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HYPERVELOCITY ORBITAL INTERCEPT GUIDANCE

by

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B.S., United States Air Force Academy, 1974
M.S., Air Force Institute of Technology, 1982

A thesis submitted to the
Faculty of the Graduate School of the
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of the requirements for the degree of
Doctor of Philosophy
Department of Electrical Engineering
1988
This thesis for the Doctor of Philosophy degree by
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has been approved for the
Department of
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by

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John M. Liebetreu

Date 14 April 1988
To my wife, Michele, for her unfailing love and support
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First and foremost, I would like to thank God for giving man the ability, limited though it may be, to see and understand the workings of His wisdom and His hand in all of creation. My thanks go to Dr. Charles E. Fosha, Jr., my thesis advisor, and Dr. Robert B. Asher, for their guidance on this project. I would also like to thank Dr. John M. Liebetreu for his careful review of this manuscript as a second reader. In addition, I wish to thank the other members of my examining committee, Drs. Saber Elaydi, Ronald M. Sega, Renjeng Su, Mark A. Wickert, and Rodger E. Ziemer for their helpful suggestions.
Terminal guidance of a hypervelocity exo-atmospheric orbital interceptor with free end-time is examined. The pursuer is constrained to lateral thrusting with the evader modeled as an ICBM in its final boost phase. Proportional navigation, optimal control using certainty equivalence, dual control, and control with optimum thrust spacing are all examined. Also, a new approach called certainty control is developed for this problem. This algorithm constrains the final state to a function of projected estimate error to reduce control energy expenditure. All methods model the trajectories using splines and employ eight state Extended Kalman Filters with line-of-sight and range updates. The relative effectiveness of these control strategies is illustrated by applying them to various intercept problems.
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Orbital interceptor performance can be enhanced by using a terminal guidance law that incorporates the orbital dynamics of the pursuer and evader plus the error knowledge of their estimates. The purpose of this research is to develop a guidance scheme for a hypervelocity, exo-atmospheric orbital vehicle in the final thirty seconds of flight that minimizes lateral thrusting while attempting to intercept a boosting missile. Much work has been done on the most common form of intercept guidance, proportional navigation, and its variations. This type of navigation assumes that the force of gravity acts equally on the pursuer and evader and can therefore be ignored in the relative dynamics. For orbital intercepts with large initial ranges the force of gravity will affect the relative trajectory and should be included in the equations of motion. To date, analytic solutions for such intercepts exist only when the pursuer's impact conditions are prespecified.

The general guidance schemes studied in this research attempt to minimize lateral velocity changes by varying the impact conditions through the use of splines. The pursuer is modeled as
a satellite with lateral thrusting capability using two-body orbital dynamics. The evader is modeled as an Intercontinental Ballistic Missile (ICBM) in its final boost phase, prior to burnout. The relative trajectory is propagated numerically to predicted impact time and then approximated by splines, eliminating the need to repeatedly propagate new trajectories when present conditions are varied. A search is then made for a new impact time and point that will minimize present interceptor velocity changes and final miss distance.

Six different variations of the general scheme are derived. The first scheme, presented in Chapter VI, uses a variable weighting factor and the principle of certainty equivalence to reduce velocity changes at the expense of final accuracy. The second scheme, also presented in Chapter VI, is a specialized version of the first, determining the velocity changes for zero miss. The third guidance algorithm in Chapter VI ignores the effects of gravity on the relative trajectory while attempting a zero miss solution. The fourth scheme, presented in Chapter VII, optimizes thrust spacing for a zero miss solution. The fifth scheme employs dual control techniques to reduce estimation error and is also presented in Chapter VII. The last algorithm is a new control approach that constrains the predicted miss distance to a function of final estimator error and is presented in Chapter VIII. Chapter IX summarizes the control strategies.
Target tracking is accomplished with a ranging device and line-of-sight sensors for in-plane and out-of-plane measurements. Noise corrupted data is processed through an eight state extended Kalman Filter with serial updates occurring every tenth of a second. The Kalman Filter equations are contained in Chapter X.
CHAPTER II

REVIEW OF LITERATURE

Much work has been done in the area of air-to-air intercept guidance. Guelman has derived a closed form solution for pure proportional navigation [1],[2] which is implemented in Chapter VI. Perturbation methods have been employed by Sridhar and Gupta [3]. Design procedures using optimal and stochastic control techniques abound [4]-[14] with variations of these techniques implemented in Chapters VI and VII. In the works cited above, the force of gravity is assumed to act equally on the pursuer and evader and is ignored in the relative dynamics. This 'flat earth' assumption is adequate for air-to-air encounters, but not for space-to-space. For orbital intercepts with large initial ranges the force of gravity will affect the relative trajectory and should be included in the equations of motion.

The literature for space-to-space guidance reveals many numerical approaches for determining present velocity for future rendezvous [15]-[21]. To date, analytic solutions for such intercepts exist only when the pursuer's impact conditions are pre-specified [19]. These works do not address hypervelocity intercept involving seconds, but are concerned with a much slower...
rendezvous process involving hours or even days. Also, most of
the literature reviewed assumed a passive, non-thrusting target.
The literature that addressed thrusting targets was concerned with
evasive maneuvering or 'gaming', the most recent being the paper
by Menon and Calise [21]. A Defense Technology Information Center
literature search revealed that the few papers addressing this
problem are classified and therefore unavailable to the public.

The guidance schemes presented here attempt to minimize
lateral velocity changes by varying the impact conditions through
the use of splines. Splines were used by Johnson [16] in
presenting a possible Earth-Mars transfer guidance algorithm.
Dickmanns and Wells have used third order polynomials for general
trajectory optimization [22], as well as Hargraves and Paris [23].
The splines eliminate the need to repeatedly propagate new
trajectories when conditions are varied, resulting in faster
searches. This feature makes them attractive for a hypervelocity
orbital intercept where a fast and reasonably accurate numerical
search is needed. Spline approximations are presented in Chapter
V.
CHAPTER III

SYSTEM MODELING

In this chapter, the equations of motion for the evader and pursuer are developed, along with the necessary coordinate transformation for pursuer thrusting. Atmospheric drag will not be considered in the dynamics because the interceptor is assumed exo-atmospheric. Also, due to the pursuer’s lateral thrusting limitation, the longitudinal axis will be assumed parallel to the pursuer’s initial velocity vector.

It is convenient to transform the present coordinate frame to align the x axis with the pursuer’s initial velocity vector. This is done by first rotating about the y axis until the z component of velocity is eliminated,

![Figure 3-1. Rotation of coordinate frame about the y axis.](image)

\[ V_p \]
resulting in the following orthogonal transformation matrix:

\[
[T_1] = \begin{bmatrix}
\frac{\dot{x}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} & 0 & \frac{\dot{z}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} \\
0 & 1 & 0 \\
-\frac{\dot{z}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} & 0 & \frac{\dot{x}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}}
\end{bmatrix}
\] (3-1)

The second rotation is about the new z axis \( (z') \), eliminating the y component of velocity.

Figure 3-2. Rotation of the coordinate frame about the z axis.
This rotation yields the transformation matrix:

\[
V_p = \sqrt{\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2}
\]  

(3-2)

\[
{T_2} = \begin{bmatrix}
\frac{\dot{x}_p^2 + \dot{z}_p^2}{V_p} & \frac{\dot{y}_p}{V_p} & 0 \\
-\frac{\dot{y}_p}{V_p} & \frac{\dot{x}_p^2 + \dot{z}_p^2}{V_p} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(3-3)

Multiplying the two matrices in the proper order produces the overall transformation matrix:

\[
{T} = {T_2}{T_1}
\]  

(3-4)

The pursuer is modeled as a satellite traveling in excess of twelve kilometers per second with lateral thrusting capability using two-body orbital dynamics. Thrusting is prohibited along the longitudinal (x) axis to prevent sensor contamination and to satisfy the structural constraints of having forward sensors and a large aft booster to achieve hypervelocity speed. The equations of motion are:
\[ \ddot{x}_p = -\frac{\mu x_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} \quad (3-5) \]

\[ \ddot{y}_p = -\frac{\mu y_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} + a_y \quad (3-6) \]

\[ \ddot{z}_p = -\frac{\mu z_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} + a_z \quad (3-7) \]

where \(a_y\) and \(a_z\) are the lateral thrust accelerations, \(\mu\) is the earth's gravitational constant, and the double dots denote the second derivative with respect to time.

The evader is modeled as an Intercontinental Ballistic Missile (ICBM) in its final boost phase using two-body orbital dynamics. For tracking purposes the intercept must occur prior to burnout. Acceleration due to thrusting is computed in the direction of the booster's velocity vector. The equations of motion are:

\[ A = \frac{A_0}{1 - \dot{m}_0 t} \quad (3-8) \]
\[ \ddot{x}_E = \frac{-\mu x_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{x}_E}{(x_E^2 + y_E^2 + z_E^2)^{1/2}} \tag{3-9} \]

\[ \ddot{y}_E = \frac{-\mu y_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{y}_E}{(x_E^2 + y_E^2 + z_E^2)^{1/2}} \tag{3-10} \]

\[ \ddot{z}_E = \frac{-\mu z_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{z}_E}{(x_E^2 + y_E^2 + z_E^2)^{1/2}} \tag{3-11} \]

where \( A \) is the present acceleration, \( A_0 \) the initial acceleration, \( \dot{m}_0 \) the initial mass flow rate divided by mass, and \( t \) the time since ignition. The single dot denotes the first derivative with respect to time.
CHAPTER IV

PROBLEM STATEMENT AND TRUTH MODEL

Time-to-go and pursuer velocity changes are the control parameters that must be varied to minimize miss distance and fuel expended (i.e. velocity changes). This can be done by establishing a time remaining until intercept (time-to-go), propagating the equations of motion forward, and determining the miss distance. An iterative process can then be used to find the pursuer velocity needed to bring the miss distance to zero. The difference between current velocity and that needed for intercept, known as velocity-to-go, must be minimized. To accomplish this, the time-to-go is varied and the above procedure repeated until a minimum velocity-to-go is found.

The computation of needed velocity is time consuming because the equations of motion are nonlinear and do not lend themselves to closed form solution. These equations must be propagated numerically to intercept time whenever the initial velocity is varied. The above method will serve as the basis (truth) model for this control problem using the numerical techniques found in Maron [24].

A Newton-Raphson method for solving nonlinear systems is
employed to determine the proper values of the control parameters.

Let

\[ \bar{u} = \begin{bmatrix} \Delta V_y \\ \Delta V_z \\ \bar{t}_{go} \end{bmatrix} \]  

(4-1)

be a solution of the nonlinear system

\[
\begin{align*}
\begin{bmatrix}
    f_1(\bar{u}) \\
    f_2(\bar{u}) \\
    f_3(\bar{u})
\end{bmatrix} &= \begin{bmatrix}
    x_E(\bar{t}_{go}) - x_p(\bar{t}_{go}) \\
    y_E(\bar{t}_{go}) - y_p(\bar{t}_{go}) - \Delta V_y \bar{t}_{go} \\
    z_E(\bar{t}_{go}) - z_p(\bar{t}_{go}) - \Delta V_z \bar{t}_{go}
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

(4-2)

where \( t_{go} \) is the time-to-go and the pursuer's velocity changes are \( \Delta V_y \) and \( \Delta V_z \). The effect of small velocity changes in (4-2) can be considered linear because the pursuer is assumed to travel at hypervelocity, resulting in a near straight-line trajectory. Any error caused by this assumption will be accounted for in the succeeding iteration when the proposed velocity change is incorporated in the nonlinear dynamics.

The initial control values must be incrementally changed to satisfy (4-2). A linear approximation of the \( f \) vector for
changes in \( y \) will yield approximate increments of the control parameters.

The linearized system becomes

\[
\begin{bmatrix}
d\Delta V_y \\
d\Delta V_z \\
dt_{go}
\end{bmatrix} =
\begin{bmatrix}
f_1(u) \\
f_2(u) \\
f_3(u)
\end{bmatrix}
\]  

where \( J \) is the Jacobian matrix of the \( f \) vector evaluated at \( y \):

\[
[J] =
\begin{bmatrix}
\frac{\partial f_1(u)}{\partial \Delta V_y} & \frac{\partial f_1(u)}{\partial \Delta V_z} & \frac{\partial f_1(u)}{\partial t_{go}} \\
\frac{\partial f_2(u)}{\partial \Delta V_y} & \frac{\partial f_2(u)}{\partial \Delta V_z} & \frac{\partial f_2(u)}{\partial t_{go}} \\
\frac{\partial f_3(u)}{\partial \Delta V_y} & \frac{\partial f_3(u)}{\partial \Delta V_z} & \frac{\partial f_3(u)}{\partial t_{go}}
\end{bmatrix}
\]  

(4-4)

Computing the partial derivatives yields
\[
[J] = \begin{bmatrix}
0 & 0 & \{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\} \\
-t_{go} & 0 & \{\dot{y}_E(t_{go}) - \dot{y}_p(t_{go}) - \Delta V_y\} \\
0 & -t_{go} & \{\dot{z}_E(t_{go}) - \dot{z}_p(t_{go}) - \Delta V_z\}
\end{bmatrix}
\] (4-5)

To determine changes in the \( \mathbf{u} \) vector, \( \mathbf{f} \) is multiplied by the negative inverse of \( \mathbf{J} \)

\[
\begin{bmatrix}
d\Delta V_y \\
d\Delta V_z \\
dt_{go}
\end{bmatrix} = -[J]^{-1}
\begin{bmatrix}
f_1(u) \\
f_2(u) \\
f_3(u)
\end{bmatrix}
\] (4-6)

\[
[J]^{-1} = \begin{bmatrix}
\dot{y}_E(t_{go}) - \dot{y}_p(t_{go}) - \Delta V_y \\
\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}t_{go} \\
\dot{z}_E(t_{go}) - \dot{z}_p(t_{go}) - \Delta V_z \\
\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}t_{go} \\
1/\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}
\end{bmatrix}
\] (4-7)
To find the control parameters the following procedure should be used. First, establish a time-to-go with zero velocity changes, a good choice being the time-to-go that yields the point of closest approach. This time-to-go is determined by propagating the orbits forward until a minimum relative distance is reached. Because the evader is assumed to be in its final boost phase throughout the intercept, this time-to-go will be less than or equal to time until ICBM thrust termination. Second, propagate the dynamic equations (3-5 thru 3-11) forward to the intercept time and determine the $f$ vector from (4-2). Changes to the control parameters are then obtained from (4-6). The velocity changes are applied to the pursuer's initial conditions and the procedure is repeated with the updated time-to-go until convergence occurs. The resulting control parameters will drive the miss distance to zero with minimum velocity changes. The difference between needed and present velocity are sufficient to determine the pursuer's thrust profile.
CHAPTER V

SPLINE APPROXIMATIONS

As discussed in Chapter IV, numerical propagation of the dynamic equations is very time consuming. It would be convenient to approximate the relative trajectory by a polynomial, eliminating the need for repeated propagation. Cubic splines lend themselves well to this application [16], [22], [23]. The current and final states can be used to generate cubic splines along each axis of the form

\[ x(t) = At^3 + Bt^2 + Ct + D \]  \hspace{1cm} (5-1)

By setting the current time to zero, \( D \) and \( C \) become the current position and velocity respectively, with time-to-go being the intercept time. Changes in velocity will be reflected only in the \( C \) coefficient and the final state can be easily determined for any intercept time. With this formulation, the determination of the spline coefficients is relatively simple. The current state gives \( D \) and \( C \) with no computations:

\[ D = x(o) \]  \hspace{1cm} (5-2)
\[ C = \dot{x}(o) \quad (5-3) \]

The \( A \) and \( B \) coefficients can be computed using the final states and (5-1) as follows:

\[ x(t_{go}) = At_{go}^3 + Bt_{go}^2 + Ct_{go} + D \quad (5-4) \]

\[ \dot{x}(t_{go}) = 3At_{go}^2 + 2Bt_{go} + C \quad (5-5) \]

Because there are only two unknowns in the above two equations, algebraic manipulation yields:

\[ A = \frac{2[x(o) - x(t_{go})]}{t_{go}^3} + \frac{[\dot{x}(o) + \dot{x}(t_{go})]}{t_{go}^2} \quad (5-6) \]

\[ B = \frac{3[x(t_{go}) - x(o)]}{t_{go}^2} + \frac{[2\dot{x}(o) + \dot{x}(t_{go})]}{t_{go}} \quad (5-7) \]

Figures depicting distance errors associated with these approximations are provided in Appendix A.

There is an added versatility in using splines. Should the system model be changed, only the spline coefficients need be changed. The search algorithms based on the splines will remain the same, operating with the new coefficients. This is very beneficial as it is far simpler to recompute the coefficients than
to alter the algorithms.

To ensure accuracy, new spline coefficients are computed every cycle time. To accomplish this, the truth model is propagated forward to predicted impact time to obtain the needed final states. By using these updated final states every iteration, propagated roundoff error is eliminated in the spline coefficient computations.
CHAPTER VI

OPTIMAL CONTROL FORMULATION USING CERTAINTY EQUIVALENCE

Changes in pursuer lateral velocity will affect final position, velocity and time. These effects can be easily computed with the relative trajectory modeled by splines in the coordinate system discussed in Chapter II. The optimal control problem is to find the intercept time that minimizes changes in pursuer velocity while ensuring a hit. Techniques to solve such problems are addressed by Bryson and Ho [25] and summarized in the following paragraphs.

PLAN A

To solve this problem, a relative spline equation is formed for each axis and a cost function is established. The cost function ($L$) incorporates velocity changes and miss distance multiplied by a weighting factor ($K$) and is represented as

$$L = \frac{k(x_1^2 + x_2^2 + x_3^2)}{2} + \frac{(\Delta v_y^2 + \Delta v_z^2)}{2}$$

(6.1)
where the final relative state vector is determined from the spline equations:

\[
\begin{align*}
\mathbf{x}_1 &= x(t_{go}) = \begin{bmatrix} A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x \\ A_y t_{go}^3 + B_y t_{go}^2 + (C_y - \Delta V_y) t_{go} + D_y \\ A_z t_{go}^3 + B_z t_{go}^2 + (C_z - \Delta V_z) t_{go} + D_z \end{bmatrix} \\
\mathbf{x}_2 &= y(t_{go}) = \begin{bmatrix} x(t_{go}) + B_t t_{go} + (C_t - \Delta V_t) t_{go} + D_t \end{bmatrix} \\
\mathbf{x}_3 &= z(t_{go}) = \begin{bmatrix} y(t_{go}) + 2c - 2 \end{bmatrix}
\end{align*}
\]

The cost function must now be minimized with respect to the control vector \( \mathbf{u} \):

\[
\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} t_{go} \\ \Delta V_y \\ \Delta V_z \end{bmatrix}
\]

As stated in Bryson and Ho [25], it should be possible to find a set of controls such that

\[
\frac{\delta L}{\delta \mathbf{u}} = 0
\]

(6·4)
Three equations arise from (6-4) with three unknowns, expressed here in vector form as

\[
\mathbf{h} = \begin{bmatrix}
  h_1 \\
  h_2 \\
  h_3
\end{bmatrix} = \begin{bmatrix}
  K(x_1 \dot{x}_1 + x_2 \dot{x}_2 + x_3 \dot{x}_3) \\
  \Delta V_y - Kx_2 \dot{t}_{go} \\
  \Delta V_z - Kx_3 \dot{t}_{go}
\end{bmatrix} = 0 \quad (6-5)
\]

with \( \dot{x} \) being

\[
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \dot{x}_3
\end{bmatrix} = \begin{bmatrix}
  3A_t \dot{t}_{go}^2 + 2B_t \dot{t}_{go} + C_x \\
  3A_y \dot{t}_{go}^2 + 2B_y \dot{t}_{go} + C_y - \Delta V_y \\
  3A_z \dot{t}_{go}^2 + 2B_z \dot{t}_{go} + C_z - \Delta V_z
\end{bmatrix} \quad (6-6)
\]

As in the truth model, a Newton-Raphson method from Maron [24] is used to solve (6-5). It is important to note that this formulation differs from the truth model in two areas. First, a weighting factor has been introduced that allows a trade-off between miss distance and velocity changes. Zero miss distance is associated with infinite \( K \), while zero \( K \) produces no velocity change. Second, the splines eliminate the need for repeated trajectory propagation, significantly reducing control parameter search time.
The Jacobian for (6-5) is

\[
[J] = \begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & 0 \\
J_{31} & 0 & J_{33}
\end{bmatrix}
\]  \tag{6-7}

where

\[
J_{11} = K(x_1 [6A_x t_{go} + 2B_x] + x_2 [6A_y t_{go} + 2B_y] + \\
x_3 [6A_z t_{go} + 2B_z] + \dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2)
\]  \tag{6-8}

\[
J_{12} = J_{21} = -K(x_2 + \dot{x}_2 t_{go})
\]  \tag{6-9}

\[
J_{13} = J_{31} = -K(x_3 + \dot{x}_3 t_{go})
\]  \tag{6-10}

\[
J_{22} = J_{33} = 1 + K t_{go}^2
\]  \tag{6-11}

Changes in the control vector are determined by

\[
dy = \begin{bmatrix}
dt_{go} \\
d\Delta V_y \\
d\Delta V_z
\end{bmatrix} = -[J]^{-1} \begin{bmatrix}
h_1 \\
h_2 \\
h_3
\end{bmatrix}
\]  \tag{6-12}
To execute this procedure, initialize time-to-go (preferably to the point of closest approach) and determine the spline coefficients for this initial trajectory. Compute the $x$, $\dot{x}$, and $h$ vectors, in that order. Update the $y$ vector using (6-12) and test for convergence. If convergence is not achieved recompute the above vectors and test again.

**PLAN B**

The formulation in this plan uses splines to determine the control parameters for a zero miss solution. This is a specialized version of Plan A where the weighting factor is set to infinity ($K=\infty$). Because the only control for miss in the longitudinal direction is time-to-go, $x_1$ in (6-2) is set equal to zero,

$$x_1 = A x_{t_{go}}^3 + B x_{t_{go}}^2 + C x_{t_{go}} + D_x = 0$$  \hspace{1cm} (6-14)
and $t_{go}$ solved using numerical techniques. With time-to-go established, (6-2) is again used with $x_2$ and $x_3$ equal to zero, yielding equations for the velocity changes:

$$
\Delta V_y = A_y t_{go}^2 + B_y t_{go} + C_y + D_y/t_{go} \tag{6-15}
$$

$$
\Delta V_z = A_z t_{go}^2 + B_z t_{go} + C_z + D_z/t_{go} \tag{6-16}
$$

This plan is computationally less burdensome than Plan A because the complexity of the search is reduced.

**PLAN C**

Within a few seconds of intercept the acceleration due to gravity will be nearly identical for the pursuer and evader. Also, the booster, still thrusting in the final boost phase, will travel in a near straight line along its velocity vector. Ignoring gravity terms in the relative dynamics leads to a simpler and faster solution, reducing the guidance to proportional navigation [1]. The relative trajectories are expressed as

$$
D_A = \frac{A_0}{\dot{m}_0^2} \sum_{i=2}^{\infty} \frac{(\dot{m}_0 t_{go})^i}{i(i-1)} \tag{6-17}
$$
\[ x(t_{go}) = \left\{ [x_E(0) - x_p(0)] + [\dot{x}_E(0) - \dot{x}_p(0)] \right\} t_{go} + \frac{D_A \dot{x}_E(0)}{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)} \] 

(6-18)

\[ y(t_{go}) = \left\{ [y_E(0) - y_p(0)] + [\dot{y}_E(0) - \dot{y}_p(0) - \Delta V_y] \right\} t_{go} + \frac{D_A \dot{y}_E(0)}{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)} \] 

(6-19)

\[ z(t_{go}) = \left\{ [z_E(0) - z_p(0)] + [\dot{z}_E(0) - \dot{z}_p(0) - \Delta V_z] \right\} t_{go} + \frac{D_A \dot{z}_E(0)}{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)} \] 

(6-20)

where \( D_A \) is the distance associated with thruster acceleration in the direction of booster velocity. As in Plan B, time-to-go is computed for zero miss on the x axis using (6-18), and then the velocity changes can be found from (6-19) and (6-20).
CERTAINTY EQUIVALENCE

It should be noted that all these techniques use the principle of certainty equivalence, where expected values from a state estimator are substituted for random variables [26]. The pursuer's states are assumed known, but because the evader's states must be estimated, the resulting system is stochastic. Optimal control formulation is based on a system that is deterministic. In applying the certainty equivalence principle, the stochastic system is replaced by a deterministic one, using the expected values of the random variables from the estimator.

