(j),j=1,7)
C Input weighting matrix W
read(5,*) iflag
if(iflag.ne.1) then
  do 31 j=1,6
    do 30 k=1,n(j)
      30 w(k,j)=1.
    31 continue
  else
    do 32 j=1,6
      read(5,1000) (w(k,j),k=1,n(j))
    32 continue
endif
C Call optimization package ZXSSQ
indi=1
C (Save the old parameters)
  do 40 k=1,7
    40 old(k)=para(k)
c
  call zxssq(func.mm,7,nsig,eps,delta,maxfn,iopt,
* parm,para,ssq.f,q,ixjac,g,work,infer,ier)
c Print out output data
  write(6,2001) (para(k),k=1,7)
c SOS : sum of squares of residuals
C SOSOR : sum of squares of observed responses (ystar)
sos=0.
sosr=0.
do 50 i=1,6
  sosri=0.
  50 sossi=0.

24
do 200 j = 1, 6
  do 100 k = 1, n(j)
    diff = y(k, j) - ystar(k, j)
  sos(j) = sos(j) + diff**2
  sosorr(j) = sosorr(j) + ystar(k, j)**2
  sos = sos + sos(j)
  sosor = sosor + sosorr(j)
  100 continue
  sos = sos + sos(j)
  sosor = sosor + sosorr(j)
  200 continue

Overall estimators
	rmsos = sqrt(sos/mm)
	rmsy = sqrt(sosor/mm)

rel = rmsos/rmsy

Test-j estimators

do 210 j = 1, 6
  rmsso(j) = sqrt(soss6j)/n6j)
  rmsyy6j) = sqrt(sosorr(j)/n6j))
  rerr(j) = rmsso(j)/rmsyy(j)
  210 continue

write(6, 2007) sos
write(6, 2004) rmsnos
write(6, 2005) rel

m1 = n(1) + n(2)
  m2 = m1 + n(3)
  m3 = m2 + n(4)
  m4 = m3 + n(5)

c.... Testj

do 220 j = 1, 6
  write(6, 2007) sossU)
  write(6, 2004) rmsso(j)
  write(6, 2005) rrelloj)
  220 continue

c.... Compute the condition number for G = = Q sup T * Q = = 1/2 H

c  Cond(G) = (max lambda) / (min lambda)
c  call eigrs(g, 7.0, eigval, eigvec, 7, wk, ierr)
c  if (eigval(1).eq.0.0d0) stop 'zero eigval'
c  cond = eigval(7)/eigval(1)
c  write(6, 31000) cond
  return

1000 format(8F10.0)
2000 format//
  * 20A,'THE INITIAL GUESS FOR PARAMETERS'//
  * 10X,'ALPHA'.7X,'THETA'.8X,'GAMA'.8X,'BETA'.
  * 11X,'R'.11X,'D'.11X,'W'/
  * /5X,7F12.6)
2001 format//
  * 20A,'THE OPTIMAL VALUES OF PARAMETERS ARE'//
  * 10X,'ALPHA'.7X,'THETA'.8X,'GAMA'.8X,'BETA'.
  * 11X,'R'.11X,'D'.11X,'W'/
  * /5X,7F12.6)
subroutine func(para,in,ip,f)

Function evaluation (stress response) and residual computation.

implicit double precision(a-h,o-z)

common/fix/bulkm,shearm,zm
common/prop/itype,tcut,fcut
common/aa/n(6),mm
common/bb/indi

c.... 500: max. no. of data pts in each test
6 6 strain components
6 6 tests
common/ab/del(500,6,6)
common/ha/old(7)
common ystar(500,6),w(500,6),y(500,6)
dimension para(1),r(1),delp(7),ytemp(500),deltem(500,6)

c.... Preserve total increments
do 10 i=1,7
10 delp(i)=para(i)-old(i)

c.... Check if constraints are violated:
para(1) > 0 required
if (para(1).le.0.d0) then
  go to 30
para(3) > 0 required
elseif (para(3).le.0.d0) then
  go to 30
para(3) < para(1) required
elseif (para(3).gt.para(1)) then
  go to 30
para(3) > 0.1 * para(1) preferred
elseif (para(3).lt.0.1*para(1)) then
  go to 30
para(2) > 0 required
elseif (para(2).lt.0.d0) then
  go to 30
para(4) >= 0.21 preferred
elseif (para(4).lt.0.21d0) then
  go to 30
para(4) <= 2 preferred
elseif (para(4).gt.2.0d0) then
  go to 30
para(5) >= 1.6 preferred
elseif (para(5).lt.1.6d0) then
  go to 30
para(6) and para(7) > 0 required
elseif (para(6).le.0.d0 or para(7).le.0.d0) then
  go to 30
else
  c  if O.K.
    go to 50
  endif

  c.... Half the increments for parameters if constraints are violated.
  30  do 40 i = 1,7
    delp(i) = delp(i)/2.
  40  para(i) = old(i) + delp(i)
    go to 20

  c.... Update the old parameters
  50  do 60 i = 1,7
    old(i) = para(i)
  60  c
    do 70 j = 1,6
      if (indi.eq.1) go to 63
      do 62 k = 1,n(j)
        do 61 kk = 1,6
          deltem(k,kk) = del(k,kk,j)
        61  continue
        62  continue
      63  call main(ytemp,n(j),para,deltem,j,indi)
        do 65 k = 1,n(j)
          y(k,j) = ytemp(k)
        65  ytemp(k) = 0.0
        do 64 kk = 1,6
          del(k,kk,j) = deltem(k,kk)
        64  deltem(k,kk) = 0.0
        65  continue
        65  continue
      70  continue
      indi = indi + 1
      c
      kn = 0
      do 200 j = 1,6
        do 100 i = 1,n(j)
          k = kn + i
          f(k) = (ystar(i,j) - y(i,j))*sqrt(w(i,j))
        100  continue
        kn = kn + n(j)
      200  continue
    return
end
 subroutine main(y,n,para,del,ino,ind)
  implicit double precision(a-h,o-z)
  common/state/sig0(6)
  common/sta/geop,xint
  Y : the response vector
  PARA : the parameter vector
  dimension del(500,6),sig(6)
  dimension y(1),para(1)
  common/fix/bulkm,shearm,zm
  Material parameters :
  common/prop/ltype,tcut,fcut
  common/elas/bulk,shear
  common/parl/alpha,theta,gama,beta,r
  common/par2/d,w,z
  Definition for parameters ( just for convenience ).
  bulk=bulkm
  shear=shearm
  alpha=para(1)
  theta=para(2)
  gama=para(3)
  beta=para(4)
  r=para(5)
  d=para(6)
  w=para(7)
  z=zm
  IND : flag, if ind=1 , read strain increment data
  INO : identifier for test # ino ( 1-6 ).
    if(ind.ne.1) go to 200
    if(ino.ne.1) go to 50
  Read common input data: ltype,tcut,sig0,geop,xint
  Read material type and tension cutoff criterion
  TCUT is in terms of live stresses.
  read(5,*) ltype,tcut
  Input the initial states of stress and strain
  read(5,*) (sig0(k),k=1,6)
  Input the geostatic pressure and XINT( the initial cap)
  read(5,*) geop,xint
  Strain controlled CAP model
  Read input data del and initial strain.
    do 100 i=1,n
      read(5,*) (del(i,k),k=1,6)
  Call preprocessor INITEL to calculate elint from
  given xint and fcut(in total stress)
  XINT := the initial X value for the initial cap.
  Z := the X value for the characteristic initial cap.
  i.e. the X value when EVP = 0.
  200 if(ino.ne.1) go to 210
    call initel(xint,elint,nocon1,nocon2)
  Assign initial state of stress and strain accordingly
210  continue
    do 220 k = 1, 6
        sig(k) = sig0(k)
    220  continue
    el = elint

    c.... Call 3-D strain history driver
    call drv3D(n, y, del, el, sig)
    return
    end
subroutine drv3D(n,y,del,el,sig)

