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## THREE-DIMENSIONAL RECURSIVE FILTER OBJECTIVE ANALYSIS OF METEOROLOGICAL FIELDS

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## 1. INTRODUCTION

Thiebaut and Pedder (1987) have categorized types of objective analysis used in the atmospheric sciences into three classes: surface fitting, empirical linear interpolation, and statistical objective analysis. The recursive filter method described here falls in the second lot, and is thus something of a throwback from the currently fashionable third category. However, the recursive filter is faster and more economical and thus more practical for applications with small computers and limited time. For this reason, it has been implemented on the University of Wisconsin McIDAS to provide gridpoint analyses from the real time data bases available on that system. Why another empirical fitting technique when other excellent methods exist (e.g., Cressman, 1959 or Barnes, 1973)? We believe that the recursive filter technique provides rather unique capabilities for handling domain boundaries and background fields, and for locally variable scaling dictated by observation quantity and quality. Also, it provides gridpoint estimates of the reliability of the final analysis as a by-product (somewhat like the more elegant statistical schemes). This is useful for both qualitative and quantitative application of the analyses.

The basic algorithm has been discussed previously in Hayden and Purser (1984) with a number of examples to elucidate the capability of the system. In this paper, we amplify on some special features of the two-dimensional system and also discuss and present examples of the extension to three dimensions.

## 2. THE TWO-DIMENSIONAL RECURSIVE FILTER OBJECTIVE ANALYSIS ALGORITHM

This section treats the analysis algorithm as the sum of five parts. The first part describes the recursive filter which is simply a technique for smoothing the information at individual gridpoints to surrounding gridpoints. The treatment of boundary points is included. The second part describes how random data and quality indicators are interpolated to regular gridpoints, so that the recursive filter can be applied. The third part explains the quality control algorithm and the update of the background field. The fourth part discusses the use of an initial background field, and the fifth part describes successive correction iterations which complete the analysis.

## 2.1 The Recursive Filter

The recursive filter is a smoother which is applied over a grid. The basic algorithm is described by:

$$B_n = \alpha B_{n-1} + (1-\alpha) A_n \quad (1)$$

According to (1), for gridpoint  $n$ , a new estimate  $B$  is obtained from a weighted average of the previous value  $A$  and the new estimate at the preceding gridpoint ( $n-1$ ). The weighting parameter can be considered a smoothing parameter since it controls the influence of the preceding gridpoints. This is more easily seen when (1) is written in the recursive form:

$$B_n = (1-\alpha) (A_n + \alpha A_{n-1} + \dots + \alpha^m A_{n-m}) \quad (2)$$

In practice, (1) is applied repeatedly across the rows and columns of the grid in alternating directions to provide symmetry. The aggregate smoothing is equivalent (Purser and McQuigg, 1982) to convolving the field with a Gaussian filter of the form:

$$G(x) = e^{-x^2/4\pi\lambda^2} / ((2\pi)(2\pi\lambda))^{\frac{1}{2}} \quad (3)$$

where  $L$  is the number of passes of the filter and  $\lambda$  is related to the smoothing parameter  $\alpha$  according to

$$\lambda = 1/\log \alpha; \quad \lambda^2 = \alpha / (1-\alpha)^2 \quad (4)$$

The intrinsic scale of the filter is given by the standard deviation (in units of grid intervals):

$$\sigma = (2L)^{\frac{1}{2}} \lambda \quad (5)$$

which can be related to a characteristic wavelength  $R$  if the gridlength is given by  $\delta$ :

$$\sigma^2 = (R/\delta)^2 = 2L\lambda^2 = 2L\alpha / (1-\alpha)^2 \quad (6)$$

In using the recursive filter analysis it is convenient to think in terms of characteristic wavelengths, and this is the parameter specified by the user. The actual smoothing parameter  $\alpha$  of (1) is internally computed according to (6). It is interesting to note that the accrued effect of the multiple passes is equivalent to a single pass

of a "Barnes" system which employs a Gaussian filter directly (Barnes, 1964; Koch et al., 1984).

