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**AUTHORITY**

COLLISION COURSE GUIDANCE FOR DISCRETELY MANEUVERABLE PROJECTILES

FINAL TECHNICAL REPORT

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1.0 INTRODUCTION

This report documents work performed for a Small Business Innovative Research contract that was awarded by the U.S. Army Armament, Munitions and Chemical Command. The Contracting Officers' Representative was initially Mr. Thomas Hutchings who was replaced by Mr. Grant Manning. Their assistance allowed this program to run smoothly.

1.1 Objective

The objective of this program was to develop a guidance law for a projectile that has discrete maneuver capability. This type of guidance is motivated by the Command Adjusted Trajectory (CAT) program. The CAT fire control system command guides a tank-tired projectile, which is controlled by firing small squib motors. Since there are a finite number of squibs on a projectile, the guidance law should minimize the use of divert maneuvers while achieving acceptable miss distance. The primary target of the CAT system is a helicopter; however, the fire control system must be also be capable of guiding projectiles in an anti-armor vehicle mission.

1.2 Approach

A guidance law used for exoatmospheric interceptors was adapted for this application. Some exoatmospheric interceptors have two key parameters in common with the CAT system. Both the interceptor and the CAT projectile have discrete maneuver capability and the target in both systems is potentially maneuvering. An optimization procedure was developed to determine the minimum number and size of discrete divert maneuvers that are required during the midcourse flight of an exoatmospheric interceptor. This procedure was used to establish the initial guidance configuration for the CAT application. Following the sizing work, Monte Carlo analyses of the guidance system were performed with a 6-DOF simulation of the CAT system. Measures of merit for this evaluation were miss distance and the number of squib motors used.

The first two sections of this report provide background information. A description of the CAT system is presented in the first section, followed by an overview of intercept guidance systems and guidance laws. This section includes analytical comparisons of popular intercept guidance laws and an introduction to collision course guidance. The divert maneuver sizing procedure is then summarized along with results of this procedure for the CAT application. The remaining sections cover the simulation results, conclusions and recommendations.
2.0 CAT SYSTEM DESCRIPTION

The CAT system described in this section is not necessarily the system that is being developed. During the period of performance of this program, proposals for a CAT concept and prototype development were under evaluation. The CAT system in this section is representative of several CAT concepts. Key concerns of the guidance law are a maneuvering target and discrete divert capability of the projectile.

A representation of a helicopter engagement is shown in Figure 1. Both forward looking infrared (FLIR) and laser sensors on board the tank have an operational range of 6 km. After the fire control system (FCS) establishes track of the target, an intercept point is predicted and a projectile is fired. The sensors continue to track the target and, when the projectile enters the sensors field of view, track of the projectile is established. Based on the target and projectile track information the FCS transmits guidance commands to the projectile. The only control available to the fire control system is the time and direction to fire a squib motor. Following receipt of a guidance command, the rolling projectile fires a squib when a motor is in the roll orientation specified in the guidance command. This procedure is repeated by the fire control system until shortly before intercept.

The current CAT projectile is a modified 120 mm caliber projectile. As shown in Figure 2, the squib motors are located toward the front of the projectile. There are four motors with a bank of squibs that can fire into each motor. This particular projectile has 20 squibs per bank for a total of 80 squibs.

![Figure 1. CAT Helicopter Scenario](image-url)
The kinematic capability of the projectile is shown in Figure 3. The design scenario selected for this program is the engagement of a maneuvering helicopter at 4 km. The maximum lateral acceleration of the helicopter is 1/3 g. In the horizontal plane the helicopter maneuvers in a circular pattern superimposed on a constant lateral velocity, and in the vertical direction it maneuvers in a sinusoidal motion. The periods of the horizontal and vertical motion are the same. Throughout this report, the x-y plane is horizontal with the x-axis initially pointing toward the target and the z-axis is positive vertical as shown in Figure 1. The helicopter motion in the y and z axes is given by (in meters):

\[ y = \frac{G}{3} \left( \frac{20}{2\pi} \right) \left[ T - \frac{20}{2\pi} \sin \left( \frac{2\pi T}{20} + f(y) \right) \right] \]

\[ z = 50 + 10 \sin \left( \frac{2\pi T}{20} + \phi(z) \right) \]

where:

- T is time
- \( \phi(y), \phi(z) \) are independent uniform random numbers from 0 to \( 2\pi \)}

**Figure 2. Projectile Profile**

**Figure 3. Field of Fire**
3.0 INTERCEPT GUIDANCE LAWS

The classical intercept guidance laws of pursuit and proportional navigation and their derivatives have several characteristics in common. The commands generated by these guidance laws are acceleration commands or some other parameter that is directly related to acceleration. The commands tend to operate in the continuous domain rather than the discrete and, either implicitly or explicitly, miss distance and time to go are predicted. The differences are in the assumptions on target and interceptor motion that are used in predicting miss distance and time to go.

3.1 Pursuit Guidance

In pursuit guidance the command is proportional to the difference between the current interceptor heading and the current line of sight to the target. The basic form of pursuit guidance would command acceleration, angle of attack or some other parameter to be proportional to this difference in angles.

\[ C = K \Delta \phi \]

where:
- \( C \) is the rate of change of the projectile heading
- \( a_c \) is the acceleration command
- \( V_p \) is the projectile velocity
- \( K_p \) is the pursuit guidance gain
- \( \Delta \phi \) is the angular difference between heading and line of sight
- \( T_{GO} \) is the time to go

A more advanced form would command the rate of change of projectile heading to reduce this difference in the remaining time to go.

\[ \dot{\beta} = \frac{a_c}{V_p} = \frac{K_p \Delta \phi}{T_{GO}} \]

\[ a_c = \frac{K_p \Delta \phi V_p}{T_{GO}} \]

For a small difference between heading and line of sight to the target and a small target velocity relative to the velocity of the projectile, the commanded acceleration is proportional to predicted miss distance and
inversely proportional to the square of time to go.

\[ V_p = \frac{R}{T_{GO}} \]

\[ MDX = R \Delta \phi \]

\[ a_c = K_p \frac{\Delta \phi R}{T_{GO}^2} = K_p \frac{MDX}{T_{GO}^2} \]

where:

- R is the distance to the target
- MDX is the predicted miss distance.

The main assumption in pursuit guidance is that the target velocity is small relative to the projectile velocity.

Lead pursuit guidance is a variation of pursuit guidance which accounts for target velocity. In lead pursuit guidance, the guidance command is proportional to the current projectile heading and the line of sight to the predicted intercept point. Again, the acceleration command can be shown to be proportional to the predicted miss distance and inversely proportional to the square of time to go.

\[ a_c = K_{LP} \frac{\Delta \alpha V_p}{T_{GO}} = K_{LP} \frac{\Delta \alpha R}{T_{GO}^2} \]

\[ a_c = K_{LP} \frac{MDX}{T_{GO}^2} \]

where:

- \( K_{LP} \) is the lead pursuit guidance gain
- \( \Delta \alpha \) is the angular difference between heading and the line of sight to the predicted intercept point.

Some forms of lead pursuit guidance include projectile and target accelerations in predicting miss distance. The difference between pursuit and lead pursuit guidance is the set of assumptions that are used in determining the predicted miss distance.

### 3.2 Proportional Navigation Guidance

A popular intercept guidance law is proportional navigation. As show in Figure 4, the guidance command results in the rate of change of the relative velocity being proportional to the rate of change of the relative range.

\[ \gamma = \frac{a_c}{V_p} = K_{PN} \dot{\sigma} \]
where:

\( \gamma \) is the angular rate of change of the relative velocity

\( \sigma \) is the line of sight

\( K_{PN} \) is the proportional navigation guidance gain

The following equations show that this guidance law also results in the acceleration command being proportional to the predicted miss distance and inversely proportional to the square of time to go.

\[
a_c = K_{PN} \sigma V_R
\]

\[
\dot{\sigma} = \frac{V_R \sin \psi}{R}
\]

where:

\( \psi \) is the angle between the line of sight and the relative velocity

for small \( \psi \):

\[
T_{GO} = \frac{R}{V_R}
\]

\[
a_c = K_{PN} \frac{MDX}{T_{GO}^2}
\]

While constant velocities are assumed in the original proportional navigation, a variation (predictive proportional navigation) includes accelerations in predicting miss distance and time to go. The primary difference between this and lead pursuit is that this guidance law does not assume that the projectile velocity is much greater than the target velocity.