This routine is a 3-D strain history driver and the variable increments are deps stored in array del.

implicit double precision(a-h,o-z)
common/sta/geop
dimension del(500,6),sig(1),deps(6),y(1)
common/prop/ltype,tcut,fcut
common/elas/bulk,shear
common/par1/alpha,theta,gama,beta,r
common/par2/d,w,z

do 200 i=1,n
do 100 k=1,6
deps(k)=del(i,k)
continue
call cap(sig,deps,geop,el,mtype,it,mocon,sj1,sj2,x1,evpi
,ej1,ej2d,f1ej1)
y(i)=sig(3)
200 continue
return
end
subroutine cap(sig, deps, geom, mtype, it, nocon, sj1, sj2, xl, evpi, ej1, ej2, f1, f1j)

For full three-dimensional stresses and strains computations by using the CAP model

Strain controlled algorithm.

Stresses and strains are sig and eps, respectively.

geop = geostatic pressure (overburden stress)

el = hardening parameter

type: 0 = tension cutoff, 1 = elastic, 2 = failure,

3 = cap mode, 4 = cone mode

nocon = 1 indicates no convergence under max iterations

(nit) restriction. Otherwise = 0

eps = error tolerance parameter

ltype: 1 = soil, 2 = rock

implicit double precision(a-h,o-z)

common/prop/ltype,tcut,fcut

common/elas/bulk,shear

common/parl/alpha,theta,gama,beta,r

common/par2/d,w,z

dimension sig(1),deps(l),s(6),de(6)

data eps/l.d-6/

Statement function for exponential with negatively large

argument for large caps

exps(z)=dexp(dmax1(-500.,z))

Failure envelope function for sj2

f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
d1(sj1)=theta+ gamabetaexps(-beta*sj1)

Cap statement functions for f2 functional forms

capi(t) = intersection point of f1 & f2,

x(k) = intersection of f2 & j1 axis

capi(el)=dmax1(0.0,el)

ra(capi)=r

ex(capi)=dmax1(0.,el+ra(capi))*f1(el))
evp(xl)=w*(1.0-exps(d*(z-xl)))/ra(capi)
f2(sj1, xl, capi)=dsqrt((xl-capi)^2-(sj1-capi)^2)**2/(ra(capi))

Elastic moduli functions

bmod(sj1, ev) = bulk

smod(sj2, ev) = shear

it=0

nocon=0
dev=deps(1)+deps(2)+deps(3)
devb3=dev/3.0

do 1 k=1,3

de(k)=deps(k)-devb3

do 2 k=4,6

de(k)=deps(k)

press=(sig(1)+sig(2)+sig(3))/3.0

do 3 k=1,3

s(k)=sig(k)-press
do 4 k=4,6
4   s(k)=sig(k)
sj1t=3.*(press+geop)
temp=0.
do 11 k=1,3
11  temp=temp+0.5*s(k)*s(k)
do 12 k=4,6
12  temp=temp+s(k)*s(k)
sj2l=dsqrt(temp)
capi=capi(el)
xl=x(el)
evpi=evp(xl)
c.... Elastic material properties
   threek=3.*bmod(sj1t,evpi)
g=smod(sj2l,evpi)
twog=2.*g
c.... Elastic trial
   sj1=threek*dev+sj1t
do 13 k=1,6
13  s(k)=s(k)+twog*de(k)
ratio=1.0
   mtype=1
c.... Tension limit test
   tencut=dmax1(fcut,tcut+3.*geop)
   if(sj1.gt.tencut) go to 10
   sj1=tencut
   ratio=0.0
   sj2=0.
   mtype=0
c If no contraction
   if(type.eq.2.or.e1.le.0.0d0) go to 200
c.... Tension dilatancy coding for soils with cl.ge.0.0
   if(cl.le.0.0d0) go to 200
c.... Dilatancy controlled by contracting cap up to el.ge.0.0
   ell=dmin1(0.0,el-eps*fl(el))
xll=x(ell)
denom=evp(xll)-evpi
   if(denom.lt.0.0d0) go to 5
   el=0.0
go to 200
5   devp=dev-(sj1-sj1t)/threek
   denom=dmin1(denom,devp)
el=el+devp*(ell-el)/denom
   el=dmax1(0.0,el)
go to 200
c.... Check if failure envelope mode is invoked
10  continue
   temp=0.
do 14 k=1,3
14  temp=temp+0.5*s(k)*s(k)
do 15 k=4,6
15  temp=temp+s(k)*s(k)
c Calc. J2E
   sj2=dsqrt(temp)
sj2e=sj2
If cap mode
  if(sj1.gt.capi) go to 40
  ej2d=sj2

TMISES is the sj2 value at the corner point(tmises>=fj1)
  tmises=f2(capi,xl,capi)
  ej1=sj1
  fj1=f1(sj1)
  fj1ej1=fj1
  fe=sj2-dmin1(fj1,tmises)

If elastic
  if(fe.le.0.0d0) go to 200

If k0<0 (small cap), no contraction allowed.

k=k0 and J1=J1E (von Mises transition)
  if (el.it.0.0d0) then
    mtype=2
    go to 30
  endif
  ...

Failure envelope surface calculation (f1)
  mtype=2
  elold=el

Iterate to find new k & J1...
call proj(deps,el,sj1,sj2,con,threek,g)

Consistency check for cap model:
  if(ltype.eq.2.or.elold.eq.0.d0) el=elold
  if(sj1.gt.el) el=sj1
  if(ltype.eq.1.and.elold.gt.0.0d0) then
    if(dabs(el-sj1).le.1.d-6) mtype=4
    el = max(el,0.0d0)
  endif
  el = max(el,0.0d0)
  fj1=f1(sj1)
  sj2=dmin1(fj1,tmises)
  ratio=sj2/sj2e
  go to 200

CAP mode calculation
  if(sj1.gt.xl) go to 50
  ...