The preceding discussion implicitly assumes convolutions extending over an unlimited number of gridpoints along a line. In reality, boundaries intervene. It is possible, however, by selecting suitable starting conditions for each application of the elementary filter (1), to simulate the effect of an infinite continuation of the grid, thereby mitigating spurious boundary effects. Because the specification becomes increasingly complex with successive passes, boundary conditions are defined in the algorithm for only the zeroth through second passes, after which the formulation for the second pass is repeated. The derivation is included as Appendix A.

## 2.2 Initial Grid Specification

The discussion above has treated the recursive filter acting on a gridpoint field. In practice, observations do not coincide with gridpoints, and it is necessary to transfer the data before applying the recursive filter. This is accomplished using the adjoint of simple bi-linear interpolation to distribute each observation to the surrounding gridpoints (see Fig. 1). Note that  $RX+RXI = RY+RYI = 1$ .

$$\begin{array}{c}
 A \quad \uparrow \quad B \\
 \quad \quad RY \\
 \quad \quad \downarrow \\
 \leftarrow RX \rightarrow O \leftarrow RIX \rightarrow \\
 \quad \quad \uparrow \\
 \quad \quad RYI \\
 C \quad \downarrow \quad D \\
 \\
 A^1 = A + RIX * RYI * O \\
 B^1 = B + RX * RYI * O \\
 C^1 = C + RIX * RY * O \\
 D^1 = D + RX * RY * O
 \end{array}$$

Fig. 1. Bilinear weighting to distribute observations (O) to gridpoints (A,B,C,D).

Two grids are created. The first represents the observation weight which is a function of both number and quality (to be described below) of data influencing each gridpoint. The second represents the difference between the observation(s) and some background estimate (bi-linearly interpolated to the observation), multiplied by the observation weight. These grids are cumulative in the sense that the contributions of multiple observations within a grid square are added at the surrounding gridpoints. A multiple pass recursive filter is applied to both grids with smoothing specified as described previously.

## 2.3 Quality Control and Background Update

Observation quality is assessed according to the function

$$Q = 1/((1+(D/T)^4)); D = |O-A| \quad (7)$$

where O is the observed value, A is the value interpolated in the background field, and T is a tolerance which defaults to twice the standard deviation of the observations, but which can also

be specified by the user. Note that an observation which exceeds the tolerance is not "rejected", but only down-weighted.  $QW=Q*W$ , where W is a relative reliability assigned to the observation, is the quantity which is distributed to and accumulated at the gridpoints to form the grid of observation weight.

The analysis background field is updated after the accumulated QW and  $QW*(O-A)$  grids which have been operated on by the recursive filter. The new value at a gridpoint is given by

$$A^1 = A + C*QW(O-A)/C*QW \quad (8)$$

where C\* represents the application of the multiple pass recursive filter.

The field of the denominator of (8) is the grid of analysis quality weights mentioned in the introduction. Note that this has the implicit scaling of the recursive filter combined with the implicit quality of the observations. The weight at any gridpoint represents a local scaling (in two dimensions) which may be written as

$$R_n = 1/(C*QW)_n \quad (9)$$

## 2.4 Initial Background

The background field to initiate the analysis is a uniform grid where each gridpoint is assigned the average value of the observations. Thus, there is no requirement for a prior analysis or forecast of the analysis variable. There is, however, provision for a prior estimate, if it is available; but it is not used in the traditional manner, as an initial grid. Rather, uniformly spaced "data" are taken from the analysis/forecast, assigned a reliability, and treated exactly as other observations. The density of these "data" can be selected by the user, but defaults to a value for every fourth gridpoint. There is no unique advantage to this over the traditional approach, but it is simple way of insuring that both observational and background information are treated analogously. The reliability of the background data may be assigned by the user, but defaults to a value such that the sum of the background data reliability is equal to the sum of the observational data reliability.