There is a drawback to this technique in the sense that imperfect knowledge of the present state produces needless thrusting. Any errors in the present state estimate cause errors in the predicted final state. This results in the computation of velocity changes based on the incorrect final state. Future iterations produce similar results requiring the pursuer to thrust excessively.

This excessive thrusting can be reduced using stochastic control techniques. Three formulations are examined in the following chapters. The first determines the optimum spacing of corrective thrusts for Plan B. The second uses dual control methods based on predicted error knowledge, such as filter covariance. The third constrains the miss distance to a function of predicted error knowledge, at the expense of accuracy.
CHAPTER VII

STOCHASTIC CONTROL

OPTIMUM SPACING OF CORRECTIVE THRUSTS

Corrective thrusting in the presence of state estimate errors can be optimally spaced to reduce fuel [27]. A control effectiveness ratio ($\rho$) is established to determine the spacing between thrusts. This ratio directly yields thrust times when control effectiveness is a linear function of time.

For this formulation, the number of corrective thrusts ($N$) must be chosen to minimize the sum of thrusts ($S_N$), which is total $\Delta V$. The behavior of $\Delta V$ and miss distance as a function of $\rho$ can be produced through digital computation and is done as part of the simulation to determine the best value of $\rho$ for Plan B.

To enhance understanding this technique, assume the control effectiveness ratio is two ($\rho=2$). This implies that corrective thrusting should take place when the control has half $(1/\rho)$ the effect of the previous corrective thrust. If control effectiveness is a near-linear function of time, as is the case for a hypervelocity vehicle, then it will be halved at about half the time to impact since the last thrust. Thrusting will take
place at the start of the intercept, at one-half time-to-go, one fourth time-to-go, one-eighth time-to-go and so on. With $\rho=3$ the optimum thrust timing always occurs at a third of the time-to-go since the last correction. When the spacing is less than the estimator's cycle time, impact is imminent and thrust is terminated.

DUAL CONTROL FORMULATION

Optimal control solutions require perfect knowledge of the states, but in reality the information provided to the controller is only an estimate. As stated by Aoki [28], a theory of control should take into account the 'imperfectness' of information. This explains the need to incorporate statistical decision theory in control formulation. A solution that uses imperfect information will be sub-optimal, but it is desirable for such a solution to have the intrinsic characteristics of optimality [5]. Recognition that the control affects not only the state but also its uncertainty leads to a form of stochastic control known as dual control. This method not only drives the system to some final state, but attempts to improve state uncertainty along the way. The result is often greater accuracy and/or reduced fuel consumption.

A dual control method for controlling stochastic nonlinear systems with free end-time was developed by Tse and
Bar-Shalom [5]. This method differs from the optimal control formulation presented in Chapter VI. Instead of minimizing the cost function $L$ of (6-1), the expected value of the cost function ($E\{L\}$) is minimized. To accomplish this, the final states and their covariances must be computed. This can be done by running the Extended Kalman Filter forward to predicted intercept time, as suggested by Tse, Bar-Shalom and Meier [4].

The solution involves establishing an expected cost function consisting of miss distance and covariance of each axis, along with the control. The cost function $L$ from (6-1) is repeated here for convenience:

$$L = K \frac{\left( x_f^2 + y_f^2 + z_f^2 \right)}{2} + \frac{(\Delta V_y^2 + \Delta V_z^2)}{2} \quad (7-1)$$

Assuming the estimates of the filter are Gaussian, the expected value is:

$$E\{L\} = K \left[ \frac{\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2}{2} + \frac{\dot{x}_f^2 + \dot{y}_f^2 + \dot{z}_f^2}{2} \right]$$

$$+ E \left[ \frac{\Delta V_y^2 + \Delta V_z^2}{2} \right] \quad (7-2)$$
The expected cost of \((7-2)\) is conditioned on the controls. Two cases must be examined: the cost associated with the certainty equivalence (CE) solution (Plan A of Chapter VI) and the cost of deviating from that solution to improve the estimate. In this manner, the approximate best cost-to-go includes both estimation and control performance.

The expected cost of the CE solution is easily computed by determining the controls from Plan A and then running the Extended Kalman Filter forward to predicted impact time assuming measurement updates. The final filter data is then inserted into \((7-2)\) to find the expected cost.

Finding the expected cost of deviating from the CE solution is computationally burdensome. The thrust direction that yields the greatest estimate improvement must first be determined. Thrusting in this direction will cause the expected miss distance to grow due to departure from the nominal (CE) path. It is therefore necessary to determine a new nominal path based on the deviation and include the control energy required for this path in the deviation cost estimate. Failure to do so may result in large expected miss distances that erroneously inflate the cost associated with deviation, causing the CE control of Plan A to always be chosen.
CHAPTER VIII

CERTAINTY CONTROL

As stated earlier, if the estimate is near perfect then optimal control should be used. For a less accurate estimate, dual control attempts to improve the measurement certainty, and thus the estimate, by expending control energy. This has been shown to work well if the certainty is a function of the control parameters [5]. Because range is included as a measurement, lateral deviations should not noticeably improve the estimate. For this reason, dual control techniques are not expected to work better than certainty equivalence formulations.

If the controls associated with cost do not affect state estimate certainty, fuel may be conserved by using that certainty to reduce the controls. By linking the controls to the certainty of the estimate, a near perfect estimate would yield the optimal control, with reduced control resulting from a poor estimate. To accomplish this, the predicted final states are constrained by a function of their variances at the final time. This form of control will be called certainty control and is implemented by establishing the cost function.
subject to the constraint:

\[ f = \frac{x_f^2 + y_f^2 + z_f^2 - K[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2]}{2} \leq 0 \]  

where \( K \) is a weighting factor. The final state estimates \((x_f, y_f, z_f)\) and their deviations \((\sigma_{xf}, \sigma_{yf}, \sigma_{zf})\) are determined by running the filter forward to predicted impact time without updates and then representing their time history with splines:

\[ x_s = A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x \]  

\[ y_s = A_y t_{go}^3 + B_y t_{go}^2 + C_y t_{go} + D_y \]  

\[ z_s = A_z t_{go}^3 + B_z t_{go}^2 + C_z t_{go} + D_z \]  

\[ x_f = x_s \]  

\[ y_f = y_s - \Delta V_y t_{go} \]  

\[ z_f = z_s - \Delta V_z t_{go} \]
\[
\sigma_{xf} = A_{\sigma x} t^3 g_0 + B_{\sigma x} t^2 g_0 + C_{\sigma x} t g_0 + D_{\sigma x} \tag{8-9}
\]

\[
\sigma_{yf} = A_{\sigma y} t^3 g_0 + B_{\sigma y} t^2 g_0 + C_{\sigma y} t g_0 + D_{\sigma y} \tag{8-10}
\]

\[
\sigma_{zf} = A_{\sigma z} t^3 g_0 + B_{\sigma z} t^2 g_0 + C_{\sigma z} t g_0 + D_{\sigma z} \tag{8-11}
\]

Conceptually, the constraint produces a deviation sphere about the predicted impact point. If the predicted miss is inside or touching the sphere, thrusting is not necessary. If the predicted miss is outside the sphere, minimum thrusting is determined to bring the miss to the surface of the sphere. As the estimates improve, the constraint tightens and the sphere shrinks. The spline representations allow this stochastic problem to be solved deterministically. The constraint is adjoined to the cost function to form the Hamiltonian [29]:

\[
H = L + \lambda f \tag{8-12}
\]

The partials of \( H \) with respect to the controls must equal zero:

\[
\frac{\partial H}{\partial \Delta V_y} = \Delta V_y - \lambda y_f t g_0 = 0 \tag{8-13}
\]

\[
\frac{\partial H}{\partial \Delta V_z} = \Delta V_z - \lambda z_f t g_0 = 0 \tag{8-14}
\]
\[
\frac{\partial H}{\partial t_{go}} = \lambda (x_f \dot{x}_f + y_f \dot{y}_f + z_f \dot{z}_f - K[\sigma_{xf} \dot{\sigma}_{xf} + \sigma_{yf} \dot{\sigma}_{yf} + \sigma_{zf} \dot{\sigma}_{zf}]) = 0 \quad (8-15)
\]

with the dot term expansions computed in Appendix B.

Equations 8-2, 8-13, 8-14, and 8-15 constitute four equations with four unknowns, which can be reduced to two equations and two unknowns using (8-7) and (8-8). Substituting (8-7) into (8-13) yields

\[
\Delta V_y = \frac{\lambda y_s t_{go}}{1 + \lambda t^2_{go}} \quad (8-16)
\]

\[
y_f = \frac{y_s}{1 + \lambda t^2_{go}} \quad . \quad (8-17)
\]

In a similar manner, substituting (8-8) into (8-14) yields

\[
\Delta V_z = \frac{\lambda z_s t_{go}}{1 + \lambda t^2_{go}} \quad (8-18)
\]

\[
z_f = \frac{z_s}{1 + \lambda t^2_{go}} \quad . \quad (8-19)
\]
Equations 8-2 and 8-15 can now be solved in terms of $\lambda$ and $t_{go}$. Once known, $\Delta V_y$ and $\Delta V_z$ can be determined from (8-16) and (8-18). The parameters $\lambda$ and $t_{go}$ can be found by numerical techniques using the Jacobian:

$$
\begin{bmatrix}
\frac{dt_{go}}{d\lambda} \\
\frac{-f_2}{-f_1}
\end{bmatrix} = \begin{bmatrix}
-f_1 \\
-f_2
\end{bmatrix}
$$

(8-20)

$$
f_1 = x_f^2 + y_f^2 + z_f^2 - \bar{r}\left[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2\right]
$$

(8-21)

$$
f_2 = x_f \dot{x}_f + y_f \dot{y}_f + z_f \dot{z}_f - 
$$

$$
K[\sigma_{xf} \dot{x}_f + \sigma_{yf} \dot{y}_f + \sigma_{zf} \dot{z}_f]
$$

(8-22)

with the elements of the Jacobian matrix computed in Appendix B.

Should the states be perfectly known, the $\sigma$ terms will be zero. In this case, the equations for certainty control reduce to the optimal control formulation for Plan B. Should the estimate be poor, the $\sigma$ terms will be large and the inequality constraint of (8-2) is met with very little (if any) change in velocity. This demonstrates the principle of certainty control, where the certainty of the estimate affects control energy expenditure.
CHAPTER IX

SUMMARY OF CONTROL STRATEGIES

In this chapter a brief summary of all the control strategies is presented. It is intended to give the reader a basis for quick comparison. The cost function of each algorithm is given, along with the requirements for computation.

Plan A is an optimal control, certainty equivalence formulation that minimizes the cost function:

\[ L = \frac{K(x_f^2 + y_f^2 + z_f^2) + (\Delta V_y^2 + \Delta V_z^2)}{2} \]

This algorithm requires an estimate of the final relative states.

Plan B is a certainty equivalence formulation that minimizes the cost function:

\[ L = \frac{(\Delta V_y^2 + \Delta V_z^2)}{2} \]
subject to the constraint

\[ f = x_f^2 + y_f^2 + z_f^2 = 0. \]

This algorithm requires an estimate of the final relative states.

Plan C is a certainty equivalence formulation with the same cost function and requirements as Plan B. The difference between these strategies is that gravity is ignored in the dynamic equations used to estimate the final relative states.

The optimal spacing of corrective thrusts also uses the same cost function and requirements as Plan B. In this strategy, however, the pursuer is not permitted to thrust every cycle time. Thrust timing is controlled by selecting a control effectiveness ratio to minimize control energy expenditure.

Dual Control is a stochastic control formulation that attempts to improve the estimate, and thus accuracy, by minimizing the cost function:
This algorithm requires estimates of the final relative states, their filter variances, and the relationship between control and variance.

Certainty Control is a new stochastic control formulation that reduces the control based on the certainty of the estimate by minimizing the cost function

\[ L = \frac{(\Delta V^2_y + \Delta V^2_z)}{2} \]

subject to the constraint

\[ \frac{x_f^2 + y_f^2 + z_f^2}{2} - K\left[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2\right] \leq 0. \]

This algorithm requires estimates of the final relative states and their filter variances.
CHAPTER X

EXTENDED KALMAN FILTERING

Optimal estimates of the pursuer and evader are needed for the search algorithms to converge properly. Due to the nature of the dynamics and sensors, the relative position and velocity must be estimated from sampled nonlinear measurements. The estimation problem for a nonlinear system having continuous dynamics and discrete-time measurements is addressed by Gelb [30]. The Extended Kalman Filter (EKF) was chosen over other estimation methods because the optimal estimate is determinate. That is, the dynamics and observations of the pursuer and evader can be well predicted in the presence of Gaussian noise.

A summary of the continuous-discrete EKF algorithm from Gelb [30] follows. The equations for the state dynamics and measurements, as well as the computations of the partial derivatives, can be found in Appendix C. The system model is a continuous model of the state dynamics with white Gaussian noise \{w(t)\} added.

\[ \dot{x}(t) = f(x(t), t) + w(t) \] (10-1)
\[ w(t) \sim \mathcal{N}(0, \mathbf{Q}(t)) \] (10-2)

where \( w \) is a Gaussian (normal) random vector with mean \( 0 \) and covariance matrix \( \mathbf{Q} \). The measurement model is discrete and corrupted by white Gaussian noise \( y_k \):

\[ z_k = h_k(x(t_k)) + y_k \] (10-3)

\[ y_k \sim \mathcal{N}(0, \mathbf{R}_k) \] (10-4)

In either case, the noise is assumed uncorrelated for all \( t(k) \).

The state estimate, denoted by a hat, is propagated from (10-1) with

\[ \dot{\hat{x}} = f(\hat{x}(t), t) \] (10-5)

and the error covariance \( \mathbf{P}(t) \) is propagated by

\[ \dot{\mathbf{P}}(t) = \mathbf{F}(\hat{x}(t), t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T(\hat{x}(t), t) + \mathbf{Q}(t) \] (10-6)

\[ \mathbf{F}(\hat{x}(t), t) = \left[ \frac{\partial f(\hat{x}(t), t)}{\partial \hat{x}(t)} \right] \bigg|_{\hat{x}(t) = \hat{x}(t)} \] (10-7)
The measurements determine the gain matrix $K_k$ through the equations

$$H_k(x_k(-)) = \frac{\partial h_k(x(t_k))}{\partial x(t_k)} \left| \begin{array}{c} x(t_k) = x_k(-) 
\end{array} \right.$$

$$k_k = p_k(-)H_k^T(x_k(-))[H_k(x_k(-))p_k(-)H_k^T(x_k(-)) + R_k]^{-1}$$

where the (-) symbolizes prior to update and the (+) after update.

With the gain matrix computed, the state estimate and error covariance can be updated by the following equations:

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k[z_k - h_k(\hat{x}_k(-))]$$

$$P_k(+) = [I - K_kH_k(\hat{x}_k(-))]P_k(-)$$

It is advantageous to process measurements one at a time. This method, called serial updating [31], eliminates the requirement to compute a matrix inverse, thereby reducing computer load and avoiding the computational problems associated with inverting an ill-conditioned matrix. Also, measurements may be skipped without reformulating the filter equations, allowing greater flexibility in examining various tracking schemes. The simultaneous measurement components of the vector $z_k$ can be considered serially over a very short time span.
CHAPTER XI

COMPUTER SIMULATION

A menu-driven program that simulates all the algorithms was written in FORTRAN 77 and run on a VAX 8600. The code for this program can be found in Appendix D. Six cases are examined to determine the accuracy and efficiency of each algorithm. In all cases the propagation (w) and measurement (v) noise properties associated with the filter are:

\[ w_{x, y, z}(t) \sim N(0, 2.21516 \times 10^{-18} \text{ m}^2 \text{s}^{-1}) \]

\[ w_{x, y, z}(t) \sim N(0, 5.52049 \times 10^{-20} \text{ m}^4 \text{s}^{-3}) \]

\[ w_A(t) \sim N(0, 4.29831 \times 10^{-12} \text{ m}^2 \text{s}^{-5}) \]

\[ w_m(t) \sim N(0, 2.493241 \times 10^{-7} \text{ m}^2 \text{s}^{-3}) \]

\[ v_{\theta, \gamma}(k) \sim N(0, 1.0 \times 10^{-8}) \]
where $\theta$ is the out-of-plane line-of-sight angle, $\gamma$ the in-plane line-of-sight angle and $R$ is range.

The startup variances are:

\[ \sigma_{xx}^2 = \sigma_{yy}^2 = \sigma_{zz}^2 = 100 \, m^2 \]

\[ \sigma_{xx}^2 = \sigma_{yy}^2 = \sigma_{zz}^2 = 10 \, \frac{m^2}{s^2} \]

\[ \sigma_{AA}^2 = \frac{1}{s^4} \]

\[ \sigma_{mm}^2 = 2.493241 \times 10^{-6} \, \frac{1}{s^2} \]

The pursuer's initial conditions for all cases are

\[ x_p = -359899.441 \, m \]

\[ \dot{x}_p = 11991.950 \, \frac{m}{s} \]

\[ y_p = 6727335.870 \, m \]
\[ \dot{y}_p = 158.764 \quad m/s \]

\[ z_p = 0.0 \quad m \]

\[ \dot{z}_p = 0.0 \quad m/s \]

with a lateral acceleration range of 3-60 m/s\(^2\) in each axis.

The booster's characteristics are modeled as

\[ A_o = 3.15788 \quad m/s^2 \]

\[ \dot{m}_o = 0.01579 \quad 1/s \]

with time-to-go equaling 30 seconds.

A time lag of one tenth second is used for all algorithms when computing velocity changes. It is unrealistic to assume the filter can process measurements, the controller determine thrust commands, and the thrusters respond to those commands all instantaneously. One cycle time is chosen to allow the velocity changes computed in the previous cycle to be implemented in the present cycle. The controller routines are built to take this lag into account. Also, thrusting is not permitted during the first three seconds of an intercept to account for target acquisition.
The following page shows the evader initial condition for six cases. Case I represents a head on, in-plane intercept. Case II represents a head on, $10^\circ$ out-of-plane intercept. Case III represents a head on, $20^\circ$ out-of-plane intercept. Case IV represents an in-plane tail chase. Case V represents a $10^\circ$ out-of-plane tail chase. Case VI represents a $20^\circ$ out-of-plane tail chase.
Table 11-1. Evader Initial Conditions

<table>
<thead>
<tr>
<th></th>
<th>CASE I</th>
<th>CASE II</th>
<th>CASE III</th>
<th>CASE IV</th>
<th>CASE V</th>
<th>CASE VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_E$ (m)</td>
<td>205720.173</td>
<td>202600.41</td>
<td>193342.193</td>
<td>-205720.173</td>
<td>-202600.401</td>
<td>-193342.193</td>
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<td>$\dot{x}_E$ (m/s)</td>
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<td>-6695.912</td>
<td>-6390.106</td>
<td>6805.072</td>
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<td>6396.106</td>
</tr>
<tr>
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<td>6639834.472</td>
<td>6639829.206</td>
<td>6638526.297</td>
<td>6639834.427</td>
<td>6639829.206</td>
</tr>
<tr>
<td>$\dot{y}_E$ (m/s)</td>
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<td>3058.185</td>
<td>3058.501</td>
<td>3101.494</td>
<td>3058.185</td>
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</tr>
<tr>
<td>$z_E$ (m)</td>
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<td>-76213.728</td>
<td>0.0</td>
<td>-38683.931</td>
<td>-76213.728</td>
</tr>
<tr>
<td>$\dot{z}_E$ (m/s)</td>
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<td>1282.071</td>
<td>2523.104</td>
<td>3.0</td>
<td>1282.71</td>
<td>2523.104</td>
</tr>
</tbody>
</table>
CHAPTER XII

RESULTS

A history of the miss and velocity changes with respect to multipliers is generated for those algorithms requiring multipliers (see Figures 12-1 through 12-24). A specific multiplier is then chosen for each plan for inclusion in the tables that follow.

One hundred Monte Carlo runs are generated per plan per case. The mean of each set of runs is adequate for judging relative performance. This performance is recorded in the six tables that follow the figures.

Appendix E contains the in-plane thrust profiles for Cases I and V of all plans. In the appendix, each profile uses the same random seed for startup to show the effect of estimate uncertainty on the various control strategies.
Figure 12-1. Performance of Plan A for Case I.
Figure 12-2. Performance of Plan A for Case II.
Figure 12-3. Performance of Plan A for Case III.
Figure 12-4. Performance of Plan A for Case IV.
Figure 12-5. Performance of Plan A for Case V.
Figure 12-6. Performance of Plan A for Case VI.
Figure 12-7. Performance of Optimum Thrust Spacing for Case I.
Figure 12-8. Performance of Optimum Thrust Spacing for Case II.
Figure 12.9. Performance of Optimum Thrust Spacing for Case III.
Figure 12-10. Performance of Optimum Thrust Spacing for Case IV.
Figure 12.11. Performance of Optimum Thrust Spacing for Case V.
Figure 12-12. Performance of Optimum Thrust Spacing for Case VI.
Figure 12-13. Performance of Dual Control for Case I.
Figure 12-14. Performance of Dual Control for Case II.
Figure 12-15. Performance of Dual Control for Case III.
Figure 12-16. Performance of Dual Control for Case IV.
Figure 12-17. Performance of Dual Control for Case V.
Figure 12-18. Performance of Dual Control for Case VI.
Figure 12-19. Performance of Certainty Control for Case I.
Figure 12-20. Performance of Certainty Control for Case II.
Figure 12-21. Performance of Certainty Control for Case III.
Figure 12-22. Performance of Certainty Control for Case IV.
Figure 12-23. Performance of Certainty Control for Case V.
Figure 12-24. Performance of Certainty Control for Case VI.
Table 12-1. Case I Performance.
(Head On, In-plane Intercept)

<table>
<thead>
<tr>
<th>Case Description</th>
<th>Mean Miss (Meters)</th>
<th>Standard Deviation Miss (Meters)</th>
<th>Mean AV (m/s)</th>
<th>Standard Deviation AV (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAN A (K=10)</td>
<td>.465</td>
<td>.230</td>
<td>81.93</td>
<td>5.64</td>
</tr>
<tr>
<td>PLAN B</td>
<td>.362</td>
<td>.173</td>
<td>88.29</td>
<td>7.00</td>
</tr>
<tr>
<td>PLAN C</td>
<td>.362</td>
<td>.174</td>
<td>90.76</td>
<td>6.86</td>
</tr>
<tr>
<td>OPTIMUM THRUST SPACING (p=1.75)</td>
<td>.363</td>
<td>.175</td>
<td>36.63</td>
<td>8.14</td>
</tr>
<tr>
<td>DUAL CONTROL (K=10)</td>
<td>.465</td>
<td>.230</td>
<td>81.93</td>
<td>5.64</td>
</tr>
<tr>
<td>CERTAINTY CONTROL (K=.4)</td>
<td>.399</td>
<td>.215</td>
<td>21.61</td>
<td>3.79</td>
</tr>
<tr>
<td>TRUTH WITH NOISE</td>
<td>.527</td>
<td>.264</td>
<td>81.44</td>
<td>6.76</td>
</tr>
<tr>
<td>TRUTH WITHOUT NOISE</td>
<td>0</td>
<td>NA</td>
<td>7.31</td>
<td>NA</td>
</tr>
</tbody>
</table>
Table 12-2. Case II Performance.
(Head On, 10° Out-of-Plane Intercept)

