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NAVAL OCEAN SYSTEMS CENTER San Diego, California 92152-5000

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Daylight and Twilight Sky Radiance and Terrestrial Irradiance

C.H. Waldman

Titan Systems, Inc.



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<p>When solar radiation is transmitted through the atmosphere it may be reflected, transmitted, or absorbed. The total effect of the atmosphere upon a beam of sunlight passing through it is to reduce its intensity by amounts varying with latitude, season, and degree of cloudiness. The atmosphere weakens solar energy through 1. molecular and small particle scattering, 2. diffuse reflection by larger particles, and 3. selective absorption as due to ozone and water vapor. The scattering and reflecting processes send a part of the solar energy back into space, but some of it reaches the earth's surface as diffuse sunlight.</p> <p>Titan Systems, Inc., developed a model for the sky radiance and terrestrial irradiance and a computer program was written to carry out the calculation.</p>					
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1. Introduction

When solar radiation is transmitted through the atmosphere it may be reflected, transmitted, or absorbed. The total effect of the atmosphere upon a beam of sunlight passing through it is to reduce its intensity by amounts varying with latitude, season, and degree of cloudiness. The atmosphere weakens solar energy through (1) molecular and small particle scattering, (2) diffuse reflection by larger particles, and (3) selective absorption as due to ozone and water vapor. The scattering and reflecting processes send a part of the solar energy back into space, but some of it reaches the earth's surface as diffuse sunlight.

Titan Systems, Inc. is interested to develop a predictive model of monochromatic flux density reaching the earth's surface. The twilight time, when direct sunlight is no longer available, is considered to be of paramount importance for the model.

A model was developed for the sky radiance and terrestrial irradiance and a computer program was written to carry out the calculation. A technical discussion of the problem is presented in Section 2 and the computer program is described in Section 3. The results for a specified set of conditions are given in Section 4. Finally, recommendations for improvements to the model are set forth in Section 5. A list of nomenclature and references are given in Sections 6 and 7, respectively.

2. Technical Discussion

According to McCartney (1975) ...

"Rayleigh scattering by molecules represents the irreducible minimum of scattering along an atmospheric path. At lower altitudes, Mie scattering by particles always predominates, but on average decreases more rapidly with altitude than does Rayleigh scattering. This is to be expected, because representative scale heights of the haze aerosol are near 1 km, while scale heights of the permanent gas envelope are in the range of 6 to 9 km and greater. Thus the haze atmosphere has a limited vertical extent, except for several tenuous layers of particles such as the one near 20 km. In contrast, the Rayleigh or molecular atmosphere continues upwards to great altitudes, but its transparency is sufficient that lines of sight have lengths limited by

geometric, not optical, factors. The Rayleigh atmosphere is a useful starting point for studying the optical complexities of the real atmosphere."

McCartney's analysis of the intensity of scattered flux will be followed. Although the spherical geometry of Figure 1 is generally applicable, for simplicity here the earth and its atmospheric envelope are assumed to be plane-parallel, as shown in Figure 2. This assumption does not cause errors greater than 2% for all solar zenith angles less than 80 deg. according to the author. Clearly, this is not valid for analysis of the twilight sky. But it is easy to understand physically and will provide the framework for the twilight sky analysis.

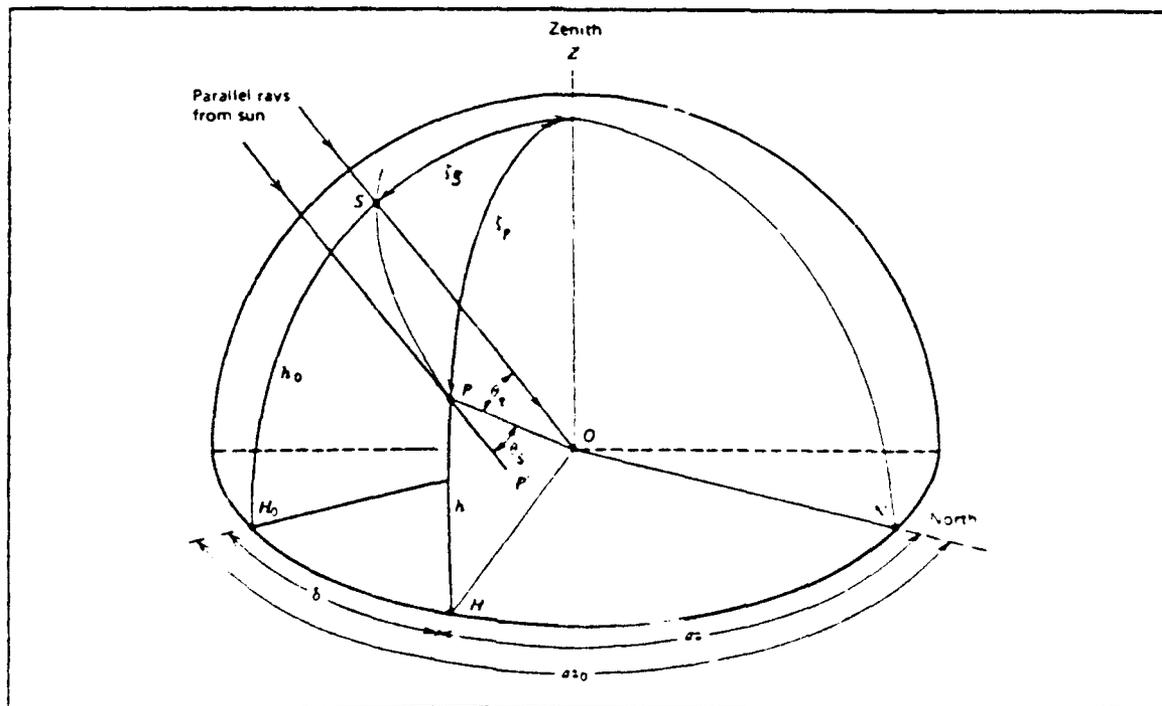


Figure 1. Geometry of atmospheric scattering of sunlight (after McCarthy [1975])