## 2.5 Successive Approximation

The previous paragraphs have defined the basic elements of an analysis where the influence of data is spread according to a characteristic wavelength R, with a quality control related to a tolerance T. The complete recursive analysis algorithm is comprised of a number of iterations, each using the result of the previous iteration as the background. On each iteration, the characteristic scaling is changed. It would be possible to vary the scaling isotropically by reducing R from some initial R0 to successively smaller values. In this way, R would play much the same role as the "radius of influence" of other successive correction schemes (e.g., Cressman, 1959). However, (9) allows the option of varying R locally to reflect both observation density and quality as determined from the previous iteration. In practice, the two concepts are combined such that (9) is used, except that it is bounded to be no larger than a predetermined, successively

shrinking maximum and no smaller than a predetermined minimum (which must be at least two grid-lengths). Thus, observation density, quality, and a nominal "radius of influence" share in the local smoothing of the observations. In this way, erroneous data are controlled, and there is no attempt to analyze scales not resolved by the local observation density. Details of how the scaling is bounded, and how the quality control is varied on successive iterations are given in Appendix B.

### 3. THE THREE DIMENSIONAL EXTENSION

The extension to three dimensions is fairly straightforward. On a single smoothing pass, the recursive filter is operated backwards and forwards along rows and columns as discussed above, but at a number of vertical levels. The filter is then passed up and down in the vertical dimension at each horizontal gridpoint. This procedure is repeated for the specified number of smoothing passes. The maximum horizontal characteristic wavelength  $R_0$  is varied for each level since the atmosphere tends to show smaller scales at lower levels. The horizontal smoothing at each level is related to the vertical smoothing through a vector (ZMET) which may be defined by the operator, but which defaults to a value such that the vertical scale is about a hundredth of the horizontal scale. Interpolation to the observation location along the vertical currently uses a six point (where possible) Lagrangian interpolation, since this appears to improve the convergence of the analysis as compared to the one dimensional bi-linear relationship shown in Fig. 1. However, the observation corrections are distributed to the surrounding levels using the bi-linear formulation, since experiments using the adjoint of the Lagrangian interpolation have not been satisfactory, perhaps because of the varying signs of the coefficients. We do not feel that this matter is completely resolved. If the horizontal versus vertical scaling were correctly specified, it seems that the simple bi-linear relationships should work in all dimensions.

A background field may be applied in the three dimensional analysis by the same technique described above. Background "data" are taken from whatever analyses/forecasts are available. In the three dimensional case, there is a clear advantage to this technique over the traditional method since analyses/forecasts do not have to be available for all vertical levels, nor do they need to be in the same vertical dimension to be readily used.

### 4. EXAMPLES

Figures 2 and 3 present examples of the three-dimensional analysis results at 500 mb for the situation of 00 UT, 28 January 1986. Figure 2 shows the geopotential derived for analysis iteration 1, 4, and 8. Figure 3 shows the corresponding (normalized) reliability field. The analysis was done at ten pressure levels with a horizontal gridlength of 111 km. A prior analysis was used to provide background data of uniform reliability at every fourth gridpoint. The figures show the "successive approximation" feature of the system, especially with respect to how the reliability field encloses the observa-

tions. Note that the reliability field becomes uniform over the Atlantic as the iterations increase, reflecting that the analysis is a product of only background data. A slight fall-off occurs at the edge of the grid, and more at the corner, reflecting that the boundaries prohibit uniform, isotropic data coverage. Figures 2 and 3 can be compared with Fig. 4 which represents the eight iteration analysis without background data. The reliability field shows more concentration around the radiosonde stations and a much sharper fall-off. The analysis in the southeast over the ocean is designated as completely unreliable, and the geopotential here versus that shown in Fig. 2 suggests that this appraisal is accurate.