<table>
<thead>
<tr>
<th>Plan</th>
<th>Mean Miss (Meters)</th>
<th>Standard Deviation (Meters)</th>
<th>Mean Δv (m/s)</th>
<th>Standard Deviation (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAN A (K=10)</td>
<td>.502</td>
<td>.224</td>
<td>83.82</td>
<td>6.99</td>
</tr>
<tr>
<td>PLAN B</td>
<td>.360</td>
<td>.171</td>
<td>90.39</td>
<td>7.24</td>
</tr>
<tr>
<td>PLAN C</td>
<td>.360</td>
<td>.171</td>
<td>93.07</td>
<td>7.39</td>
</tr>
<tr>
<td>OPTIMUM THRUST SPACING (ρ=1.75)</td>
<td>.361</td>
<td>.171</td>
<td>37.19</td>
<td>8.50</td>
</tr>
<tr>
<td>DUAL CONTROL (K=10)</td>
<td>.502</td>
<td>.224</td>
<td>83.82</td>
<td>6.99</td>
</tr>
<tr>
<td>CERTAINTY CONTROL (K=.4)</td>
<td>.386</td>
<td>.191</td>
<td>23.21</td>
<td>4.18</td>
</tr>
<tr>
<td>TRUTH WITH NOISE</td>
<td>.545</td>
<td>.264</td>
<td>83.69</td>
<td>7.18</td>
</tr>
<tr>
<td>TRUTH WITHOUT NOISE</td>
<td>0</td>
<td>NA</td>
<td>7.54</td>
<td>NA</td>
</tr>
</tbody>
</table>
(Head On, 20° Out-of-Plane Intercept)

<table>
<thead>
<tr>
<th>MEAN MISS (METERS)</th>
<th>STANDARD DEVIATION (METERS)</th>
<th>MEAN ΔV (M/S)</th>
<th>STANDARD DEVIATION (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAN A (K=10)</td>
<td>.506</td>
<td>.225</td>
<td>90.92</td>
</tr>
<tr>
<td>PLAN B</td>
<td>.358</td>
<td>.168</td>
<td>97.53</td>
</tr>
<tr>
<td>PLAN C</td>
<td>.358</td>
<td>.168</td>
<td>99.97</td>
</tr>
<tr>
<td>OPTIMUM THRUST SPACING (ρ=1.75)</td>
<td>.358</td>
<td>.168</td>
<td>39.87</td>
</tr>
<tr>
<td>DUAL CONTROL (K=10)</td>
<td>.506</td>
<td>.225</td>
<td>90.92</td>
</tr>
<tr>
<td>CERTAINTY CONTROL (K=.4)</td>
<td>.372</td>
<td>.185</td>
<td>24.45</td>
</tr>
<tr>
<td>TRUTH WITH NOISE</td>
<td>.534</td>
<td>.293</td>
<td>90.77</td>
</tr>
<tr>
<td>TRUTH WITHOUT NOISE</td>
<td>0</td>
<td>NA</td>
<td>7.78</td>
</tr>
</tbody>
</table>
Table 12-4. Case IV Performance.
(In-Plane Tail Chase)

<table>
<thead>
<tr>
<th></th>
<th>Mean Miss (Meters)</th>
<th>Standard Deviation (Meters)</th>
<th>Mean (\Delta V) (M/S)</th>
<th>Standard Deviation (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plan A (K=10)</strong></td>
<td>.204</td>
<td>.105</td>
<td>110.87</td>
<td>9.89</td>
</tr>
<tr>
<td><strong>Plan B</strong></td>
<td>.126</td>
<td>.058</td>
<td>113.72</td>
<td>9.81</td>
</tr>
<tr>
<td><strong>Plan C</strong></td>
<td>.126</td>
<td>.057</td>
<td>111.82</td>
<td>9.25</td>
</tr>
<tr>
<td><strong>Optimum Thrust Spacing ((p=1.75))</strong></td>
<td>.124</td>
<td>.057</td>
<td>35.06</td>
<td>10.34</td>
</tr>
<tr>
<td><strong>Dual Control (K=10)</strong></td>
<td>.204</td>
<td>.105</td>
<td>110.87</td>
<td>9.89</td>
</tr>
<tr>
<td><strong>Certainty Control (K=.4)</strong></td>
<td>.150</td>
<td>.081</td>
<td>26.94</td>
<td>6.78</td>
</tr>
<tr>
<td><strong>Truth with Noise</strong></td>
<td>.376</td>
<td>.221</td>
<td>105.71</td>
<td>8.93</td>
</tr>
<tr>
<td><strong>Truth without Noise</strong></td>
<td>0</td>
<td>NA</td>
<td>8.75</td>
<td>NA</td>
</tr>
</tbody>
</table>
Table 12-5. Case V Performance.

(10° Out-of-Plane Tail Chase)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Mean Miss (Meters)</th>
<th>Standard Deviation (Meters)</th>
<th>Mean AV (M/S)</th>
<th>Standard Deviation (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLAN A (K=10)</td>
<td>.190</td>
<td>.100</td>
<td>129.61</td>
<td>13.29</td>
</tr>
<tr>
<td>PLAN B</td>
<td>.126</td>
<td>.061</td>
<td>132.65</td>
<td>13.32</td>
</tr>
<tr>
<td>PLAN C</td>
<td>.126</td>
<td>.061</td>
<td>129.47</td>
<td>12.26</td>
</tr>
<tr>
<td>OPTIMUM THRUST SPACING (p=1.75)</td>
<td>.126</td>
<td>.059</td>
<td>39.96</td>
<td>12.85</td>
</tr>
<tr>
<td>DUAL CONTROL (K=10)</td>
<td>.190</td>
<td>.100</td>
<td>129.61</td>
<td>13.29</td>
</tr>
<tr>
<td>CERTAINTY CONTROL (K=.4)</td>
<td>.136</td>
<td>.076</td>
<td>29.74</td>
<td>9.10</td>
</tr>
<tr>
<td>TRUTH WITH NOISE</td>
<td>.379</td>
<td>.204</td>
<td>123.57</td>
<td>11.61</td>
</tr>
<tr>
<td>TRUTH WITHOUT NOISE</td>
<td>0</td>
<td>NA</td>
<td>9.52</td>
<td>NA</td>
</tr>
</tbody>
</table>
Table 12-6. Case VI Performance.

(20° Out-of-Plane Tail Chase)

<table>
<thead>
<tr>
<th>Plan</th>
<th>Mean Miss (Meters)</th>
<th>Standard Deviation (Meters)</th>
<th>Mean ΔV (M/S)</th>
<th>Standard Deviation (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (K=10)</td>
<td>0.161</td>
<td>0.079</td>
<td>160.42</td>
<td>14.89</td>
</tr>
<tr>
<td>B</td>
<td>0.135</td>
<td>0.064</td>
<td>162.73</td>
<td>14.85</td>
</tr>
<tr>
<td>C</td>
<td>0.136</td>
<td>0.064</td>
<td>157.91</td>
<td>13.50</td>
</tr>
<tr>
<td>Optimum Thrust Spacing (ρ=1.76)</td>
<td>0.135</td>
<td>0.065</td>
<td>46.67</td>
<td>15.90</td>
</tr>
<tr>
<td>Dual Control (K=10)</td>
<td>0.161</td>
<td>0.079</td>
<td>160.42</td>
<td>14.89</td>
</tr>
<tr>
<td>Certainty Control (K=.8)</td>
<td>0.171</td>
<td>0.087</td>
<td>30.11</td>
<td>10.40</td>
</tr>
<tr>
<td>Truth with Noise</td>
<td>0.396</td>
<td>0.233</td>
<td>151.56</td>
<td>13.37</td>
</tr>
<tr>
<td>Truth without Noise</td>
<td>0</td>
<td>NA</td>
<td>10.13</td>
<td>NA</td>
</tr>
</tbody>
</table>
As predicted, the dual control’s performance is no better than the certainty equivalence formulation of Plan A. This is due to the fact that range is included as a measurement, causing the control to have virtually no effect on improving filter variance. Plan B is more accurate than Plan A, but more costly in energy. Again, this result is expected because the formulation of Plan B is based on infinite miss penalty \((K=m)\) for Plan A. By optimally spacing the thrusts of Plan B, energy expenditure is considerably reduced with little or no sacrifice in accuracy.

Plan C is just as accurate as Plan B, with slightly greater cost resulting from large initial intercept range. This extra cost is attributed to the negligible gravity assumption used in the formulation of Plan C. For the smaller ranges associated with a tail chase, Plan C was actually less costly than Plan B.

In every case, certainty control yields the least energy expenditure. This result is not surprising, as the formulation of certainty control is based on reducing control energy in the presence of poor estimates. This form of control works best because filter variance is range dependent. As range decreases, the control constraint tightens, and accuracy increases. Therefore, less fuel is used when range is great, with refinements made as impact nears.

The last two entries (truth with and without noise) do not use splines. Trajectory changes are computed as outlined in
Chapter IV. This data is included as a baseline reference of performance.

Because ranging is an active and costly process, various tracking schemes were examined to determine if ranging is needed. This was easily done in the simulation because of the serial updating discussed in Chapter X. Using only line-of-sight measurement angles, all algorithms are less accurate and/or require more velocity changes. It is of interest to note that the dual control guidance scheme, true to its nature, did expend control energy to improve the estimate. The improvement was very slight because of the pursuer's speed and lateral thrusting limits. Allowing one range update at midcourse also proved costly for all guidance schemes.

An attempt was made to reduce the order of the filter in the hopes of reducing processing time. The result was a serious degradation of performance for all algorithms. The system model is very sensitive to the evader booster characteristics, $A$ and $\dot{m}$, which are estimated by the eight state filter. Failure of the filtering process to refine initial booster estimates allows greater acceleration errors to be passed on to the evader estimates, significantly reducing end-game accuracy.
CHAPTER XIII

CONCLUSIONS AND AREAS OF FURTHER RESEARCH

In this research, six guidance schemes were examined to determine their capability to minimize lateral velocity changes of a hypervelocity orbital intercept vehicle. Proportional navigation, optimal control using certainty equivalence, dual control, control with optimum thrust spacing, and certainty control were all examined. Certainty control was shown to be the most energy efficient.

Certainty control constrains the final condition to a function of final estimator accuracy in the absence of updates. This general approach is not limited to hypervelocity vehicles, and would suggest other applications of this form to stochastically control intercepts.

This control requires a measure of final estimator accuracy which was achieved by running the Extended Kalman Filter forward to intercept time without updates. This time consuming process could be eliminated if filter variances could be estimated by some function (polynomial or otherwise). Also, the constraint multiplier was assumed constant for this formulation. Perhaps a multiplier that was range or time dependent would further reduce
Interceptor thrusting.

In summary, the approach identified by this research not only improves the efficiency of hypervelocity intercept, but can be applied to a broad range of stochastic problems where control energy does not improve filter accuracy. It is also possible to combine the effects of dual and certainty control in certain cases by initially using dual control to improve estimator accuracy and then switching to certainty control. End-game accuracy may be improved by switching from certainty control to a certainty equivalence formulation just prior to impact.


APPENDIX A

SPLINE APPROXIMATION ERRORS

Because the splines discussed in Chapter V are only approximations, there will be a small difference from the true trajectories modeled by them. To examine these errors, six figures are generated from a worst case scenario. Case I is considered the worst because of the high relative velocities. From this case profiles are generated for no velocity change, a velocity change of one meter per second, maximum $\Delta V_y$, and maximum $\Delta V_z$.

Figures A-1 and A-2 show no error at predicted impact time. This is expected because the splines are constrained to match final position and velocity. Figure A-3 reflects the error caused by a one meter per second in-plane velocity change decreasing as time-to-go decreases. Figure A-4 shows a similar effect for an out-of-plane velocity change. Figure A-5 shows the effect of maximum $\Delta V_y$ on trajectory error in the region of impact. Figure A-6 shows a similar effect for maximum out-of-plane thrusting.
Figure A-1. Distance error of spline trajectory vs. time for zero velocity change.
Figure A-2. Distance error of spline trajectory vs. time for zero velocity change.
Figure A-3. Distance error of spline trajectory vs. time for $\Delta v_y = 1 \text{ m/s}$.
Figure A-4. Distance error of spline trajectory vs. time for $\Delta V_z = 1$ m/s.
Figure A-5. Distance error of spline trajectory vs. time for maximum $\Delta V_y$ ($\Delta V_y = 6$ m/s).
Figure A-6. Distance error of spline trajectory vs. time for maximum $\Delta V_z$ ($\Delta V_z = 6 \text{ m/s}$).
APPENDIX B

DERIVATION OF CERTAINTY CONTROL EQUATIONS

The dot terms for (8-15) are computed as follows:

\[ \dot{x}_f = 3A_x t^2_{go} + 2B_x t_{go} + C_x \quad (B-1) \]

\[ \dot{y}_f = 3A_y t^2_{go} + 2B_y t_{go} + C_y - \Delta V_y \quad (B-2) \]

\[ \dot{z}_f = 3A_z t^2_{go} + 2B_z t_{go} + C_z - \Delta V_z \quad (B-3) \]

\[ \dot{x}_f = 3A_{sx} t^2_{go} + 2B_{sx} t_{go} + C_{sx} \quad (B-4) \]

\[ \dot{y}_f = 3A_{sy} t^2_{go} + 2B_{sy} t_{go} + C_{sy} \quad (B-5) \]

\[ \dot{z}_f = 3A_{sz} t^2_{go} + 2B_{sz} t_{go} + C_{sz} \quad (B-6) \]

The Jacobian matrix elements for (8-20) are:

\[ J_{11} = \frac{\partial f_1}{\partial t_{go}} = f_2 \quad (B-7) \]
\begin{align*}
J_{12} &= \frac{\partial f_1}{\partial \lambda} = y_f \frac{\partial y_f}{\partial \lambda} + z_f \frac{\partial z_f}{\partial \lambda} \tag{B-8} \\
\frac{\partial y_f}{\partial \lambda} &= -y_s t_{go}^2 \tag{B-9} \\
\frac{\partial z_f}{\partial \lambda} &= -z_s t_{go}^2 \tag{B-10} \\
J_{21} &= \frac{\partial f_2}{\partial t_{go}} = x_f \ddot{x}_f + \dot{x}_f^2 + y_f \ddot{y}_f + \dot{y}_f^2 + z_f \ddot{z}_f + \dot{z}_f^2 - K[\sigma_f \ddot{\sigma}_f + \dot{\sigma}_f^2 + \sigma_y \ddot{\sigma}_y + \dot{\sigma}_y^2 + \sigma_z \ddot{\sigma}_z + \dot{\sigma}_z^2] \tag{B-11} \\
\ddot{x}_f &= 6A \sigma_x t_{go} + 2B_x \tag{B-12} \\
\ddot{y}_f &= 6A \sigma_y t_{go} + 2B_y - \lambda(\dot{y}_f t_{go} + y_f) \tag{B-13} \\
\ddot{z}_f &= 6A \sigma_z t_{go} + 2B_z - \lambda(\dot{z}_f t_{go} + z_f) \tag{B-14} \\
\ddot{\sigma}_x &= 6A \sigma_x t_{go} + 2B \sigma_x \tag{B-15} \\
\ddot{\sigma}_y &= 6A \sigma_y t_{go} + 2B \sigma_y \tag{B-16} \\
\ddot{\sigma}_z &= 6A \sigma_z t_{go} + 2B \sigma_z \tag{B-17}
\end{align*}
\[ J_{22} = \frac{\partial f_2}{\partial \lambda} = \frac{\partial y_f}{\partial \lambda} \dot{y}_f + y_f \frac{\partial \dot{y}_f}{\partial \lambda} + \frac{\partial z_f}{\partial \lambda} \dot{z}_f + z_f \frac{\partial \dot{z}_f}{\partial \lambda} \]  \hspace{1cm} (B-18)

\[ \frac{\partial \dot{y}_f}{\partial \lambda} = -\frac{y_s t g_0}{(1 + \lambda t^2_{g_0})^2} \]  \hspace{1cm} (B-19)

\[ \frac{\partial \dot{z}_f}{\partial \lambda} = -\frac{z_s t g_0}{(1 + \lambda t^2_{g_0})^2} \]  \hspace{1cm} (B-20)
APPENDIX C

EXTENDED KALMAN FILTER EQUATIONS

The EKF states are defined from (3-5) through (3-11) as follows:

\[ x_1 = x_E - x_p \]  \hspace{1cm} \text{(C-1)}
\[ x_2 = \dot{x}_E - \dot{x}_p \]  \hspace{1cm} \text{(C-2)}
\[ x_3 = y_E - y_p \]  \hspace{1cm} \text{(C-3)}
\[ x_4 = \dot{y}_E - \dot{y}_p \]  \hspace{1cm} \text{(C-4)}
\[ x_5 = z_E - z_p \]  \hspace{1cm} \text{(C-5)}
\[ x_6 = \dot{z}_E - \dot{z}_p \]  \hspace{1cm} \text{(C-6)}
\[ x_7 = A \]  \hspace{1cm} \text{(C-7)}
\[ x_8 = \dot{m} \]  \hspace{1cm} \text{(C-8)}

Determining the \( F \) matrix components from (10-7) yields

\[ F = \sqrt{x_E^2 + y_E^2 + z_E^2} \]  \hspace{1cm} \text{(C-9)}
\[ \dot{F} = \sqrt{\dot{x}_E^2 + \dot{y}_E^2 + \dot{z}_E^2} \]  \hspace{1cm} \text{(C-10)}
\[ F_{12} = F_{34} = F_{56} = 1 \] (C-11)

\[ F_{21} = \frac{-\mu}{E^3} + \frac{3x_\mu}{E^5} \] (C-12)

\[ F_{22} = \frac{-x_\mu^2 A}{E^3} + \frac{A}{E} \] (C-13)

\[ F_{23} = F_{41} = \frac{3x_\nu y_\nu^\mu}{E^5} \] (C-14)

\[ F_{24} = F_{42} = \frac{-x_\nu y_\nu^\mu}{E^3} \] (C-15)

\[ F_{25} = F_{61} = \frac{3x_\nu^2 y_\nu^\mu}{E^5} \] (C-16)

\[ F_{26} = F_{62} = \frac{-x_\nu^2 y_\nu^\mu}{E^3} \] (C-17)

\[ F_{27} = \frac{x_\nu}{E} \] (C-18)

\[ F_{43} = \frac{-\mu}{E^3} + \frac{3y_\mu^2}{E^5} \] (C-19)
\[ F_{44} = \frac{-\gamma_E^2 A}{E^3} + \frac{A}{E} \quad (C-20) \]

\[ F_{45} = F_{63} = \frac{3y_E z_E \mu}{E^5} \quad (C-21) \]

\[ F_{46} = F_{64} = \frac{-\dot{y}_E \dot{z}_E A}{E^3} \quad (C-22) \]

\[ F_{47} = \frac{\dot{y}_E}{E} \quad (C-23) \]

\[ F_{65} = \frac{-\mu}{E^3} + \frac{3z_E^2 \mu}{E^5} \quad (C-24) \]

\[ F_{66} = \frac{-\dot{z}_E^2 A}{E^3} + \frac{A}{E} \quad (C-25) \]

\[ F_{67} = \frac{\dot{z}_E}{E} \quad (C-26) \]

\[ F_{77} = \dot{m} \quad (C-27) \]

\[ F_{78} = A \quad (C-28) \]
\[ F_{88} = 2m \]  \hspace{1cm} (C-29)

All other elements are zero.

The measurements of range and line-of sight angles are:

\[ z_{1k} = \sqrt{x_1^2 + x_3^2 + x_5^2} + V_{Rk} \]  \hspace{1cm} (C-30)

\[ z_{2k} = \tan^{-1}(x_5/x_1) + V_{\theta k} \]  \hspace{1cm} (C-31)

\[ z_{3k} = \tan^{-1}(x_3/x_1) + V_{\gamma k} \]  \hspace{1cm} (C-32)

The \( \mathbf{H}_k \) vectors for serial update from (10-8) are:

\[ R = \sqrt{x_1^2 + x_3^2 + x_5^2} \]  \hspace{1cm} (C-33)

\[ H_{1k1} = x_1/R \]  \hspace{1cm} (C-34)

\[ H_{1k3} = x_3/R \]  \hspace{1cm} (C-35)

\[ H_{1k5} = x_5/R \]  \hspace{1cm} (C-36)

\[ H_{2k1} = \frac{-x_5}{(x_1^2 + x_5^2)} \]  \hspace{1cm} (C-37)
\[ H_{2k5} = \frac{x_1}{(x_1^2 + x_5^2)} \]  
\[ (C-38) \]

\[ H_{3k1} = \frac{-x_3}{(x_1^2 + x_3^2)} \]  
\[ (C-39) \]

\[ H_{3k3} = \frac{x_1}{(x_1^2 + x_3^2)} \]  
\[ (C-40) \]

All other elements are zero.
APPENDIX D

COMPUTER SIMULATION PROGRAM

Contained here are the routines used to simulate the guidance algorithms of Chapters IV through IX. The program (main code) is separate from the supporting routines (subroutines) with the following labeling:

1. KEVSIM - This is the main program for the hyper-velocity orbital intercept.

2. TOOL 1 - Contained here are the subroutines needed for orbit propagation.

3. TOOL 2 - The coordinate transformation matrix subroutines are found here.

4. TOOL 3 - The Extended Kalman Filter subroutines are kept here.

5. TOOL 4 - The subroutines for all the guidance algorithms plus the truth model are here.

The source code for the above can be found on the following pages. All the code is written in FORTRAN 77.
C PROGRAM KEWSIM

C THIS PROGRAM IS A HYPERVELOCITY ORBITAL INTERCEPT SIMULATION THAT FINDS THE VELOCITY CHANGES WITH AN EIGHT, SIX OR THREE STATE EXTENDED KALMAN FILTER AND THREE MEASUREMENTS.