Considering Figure 2, the value of the sky radiance in direction OP is to be determined. The point P at altitude Z , lies within the atmosphere, and the observer is at point O , whose altitude may be taken as zero. (This is not necessarily sea level, it may be any reference altitude) The analysis will be

done in terms of atmospheric reduced height.* The reduced height of the atmosphere above P is denoted Z' , and the total reduced height above O by Z_0 . All the solar rays incident on the atmosphere are considered parallel, and the relative bearing (or azimuth) of the line OP from the line OS is denoted by δ . The zenith angles of the sun and the point P are ζ_s and ζ_p , respectively. Only a narrow spectral interval of radiant flux is considered, and the wavelength is taken to be in a spectral region free of absorption.

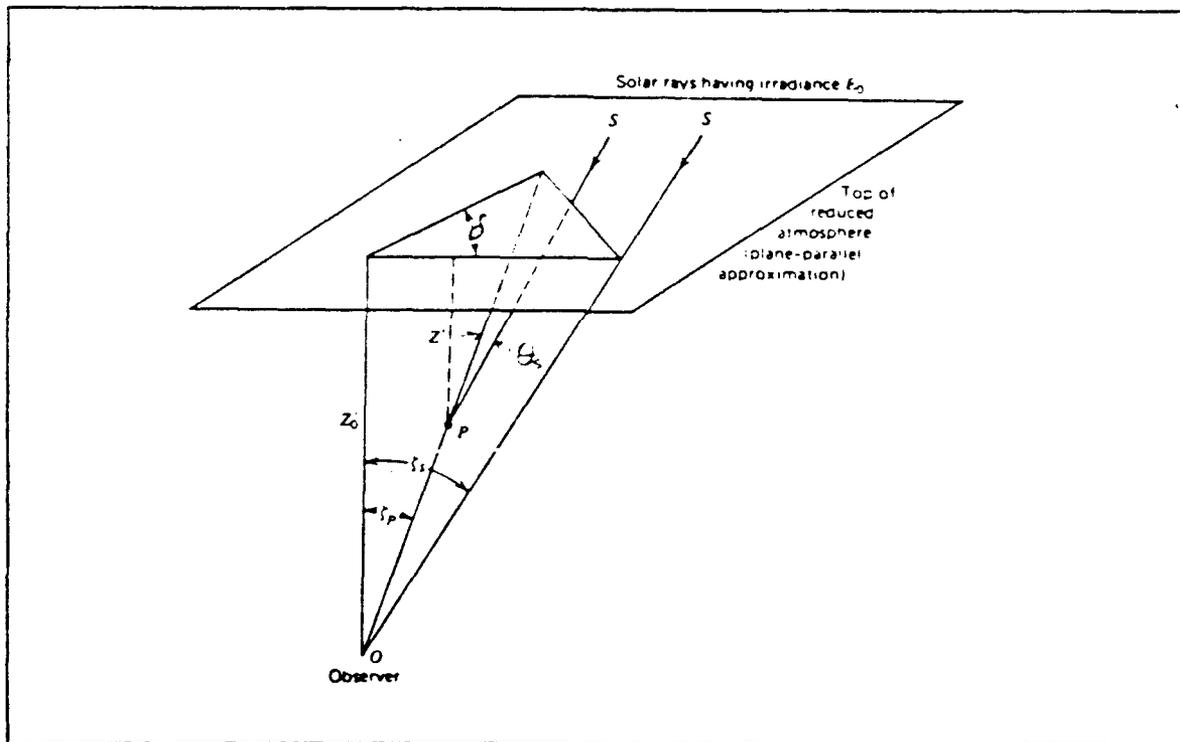


Figure 2. Geometry of atmospheric scattering of sunlight (after McCartney [1975]).

*In effect, the atmosphere is replaced by a uniform density atmosphere which has the same pressure and, in fact, total number of molecules along a vertical path, as the true atmosphere; thus the reduced height atmosphere is finite in extent. The molecular optical thickness of a vertical path from a given altitude to the top of the atmosphere is the product of the reduced height for that altitude and the volume total scattering coefficient relevant to the density of the reduced atmosphere. The reduced height at a given altitude can be shown to be $Z' = P/\rho g$.

The solar flux, having the irradiance E_{λ_0} at the top of the atmosphere, is attenuated by scattering as it travels the slant path through the reduced height Z' to reach point P. The irradiance of a unit volume at P is

$$E_{\lambda} = E_{\lambda_0} \exp(-\beta_m Z' \sec \zeta_s) \quad (1)$$

The scattered intensity in the direction PO from a differential element of path at P having a unit cross section and a length

$$ds = \sec \zeta_p dZ' \quad (2)$$

and characterized by the angular scattering coefficient $\beta_m(\theta_s)$ is given by

$$dI_{\lambda} = E_{\lambda_0} \exp(-\beta_m Z' \sec \zeta_s) \beta_m(\theta_s) \sec \zeta_p dZ' \quad (3)$$

The attenuation along the path PO is given by

$$\exp[-\beta_m (Z_0' - Z') \sec \zeta_p] \quad (4)$$

so that the differential intensity perceived at O is

$$dI_{\lambda} = E_{\lambda_0} \exp(-\beta_m Z' \sec \zeta_s) \beta_m(\theta_s) \exp[-\beta_m (Z_0' - Z') \sec \zeta_p] \sec \zeta_p dZ' \quad (5)$$

The intensity at O due to all elements along PO is given by

$$I_{\lambda} = E_{\lambda_0} \beta_m(\theta_s) \int_0^{Z_0'} \exp(-\beta_m Z' \sec \zeta_s) \exp[-\beta_m (Z_0' - Z') \sec \zeta_p] \sec \zeta_p dZ' \quad (6)$$

where it has been implicitly assumed that the (total and angular) scattering coefficients are constant in the atmosphere.

