SCALE DRIFT IN ON-LINE CALIBRATION

Martha L. Stocking

This research was sponsored in part by the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-83-K-0457

Contract Authority Identification No.
NR 150-520

Frederic M. Lord, Principal Investigator

Educational Testing Service
Princeton, New Jersey

May 1988

Reproduction in whole or in part is permitted
for any purpose of the United States Government.

Approved for public release; distribution unlimited.
Recent advances in psychometrics and computer technology encourage the development of model-based methods of individualized testing on a microcomputer, where each examinee receives short tests and the number of pretest items that can be administered is severely restricted. On-line (i.e., data is collected on operational equipment) methods for calibrating pretest items in this setting face new challenges. In the context of adaptive testing, two LOGIST-based methods of on-line calibration were developed and are described. These two methods are tried out and compared in a simulation study using data for the CAT-ASVAB.
Scale Drift in On-Line Calibration

Martha L. Stocking

Educational Testing Service

Princeton, New Jersey

May 1988
Abstract

Recent advances in psychometrics and computer technology encourage the development of model-based methods of individualized testing on a microcomputer, where each examinee receives short tests and the number of pretest items that can be administered is severely restricted. On-line (i.e., data is collected on operational equipment) methods for calibrating pretest items in this setting face new challenges. In the context of adaptive testing, two LOGIST-based methods of on-line calibration were developed and are described. These two methods are tried out and compared in a simulation study using data for the CAT-ASVAB.

Keywords: Adaptive testing
Item pools
IRT scales
Calibration
Introduction

Conventional mode of testing, where every examinee takes a single test form (or parallel versions of it) in large administrations using paper and pencil as the delivery and recording medium, belong to the past. Recent advances in both psychometrics and computer technology seem to point in the direction of model-based methods of individualized testing on a microcomputer, where each examinee receives short tests and testing is generally done on a demand basis. Much has been written about adaptive testing in this latter context. But other new methods of testing, for example model-based mastery testing or work-sample testing, may also be designed as short tests administered by computer to small samples of examinees.

Traditional methods of estimating parameters for items, particularly model-based parameters as opposed to conventional psychometric parameters such as proportions-correct, depend upon relatively large numbers of examinees taking relatively large numbers of items. This is especially true for the more frequently used complex models, such as the 3-parameter logistic model. Model-based testing requires, moreover, item parameter estimates in order to design tests. The challenge is to develop new methods of item parameter estimation (calibration) that capitalize on
computerized testing modes without destroying the advantages of short computerized tests. These methods are generally referred to as 'on-line' calibration methods because the data collected for calibration of new items comes from the administration of these items on operational equipment as part of an examinee's testing session.

In some contexts, the acquisition of sufficient numbers of examinees for adequate parameter estimation in on-line calibration may be problematical. In these situations, it may be possible to accumulate a reasonable number of examinees over a realistic time frame. If this is not feasible, then on-line calibration methods that are more optimal for this situation than those described here need to be developed.

Even if the number of examinees is sufficient, a central problem, in any application of on-line calibration, is how to obtain adequate calibrations of pretest items when both the length of the computerized test is very short and the number of extra items that can be administered in pretest mode to an examinee is severely restricted. The current context of the particular methods of on-line calibration reported here is adaptive testing for the Armed Services Vocational Aptitude Battery (ASVAB). The results, however, are more general and apply to other modes of
computerized testing that rely on recent advances in psychometric theory.

Remaining sections of this paper will describe the current project and its design, two methods of on-line calibration based on the estimation procedures in the LOGIST computer program (Wingersky, 1983) and the efficacy of these methods in the context of adaptive testing, and how these and other similar on-line calibration methods might be improved in the context of adaptive testing as well as other types of computerized testing.

The Current Project

A number of agencies of the Department of Defense funded a three-year project to develop and evaluate different methods of on-line calibration for the computerized adaptive Armed Services Vocational Aptitude Battery (CAT-ASVAB). Although exploration of general classes of item calibration methods was encouraged in this context, final results were constrained to be parameterized as the 3-parameter logistic model of item response functions. The final phase of this project consisted of a series of 'high-fidelity' simulations of adaptive testing and item pool refreshment. The simulations are called 'high-fidelity' for two reasons. First, the true item response functions and ability densities, both developed by Levine (1987) are nonparametric, and therefore more
realistic than more restrictive parametric models. Second, the study design represents a realistic scenario for on-line calibrations for the adaptive ASVAB as well as many other types of computerized tests. Two LOGIST-based methods of on-line calibration were investigated. As part of the same project, Darrell Bock (1987), Michael Levine (1987), and Fumiko Samejima (1987) investigated other methods of on-line calibration.

The results presented here provide some general guidance for effective computerized test item pool construction and maintenance. In addition, one LOGIST-based method is superior to the other in terms of statistical properties. However, the superior method is more expensive to implement in terms of the numbers of items required, and either the numbers of examinees required or the numbers of items per examinee.

The Study Design

Figure 1 displays the overall design of the study. Although four experimenters participated, only the details of the LOGIST-based methods indicated on the right side of Figure 1 will be discussed in detail below. Using data provided by Vale (Prestwood, Vale, Massey and Welsh, 1985), nonparametric item response functions and ability densities were produced by Levine (1987) to serve as true item response functions and ability
densities. Also using Vale's data, LOGIST was run to provide parameter estimates for subsequent use. Items were selected for an initial pool and calibrated by each experimenter. Then four rounds adaptive test simulations and item pool refreshment were conducted.

The Original Data (Block 1 of Figure 1)

Vale, et al., developed over 2000 experimental items to be considered as candidates for the original ASVAB adaptive testing item pool. A subset of these items, developed for the Word Knowledge (WK) subtest, was used in this study. The WK subtest consists of a single item type, synonyms, and is designed to measure the understanding of words typically used in social studies and everyday life, human relationships, science and nature, and arts and humanities (Prestwood, et al., 1985). The focus of the Vale item development effort was to write and calibrate similar items that spanned a wider range of difficulty than those found in operational use.

Vale developed a total of 258 such WK items. For the purpose of obtaining item parameter estimates he obtained a sample of \( N = 8171 \) candidates for military service from Military Entrance Processing Stations who had also taken the conventional ASVAB test battery. This set of data, that is, the responses of 8171
individuals to the 'experimental' Word Knowledge items as well as the conventional ASVAB, forms the basis of the current study.

A calibration of the experimental WK items using the computer program LOGIST was performed. This calibration is noted in the top block in Figure 1. Table 1 shows summary statistics for the item parameter estimates obtained from LOGIST. The items were designed to be different from a conventional test aimed at the average examinee; the purpose was to obtain items for an adaptive test item pool. The items are supposed to be very discriminating, with low guessing parameters, and span a wide range of difficulty. As seen in Table 1, the items are indeed more discriminating than is customarily seen. Although the items span a wide range of difficulty, there are more easier items relative to the number of harder items.