C LINK AS FOLLOWS:
C LINK KEWSIM, TOOL1, TOOL2, TOOL3, TOOL4

C PROGRAM DICTIONARY

C A  EVADER ACCELERATION DUE TO THRUSTING
C AD  DUMMY ACCELERATION
C ASIGX  SIGMA X AXIS SPLINE COEFFICIENT OF T**3
C ASIGY  SIGMA Y AXIS SPLINE COEFFICIENT OF T**3
C ASIGZ  SIGMA Z AXIS SPLINE COEFFICIENT OF T**3
C AT  DUMMY ACCELERATION
C AX  X AXIS SPLINE COEFFICIENT OF T**3
C AY  Y AXIS SPLINE COEFFICIENT OF T**3
C AZ  Z AXIS SPLINE COEFFICIENT OF T**3
C BSIGX  SIGMA X AXIS SPLINE COEFFICIENT OF T**2
C BSIGY  SIGMA Y AXIS SPLINE COEFFICIENT OF T**2
C BSIGZ  SIGMA Z AXIS SPLINE COEFFICIENT OF T**2
C BX  X AXIS SPLINE COEFFICIENT OF T**2
C BY  Y AXIS SPLINE COEFFICIENT OF T**2
C BZ  Z AXIS SPLINE COEFFICIENT OF T**2
C COUNT  ITERATION FINAL COUNT
C COV  COVARIANCE MATRIX
C CVD  COVARIANCE MATRIX TRACE ELEMENTS
C CVDU  DUMMY COVARIANCE MATRIX
C CVDUAL  DUMMY COVARIANCE MATRIX
C CSIGX  SIGMA X AXIS SPLINE COEFFICIENT OF T
C CSIGY  SIGMA Y AXIS SPLINE COEFFICIENT OF T
C CSIGZ  SIGMA Z AXIS SPLINE COEFFICIENT OF T
C CX  X AXIS SPLINE COEFFICIENT OF T
C CY  Y AXIS SPLINE COEFFICIENT OF T
C CZ  Z AXIS SPLINE COEFFICIENT OF T
C DDX  CHANGE IN X VELOCITY (INERTIAL FRAME)
C DDY  CHANGE IN Y VELOCITY (INERTIAL FRAME)
C DDZ  CHANGE IN Z VELOCITY (INERTIAL FRAME)
C DELTAY  CHANGE IN Y VELOCITY (BODY FRAME)
C DELTAZ  CHANGE IN Z VELOCITY (BODY FRAME)
C DH  DUMMY STEP SIZE
C DSIGX  SIGMA X AXIS SPLINE COEFFICIENT
C DSIGY  SIGMA Y AXIS SPLINE COEFFICIENT
C DSIGZ  SIGMA Z AXIS SPLINE COEFFICIENT
C DTGO  DUMMY TIME-TO-GO
C DUM  DUMMY VARIABLE
C DV  INCREMENTAL VELOCITY CHANGE
C DX  X AXIS SPLINE COEFFICIENT
C DY  Y AXIS SPLINE COEFFICIENT
C DZ  Z AXIS SPLINE COEFFICIENT
C FILTER  NUMBER OF FILTER STATES
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
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<td>GAUSSIAN POINT</td>
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<td>H</td>
<td>STEP SIZE</td>
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<td>I</td>
<td>ITERATION COUNTER</td>
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<td>CONSTRAINT BASED ON FINAL COVARIANCE</td>
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<td>MDOTD</td>
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<td>DUMMY MASS FLOW RATE OF EVADER</td>
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<td>OPT</td>
<td>OPTION</td>
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<td>1</td>
<td>WITHOUT KALMAN FILTER</td>
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<td>2</td>
<td>WITH KALMAN FILTER</td>
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<td>3</td>
<td>WITH KALMAN FILTER + PRINTOUT</td>
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<tr>
<td>PLAN</td>
<td>PLAN OPTION</td>
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<tr>
<td>1</td>
<td>PLAN A</td>
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<td>2</td>
<td>PLAN B</td>
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<td>3</td>
<td>PLAN C</td>
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<td>4</td>
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<td>5</td>
<td>CERTAINTY CONTROL</td>
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<td>6</td>
<td>TRUTH MODEL</td>
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<tr>
<td>Q</td>
<td>PROPAGATION NOISE VARIANCE</td>
</tr>
<tr>
<td>RANGE</td>
<td>RANGE MEASUREMENT OF EVADER FROM PURSUER</td>
</tr>
<tr>
<td>RES</td>
<td>MEASUREMENT RESIDUALS</td>
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<tr>
<td>RHO</td>
<td>CONTROL EFFECTIVENESS RATIO</td>
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<td>R3</td>
<td>MEASUREMENT NOISE VARIANCE</td>
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<tr>
<td>SEED</td>
<td>RANDOM NUMBER SEED</td>
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<td>SFILTR</td>
<td>SIMULATION FILTER</td>
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<tr>
<td>SFLAG</td>
<td>SEARCH CONVERGENCE FLAG</td>
</tr>
<tr>
<td>SIGMAM</td>
<td>STANDARD DEVIATION OF MEASUREMENTS</td>
</tr>
<tr>
<td>SIGT0</td>
<td>INITIAL X,Y,Z DEVIATIONS AND THEIR RATES</td>
</tr>
<tr>
<td>SIGTF</td>
<td>FINAL X,Y,Z DEVIATIONS AND THEIR RATES</td>
</tr>
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<td>SIMCNT</td>
<td>SIMULATION COUNTER</td>
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<tr>
<td>SKFLAG</td>
<td>SIMULATION KALMAN GAIN CONVERGENCE FLAG</td>
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<tr>
<td>SCN</td>
<td>SIMULATION NUMBER</td>
</tr>
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<td>SPLAN</td>
<td>SIMULATION PLAN</td>
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<tr>
<td>SRANGE</td>
<td>SIMULATION RANGE</td>
</tr>
<tr>
<td>SSFLAG</td>
<td>SIMULATION SEARCH CONVERGENCE FLAG</td>
</tr>
<tr>
<td>SVTOT</td>
<td>SIMULATION TOTAL VELOCITY CHANGE</td>
</tr>
<tr>
<td>SW</td>
<td>INTEGER SWITCH FOR FUNCTION 'GAUSS'</td>
</tr>
<tr>
<td>T</td>
<td>TIME</td>
</tr>
<tr>
<td>TD</td>
<td>DUMMY TIME</td>
</tr>
<tr>
<td>TGO</td>
<td>TIME-TO-GO (UNTIL IMPACT)</td>
</tr>
<tr>
<td>THETA</td>
<td>OBSERVED LOS ANGLE (OUT OF PLANE)</td>
</tr>
<tr>
<td>TMAT</td>
<td>TRANSFORMATION MATRIX</td>
</tr>
<tr>
<td>TOL</td>
<td>RANGE TOLERANCE FOR SEARCH ROUTINE</td>
</tr>
</tbody>
</table>
TSTART   START TIME FOR CONTROL
UPDATE   UPDATE FLAG FOR EKF
       0 - NO UPDATE
       1 - UPDATE
       2 - UPDATE WITH RESIDUALS SET TO ZERO
VAR      VARIANCE OF MEASUREMENT RESIDUALS
VEL      EVADER VELOCITY
VTOT     TOTAL VELOCITY CHANGE
XDUAL    DUMMY XHAT VECTOR
XDUALD   DUMMY XHAT VECTOR
XE       STATE VECTOR OF EVADER
XED      DUMMY STATE VECTOR OF EVADER
XEDD     DUMMY STATE VECTOR OF EVADER
XEEST    ESTIMATED STATE VECTOR OF EVADER
XET      TRANSFORMED STATE VECTOR OF EVADER
XHAT     ESTIMATED STATES
       1 - RELATIVE X POSITION
       2 - RELATIVE X VELOCITY
       3 - RELATIVE Y POSITION
       4 - RELATIVE Y VELOCITY
       5 - RELATIVE Z POSITION
       6 - RELATIVE Z VELOCITY
       7 - A
       8 - MDOT
XP       STATE VECTOR OF PURSUER
XPD      DUMMY STATE VECTOR OF PURSUER
XPDD     DUMMY STATE VECTOR OF PURSUER
XPEST    ESTIMATED STATE VECTOR OF PURSUER
XPT      TRANSFORMED STATE VECTOR OF PURSUER
XR       RELATIVE EVADER STATE VECTOR

DECLARE VARIABLES
    REAL*8 A,MDOT,T,TIME,H,XE(6),XP(6),XED(6),XPD(6)
    REAL*8 TGO,MING,RES(3),VAR(3),MEAN(3),XHAT(8)
    REAL*8 DELTAY,DELTAZ,DV,K,DH,TD,TMAT(3,3),MAXG
    REAL*8 AX,BX,CX,DX,AZ,BZ,CZ,DZ,G2
    REAL*8 DDW,DDY,DDZ,XET(6),XPT(6),SIGMAM(3),MISS2
    REAL*8 COV(8,8),Q(8,8),R3(3,3),XEEST(6),XPEST(6)
    REAL*8 MAXDV,MINDV,VTOT,XR(6),RANGE,THETA,GAMMA
    REAL*8 MDOTT,MDOTD,SIGT0(6),SIGTF(6)
    REAL*8 GAUSS,AD,AT,TOL,DUM,TSTART,KDEVF,RHO
    REAL*8 SRANGE,SVTOT,ASIGZ,BSIGZ,CSIGZ,DSIGZ
    REAL*8 ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,BSIGY,CSIGY
    REAL*8 DSIGY,CVD(6),XPDD(6),CVDD(8,8),XEDD(6)
    REAL*8 XDUAL(8),XDUALD(8),CVDUAL(8,8),DTGO
    INTEGER I,J,JUP,COUNT,SIMCNT,SEED,OPT,SW,PLAN
    INTEGER SNUM,UPDATE,SSFLAG
    INTEGER KFLAG,SFLAG,FILTER,SPLEN,SFILTR,SKFLAG
**INITIALIZATIONS**

READ IN INITIAL CONDITIONS FOR DYNAMICS

```
C READ IN INITIAL CONDITIONS FOR DYNAMICS

READ IN INITIAL CONDITIONS FOR DYNAMICS
1 FORMAT(2X,3F14.3)
6 FORMAT(2X,F8.2)
8 FORMAT(2X,F9.5)

OPEN(UNIT=2,NAME= 'ENGR.THESIS.SALFANO.INIT.DAT',
+ TYPE='OLD',READONLY)
 READ(2,1)XE(1),XE(3),XE(5)
 READ(2,1)XE(2),XE(4),XE(6)
 READ(2,1)XP(1),XP(3),XP(5)
 READ(2,1)XP(2),XP(4),XP(6)
 READ(2,6)TGO
 READ(2,8)A
 READ(2,8)MDOT

CLOSE(2)
 PRINT *,
 PRINT *, ENTER TIME STEP VALUE'
 READ *,H
 PRINT *,
 PRINT *, ENTER CONTROL/CONSTRAINT MULTIPLIER'
 READ *,K
 PRINT *,
 PRINT *, ENTER CONTROL EFFECTIVENESS RATIO'
 READ *,RHO

IF (RHO .LT. 1.0) RHO=1.0
```

READ IN FILTER MEASUREMENT STANDARD DEVIATIONS
AND ASSIGN COVARIANCES TO R MATRIX DIAGONAL

```
C READ IN FILTER MEASUREMENT STANDARD DEVIATIONS
C AND ASSIGN COVARIANCES TO R MATRIX DIAGONAL

7 FORMAT(F14.10)

OPEN(UNIT=3,NAME= 'ENGR.THESIS.SALFANO.FILTER.REL',
+ TYPE='OLD',READONLY)
 READ(3,7)SIGMAM(1)
 READ(3,7)SIGMAM(2)
 READ(3,7)SIGMAM(3)

CLOSE(3)
 XR(1)=XE(1)-XP(1)
 XR(3)=XE(3)-XP(3)
 XR(5)=XE(5)-XP(5)

RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
 R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
 R3(2,2)=SIGMAM(2)*SIGMAM(2)
 R3(3,3)=SIGMAM(3)*SIGMAM(3)

C READ IN THE NEXT SEED

OPEN(UNIT=4,NAME= 'SIM.STATS',TYPE='OLD',READONLY)

10 FORMAT(2X,I3,2X,I14)

READ(4,10)SNUM,SEED
```
PRINT *, 'WHAT FILTER DO YOU CHOOSE?'
PRINT *, ' 8 - EIGHT STATE EKF'
PRINT *, ' 6 - SIX STATE EKF'
PRINT *, ' 60 - SIX STATE EKF WITHOUT GRAVITY'
READ *, FILTER

C ZERO OUT OFF DIAGONAL FILTER MATRIX COMPONENTS
   DO 40 I=2,8
      JUP=I-1
      DO 40 J=1,JUP
         COV(I,J)=0.0
         COV(J,I)=0.0
         Q(I,J)=0.0
         Q(J,I)=0.0
   40 CONTINUE

C ESTABLISH ACCELERATION AND
C MDOT PROPAGATION VARIANCES
   DUM=.1*MDOT*H
   Q(8,8)=DUM*DUM/H
   Q(7,7)=A*A*DUM*DUM/H

C COMPUTE AND INITIALIZE PROPAGATION VARIANCES
C COMPUTE TRANSFORMATION MATRIX
   CALL COMPTV(XP,TMAT)

C TRANSFORM ESTIMATED STATE VARIABLES
   CALL TRANSFWD(XP(1),XP(3),XP(5),
                  TMAT,XPT(1),XPT(3),XPT(5))
   CALL TRANSFWD(XP(2),XP(4),XP(6),
                  TMAT,XPT(2),XPT(4),XPT(6))
   CALL TRANSFWD(XE(1),XE(3),XE(5),
                  TMAT,XET(1),XET(3),XET(5))
   CALL TRANSFWD(XE(2),XE(4),XE(6),
                  TMAT,XET(2),XET(4),XET(6))

C INITIALIZE STATE VECTORS IN NEW FRAME
   DO 50 I=1,6
      XP(I)=XPT(I)
      XE(I)=XET(I)
      XED(I)=XET(I)
   50 CONTINUE

C ESTABLISH DUMMY TIME STEP
   DH=H/256

C PROPAGATE DUMMY VARIABLES FORWARD ONE STEP
   TD=0.0
   AD=A
   MDOTD=MDOT
   DO 60 I=1,256
      CALL RK4SYSE(TD,XED,DH,AD,MDOTD)
      TD=TD+DH
   60 CONTINUE
C PROPAGATE TRANSFORMED VARIABLES FORWARD ONE STEP
T=0.0
IF (FILTER .EQ. 8) THEN
    AT=A
    MDOTT=MDOT
    CALL RK4SYSE(T,XET,H,AT,MDOTT)
ENDIF
IF (FILTER .EQ. 6) THEN
    AT=A+SQRT(H*Q(7,7))
    MDOTT=MDOT+SQRT(H*Q(8,8))
    CALL RK4SYSE(T,XET,H,AT,MDOTT)
ENDIF
IF (FILTER .EQ. 60) THEN
    XET(1)=XET(1)+H*XET(2)
    XET(3)=XET(3)+H*XET(4)
    XET(5)=XET(5)+H*XET(6)
    AT=A+SQRT(H*Q(7,7))
    DUM=1.0+H*AT/SQRT(XET(2)**2+XET(4)**2+
         XET(6)**2)
    XET(2)=XET(2)*DUM
    XET(4)=XET(4)*DUM
    XET(6)=XET(6)*DUM
ENDIF

C COMPUTE REMAINING Q DIAGONAL COMPONENTS
Q(1,1)=((XET(1)-XED(1))**2+(XET(3)-XED(3))**2+ 
        (XET(5)-XED(5))**2)/3.0/H
Q(3,3)=Q(1,1)
Q(5,5)=Q(1,1)
Q(2,2)=((XET(2)-XED(2))**2+(XET(4)-XED(4))**2+ 
        (XET(6)-XED(6))**2)/3.0/H
Q(4,4)=Q(2,2)
Q(6,6)=Q(2,2)
Q(7,7)=(AT-AD)*(AT-AD)/H

C ASSIGN STARTUP COVARIANCES
COV(1,1)=100.0
COV(2,2)=SQRT(COV(1,1))
COV(3,3)=COV(1,1)
COV(4,4)=COV(2,2)
COV(5,5)=COV(1,1)
COV(6,6)=COV(2,2)
COV(7,7)=(0.1*A)**2
COV(8,8)=(0.1*MDOT)**2

C INITIALIZE VARIABLES
TOL=0.0001
TSTART=3.0
SFLAG=0
KFLAG=0
SW=0
DELTAY=0.0
DELTAZ=0.0
VTOT=0.0
MAXG=6.0
MING=0.05*MAXG
MAXDV=MAXG*10.0*H
MINDV=MING*10.0*H
COUNT=100

C ASK USER TO CHOOSE CONTROL METHOD
PRINT *,
PRINT *, WHAT CONTROL METHOD DO YOU CHOOSE ?'
PRINT *, 1 - PLAN A'
PRINT *, 2 - PLAN B'
PRINT *, 3 - PLAN C'
PRINT *, 4 - DUAL CONTROL'
PRINT *, 5 - CERTAINTY CONTROL'
PRINT *, 6 - TRUTH MODEL'
READ *,PLAN
UPDATE=1
IF (PLAN .EQ. 4) UPDATE=2
IF (PLAN .EQ. 5) UPDATE=0

C ASK USER FOR NOISE OPTION
PRINT *,
PRINT *, CHOOSE YOUR OPTION'
PRINT *, 1 - NO NOISE'
PRINT *, 2 - NOISE'
PRINT *, 3 - NOISE + SCREEN PRINTOUT'
PRINT *, 4 - NOISE + DATAFILE PRINTOUT'
READ *,OPT

C INITIALIZE ESTIMATED VARIABLES
DO 100 I=1,6
    XPEST(I)=XP(I)
    IF (OPT .EQ. 1) THEN
        XEEST(I)=XE(I)
    ELSE
        XEEST(I)=XE(I)+SQRT(COV(I,I))*GAUSS(SEED,SW,G2)
    ENDIF
    XHAT(I)=XEEST(I)-XPEST(I)
100 CONTINUE
IF (OPT .EQ. 1) THEN
    XHAT(7)=A
    XHAT(8)=MDOT
ELSE
    XHAT(7)=A+SQRT(COV(7,7))*GAUSS(SEED,SW,G2)
    XHAT(8)=MDOT+SQRT(COV(8,8))*GAUSS(SEED,SW,G2)
ENDIF

C OPEN UNITS FOR WRITING OUTPUT DATA
5 FORMAT(F13.7,2X,F13.7)
IF (OPT .EQ. 4) THEN
    OPEN(UNIT=11,FILE='RES1.DAT',STATUS='NEW',
        IOSTAT=ISTAT)
OPEN(UNIT=12, FILE='RES2.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=13, FILE='RES3.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=14, FILE='RES4.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=15, FILE='RES5.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=16, FILE='RES6.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=17, FILE='RES7.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=18, FILE='RES8.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=19, FILE='DELTAY.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=20, FILE='DELTAYZ.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=21, FILE='COV1.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=22, FILE='COV2.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=23, FILE='COV3.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=24, FILE='COV4.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=25, FILE='COV5.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=26, FILE='COV6.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=27, FILE='COV7.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=28, FILE='COV8.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=29, FILE='MISS.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=30, FILE='TOL.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=31, FILE='COVM1.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=32, FILE='COVM2.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=33, FILE='COVM3.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=34, FILE='COVM4.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=35, FILE='COVM5.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=36, FILE='COVM6.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=37, FILE='COVM7.DAT', STATUS='NEW', IOSTAT=ISTAT)
OPEN(UNIT=38, FILE='COVM8.DAT', STATUS='NEW', IOSTAT=ISTAT)
ENDIF
**BEGIN SIMULATION LOOP**

DO 990 SIMCNT=1,50000

C PRINT RESIDUALS, VELOCITY CHANGES, AND COVARIANCES TO DATAFILES

IF (OPT .EQ. 4) THEN
    WRITE(11,5), T, XEEST(1) - XE(1)
    WRITE(12,5), T, XEEST(2) - XE(2)
    WRITE(13,5), T, XEEST(3) - XE(3)
    WRITE(14,5), T, XEEST(4) - XE(4)
    WRITE(15,5), T, XEEST(5) - XE(5)
    WRITE(16,5), T, XEEST(6) - XE(6)
    WRITE(17,5), T, XHAT(7) - A
    WRITE(18,5), T, XHAT(8) - MDOT
    IF ((DELTAY .NE. 0.0) .OR. (SIMCNT .EQ. 1))
        WRITE(19,5), T, DELTAY
    ELSE
        WRITE(19,5), T, DELTAY
    ENDIF
    WRITE(20,5), T, DELTAZ
    WRITE(21,5), T, SQRT(COV(1,1))
    WRITE(22,5), T, SQRT(COV(2,2))
    WRITE(23,5), T, SQRT(COV(3,3))
    WRITE(24,5), T, SQRT(COV(4,4))
    WRITE(25,5), T, SQRT(COV(5,5))
    WRITE(26,5), T, SQRT(COV(6,6))
    WRITE(27,5), T, SQRT(COV(7,7))
    WRITE(28,5), T, SQRT(COV(8,8))
    WRITE(29,5), T, SQRT(MISS2)
    IF (KDEVF .GT. 0.0) WRITE(30,5), T, KDEVF
    WRITE(31,5), T, -SQRT(COV(1,1))
    WRITE(32,5), T, -SQRT(COV(2,2))
    WRITE(33,5), T, -SQRT(COV(3,3))
    WRITE(34,5), T, -SQRT(COV(4,4))
    WRITE(35,5), T, -SQRT(COV(5,5))
    WRITE(36,5), T, -SQRT(COV(6,6))
    WRITE(37,5), T, -SQRT(COV(7,7))
    WRITE(38,5), T, -SQRT(COV(8,8))
ENDIF

C TEST TO SEE IF TIME IS UP

IF (TGO .LE. H) GOTO 995

C REASSIGN DUMMY VARIABLES

DO 105 I=1,6
    XP(D(I)) = XPEST(I)
    XED(I) = XEEST(I)
105    CONTINUE
IF ((UPDATE .NE. 1) .AND. (T .GE. TSTART)) THEN
  DO 106 I=1,8
    XDUAL(I)=XHAT(I)
  DO 106 J=1,8
    CVDUAL(I,J)=COV(I,J)
  106 CONTINUE
ENDIF

C PROPAGATE DUMMY VARIABLES FORWARD ONE STEP
TD=T
IF ((UPDATE .NE. 1) .AND. (T .GE. TSTART)) THEN
  RANGE=SQR(T(XDUAL(1)*XDUAL(1)+XDUAL(3)*XDUAL(3)+
           XDUAL(5)*XDUAL(5))
  R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
  IF (FILTER .EQ. 8) CALL EKF8(XDUAL,XED,XPD,TD,
          H,CVDUAL,Q,R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
  ELSE
    Q(2,2)=1.0+XDUAL(7)*H*Q(8,8)
    Q(4,4)=Q(2,2)
    Q(6,6)=Q(2,2)
    CALL EKF6(XDUAL,XED,XPD,TD,H,CVDUAL,Q,
              R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
  ENDIF
  AD=XDUAL(7)
  MDOTD=XDUAL(8)
ELSE
  AD=XHAT(7)
  MDOTD=XHAT(8)
  CALL RK4SYSP(T&.w,XPD,H)
  CALL RK4SYSE(TD,XED,H,AD,MDOTD)
ENDIF
TD=TD+H
TGO=TGO-H

C INITIALIZE TRANSFORMED VARIABLES TO DUMMY VARIABLES
AT=AD
MDOTT=MDCTD
DO 110 I=1,6
  XET(I)=XED(I)
  XPT(I)=XPD(I)
110 CONTINUE
IF (PLAN .EQ. 4) THEN
  DO 111 I=1,8
    XDUALD(I)=XDUAL(I)
  DO 111 J=1,8
    CVDD(I,J)=CVDU(I,J)
111 CONTINUE
DO 112 I=1,6
  XEDD(I)=XED(I)
  XPDD(I)=XPD(I)
112 CONTINUE
ENDIF
C STORE INITIAL VALUES OF DEVIATIONS FOR
C SPLINE COMPUTATION
IF ((PLAN .EQ. 5) .AND.
   + (T .GE. TSTART)) THEN
   SIGTO(1)=SQRT(CVDUAL(1,1))
   SIGTO(2)=(SIGTO(1)-SQRT(COV(1,1)))/H
   SIGTO(3)=SQRT(CVDUAL(3,3))
   SIGTO(4)=(SIGTO(3)-SQRT(COV(3,3)))/H
   SIGTO(5)=SQRT(CVDUAL(5,5))
   SIGTO(6)=(SIGTO(5)-SQRT(COV(5,5)))/H
ENDIF

C ESTABLISH SPLINE TIME STEP
IF (TGO/COUNT .LT. H) COUNT=COUNT-1
IF (COUNT .LT. 1) COUNT=1
DH=TGO/COUNT

C PROPAGATE DUMMY VARIABLES FORWARD
C TO PREDICTED IMPACT TIME
IF ((PLAN .EQ. 5) .AND.
   + (T .GE. TSTART)) THEN
   DO 113 I=1,COUNT
   IF (I .EQ. COUNT) THEN
   CVD(1)=CVDUAL(1,1)
   CVD(3)=CVDUAL(3,3)
   CVD(5)=CVDUAL(5,5)
   ENDIF
   RANGE=SQRT(XDUAL(1)*XDUAL(1)+XDUAL(3)*XDUAL(3)+XDUAL(5)*XDUAL(5))
   R3(1,1)=SIGMAN(1)*RANGE*SIGMAN(1)*RANGE
   IF (FILTER .EQ. 8) THEN
   CALL EKF8(XDUAL,XED,XPD,TD,DH,CVDUAL,Q,R3,
   + 0.0,0.0,0.0,KFLAG,RES,UPDATE)
   ENDIF
   IF (FILTER .EQ. 6) THEN
   Q(2,2)=1.0+XDUAL(7)*DH*Q(8,8)
   Q(4,4)=Q(2,2)
   Q(6,6)=Q(2,2)
   CALL EKF6(XDUAL,XED,XPD,TD,DH,CVDUAL,Q,
   + R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
   ENDIF
   TD=TD+DH
113 CONTINUE
SIGTF(1)=SQRT(CVDUAL(1,1))
SIGTF(2)=(SIGTF(1)-SQRT(CVD(1)))/DH
SIGTF(3)=SQRT(CVDUAL(3,3))
SIGTF(4)=(SIGTF(3)-SQRT(CVD(3)))/DH
SIGTF(5)=SQRT(CVDUAL(5,5))
SIGTF(6)=(SIGTF(5)-SQRT(CVD(5)))/DH
KDEVF=SQRT(K*(CVDUAL(1,1)+CVDUAL(3,3)+
   CVDUAL(5,5)))
ELSE
DO 115 I=1,COUNT
    CALL RK4SYSP(TD,XPD,DH)
    CALL RK4SYSE(TD,XED,AD,MDOTD)
    TD=TD+DH
115 CONTINUE
ENDIF