Since the total scattering coefficient and the zenith angles are independent of

the altitude, the above integration can be readily carried out in closed form. First rewrite Eq. (6) as follows

$$I_{\lambda} = E_{\lambda 0} \beta_m(\theta_s) \exp(-\beta_m Z_0' \sec \zeta_p) \sec \zeta_p \int_0^{Z_0'} \exp[-\beta_m (\sec \zeta_s - \sec \zeta_p) Z'] dZ' \quad (7)$$

The integration yields

$$I_{\lambda} = E_{\lambda 0} \frac{\beta_m(\theta_s)}{\beta_m} \left[\frac{\exp(-\beta_m Z_0' \sec \zeta_s) - \exp(-\beta_m Z_0' \sec \zeta_p)}{1 - \sec \zeta_s / \sec \zeta_p} \right] \quad (8)$$

for the intensity of scattered solar flux at the bottom of a plane-parallel atmosphere in a specific direction. Because the area (i.e., cross section) of the path was incorporated in the analysis this is not, strictly speaking, an intensity (i.e., W/sr) but rather is a radiance (i.e., W/m²·sr).

Equation (8) was derived for a plane-parallel atmosphere and is not valid in two limiting cases: observation along a horizontal line of sight ($\zeta_p = \pi/2$) and sunset ($\zeta_s = \pi/2$). In the former case the sky radiance becomes zero because the optical path becomes infinite while in the latter case the sky radiance becomes zero immediately upon sunset because there is no light at the top of the atmosphere. Naturally, these would not be true for a spherical atmosphere.

We can formally derive an expression for the sky radiance for a spherical earth by accounting properly for the optical air mass and solar irradiance at the top of the atmosphere. First consider the optical air mass, m . For a plane-parallel atmosphere

$$m(\zeta_p) = \sec \zeta_p \quad (9)$$

while for spherical earth (refer to Figure 3) we can use an empirical expression given by **Rozenberg** [1966], viz.,

$$m(\zeta_p) = (\cos \zeta_p + 0.025 e^{-11 * \cos \zeta_p})^{-1} \quad (10)$$

Since this expression depends only upon ζ_p and is independent of Z' we can formally replace $\sec \zeta_p$ in Eq. (8) by $m(\zeta_p)$.

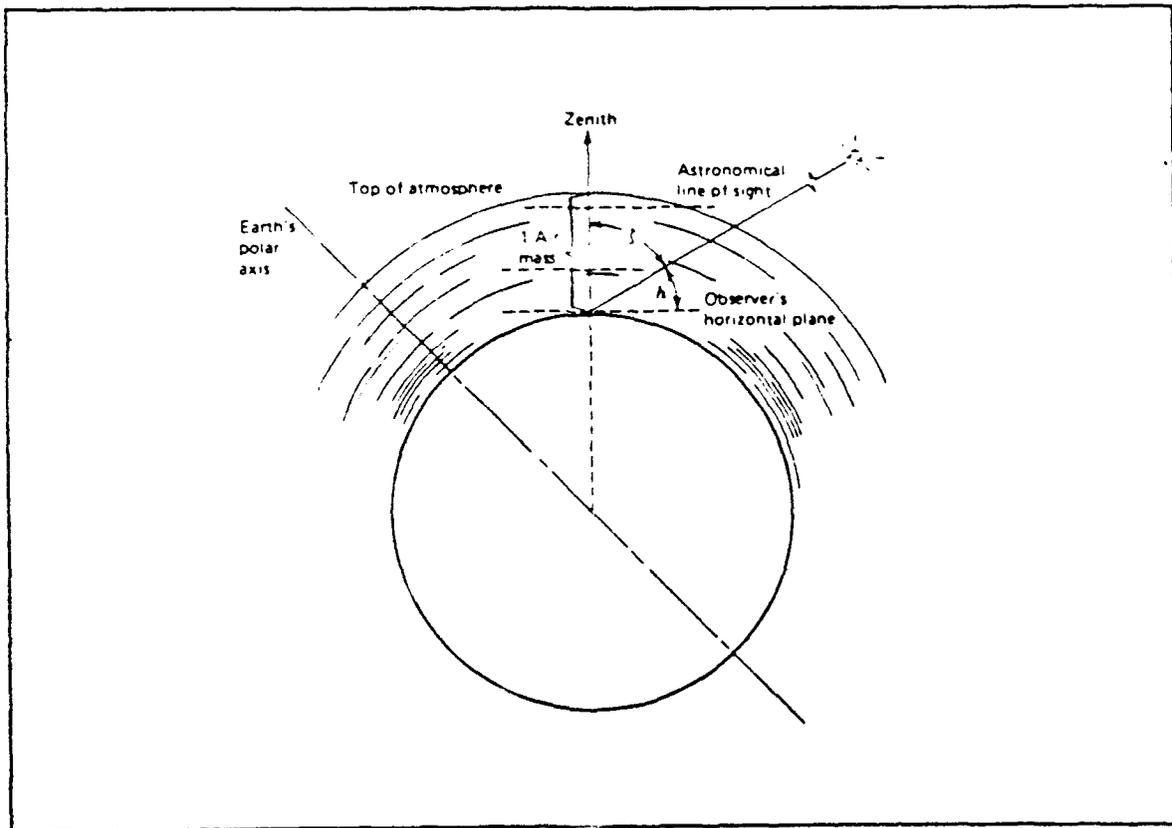


Figure 3. Geometry of the Optical Air Mass (after McCartney [1975])