-----------------------------
Insert Figure 1 and Table 1 about here
-----------------------------

Summary statistics for the ability estimates of the calibration sample are also shown in Table 1. Although this sample of people was the only one available for the purpose of calibrating these experimental items, the sample may not be completely appropriate. Examinees were informed that their scores
on the experimental items did not count. It is therefore possible that the motivation of individuals was not the same as motivation presumably will be in the intended population.

The Definition of Truth (Block 1 of Figure 1)

Using the data provided by Vale, Levine produced nonparametric item response functions and ability densities to be used as true item response functions and ability densities in this study. Figure 2 shows the true item response functions for some typical items. In general, these true item response functions are nonmonotonic.

-----------------------------
Insert Figure 2 about here
-----------------------------

In Vale's data collection design, no examinee was administered all 258 experimental items. Rather, the items were broken up into three blocks of 86 items that were roughly parallel to each other. Each block was administered to a subset of examinees. Therefore Levine produced three nonparametric ability densities. To produce a description of a single ability density, Davis (1987) averaged the three densities, and then interpolated, integrated, and normalized the resulting function. All simulated examinees (simulees) are drawn from the resulting distribution.
Initial Item Selection (Block 2 of Figure 1)

An initial set of 100 items was drawn by Davis from the 258 experimental items to form the first adaptive test item pool. Research has shown (Hulin, Drasgow and Parsons, 1983) that item pools much larger than this are not necessary for short adaptive tests. To select the 100 items, Davis used the LOGIST parameter estimates summarized in Table 1 to compute a table of items sorted by their estimated information functions at various levels of ability. Items yielding successively decreasing amounts of information were selected until 100 unique items were obtained.

There were two constraints on the process used by Davis. First, 10 items that were judged to be excessively non-3PL in shape on the basis of observed data were eliminated from the 258 before selection was done. These non-3PL items exhibited either severe nonmonotonicities or broad plateaus in the mid-range of ability. Second, the ability metric was divided into intervals of 1.0 between -3.0 and +3.0, and the numbers of items selected from each of the two most extreme intervals was constrained to be no more that 10, while 20 items were selected from each of the remaining intervals. This latter constraint was to control for the fact that there are proportionally too many easy items in Vale’s original item set.
The Simulated Initial Calibration (Block 2 of Figure 1)

Up to this point in the study design, Vale's WK data have been used to develop a definition of truth for the simulations, and to provide estimated parameters for the purpose of selecting the initial 100 item pool. But these parameter estimates are no longer appropriate for subsequent steps. They represent 3PL estimates of the true item response functions that are hypothesized to underlie the responses collected by Vale from live examinees. These item response functions are not the same as those generated by Levine to be used as the (non-3PL) definition of truth for this study. Instead, it is necessary to obtain 3PL parameter estimates from data where Levine's true item response functions generate responses to items from simulees. This step is a simulated initial calibration of the 100 item pool.

Davis divided the 100 items into four 25-item subsets, each of which had approximately the same estimated test information function based on LOGIST parameter estimates. Each of N = 6000 simulees was administered two subsets of 25 items in an overlapping design that provided 3000 simulee responses per item. Responses to the items were generated using the true (non-3PL) item response functions and simulee abilities. LOGIST was run on these data to provide estimated item parameters for the 100 items.
in the Round 0 adaptive test item pool. These parameters estimates were then returned to Davis for the next step in the simulation study.

A Typical Round in the Simulation Study (Blocks 3 through 6, Figure 1)

A typical Round in the simulation study consisted of the simulation of an adaptive test (by Davis), the selection and seeding of candidate new items for the adaptive test item pool (by Davis), the identification of items to be removed (by Davis), and the calibration and selection of new items to be included in the next Round of simulations (by individual experimenters). Davis performed his functions separately by experimenter, and, for the results reported here, separately by the two LOGIST-based methods of on-line calibration. Thus the items in the pool, the item parameter estimates, and adaptive test simulations may differ for each experimental method at each Round. This process was repeated for a total of four Rounds.

For each Round, Davis simulated the administration of a 15-item fixed-length adaptive test to a sample of N = 15,000 simulees drawn at random from the composite ability density. The same sample of simulees was used for each experimental method within a Round. Owen's (1975) Bayesian procedure was used to update
ability estimates during the adaptive test. The next item to be administered during the test was chosen to be that item that was most informative at the estimated ability, except for the imposition of efforts to control item exposure. 'Exposure' parameters controlled the probabilities with which an optimally-selected item was actually administered to a simulee. For the first item, this probability was .2, for the second item the probability was .25, for the third it was .33, for the fourth it was .5, and for the fifth through the fifteenth, this probability was 1.0.

Item usage data was collected by Davis for each item, and accumulated across each Round in the simulations. At each Round, the 25 most used items of the current 100 item pool were identified as items that must be replaced for the next Round. In addition, for each Round, Davis identified a pool of candidate 'new' items by randomly selecting 50 items from the full set of Vale's original 258 items. Within a Round, the same 50-item set of candidate new items was used for each experimental Method. Response data were collected for these items by administering each simulee 5 randomly selected items from this set of 50. On average, each new item had about 1500 responses to be used in the subsequent estimation of item parameters by each experimenter.
Responses to these new items played no role in item selection or ability estimation during the course of the adaptive test.

The Final 'Half' Round (Block 7 of Figure 1)

At the end of four Rounds, the original 100-item pool has been 'refreshed' four times. In order to examine the cumulative effects of these four Rounds on simulee ability, Davis conducted a final half-Round, consisting of just the simulation of an adaptive test using this final item pool.

Two LOGIST-based Methods of On-line Calibration

A Note on Methods That Do NOT Work

Lord (1984) described a straightforward approach to on-line calibration that did not work. Since the logic underlying this approach is so appealing, it is instructive to examine it. The basic idea of this approach was to simultaneously calibrate the candidate new items and recalibrate the entire adaptive test item pool. Since the results of such a calibration would not be on the same scale as the original item pool, it would be necessary to determine a transformation of these results to that scale. Using the relationship between the item parameter reestimates for the adaptive test pool and the original item parameter estimates for these same items, a suitable scaling transformation could be determined by minimizing the difference between the test
characteristic curves for the two different sets of estimates, as in Stocking and Lord (1983). After such a transformation had been applied, the calibration results for the candidate new items would also be on the scale of the adaptive test item pool. This basic plan of simultaneous calibration of candidate new items and the current adaptive test item pool, along with a transformation, would then be repeated for each Round of the simulation study.