C COMPUTE FINAL ESTIMATED RELATIVE STATES
DO 120 I=1,6
    XR(I)=XED(I)-XPD(I)
120 CONTINUE

C COMPUTE ESTIMATED MISS DISTANCE SQUARED
MISS2=XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5)

C UPDATE DUMMY TIME-TO-GO
DTGO=TD-T-H

C COMPUTE VELOCITY CHANGES
IF ((TGO .GT. H) .AND. (T .GE. TSTART)) THEN
    C COMPUTE RELATIVE SPLINE COEFFICIENTS
    CALL SPLINE(XET(1)-XPT(1),XET(2)-XPT(2),XR(1),XR(2),DTGO,AX,BX,CX,DX)
    CALL SPLINE(XET(3)-XPT(3),XET(4)-XPT(4),XR(3),XR(4),DTGO,AY,BY,CY,DY)
    CALL SPLINE(XET(5)-XPT(5),XET(6)-XPT(6),XR(5),XR(6),DTGO,AZ,BZ,CZ,DZ)
ENDIF

C COMPUTE OPTIMAL CHANGES IN VELOCITY AND IMPACT TIME
IF (PLAN .EQ. 5) THEN
    CALL SPLINE(SIGTO(1),SIGTO(2),SIGTF(1),SIGTF(2),DTGO,ASIGX,BSIGX,CSIGX,DSIGX)
    CALL SPLINE(SIGTO(3),SIGTO(4),SIGTF(3),SIGTF(4),DTGO,ASIGY,BSIGY,CSIGY,DSIGY)
    CALL SPLINE(SIGTO(5),SIGTO(6),SIGTF(5),SIGTF(6),DTGO,ASIGZ,BSIGZ,CSIGZ,DSIGZ)
ENDIF

C COMPUTE OPTIMAL CHANGES IN VELOCITY AND IMPACT TIME
IF (PLAN .EQ. 1) THEN
    CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,SFLAG)
ENDIF

IF (PLAN .EQ. 2) THEN
    CALL SEARCHB(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,TGO,DELTAY,DELTAZ,TOL,SFLAG)
ENDIF
IF (PLAN .EQ. 3) THEN
    DO 135 I=1,6
        XR(I)=XET(I)-XPT(I)
    CONTINUE
    CALL SEARCHC(XR,XET(2),XET(4),XET(6),AT,
                MDOT,TGO,DELTAY,DELTAZ,TOL,SFLAG)
ENDIF

IF (PLAN .EQ. 4) THEN
    CALL SEARCHD(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
                CX,DZ,K,TGO,DELTAY,DELTAZ,TOL,SFLAG,
                XD,JALD,XEDD,XPDD,CVDD,MADV,COUNT,
                Q,R3,SIGMA,H)
ENDIF

IF (PLAN .EQ. 5) THEN
    CALL SEARCHC(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
                CZ,DZ,ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,BSIGY,
                CSIGY,DSIGY,ASIGZ,BSIGZ,CSIGZ,DSIGZ,K,TGO,
                !DELTAY,DELTAZ,TOL,SFLAG)
ENDIF

IF (PLAN .EQ. 6) THEN
    CALL SEARCHT(XPT,XET,AT,MDOTT,H,T,TGO,
                 DELTAY,DELTAZ,TOL,SFLAG,COUNT)
ENDIF

C BOUND VELOCITY CHANGES
IF (ABS(DELTAY) .LT. MINDV) THEN
    DELTAY=0.0
ELSE
    IF (DELTAY .GT. MAXDV) DELTAY=MAXDV
    IF (DELTAY .LT. -MAXDV) DELTAY=-MAXDV
ENDIF

IF (ABS(DELTAZ) .LT. MINDV) THEN
    DELTAZ=0.0
ELSE
    IF (DELTAZ .GT. MAXDV) DELTAZ=MAXDV
    IF (DELTAZ .LT. -MAXDV) DELTAZ=-MAXDV
ENDIF

DV=ABS(DELTAY)+ABS(DELTAZ)
IF (DV .GE. MAXDV) THEN
    TSTART=T
ELSE
    TSTART=T+(RHC-1.0)*TGO/RHO
ENDIF

ELSE

    DELTAY=0.0
    DELTAZ=0.0
    DV=0.0
ENDIF

IF (DV .GE. MINDV) WRITE(*,1)DELTAY,DELTAZ,TGO

C PROPAGATE REAL VARIABLES AND PURSUER ESTIMATE
C FORWARD ONE STEP (I.M.U. ASSUMED PERFECT)
CALL RK4SYS$(T,XP,H)
CALL RK4SYS$(T,XE,H,A,MDOT)
DO 210 I=1,6
   XPEST(I)=XP(I)
210 CONTINUE

C COMPUTE SENSOR MEASUREMENTS (PLUS NOISE)
XR(1)=XE(1)-XP(1)
XR(3)=XE(3)-XP(3)
XR(5)=XE(5)-XP(5)
RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
R3(1,1)=RANGE*SIGMAM(1)*RANGE*SIGMAM(1)
THETA=ATAN(XR(5)/XR(1))
GAMMA=ATAN(XR(3)/XR(1))
IF (OPT .NE. 1) THEN
   RANGE=RANGE*(1.0+GAUSS(SEED,SW,G2)*SIGMAM(1))
   THETA=THETA+GAUSS(SEED,SW,G2)*SIGMAM(2)
   GAMMA=GAMMA+GAUSS(SEED,SW,G2)*SIGMAM(3)
ENDIF

C GET FILTER ESTIMATES
IF (FILTER .EQ. 8) CALL EKFB(XHAT,XEEST,XPEST,T,H,COV,Q,R3,RANGE,THETA,GAMMA,KFLAG,RES,1)
IF (FILTER .EQ. 6) THEN
   Q(2,2)=1.0+XHAT(7)*H*Q(8,8)
   Q(4,4)=Q(2,2)
   Q(6,6)=Q(2,2)
   CALL EKF6(XHAT,XEEST,XPESTIT,H,COV,QR3,RANGE,THETA,GAMMA,KFLAG,RES,1)
ENDIF

IF (FILTER .EQ. 60) CALL EKF60(XHAT,XEEST,XPEST,T,H,COV,Q,R3,RANGE,THETA,GAMMA,KFLAG,RES)

C UPDATE EVADER ESTIMATE USING RELATIVE ESTIMATE
DO 200 I=1,6
   XEEST(I)=XPEST(I)+XHAT(I)
200 CONTINUE

C RECURSIVELY COMPUTE MEAN AND VARIANCE OF
C MEASUREMENT RESIDUALS
IF (SIMCNT .EQ. 1) THEN
   VAR(1)=RES(1)*RES(1)
   VAR(2)=RES(2)*RES(2)
   VAR(3)=RES(3)*RES(3)
ELSE
   VAR(1)=VAR(1)*(SIMCNT-2)/(SIMCNT-1)+
   (MEAN(1)-RES(1))*(MEAN(1)-RES(1))/SIMCNT
ENDIF
VAR(2) = VAR(2) * (SIMCNT - 2) / (SIMCNT - 1) + 
      (MEAN(2) - RES(2)) * (MEAN(2) - RES(2)) / SIMCNT 
VAR(3) = VAR(3) * (SIMCNT - 2) / (SIMCNT - 1) + 
      (MEAN(3) - RES(3)) * (MEAN(3) - RES(3)) / SIMCNT 
ENDIF
MEAN(1) = (MEAN(1) * (SIMCNT - 1) + RES(1)) / SIMCNT 
MEAN(2) = (MEAN(2) * (SIMCNT - 1) + RES(2)) / SIMCNT 
MEAN(3) = (MEAN(3) * (SIMCNT - 1) + RES(3)) / SIMCNT 
C PRINT ESTIMATED AND TRUE STATES AND COVARIANCES
IF (OPT .EQ. 3) THEN
  PRINT *, 'ESTIMATED STATE, TRUE STATE, ERROR'
  PRINT *, 'AND COVARIANCE'
  DO 220 I=1,6
    PRINT *, XHAT(I), XE(I) - XP(I), 
    XHAT(I) - XE(I) + XP(I)
    PRINT *, ', COV(I,I)
  220 CONTINUE
ENDIF
C APPLY VELOCITY CHANGES
XP(4) = XP(4) + DELTAY
XP(6) = XP(6) + DELTAZ
XPEST(4) = XP(4)
XPEST(6) = XP(6)
XHAT(4) = XHAT(4) - DELTAY
XHAT(6) = XHAT(6) - DELTAZ
VTOT = VTOT + DV
C UPDATE TIME
T = T + H
990 CONTINUE
C
C ***********************
C * END SIMULATION LOOP *
C ***********************
995 CONTINUE
C PRINT SQUARE ROOT OF COVARIANCE DIAGONAL
PRINT *,'
PRINT *, 'DEVIATIONS, ERROR'
DO 800 I=1,6
  PRINT *, SQRT(COV(I,I)), XEEST(I) - XE(I)
800 CONTINUE
PRINT *, SQRT(COV(7,7)), XHAT(7) - A
PRINT *, SQRT(COV(8,8)), XHAT(8) - MDOT
C PROPAGATE REAL DATA TO FINAL PREDICTED IMPACT TIME
    DH=TGO
    CALL RK4SYS(T,XP,DH)
    CALL RK4SYSE(T,XE,DH,A,MDOT)
    T=T+DH

C ITERATE TO FIND POINT OF CLOSEST APPROACH
    DH=H
    DO 310 J=1,25
       DO 300 I=1,6
           XR(I)=XE(I)-XP(I)
       300 CONTINUE
       IF (ABS(DH) .LE. 0.0000001) GOTO 320
       DH=XR(1)*XR(2)+XR(3)*XR(4)+XR(5)*XR(6)
       DH=-(DH)/(XR(2)*XR(2)+XR(4)*XR(4)+XR(6)*XR(6))
       CALL RK4SYS(T,XP,DH)
       CALL RK4SYSE(T,XE,DH,A,MDOT)
       T=T+DH
    310 CONTINUE

320 CONTINUE

C PRINT CONVERGENCE MESSAGE
    PRINT *, 'SEARCH NON-CONVERGENCE =', SFLAG
    PRINT *, 'GAIN NON-CONVERGENCE =', KFLAG

C PRINT TIME AND MISS OF CLOSEST APPROACH
    PRINT *, 'IMPACT TIME :', T
    RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
    PRINT *, 'TOTAL VELOCITY CHANGE :', VTOT
    PRINT *, 'MISS DISTANCE : RANGE
    PRINT *,XR(1),XR(3),XR(5)

C CLOSE OUTPUT DATA FILES
    IF (OPT .EQ. 4) THEN
       DO 340 I=1,38
           CLOSE(I)
       340 CONTINUE
    ENDIF

C READ IN PREVIOUS SIMULATION DATA, SORT ON RANGE, C AND WRITE TO NEW FILE
    OPEN(UNIT=5,FILE='SIM.STATS',STATUS='NEW', +
           IOSTAT=ISTAT)
    WRITE(5,10)(SNUM+1),SEED
    SRANGE=0.0
    RANGE=SIGN(RANGE,XR(3))
    J=0
DO 350 I=1,SNUM
    READ(4,9)SRANGE,SVTOT,SPLAN,SFILTR,SSFLAG,SKFLAG
    IF ((J .EQ. 0) .AND. (RANGE .LE. SRAANGE)) THEN
        WRITE(5,9)RANGE,VTOT,PLAN,FILTER,SFLAG,KFLAG
        J=1
    ENDIF
    WRITE(5,9)SRANGE,SVTOT,SPLAN,SFILTR,SSFLAG,SKFLAG
    CONTINUE
    IF (J .EQ. 0)
        WRITE(5,9)RANGE,VTOT,PLAN,FILTER,SFLAG,KFLAG
    CLOSE(4)
    CLOSE(5)
END
THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR 
ORBIT PROPAGATION 
IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM 

SUBROUTINE DICTIONARY 

A EVADER ACCELERATION DUE TO THRUSTING 
AD DUMMY ACCELERATION 
COUNT ITERATION FINAL COUNT 
MDOT UNITIZED MASS FLOW RATE OF EVADER 
MDOTD DUMMY MASS FLOW RATE OF EVADER 
H STEP SIZE 
I ITERATION COUNTER 
T TIME 
TFINAL FINAL TIME 
XE STATE VECTOR OF EVADER 
XP STATE VECTOR OF PURSUER 

SUBROUTINE XPSYSP(X,F) 

THIS SUBROUTINE EVALUATES THE FUNCTIONS FOR RK4SYSP 
(ORBITAL DYNAMICS FOR TWO BODY PROBLEM FOR PURSUER) 
(X VECTOR = [X XDOT Y YDOT Z ZDOT]) 
(F VECTOR = [XDOT XDOUBLEDOT YDOT YDOUBLEDOT ZDOT ZDOUBLEDOT]) 

REAL*8 X(6),F(6),RSQRD,CONST 
RSQRD=X(1)*X(1)+X(3)*X(3)+X(5)*X(5) 
CONST=-3.986012E14/RSQRD/SQRT(RSQRD) 
F(1)=X(2) 
F(2)=CONST*X(1) 
F(3)=X(4) 
F(4)=CONST*X(3) 
F(5)=X(6) 
F(6)=CONST*X(5) 
END 

SUBROUTINE RK4SYSP(T,X,H) 

THIS SUBROUTINE IS A ONE STEP RUNGE-KUTTA 
4TH ORDER INTEGRATOR FOR THE PURSUER DYNAMICS
REAL*8 T,X(6),H,F1(6),F2(6),F3(6)
REAL*8 F4(6),H2,DUMX(6)
INTEGER I

H2=0.5*H

C FIND F1
   CALL XPSYSP(X,F1)

C FIND F2
   DO 100 I=1,6
      DUMX(I)=X(I)+H2*F1(I)
   100 CONTINUE
   CALL XPSYSP(DUMX,F2)

C FIND F3
   DO 200 I=1,6
      DUMX(I)=X(I)+H2*F2(I)
   200 CONTINUE
   CALL XPSYSP(DUMX,F3)

C FIND F4
   DO 300 I=1,6
      DUMX(I)=X(I)+H*F3(I)
   300 CONTINUE
   CALL XPSYSP(DUMX,F4)

C UPDATE THE STATE
   DO 400 I=1,6
      X(I)=X(I)+H*(F1(I)+F2(I)+F2(I)+F3(I)+F3(I)+F4(I))/6.0
   400 CONTINUE

END

SUBROUTINE EULERP(T,X,H)

C THIS SUBROUTINE IS A ONE STEP EULER INTEGRATOR
C FOR THE PURSUER DYNAMICS

REAL*8 T,X(6),F1(6),H
INTEGER I

C FIND F1
   CALL XPSYSP(X,F1)

C UPDATE THE STATE
   DO 400 I=1,6
      X(I)=X(I)+H*F1(I)
   400 CONTINUE

END
SUBROUTINE XPSYSE(X,F,A,MDOT,T)
C THIS SUBROUTINE EVALUATES THE FUNCTIONS FOR RK4SYSE
C (ORBITAL DYNAMICS FOR TWO BODY PROBLEM FOR EVADER
C USING THE ROCKET EQUATION:
C \( A = A_0 / (1 - \text{MDOT} \times T) \)
C (X VECTOR = [X XDOT Y YDOT Z ZDOT])
C (F VECTOR = [XDOT XDOUBLEDOT YDOT YDOUBLEDOT ZDOT
C ZDOUBLEDOT ADOT])
REAL*8 X(6),F(8),RSQRD,CONST,MDOT,T,A,V
RSQRD=X(1)*X(1)+X(3)*X(3)+X(5)*X(5)
CONST=-3.986012E14/RSQRD/SQRT(RSQRD)
V=SQRRT(X(2)*X(2)+X(4)*X(4)+X(6)*X(6))
F(1)=X(2)
F(2)=CONST*X(1)+A*X(2)/V
F(3)=X(4)
F(4)=CONST*X(3)+A*X(4)/V
F(5)=X(6)
F(6)=CONST*X(5)+A*X(6)/V
F(7)=A*MDOT
F(8)=MDOT*MDOT
END

SUBROUTINE RK4SYSE(T,X,H,A,MDOT)
C THIS SUBROUTINE IS A ONE STEP RUNGE-KUTTA
C 4TH ORDER INTEGRATOR FOR THE EVADER DYNAMICS
REAL*8 T,X(6),H,F1(8),F2(8),F3(8),F4(8)
REAL*8 H2,DUMX(6),A,MDOT,AD,MDOTD
INTEGER I

H2=0.5*H

C FIND F1
CALL XPSYSE(X,F1,A,MDOT,T)

C FIND F2
DO 100 I=1,6
   DUMX(I)=X(I)+H2*F1(I)
100 CONTINUE
AD=A+H2*F1(7)
MDOTD=MDOT+H2*F1(8)
CALL XPSYSE(DUMX,F2,AD,MDOTD,T)
C FIND F3
   DO 200 I=1,6
     DUMX(I)=X(I)+H2*F2(I)
 200 CONTINUE
   AD=A+H2*F2(7)
   MDOTD=MDOT+H2*F2(8)
   CALL XPSYSE(DUMX,F3,AD,MDOTD,T)

C FIND F4
   DO 300 I=1,6
     DUMX(I)=X(I)+H*F3(I)
 300 CONTINUE
   AD=A+H*F3(7)
   MDOTD=MDOT+H2*F3(8)
   CALL XPSYSE(DUMX,F4,AD,MDOTD,T)

C UPDATE THE STATE
   DO 400 I=1,6
     X(I)=X(I)+H*(F1(I)+F2(I)+F2(I)+F3(I)+F3(I)+F4(I))/6.0
   400 CONTINUE
   A=A+H*(F1(7)+F2(7)+F2(7)+F3(7)+F3(7)+F4(7))/6.0
   MDOT=MDOT+H*(F1(8)+F2(8)+F2(8)+F3(8)+F3(8)+F4(8))/6.0
END

SUBROUTINE EULERE(T,X,H,AD,MDOT)

C THIS SUBROUTINE IS A ONE STEP EULER INTEGRATOR
C FOR THE EVADER DYNAMICS

REAL*8 T,X(6),F1(8),H,A,MDOT
INTEGER I

C FIND F1
   CALL XPSYSE(X,F1,A,MDOT,T)

C UPDATE THE STATE
   DO 400 I=1,6
     X(I)=X(I)+H*F1(I)
 400 CONTINUE
   A=A+H*F1(7)
   MDOT=MDOT+H*F1(8)
END
C  TOOL2
C  THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR
C  COORDINATE TRANSFORMATIONS
C  IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM
C
C  SUBROUTINE DICTIONARY
C
C  XHAT   ESTIMATED RELATIVE STATE VECTOR
C  XP     STATE VECTOR OF PURSUER
C  XPD    DUMMY STATE VECTOR OF PURSUER
C  TMAT   TRANSFORMATION MATRIX
C  TMATA  TRANSFORMATION MATRIX

SUBROUTINE COMPTLOS(XHAT,TMAT)
C  THIS SUBROUTINE COMPUTES THE MATRIX (TMAT) THAT
C  TRANSFORMS THE REFERENCE FRAME TO THE LOS FRAME
C  WHERE THE X AXIS OF THE LOS FRAME LIES ALONG THE
C  RELATIVE POSTION VECTOR.

REAL*8 XHAT(6),TMAT(3,3),A,R,AR

R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+    XHAT(5)*XHAT(5))
A=SQRT(XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5))
IF (A .LT. .00001) THEN A=.00001
AR=A*R
TMAT(1,1)=XHAT(1)/R
TMAT(1,2)=XHAT(3)/R
TMAT(1,3)=XHAT(5)/R
TMAT(2,1)=XHAT(1)*XHAT(3)/AR
TMAT(2,2)=A/R
TMAT(2,3)=XHAT(3)*XHAT(5)/AR
TMAT(3,1)=XHAT(5)/A
TMAT(3,2)=0.0
TMAT(3,3)=XHAT(1)/A

END

SUBROUTINE COMPTV(XP,TMAT)
C  THIS SUBROUTINE COMPUTES THE MATRIX (TMAT) THAT
C  TRANSFORMS THE REFERENCE FRAME TO THE BODY FRAME
C  WHERE THE X AXIS OF THE BODY FRAME LIES ALONG
C  THE PURSUER'S VELOCITY VECTOR AND THE PURSUER'S
C  RADIUS VECTOR IS IN THE NEW XY PLANE.

REAL*8 XP(6), XPD(6), TMATA(3,3), TMAT(3,3)
REAL*8 A, B, V, AV

V = SQRT(XP(2)*XP(2) + XP(4)*XP(4) + XP(6)*XP(6))
A = SQRT(XP(2)*XP(2) + XP(6)*XP(6))
IF (A .LT. .00001) THEN A = .00001
AV = A*V
TMAT(1,1) = XP(2)/V
TMAT(1,2) = XP(4)/V
TMAT(1,3) = XP(6)/V
TMAT(2,1) = -XP(2)*XP(4)/AV
TMAT(2,2) = A/V
TMAT(2,3) = -XP(4)*XP(6)/AV
TMAT(3,1) = -XP(6)/A
TMAT(3,2) = 0.0
TMAT(3,3) = XP(2)/A

CALL TRANSFWD(XP(1), XP(3), XP(5), TMAT, XPD(1),
               XPD(3), XPD(5))
B = SQRT(XPD(3)*XPD(3) + XPD(5)*XPD(5))
IF (B .LT. .0001) RETURN
TMATA(2,1) = (TMAT(2,1)*XPD(3) + TMAT(3,1)*XPD(5))/B
TMATA(3,1) = (TMAT(3,1)*XPD(3) - TMAT(2,1)*XPD(5))/B
TMATA(2,2) = (TMAT(2,2)*XPD(3) + TMAT(3,2)*XPD(5))/B
TMATA(3,2) = (TMAT(3,2)*XPD(3) - TMAT(2,2)*XPD(5))/B
TMATA(2,3) = (TMAT(2,3)*XPD(3) + TMAT(3,3)*XPD(5))/B
TMATA(3,3) = (TMAT(3,3)*XPD(3) - TMAT(2,3)*XPD(5))/B
TMAT(2,1) = TMATA(2,1)
TMAT(3,1) = TMATA(3,1)
TMAT(2,2) = TMATA(2,2)
TMAT(3,2) = TMATA(3,2)
TMAT(2,3) = TMATA(2,3)
TMAT(3,3) = TMATA(3,3)

END

SUBROUTINE TRANSFWD(X, Y, Z, TMAT, XT, YT, ZT)
C THIS ROUTINE TAKES THE X, Y, Z VECTOR AND USES
C THE TRANSFORMATION MATRIX TMAT TO FORM THE
C VECTOR XT, YT, ZT

REAL*8 X, Y, Z, TMAT(3, 3), XT, YT, ZT
INTEGER I

XT = TMAT(1, 1)*X + TMAT(1, 2)*Y + TMAT(1, 3)*Z
YT = TMAT(2, 1)*X + TMAT(2, 2)*Y + TMAT(2, 3)*Z
ZT = TMAT(3, 1)*X + TMAT(3, 2)*Y + TMAT(3, 3)*Z

END
SUBROUTINE TRANSBKWD(X, Y, Z, TMAT, XT, YT, ZT)

C THIS ROUTINE TAKES THE X, Y, Z VECTOR AND USES THE
C INVERSE OF THE TRANSFORMATION MATRIX TMAT TO
C FORM THE VECTOR XT, YT, ZT

REAL*8 X, Y, Z, TMAT(3), XT, YT, ZT
INTEGER I

XT = TMAT(1, 1) * X + TMAT(2, 1) * Y + TMAT(3, 1) * Z
YT = TMAT(1, 2) * X + TMAT(2, 2) * Y + TMAT(3, 2) * Z
ZT = TMAT(1, 3) * X + TMAT(2, 3) * Y + TMAT(3, 3) * Z