Next consider the solar irradiance of a spherical earth. McCartney (1975) has considered the problem of evaluating terrestrial paths at high altitudes and zenith angles greater than about 75°. The solution is provided by the Chapman function which is defined as follows

$$Ch(x, \zeta) = x \sin \zeta \int_0^{\zeta} \exp(x - x \sin \zeta \csc \psi) \csc^2 \psi \, d\psi \quad (11)$$

where $x = (Z + R_e)/H_p$ and $H_p = RT/(g - R\gamma)$. The parameter x is seen to depend upon the altitude of the observed point. Using the Chapman function in the derivation for the sky radiance would commence with the solar irradiance at P given by

$$E_\lambda = E_{\lambda_0} \exp(-\beta_m Z' \text{Ch}(x, \zeta_s)) \quad (12)$$

This expression accounts for solar irradiance on the atmosphere of a spherical earth but does not account for refraction of light by the atmosphere.

With Eq. (12) we can formally replace $\sec \zeta_s$ in the analysis by $\text{Ch}(x, \zeta_s)$. Eq. (6) then becomes

$$I_\lambda = E_{\lambda_0} \beta_m(\theta_s) \int_0^{Z_0'} \exp(-\beta_m Z' \text{Ch}(x, \zeta_s)) \exp[-\beta_m (Z_0' - Z') m(\zeta_p)] m(\zeta_p) dZ' \quad (13)$$

Tabulated values of the Chapman function show that it is only weakly dependent upon altitude and may be treated as a constant as far the integration is concerned. Then the resultant sky radiance is identical to the previous integration with $\sec \zeta_s$ and $\sec \zeta_p$ replaced by $\text{Ch}(x, \zeta_s)$ and $m(\zeta_p)$, respectively.

$$I_\lambda = E_{\lambda_0} \frac{\beta_m(\theta_s)}{\beta_m} \left[\frac{\exp(-\beta_m Z_0' \text{Ch}(x, \zeta_s)) - \exp[-\beta_m Z_0' m(\zeta_p)]}{1 - \text{Ch}(x, \zeta_s) / \sec \zeta_p} \right] \quad (14)$$

A more detailed treatment of the sky radiance, including the effects of refraction, is given by Rozenberg [1966]. Rozenberg's results agree with those obtained above when refraction is neglected in his analysis. Unfortunately, the refraction terms are relatively intractable and could not be readily adopted for a practical calculation.

With the sky radiance specified we can now develop the monochromatic flux density or monochromatic irradiance of radiant energy. This is defined as the

normal component of I_λ (the monochromatic intensity or radiance, i.e., energy per unit area per unit time per unit solid angle per unit frequency interval) integrated over the entire hemispherical solid angle as shown in Figure 4. Liou [1980] gives the following expression for the flux density

$$F_\lambda = \int_{\omega} I_\lambda \cos(\theta) d\omega = \int_0^{2\pi} \int_0^{\pi/2} I_\lambda(\theta, \phi) \cos(\theta) \sin(\theta) d\theta d\phi \quad (15)$$

where (θ, ϕ) are the line-of-sight zenith and azimuth relative to the solar vertical, respectively, and are equivalent to (ζ_p, δ) of Figure 1.

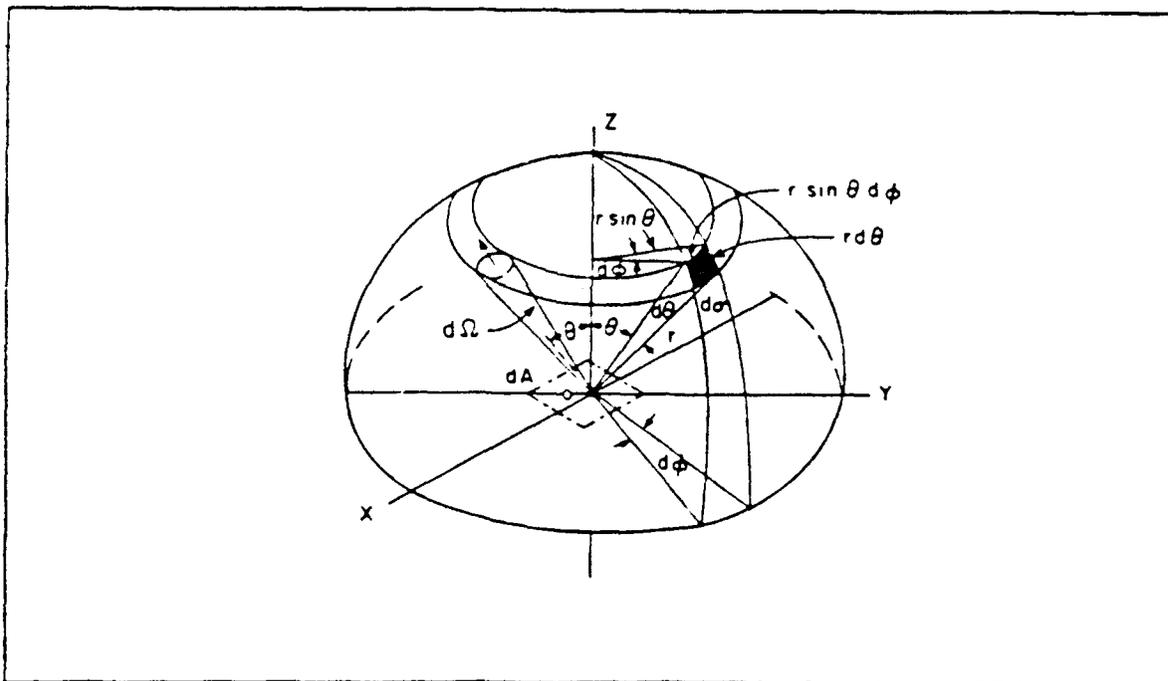


Figure 4. Illustration of solid angle and representation in polar coordinates (after Liou [1980]).