If elastic
  if(sj2.le.f2(sj1,xl,capi)) go to 200
  mtype=3
  call proj(deps,el,sj1,sj2,con,threek,g)
  ratio=0.0
  if(sj2e.ne.0.0d0) ratio=sj2/sj2e
  continue

Update dev. stresses...
do 300 k=1.6
  s(k)=s(k)*ratio
  press=sj1/3.-geop

Calc. live stresses
  do 400 k=1.3
400 sig(k)=s(k)+press
   do 410 k=4,6
410 sig(k)=s(k)
c.... calc. X and vol. plastic strain.
   xl=x(el)
   evpi=evp(xl)
   return
end

subroutine proj(deps,el,sj1,mtype,sj2,nocon,it,threek,g)
  c
  ********************************
  c.... Subprogram to calc. the k and J1 iteratively by modified
  c  Regula Falsi Secant Method.
  implicit double precision(a-h,o-z)
  common/prop/ltype,tcut,fcut
  common/elas/bulk,shear
  common/param/alpha,theta,gama,beta,r
  common/param2/d,w,z
  dimension deps(1)
  data nit/600/
  data eps/1.d-6/
  c.... Statement function for exponential with negatively large
  c.... argument for large caps
  exps(z)=dexp(dmax1(-500.,z))
  c.... Failure envelope function for sj2
  f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
  d1(sj1)=theta+gama*beta*exps(-beta*sj1)
  c.... Cap statement functions for f2 functional forms
  c.... capl(l(k) : intersection point of f1 & f2,
  c.... x(k) : intersection of f2 & j1 axis
  capl(el)=dmax1(0.,el)
  ra(capi)=r
  x(el)=dmax1(0.,el+ra(capl(el))*f1(el))
  evp(xl)=w*(1.0-exps(d*(z-xl)))
  f2(sj1,xl,capi)=dsqrt((xl-capi)**2-(sj1-capi)**2)
  * (sj1-capi)**2)
  c.... Elastic moduli functions
  bmod(sj1,ev)=bulk
  smod(sj2,ev)=shear
  ********************************
  c
  nocon=0
  it=0
  sj1e=sj1
  sj2e=sj2
  xl=x(el)
  evpi=evp(xl)
  c.... Convergence criterion
  conv=eps*0.1
  c.... Failure mode
  c  Initial guess
  if (mtype.eq.2) then
ell = sjle
elr = el
else
go to 45
endif
c.... tcut > -1
xll = x(ell)
devpl = evp(xll)-evpi
sji1 = sj1e-threek*devpl
ql = -(sji1+1.)/(sj1e+1.)
c
xlr = xl
devpr = evp(xlr)-evpi
sji1 = sj1e-threek*devpr
qr = (xlr-sji1)/(xlr-jile)
go to 47
c.... Cap mode
45 ell = el
elr = sjle
if(sj1e.ge.xl) ql = (el-sj1e)/(el-xl)
if(sj1e.lt.xl) ql = 2.*sj2e/(sj2e+f2(sjle, xl, capi))-1.0
xr = x(elr)
sji1 = sj1e-threek*(evp(xr)-evpi)
qr = (xr-sji1)/(elr-xr)
47 qold = 0.0
c.... Modified Regula Falsi Method
do 80 it = 1, nit
c Secant method
el = (qr*ell-ql*elr)/(qr-ql)
xl = x(el)
devp = evp(xl)-evpi
sji = sjle-threek*devp
capi = capl(el)
if(mtype.eq.3) go to 48
c If Failure mode
c el >= -1
if(sj1.gt.el) qc = -(sj1+1.)/(el+1.)
if(sj1.le.sj1e) qc = (xl-sj1)/(xl-sj1e)
if(sj1.gt.el.or.sj1.le.sj1e) go to 60
sji2 = f1(sj1)
go to 49
c If Cap mode
48 continue
if(sj1.ge.xl) qc = (el-sj1)/(el-xl)
if(sj1.le.capi) qc = (xl-sj1)/(capi-xl)
if(sj1.ge.xl.or.sj1.le.capi) go to 60
sji2 = f2(sj1, xl, capi)
49 if (mtype.eq.2) then
c**** If cone mode (at corner pt.) inside failure mode********
if(dabs(el-sj1).le.1.d-6) then
c.... Correct treatment
slope = (sj1-sj1e)/(sj2e-sj2)*g/(3.*threek)
desp = devp/(3.*slope)
else
35
desp = devp/(3.*d1(sj))
endif
else
desp = devp/(3.*d2(sj1,xl,capi))
endif
a = sj2-g*desp
error = sj2e-a
qc = error/(sj2e+a)

Convergence criteria
if(dabs(error).le.conv) go to 90

if(qc.gt.0.0d0) go to 70
if (mtype.eq.3) then
  c k too large
  e1r = el
  qr = qc
  if(qcold.lt.0.0d0) ql = 0.5*ql
  else
  c k too small
  e1l = el
  ql = qc
  if(qcold.lt.0.0d0) qr = 0.5*qr
endif
go to 80
endif

if (mtype.eq.3) then
  c k too small
  e1l = el
  ql = qc
  if(qcold.gt.0.0d0) qr = 0.5*qr
  else
  c k too large
  e1r = el
  qr = qc
  if(qcold.gt.0.0d0) ql = 0.5*ql
endif
70 go to 80

if (mtype.eq.3) then
  c k too small
  e1l = el
  ql = qc
  if(qcold.gt.0.0d0) qr = 0.5*qr
  else
  c k too large
  e1r = el
  qr = qc
  if(qcold.gt.0.0d0) ql = 0.5*ql
endif
80 qcold = qc

If no convergence within NIT iterations:
  nocon1 = 1
  If cap mode:
    if (mtype.eq.3) then
      sj1 = dmin1(sj1,xl)
      if(sj1.lt.capi) sj1 = capi
      sj2 = dmin1(sj2e,f2(sj1,xl,capi))
    else
      sj1 = dmin1(sj1,el)
    endif
  endif
89 continue
90 return
end

subroutine initel(xint,elint,nocon1,nocon2)
c                          ***********************
c.... This routine uses secant method to find initial
c.... value of el(hardening parameter) for a given

c.... Also, it solves FCUT, the intersection of Fl and

c.... J1-axis.
implicit double precision(a-h,o-z)
common/prop/ltpe,tcut,fcut
common/elas/bulk,shear
common/par1/alpha,theta,gama,beta.r
common/par2/d,w,z
data eps,nit/1.d-6,60/
c.... Statement function for exponential with negatively
large argument for large caps
exps(z)=dexp(dmax1(-500.,z))
c.... Failure envelope function for sj2
f1(sj1)=alpha-gama*exps(-beta*sj1)+theta*sj1
c.... Cap statement functions
  capl(el)=dmax1(0.0,el)
  ra(capl)=r
  x(el)=el+ra(capl))*f1(el)
c.... Elastic modulus function
  bmod(sj1,el)=bulk
c.... Find initial el
  Solve f(k)=x(k)-xint=0, not related to Z.
c.... xint is reset so that within the convergence criteria
xint is positive (because we assume x>0, l(k)>=0.)
c.... nocon1 = 1 means no convergence for initial el iteration
  nocon2 = 1 means no convergence for fcut iteration
c
  xint=dmax1(xint,eps*0.0001*bmod(0.,0.))
c.... Make initial guess k0
  el0=xint*0.1
  fl0=x(el0)-xint
c.... Make second initial guess k1
  el1=(xint-0.1*dmax1(dabs(xint),f1(xint)))*0.05
c.... Set up convergence criterion
  conv=dmin1(1.d-7,xint)
c.... Secant iteration
  do 100 it=1,nit
    fl1=x(el1)-xint
    if(dabs(fl1).lt.conv.or.dabs(el1-el0).lt.conv) go to 200
    el2=el1-fl1*(el1-el0)/(fl1-fl0)
el0=el1
    fl0=fl1
    el1=el2
  100 continue
  nocon1 = 1
200  elint=el1
c.... Find fcut
c  Solve f1(fcut)=0