END
C TOOL3
C THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR
C EXTENDED KALMAN FILTERING
C IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM
C
C SUBROUTINE DICTIONARY
C
C ACCDEN   DENOMINATOR OF EVADER ACCELERATION TERM
C ACCEL    PRESENT EVADER THRUSTING ACCELERATION
C COV      COVARIANCE MATRIX
C DET      THE DETERMINANT OF THE MATRIX 'MAT'
C DMAT     DUMMY MATRIX
C DCMAT    DUMMY COLUMN MATRIX
C DT       TIME STEP
C DUM      DUMMY VARIABLE
C E        RADIUS OF EVADER
C E2       E**2
C E3       E**3
C E5       E**5
C EDOT     VELOCITY OF EVADER
C EDOT2    EDOT**2
C EDOT3    EDOT**3
C F        MATRIX OF STATE PARTIAL DERIVATIVES
C GAIN     GAIN MATRIX
C GAMMA    OBSERVED LINE-OF-SIGHT ANGLE (IN PLANE)
C H        MATRIX OF MEASUREMENT PARTIALS
C HYP13    XHAT(1)**2 + XHAT(3)**2
C HYP15    XHAT(1)**2 + XHAT(5)**2
C I        COUNTER
C J        COUNTER
C JUP      UPPER LIMIT ON 'J' COUNTER
C K        COUNTER
C KFLAG    GAIN CONVERGENCE FLAG
C MEAN     GAUSSIAN MEAN
C          1 - WITHOUT FILTER
C          2 - WITH FILTER
C          3 - WITH FILTER + PRINTOUT
C PDOT     TIME DERIVATIVE OF COVARIANCE MATRIX
C POLD     PROPAGATED COVARIANCE MATRIX
C Q        VARIANCE OF FILTER STATE NOISE
C R        ESTIMATE OF RANGE OF EVADER FROM PURSUER
C RES      MEASUREMENT RESIDUALS
C R3       VARIANCE OF MEASUREMENT NOISE
C RANGE    MEASUREMENT OF RELATIVE RANGE
C SEED     SEED FOR RANDOM NUMBER GENERATOR
C SIGMA    GAUSSIAN STANDARD DEVIATION
C SUM      SUM OF UNIFORMLY DISTRIBUTED NUMBERS
C T        TIME
C THETA    OBSERVED OUT-OF-PLANE LOS ANGLE
C U        GRAVITATIONAL CONSTANT
UPDATE FLAG FOR UPDATING EKF
0 NO UPDATE
1 UPDATE
2 UPDATE WITH RESIDUALS EQUAL TO ZERO
V ESTIMATE OF RELATIVE RANGE RATE
VAR VARIANCE OF MEASUREMENT
XEEST ESTIMATED EVADER VECTOR
XHAT ESTIMATE OF X VECTOR (BODY FRAME)
1 RELATIVE X POSITION
2 RELATIVE X VELOCITY
3 RELATIVE Y POSITION
4 RELATIVE Y VELOCITY
5 RELATIVE Z POSITION
6 RELATIVE Z VELOCITY
7 PRESENT ACCELERATION OF EVADER
8 BOOSTER MASS FLOW RATE
XPEST ESTIMATED PURSUER VECTOR

FUNCTION GAUSS(SEED,SW,G2)
C CREATES A POINT HAVING A GAUSSIAN DISTRIBUTION WITH
C MEAN=0.0
C SIGMA=1.0
REAL MTH$RANDOM,A,B
REAL*8 GAUSS,G2
INTEGER SEED,SW,I
IF (SW .GT. 0) THEN
  SW=0
  GAUSS=G2
ELSE
  SW=1
  A=SQRAT(-2.0*ALOG(MTH$RANDOM(SEED)))
  B=2.0*ACOS(-1.0)*(MTH$RANDOM(SEED))
  G2=A*COS(B)
  GAUSS=A*SIN(B)
ENDIF
END

SUBROUTINE EKF8(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,
+ RANGE,THETA,GAMMA,KFLAGRES,RES,UPDATE)
C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION AND MASS FLOW
C RATE OF THE EVADER USING AN EIGHT STATE EXTENDED
C KALMAN FILTER WITH SERIAL UPDATES OF RANGE AND
C TWO LINE-OF-SIGHT ANGLES.
C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,8)
REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
REAL*8 EDOT3,T,HYP15,HYP13,H(8)
REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
REAL*8 POLD(8,6),DMAT,DCMAT(8),GAIN(8)
INTEGER I,J,JUP,K,KFLAG, UPDATE

C INITIALIZE DUMMY VARIABLES
U=3.986012E14
E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
+ XEEST(5)*XEEST(5)
E=SQRTE(E2)
E3=E2*E
E5=E3*E2
EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
+ XEEST(6)*XEEST(6)
EDOT=SQRTE(EDOT2)
EDOT3=EDOT*EDOT2

C COMPUTE F MATRIX
DO 200 I=1,8
   F(1,I)=0.0
   F(3,I)=0.0
   F(5,I)=0.0
   F(7,I)=0.0
   F(8,I)=0.0
200 CONTINUE

F(1,2)=1.0
F(3,4)=1.0
F(5,6)=1.0
F(2,1)=(-1.0+3.0*XEEST(1)*XEEST(1)/E2)*U/E3
F(2,2)=(1.0-XEEST(2)*XEEST(2)/EDOT2)*XHAT(7)/EDOT
F(2,3)=3.0*U*XEEST(1)*XEEST(3)/E5
F(2,4)=XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
F(2,5)=3.0*U*XEEST(1)*XEEST(5)/E5
F(2,6)=XEEST(2)*XHAT(7)*XEEST(6)/EDOT3
F(2,7)=XEEST(2)/EDOT
F(2,8)=0.0
F(4,1)=F(2,3)
F(4,2)=F(2,4)
F(4,3)=(-1.0+3.0*XEEST(3)*XEEST(3)/E2)*U/E3
F(4,4)=(1.0-XEEST(4)*XEEST(4)/EDOT2)*XHAT(7)/EDOT
F(4,5)=3.0*U*XEEST(3)*XEEST(5)/E5
F(4,6)=XEEST(4)*XHAT(7)*XEEST(6)/EDOT3
F(4,7)=XEEST(4)/EDOT
F(4,8)=0.0
\[
\begin{align*}
F(6,1) &= F(2,5) \\
F(6,2) &= F(2,6) \\
F(6,3) &= F(4,5) \\
F(6,4) &= F(4,6) \\
F(6,5) &= (-1.0 - 3.0 \times \text{XEEST}(5) \times \text{XEEST}(5)/E2) \times U/E3 \\
F(6,6) &= (1.0 - \text{XEEST}(6) \times \text{XEEST}(6)/EDOT2) \times \text{XHAT}(7)/EDOT \\
F(6,7) &= \text{XEEST}(6)/EDOT \\
F(6,8) &= 0.0 \\
F(7,7) &= \text{XHAT}(8) \\
F(7,8) &= \text{XHAT}(7) \\
F(8,8) &= \text{XHAT}(8) + \text{XHAT}(8)
\end{align*}
\]

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER’S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)
DO 300 I=1,8
DO 300 J=I,1
PDOT(I,J)=Q(I,J)
DO 300 K=1,8
PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
\quad \text{COV}(I,K) \times F(J,K)
300 CONTINUE
DO 310 I=1,8
DO 310 J=I,1
POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
310 CONTINUE
C REASSIGN COV AND ZERO OUT H MATRIX
DO 320 I=1,8
COV(I,I)=POLD(I,I)
H(I)=0.0
JUP=I-1
DO 320 J=1,JUP
COV(I,J)=POLD(I,J)
COV(J,I)=POLD(I,J)
320 CONTINUE
C PROPAGATE STATE ESTIMATE FORWARD ONE STEP
CALL RK4SYS(T,\text{XEEST},DT,\text{XHAT}(7),\text{XHAT}(8))
IF (UPDATE .NE. 1) CALL RK4SYS(T,\text{XPEST},DT)
DO 330 I=1,6
\quad \text{XHAT}(I)=\text{XEEST}(I)-\text{XPEST}(I)
330 CONTINUE
IF (UPDATE .NE. 0) THEN
C PERFORM RANGE UPDATE
\quad R=\sqrt{\text{XHAT}(1) \times \text{XHAT}(1)+\text{XHAT}(3) \times \text{XHAT}(3)+
\quad \text{XHAT}(5) \times \text{XHAT}(5)}
\quad H(1)=\text{XHAT}(1)/R
\quad H(3)=\text{XHAT}(3)/R
\quad H(5)=\text{XHAT}(5)/R
RES(1) = RANGE - R  
IF (UPDATE .NE. 1) RES(1) = 0.0  
CALL UPDATE8 (XHAT, H, COV, R3(1,1), RES(1), KFLAG)

C PERFORM THETA UPDATE  
HYP15 = XHAT(1)*XHAT(1) + XHAT(5)*XHAT(5)  
H(1) = -XHAT(5)/HYP15  
H(3) = 0.0  
H(5) = XHAT(1)/HYP15  
RES(2) = THETA - ATAN (XHAT(5)/XHAT(1))  
IF (UPDATE .NE. 1) RES(2) = 0.0  
CALL UPDATE8 (XHAT, H, COV, R3(2,2), RES(2), KFLAG)

C PERFORM GAMMA UPDATE  
HYP13 = XHAT(1)*XHAT(1) + XHAT(3)*XHAT(3)  
H(1) = -XHAT(3)/HYP13  
H(3) = XHAT(1)/HYP13  
H(5) = 0.0  
RES(3) = GAMMA - ATAN (XHAT(3)/XHAT(1))  
IF (UPDATE .NE. 1) RES(3) = 0.0  
CALL UPDATE8 (XHAT, H, COV, R3(3,3), RES(3), KFLAG)

ENDIF
END

SUBROUTINE UPDATE8 (XHAT, H, COV, VAR, RES, KFLAG)

C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE  
C EIGHT STATE EXTENDED KALMAN FILTER  
REAL*8 XHAT(8), H(8), COV(8,8), VAR, RES  
REAL*8 DMAT, DCMAT(8), GAIN(8), POLD(8,8)  
INTEGER I, J, JUP, KFLAG

C INITIALIZE COUNT  
COUNT = 0

C COMPUTE MATRIX (1X1) FOR INVERSION  
DO 110 I = 1, 8  
    DCMAT(I) = 0.0  
    DO 110 J = 1, 8  
        DCMAT(I) = DCMAT(I) + COV(I, J)*H(J)  
    110 CONTINUE  
DMAT = VAR  
DO 120 I = 1, 8  
    DMAT = DMAT + DCMAT(I)*H(I)  
120 CONTINUE
C COMPUTE GAIN MATRIX
   DO 140 I=1,8
      GAIN(I)=0.0
   DO 130 J=1,8
      GAIN(I)=GAIN(I)+COV(I,J)*H(J)
   CONTINUE
   GAIN(I)=GAIN(I)/DMAT
   140 CONTINUE

C UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C PREVENT NEGATIVE DIAGONAL COVARIANCES
   DO 150 I=1,8
      DCMAT(I)=0.0
   DO 150 J=1,8
      DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
   CONTINUE
   DO 160 I=1,8
      POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
      IF (POLD(I,I) .LT. 0.0) THEN
         IF (COUNT .LT. 10) THEN
            DO 155 J=1,8
               GAIN(J)=GAIN(J)/2.0
            CONTINUE
            COUNT=COUNT+1
            KFLAG=KFLAG+9
            GOTO 150
         ELSE
            KFLAG=KFLAG+990
            RETURN
         ENDIF
      ENDIF
   ENDIF
   JUP=I-1
   DO 160 J=1,JUP
      POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
   CONTINUE

C UPDATE ESTIMATE
   DO 170 I=1,8
      XHAT(I)=XHAT(I)+GAIN(I)*RES
   CONTINUE

C UPDATE COV MATRIX
   DO 175 I=1,8
      COV(I,I)=POLD(I,I)
   JUP=I-1
   DO 175 J=1,JUP
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
   CONTINUE

END
SUBROUTINE EKF6(XHAT, XEEST, XPEST, T, DT, COV, Q, R3, + RANGE, THETA, GAMMA, KFLAG, RES, UPDATE)

C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C USING A SIX STATE EXTENDED KALMAN FILTER WITH
C SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8), COV(8,8), RANGE, THETA, GAMMA, THETA, GAMMA, R(8,8)
REAL*8 E, E2, E3, E5, U, EDOT, EDOT2, Q(8,8), RES(3)
REAL*8 EDOT3, T, HYP15, HYP13, H(8)
REAL*8 XEEST(6), XPEST(6), DT, PDOT(8,8), R, R(3,3)
REAL*8 POLD(8,8), DMAT, DCMAT(8), GAIN(8)
INTEGER I, J, JUP, K, KFLAG, UPDATE

C INITIALIZE DUMMY VARIABLES
U=3.986012E14
E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+ + XEEST(5)*XEEST(5)
E=SQR(T(E2))
E3=E2*E
E5=E3*E2
EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+ + XEEST(6)*XEEST(6)
EDOT=SQR(T(EDOT2))
EDOT3=EDOT*EDOT2

C COMPUTE F MATRIX
DO 200 I=1,8
  F(1,I)=0.0
  F(3,I)=0.0
  F(5,I)=0.0
  F(7,I)=0.0
  F(8,I)=0.0
200 CONTINUE

F(1,2)=1.0
F(3,4)=1.0
F(5,6)=1.0
F(2,1)=(-1.0+3.0*XEEST(1)*XEEST(1)/E2)*U/E3
F(2,2)=(1.0-XEEST(2)*XEEST(2)/EDOT2)*XHAT(7)/EDOT
F(2,3)=3.0*U*XEEST(1)*XEEST(3)/E5
F(2,4)= -XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
F(2,5)=3.0*U*XEEST(1)*XEEST(5)/E5
F(2,6)= -XEEST(2)*XHAT(7)*XEEST(6)/EDOT3
F(2,7)=0.0
F(2,8)=0.0
F(4,1)=F(2,5)
F(4,2)=F(2,4)
F(4,3)=(-1.0+3.0*XEEST(3)*XEEST(3)/E2)*U/E3
\[
{F}(4,4) = (1.0 - {XEEST}(4) * {XEEST}(4) / {EDOT}2) * {XHAT}(7) / {EDOT}
\]
\[
{F}(4,5) = 3.0 * U * {XEEST}(3) * {XEEST}(5) / {E}5
\]
\[
{F}(4,6) = - {XEEST}(4) * {XHAT}(7) * {XEEST}(6) / {EDOT}3
\]
\[
{F}(4,7) = 0.0
\]
\[
{F}(4,8) = 0.0
\]
\[
{F}(6,1) = {F}(2,5)
\]
\[
{F}(6,2) = {F}(2,6)
\]
\[
{F}(6,3) = {F}(4,5)
\]
\[
{F}(6,4) = {F}(4,6)
\]
\[
{F}(6,5) = (-1.0 + 3.0 * {XEEST}(5) * {XEEST}(5) / {E}2) * U / {E}3
\]
\[
{F}(6,6) = (1.0 - {XEEST}(6) * {XEEST}(6) / {EDOT}2) * {XHAT}(7) / {EDOT}
\]
\[
{F}(6,7) = 0.0
\]
\[
{F}(6,8) = 0.0
\]

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)
DO 300 I = 1, 6
   DO 300 J = 1, I
      PDOT(I, J) = Q(I, J)
      DO 300 K = 1, I
         PDOT(I, J) = PDOT(I, J) + F(I, K) * COV(K, J) + COV(I, K) * F(J, K)
      CONTINUE
   CONTINUE
DO 310 I = 1, 6
   DO 310 J = 1, I
      POLD(I, J) = COV(I, J) + (PDOT(I, J) * DT)
   CONTINUE

C REASSIGN COV AND ZERO OUT H MATRIX
DO 320 I = 1, 6
   COV(I, I) = POLD(I, I)
   H(I) = 0.0
   JUP = I - 1
   DO 320 J = 1, JUP
      COV(I, J) = POLD(I, J)
      COV(J, I) = POLD(I, J)
   CONTINUE

C PROPAGATE STATE ESTIMATE FORWARD ONE STEP
CALL RK4S YE(T, XEEST, DT, XHAT(7), XHAT(8))
IF (UPDATE .NE. 1) CALL RK4S YSP(T, XPES?, DT)
DO 330 I = 1, 6
   XHAT(I) = XEEST(I) - XPEST(I)
CONTINUE

IF (UPDATE .NE. 0) THEN

C PERFORM RANGE UPDATE
   R = SQRT(XHAT(1) * XHAT(1) + XHAT(3) * XHAT(3) +
       XHAT(5) * XHAT(5))
   H(1) = XHAT(1) / R
   H(3) = XHAT(3) / R
   H(5) = XHAT(5) / R
RES(1) = RANGE - R
IF (UPDATE .NE. 1) RES(1) = 0.0
CALL UPDATE6(XHAT, H, COV, R3(1,1), RES(1), KFLAG)

C PERFORM THETA UPDATE
HYP15 = XHAT(1) * XHAT(1) + XHAT(5) * XHAT(5)
H(1) = -XHAT(5) / HYP15
H(3) = 0.0
H(5) = XHAT(1) / HYP15
RES(2) = THETA - ATAN(XHAT(5) / XHAT(1))
IF (UPDATE .NE. 1) RES(2) = 0.0
CALL UPDATE6(XHAT, H, COV, R3(2,2), RES(2), KFLAG)

C PERFORM GAMMA UPDATE
HYP13 = XHAT(1) * XHAT(1) + XHAT(3) * XHAT(3)
H(1) = -XHAT(3) / HYP13
H(3) = XHAT(1) / HYP13
H(5) = 0.0
RES(3) = GAMMA - ATAN(XHAT(3) / XHAT(1))
IF (UPDATE .NE. 1) RES(3) = 0.0
CALL UPDATE6(XHAT, H, COV, R3(3,3), RES(3), KFLAG)

ENDIF
END

SUBROUTINE EKF60(XHAT, XEEST, XPEST, T, DT, COV, Q, R3,
+ RANGE, THETA, GAMMA, KFLAG, RES)

C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C USING A SIX STATE EXTENDED KALMAN FILTER WITH
C SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C (GELD, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8), COV(8,8), RANGE, THETA, GAMMA, F(8,8)
REAL*8 E, E2, E3, E5, U, EDOT, EDOT2, Q(8,8), RES(3)
REAL*8 EDOT3, T, HYP15, HYP13, H(8)
REAL*8 XEEST(6), XPEST(6), DT, PDOT(8,8), R, R3(3,3)
REAL*8 POLD(8,8), DMAT, DCMAT(8), GAIN(8)
INTEGER I, J, JUP, K, KFLAG

C INITIALIZE DUMMY VARIABLES
U = 3.986012E14
E2 = XEEST(1) * XEEST(1) + XEEST(3) * XEEST(3) +
+ XEEST(5) * XEEST(5)
E = SQRT(E2)
E3 = E2 * E
E5 = E3 * E2
EDOT2 = XEEST(2) * XEEST(2) + XEEST(4) * XEEST(4) +
+ XEEST(6) * XEEST(6)
EDOT = SQRT(EDOT2)
EDOT3 = EDOT * EDOT2

C COMPUTE F MATRIX
DO 200 I = 1, 8
   DO 200 J = 1, 8
      F(I,J) = 0.0
   200 CONTINUE
F(1,2) = 1.0
F(3,4) = 1.0
F(5,6) = 1.0

F(2,2) = (1.0 - XEEST(2)*XEEST(2)/EDOT2) * XHAT(7)/EDOT
F(2,4) = -XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
F(2,6) = -XEEST(2)*XHAT(7)*XEEST(6)/EDOT3

F(4,2) = F(2,4)
F(4,4) = (1.0 - XEEST(4)*XEEST(4)/EDOT2) * XHAT(7)/EDOT
F(4,6) = -XEEST(4)*XHAT(7)*XEEST(6)/EDOT3

F(6,2) = F(2,6)
F(6,4) = F(4,6)
F(6,6) = (1.0 - XEEST(6)*XEEST(6)/EDOT2) * XHAT(7)/EDOT

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)
DO 300 I = 1, 6
   DO 300 J = 1, I
      PDOT(I,J) = Q(I,J)
   DO 300 K = 1, 6
      PDOT(I,J) = PDOT(I,J) + F(I,K) * COV(K,J) +
      COV(I,K) * F(J,K)
   300 CONTINUE

DO 310 I = 1, 6
   DO 310 J = 1, I
      POLD(I,J) = COV(I,J) + (PDOT(I,J) * DT)
   310 CONTINUE

C REASSIGN COV AND ZERO OUT H MATRIX
DO 320 I = 1, 6
   COV(I,I) = POLD(I,I)
   H(I) = 0.0
   JUP = I - 1
   DO 320 J = 1, JUP
      COV(I,J) = POLD(I,J)
      COV(J,I) = POLD(I,J)
   320 CONTINUE

C PROPAGATE STATE ESTIMATES FORWARD ONE STEP
XEEST(1) = XEEST(1) + DT * XEEST(2)
XEEST(3) = XEEST(3) + DT * XEEST(4)
XEEST(5) = XEEST(5) + DT * XEEST(6)
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```
XEST(2)=XEST(2)*(1.0+DT*XHAT(7)/EDOT)
XEST(4)=XEST(4)*(1.0+DT*XHAT(7)/EDOT)
XEST(6)=XEST(6)*(1.0+DT*XHAT(7)/EDOT)
XHAT(7)=XHAT(7)/(1.0-XHAT(8)*DT)
XHAT(8)=XHAT(8)/(1.0-XHAT(8)*DT)

DO 330 I=1,6
    XHAT(I)=XEST(I)-XPEST(I)
330 CONTINUE

C PERFORM RANGE UPDATE
    R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
           XHAT(5)*XHAT(5))
    H(1)=XHAT(1)/R
    H(3)=XHAT(3)/R
    H(5)=XHAT(5)/R
    RES(1)=RANGE-R
    CALL UPDATE6(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

C PERFORM THETA UPDATE
    HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
    H(1)=-XHAT(5)/HYP15
    H(3)=0.0
    H(5)=XHAT(1)/HYP15
    RES(2)=THETA-ATAN(XHAT(3)/XHAT(1))
    CALL UPDATE6(XHAT,H,COV,R3(2,2),RES(2),KFLAG)

C PERFORM GAMMA UPDATE
    HYP13=XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)
    H(1)=-XHAT(3)/HYP13
    H(3)=XHAT(1)/HYP13
    H(5)=0.0
    RES(3)=GAMMA-ATAN(XHAT(3)/XHAT(1))
    CALL UPDATE6(XHAT,H,COV,R3(3,3),RES(3),KFLAG)

END

SUBROUTINE UPDATE6(XHAT,H,COV,VAR,RES,KFLAG)

C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE
C SIX STATE EXTENDED KALMAN FILTER