The problem will be completely specified once we have defined the angular and total scattering coefficients. McCartney [1975] derives the angular scattering coefficient (molecular) for the case of Rayleigh scattering in the atmosphere; for unpolarized light

$$\beta_m(\theta_s) = \frac{\pi^2 (n^2 - 1)^2}{2N\lambda^4} (1 + \cos^2\theta_s) \quad (16)$$

where the scattering angle, θ_s , can be derived from the spherical geometry shown in Figure 1 and is given by

$$\cos \theta_s = \cos \zeta_s \cos \zeta_p + \sin \zeta_s \sin \zeta_p \cos \delta \quad (17)$$

The total scattering coefficient is found by integrating the angular coefficient over the spherical solid angle, thus

$$\beta_m = \int_0^{4\pi} \beta_m(\theta_s) d\omega = \frac{8\pi^3(n^2-1)^2}{3N\lambda^4} \quad (18)$$

The scattering coefficients in Eqs. (17) and (18) exhibit the well-known λ^4 behavior associated with Rayleigh scattering.

3. Computer Program

A computer program was written in UCSD Pascal to carry out the double integration in Eq. (15) of the monochromatic radiance given in Eq. (14). The solution $F_\lambda = F_\lambda(\zeta_s)$ is a one-parameter family depending only on the solar zenith. The equation for the Chapman function, Eq. (11) is integrated once for each value of the solar zenith (except that $\sec \zeta_s$ is used for $\zeta_s < 75^\circ$).

All the integrations were carried out by Simpson's integration method. This method requires the interval to be divided into an even number of steps. The general formula for the integral is given by

$$\left[f_0 + f_n + 4 \sum_{j \text{ odd}}^{n-1} f_j + 2 \sum_{j \text{ even}}^{n-2} f_j \right] \Delta x / 3 \quad (19)$$

The algorithm first chooses the number of steps, n to be equal to 2 and automatically doubles the number of steps until the result is within a specified tolerance ($1 \cdot 10^{-5}$ in the present calculations). At each step of the successive approximations the function is evaluated only at the odd positions. The sum for the even positions is obtained from the sum of all previous interior positions, both even and odd. The integration routine is called recursively to carry out the double integration.

UCSD Pascal does not permit a function or procedure to be passed as a parameter to another routine. This problem was solved by defining a TYPE to specify the integrand (e.g., Chapman, azimuth, or zenith integration). Calculation of a single azimuth point requires the complete integration over the (line-of-sight) zenith range $(0, 2\pi)$, each point of which is defined in terms of the sky radiance.

The program is not interactive; operation of the program requires no input from the user. The range of solar zenith angles is specified in the program and all physical constants and parameters are declared at the head of the program. Naturally, the program can be changed and recompiled.

A complete, annotated program listing is appended to this report.

4. Results

One set of calculations of monochromatic flux density (or irradiance) vs. solar zenith was carried out. The solar zenith was varied from 0° to nightfall, which is defined in terms of atmospheric opacity as follows

$$\text{opacity} = 10 \log (\text{irradiance at noon/irradiance}) = 80 \text{ decibels}$$

The wavelength and solar irradiance at the top of the atmosphere were taken as $\lambda = 459.3 \text{ nm}$ and $E_{\lambda_0} = 2.00 \text{ W/m}^2 \cdot \text{nm}$. All physical parameters used in the calculation were evaluated at sea level; the complete list of parameters is presented below:

<u>Quantity</u>	<u>Value</u>
λ	459.3 nm
E_{λ_0}	2.00 W/m ² ·nm
Z_0	8.44 km
n	1.000293
N	$2.68719 \cdot 10^{25} \text{ m}^{-3}$
x	612.5

The results showed that under these conditions night falls at a solar zenith of $\zeta_s = 103^\circ$, or about 52 minutes after sunset. This result is predicated on the assumption that the solar zenith is zero at the solar noon.

The complete tabulated results for Chapman function, flux density, and opacity are appended to this report.

5. Recommendations

There are several improvements which could be made to the model described in Section 2. Some of these are noted below:

<u>Phenomenon</u>	<u>References</u>
refraction of light	McCartney [1975] Rozenberg [1966]
shadow of earth on atmosphere	Rozenberg [1966]
airglow	Henderson [1977] Fleagle [1963]
selective absorption	Liou [1980]
particle (haze) scattering	McCartney [1975]
multiple scattering	McCartney [1975]
diffuse scattering by clouds	Liou [1980] McCartney [1975]

Additionally, the program can be adapted to permit calculation of the solar zenith for a given latitude, day of the year, and time of day which would permit a more realistic calculation for a particular locale. Figure 5 shows the geometry for calculating the solar zenith. Liou [1980] gives this result

$$\cos \theta_s = \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos h \quad (20)$$

where

- θ_s = solar zenith
- λ = latitude of the observation point
- h = hour angle (this is zero at solar noon and increases 15° for every hour before or after solar noon)
- δ = solar inclination