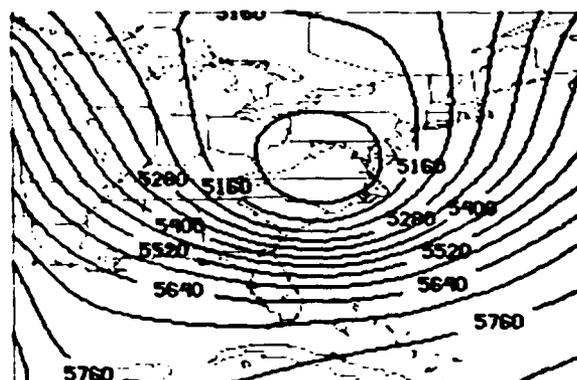
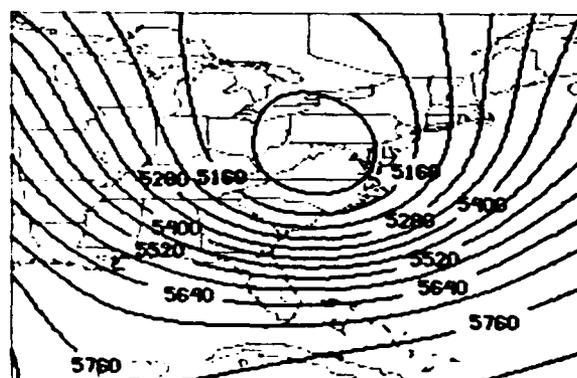
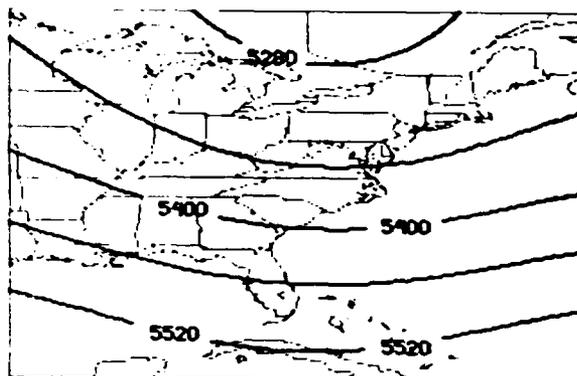


Fig. 2. 500 mb geopotential, 00UT 28 January 1986 top to bottom: one, four and eight analysis iterations.

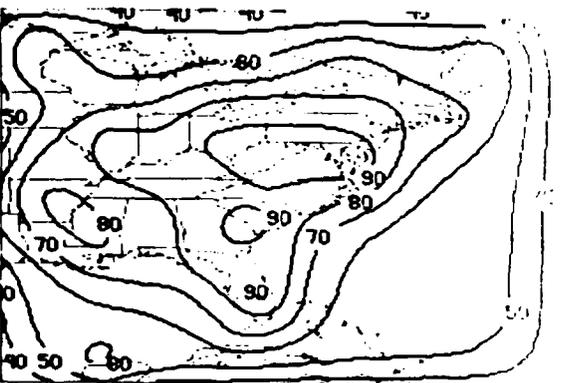
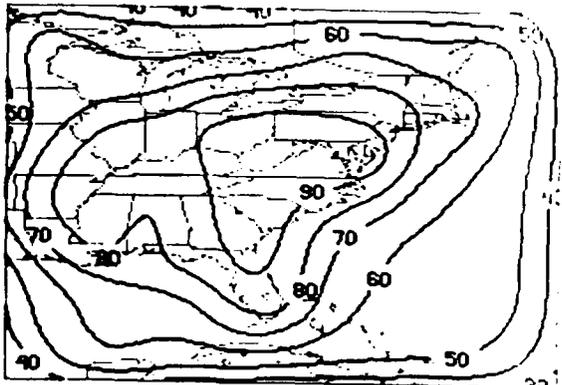
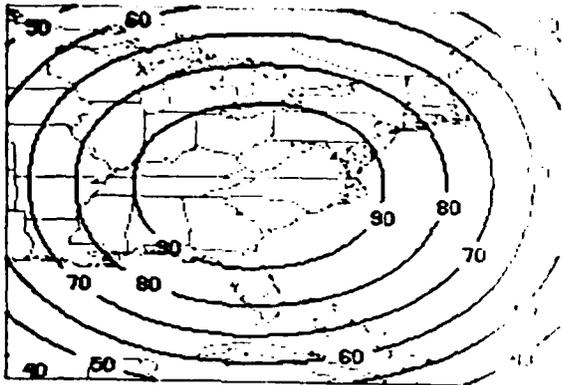


Fig. 3. Reliability fields associated with analyses of Fig. 2.