In addition to using predicted miss distance and time to go either implicitly or explicitly, the preceding guidance laws are generally used to guide interceptors that accept and respond to continuously variable
acceleration commands. However, the projectile only has discrete maneuver capability. The projectile maneuver can be considered to be an impulse of velocity. To transform the continuous acceleration command guidance laws to a discrete command, the equivalent commanded change in velocity can be determined. These continuous acceleration command guidance laws assume that an acceleration will be achieved over the duration of a guidance update interval, which leads to a method of transforming the continuous guidance law to a discrete form. (Guidance update interval is the inverse of the frequency that guidance commands are computed or updated.) As shown in Figure 5, the desired divert velocity (change in velocity) is determined to be the product of the commanded acceleration and the guidance update interval.

\[
\Delta V_D = a_c \Delta t_G
\]

\[
\Delta V_D = K \frac{MDX}{T_{GO}} \Delta t_G
\]

where:

- \( \Delta V_D \) is the desired change in velocity
- \( \Delta t_G \) is the guidance update interval

A version of this type of guidance law would command a squib to be fired when the desired divert velocity is greater than or equal to the divert velocity which is achieved by firing a squib.

### 3.3 Collision Course Guidance

Collision course guidance is a discrete guidance law that has been used for guidance of exoatmospheric interceptors with discrete maneuver capability. The only control available to the guidance system was the time of motor ignition and the direction to fire the motor. The first application of this guidance law was for an interceptor with a single fixed burn motor. Collision course derives this name from the criterion used to determine the time of motor ignition. The time and direction of motor ignition are selected so that at the end of the motor burn the interceptor is on a collision course with the predicted target position. This is shown graphically in Figure 6.
More recently this form of guidance has been proposed for the midcourse flight of exoatmospheric kill vehicles. These kill vehicle systems have two qualities that are similar to the CAT system. The kill vehicle has discrete maneuver capability during midcourse and the targets capability to maneuver dominates the uncertainty of predicting the target position.

Typically, collision course guidance has a much higher gain than the acceleration guidance laws. This can be observed by comparing the desired divert velocities generated by the guidance laws. The desired divert velocity for collision course is:

\[
\Delta V_{cc} = \frac{MDX}{T_G}
\]

and from Figure 5 the desired divert velocity for the continuous acceleration guidance laws is:

\[
\Delta V_a = K \frac{MDX}{T_G^2} \Delta T_G
\]

Therefore, the ratio of desired divert velocities is:

\[
\frac{\Delta V_a}{\Delta V_{cc}} = K \frac{\Delta T_G}{T_G}
\]

A typical guidance gain (K) for proportional navigation guidance is in the range of 3 to 4. The ratio of desired divert velocities is shown in Figure 7 as a function of time to go for a guidance gain of 4. This shows that the desired divert velocity as determined by the acceleration type guidance laws is much less for any time to go that is greater than about 1.0 second. For guidance systems that use proportional navigation, use of control energy or fuel is increased as the guidance gain is increased because the guidance system responds to the noise. If this is true then collision course guidance might be expected to use more control energy than a discrete adaptation of proportional navigation. However, if the uncertainty in predicting the target position is dominated by the maneuver uncertainty of the target rather than noise, then collision course guidance may
use less control energy by recognizing and responding to the maneuver sooner and, therefore, making better use of the time to go lever arm. This is the hypothesis that is tested in the subsequent sections.