6. Nomenclature

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
E_λ	monochromatic irradiance	$W/m^2 \cdot nm$
E_{λ_0}	E_λ at the top of the atmosphere	$W/m^2 \cdot nm$
F_λ	monochromatic flux density (irradiance)	W/m^2
g	acceleration of gravity	m^2/sec^2
H_p	density scale height of molec. atmos.	km
I_λ	monochromatic radiance	$W/m^2 \cdot nm \cdot sr$
m	optical air mass	
n	refractive index	
N	number of molecules per unit volume	m^{-3}
R	gas constant	$m^2/sec^2 \cdot ^\circ K$
R_e	mean radius of the earth	km
s	distance	km
T	temperature	$^\circ K$
Z	altitude	km
Z'	reduced height of atmos. (ref. altitude)	
Z_0'	reduced height of atmos. (sea level)	
$\beta_m(\theta_s)$	volume angular scattering coefficient (molecular), unpolarized light	$km^{-1} sr^{-1}$
β_m	volume total scattering coefficient (molecular)	km^{-1}
γ	atmos. lapse rate of temperature	$^\circ K km^{-1}$
δ	azimuth relative to solar vertical	rad
ζ	zenith	rad
ζ_p	zenith of line of sight at observer	rad
ζ_s	zenith of sun at observer	rad
θ_s	observation angle of scattering	rad
λ	wavelength	nm
x	parameter of the Chapman function	
ω	solid angle	sr

7. References

Fleagle, R.G. and Businger, J.A. (1963) An Introduction to Atmospheric Physics, Academic Press, New York. [QC880.F593]

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Kreider, J.F. and Kreith, F. (1981) Solar Energy Handbook, McGraw-Hill, New York. [TJ810.H35]

Liou, K.N. (1980) An Introduction to Atmospheric Radiation, Academic Press, New York. [QC912.3.L56]

McCartney, E.J. (1976) Optics of the Atmosphere. Scattering by Molecules and Particles, J. Wiley & Sons, New York. [QC976.S3M3]

Rozenberg, G.V. (1966) Twilight. A Study in Atmospheric Optics, Plenum Press, New York. [QC976.T9R893 1966]

SKY RADIANT FLUX

Monday, 14 January 1985 4:09 PM

PAGE 1

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(*****
*
*   PROGRAM TITLE:   Calculation of Sky Radiant Flux
*   WRITTEN BY:     Cye H. Walzman, Ph.D.
*
*   DATE WRITTEN:   8 January 1985
*
*   WRITTEN FOR:    Titan, Inc.: Gary Lee, Ph.D., Program Mgr.
*
*   PROGRAM SUMMARY: This program calculates the monochromatic
*                   flux density (or irradiance) of radiant
*                   energy by integrating the monochromatic
*                   intensity or radiance I (power/area*freq*sr)
*                   over the spherical solid angle encompassing
*                   the sky hemisphere.
*
*   INPUT & OUTPUT FILES:   DESCRIPTION
*
*       SKY.FILE.TEXT       Output file of tabulated results
*
*****

```

(Notes on the integration scheme: A double Simpson's integration is performed over the azimuth and zenith angles. The integration procedure is called recursively in order to do this. Since UCSD Pascal doesn't permit a FUNCTION or PROCEDURE to be passed as a parameter to another routine we circumvent this problem by using a defined TYPE to specify the integrand. When the Simpson's procedure calls the function for the integrand it chooses the correct function in a CASE statement. Calculation of an azimuth point requires the complete integration over the (line of sight) zenith, hence that "function" is, in fact, another Simpson's integration of the intensity.)

```

(xL TITAN:SKY.LST.TEXT)
PROGRAM SkyRadiantFlux;

```

```

CONST  ( solar and terrestrial properties )
lambda = 459.3;      ( wavelength (blue/green), nm )
E_zero = 2.00;      ( extraterrestrial solar irradiance at
                    frequency lambda, W/(m^2*nm) )
Z_zero = 8.44;      ( reduced height of atmosphere/sea level, km )
n = 1.000293;      ( refractive index of air (STP) )
N_mol = 2.68719E25; ( Loschmidt's number (STP), molecules/m^3 )
chi = 612.5;       ( parameter of the Chapman function )
night_def = 80.0;  ( def of night opacity, decibels )

( miscellaneous and system constants )
pi = 3.141593;     ( a transcendental number )
convert = 1.745329E-2; ( degrees to radians )
tol = 1.0E-5;      ( tolerance for numerical integration )
bell = 7;         ( ASCII character for bell sound )
FF = 12;          ( ASCII character for form feed )
debugging = FALSE; ( turns debugging on/off )
outfilename = 'TITAN:SKY.FILE.TEXT';

( limits of integration )
az_1st = 0.0;      ( lower limit azimuth )
az_last = 6.283185; ( upper limit azimuth, 2*pi )
zen_1st = 0.0;     ( lower limit zenith (line of sight) )
zen_last = 1.570796; ( upper limit zenith (line of sight), pi/2 )

TYPE  ( descriptions of integrands for Simpson's integration )
integrand = (chapman, azimuth, zenith);

VAR  ( input/output parameters, input is 1 line in SKY.DATA.TEXT )
zeta_S,      ( zenith of sun at observer, radians;
              this could also be calculated from the
              latitude, day of year, & time of day )
flux: REAL;  ( result: monochromatic flux density )

( independent variables )
zeta_P,      ( zenith of line of sight at observer, rad )
delta: REAL; ( azimuth relative to solar vertical, rad )

( scattering and refraction variables )

```

```

theta_s,          ( scattering angle, radians )
beta,             ( angular scattering coeff, 1/(km*sr) )
beta_total,      ( total scattering coefficient, 1/km )
chapman_fn,      ( Chapman function calc'd for chi & zeta_S )
beta_Z: REAL;    ( calc'd product of beta_total * Z_zero )