Figures 5 and 6 represent a temperature analysis at 750 mb. Since this is not a standard reporting level, the analysis is a product of vertical interpolation and significant level reports. Temperature contours are shown in Fig. 5 for an analysis which used the significant level data. The field of differences from an analysis which did not use the significant level data is also shown. It can be seen that the significant level data has sharpened the analysis, making changes of greater than 2 K in the vicinity of Florida. Figure 6 shows the reliability fields associated with the two analyses. The one without significant level data shows that the result is almost entirely composed of the background data, though there is a slight

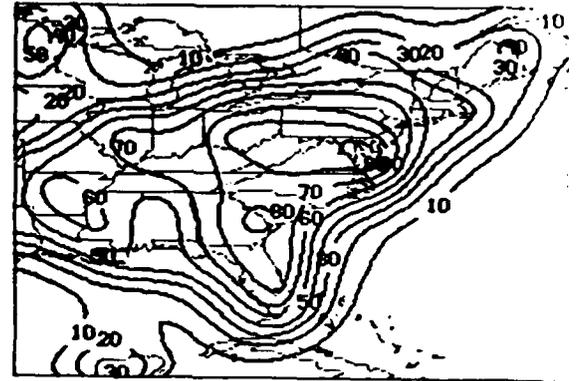
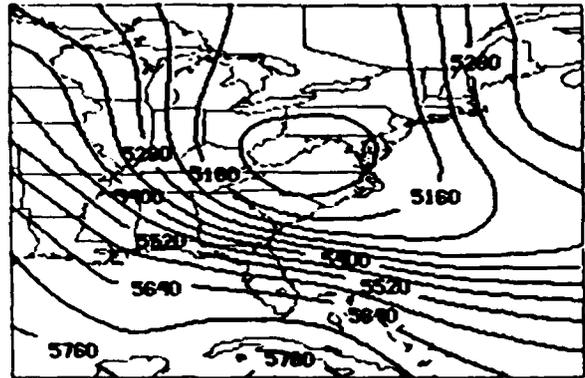


Fig. 4. Geopotential and reliability analyses of Fig. 2 without background estimate.

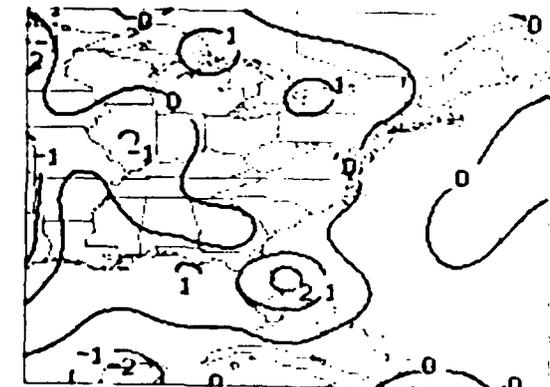
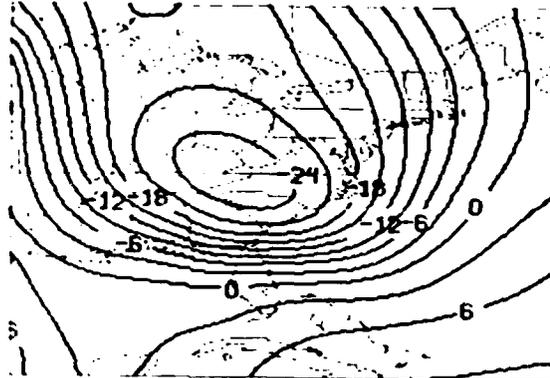


Fig. 5. Top: 750 mb temperature analysis using significant level data. Bottom: difference from analysis without significant level data.