As discussed in this section all intercept guidance laws predict miss distance and time to go. The more advanced guidance laws have the better predictors. A block diagram of a command guidance system is shown in Figure 8. In this diagram the guidance law is divided into two components: an intercept predictor and a command generator. Although state of the art target and projectile predictors are used, the emphasis of this guidance law development program is to develop the command generator which transforms predicted miss distance and time to go into guidance commands.

Figure 7. Relative Gain of Guidance Laws

Figure 8. Command Guidance System
4.0 SIMULATION

Before continuing with the development of the guidance law, a brief description of the Monte Carlo simulation that was used to evaluate the guidance laws is presented here. The structure of this event-based simulation is similar to the block diagram in Figure 8 and includes a 6 degree-of-freedom (DOF) model of the projectile. A sample size of 50 replications was used for the Monte Carlo results. The target model can either fly the circular pattern previously discussed or a tabular acceleration history. The sensor is a functional model of an FLIR and laser. To simplify filter processing the sensor measurements from the two sensors are synchronized and operate at 20 hz. The target filter is a nine-state Kalman filter: three position, three velocity and three acceleration states. The dynamic or plant acceleration noise in the filter is assumed to be correlated with a first order lag that has a time constant of 2 seconds. An extended Kalman filter was used for estimating projectile states. The projectile model used in the filter includes provisions for ballistic dispersion which is the out-of-plane motion that results from the interaction of gravity and aerodynamic forces on a spinning projectile. A maximum error (no noise) of 2 meters is achieved by the projectile model over the field of fire.

An example of the target filter performance is contained in Appendix A and projectile filter performance is contained in Appendix B. The target filter statistics indicate that it takes several seconds for the filter to reach steady state. Therefore, the fire control system did not fire the projectile until 3 seconds after track initiation of the target. The projectile was assumed to enter the field of view of the sensors at one-half second after firing. To allow several measurements to be processed by the projectile filter before guidance commands were computed, guidance calculations were not initiated until 0.75 seconds after firing the projectile.

5.0 DIVERT SIZING

To establish the divert required for exoatmospheric kill vehicles (KVs) during midcourse, a procedure was developed to determine the minimum divert capability required of the KV. Divert capability is specified as the number and size of divert maneuvers. This procedure was applied to the CAT projectile and the results are presented in this section. The intent is not to redesign the CAT projectile. Rather, the intent is to determine the best use of the CAT projectile maneuver capability. Divert capability is specified as an incremental velocity. In the next section divert velocity is translated into a parameter that is more meaningful to the CAT guidance system.

The midcourse flight of a kill vehicle is similar to the CAT projectile. The purpose of guidance during midcourse is to steer the kill vehicle to a place where the seeker of the KV can acquire the target and then allow the homing kill vehicle to steer out the remaining miss distance. Statistics of predicted miss distance
uncertainty is a monotonically decreasing function of time. If the last midcourse correction occurs after the
statistics of miss distance have reached the allowable maximum, then the achieved miss distance will be
within the allowed limit without further corrections by the homing vehicle. For the CAT projectile this
simply means that the last squib should be fired shortly before intercept and the entire flight of the projectile
is similar to the midcourse portion of an exoatmospheric kill vehicle and the divert sizing procedure
developed for these kill vehicles should be applicable to the CAT projectile. For purposes of sizing the divert
maneuver, the last maneuver is scheduled at 0.25 seconds before intercept.

Uncertainties in predicting the intercept point contribute to either miss distance, control fuel use or both.
There are several guidance system errors indicated in Figure 8 that contribute to predicting the intercept
point. These include target and interceptor state estimation errors and dynamic errors. Projectile dynamic
errors are differences between the true projectile and the model in the filter and predictor. These errors
include aerodynamic parameter variations, aerodynamic disturbances, and projectile responses to guidance
commands that are not included in the model. The major contributor to control fuel use is the capability of
the target to maneuver. The other errors contribute primarily to miss distance rather than control fuel use.