( miscellaneous variables )
opacity,         ( relative irradiance (noon ref), decibels;
                 this is the relative capacity of matter to
                 obstruct the transmission of radiant energy )
noon_flux: REAL; ( monochromatic radiance at noon, W/m^2*nm )
choice: integrand;
i: INTEGER;
Chap_quad,exp_S: REAL;
infiTe, outfiTe: TEXT;
separatorline,spaces: STRING;

PROCEDURE Integrate(choice: integrand; lower,upper,tol: REAL;
VAR sum:REAL);FORWARD;

FUNCTION EXPO(x:REAL):REAL;
( checks for exponent to be within bounds/max real = 10^308 )

BEGIN
  IF x < -700 THEN
    EXPO := 0.0
  ELSE
    EXPO := EXP(x);
  END (EXPO);

FUNCTION ScatteringFn(solar_zenith, obs_pt_zenith, azimuth: REAL):REAL;
( calculates the ratio of angular to total scattering function as a
function of the scattering angle )
VAR beta_ratio,cos_of_theta: REAL;

BEGIN
  cos_of_theta := COS(solar_zenith) * COS(obs_pt_zenith)
                + SIN(solar_zenith) * SIN(obs_pt_zenith) * COS(azimuth);
  beta_ratio := (1 + SQR(cos_of_theta)) * 3 / (8 * pi);
  IF debugging THEN BEGIN
    WRITELN('COS(theta_s) ',cos_of_theta);
    WRITELN('beta_ratio = ',beta_ratio);
  END;
  ScatteringFn := beta_ratio;
END (ScatteringFn);

FUNCTION ChapFX(chi,zeta,psi: REAL):REAL;
( calculates the kernel for the Chapman function integration )
VAR kernel:REAL;

BEGIN
  IF psi = 0 THEN
    kernel := 0
  ELSE
    kernel := EXPO(chi * (1 - SIN(zeta) / SIN(psi))) / SQR(SIN(psi));
  ChapFX := kernel;
END (ChapFX);

FUNCTION Intensity(delta, zeta_P: REAL):REAL;
( calculates the intensity function already integrated in closed for a
plane parallel atmosphere with curvature corrections for air mass and
solar irradiance )
VAR kernel, radiance, air_mass, exp_P, cos_zeta: REAL;

BEGIN
  cos_zeta := COS(zeta_P);
  air_mass := 1 / (cos_zeta + 0.025 * EXP(-11 * cos_zeta));
  exp_P := EXPO(-beta_Z * air_mass);
  IF air_mass = chapman_fn THEN
    radiance := E_zero * ScatteringFn(zeta_S, zeta_P, delta)
              * beta_Z * chapman_fn * exp_P
  ELSE
    radiance := E_zero * ScatteringFn(zeta_S, zeta_P, delta)
              * (exp_S - exp_P) / (1 - chapman_fn/air_mass);
  ( multiply by SIN * COS to get kernel for flux density integration )
  kernel := radiance * COS(zeta_P) * SIN(zeta_P);
  Intensity := kernel;
END (Intensity);

```

```

FUNCTION FX(choice: integrand; x: REAL):REAL;
( "choice" gives the correct function for Simpson's integration )
VAR result: REAL;

BEGIN
  CASE choice OF
    azimuth: BEGIN
      delta := x;
      Integrate(zenith, zen_1st, zen_last, tol, result);
    END;
    zenith: BEGIN
      zeta_P := x;
      IF debugging THEN BEGIN
        WRITELN('delta = ', delta);
        WRITELN('zeta_P = ', zeta_P);
      END;
      result := Intensity(delta, zeta_P);
    END;
    chapman: result := ChapFX(chi, zeta_S, x);
  END (of CASEs);
  FX := result;
END (FX);

```

```

PROCEDURE Integrate(choice: integrand; lower, upper, tol: REAL; VAR sum: REAL);
( numerical integration by Simpson's method of FX(x) for x = [lower, upper] )
VAR
  steps: INTEGER;
  x, delta_x, even_sum, odd_sum, end_sum, sum1: REAL;

```

```

BEGIN
  x := lower;
  steps := 2;
  delta_x := (upper - lower) / steps;
  odd_sum := FX(choice, lower + delta_x);
  even_sum := 0;
  end_sum := FX(choice, lower) + FX(choice, upper);
  sum := (end_sum + 4 * odd_sum) * delta_x / 3;
  IF debugging THEN
    BEGIN
      CASE choice OF
        azimuth: WRITELN(steps:6, x:10:3, spaces, sum:10:3);
        zenith: WRITELN(spaces, steps:6, x:10:3, sum:10:3);
      END (of CASEs);
    END;
  REPEAT
    steps := steps * 2;
    sum1 := sum;
    delta_x := (upper - lower) / steps;
    even_sum := even_sum + odd_sum;
    odd_sum := 0;
    FOR i := 1 TO steps DIV 2 DO
      BEGIN
        x := lower + delta_x * (2 * i - 1);
        odd_sum := odd_sum + FX(choice, x);
      END;
    sum := (end_sum + 4 * odd_sum + 2 * even_sum) * delta_x / 3;
    IF debugging THEN
      BEGIN
        CASE choice OF
          azimuth: WRITELN(steps:6, x:10:3, spaces, sum:10:3);
          zenith: WRITELN(spaces, steps:6, x:10:3, sum:10:3);
        END (of CASEs);
      END;
  UNTIL ABS(sum - sum1) <= ABS(tol * sum);
END (Integrate);

```