REAL*8 XHAT(8),H(8),COV(8,8),VAR,RES
REAL*8 DMAT,DCMAT(8),GAIN(8),POLD(8,8)
INTEGER I,J,JUP,KFLAG

C INITIALIZE COUNT
    COUNT=0
```
C COMPUTE MATRIX (1X1) FOR INVERSION
  DO 110 I=1,6
    DCMAT(I)=0.0
  DO 110 J=1,6
    DCMAT(I)=DCMAT(I)+COV(I,J)*H(J)
  110  CONTINUE
DMAT=VAR
  DO 120 I=1,6
    DMAT=DMAT+DCMAT(I)*H(I)
  120  CONTINUE
C COMPUTE GAIN MATRIX
  DO 140 I=1,6
    GAIN(I)=0.0
  DO 130 J=1,6
    GAIN(I)=GAIN(I)+COV(I,J)*H(J)
  130  CONTINUE
  GAIN(I)=GAIN(I)/DMAT
  140  CONTINUE
C UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C PREVENT NEGATIVE DIAGONAL COVARIANCES
  DO 150 I=1,6
    DCMAT(I)=0.0
  DO 150 J=1,6
    DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
  150  CONTINUE
  DO 160 I=1,6
    POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
    IF (POLD(I,I).LT.0.0) THEN
      IF (COUNT.LT.10) THEN
        DO 155 J=1,6
          GAIN(J)=GAIN(J)/2.0
        155  CONTINUE
        COUNT=COUNT+1
        KFLAG=KFLAG+1
        GOTO 150
      ELSE
        KFLAG=KFLAG+990
        RETURN
      ENDIF
    ENDF
  ENDIF
  JUP=I-1
  DO 160 J=1,JUP
    POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
  160  CONTINUE
C UPDATE ESTIMATE
  DO 170 I=1,6
    XHAT(I)=XHAT(I)+GAIN(I)*RES
  170  CONTINUE
C UPDATE COV MATRIX
DO 175 I=1,6
  COV(I,I)=POLD(I,I)
  JUP=I-1
  DO 175 J=1,JUP
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
175 CONTINUE

END

SUBROUTINE EKF3(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,+
                 RANGE,THETA,GAMMA,KFLAG,RES)

C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C USING A THREE STATE EXTENDED KALMAN FILTER WITH
C SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,3)
REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
REAL*8 EJOT3,T,HYP15,HYP13,H(8)
REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
REAL*8 POLD(8,8),DMAT,DCMAT(8),GAIN(8)
INTEGER I,J,JUP,K,KFLAG

C INITIALIZE DUMMY VARIABLES
  U=3.986012E14
  E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
     XEEST(5)*XEEST(5)
  E=SQRT(E2)
  E3=E2*E
  E5=E3*E2
  EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
         XEEST(6)*XEEST(6)
  EDOT=SQRT(EDOT2)
  EDOT3=EDOT*EDOT2

C COMPUTE F MATRIX
DO 200 I=1,8
   DO 200 J=1,8
      F(I,J)=0.0
200 CONTINUE
  F(1,1)=DT*(-1.0+3.0*XHAT(1)*XHAT(1)/E2)*U/E3
  F(1,3)=DT*3.0*U*XHAT(1)*XHAT(3)/E5
  F(1,5)=DT*3.0*U*XHAT(1)*XHAT(5)/E5
F(3,1)=F(1,3)
F(3,3)=DT*(-1.0+3.0*XEEST(3)*XEST(3)/E2)*U/E3
F(3,5)=DT*3.0*U*XEEST(3)*XEEST(5)/E5

F(5,1)=F(1,5)
F(5,3)=F(3,5)
F(5,5)=DT*(-1.0+3.0*XEEST(5)*XEST(5)/E2)*U/E3

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)

C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)

DO 300 I=1,5,2
   DO 300 J=1,1,2
      PDOT(I,J)=Q(I,J)
      DO 300 K=1,5,2
         PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
               +COV(I,K)*F(J,K)
      300 CONTINUE
   DO 310 I=1,5,2
   DO 310 J=1,1,2
      POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
   310 CONTINUE

C REASSIGN COV AND ZERO OUT H MATRIX

DO 320 I=1,5,2
   COV(I,I)=POLD(I,I)
   H(I)=0.0
   H(I+1)=0.0
   JUP=I-2
   DO 320 J=1,JUP,2
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
   320 CONTINUE

C PROPAGATE STATE ESTIMATES FORWARD ONE STEP

CALL RK4SYSE(T,XEEST,DT,XHAT(7),XHAT(8))
DO 330 I=1,6
   XHAT(I)=XEEST(I)-XPEST(I)
330 CONTINUE

C PERFORM RANGE UPDATE

R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
       +XHAT(5)*XHAT(5))
H(1)=XHAT(1)/R
H(3)=XHAT(3)/R
H(5)=XHAT(5)/R
RES(1) RANGE-R
CALL UPDATE3(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

C PERFORM THETA UPDATE

HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
H(1)=-XHAT(5)/HYP15
H(3)=0.0
H(5)=XHAT(1)/HYP15
RES(2) = THETA - ATAN(XHAT(5)/XHAT(1))
CALL UPDATE3(XHAT,H,COV,R3(2,2),RES(2),KFLAG)

C PERFORM GAMMA UPDATE
HYP13 = XHAT(1)*XHAT(1) + XHAT(3)*XHAT(3)
H(1) = -XHAT(3)/HYP13
H(3) = XHAT(1)/HYP13
H(5) = 0.0
RES(3) = GAMMA - ATAN(XHAT(3)/XHAT(1))
CALL UPDATE3(XHAT,H,COV,R3(3,3),RES(3),KFLAG)

END

SUBROUTINE UPDATE3(XHAT,H,COV,VAR,RES,KFLAG)

C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE
C THREE STATE EXTENDED KALMAN FILTER

REAL*8 XHAT(8),H(8),COV(8,8),VAR,RES
REAL*8 DMAT,DCMAT(8),GAIN(8),POLD(8,8)
INTEGER I,J,JUP,KFLAG

C INITIALIZE COUNT
COUNT = 0

C COMPUTE MATRIX (1X1) FOR INVERSION
DO 110 I=1,5,2
    DCMAT(I) = 0.0
DO 110 J=1,5,2
    DCMAT(I) = DCMAT(I) + COV(I,J)*H(J)
110 CONTINUE

DMAT = VAR
DO 120 I=1,6
    DMAT = DMAT + DCMAT(I)*H(I)
120 CONTINUE

C COMPUTE GAIN MATRIX
DO 140 I=1,5,2
    GAIN(I) = 0.0
    GAIN(I+1) = 0.0
DO 130 J=1,5,2
    GAIN(I) = GAIN(I) + COV(I,J)*H(J)
130 CONTINUE
GAIN(I) = GAIN(I)/DMAT
140 CONTINUE
C UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C PREVENT NEGATIVE DIAGONAL COVARIANCES
DO 150 I=1,5,2
   DCMAT(I)=0.0
   DO 150 J=1,5,2
      DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
   150 CONTINUE

DO 160 I=1,5,2
   POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
   IF (POLD(I,I) .LT. 0.0) THEN
      IF (COUNT .LT. 10) THEN
         DO 155 J=1,5,2
            GAIN(J)=GAIN(J)/2.0
         155 CONTINUE
         COUNT=COUNT+1
         KFLAG=KFLAG+1
         GOTO 150
      ELSE
         KFLAG=KFLAG+990
         RETURN
      ENDIF
   ENDIF
   JUP=I-2
   DO 160 J=1,JUP,2
      POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
   160 CONTINUE

C UPDATE ESTIMATE
XHAT(1)=XHAT(1)+GAIN(1)*RES
XHAT(3)=XHAT(3)+GAIN(3)*RES
XHAT(5)=XHAT(5)+GAIN(5)*RES

C UPDATE COV MATRIX
DO 175 I=1,5,2
   COV(I,I)=POLD(I,I)
   JUP=I-2
   DO 175 J=1,JUP,2
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
   175 CONTINUE

END
SUBROUTINE DICTIONARY

A  PRESENT ACCELERATION OF EVADER
ACOEF  A SPLINE COEFFICIENT
AD  DUMMY ACCELERATION
ASIGX  A SPLINE COEFFICIENT OF SIGMAX
ASIGY  A SPLINE COEFFICIENT OF SIGMAY
ASIGZ  A SPLINE COEFFICIENT OF SIGMAZ
AX  A SPLINE COEFFICIENT OF X
AY  A SPLINE COEFFICIENT OF Y
AZ  A SPLINE COEFFICIENT OF Z
BCOEF  B SPLINE COEFFICIENT
BSIGX  B SPLINE COEFFICIENT OF SIGMAX
BSIGY  B SPLINE COEFFICIENT OF SIGMAY
BSIZG  B SPLINE COEFFICIENT OF SIGMAZ
BX  B SPLINE COEFFICIENT OF X
BY  B SPLINE COEFFICIENT OF Y
BZ  B SPLINE COEFFICIENT OF Z
CCOEF  C SPLINE COEFFICIENT
CFLAG  CONVERGENCE FLAG
COST1  DUAL CONTROL COST
COST2  DUAL CONTROL COST
CSIGX  C SPLINE COEFFICIENT OF SIGMAX
CSIGY  C SPLINE COEFFICIENT OF SIGMAY
CSIGZ  C SPLINE COEFFICIENT OF SIGMAZ
CVDD  COVARIANCE MATRIX FOR EKF
CVDR  COVARIANCE MATRIX FOR EKF
CVDY  COVARIANCE MATRIX FOR EKF
CVDZ  COVARIANCE MATRIX FOR EKF
CVD2  COVARIANCE MATRIX FOR EKF
CVDIFY  SELECTED COVARIANCE DIFFERENCE
CVDIFZ  SELECTED COVARIANCE DIFFERENCE
CVTOT  SELECTED COVARIANCE TOTAL
CX  C SPLINE COEFFICIENT OF X
CY  C SPLINE COEFFICIENT OF Y
CZ  C SPLINE COEFFICIENT OF Z
DA  DISTANCE DUE TO BOOSTER ACCELERATION
DCOEF  D SPLINE COEFFICIENT
DDY  CHANGE IN Y VELOCITY
DDZ  CHANGE IN Z VELOCITY
DELTAY  CHANGE IN Y VELOCITY
DELTAZ  CHANGE IN Z VELOCITY
DEN  DENOMINATOR (FOR JACOBIAN DETERMINANT)
DSIGX  D SPLINE COEFFICIENT OF SIGMAX
DSIGY  D SPLINE COEFFICIENT OF SIGMAY
DSIGZ  D SPLINE COEFFICIENT OF SIGMAZ
C DTGO CHANGE IN TIME-TO-GO
C DUMMY DUMMY VARIABLE
C DVY Y VELOCITY CHANGE FOR DEVIATION
C DVYA Y VELOCITY CHANGE FOR DEVIATION
C DVZ Z VELOCITY CHANGE FOR DEVIATION
C DVZA Z VELOCITY CHANGE FOR DEVIATION
C DX D SPLINE COEFFICIENT OF X
C DY D SPLINE COEFFICIENT OF Y
C DZ D SPLINE COEFFICIENT OF Z
C FSIGX SIGMA X SPLINE COEFF FOR Y VEL CHANGE
C FSIGY SIGMA Y SPLINE COEFF FOR Y VEL CHANGE
C FSIGZ SIGMA Z SPLINE COEFF FOR Y VEL CHANGE
C F1 NONLINEAR SYSTEM EQUATION VALUE
C F2 NONLINEAR SYSTEM EQUATION VALUE
C F3 NONLINEAR SYSTEM EQUATION VALUE
C GSIGX SIGMA X SPLINE COEFF FOR Z VEL CHANGE
C GSIGY SIGMA Y SPLINE COEFF FOR Z VEL CHANGE
C GSIGZ SIGMA Z SPLINE COEFF FOR Z VEL CHANGE
C H ITERATION TIME STEP SIZE
C I COUNTER
C J COUNTER
C J11 JACOBIAN MATRIX 1,1 ELEMENT
C J12 JACOBIAN MATRIX 1,2 ELEMENT
C J13 JACOBIAN MATRIX 1,3 ELEMENT
C J22 JACOBIAN MATRIX 2,2 ELEMENT
C LAMBDA LAGRANGE MULTIPLIER
C MAXDV MAXIMUM PERMISSIBLE VELOCITY CHANGE
C MISS MISS DISTANCE
C MDCT UNITIZED MASS FLOW RATE OF EVADER
C OLDTGO PREVIOUS TIME-TO-GO
C RADIUS QUANTITY INSIDE RADICAL SIGN
C RANGE RELATIVE RANGE
C SIG DEVIATION AT FINAL TIME
C SIGMAM MEASUREMENT DEVIATIONS
C SIGX X DEVIATION AT FINAL TIME
C SIGY Y DEVIATION AT FINAL TIME
C SIGZ Z DEVIATION AT FINAL TIME
C SIGXDT TIME DERIVATIVE OF SIGX
C SIGYDT TIME DERIVATIVE OF SIGY
C SIGZDT TIME DERIVATIVE OF SIGZ
C TD DUMMY TIME
C TGO TIME-TO-GO
C TG03 3.0*TGO
C TG06 6.0*TGO
C TLIM TIME STEP LIMIT
C TOL MISS TOLERANCE
C VA VELOCITY DUE TO BOOSTER ACCELERATION
C VE EVADER VELOCITY MAGNITUDE
C VX EVADER UNITIZED X VELOCITY
C VXE EVADER X VELOCITY
C VY EVADER UNITIZED Y VELOCITY
C VYE EVADER Y VELOCITY
C VZ EVADER UNITIZED Z VELOCITY
SUBROUTINE SPLINE4(XO,X1,X2,X3,TGO,
   + ACOEF,BCOEF,Ccoef,DCOEF)

C THIS SUBROUTINE CREATES THE SPLINE COEFFICIENTS
C FOR THE EQUATION:
C \[ ACOEF \cdot T^3 + BCOEF \cdot T^2 + CCOEF \cdot T + DCOEF \]
C GIVEN FOUR EQUISPACED POINTS AND TIME-TO-GO.
C PRESENT TIME IS EQUAL TO ZERO.

REAL*8 XO,X1,X2,X3,TGO,ACOEF,BCOEF,Ccoef,DCOEF

ACOEF=(-4.5*X0 +13.5*X1 -13.5*X2 +4.5*X3)/
   + (TGO*TGO*TGO)
Bcoef=(+9.0*X0 -22.5*X1 +18.0*X2 -4.5*X3)/
   + (TGO*TGO)
Ccoef=(-5.5*X0 +9.0*X1 -4.5*X2 +1.0*X3)/TGO
DCOEF=X0

END
SUBROUTINE SPLINE(XO,X0DOT,XF,XFDOT,TGO,
ACOEF,BCOEF,CCOEF,DCOEF)

C THIS SUBROUTINE CREATES THE SPLINE COEFFICIENTS
C FOR THE EQUATION:
C \( ACOEF*T^3 + BCOEF*T^2 + CCOEF*T + DCOEF \)
C GIVEN INITIAL POSITION AND VELOCITY AND
C FINAL POSITION, VELOCITY, AND TIME-TO-GO.
C PRESENT TIME IS EQUAL TO ZERO.

REAL*8 XO,X0DOT, XF, XFDOT, TGO
REAL*8 ACOEF, BCOEF, CCOEF, DCOEF

DCOEF=XO
CCOEF=X0DOT
ACOEF=({2.0*(XO-XF)/TGO+X0DOT+XFDOT)/TGO/TGO
BCOEF=({3.0*(XF-XO)/TGO-2.0*X0DOT-XFDOT)/TGO

END

SUBROUTINE SEARCHA(AX,BX,CX,DX,AY,BY,CY,DX,
+ AZ,BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,CFLAG)

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C \( L = K*(XF**2+XY**2+XZ**2)/2 + \)
C \( \frac{(DELTAX**2 + DELTAY**2)}{2} \)
C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY
C CHANGES (DELTAY AND DELTAZ) TO BRING THE MISS
C DISTANCE WITHIN SOME TOLERANCE.
C XF, YF and ZF ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX, BX, ..., DZ AT THE FINAL TIME.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME FOR NON-LINEAR SYSTEMS.
C (PP. 176-179 OF MARON, 'NUMERICAL ANALYSIS,
C A PRACTICAL APPROACH')

REAL*8 AX,BX,CX,DX,AY,BY,CY,DX,
+ AZ,BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ, XF, YF, ZF, F1,F2,F3
REAL*8 XFDOT,YFDOT,ZFDOT,J11,J12,J13,J22,TGO3
REAL*8 MISS,TOL,TG06,OLDTGO,SCALE,DEN,DTGO,DDZ
INTEGER I,CFLAG

C INITIALIZE VELOCITY CHANGES AND OLD FINAL TIME
DELTAY=0.0
DELTAZ=0.0
OLDTGO=TGO

C BEGIN SEARCH LOOP
DO 10 I=1,25
C COMPUTE FINAL RELATIVE POSITIONS
XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
YF=((AY*TGO+BY)*TGO+CY-DELTAY)*TGO+DY
ZF=((AZ*TGO+AZ)*TGO+CZ-DELTAZ)*TGO+DZ
MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS (THIS IS RANGE RELATIVE)
IF (MISS .LE. TOL) RETURN

C COMPUTE FINAL RELATIVE VELOCITIES
TG03=TGO+TGO+TGO
XFDOT=(AX*TGO3+BX+BX)*TGO+CX
YFDOT=(AY*TGO3+BY+BY)*TGO+CY-DELTAY
ZFDOT=(AZ*TGO3+AZ+AZ)*TGO+CZ-DELTAZ

C COMPUTE NONLINEAR SYSTEM EQUATIONS
F1=K*(XF*XFDOT+XF*YFDOT+ZF*ZFDOT)
F2=DELTAY-K*YF*TGO
F3=DELTAZ-K*ZF*TGO

C COMPUTE NECESSARY ELEMENTS OF JACOBIAN MATRIX
TG06=TGO3+TGO3
J11=K*(XF*(AX*TGO6+BX+BX)+YF*(AY*TGO6+BY+BY)+
+ ZF*(AZ*TGO6+BZ+BZ)+XFDOT*XFDOT+
+ YFDOT*YFDOT+ZFDOT*ZFDOT)
J12=-K*(YF+YFDOT*TGO)
J13=-K*(ZF+ZFDOT*TGO)
J22=1.0+K*TGO*TGO

C COMPUTE CHANGES IN CONTROL PARAMETERS
DEN=(J11+J22-J13-J12-J12)/DEN
DTGO=-(F1*J22-J12*F2-J13*F3)/DEN
DDY=-(J12+F1+(J11-J13-J12)*J22)*F2+
+ F3*J12*J13/J22)/DEN
DDZ=-(J13*F1+F2*J12*J13*J22+/
+ (J11-J12*J12/J22)*F3)/DEN

C UPDATE CONTROL PARAMETERS
TGO=TGO+DTGO
DELTAY=DELTAY+DDY
DELTAZ=DELTAZ+DDZ

C TEST FOR CONVERGENCE OF FINAL TIME
IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN
10 CONTINUE

C INCREMENT CONVERGENCE FLAG
CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
DELTAY=0.0
DELTAZ=0.0
TGO=OLDTGO
END
SUBROUTINE SEARCHB(AX,BX,CX,DX,AY,BY,CY,DY, 
+ AZ,BZ,CZ,DZ,TGO,DELTAY,DELTAZ,TOL,CFLAG)

C THIS SUBROUTINE FINDS THE TIME-TO-GO AND VELOCITY 
C CHANGES WITH MISS DISTANCE WITHIN SOME EXTERNAL 
C TOLERANCE.
C THIS IS DONE BY VARYING THE TIME-TO-GO (TGO) TO 
C VARY THE X MISS DISTANCE.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON 
C SEARCH SCHEME (PP. 48-53 OF MARON, 'NUMERICAL 
C ANALYSIS, A PRACTICAL APPROACH') 
C XF, AND XFDOT ARE COMPUTED FROM THE SPLINE 
C COEFFICIENTS AX, BX, CX, DX AT THE FINAL TIME. 
C ONCE THE FINAL TIME IS KNOWN THE VELOCITY CHANGES 
C (DELTAY AND DELTAZ) ARE COMPUTED.

REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,DTGO 
REAL*8 TGO,DELTAY,DELTAZ,XFDOT,SCALE,XF,YF,ZF 
REAL*8 MISS,TOL,DDY,DDZ,OLDTGO 
INTEGER I,CFLAG

C

INITIALIZE VARIABLES
DELTAY=0.0 
DELTAZ=0.0 
OLDTGO=TGO

C BEGIN SEARCH LOOP
DO 10 I=1,10

C COMPUTE FINAL POSITIONS AND MISS DISTANCE 
XF=((AX*TGO+BX)*TGO+CX)*TGO+DX 
YF=((AY*TGO+BY)*TGO+CY-DELTAY)*TGO+DY 
ZF=((AZ*TGO+BZ)*TGO+CZ-DELTAZ)*TGO+DZ 
MISS=SQR(TXF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS 
IF (MISS .LE. TOL) RETURN

C COMPUTE CHANGE IN TIME-TO-GO 
XFDOT=(AX* (TGO+TGO)*TGO+BX)*TGO+CX 
DTGO= XF/XFDOT 
TGO=TGO+DTGO

C COMPUTE CHANGES IN VELOCITIES 
YF=((AY* (TGO+BY)*TGO+CY-DELTAY)*TGO+DY 
ZF=((AZ* (TGO+BZ)*TGO+CZ-DELTAZ)*TGO+DZ 
DDY=(AY* (TGO+BY)*TGO+CY-DELTAY+DY/TGO 
DELTAY=DELTAY+DDY 
DDZ=(AZ* (TGO+BZ)*TGO+CZ-DELTAZ+DZ/TGO 
DELTAZ=DELTAZ+DDZ

C TEST FOR CONVERGENCE OF FINAL TIME 
IF (ABS(DTGO) .LT. (.00001*TGO)) RETURN
CONTINUE

C INCREMENT CONVERGENCE FLAG
CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
DELTAY=0.0
DELTAZ=0.0
TGO=OLDTGO

END

SUBROUTINE SEARCHC(XR,VXE,VYE,VZE,A,MDOT,
+ TGO,DELTAY,DELTAZ,TOL,CFLAG)

C THIS SUBROUTINE DETERMINES TIME-TO-GO AND VELOCITY
C CHANGES FOR PLAN C WITH MISS DISTANCE WITHIN SOME
C TOLERANCE.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME (PP. 48-53 OF MARON, 'NUMERICAL
C ANALYSIS, A PRACTICAL APPROACH')
C ONCE THE TIME-TO-GO IS KNOWN THE VELOCITY CHANGES
C (DELTAY AND DELTAZ) ARE COMPUTED.

REAL*8 DTGO,XR(6),VXE,VYE,VZE,A,MDOT,DA,VE
REAL*8 TF,DELTAY,DELTAZ,XF,YF,ZF,XFDOT,SCALE
REAL*8 OLDTGO,VX,VY,VZ,MISS,TGO,TOL,MT
INTEGER I,J,CFLAG

C COMPUTE EVADER UNITIZED VELOCITY VECTOR COMPONENTS
VE=SQRT(VXE*VXE+VYE*VYE+VZE*VZE)
VX=VXE/VE
VY=VYE/VE
VZ=VZE/VE

C INITIALIZE VARIABLES
DELTAY=0.0
DELTAZ=0.0
OLDTGO=TGO

C COMPUTE DISTANCE DUE TO BOOSTER ACCELERATION
DA=0.0
MT=MDOT*TGO
DO 15 J=2,50
   DA=DA+MT**J/(J*(J-1))
15 CONTINUE
   DA=DA*A/(MDOT*MDOT)

C BEGIN SEARCH LOOP
DO 30 I=1,10
C COMPUTE FINAL RELATIVE POSITION AND MISS DISTANCE
XF=XR(1)+XR(2)*TGO+DA*VX
YF=XR(3)+(XR(4)-DELTAY)*TGO+DA*VY
ZF=XR(5)+(XR(6)-DELTAZ)*TGO+DA*VZ
MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS
IF (MISS .LE. TOL) RETURN

C COMPUTE CHANGE IN TIME-TO-GO
XFDOT=XR(2)+DA*VX/TGO
DTGO=-XF/XFDOT
TGO=TGO+DTGO

C COMPUTE CHANGES IN VELOCITIES
DA=0.0
MT=MDOT*TGO
DO 25 J=2,50
   DA=DA+MT**J/(J*(J-1))
25 CONTINUE
   DA=DA*A/(MDOT*MDOT)
DELTAY=(DA*VY+XR(3))/TGO+XR(4)
DELTAZ=(DA*VZ+XR(5))/TGO+XR(6)

C TEST FOR CONVERGENCE OF TIME-TO-GO
IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN

30 CONTINUE

C INCREMENT CONVERGENCE FLAG
CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
DELTAY=0.0
DELTAZ=0.0