A problem with this approach is that some of the items in the adaptive test item pool may not have been administered frequently enough to provide adequate data for subsequent recalibration. To solve this problem, those items that were infrequently used in the adaptive test were administered nonadaptively to a sufficient number of examinees for adequate calibration. Thus, in Lord’s design, there were three types of items to be calibrated, distinguishable by the nature of the data available for calibration purposes. The first type consisted of candidate new items for which only nonadaptive responses were collected. The second type consisted of items already in the pool, but whose response rate was sufficiently low in the adaptive test that some nonadaptive administrations had been performed to increase the number of responses per item. The third type consisted of items already in the pool, but whose response rate was sufficiently high
that only adaptive test responses were required for adequate calibration.

The third type of item, that is, those items for which only adaptive responses were obtained, causes severe problems when attempting to estimate parameters for the 3PL. Since their response rate is high in the adaptive test, these are the more discriminating items in the pool. The more discriminating an item, the greater the change in the probability of a correct answer for small changes in ability close to the item difficulty. Since the adaptive test works well, these items are usually administered to simulees with very similar levels of ability. Thus the distribution of ability for those simulees administered this kind of item becomes more concentrated the better the adaptive test works. These highly discriminating items divide this concentrated ability distribution into two classes: those simulees whose ability lies slightly below the item difficulty and therefore have only a small probability of responding correctly, and those simulees whose ability lies slightly above the item difficulty and therefore have a high probability of responding correctly. The observed data from which item parameters are to be estimated are the actual item responses from the simulees. These data also tend to be divided into two classes by the highly
discriminating item -- those who respond correctly and those who respond incorrectly. These responses are not sufficient to estimate item discrimination or the lower asymptote. This situation is illustrated for a few items in Figure 3 (taken from Lord, 1984), where the boxes represent observed proportions correct for particular ability groups and are plotted proportional in size to the number of cases in the group. In the extreme case, for an infinitely long adaptive test in which the estimated ability becomes indistinguishable from the true ability, the response data from which three item parameters must be estimated collapses into a single point located at the item difficulty, with the observed proportion of correct responses halfway between chance and 1.0.

Insert Figure 3 about here

Calibration procedures such as LOGIST, that do not utilize other information that might be available about the item parameters, cannot perform adequately on items such as these for which only adaptive responses are available. Mislevy (personal communication, 1987) conjectures that any other currently available estimation procedure would encounter similar
difficulties in trying to estimate 2PL or 3PL parameters from these adaptive responses, unless these adaptive responses were augmented by additional information, such as prior distributions on item parameters or responses from examinees over a broader range of ability.

The solution is not to discard the items in this set from the calibration and simply proceed with the two remaining sets of items. This is equivalent to discarding the better (more discriminating) items available and using only the poorer (less discriminating) items in the pool as a scale anchor. Instead it appears that one solution could lie in the direction of avoiding the direct use of adaptive test responses by summarizing the information available from the adaptive test when estimating parameters for the new items. Both methods described below move in this direction.

Method A

For this method, adaptive test item responses and parameter estimates for items already in the adaptive test item pool are used to compute a maximum likelihood estimate of simulee ability. This estimated ability serves as a summary of information available from the adaptive test. A LOGIST calibration run is then performed in which these ability estimates are fixed, and
nonadaptive responses to the new seeded items are used to estimate parameters for the 50 candidate new items. The top half of Figure 4 shows the details of this method across all four Rounds, starting with Round 1.

Because the ability estimates are fixed in the calibration of the new items, and no rescaling is done within the LOGIST run, the ability estimates determine the scale on which the item parameter estimates are reported. Since the ability estimates themselves are on the scale of the adaptive test item pool, the estimates for the candidate new items will be also.

Method B

This method requires a set of 'anchor' items to be seeded, along with the new items. The purpose of the anchor items is to try to improve upon Method A. Method A depends entirely upon treating estimates of ability as if they were, in fact, true abilities in order to maintain the scales of subsequent item pools. In so doing, errors will be made because estimated abilities differ from true abilities. The anchor items will be used to attempt to correct for scale drift that may result from
the use of these imperfect ability estimates for scale maintenance.

It seems reasonable to select as anchors items that are representative of those in the adaptive test item pool in terms of difficulty. This is because scale transformations that are derived from the anchor items will be applied to all items to be considered as candidates for item pools. In addition, the quality of the anchor items, in terms of item information, should be no worse than typical items produced for the adaptive test item pool, since it makes no sense to develop scale transformations based on items of poor quality.

For purposes of this study, 25 anchor items were defined to be exactly like 25 items selected from the original 100-item pool. The first step in the selection process was to eliminate from further consideration 9 items of the 100-item pool that were judged to be poorly fit by the 3PL in Round 0. The remaining 91 items were grouped on the basis of estimated difficulty into 5 equal intervals between -2.0 and 2.0. Five items were then selected randomly from each interval to be the anchor items.

Nonadaptive responses to 5 randomly selected new items and 5 randomly selected anchor items were collected from each simulee. Since there are half as many anchor items as there are new items,
each new item received about 1500 responses and each anchor item received about 3000 responses. The alternative design in which each simulee received only 5 seeded items, either new or anchor items, requires more simulees before an on-line calibration can occur, and was judged inconvenient to implement for the purposes of this study. Thus, the doubling of response rate for anchor items is not a requirement of this Method, but an artifact of the chosen design.

As in Method A, adaptive test information is summarized in a maximum likelihood estimate of simulee ability. A LOGIST calibration is done, fixing the ability estimates and estimating item parameters for the 50 new items and the 25 anchor items. Using the reestimates for the anchor items, and their initial estimates from Round 0, a scaling transformation is chosen to minimize the difference between the two test characteristic curves (Stocking & Lord, 1983). Using this transformation the results of this LOGIST run are placed on the scale of the Round 0 item pool. The bottom half of Figure 4 shows the details of Method B across all four Rounds, beginning with Round 1.

The Selection of New Items

Although the calibration of new items at each Round differed for the two Methods studied, the same algorithm was used to select
from the 50 candidate items the 25 new items to be included in the next Round's item pool. The difference between the estimated test information function for the Round 0 item pool and the estimated test information function for the 75 items to remain from the current pool was defined as a 'target' information function. The use of this difference as a target insures that items will be selected to maximize the resemblance of the next Round's item pool to the Round 0 item pool.