```

PROCEDURE Initialize;
( various and sundry business to take care of )

```

```

BEGIN
  ( calc total scattering coefficient in 1/km )
  beta_total := 9 * pi * SQR(pi) * SQR(SQR(n)-1) * 1.0E39 /
    (3 * N_mol * SQR(SQR(lambda)));
  ( calc the Chapman function for the given solar zenith )
  IF zeta_S < 1.3 (74.5 degrees) THEN
    chapman_fn := 1 / COS(zeta_S)
  ELSE
    BEGIN
      Integrate(chapman, 0, zeta_S, tol, Chap_quad);
      chapman_fn := chi * SIN(zeta_S) * Chap_quad;
    END;

```

```

END;
beta_2 := beta_total * 2 zero;
exp_S := EXPO(-beta_2 * chapman_fn);
IF Debugging THEN BEGIN
  WRITELN('beta_total = ',beta_total);
  WRITELN('chapman_fn = ',chapman_fn);
  WRITELN('beta_2 = ',beta_2);
  WRITELN('exp_S = ',exp_S);
END;
separatorline := '-';
spaces := ' ';
FOR i := 1 TO 79 DO
  separatorline := CONCAT(separatorline,'-');
END (Initialize);

```

```

PROCEDURE PageHeader;
( write input data to output file and titles for tabulated output )

```

```

BEGIN
  WRITELN(outfile); WRITELN(outfile); WRITELN(outfile);
  WRITELN(outfile,'Calculation of Sky Radiant Flux');
  WRITELN(outfile); WRITELN(outfile);
  WRITE (outfile,'Solar Zenith':15,'Chapman ':18,'Mono. Flux Density':22);
  WRITELN(outfile,' Opacity ':15);
  WRITE (outfile,' (degrees) ':15,'Function':18,' (Watts/m^2*nm) ':22);
  WRITELN(outfile,'(decibels)':15);
  WRITE (outfile,'-----':15,'-----':18,'-----':22);
  WRITELN(outfile,'-----':15);
  ( and write it out the the screen as well... )
  WRITELN; WRITELN; WRITELN;
  WRITELN('Calculation of Sky Radiant Flux');
  WRITELN; WRITELN;
  WRITE ('Solar Zenith':15,'Chapman ':18,'Mono. Flux Density':22);
  WRITELN(' Opacity ':15);
  WRITE (' (degrees) ':15,'Function':18,' (Watts/m^2*nm) ':22);
  WRITELN(' (decibels)':15);
  WRITE ('-----':15,'-----':18,'-----':22);
  WRITELN('-----':15);
END (PageHeader);

```

```

BEGIN (SkyRadiantFlux)
  REWRITE(outfile, outfilename);
  PageHeader;
  zeta_S := 0;
  opacity := 0;
  WHILE opacity <= night_def DO BEGIN
    Initialize;
    Integrate(azimuth, az_1st, az_last, tol, flux);
    IF zeta_S = 0 THEN noon_flux := flux;
    opacity := 10 * LOG(noon_flux/flux);
    WRITELN(outfile,zeta_S/convert:15:2,chapman_fn:18:3,flux:22:9,
      opacity:15:2);
    WRITELN(zeta_S/convert:15:2,chapman_fn:18:3,flux:22:9,opacity:15:2);
    zeta_S := zeta_S + 1 * convert;
  END (WHILE);
  CLOSE(outfile, LOCK);
END (SkyRadiantFlux).

```

MONOCHROMATIC IRRADIANCE VS. ZENITH

Monday, 14 January 1985 4:07 PM

PAGE 1

E (Solar irradiance) = 2.00 W/m²*nm at (wavelength) = 459.3 nm

Solar Zenith (degrees)	Chapman Function	Mono. Flux Density (Watts/m ² *nm)	Opacity (decibels)
0.00	1.000	0.277675551	0.00
5.00	1.004	0.277429912	0.00
10.00	1.015	0.276693685	0.02
15.00	1.035	0.275468486	0.03
20.00	1.064	0.273755427	0.04
25.00	1.103	0.271552579	0.10
30.00	1.155	0.268950962	0.14
35.00	1.221	0.265628529	0.19
40.00	1.305	0.261841014	0.25
45.00	1.414	0.257407733	0.23
50.00	1.556	0.252188539	0.42
55.00	1.743	0.245944344	0.53
60.00	2.000	0.238264905	0.66
65.00	2.366	0.229426001	0.85
70.00	2.924	0.215079046	1.11
75.00	3.782	0.197046060	1.49
74.00	4.033	0.192224544	1.60
77.00	4.321	0.186924108	1.72
78.00	4.653	0.181072870	1.85
79.00	5.040	0.174586476	2.02
80.00	5.496	0.167366458	2.20
81.00	6.040	0.159299163	2.41
82.00	6.700	0.150256251	2.67
83.00	7.515	0.140098668	2.97
84.00	8.539	0.128688104	3.34
85.00	9.961	0.115911849	3.79
86.00	11.616	0.101732926	4.36
87.00	14.027	0.086277843	5.08
88.00	17.491	0.069965942	5.99
89.00	22.689	0.053632170	7.14
90.00	31.037	0.038481520	8.58
91.00	45.449	0.025662122	10.34
92.00	72.638	0.015735736	12.47
93.00	129.575	0.008696670	15.04
94.00	264.030	0.004230336	18.17
95.00	627.384	0.001771989	21.95
96.00	1765.305	0.000628543	26.45
97.00	5935.977	0.000186841	31.72
98.00	23955.375	0.000046309	37.78
99.00	116186.717	0.000009553	44.63
100.00	677317.886	0.000001640	52.29
101.00	4743964.314	0.000000234	60.74
102.00	39898639.363	0.000000028	69.98
103.00	402682251.982	0.000000003	80.02