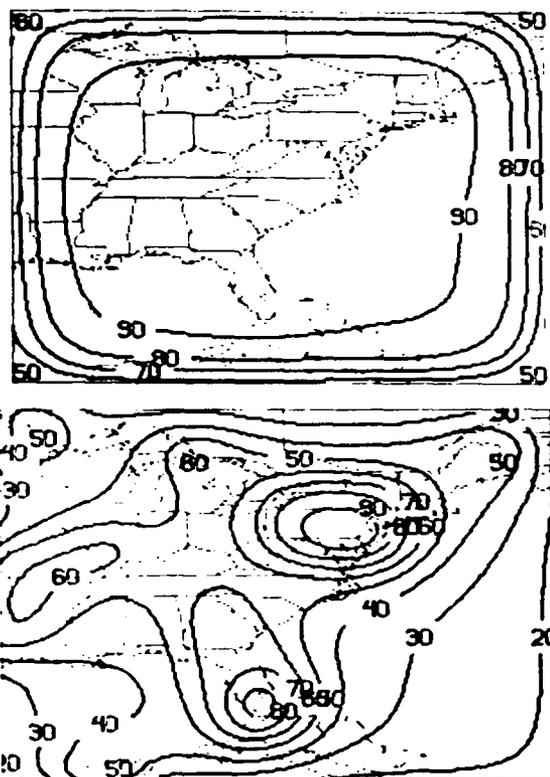


Fig. 6. Reliability fields of 750 mb analysis without (top) and with (bottom) significant level data.

skewing over the land. One can also see that the reliability field with significant data is lower over the data sparse Atlantic than for the 500 mb geopotential analysis (Fig. 3). This is because the background data reliability is adjusted according to the density of observations as explained above.

#### 5. SUMMARY

This paper has described the features of a three-dimensional objective method for deriving meteorological analyses. While the analysis is not optimal in any rigorous statistical sense, it is highly versatile and efficient computationally. For example, the analysis described in section 4 comprised 961 horizontal gridpoints at ten levels with 1468 observations. Using eight analysis iterations, it was accomplished in 15 seconds on the McIDAS (an IBM 4381). The three-dimensional form of the analysis allows observations at significant levels to be easily incorporated (especially important for cloud drift winds). The user is able to specify the relative weight of observations and the background field independently at each level. A simple quality control procedure allows the gross error of observations to be identified and rendered ineffective by severely down-weighting these observations before they can impact the analysis. Apart from efficiency, the main strength of the scheme is its ability to handle the presence of data-dense and sparse areas in the same domain, producing fields of reliability which are useful in the application of the analyses.

#### 6. APPENDIX A

The boundary problem arises in specifying the updated value of the leftmost gridpoint for a right-moving filter, and vice versa, for multiple passes, since there is an implied "tail" extending beyond the grid from preceding passes. For the zeroth pass, no special treatment is required since there is no "tail" and the initial adjustment is just

$$B_1 = (1-\alpha) A_1 \quad (A1)$$

On succeeding passes, the tail can be treated explicitly. However, the situation becomes increasingly complicated, and as mentioned in the main text, the analysis algorithm specifies starting conditions for only the first and second passes, after which the relationship for the second pass is repeated.

In general, the starting conditions for a right-moving filter can be derived by considering the aggregate of multiple applications of a left-moving filter which is expressed by:

$$A_n^P = \alpha_1 A_{n+1}^P + \alpha_2 A_{n+2}^P + \dots + \alpha_p A_{n+p}^P + (1 - \sum_{i=1}^p \alpha_i) A_n^O \quad (A2)$$

where  $n$  is a gridpoint number,  $p$  is the pass number, the  $\alpha_i$  are some combination of the basic smoothing parameter  $\alpha$ , and the  $A_n$  are the gridpoint values. Note that for gridpoint 0 (to the left of the row) (A2) reduces to

$$A_0^P = \alpha_1 A_1^P + \alpha_2 A_2^P + \dots + \alpha_p A_p^P \quad (A3)$$

since the initial value  $A_0^O$  is not defined outside the grid.  $A_0^P$  is precisely what is required to begin the right-moving filter at gridpoint 1. Thus, the problem reduces to specifying the  $\alpha_i$ .

For  $p=1$ , (A2) reduces to

$$A_n^1 = \alpha_1 A_{n+1}^1 + (1-\alpha_1) A_n^O \quad (A4)$$

and thus  $\alpha_1 = \alpha$ . For  $p=2$ , there are the basic relationships:

$$\begin{aligned} A_n^1 &= \alpha A_{n+1}^1 + (1-\alpha) A_n^O \\ A_n^2 &= \alpha A_{n+1}^2 + (1-\alpha) A_n^1 \\ A_{n+1}^2 &= \alpha A_{n+2}^2 + (1-\alpha) A_{n+1}^1 \end{aligned} \quad (A5)$$

Eliminating  $A_n^1$  and  $A_{n+1}^1$ :

$$A_n^2 = 2\alpha A_{n+1}^2 - \alpha^2 A_{n+2}^2 + (1-\alpha)^2 A_n^O$$

which is of the form (A2) where  $\alpha_1 = 2\alpha$  and  $\alpha_2 = -\alpha^2$ .