Three methods of selecting item sets or 'drafts' to match the target information function were tried. The first method selected items to minimize the maximum difference between the target and the draft test information functions across the ability metric from -3.0 to +3.0. The second method selected items with the greatest area under their item information functions within ability levels that appeared important based on the target information function. A third method was a combination of these two: a draft set of 25 items was selected on the basis of the area under the item information functions and then attempts were made to improve on this draft by discarding some items and selecting others that minimized the maximum difference between the target and the draft test information functions.
None of these methods of selecting replacement items worked automatically without intervention; the process can best be described as more of an art than a science. The 25 replacement items (and the selection method) were ultimately chosen on the basis of a subjective criterion: item sets with information functions closer to the target over middle ranges of ability were preferable to item sets with information functions more distant from the target in the middle but closer at the extremes. The first method was never judged to produce the best set of replacement items; it frequently resulted in the selection of items with more extreme difficulties over items with more moderate difficulties. This has the effect of matching or exceeding the target curve at more extreme ability levels where there are fewer simulees and where the reduction in information over the Round 0 item pool is least. The second method, selecting on the basis of the area under the item information functions between fairly narrow limits of ability, was most frequently judged the best, that is, the information functions were as close as possible to the target curve for middle levels of ability. The results of the third method were occasionally selected as well. Both of the more desirable methods required tinkering with the ability limits within which a match to the target was desired.
Results, Part 1: The True Score Metric

In examining the results of on-line calibration by any Method, it seems important to first focus attention on global effects in a metric that approximates one in which examinee scores will be reported. A subsequent section will examine in more detail the operation of each Method in the IRT metric.

The number-right true score metric for the Round 0 pool was chosen for this series of comparisons because the Round 0 pool is the only item pool that is constant across Methods, and, indeed, across all other experimenters in this project. In this context, the items in the Round 0 pool may be viewed as a "reference" test that is common to all experimenters. A true score on this common reference test is the score a simulee would have obtained if administered the entire Round 0 pool as a conventional test.

The conditional RMSE between estimated number-right true score and true score, and the conditional bias in estimated number-right true score were computed for each Method after each Round. True scores were computed using true simulee abilities for a Round and the true IRF's for the Round 0 items. Estimated number-right true scores were computed using simulee abilities estimated from the adaptive test at each Round, and the estimated IRF's for the Round 0 items. For small intervals of true score,
the square root of the average squared difference between the true scores and their estimates is considered to be the conditional RMSE function. Similarly, the average difference between the estimated and true scores for the same small intervals of true score is considered to be the conditional bias function.

The conditional RMSE and the conditional bias measures contain errors attributable to two sources that, because of the study design, cannot be examined separately. Both incorporate any scale drift due to the sequence of on-line calibrations. Because the ability estimates at each Round are from adaptive tests using different item pools, these measures also incorporate any effects arising from the changes in the item pools across Rounds.

Figure 5 compares the conditional RMSE and bias for Method A across all Rounds of the study, while Figure 6 displays the same information for Method B. The conditional RMSE and bias functions are the same for both Methods for the first Round. In these Figures, the dashed vertical lines mark the quintiles of the distribution of true scores for the final half-round of adaptive test simulation. In terms of the conditional RMSE, there is little difference between Methods at low and high true score levels. For middle true score levels, the conditional RMSE for Method A increases with every Round of adaptive testing. For
these same true score levels, the Method B RMSE tends to remain about the same after an initial increase from the first to the second Round of adaptive testing. For middle true score levels, where most of the simulees are located, Method B generally has a smaller RMSE than Method A.

-----------------------------

Insert Figures 5 and 6 about here

-----------------------------

The changes in the conditional bias functions appear to be systematic, although in different ways, across Rounds for both Methods. For Method A, the bias becomes more positive across Rounds for true scores slightly above the median and more negative for low true scores. For Method B the direction of the bias is opposite for middle true scores -- higher levels show more negative bias and lower levels show more positive bias across Rounds. Method B tends to have smaller absolute bias than Method A across all Rounds.

Figure 7 displays the conditional RMSE and bias for both methods after the end of all Rounds of adaptive test simulations. This Figure clarifies the comparisons of the two Methods. For low and high true scores, the RMSE functions are similar; at middle true scores, Method B has smaller RMSE. For the very lowest and
highest true scores, the conditional bias for the two Methods is similar; for the remaining true score levels, the bias works in opposite directions. Overall, the absolute bias is smaller for Method B.

Results, Part 2: The IRT Metric

The bias results on the true score metric seen in Figures 5, 6, and 7, may be viewed as scale drift. To interpret these results it seems necessary to explore the behavior of the two Methods in detail on the IRT metric. The various features of the simulations are complex and difficult to examine in a sequential fashion. Unfortunately, any written exposition can only proceed sequentially rather than simultaneously.

Many factors concurrently impact the four Rounds of item pool refreshment and the four-and-one-half Rounds of adaptive testing. Among these are:

--- the method of selecting the initial item pool,
--- the correlations among errors of estimation for the item parameter estimates,
the adaptive testing paradigm that selects items for administration based on an estimated ability and estimated item information,

--- the use of ability estimates in place of true abilities,

--- the precision with which new items are calibrated,

--- the method of identifying items in the current pool for replacement,

--- the method of selecting replacement items from the new items.

The effort to understand the interactions among these factors is complicated by other factors that are not central to on-line calibration, but must be dealt with nevertheless. Among these complicating factors are:

--- the necessity of establishing a single IRT scale upon which comparisons can be made,

--- the comparison of estimated parametric IRF's with true nonparametric counterparts,

--- the fact that changes in relevant variables between consecutive Rounds may be small, necessitating the examination of multiple Rounds simultaneously and across two different Methods.
The information available from each Method for each Round will be described in five different but overlapping analyses. The first such analysis deals with what, for lack of a better term, may be called a 'snapshot' of a single Round. Included in this snapshot are data about 1) the item pool at the beginning of the Round, 2) the nature of the ability estimates obtained from using this item pool to administer adaptive tests, 3) the nature of the errors made when using these ability estimates as if they were true abilities, and 4) the estimation of the parameters for the candidate new items.

The remaining analyses will look at many of these same features summarized in different ways. Given an item pool, the second analysis will explore what happens to ability estimation when this pool is used to administer adaptive tests. This analysis will take place across Rounds within Methods of on-line calibration. The third analysis will investigate what happens to the calibration of candidate new items, given that the ability estimates are used as if they are true abilities. This analysis will also take place across Rounds within Methods. The fourth analysis will focus in detail on the differences in the calibration of new items between Methods across Rounds, with particular attention paid to considering separately the effects of
the approximate scaling transformation for Method B. Finally, given the rules for elimination of items and the method of selecting new items for the adaptive test pool, the fifth analysis will explore the impact of these across Rounds and within Methods.

The Establishment of a Single IRT Metric

Throughout these discussions, comparisons of estimated quantities are made with their true counterparts. This, of course, is one of the advantages of a simulation study. Before this comparison can be made, however, true and estimated quantities must be on the same (arbitrary) metric.

It is unlikely that the metric on which the true item response functions and abilities were developed by Levine bears any simple linear relationship with the metric of LOGIST estimates. However, the assumption is made here that a linear transformation can be developed that will be a good approximation to a possibly more complex transformation. From the simulated initial calibration of the Round 0 item pool we have abilities estimated by LOGIST. Davis provided the corresponding true abilities for the N = 6000 simulees. Robust measures of location and scale (Mosteller & Tukey, 1977, p.20) were computed for both the estimated and true abilities and a linear transformation developed from them to transform true abilities to the scale of
the Round 0 calibration. This same transformation was applied to the true abilities of all other simulees in the remaining Rounds of the study.