c.... Make first initial guess for fcut
  fcut=dmin1(0.,elint)
del=f1(fcut)
if(del.eq.0.d0) go to 600
c.... Make two better initial guesses for fcut
do 300 it = 1, nit
   el0 = fcut-del
   f0 = f1(el0)
   if(f0.lt.0.d0) go to 400
   del = 10.*del
   fcut = el0
300   continue

c.... Secant iterations
400   do 500 it = 1, nit
      f1 = f1(fcut)
      if(dabs(f1).lt.conv.or.dabs(fcut-el0).lt.conv) go to 600
      el2 = fcut-f1*(fcut-el0)/(f1-f0)
      el0 = fcut
      f0 = f1
      fcut = el2
500   continue
      nocon2 = 1
600   return
end
subroutine dprint(y,n1,n2,name)

* Program for printing response y (sig.33).
  implicit double precision(a-h,o-z)
  dimension y(1)
  character*6 name

write(6,2000) name

* Print out 8 columns each time.
  do 100 j=n1,n2,8
  * JH: the right-most index.
    jh=j+7
    if(jh.gt.n2) jh=n2
    write(6,2001) (n,n=jjh)
    write(6,2002) (y(k),k=jjh)
100 continue
  return

* Format
  2000 format(///20x,a6/
   * 20x,"==")
  2001 format(//8x,8i5)
  2002 format(//8x,8d15.7)
end
APPENDIX B
EXAMPLE INPUT AND OUTPUT FOR APPENDIX A

INPUT BULK MODULUS, SHEAR MODULUS, AND INITIAL CAP (Z) PARAMETER:
2100. 1700. 0.

INPUT NO. OF OBSERVATIONS FOR 6 TESTS:
47 49 48 49 48 48

INPUT OBSERVED (EXPERIMENTAL) STRESS RESPONSES FOR 6 TESTS:

<table>
<thead>
<tr>
<th>NO. 1</th>
<th>1.</th>
<th>3.</th>
<th>5.</th>
<th>6.</th>
<th>5.</th>
<th>4.</th>
<th>3.</th>
<th>1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8</td>
<td>9.</td>
<td>8.8</td>
<td>8.6</td>
<td>8.3</td>
<td>8.</td>
<td>9.</td>
<td>10.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO. 2</th>
<th>1.</th>
<th>2.</th>
<th>4.</th>
<th>6.</th>
<th>8.</th>
<th>8.5</th>
<th>9.</th>
<th>9.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5</td>
<td>8.</td>
<td>7.5</td>
<td>7.</td>
<td>6.5</td>
<td>7.</td>
<td>7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.5</td>
<td>9.</td>
<td>9.5</td>
<td>10.</td>
<td>9.5</td>
<td>9.</td>
<td>8.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>7.5</td>
<td>6.5</td>
<td>6.</td>
<td>6.5</td>
<td>7.</td>
<td>7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>9.</td>
<td>10.</td>
<td>10.5</td>
<td>10.</td>
<td>9.</td>
<td>8.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>6.</td>
<td>5.</td>
<td>4.5</td>
<td>4.</td>
<td>3.5</td>
<td>3.</td>
<td>2.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO. 3</th>
<th>1.</th>
<th>2.</th>
<th>4.</th>
<th>6.</th>
<th>8.</th>
<th>10.</th>
<th>10.12</th>
<th>10.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.63</td>
<td>7.275</td>
<td>6.94</td>
<td>6.63</td>
<td>6.375</td>
<td>6.16</td>
<td>6.</td>
<td>5.91</td>
<td></td>
</tr>
<tr>
<td>5.89</td>
<td>5.91</td>
<td>6.</td>
<td>6.16</td>
<td>6.375</td>
<td>6.63</td>
<td>6.94</td>
<td>7.275</td>
<td></td>
</tr>
<tr>
<td>7.63</td>
<td>8.</td>
<td>8.37</td>
<td>8.73</td>
<td>9.06</td>
<td>9.36</td>
<td>9.625</td>
<td>9.84</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO. 4</th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
<th>8.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.293</td>
<td>7.646</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td>8.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>7.823</td>
<td>7.646</td>
<td>8.</td>
<td>7.292</td>
<td>6.584</td>
<td>5.172</td>
<td>4.466</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO. 5</th>
<th>1.</th>
<th>1.5</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>4.566</th>
<th>5.132</th>
<th>5.698</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.586</td>
<td>3.293</td>
<td>4.</td>
<td>3.717</td>
<td>3.434</td>
<td>3.151</td>
<td>2.869</td>
<td>2.586</td>
<td></td>
</tr>
<tr>
<td>3.293</td>
<td>4.</td>
<td>5.</td>
<td>6.</td>
<td>7.</td>
<td>7.5</td>
<td>8.</td>
<td>9.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>8.</td>
<td>6.</td>
<td>4.</td>
<td>4.708</td>
<td>5.414</td>
<td>6.122</td>
<td>6.828</td>
<td></td>
</tr>
<tr>
<td>6.828</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO. 6</th>
<th>3.6</th>
<th>3.6</th>
<th>2.92</th>
<th>2.24</th>
<th>1.56</th>
<th>.88</th>
<th>0.</th>
<th>0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.</td>
<td>.88</td>
<td>1.56</td>
<td>2.24</td>
<td>2.92</td>
<td>3.6</td>
<td>3.6</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>3.6</td>
<td>2.92</td>
<td>2.24</td>
<td>1.56</td>
<td>.88</td>
<td>0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>1.5</td>
<td>2.</td>
<td>3.</td>
<td>3.6</td>
<td>4.</td>
<td>5.</td>
<td>6.</td>
<td></td>
</tr>
</tbody>
</table>