So to begin the right-moving filter for  $p=1$

$$A_0^1 = \alpha_1 A_1^1 = \alpha A^1$$

but

$$A_1^1 = \alpha A_0^1 + (1-\alpha) A_n^0 \quad (A6)$$

so

$$A_1^1 = \frac{(1-\alpha)}{(1-\alpha^2)} A_1^0$$

and for p=2

$$A_0^2 = \alpha_1 A_1^2 + \alpha_2 A_2^2 = 2\alpha A_1^2 - \alpha^2 A_2^2$$

$$A_1^2 = \alpha A_0^2 + (1-\alpha) A_1^1$$

$$A_2^2 = \alpha A_1^2 + (1-\alpha) A_2^1$$

Eliminating  $A_0^2$  and  $A_2^2$

$$A_1^2 = \frac{1-\alpha}{(1-\alpha^2)^2} (A_1^1 - \alpha^3 A_2^1) \quad (A6)$$

For the left-moving filter, the starting conditions are symmetric since the filters commute.

#### 7. APPENDIX B

The concept of locally varying scaling which is determined by observation quantity and quality has been developed in the main text. For each gridpoint, after the first iteration of the analysis, there is a weight  $W_n$  which is implicitly related to a scale  $R_n$ , depending on the scaling of the recursive filter applied in the previous iteration

$$W_n = \frac{1}{R_n^2} \quad (B1)$$

(Note:  $R_n^3$  is used in the three-dimensional version.)  $R_n$  is used to determine the local scaling of the recursive filter, subject to bounding limits which will be explained here. First, however, it is necessary to express the way in which the scaling is formulated in the recursive filter algorithm. As presented in the main text, the characteristic wavelength of the recursive filter is given by

$$(R/\delta)^2 = 2L\alpha / (1-\alpha)^2 \quad (B2)$$

where L is the number of passes and  $\delta$  is the gridlength. If the basic recursive algorithm is reformulated such that  $\beta = 1-\alpha$ ,

$$B_n = (1-\beta) B_{n-1} + \beta A_n$$

then (B2) leads to

$$\beta = -e + \sqrt{e(e+2)} \quad (B3)$$

where

$$-e + (e(e+2))^{1/2}$$

$$e = L\delta^2/R^2 = L\delta^2W \quad (B4)$$

For the first iteration of the analysis, the  $R$  of (B4) is  $R_0$  which is chosen by the operator as the largest characteristic scale for the analysis. For the second and subsequent iterations  $R$  is replaced by  $R_f * R_n$  where  $R_n$  is given by (B1) and  $R_f$  is a tuning constant which will be explained below. The value is constrained such that

$$R_m \leq R_f * R_n \leq R_0 \quad (B5)$$

$$R_m = RL + R_s (R_0 - RL)$$

where  $RL$  (chosen by the operator) represents a limiting wavelength, and  $R_s$  controls how rapidly the characteristic wavelength is diminished on each succeeding iteration  $i$  (a factor of .33 is the default value). Equation B5 is shown schematically in Fig. B1. Note from (B5) that the scale for the entire analysis is essentially limited between  $R_0$  and  $RL$ .

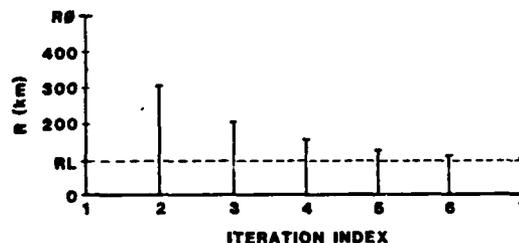


Fig. B1. Schematic of the assignment for  $R_m$  limit.

The factor  $R_f$  is included to give the operator some control over the scaling beyond that offered by choosing the upper and lower limits of  $R$ .  $R_f$  effectively sharpens or spreads the scaling according to whether it is less than or greater than one. For noisy data, a value of 2 is perhaps appropriate, whereas for very clean data, a value of 0.5 might be used.

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