All true item response functions were transformed to the Round 0 scale by applying this approximate linear transformation to Levine’s tabled ability values. Figure 8 shows the transformed true item response functions (solid lines) and the LOGIST estimates of the same item response functions (dotted lines) on the Round 0 metric for some typical items. All subsequent comparisons with true values were done using the transformed true item response functions and abilities.

Some Results From the Bivariate Normal Distribution

Some aspects of the subsequent discussions, particularly those dealing with true and estimated abilities, may at first appear confusing. This section attempts to deal in advance with these aspects by removing them temporarily from the context of on-line calibration, and placing them in the more familiar context of the bivariate normal distribution. The point of the exercise is not to imply that the joint distribution of estimated and true
abilities is bivariate normal, but rather to clarify some of the
techniques used in subsequent analyses.

Suppose we have two variables whose joint distribution is
bivariate normal, \( z_1 \) and \( z_2 \). Suppose that both variables have
means of 0 and variances of 1, and that the correlation between
the two is \( \rho < 1 \). A scatterplot of 400 random draws from such a
distribution, restricted to the range of \(-1\) to \(+1\), is shown in the
left panel of Figure 9. On the same panel, the solid line is the
45-degree line of \( z_2 - z_1 \). The line with long dashes is the
regression of \( z_2 \) on \( z_1 \), namely \( z_2 = \rho * z_1 \). The line with the
short dashes is the regression of \( z_1 \) on \( z_2 \), \( z_1 = \rho * z_2 \), or \( z_2 =
(1/\rho) * z_1 \).

\[ E(z_2 - z_1 | z_1) = E(z_2 | z_1) - z_1 = \rho * z_1 - z_1 = (\rho - 1) * z_1. \]

This function is plotted as a function of \( z_1 \) in the middle panel
of Figure 9. Because \( \rho \) is less than 1, this function has a
negative slope. Suppose that \( z_1 \) is an unobservable variable, and
the $z_2$ is an observable estimate of it. The average value of this residual for different values of the unobservable variable is, by definition, a conditional bias function.

Consider the expectation of this same residual, now conditional on $z_2$:

$$E(z_2 - z_1 | z_2) = z_2 - E(z_1 | z_2) = z_2 - \rho z_2 - (1 - \rho) z_2.$$

This function is plotted as a function of $z_2$ in the right panel of Figure 9. Because $\rho$ is less than 1, this function has a positive slope. We again suppose that $z_1$ is an unobservable variable, and $z_2$ is an observable estimate of it. The average value of this residual for different values of the observable variable can be considered to be a conditional error function. That is to say, it gives the average of the errors that will be made if the observable variable is used as if it were the unobservable variable, for different values of the observable variable.

In subsequent sections, the joint distributions of estimated and true ability will be analyzed in both ways. The expected residuals conditional on the true abilities will constitute an examination of the bias in the estimated abilities. These
conditional bias functions will usually have a negative slope.
The expected residuals conditional on the estimated abilities will constitute an examination of the error made when using estimated abilities rather than true abilities. These conditional error functions will usually have a positive slope.

Analysis 1: Snapshots

Figures 10 for Method A and 11 for Method B display the snapshots of each Round for each Method. Each subfigure (labeled 10a, 10b, . . . , or 11a, 11b, . . . ) of each Figure shows a single Round and is intended to be read left to right and top to bottom as in a page of printed text. The following section describes the derivation and interpretation of each panel of a subfigure. Subsequent sections compare each Round of the two methods of online calibration.

The Panels of a Subfigure

The first two panels in the top row of a subfigure display information about the item pool with which the Round is begun. The first panel shows the differences between the estimated and true slopes, plotted as a function of the true slopes; the second
panel shows the differences between the estimated and true
difficulties, plotted as a function of the true difficulties. In
the second panel, the difficulty of an item for which the slope
has been overestimated is indicated by a plus, and the difficulty
of an item for which the slope has been underestimated is
indicated by a circle.

In these and all other figures containing item parameter
information, the 'true' difficulty is defined as that value on the
true (transformed) ability metric that yields the same probability
of a correct response as does the estimated difficulty on the
estimated item response function. The 'true' slopes are computed
by numerical methods (Hamming, 1962, p. 318) at the location of
the true difficulty. This method of obtaining 'true' difficulty
and slope parameters for the nonparametric true IRF's is flawed.
Consider an item represented by a particular true nonparametric
IRF. If we have two different estimates of the difficulty
parameter for this item, it is possible that this procedure will
produce two different values of the 'true' difficulty parameter,
and also two different values of the 'true' slope parameter. To
examine the magnitude of this problem, the 50 new items calibrated
in Round 4 were examined. These items are the same for each
Method, and the difference between the parameter estimates, and
therefore between the 'true' parameters determined by this Method, should be largest on this final Round. The average difference between the 'true' difficulties was found to be .02, and the average difference between the 'true' slopes was found to be .004. These differences are small.

This method has an additional disadvantage. In this and subsequent figures, not all relevant items may appear. The nonparametric item response function is described by tabled function values over a finite interval of ability, and occasionally it is not possible to find the 'true' difficulty and slope values within this table. Appendix 1 contains a list, by Figure, of all items that should, but do not, appear in plots of this nature throughout this paper, along with the values of their estimated item parameters.

In spite of these flaws, this method of determining 'true' parameters from nonparametric item response functions, although crude, seems to work well enough to give useful insights into differences between the two Methods of on-line calibration.

The third panel in the top row of each snapshot displays the median difference between estimated and true ability as a function of the median true ability for small intervals of true ability. The median difference rather than the average difference is used
because some estimated abilities may be outlying values that distort averages. This panel may be viewed as a plot of an approximate bias function for estimated ability resulting from the use of this item pool to administer an adaptive test.

The first panel on the bottom row of each snapshot displays the median difference between estimated and true ability as a function of the median estimated ability for small intervals of estimated ability. This may be viewed as approximating the errors made when using an estimated ability as if it were a true ability. For example, in Figure 10a, we see that when we use an estimated ability of 2., that this estimated ability is larger than the true ability by about .1; when we use an estimated ability of -3, this estimated ability is lower than the true ability by about .8.

The final two panels on the bottom row of each snapshot display the residual plots for the estimates of the new items in a Round. The estimation of these item parameters is accomplished using the estimated abilities as if they were true abilities.