INPUT CONVERGENCE CRITERION AND MAXIMUM NO. OF FUNCTION EVALUATIONS:
5 .1.d-11 0.000001 500 2

INPUT PARAMETERS FOR MARQUARDT-LEVENBERG ALGORITHM:
INPUT INITIAL GUESS FOR MATERIAL PARAMETERS:
3.2 .09 1.0 .49 4. 0.004 .2
INPUT OPTION FOR WEIGHTING MATRIX (0 : WEIGHT = IDENTITY):
0
INPUT OPTION FOR SOIL (1) OR ROCK (2); AS WELL AS TENSION CUTOFF VALUE
1 -0.3
INPUT INITIAL STRESS STATE:
0. 0. 0. 0. 0. 0.
INPUT GEOSTATIC OVERBURDEN PRESSURE AND INITIAL CAP POSITION:
0. 16.
INPUT STRAIN HISTORY OF 6 TESTS:
NO. 1
-0.0000020 -0.0000587 -0.0000635 0. 0. 0.
0.0002395 0.0002086 0.0001715 0. 0. 0.
0.0004127 0.0004370 0.0004799 0. 0. 0.
0.0003060 0.0003555 0.0003940 0. 0. 0.
-0.0001604 -0.0002206 -0.0001891 0. 0. 0.
-0.0003913 -0.0003993 -0.0003953 0. 0. 0.
-0.0001187 -0.0001047 -0.0001095 0. 0. 0.
-0.0000720 -0.0000330 -0.0000827 0. 0. 0.
0.0001420 0.0000992 0.0000880 0. 0. 0.
0.0004722 0.0004828 0.0004767 0. 0. 0.
0.0005390 0.0006185 0.0006685 0. 0. 0.
0.0004835 0.0005567 0.0005853 0. 0. 0.
-0.0001502 -0.0002710 -0.0001843 0. 0. 0.
-0.0001722 -0.0002279 -0.000293 0. 0. 0.
-0.0001348 -0.0001654 -0.0001715 0. 0. 0.
-0.0004710 -0.0004949 -0.0005204 0. 0. 0.
-0.0001411 -0.0001104 -0.0001218 0. 0. 0.
-0.0001776 -0.0001480 -0.0002248 0. 0. 0.
0.0004733 0.0004259 0.0004466 0. 0. 0.
0.0004973 0.0005316 0.0005792 0. 0. 0.
0.0004338 0.0005567 0.0005685 0. 0. 0.
-0.0011258 0.0011877 0.0012631 0. 0. 0.
0.0000354 -0.0000029 0.0000332 0. 0. 0.
0.0000676 0.0000314 0.0000235 0. 0. 0.
-0.0000763 -0.0000882 0.0001768 0. 0. 0.
0.0000006 -0.0000269 0.0001764 0. 0. 0.
0.0000336 0.0000646 -0.000006 0. 0. 0.
0.0000244 0.0000372 -0.0000439 0. 0. 0.
0.0000506 0.0000774 -0.0000951 0. 0. 0.
0.0000724 0.0000719 -0.0001010 0. 0. 0.
-0.0001322 -0.0001861 0.0003957 0. 0. 0.
-0.0001670 -0.0002238 0.0009633 0. 0. 0.
-0.0002107 -0.0002359 0.0014727 0. 0. 0.
0.0000704 0.0001000 -0.0000485 0. 0. 0.
0.0002083 0.0002957 -0.0004022 0. 0. 0.
0.0001969 0.0002688 -0.0003118 0. 0. 0.
-0.0002632 -0.0003922 0.0006584 0. 0. 0.
-0.0003644 -0.0004242 0.0016212 0. 0. 0.
-0.0003102 -0.0003345 0.0019130 0. 0. 0.
0.0001826 0.0002295 -0.0000325 0. 0. 0.
0.0003423 0.0004182 -0.0004839 0. 0. 0.
0.0003900 0.0004675 -0.0005386 0. 0. 0.
NO. 2