TGO=OLDTGO

END

SUBROUTINE SEARCHCC(AX,BX,CX,DX,Ay,By,Cy,Dy,+
+ Az,Bz,Cz,Dz,Asigx,bsigx,csigx,dssigx,asigy,+
+ BsigY,csigY,dssigY,asigz,bsigZ,csigZ,dssigZ,+
+ K,TGO,DELTAY,DELTAZ,TOL,CFLAG)

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C L = (DELTAX**2 + DELTAY**2)/2
C SUBJECT TO THE CONSTRAINT :
C XF*XF+YF*YF+ZF*ZF <= K*(SIGMAF*SIGMAF)
D- 50

C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY
C CHANGES (DELTAY AND DELTAZ).
C XF, YF AND ZF ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX, BX, ..., DZ AT THE FINAL TIME,
C AS ARE THE FINAL SIGMAS.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME FOR NON-LINEAR SYSTEMS.
C (PP. 176-179 OF MARON, 'NUMERICAL ANALYSIS,
C A PRACTICAL APPROACH')

REAL*8 AX, BX, CX, DX, AY, BY, CY, DY, AZ, BZ, CZ, DZ
REAL*8 DTGO, TLIM, SCALE, DEN, OLDTGO, KSIG
REAL*8 K, TGO, DELTAY, DELTAZ, XF, YF, ZF, P1, F2, F3
REAL*8 XFDOT, YFDOT, ZFDOT, J11, J12, J21, J22, TGO3
REAL*8 ASIGX, BSIGX, CSIGX, DSIGX, ASIGY, BSIGY
REAL*8 CSIGY, DSIGY, DLAM, PERT, MISS, F1T, F2T, F1L, F2L
REAL*8 TOL, ASIGZ, BSIGZ, CSIGZ, DSIGZ, LAMBD
REAL*8 SIGX, SIGY, SIGZ, SIGXDT, SIGYDT, SIGZDT
REAL*8 DYDL, LZDL, DYDTDL, DZDTDL, TGO6, YS, ZS
REAL*8 SIGXDD, SIGYDD, SIGZDD, XFD2, YFD2, ZFD2
INTEGER I, J, CFLAG

C INITIALIZE PARAMETERS
PERT = 0.0000001
OLDTGC = TGO
TLIM = 0.01 * TGO
DELTAY = 0.0
DELTAZ = 0.0

C DETERMINE IF CONSTRAINT CAN BE SATISFIED
C WITHOUT THRUSTING
DO 5 I = 1, 10

C COMPUTE FINAL RELATIVE POSITIONS AND DEVIATIONS
XF = ((AX * TGO + BX) * TGO + CX) * TGO + DX
YF = ((AY * TGO + BY) * TGO + CY) * TGO + DY
ZF = ((AZ * TGO + BZ) * TGO + CZ) * TGO + DZ
SIGX = ((ASIGX * TGO + BSIGX) * TGO + CSIGX) * TGO + DSIGX
SIGY = ((ASIGY * TGO + BSIGY) * TGO + CSIGY) * TGO + DSIGY
SIGZ = ((ASIGZ * TGO + BSIGZ) * TGO + CSIGZ) * TGO + DSIGZ

C COMPUTE FINAL RELATIVE VELOCITIES
C AND DEVIATION DERIVATIVES
TGO3 = TGO + TGO + TGO
XFDOT = (AX * TGO3 + BX) * TGO + CX
YFDOT = (AY * TGO3 + BY) * TGO + CY
ZFDOT = (AZ * TGO3 + BZ) * TGO + CZ
SIGXDT = (ASIGX * TGO3 + BSIGX) * TGO + CSIGX
SIGYDT = (ASIGY * TGO3 + BSIGY) * TGO + CSIGY
SIGZDT = (ASIGZ * TGO3 + BSIGZ) * TGO + CSIGZ
C COMPUTE NONLINEAR SYSTEM EQUATIONS
   F1 = (XF*XF + YF*YF + ZF*ZF - K*(SIGX*SIGX + SIGY*SIGY + SIGZ*SIGZ))/2.0
C DETERMINE IF CONSTRAINT IS SATISFIED
   IF (F1 .LT. TOL) RETURN
   F2 = XF*XFDOT + YF*YFDOT + ZF*ZFDOT - K*(SIGX*SIGXDT + SIGY*SIGYDT + SIGZ*SIGZDT)
   F3 = XF*XFDOT + YF*YFDOT + ZF*ZFDOT - K*(SIGX*SIGXDT + SIGY*SIGYDT + SIGZ*SIGZDT)
C COMPUTE NEW TGO
   DTGO = -F2/F3
   TGO = TGO + DTGO
C TEST FOR CONVERGENCE OF FINAL TIME
   IF (ABS(DTGO) .LT. (TOL*TLIM)) GOTO 6
5 CONTINUE
6 CONTINUE
C INITIALIZE LAMBDA
   MISS = SQRT(XF*XF + YF*YF + ZF*ZF)
   KSIG = SQRT(K*(SIGX*SIGX + SIGY*SIGY + SIGZ*SIGZ))
   LAMBDAM = (1.0 - KSIG/MISS)/TGO/TGO
C BEGIN SEARCH LOOP
   DO 15 I = 1, 20
C COMPUTE VELOCITY CHANGES
   YS = ((AY*TGO + BY)*TGO + CY)*TGO + DY
   ZS = ((AZ*TGO + BZ)*TGO + CZ)*TGO + DZ
   DEN = 1.0 + LAMBDAM*TGO*TGO
   DELTAY = YS*TGO + LAMBDAM/DEN
   DELTAZ = ZS*TGO + LAMBDAM/DEN
C COMPUTE FINAL RELATIVE POSITIONS AND DEVIATIONS
   XF = ((AX*TGO + BX)*TGO + CX)*TGO + DX
   YF = YS - DELTAY * TGO
   ZF = ZS - DELTAZ * TGO
   SIGX = ((ASIGX*TGO + BSIGX)*TGO + CSIGX)*TGO + DSIGX
   SIGY = ((ASIGY*TGO + BSIGY)*TGO + CSIGY)*TGO + DSIGY
   SIGZ = ((ASIGZ*TGO + BSIGZ)*TGO + CSIGZ)*TGO + DSIGZ
C COMPUTE FINAL RELATIVE VELOCITIES AND DEVIATION DERIVATIVES
   TGO3 = TGO + TGO + TGO
   XFDOT = (AX*TGO3 + BX + BX)*TGO + CX
   YFDOT = (AY*TGO3 + BY + BY)*TGO + CY - DELTAY
   ZFDOT = (AZ*TGO3 + BZ + BZ)*TGO + CZ - DELTAZ
   SIGXDT = (ASIGX*TGO3 + BSIGX + BSIGX)*TGO + CSIGX
   SIGYDT = (ASIGY*TGO3 + BSIGY + BSIGY)*TGO + CSIGY
   SIGZDT = (ASIGZ*TGO3 + BSIGZ + BSIGZ)*TGO + CSIGZ
C COMPUTE NONLINEAR SYSTEM EQUATIONS
F1=(XFXF+YFYF+ZFZF-K*(SIGXSIGX+
+SIGYSIGY+SIGZSIGZ))/2.0
F2=XFXFD+YFYFD+ZFZFD-K*
+(SIGXSIGXDT+SIGYSIGYDT+SIGZSIGZDT)

C TEST FOR CONVERGENCE OF CONSTRAINTS
IF ((ABS(F1).LT.TOL).AND.
+(ABS(F2).LT.TOL)) RETURN

C COMPUTE NECESSARY PARTIALS
DYDTDL=-YF*TGO/DEN
DZDTDL=-ZF*TGO/DEN
DYDL=DYDTDL*TGO
DZDL=DZDTDL*TGO
TGO6=TGO3+TGO3
XFDD=AX*TGO6+BX+BX
YFDD=AY*TGO6+BY+BY-LAMBDA*(YFDOT*TGO+YF)
ZFDD=AZ*TGO6+BZ+BZ-LAMBDA*(ZFDOT*TGO+ZF)
SIGXDD=ASIGX*TGO6+BSIGX+BSTGX
SIGYDD=ASIGY*TGO6+BSIGY+BSIGY
SIGZDD=ASIGZ*TGO6+BSIGZ+BSIGZ

C COMPUTE NECESSARY ELEMENTS OF JACOBIAN MATRIX
J11=F2
J12=YF*DYDL+ZF*DZDL
J21=XF*XFD+XFDOT*XFDT+YF*YFD+YFDOT*YFDOT+
+ZF*ZFDD+ZFDOT*ZFDOT
J21=J21-K*(SIGXSIGXDD+SIGXDTSIGXDT+
+SIGYSIGYDD+SIGYDTSIGYDT+
+SIGZSIGZDD+SIGZDTSIGZDT)
J22=DYDL*YFD+YF*DYDTDL+DZDL*ZFDOT+ZF*DZDTDL

C COMPUTE CHANGES IN CONTROL PARAMETERS
DEN=(J11*J22-J12*J21)
DTGO=-(F1*J22+F2*J12)/DEN
DLAM=(F1*J21-F2*J11)/DEN
SCALE=ABS(DTGO/TLIM)
IF (SCALE.GT.1.0) THEN
DTGO=DTGO/SCALE
DLAM=DLAM/SCALE
ENDIF

C UPDATE CONTROL PARAMETERS
TGO=TGO+DTGO
LAMBDA=LAMBDA+DLAM

15 CONTINUE

C INCREMENT CONVERGENCE FLAG
CFLAG=CFLAG+1
ZERO OUT VELOCITY CHANGES AND RESET TIME

DELTA_Y = 0.0
DELTA_Z = 0.0
TGO = OLD TGO

SUBROUTINE SEARCHD(AX,BX,CX,DX,AY,BY,CY,DY,AZ,
+ BZ,CZ,DZ,K,TGO,DELTA_Y,DELTA_Z,TOL,CFLAG,XDUALD,
+ XEDD,XFD2,CVD2,MADV,COUNT,Q,R3,SIGMAM,H)

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C
L = E[K*(XF**2+XY**2+XZ**2)/2 +
  (DELTAX**2 + DELTAY**2)/2]
C
C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY
C CHANGES (DELTA_Y AND DELTA_Z) TO BRING THE MISS
C DISTANCE WITHIN SOME TOLERANCE.
C XF,YF AND ZF ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX,BX,...,DZ AT THE FINAL TIME.

REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ
REAL*8 K,TGO,DELTA_Y,DELTA_Z,DH,Q(8,8),RES(3)
REAL*8 DVY,DVZ,XDUALD(8),CVD(8,8),XPD(6)
REAL*8 MISS,TOL,XEDD(6),R3(3,3),TD,RANGE,DVYA
REAL*8 SIGMAM(3),COST1,COST2,CVP,TOTDV,TVYA
REAL*8 XDUAL2(8),CVD2(8,8),XED2(6),XPD2(6)
REAL*8 CVDY(8,8),CVDZ(8,8),XEDY(6),XEDZ(6)
REAL*8 XPDY(6),XPDZ(6),XDUALY(8),XDUALZ(8)
REAL*8 XF,YF,ZF,XFDT,YFDT,ZFDT,MADV,DTGO
REAL*8 CVDR(8,8),XEDR(6),XPD(6),XDUALR(8)
REAL*8 DEN,CVDIFY,CVDIFZ,RADICL,RTGO
INTEGER I,J,CFLAG,COUNT,KFLAG

C COMPUTE CERTAINTY EQUIVALENCE SOLUTION
CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,
+ AZ,BZ,CZ,DZ,K,TGO,DELTA_Y,DELTA_Z,TOL,CFLAG)

C ASSIGN DUMMY FILTER VARIABLES
DO 100 I=1,8
  XDUAL2(I)=XDUALD(I)
  XDUALY(I)=XDUALD(I)
  XDUALZ(I)=XDUALD(I)
  XDUALR(I)=XDUALD(I)
DO 100 J=1,8
  CVD2(I,J)=CVD(I,J)
  CVDY(I,J)=CVD(I,J)
  CVDZ(I,J)=CVD(I,J)
  CVDR(I,J)=CVD(I,J)
100 CONTINUE
DO 110 I=1,6
   XED2(I)=XEDD(I)
   XEDY(I)=XEDD(I)
   XEDZ(I)=XEDD(I)
   XEDR(I)=XEDD(I)
   XP2(I)=XPDD(I)
   XPY(I)=XPDD(I)
   XP2(I)=XPDD(I)
   XPDR(I)=XPDD(I)
110 CONTINUE

C DETERMINE BEST DIRECTION TO IMPROVE THE ESTIMATE

DVY=MAXDV
DVZ=MAXDV
XP2(4)=XP2(4)+DVY
XP2(6)=XP2(6)+DVZ
XDUALY(4)=XDUALY(4)-DVY
XDUALZ(6)=XDUALZ(6)-DVZ

DH=TGO/COUNT
DO 300 I=1,COUNT
   RANGE=SQRT(XDUALR(1)*XDUALR(1)+XDUALR(3)*XDUALR(3)+XDUALR(5)*XDUALR(5))
   R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
   CALL EKF8(XDUALR,XEDR,XPDR,TD,DH,CVDR,Q,R3,
            0.0,0.0,0.0,KFLAG,RES,2)
   RANGE=SQRT(XDUALY(1)*XDUALY(1)+XDUALY(3)*XDUALY(3)+XDUALY(5)*XDUALY(5))
   R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
   CALL EKF8(XDUALY,XEDY,XPDY,TD,DH,CVDY,Q,R3,
            0.0,0.0,0.0,KFLAG,RES,2)
   RANGE=SQRT(XDUALZ(1)*XDUALZ(1)+XDUALZ(3)*XDUALZ(3)+XDUALZ(5)*XDUALZ(5))
   R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
   CALL EKF8(XDUALZ,XEDZ,XPDZ,TD,DH,CVDZ,Q,R3,
            0.0,0.0,0.0,KFLAG,RES,2)
300 CONTINUE

CVTOT=CVDR(1,1)+CVDR(3,3)+CVDR(5,5)
CVDIFY=CVTOT-CVDY(1,1)-CVDY(3,3)-CVDY(5,5)
CVDIFZ=CVTOT-CVDZ(1,1)-CVDZ(3,3)-CVDZ(5,5)
DEN=CVDIFY*CVDIFY+CVDIFZ*CVDIFZ
IF (DEN .GT. 1.0E-20) THEN
   DEN=SQRT(DEN)
   DVY=DVY*CVDIFY/DEN
   DVZ=DVZ*CVDIFZ/DEN
ELSE
   RETURN
ENDIF

C ENSURE C.E. SOLUTION IS WITHIN THRUST LIMITS

RTGO=TGO-H
RADICL=RTGO*RTGO-(DELTAY+DELTAY)*RTGO*H/MAXDV
IF (RADICL .LE. 0.0) THEN
  DELTAY=MAXDV*INT(RTGO/H)/2.0
  COST1=(DELTAY+DELTAY)**2
ELSE
  COST1=((RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF
RADICL=RTGO*RTGO-(DELTAZ+DELTAZ)*RTGO/H/MAXDV
IF (RADICL .LE. 0.0) THEN
  DELTAZ=MAXDV*INT(RTGO/H)/2.0
  COST1=COST1+(DELTAZ+DELTAZ)**2
ELSE
  COST1=COST1+( (RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF

C COMPUTE COST OF CERTAINTY EQUIVALENCE SOLUTION
XDUALD(4)=XDUALD(4)-DELTAY
XDUALD(6)=XDUALD(6)-DELTAZ
XPDD(4)=XPDD(4)-DELTAY
XPDD(6)=XPDD(6)+DELTAZ
TD=0.0
IH=H
RANGE=SQR(XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))
R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
CALL EKF8(XDUALD,XEDD,XPDD,TD,DH,CVDD,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)
DTGO=TGO-DH
DH=DH/COUNT
DO 200 I=1,COUNT
  RANGE=SQR(XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))
  R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
  CALL EKF8(XDUALD,XEDD,XPDD,TD,DH,CVDD,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)
200 CONTINUE
CVTOT=CVDD(1,1)+CVDD(3,3)+CVDD(5,5)
COST1=COST1+K*(CVTOT+XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))

C DETERMINE COST OF IMPROVING THE ESTIMATE
XPDD2(4)=XPDD2(4)+DVY
XPDD2(6)=XPDD2(6)+DVZ
XDUAL2(4)=XDUAL2(4)-DVY
XDUAL2(6)=XDUAL2(6)-DVZ

C MOVE FORWARD ONE STEP
DH=H
RANGE=SQR(XDUAL2(1)*XDUAL2(1)+
+ XDUAL2(3)*XDUAL2(3)+XDUAL2(5)*XDUAL2(5))
R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
CALL EKF8(XDUAL2,XED2,XPDD2,TD,DH,CVDD,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)


C COMPUTE NEW SPLINES
CY=CY-DV Y
CZ=CZ-DV Z
XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
YF=((AY*TGO+BY)*TGO+CY)*TGO+DY
ZF=((AZ*TGO+BZ)*TGO+CZ)*TGO+DZ
XFDOT=(3.0*AX*TGO+BX)*TGO+CX
YFDOT=(3.0*AY*TGO+BY)*TGO+CY
ZFDOT=(3.0*AZ*TGO+BZ)*TGO+DZ
DTGO=TGO-H
CALL SPLINE(XDUAL2(1),XDUAL2(2),XF,XFDOT,
 + DTGO,AX,BX,CX,DX)
CALL SPLINE(XDUAL2(3),XDUAL2(4),YF,YFDOT,
 + DTGO,AY,BY,CY,DY)
CALL SPLINE(XDUAL2(5),XDUAL2(6),ZF,ZFDOT,
 + DTGO,AZ,BZ,CZ,DZ)

C COMPUTE NEW CERTAINTY EQUIVALENCE SOLUTION
CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,
 + AZ,BZ,CZ,DZ,K,DTGO,DVYA,DVZATOL,CFLAG)

C ENSURE NEW C.E. SOLUTION IS WITHIN THRUST LIMITS
RTGO=DTGO-H
RADICL=RTGO*RTGO-(DVYA+DVYA)*RTGO/H/MA XD Y
IF (RADICL .LE. 0.0) THEN
   DVYA=MAXDV*INT(RTGO/H)/2.0
   COST2=(DVYA+DVYA)**2
ELSE
   COST2=((RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF
RADICL=RTGO*RTGO-(DVZA+DVZA)*RTGO/H/MA XD Y
IF (RADICL .LE. 0.0) THEN
   DVZA=MAXDV*INT(RTGO/H)/2.0
   COST2=COST2+(DVZA+DVZA)**2
ELSE
   COST2=COST2+((RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF

C COMPUTE NEXT ITERATION COST
XDUAL2(4)=XDUAL2(4)-DVYA
XDUAL2(6)=XDUAL2(6)-DVZA
XP D2(4)=XP D2(4)+DVYA
XP D2(6)=XP D2(6)+DVZA
DH=DTGO/COUNT
DO 400 I=1,COUNT
   RANGE=SQRT(XDUAL2(1)*XDUAL2(1)+
   + XDUAL2(3)*XDUAL2(3)+XDUAL2(5)*XDUAL2(5))
   R3(1,1)=SIGMAM(1)*RANGE*SIGKAM(1)*RANGE
   CALL EKF8(XDUAL2,XED2,XP D2,T D,DH,CVD2,Q,R3,
   + 0.0,0.0,0.0,KFLAG,RES,2)
400 CONTINUE
CVTOT=CVD2(1,1)+CVD2(3,3)+CVD2(5,5)
\[
\text{COST2} = \text{COST2} + K \times (\text{CVTOT} + \text{XDUAL2}(1) \times \text{XDUAL2}(1) + \\
+ \text{XDUAL2}(3) \times \text{XDUAL2}(2) + \text{XDUAL2}(5) \times \text{XDUAL2}(5)) + \\
+ \text{DVY} \times \text{DVY} + \text{DVZ} \times \text{DVZ}
\]

\text{C CHOOSE THE LEAST COST OPTION}
\text{IF (COST1 .GT. COST2) THEN}
\begin{align*}
\text{PRINT } *, \text{ 'DEViating FROM NOMINAL PATH'} \\
\text{DELTAY} = \text{DVY} \\
\text{DELTAZ} = \text{DVZ} \\
\text{TGO} = \text{DTGO} + H
\end{align*}
\text{ENDIF}
\text{END IF}
\text{END}

\text{END}

\text{SUBROUTINE SEARCHT(XP,XE,A,MDOT,H,T,TGO,}
\text{ + DELTAY,DELTAZ,TOL,CFLAG,COUNT)}
\text{C THIS SUBROUTINE VARIES THE TIME-TO-GO (TGO) AND THE}
\text{C VELOCITY CHANGES (DELTAY AND DELTAZ) WHILE BRINGING}
\text{C MISS DISTANCE WITHIN A PRESPECIFIED TOLERANCE.}
\text{C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON}
\text{C SEARCH SCHEME FOR NON-LINEAR SYSTEMS. (PP. 176-179}
\text{C OF MARON, 'NUMERICAL ANALYSIS, A PRACTICAL APPROACH')}
\text{C XF,YF,ZF,XFDOT,YFDOT, AND ZFDOT ARE COMPUTED}
\text{C NUMERICALLY.}
\text{REAL*8 XP(6),XE(6),XPD(6),XED(6),T,TGO,DELTAY}
\text{REAL*8 XF,YF,ZF,SCALE,XFDOT,YFDOT,ZFDOT,TD,DTGO}
\text{REAL*8 A,MDOT,TOL,AD,MDOTD,OLDTGO,DH,DELTAZ,H}
\text{REAL*8 DDY,DDZ}
\text{INTEGER I,J,COUNT,CFLAG}
\text{C INITIALIZE VELOCITY CHANGES}
\text{DELTAY} = 0.0
\text{DELTAZ} = 0.0
\text{OLDTGO} = TGO
\text{C BEGIN SEARCH LOOP}
\text{DO 10 J=1,10}
\text{C INITIALIZE DUMMY VARIABLES}
\text{DO 110 I=1,6}
\text{XED(I)} = \text{XE(I)}
\text{XPD(I)} = \text{XP(I)}
\text{110 CONTINUE}
\text{XPD(4)} = \text{XPD(4)} + \text{DELTAY}
\text{XPD(6)} = \text{XPD(6)} + \text{DELTAZ}
\text{AD} = A
\text{MDOTD} = \text{MDOT}
C PROPAGATE DUMMY VARIABLES FORWARD TO IMPACT TIME
TD=T
DH=TGO/COUNT
DO 115 I=1,COUNT
   CALL RK4SYSF(TD,XPD,DH)
   CALL RK4SYSB(TD,XED,DH,AD,MDOTD)
   TD=TD+H
115 CONTINUE

C ASSIGN FINAL RELATIVE POSITIONS
XF=XED(1)-XPD(1)
YF=XED(3)-XPD(3)
ZF=XED(5)-XPD(5)
MISS=SQR(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS
IF (MISS .LE. TOL) RETURN

C ASSIGN FINAL RELATIVE VELOCITIES
XFDOT=XED(2)-XPD(2)
YFDOT=XED(4)-XPD(4)
ZFDOT=XED(6)-XPD(6)

C COMPUTE CHANGES IN CONTROL PARAMETERS AND UPDATE
DTGO= XF/XFDOT
DDY=(DTGO*YFDOT+YF)/TGO
DDZ=(DTGO*ZFDOT+ZF)/TGO
TGO=TGO+DTGO
DELTA Y=DELTA Y+DDY
DELTA Z=DELTA Z+DDZ

C TEST FOR CONVERGENCE OF TIME-TO-GO
IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN

10 CONTINUE

C INCREMENT CONVERGENCE FLAG
CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
DELTA Y=0.0
DELTA Z=0.0
TGO=OLDTGO

END
APPENDIX E

IN-PLANE THRUST PROFILES

This appendix contains the in-plane thrust profiles for Cases I and V, showing the effect of estimate uncertainty on the various control strategies. Each profile was started with the same random seed to form a basis for comparison.
Figure E-1. In-plane thrust profile of Plan A for Case I.
Figure E-2. In-plane thrust profile of Plan B for Case I.
Figure E-3. In-plane thrust profile of Plan C for Case I.
Figure E-4. In-plane Optimum Thrust Spacing profile for Case I.
Figure E-5. In-plane thrust profile of Dual Control for Case I.
Figure E-6. In-plane thrust profile of Certainty Control for Case I.
Figure E-7. In-plane thrust profile of Truth Model for Case I.
Figure E-8. In-plane thrust profile of Plan A for Case V.
Figure E-9. In-plane thrust profile of Plan B for Case V.
Figure E-10. In-plane thrust profile of Plan C for Case V.
Figure E-11. In-plane Optimum Thrust Spacing profile for Case V.
Figure E-12. In-plane thrust profile of Dual Control for Case V.
Figure E-13. In-plane thrust profile of Certainty Control for Case V.
Figure E-14. In-plane thrust profile of Truth Model for Case V.
ABSTRACT: Terminal guidance of a hypervelocity exo-atmospheric orbital interceptor with free end-time is examined. The pursuer is constrained to lateral thrusting with the evader modeled as an ICBM in its final boost phase. Proportional navigation, optimal control using certainty equivalence, dual control, and control with optimum thrust spacing are all examined. Also, a new approach called certainty control is developed for this problem. This algorithm constrains the final state to a function of projected estimate error to reduce control energy expenditure. All methods model the trajectories using splines and employ eight state Extended Kalman Filters with line-of-sight and range updates. The relative effectiveness of these control strategies is illustrated by applying them to various intercept problems.