Round 1 Snapshots

The first Round of the simulation is represented in Figure 10a for Method A and Figure 11a for Method B. In these two figures, the starting item pool for this Round is the same -- the Round 0 pool. Likewise, the bias function and error function for
the abilities are the same. Most of the items in the Round 0 pool have over estimated slopes (first panel in top row). Items with middle difficulty are better estimated than items with more extreme difficulties (second panel in top row). This is because these item parameter estimates were obtained from a group of simulees with a typical (bell-shaped) distribution of abilities. Easier items tend to have overestimated difficulties, and also overestimated slopes. Harder items are also less well estimated than those of middle difficulty. In addition, overestimated harder items tend to have underestimated slopes while underestimated harder items tend to have overestimated slopes. These relationships between over- and underestimated slopes and over- and under estimated difficulties are consistent with the Wingersky and Lord (1984) result of a positive correlation between errors of estimation for item discriminations and difficulties for easy items and a negative correlation for hard items. These correlations are perhaps emphasized here because the distribution of ability in the sample of simulees was not broad enough to obtain accurate parameter estimates for the more extreme items.

Adaptive tests were administered to simulees using this Round 0 item pool. An ability estimate was first determined, based on responses to items previously administered. During the actual
operation of the adaptive test, this ability estimate was computed using Bayesian methods developed by Owen (1975). For both Methods of on-line calibration, a maximum likelihood ability estimate, rather than Owen’s Bayesian ability estimate, was used. Both the Owen’s Bayesian ability estimate and the maximum likelihood ability estimate weight items with higher estimated discriminations more than items with lower estimated discriminations.

In the adaptive test, items that were maximally informative at the estimated ability were then selected for administration. The information for an item is itself an estimate, and will be an overestimate of true information for items with overestimated slopes. For items of approximately the same difficulty, the items with the most overestimated slopes will be chosen more frequently than items with less overestimated slopes. Because of the correlation between errors of estimation for difficulty and discrimination, easy items with overestimated difficulties will be chosen more frequently than easy items with less overestimated difficulties. Likewise harder items with underestimated difficulties will be chosen more frequently than harder items with less underestimated difficulties. The misestimation of difficulties will be emphasized by methods of estimating ability
that weight items with higher estimated slopes more than items with lower estimated slopes.

We can expect these speculations to be confirmed by the bias plot of estimated abilities. Indeed, the bias in the ability estimates (third panel of top row in Figures 10a and 11a) does follow the same pattern as that of the difficulties of the items with overestimated slopes (middle panel of top row in Figures 10a and 11a).

In the next step of Round 1 for both Methods, the ability estimates are used as if they were true abilities and the item parameters are estimated for the candidate new items. We know that estimated abilities have a larger variance than true abilities. Because the unit of measurement of the IRT scale is usually taken to be the standard deviation of abilities, this difference in variance may be viewed as a difference in scale. Table 2 displays the variance of the true and estimated abilities for all samples of simulees for all Rounds. While the differences in the Round 1 variances may seem quite small, a better picture of the nature of the errors incurred is found in the error function panel (first panel of bottom row in Figures 10a and 11a). Higher estimated abilities are systematically overestimates of their corresponding true values, and the difference becomes larger as
the estimated ability increases. The same is true in the opposite direction for lower estimated abilities.

Up to this point in Round 1, the two Methods of on-line calibration have been identical. It is in the calibration of the new items that the Methods begin to differ. The spreading out of the estimated abilities and their subsequent use as if they were true abilities causes the slopes to be generally underestimated for the new items for Method A (middle panel, bottom row, in Figure 10a). The difficulties for the new items are not as well estimated as the difficulties for the Round 0 pool; the scatter of the residuals is greater. In addition the trend towards the overestimation of difficulty for harder items and underestimation of difficulty for easy items is consistent with the direction predicted on the basis of correlation between estimation errors for items with generally underestimated slopes.

The calibration of new items for Method B (second and third panels, bottom row, Figure 11a) is less affected by the use of estimated abilities as if they were true abilities. About half of the items have overestimated slopes. The difficulties for the
items with overestimated slopes are misestimated in a pattern consistent with the correlation of estimation errors. The difficulties for the items with underestimated slopes seem less affected. The items are the same for both Methods and the parameters are estimated from the same set of estimated abilities. What is different is the scaling transformation that is performed for Method B, using information from the anchor items. This has the effect of an approximate correction for the fact that using the estimated abilities to obtain parameter estimates is not the same as using true abilities. From this point on, the two Methods of on-line calibration will differ, and the differences will become more notable with each Round.

Round 2 Snapshots

Figure 10b for Method A and Figure 11b for Method B display the second Round of simulation results. Items have been discarded from the Round 0 pool on the basis of the frequency of use in adaptive test simulations. Because the simulees are a typical group, the most frequently used items were the discriminating items of middle difficulty. Replacements were selected from the new items calibrated in the previous Round (Figures 10a and 11a, bottom row, third panel). These items spanned a broad range of difficulty. All items of middle difficulty were selected as
replacements, as well as more extreme items with higher estimated slopes.

The Method A pool for Round 2 contains more items with underestimated slopes than the Method B pool. The well estimated middle difficulty items from Round 0 have been eliminated for both Methods, and replaced by items whose difficulties are less well estimated. For Method B, and also, but less clearly, for Method A, the pattern of residuals for the item difficulties for items with overestimated slopes is consistent with the correlation of estimation errors; the trend is less clear for items with underestimated slopes. As in Round 0, the bias in the estimated abilities follows the bias in item difficulties for those items with overestimated slopes.

For both Methods, the variance of estimated abilities is greater than the variance of true abilities, although this difference is smaller than the differences for Round 1, as shown in Table 2. But, also for both Methods, the pattern of errors made when using estimates as if they were true shows systematic errors are now made, even for middle levels of estimated ability. It seems probable that this phenomenon is the result of the fact that the item difficulties are less well estimated for both
Methods for middle difficulty items as well as the patterns of misestimation of slopes and difficulties peculiar to a Method.

For Method A, estimated abilities just above the middle are overestimates, while estimated abilities just below the middle are underestimates. Method B shows the same error pattern for estimated abilities just above the middle, and a slightly less severe underestimation for estimated abilities just below the middle. For Method A, then, the estimated abilities are more spread out than for Method B, even for middle ability estimates. This difference in patterns is a consequence of the underestimation of slopes for almost all new items selected for inclusion in this Round 2 pool by Method A.

When using these estimated abilities to estimate parameters for new items, as before the slopes for Method A are mostly underestimated while those for Method B are less so. The correlation of estimation errors is now clearly visible for Method A for those items with underestimated slopes. It is also clearly visible for Method B for those items with overestimated slopes.

Snapshots for Rounds 3, 4, and 5

The remaining subfigures of Figures 10 and 11 show the results for the remaining Rounds. All of the phenomena examined so far appear and become more exaggerated as the Rounds progress:
1) The starting pool for a Round for Method A always contains more items with underestimated slopes than the pool for Method B. As the Rounds progress, the slopes for the Method A pools become mostly underestimated. For Method B, the numbers of over- and underestimated slopes become about the same.