<p>| -0.0002378 | -0.0003693 | 0.0005416 | 0. | 0. |
| -0.0002967 | -0.0003534 | 0.0006050 | 0. | 0. |
| -0.0002038 | -0.0002265 | 0.0004095 | 0. | 0. |
| -0.0003311 | -0.0003540 | 0.0012834 | 0. | 0. |
| -0.0006453 | -0.0006992 | 0.0026053 | 0. | 0. |
| NO. 2 |
| 0.0000605 | 0.0001083 | 0.0000454 | 0. | 0. |
| 0.0002414 | 0.0002434 | 0.0001962 | 0. | 0. |
| 0.0006576 | 0.0005363 | 0.0005038 | 0. | 0. |
| 0.0009995 | 0.0008484 | 0.0008523 | 0. | 0. |
| 0.0012717 | 0.0011145 | 0.0011642 | 0. | 0. |
| -0.0000126 | 0.0000866 | 0.0004583 | 0. | 0. |
| 0.0001748 | 0.0000072 | -0.0000809 | 0. | 0. |
| 0.0002013 | 0.0000030 | -0.0001115 | 0. | 0. |
| 0.0001768 | 0.0000058 | -0.0001113 | 0. | 0. |
| 0.0001798 | 0.0000064 | -0.0001113 | 0. | 0. |
| 0.0001866 | 0.0000034 | -0.0001223 | 0. | 0. |
| 0.0001946 | 0.0000161 | -0.0001244 | 0. | 0. |
| 0.0001869 | 0.0000088 | -0.0001155 | 0. | 0. |
| 0.0002393 | 0.0000143 | -0.0001289 | 0. | 0. |
| 0.0004460 | 0.0000048 | -0.0001845 | 0. | 0. |
| -0.0000910 | 0.0000106 | 0.0001064 | 0. | 0. |
| -0.0001403 | 0.0000132 | 0.0001417 | 0. | 0. |
| -0.0001539 | 0.0000090 | 0.0001467 | 0. | 0. |
| -0.0001847 | 0.0000051 | 0.0001365 | 0. | 0. |
| -0.0003387 | 0.0000249 | 0.0002432 | 0. | 0. |
| -0.0003679 | 0.0000020 | 0.0003542 | 0. | 0. |
| -0.0005787 | 0.0000048 | 0.0017548 | 0. | 0. |
| 0.0001755 | 0.0000317 | -0.0000571 | 0. | 0. |
| 0.0001739 | 0.0000169 | -0.0000843 | 0. | 0. |
| 0.0003868 | 0.0000054 | -0.0002295 | 0. | 0. |
| 0.0003718 | 0.0000153 | -0.0002529 | 0. | 0. |
| 0.0004210 | -0.0000016 | -0.0002611 | 0. | 0. |
| 0.0005352 | 0.0000149 | -0.0003012 | 0. | 0. |
| 0.0018660 | 0.0000722 | -0.0004459 | 0. | 0. |
| 0.0010116 | 0.0000614 | -0.0003765 | 0. | 0. |
| 0.0011949 | 0.0000972 | -0.0004134 | 0. | 0. |
| 0.0012990 | 0.0001705 | -0.0006715 | 0. | 0. |
| 0.0018603 | 0.0002503 | -0.0009851 | 0. | 0. |
| 0.0020935 | 0.0004362 | -0.0016850 | 0. | 0. |</p>
<table>
<thead>
<tr>
<th>NO. 3</th>
<th>NO. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000484</td>
<td>0.0000791</td>
</tr>
<tr>
<td>0.0002150</td>
<td>0.0001569</td>
</tr>
<tr>
<td>0.0005414</td>
<td>0.0002074</td>
</tr>
<tr>
<td>0.0007421</td>
<td>0.0003594</td>
</tr>
<tr>
<td>0.0009765</td>
<td>0.0004221</td>
</tr>
<tr>
<td>-0.0009700</td>
<td>-0.0008130</td>
</tr>
<tr>
<td>0.0000007</td>
<td>0.0000199</td>
</tr>
<tr>
<td>0.0001209</td>
<td>0.0000925</td>
</tr>
<tr>
<td>0.0001261</td>
<td>0.0000845</td>
</tr>
<tr>
<td>0.0000925</td>
<td>0.0000845</td>
</tr>
<tr>
<td>0.0000561</td>
<td>0.0000561</td>
</tr>
<tr>
<td>0.0001549</td>
<td>0.0001549</td>
</tr>
<tr>
<td>0.0000986</td>
<td>-0.0000986</td>
</tr>
<tr>
<td>0.0000517</td>
<td>-0.0000517</td>
</tr>
<tr>
<td>-0.0000806</td>
<td>-0.0000806</td>
</tr>
<tr>
<td>0.0000517</td>
<td>0.0000517</td>
</tr>
<tr>
<td>-0.0000616</td>
<td>-0.0000616</td>
</tr>
<tr>
<td>-0.0000121</td>
<td>-0.0000121</td>
</tr>
<tr>
<td>-0.0000821</td>
<td>-0.0000821</td>
</tr>
<tr>
<td>-0.0000560</td>
<td>-0.0000560</td>
</tr>
<tr>
<td>-0.0000832</td>
<td>-0.0000832</td>
</tr>
<tr>
<td>-0.0000732</td>
<td>-0.0000732</td>
</tr>
<tr>
<td>-0.0000191</td>
<td>-0.0000191</td>
</tr>
<tr>
<td>-0.0000628</td>
<td>-0.0000628</td>
</tr>
<tr>
<td>-0.0000772</td>
<td>-0.0000772</td>
</tr>
<tr>
<td>-0.0000528</td>
<td>-0.0000528</td>
</tr>
<tr>
<td>-0.0000730</td>
<td>-0.0000730</td>
</tr>
<tr>
<td>-0.0000642</td>
<td>-0.0000642</td>
</tr>
<tr>
<td>-0.0000509</td>
<td>-0.0000509</td>
</tr>
<tr>
<td>-0.0000317</td>
<td>-0.0000317</td>
</tr>
<tr>
<td>-0.0000125</td>
<td>-0.0000125</td>
</tr>
<tr>
<td>-0.0000096</td>
<td>-0.0000096</td>
</tr>
<tr>
<td>0.0000333</td>
<td>0.0000333</td>
</tr>
<tr>
<td>0.0000159</td>
<td>0.0000159</td>
</tr>
<tr>
<td>0.0000992</td>
<td>0.0000079</td>
</tr>
<tr>
<td>0.0000813</td>
<td>0.0000813</td>
</tr>
<tr>
<td>-0.0000199</td>
<td>-0.0000199</td>
</tr>
<tr>
<td>0.0001928</td>
<td>0.0000791</td>
</tr>
<tr>
<td>0.0000656</td>
<td>0.0001324</td>
</tr>
<tr>
<td>-0.0027520</td>
<td>0.0000957</td>
</tr>
<tr>
<td>0.0001842</td>
<td>0.0001699</td>
</tr>
<tr>
<td>0.0004483</td>
<td>0.0001982</td>
</tr>
<tr>
<td>0.0000708</td>
<td>0.0000169</td>
</tr>
<tr>
<td>0.0008171</td>
<td>0.0000043</td>
</tr>
<tr>
<td>0.0008264</td>
<td>0.0000430</td>
</tr>
<tr>
<td>0.000534</td>
<td>0.0000264</td>
</tr>
<tr>
<td>0.0000311</td>
<td>0.0000221</td>
</tr>
<tr>
<td>0.0000355</td>
<td>0.0000115</td>
</tr>
<tr>
<td>0.0000463</td>
<td>0.0000369</td>
</tr>
<tr>
<td>-0.001915</td>
<td>0.0000281</td>
</tr>
<tr>
<td>-0.0000475</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000549</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000474</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000737</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000002</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000058</td>
<td>0.0000251</td>
</tr>
<tr>
<td>-0.0000020</td>
<td>0.0000251</td>
</tr>
<tr>
<td>0.0000286</td>
<td>0.0000286</td>
</tr>
<tr>
<td>0.0000442</td>
<td>0.0000442</td>
</tr>
<tr>
<td>0.0000552</td>
<td>0.0000552</td>
</tr>
<tr>
<td>0.0000650</td>
<td>0.0000650</td>
</tr>
<tr>
<td>0.0000605</td>
<td>0.0000605</td>
</tr>
<tr>
<td>0.0000548</td>
<td>0.0000548</td>
</tr>
<tr>
<td>0.0000582</td>
<td>0.0000582</td>
</tr>
<tr>
<td>0.0000813</td>
<td>0.0000813</td>
</tr>
<tr>
<td>0.0000428</td>
<td>0.0000428</td>
</tr>
<tr>
<td>0.0000419</td>
<td>0.0000419</td>
</tr>
<tr>
<td>-0.0003004</td>
<td>-0.0003004</td>
</tr>
<tr>
<td>0.0000690</td>
<td>0.0000690</td>
</tr>
<tr>
<td>0.0000546</td>
<td>0.0000546</td>
</tr>
<tr>
<td>0.0000068</td>
<td>0.0000068</td>
</tr>
<tr>
<td>0.0000062</td>
<td>0.0000062</td>
</tr>
<tr>
<td>-0.0000042</td>
<td>-0.0000042</td>
</tr>
<tr>
<td>0.0000366</td>
<td>0.0000366</td>
</tr>
<tr>
<td>Value 1</td>
<td>Value 2</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>0.0006292</td>
<td>0.0006308</td>
</tr>
<tr>
<td>0.0012222</td>
<td>0.0011276</td>
</tr>
<tr>
<td>0.0001410</td>
<td>0.0000790</td>
</tr>
<tr>
<td>-0.0000600</td>
<td>-0.0001391</td>
</tr>
<tr>
<td>-0.0000592</td>
<td>-0.0000187</td>
</tr>
<tr>
<td>-0.0001154</td>
<td>-0.0000927</td>
</tr>
<tr>
<td>-0.000266</td>
<td>-0.0001771</td>
</tr>
<tr>
<td>-0.00002405</td>
<td>-0.0001202</td>
</tr>
<tr>
<td>-0.0001138</td>
<td>-0.0000733</td>
</tr>
<tr>
<td>-0.0000592</td>
<td>-0.0000187</td>
</tr>
</tbody>
</table>