There are two parts to the divert sizing procedure. First, the divert velocity required to steer out the target
maneuver is determined. Then, the divert velocity required to steer out state estimation errors is determined.
These components are then combined. The procedure will be shown for a 4 km engagement of a maneuvering
helicopter that has a 1/3 g acceleration capability. The time of flight for a 4 km engagement is approximately
5.5 seconds. Although the helicopter maneuver in the CAT design scenario is a circular pattern, the divert
sizing will be shown for a target accelerating in a linear path normal to the line of sight. The guidance law
and thresholds resulting from this sizing procedure will be evaluated in the Monte Carlo simulation against
the target with the circular motion. The linearly accelerating target is more demanding of the divert system
than a circular motion because of lags in the guidance loop. With the target maneuvering in a sinusoidal or
circular path, sometimes the target accelerates in a direction that decreases miss distance.

5.1 Divert Sizing for Maneuver Uncertainty

To determine the divert required to overcome the target maneuver, perfect knowledge of the target position
and velocity at the time of prediction is assumed. No knowledge of the current or future target acceleration
is assumed. This is a no noise scenario with no projectile dynamic errors. The objective of the divert sizing
procedure is to determine the minimum divert velocity required to steer out the predicted miss distance with
the constraint that the projectile has only impulsive maneuver capability.
The problem is to minimize the total divert given by the following equation.

\[ \Delta V = \sum_{i=1}^{N} \Delta V_i \]

where:
- \( \Delta V \) is the total divert velocity
- \( N \) is the number of impulses available
- \( \Delta V_i \) is the divert velocity of the "ith" impulse

\[ \Delta V_i = \frac{MDX_{i+1} - MDX_i}{T_{i+1} - t_i} \]

where:
- \( MDX_i \) is the predicted miss distance at time \( t_i \)
- \( T_i \) is the time of intercept
- \( t_i \) is the time of the "ith" impulse

A gradient method is used to find the solution.

A sensitivity of total divert velocity to the number of impulses is shown in Figure 9. The results in this figure indicate that the minimum total divert velocity is achieved as the number of impulses approaches infinity. This is the expected result for no noise in the guidance system. The minimum total divert velocity is achieved when the predicted miss distance is steered out as soon as it is recognized. The minimum divert velocity would be achieved by matching the acceleration of the target. For a 1/3 g target and a time of flight of 5.5 seconds, the minimum required divert velocity is 18 m/sec. The divert velocity of the individual impulses may be obtained by dividing the total divert velocity by the number of impulses.

![Figure 9. Divert Velocity Required for Maneuver Uncertainty](image-url)
5.2 Divert Sizing for Estimation Errors

The following paragraphs develop the divert velocity required by impulsive maneuvers to steer out miss distance due to estimation errors. For this analysis there is an assumption that the dominant noise in predicting miss distance for the guidance system is the uncertainty associated with estimating the target states. To simplify the problem the solution is one-dimensional.

If the estimates of position and velocity are highly correlated then the uncertainty of the estimate of predicted miss distance may be determined from the following equation.

\[ \sigma_{MDX} = \sigma_p + \sigma_v T_{GC} \]

where:
- \( \sigma_p \) and \( \sigma_v \) are the standard deviation of estimated position and velocity
- \( \sigma_{MDX} \) is the standard deviation of the predicted miss distance

Since the position and velocity estimates are the outputs of a filtering process, the statistics of the difference in predicted miss distance determined at two time points is: (Reference)

\[ \sigma_{\Delta MDX_i} = \sigma_{MDX_{t1}} - \sigma_{MDX_{t2}} \]

where:
- \( \sigma_{\Delta MDX_i} \) is the standard deviation of the difference in predicted miss distance

The statistic of impulsive velocity required to respond to the change in predicted miss distance from one time to another is then:

\[ \sigma_{\Delta V_i} = \frac{\sigma_{\Delta MDX}}{T_{CO_i}} \]

The standard deviation of total divert velocity given by,

\[ \sigma_{\Delta V_T} = \sum_{i=1}^{N} \sigma_{\Delta V_i} \]

can be minimized by iterating on \( \sigma_{\Delta V_i} \).