2) The middle difficulty items are the items that are continually replaced for both Methods. For Method A, it becomes more visible that the estimates of these difficulties follow patterns predicted by the correlated estimation errors for items with underestimated slopes. Patterns for overestimated slopes are less discernable for this Method, presumably because there are so few items with overestimated slopes. For Method B, the estimates of the difficulties for the new items with overestimated slopes follow patterns predicted by the correlated estimation errors. The prediction of patterns for items with underestimated slopes is not very visible.

3) For middle levels of ability, the bias in the ability estimates follows the bias in the difficulties of the items that are most used in the adaptive test. For Method B, this tends to be the middle difficulty items with overestimated slopes. For Method A, this tends to be the middle difficulty items with underestimated slopes, simply because there are few, if any,
Method A items calibrated on-line with overestimated slopes. These biases tend to be in opposite directions, predictable on the basis of the correlated estimation errors when the items were calibrated on-line.

4) Table 2 shows that, across Rounds, the estimated abilities are more spread out than the true abilities. This difference is generally larger for Method A than for Method B, and for Method A it increases across Rounds. The Figures show that for middle levels of estimated ability, the errors made when using estimated ability as if it were true become increasingly large in opposite directions around the middle for Method A. They remain about the same for Method B. This reflects fact that the slopes of items for Method A are consistently underestimated, as well as the misestimation of middle item difficulties for both Methods.

Analysis 2: Item Pools and Bias in Ability Estimates

This analysis is offered as an aid in understanding a single aspect of the data presented in the snapshots of each Round. Figure 12 for Method A and Figure 13 for Method B display for each Round the item pool with which the Round is begun, and the approximate bias functions for the estimated abilities that result when the item pool is used in adaptive testing. These Figures are constructed by lining up the top row of three panels from each
subfigure in Figures 10 and 11. Each row in the Figures 12 and 13 represents a Round in the simulation. The top rows in the two Figures are the same; they both display the Round 0 item pool and the bias functions for the first Round of the simulation. Although no new information is presented here, it may be easier to comprehend the trends across Rounds when the panels are arranged in this manner.

Looking down the left hand column in Figure 12, it is easy to see that, for Method A, the point cloud representing the residuals of the slopes is mostly above the horizontal line in Round 1, gradually drifts downward, and is mostly below the horizontal line in the final Round. For Method B in Figure 13, this drift seems to stabilize at about the point where the slopes are evenly over and underestimated.

The middle column of panels in the two Figures shows that the changes in the residuals of the difficulties are predominantly for middle difficulty items. These are the items that get used most frequently, and therefore replaced most frequently. For Method A, these items become overestimated if they are slightly above the
middle and underestimated if they are slightly below the middle. Most of these items have underestimated slopes. For Method B, these middle difficulty items have slopes that are both over- and underestimated.

The third column of panels in the two Figures shows that the bias in the estimated abilities tends to remain the same for extreme abilities, regardless of the Method of on-line calibration or the particular Round. The bias of middle estimated abilities changes because the middle difficulty items are being replaced. For Method B, the bias tends to look like the bias in the difficulties for items of middle difficulty that have overestimated slopes. This is because it is just those items that are selected most frequently and weighted most heavily for simulees of middle ability levels in adaptive testing. For Method A this bias also tends to look like the bias for items of middle difficulty, because these are the only items available for simulees of middle ability levels. Most, if not all, of these items have underestimated slopes.

Analysis 3: Errors in Ability Estimates and the Calibration of New Items

This analysis is offered as an aid to understanding a second aspect of the data presented in the snapshots of each Round.
Figure 14 for Method A and Figure 15 for Method B display the approximate error functions when using estimated abilities as if they were true abilities, and the resulting residual plots for the new items calibrated from these estimated abilities. These Figures are constructed by lining up the bottom three panels from each snapshot subfigure across Rounds. The left panel in the top row of each Figure is the same; it displays the error function for Round 0. All remaining panels differ.

For Method A, the left column of panels in Figure 14 shows that the estimated abilities are increasingly spread out across Rounds. This is confirmed by Table 2. The middle column of panels shows that more new items have underestimated slopes across Rounds; this is because the abilities become more spread out. The right panels show the correlation between estimation errors for items with underestimated slopes becomes increasingly pronounced across Rounds. This is not surprising in view of the greater number of items with underestimated slopes.

For Method B, Table 2 and the left column of panels in Figure 15 show that the spread in estimated abilities does not change
much across Rounds. The middle column of panels shows that the slopes for the new items are about evenly over- and underestimated. The right column of panels shows that correlation between errors of estimation for items with overestimated slopes becomes more visible across Rounds.

It is possible for individual item parameter estimates to be different but the estimated item response functions produced by these estimates to be similar. This is particularly true for very easy and very hard items, where individual item parameter estimates are not well determined; quite different parameter estimates can give similar item response functions in the area of the ability distribution where simulees are located. It would be instructive to compare estimated item response functions for both Methods across Rounds. Unfortunately, only a single item (Vale number 244) was included in the Round 0 pool and was also included in the 50 candidate new items for each Round for each Method. This item was a discriminating item with higher than average difficulty. Figures 16 (Method A) and 17 (Method B) show the true item response function (solid line) and the estimates of this item response function across all Rounds. Method A estimates show more variability, both in terms of slope and location, than do Method B estimates. Method A estimated slopes tend to be too low and
estimated difficulties too high. Method B estimated slopes tend to be too high and estimated difficulties too low.

While it is not possible to directly compare the estimated item response functions for any other item across Rounds and Methods, it is possible to approach issues of the accuracy of estimation more indirectly. Using the true abilities at a Round as weights, a weighted RMSE between the estimated and true item response functions was computed. At each Round, an average weighted RMSE was then computed, where the average was taken over the new items calibrated in that Round. Table 3 shows these average weighted RMSE’s for all Rounds for both Methods. The Round 0 average weighted RMSE is the same for both Methods since the items and estimates of item parameters are identical. The average weighted RMSE for Method A increases across subsequent Rounds, while the average RMSE for Method B remains approximately the same.

Insert Table 3 about here
Analysis 4: The Calibration of New Items and the Approximate Scaling Transformation

The Round 0 item pool is, by design, identical for both Methods of on-line calibration. After the first Round the actual items included in a pool can be different for the two Methods, also by design. The differences are introduced by the elimination of overused items and the selection of the 25 new items to be included in the next item pool, processes that operated independently for the two Methods. In actuality, all items of the 100 in the final pool created for Round 1 were the same, 96 out of 100 were the same for Round 2, 95 were the same for Round 3, and 90 were the same for Round 4. Since the pools are so similar for the two Methods, the differences between the two Methods appear to originate with the approximate scaling transformation developed from the anchor items for Method B. This analysis will examine the effects of this rescaling in more detail.