NO. 5

<table>
<thead>
<tr>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000602</td>
<td>-0.000033</td>
<td>-0.000030</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.000662</td>
<td>0.0001508</td>
<td>0.0001436</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.000986</td>
<td>0.0001254</td>
<td>0.0001093</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0002091</td>
<td>0.0001187</td>
<td>0.0001403</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0004602</td>
<td>0.0005228</td>
<td>0.0005385</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000782</td>
<td>-0.0000767</td>
<td>0.0003576</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000840</td>
<td>-0.0000768</td>
<td>0.0004826</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000645</td>
<td>-0.0000581</td>
<td>0.0007508</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000853</td>
<td>-0.0000996</td>
<td>0.0008040</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0001174</td>
<td>-0.0001384</td>
<td>0.0009033</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001483</td>
<td>0.0001664</td>
<td>-0.0003725</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0.0003256</td>
<td>0.0003237</td>
<td>-0.0004044</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0002350</td>
<td>0.0000300</td>
<td>-0.0000478</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0006768</td>
<td>0.0000320</td>
<td>-0.0000767</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0008572</td>
<td>0.0000342</td>
<td>-0.0000830</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0024622</td>
<td>0.0001020</td>
<td>-0.0001551</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.002812</td>
<td>0.001777</td>
<td>0.0001875</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.001534</td>
<td>0.000517</td>
<td>0.0002252</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.000682</td>
<td>0.002975</td>
<td>-0.0000719</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.000849</td>
<td>0.004480</td>
<td>-0.0000842</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.000731</td>
<td>0.006175</td>
<td>-0.0000746</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.001584</td>
<td>0.0009155</td>
<td>-0.0001398</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.001798</td>
<td>0.013772</td>
<td>-0.0001679</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.002120</td>
<td>-0.003489</td>
<td>0.0002209</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.001899</td>
<td>-0.004123</td>
<td>0.0002372</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.001567</td>
<td>0.000869</td>
<td>0.0001063</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.003453</td>
<td>0.002330</td>
<td>0.0003364</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.005674</td>
<td>0.004565</td>
<td>0.0004510</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.007839</td>
<td>0.008205</td>
<td>0.0005003</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.007683</td>
<td>0.007891</td>
<td>0.0005170</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0015877</td>
<td>0.0017248</td>
<td>0.0001167</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0013860</td>
<td>0.0014737</td>
<td>0.0001063</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000516</td>
<td>-0.0000565</td>
<td>-0.0001214</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001216</td>
<td>-0.0001476</td>
<td>-0.0000936</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0002875</td>
<td>-0.0003229</td>
<td>-0.0003298</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000104</td>
<td>0.0000052</td>
<td>0.0002081</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001008</td>
<td>-0.0000906</td>
<td>0.0001869</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000930</td>
<td>-0.0000756</td>
<td>0.0002327</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001250</td>
<td>-0.0001354</td>
<td>0.0002326</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001757</td>
<td>-0.0000326</td>
<td>0.000376</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0002167</td>
<td>0.0000949</td>
<td>0.000693</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001410</td>
<td>0.0000656</td>
<td>0.000065</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001834</td>
<td>0.0000666</td>
<td>0.0000742</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001515</td>
<td>0.0000727</td>
<td>0.0000440</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0002026</td>
<td>0.0000717</td>
<td>0.0000534</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0002599</td>
<td>0.0000473</td>
<td>0.0000907</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0003525</td>
<td>0.0001030</td>
<td>0.0000933</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0003709</td>
<td>0.0000825</td>
<td>0.0001300</td>
<td>0.</td>
<td>0.</td>
</tr>
</tbody>
</table>

NO. 6

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0000625</td>
<td>-0.0000546</td>
<td>0.0001171</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000476</td>
<td>-0.0000387</td>
<td>0.0003012</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000532</td>
<td>-0.0000341</td>
<td>0.0003818</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000675</td>
<td>-0.0000629</td>
<td>0.0004775</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001220</td>
<td>-0.0000960</td>
<td>0.0005752</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000531</td>
<td>0.0001651</td>
<td>0.0000414</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000679</td>
<td>0.0002344</td>
<td>0.0000331</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000715</td>
<td>0.0003692</td>
<td>-0.0000080</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001069</td>
<td>0.0003996</td>
<td>0.0000018</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001335</td>
<td>0.0004520</td>
<td>0.0000100</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0001511</td>
<td>0.0001397</td>
<td>-0.0001865</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000776</td>
<td>0.0000723</td>
<td>-0.0002161</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000234</td>
<td>0.0000978</td>
<td>-0.0002795</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>0.0000688</td>
<td>0.0002153</td>
<td>-0.0003166</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>-0.0000229</td>
<td>0.0003285</td>
<td>-0.0006315</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0.0005334</td>
<td>0.0000008</td>
<td>-0.0000703</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0004023</td>
<td>-0.000019</td>
<td>-0.0001729</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0004588</td>
<td>0.0000356</td>
<td>-0.0001098</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0004159</td>
<td>-0.0000104</td>
<td>-0.0001238</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0005309</td>
<td>0.0000204</td>
<td>-0.0001665</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001729</td>
<td>-0.0001561</td>
<td>0.0000129</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000965</td>
<td>-0.0002153</td>
<td>0.0000149</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001470</td>
<td>-0.0002790</td>
<td>0.0000144</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001908</td>
<td>-0.0003715</td>
<td>0.000025</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.00003892</td>
<td>-0.0007686</td>
<td>-0.001695</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000529</td>
<td>-0.0000881</td>
<td>0.0003802</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000426</td>
<td>-0.0001118</td>
<td>0.0004453</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.00000148</td>
<td>-0.0001371</td>
<td>0.0005146</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000144</td>
<td>-0.0001193</td>
<td>0.0003962</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0000554</td>
<td>-0.0001868</td>
<td>0.0005284</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0001609</td>
<td>-0.000049</td>
<td>0.0001260</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0002267</td>
<td>0.0000196</td>
<td>0.0001397</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0002755</td>
<td>0.0000271</td>
<td>0.0000800</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.00004377</td>
<td>0.0000032</td>
<td>0.0001573</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.0009679</td>
<td>-0.0000286</td>
<td>0.0003732</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000160</td>
<td>0.0000371</td>
<td>-0.0001910</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000396</td>
<td>0.0000453</td>
<td>-0.0002006</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000427</td>
<td>0.0000660</td>
<td>-0.0002739</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000475</td>
<td>0.0000792</td>
<td>-0.0003740</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0000935</td>
<td>0.0001432</td>
<td>-0.0006179</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0004870</td>
<td>0.0003082</td>
<td>0.0002324</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001613</td>
<td>0.0001642</td>
<td>0.0001151</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001853</td>
<td>0.0002125</td>
<td>0.0001662</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0003383</td>
<td>0.0004073</td>
<td>0.0003093</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0002077</td>
<td>0.0002818</td>
<td>0.0001864</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001044</td>
<td>0.0001421</td>
<td>0.0001103</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0005185</td>
<td>0.0006779</td>
<td>0.0013489</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0007016</td>
<td>0.0011151</td>
<td>0.0017758</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
BULK MODULUS = 0.210000d+04
SHEAR MODULUS = 0.170000d+04
INITIAL CAP (Z) = 0.000000d+00

THE INITIAL GUESS FOR PARAMETERS:

ALPHA THETA GAMA BETA
3.200000 0.090000 1.000000 0.490000
R D W
4.000000 0.004000 0.200000

THE OPTIMAL VALUES OF PARAMETERS ARE:

ALPHA THETA GAMA BETA
3.865751 0.100000 1.163779 0.443505
R D W
4.433298 0.003223 0.429271

TRUE SUM OF SQUARES = 0.6758328d+03

THE TRUE ROOT-MEAN-SQUARE OF PHI = 0.1537222d+01

THE NORMALIZED RELATIVE ERROR = 0.2221052d+00

CONDITION NUMBER OF G = 0.1611413d+05
<table>
<thead>
<tr>
<th>DEPARTMENT OF DEFENSE</th>
<th>Distribution List</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEFENSE INTELLIGENCE AGENCY</td>
<td>ATTN: DB-4C</td>
</tr>
<tr>
<td>ATTN: C WIEHLE</td>
<td></td>
</tr>
<tr>
<td>2 CYS ATTN: RTS-2B</td>
<td></td>
</tr>
<tr>
<td>ATTN: S HALPERSON</td>
<td></td>
</tr>
<tr>
<td>DEFENSE NUCLEAR AGENCY</td>
<td>ATTN: OPNS</td>
</tr>
<tr>
<td>4 CYS ATTN: TITL</td>
<td></td>
</tr>
<tr>
<td>DEFENSE NUCLEAR AGENCY</td>
<td>ATTN: TDIC/FDAB</td>
</tr>
<tr>
<td>DEFENSE TECH INFO CENTER</td>
<td>12CYS ATTN:</td>
</tr>
<tr>
<td>JOINT STRAT TGT PLANNING STAFF</td>
<td>ATTN: JK (ATTN: DNA REP)</td>
</tr>
<tr>
<td>DEPARTMENT OF THE ARMY</td>
<td>U S ARMY COLD REGION RES ENGR LAB</td>
</tr>
<tr>
<td>ATTN: LIBRARY</td>
<td></td>
</tr>
<tr>
<td>U S ARMY CONSTRUCTION ENGRG RES LAB</td>
<td>ATTN: LIBRARY</td>
</tr>
<tr>
<td>U S ARMY ENGR WATERWAYS EXPER STA</td>
<td>ATTN: J BALSARA</td>
</tr>
<tr>
<td>ATTN: S KIGER</td>
<td></td>
</tr>
<tr>
<td>ATTN: TECHNICAL LIBRARY</td>
<td></td>
</tr>
<tr>
<td>DEPARTMENT OF THE NAVY</td>
<td>NAVAL POSTGRADUATE SCHOOL</td>
</tr>
<tr>
<td>ATTN: CODE 1424 LIBRARY</td>
<td></td>
</tr>
<tr>
<td>DEPARTMENT OF THE AIR FORCE</td>
<td>AIR FORCE INSTITUTE OF TECH/EN</td>
</tr>
<tr>
<td>ATTN: C BRIDGMAN</td>
<td></td>
</tr>
<tr>
<td>AIR FORCE WEAPONS LABORATORY</td>
<td>ATTN: NTE</td>
</tr>
<tr>
<td>ATTN: R HENNY</td>
<td></td>
</tr>
<tr>
<td>BALLISTIC MISSILE OFFICE</td>
<td>ATTN: MYEB D GAGE</td>
</tr>
<tr>
<td>DEPARTMENT OF ENERGY</td>
<td>LAWRENCE LIVERMORE NATIONAL LAB</td>
</tr>
<tr>
<td>ATTN: L-53 TECH INFO DEPT LIB</td>
<td></td>
</tr>
<tr>
<td>DEPARTMENT OF DEFENSE CONTRACTORS</td>
<td>AGBABIAN ASSOCIATES</td>
</tr>
<tr>
<td>ATTN: C BAEGE</td>
<td></td>
</tr>
<tr>
<td>APPLIED &amp; THEORETICAL MECHANICS, INC</td>
<td>ATTN: J M CHAMPNEY</td>
</tr>
<tr>
<td>APPLIED RESEARCH ASSOCIATES, INC</td>
<td>ATTN: N HIGGINS</td>
</tr>
<tr>
<td>APPLIED RESEARCH ASSOCIATES, INC</td>
<td>ATTN: S BLOUIN</td>
</tr>
<tr>
<td>BDM CORP</td>
<td>ATTN: J STOCKTON</td>
</tr>
<tr>
<td>BDM CORPORATION</td>
<td>ATTN: LIBRARY</td>
</tr>
<tr>
<td>BOEING CO</td>
<td>ATTN: G R BURWELL</td>
</tr>
<tr>
<td>BOEING TECH &amp; MANAGEMENT SVCS, INC</td>
<td>ATTN: AEROSPACE LIBRARY</td>
</tr>
<tr>
<td>CALIF RESEARCH &amp; TECHNOLOGY, INC</td>
<td>ATTN: LIBRARY</td>
</tr>
<tr>
<td>CALIF RESEARCH &amp; TECHNOLOGY, INC</td>
<td>ATTN: Z P LEE</td>
</tr>
<tr>
<td>CALIFORNIA, UNIVERSITY OF BERKLEY</td>
<td>2 CYS ATTN: J SIMO</td>
</tr>
<tr>
<td>2 CYS ATTN: JIANN-WEN JU</td>
<td></td>
</tr>
<tr>
<td>2 CYS ATTN: K PISTER</td>
<td></td>
</tr>
<tr>
<td>2 CYS ATTN: R TAYLOR</td>
<td></td>
</tr>
<tr>
<td>H &amp; H CONSULTANTS, INC</td>
<td>ATTN: J HALTIWANGER</td>
</tr>
<tr>
<td>KAMAN SCIENCES CORP</td>
<td>ATTN: D SEITZ</td>
</tr>
<tr>
<td>KAMAN SCIENCES CORPORATION</td>
<td>ATTN: DASIAC</td>
</tr>
<tr>
<td>KAMAN TEMPO</td>
<td>ATTN: DASIAC</td>
</tr>
<tr>
<td>KARAGOZIAN AND CASE</td>
<td>ATTN: J KARAGOZIAN</td>
</tr>
<tr>
<td>LACHEL PIEPENBURG AND ASSOCIATES</td>
<td>ATTN: D PIEPENBURG</td>
</tr>
<tr>
<td>PACIFIC SIERRA RESEARCH CORP</td>
<td>ATTN: H BRODE</td>
</tr>
<tr>
<td>R &amp; D ASSOCIATES</td>
<td>ATTN: J LEWIS</td>
</tr>
<tr>
<td>TRW SPACE &amp; DEFENSE, DEFENSE SYSTEMS</td>
<td>ATTN: R CRAMOND</td>
</tr>
<tr>
<td>WEIDLINGER ASSOC</td>
<td>ATTN: J ISBNECK</td>
</tr>
<tr>
<td>WEIDLINGER ASSOC, CONSULTING ENGRG</td>
<td>ATTN: T DEEY</td>
</tr>
<tr>
<td>WEIDLINGER ASSOC, CONSULTING ENGINEERS</td>
<td>ATTN: M BARON</td>
</tr>
</tbody>
</table>

Dist-1