For the CAT system the miss distance direction is approximately normal to the line of sight to the target. Therefore, sensor measurement errors in the miss distance direction are the angular measurements of the FLIR. At 4 km the 0.1 mrad accuracy of the FLIR translates into a measurement accuracy of 0.4 meter. For this measurement accuracy operating at 20 hz and a target acceleration uncertainty of 1/3 g, the steady state estimates of position and velocity are 0.1 meters (1σ) and 0.4 m. sec (1σ). Although this portion of the analysis assumes a non-maneuvering target, this assumption can not be used when estimating the performance of the filter.

This procedure was used to generate the sensitivity of required divert velocity to number of impulses that is shown in Figure 10. The results indicate that more total divert velocity is required for the smaller individual impulses (larger number of impulses) than for larger individual impulses. The individual impulse is like a dead zone. A control system with a small dead zone will consume more fuel than one with a large dead zone by responding to noise.

5.3 Total Divert Requirement

Figure 11 contains the sum of Figures 9 and 10 to yield the total required divert velocity as a function of the number of impulses. The sensitivity of the total required divert velocity to the impulsive correction that is shown in Figure 12 is derived from Figure 11 by dividing the total divert velocity by the number of impulses. These figures indicate that the minimum divert velocity is required when there are approximately 10 maneuvers of 3 m/sec. If the projectile had this maneuver capability then the procedure for implementing this into a collision course guidance law is: 1) predict miss distance and time to go, 2) determine required
divert velocity, and 3) command a maneuver when the required divert velocity is equal to or greater than the threshold of 3 m/sec.

### 6.0 PROJECTILE MANEUVER CAPABILITY

This section addresses two aspects of the projectile response to the firing of a squib motor. First, the previous section stated the desired maneuver capability as an incremental velocity which is not a convenient parameter for a non-powered atmospheric projectile. This is translated into an equivalent divert velocity of the projectile. Secondly, the dynamic response decreases the efficiency of sequential motor firings. This can improved by proper timing of the sequential firings.
A convenient parameter to describe projectile divert capability is the change in flight path angle which is shown in Figure 13. Makeup capability may be characterized by the product of this change in flight path angle and the range to the predicted intercept point. A pseudo divert velocity defined in the following equation is shown in Figure 14.

\[ \Delta V = \frac{\Delta y R_{\infty}}{T_{\infty}} \]
where:

\[ \Delta V \] is the pseudo projectile divert velocity per squib

\[ \Delta y \] is the change in flight path angle per squib

\[ R_0 \] is the range to go

Combining the results of Figure 12 and Figure 14, the desired maneuver increment to implement collision course guidance is a threshold of 6 squibs.

6.2 Efficiency of Sequential Firings

Since the projectile spins, the firing of a squib motor results in a damped mutation and precession motion. As shown by the angle of attack response in Figures 15 and 16, the decay time for this motion is approximately 0.5 seconds. The nutation and precession motion can be seen more easily in the cross plot of pitch and yaw angle of attack in Figure 17. The divert of the projectile is achieved by the impulse of the

![Figure 5. Pitch Angle of Attack](image)

![Figure 16. Yaw Angle of Attack](image)
motor which is amplified by jet interaction and the net aerodynamic lift. Clearly, with a decay time that is on the order of 0.5 seconds, commands sent to the projectile more frequently than 0.5 seconds will result in the motion interacting. These interactions do not affect the guidance system, provided the net divert is proportional to the number of squibs that are fired. However, the efficiency of sequential commands is sensitive to the time interval between squib firings. The response of the projectile can be viewed as the impulse response of a damped second order system. Maximum response is achieved when the system is pulsed at its natural frequency. With proper timing of squib motor firings the net divert can achieve near unity efficiency.

Sequential squib motor firings have a second degrading effect on the net divert. An out of plane divert is induced. This effect can be diminished by proper phasing of the sequential squib firing. That is, by slightly modifying the desired firing direction, the net out of plane divert can be reduced.

To achieve near unity efficiency of sequential squib motor firings, the interval between motor firings and phasing need to be controlled. Since the projectile roll rate and velocity vary considerably over the field of fire (see Figures 18 and 19), firing interval and angle need to be functions of time of flight or downrange. Time of flight was chosen as the independent variable. Sequential firing interval is shown in Figure 20 and sequential firing angle is shown in Figure 21. Note that the firing phase angle is a function of the number of sequential squibs to be fired. The guidance law determines how many squibs to fire. Then the interval
Figure 18. Projectile Roll Rate

Figure 19. Projectile Velocity

Figure 20. Sequential Firing Interval
between transmission of sequential guidance commands is determined from Figure 20 and the direction to fire sequential squibs is incremented by the angle from Figure 21.

7.0 COLLISION COURSE GUIDANCE PROCEDURE

The real time inputs to the guidance law are predicted miss distance, predicted time to go, and estimates of current projectile position and time of flight. Data base parameters that are a function of the time of flight include: 1) the achieved change in flight path angle per squib, 2) the sequential firing interval, and 3) the total firing phase. Other data base input parameters are time before intercept of last maneuver and the guidance threshold \( N_{th} \), which is the number of squib motors to be fired sequentially. The guidance law commands sequential firing of \( N_{th} \) squibs when guidance determines that at least \( N_{th} \) squibs are required to steer out the predicted miss distance.

The following steps outline the procedure that was used in the simulation.

1) Predict miss distance (MDX) and time to go \( (T_{go}) \).
2) Determine distance (R) between predicted intercept point and current projectile position.
3) Compute change in flight path angle \( (\Delta \gamma_R) \) required to hit the target.

\[
\Delta \gamma_R = \frac{MDX}{R}
\]
4) Determine change in flight path angle ($\Delta \gamma_y$) achieved by firing one squib from tabular data of Figure 13.

5) Determine number of squibs required to steer out predicted miss distance.

\[ R_N = \frac{\Delta \gamma_R}{\Delta \gamma_A} \]

6) If this number of required squibs is greater than the guidance threshold then command $N_e$ ($R_N$ rounded to the nearest integer) squibs to be fired sequentially.

\[ \text{If } R_N > N_h \text{ then } N_e = R_N + 0.5 \]
\[ \text{otherwise } N_e = 0 \]

7) If the time to go is less than the desired time of last divert maneuver, then command the integer of $R_N$ squibs to be fired.

8) When more than one squib is commanded to be fired, sequential guidance commands are transmitted to the projectile. The interval between the sequential commands is determined from tabular data of Figure 20. The roll direction to fire, specified in each successive command, is advanced by the sequential phase as determined from tabular data of Figure 21. The direction of roll advancement is counterclockwise as observed along the line of sight to the projectile.

9) If a command is issued, then wait 0.5 seconds before determining a new guidance command, otherwise, guidance computations are performed at a rate of 20 hz.

### 8.0 SIMULATION RESULTS

In this section results of the guidance system performance evaluations are presented. The measures of merit are miss distance and number of squibs used. Since the Monte Carlo sample size was 50, the 90 percent probability of these two parameters is presented. Evaluations include a comparison of collision course guidance and proportional navigation. In collision course guidance the independent variable is the threshold of number of squibs fired. For proportional navigation the discrete version that is outlined in Figure 5 is used and the independent variable is guidance gain.

Since the sizing study for establishing the threshold (number of squibs fired sequentially) for collision course guidance was performed for a linearly accelerating target, the first evaluation is for this scenario. The target begins accelerating at the time of projectile firing. Analytical results in previous sections indicated that the minimum squib usage should occur for a threshold of six squibs and, indeed, the squib usage in Figure 22 exhibits a slight minimum in this region. In Figure 12 the minimum required divert velocity (3s) is 29 m/
sec, while the 47 squib minimum in Figure 22 translates into 23.5 m/sec (90%). This assumes an average of 0.5 m/sec pseudo divert velocity per squib for the 4 km engagement (Figure 14). The desired maximum miss distance for the intercept of a helicopter is in the range of 2 to 4 meters and Figure 23 indicates that the achieved miss distance is well within this range.

Squib usage for the circular motion scenario is shown in Figure 24. This sensitivity shows that approximately equal performance is achieved over the threshold range of 2 to 6 squibs. As discussed previously, fewer squibs should be required against this target than against the linearly accelerating target. These results confirm this expectation. Miss distance achieved for this scenario is well within the required maximum as shown in Figure 25.
The next results are for predictive proportional navigation guidance. The same methods for predicting miss distance and time to go and for sending sequential commands were used for these results as for the rest of this section. Steps 2 through 6 of the collision course guidance procedure were replaced by the discrete version of proportional navigation found in Figure 5. A sensitivity of squib usage to guidance gain is shown in Figure 26. In order to perform this evaluation, the number of available squibs was increased to 100 (25 per bank). As expected, squib usage is increased as the guidance gain increases. The results also show a significant increase in squib usage over collision course guidance. Although miss distance is within the required maximum, there is an increase over collision course as shown in Figure 27.

A variable threshold was investigated for collision course and no improvement over the constant threshold was found for the 4 km engagement. However, preliminary evaluations of the 6 km engagement indicate
there is a decrease in squib usage by increasing the threshold early in flight. A decrease in squib usage (90%) from 97 to 85 squibs was observed by increasing the threshold by 25 percent early in flight. The reason a variable threshold should be more effective for the 6 km engagement is found in Figure 14. Pseudo divert velocity varies a few percent during the engagement of a target at 4 km; however, it increases by 60 percent during a 6 km engagement.

A maximum miss distance of one meter is desired against an armored vehicle. For a stationary target at 4 km an average of 11 squibs were used and a 0.65 meter (90%) miss distance was achieved. For slowly accelerating ground targets, adequate miss distance performance should be easily achievable.
9.0 CONCLUSIONS AND RECOMMENDATIONS

In this report, good correlation between analytical and Monte Carlo results are presented. Collision course guidance is shown to be superior to predictive proportional navigation for the CAT application. Collision course guidance yields near minimum squib (control fuel) usage while achieving acceptable miss distance.

Although this program did not address implementation issues, neither processing nor data base storage requirements are a concern. Target and projectile track filters along with the intercept predictor overshadow requirements for command computation. If collision course is selected for the CAT system, further analyses are required. Other engagement ranges should be evaluated. The engagement at 6 km is particularly interesting, because this one should drive the divert requirements of the projectile.

Finally, the limited evaluation of the 6 km engagement indicated that the divert capability of the projectile in the study is marginal and the squib impulse is smaller than is required. If only one recommendation could be made, it would be to increase the size of the squib impulse and decrease the number of them. For example, 30 squibs that are three times the current individual impulse appear to be better than the current 80 squibs.
APPENDIX A:
TARGET FILTER PERFORMANCE

This appendix contains target filter performance for a 4 km engagement. Measurement errors were 0.1 mrad (1σ) in angle and 1 meter (1σ) in range.
Figure A1. Target Position Estimation Errors - Range

Figure A2. Target Position Estimation Errors - Crossrange

Figure A3. Target Position Estimation Errors - Vertical
Figure A4. Target Velocity Estimation Errors - Range

Figure A5. Target Velocity Estimation Errors - Crossrange

Figure A6. Target Velocity Estimation Errors - Vertical
This appendix contains projectile filter performance for a 4 km engagement. Measurement errors were 0.1 mrad (1σ) in angle and 1 meter (1σ) in range.
Figure B1. Projectile Position Estimation Errors - Range

Figure B2. Projectile Position Estimation Errors - Crossrange

Figure B3. Projectile Position Estimation Errors - Vertical
Figure B4. Projectile Velocity Estimation Errors - Range

Figure B5. Projectile Velocity Estimation Errors - Crossrange

Figure B6. Projectile Velocity Estimation Errors - Vertical