The multiplicative and additive constants of each rescaling are shown in Table 4. All multiplicative constants are less than one, indicating that at each Round, the scale unit before transformation is too large. All additive constants are non-zero, indicating that the scale origin before transformation is not precisely correct. The importance to attach to the differences
among the multiplicative constants or the differences among the additive constants is unknown but could be investigated through a jackknife study.

------------------------

Insert Table 4 about here

------------------------

Note that the rescalings depend upon one another in a complex and indirect fashion. The rescaling for Round 1 is applied to the new items calibrated in that Round, of which 25 are selected to be included in the second item pool. To the extent that abilities in the second Round of adaptive testing are estimated using the items that have been calibrated on-line in Round 1, the calibration of the new items in the second Round will be affected by the rescaling from the first Round. Since the 25 items selected are of middle difficulty, it is certain that some abilities are estimated from some items that have been calibrated on-line. This same type of dependency propagates across Rounds.

Figure 18 shows the residuals for the estimated slopes and difficulties of the new items calibrated in each Round by each Method. Each subfigure (18a, 18b, etc.) shows a different Round, and within a Round, the items calibrated are the same for each Method. In each subfigure, the panels in the top row show the
results for Method A. The panels in the middle row show the results for Method B before the application of the approximate scaling transformation. The panels in the bottom row show the results for Method B after the application of the approximate scaling transformation.

For Round 1 (Figure 18a), Method B before transformation (middle row) is identical to Method A (top row), that is, the items are the same and the estimated parameters are also the same. As seen before, the slopes are generally underestimated. Also as seen before, the underestimation of difficulties for easy items with underestimated slopes and the overestimation of difficulties for hard items with underestimated slopes is apparent. The sampling correlation for items with overestimated slopes is not apparent. The effect of the transformation (bottom row) is to reduce the underestimation of the slopes. The overestimation of easy items with overestimated slopes and the underestimation of hard items with overestimated slopes is now visible, while the sampling correlation for items with underestimated slopes disappears.

-------------------------------

Insert Figures 18a-18d about here

-------------------------------
For Round 2 (Figure 18b) and all subsequent Rounds, Method B before transformation (middle row) is no longer identical to Method A (top row), that is, the items are the same for a Round, but the parameter estimates are different. This is because the estimated abilities used to calibrate the new items have been affected by the different parameter estimates for the items included in the final pool from the previous Round. Across Rounds, Method B untransformed slopes generally have less scatter than Method A slopes, although they are generally underestimated, as are the Method A slopes. Method B untransformed difficulties also have less scatter than Method A difficulties, and as for Method A, the sampling correlation for items with underestimated slopes is visible.

Across Rounds, the application of the scaling transformation has the effect of raising slopes, so that there are about equal numbers of slopes that are over- and under-estimated. Simultaneously, the correlation between errors of estimation for slopes and difficulties for items with overestimated slopes is enhanced, and the correlation between errors of estimation for slopes and difficulties for items with underestimated slopes is suppressed.
The middle panels of Figures 18b, 18c, and 18d show the estimation of parameters for the new items for Method B is improved when compared to Method A even before the scaling transformation is applied for the current Round. This improvement can only occur indirectly through the improvement of the ability estimates (derived from the adaptive tests that used the previous Round's final item pool) used to calibrate these new items. An alternative method of examining the improvement is shown in Table 3. In parentheses, the average RMSE for the new items in each Round for Method B before the application of the scaling transformation is shown. The RMSE's are smaller than those for Method A, and also smaller than those for Method B after transformation. The scaling transformation has the effect of reducing the bias in the estimated item parameters, at the expense of increasing the overall RMSE. But the RMSE after transformation for Method B is still smaller than the RMSE for Method A.

Analysis 5: The Impact of Elimination and Selection Rules on Item Pools

The snapshots of each Round in Figures 10 and 11 show that cycles of on-line calibration and item pool refreshment have most impact on items of middle difficulty in the item pool. The analysis presented here examines these results from a more global
perspective in terms of the estimated test information functions for the 100-item pools developed during each Round of the study.

The estimated test information function (Lord, 1980, equation 5-6) of a set of items is the maximum amount of information that can be obtained from the item set if it were administered as a conventional test. It is not the information function for an adaptive test using this item set as the item pool. The adaptive test information function for algorithms of the type used here can only be conveniently estimated from a numerical approximation using Monte Carlo results (see, for example, Lord, 1980, section 10.6). This latter information function depends upon not only the item pool, but also the algorithms incorporated in the adaptive test that deal with the selection of the first item, randomization to increase item security, the selection of the next item to be administered, the stopping rule, and the scoring method. In this discussion, the estimated test information function will be viewed as a convenient mechanism for examining the information structure of the item pool upon which the adaptive testing algorithm will operate.

Figure 19 for Method A and Figure 20 for Method B display the estimated test information functions for the 100-item pools developed during each Round of the study. The estimated test
information function for Round 0 is the same for both Figures. For both Methods, the information in the pool for middle levels of ability declines from Round 0 to Round 1; subsequent declines are less severe. Across Rounds, the information at low ability levels does not change very much and actually increases over that for Round 0 for some items. The information at high ability levels continues to decline, particularly for Method A, although it actually increases somewhat for the very highest ability levels.

Insert Figures 19 and 20 about here

These results are due to the interaction of a number of circumstances that are visible in the snapshots of Figures 10 and 11. First, the method of building the Round 0 pool makes it look better than it actually was, since the items were selected on the basis of estimated information. This estimated information came from parameter estimates derived from a LOGIST calibration of Vale's original response data, rather than the Levine-based simulated data used to obtain parameter estimates in Round 0. To the extent that Levine's true IRF's and ability distribution are adequate representations of the phenomena generating the Vale data, we would expect the calibrations in Round 0 to be similar to
the calibrations based on the Vale data. Indeed, the first panel of Figure 10a or 11a show that most Round 0 slopes are overestimated.

Second, the decline in estimated test information for middle levels of ability is also a consequence of the rule for eliminating items at each Round. In adaptive testing with a typical group, it is the informative middle difficulty items that will be selected most frequently for administration. Since items are eliminated on the basis of frequency of use, these items will be the first to be removed as the process goes from Round 0 to Round 1. This phenomenon was apparent in the snapshots. Since frequency data is accumulated across Rounds, informative easy and difficult items may be removed in subsequent Rounds.

Replacement items are selected on the basis of estimated information from a set of 50 items that have been randomly selected from the original Vale pool of 258 items. These items, as seen in the snapshots, have a broad range of difficulty. For both Methods, the estimated difficulty for items of true middle difficulty are less well estimated than Round 0 items of middle difficulty. For Method A, their slopes are generally underestimated, while for Method B, the slopes are about evenly over- and underestimated. Since replacement items are selected on
the basis of estimated information, replacement items selected for Method A will have lower estimated information than replacement items selected for Method B.

Figure 21 for Method A and Figure 22 for Method B show the estimated test information functions for the target curve, the 50 new items, and the final selection of 25 replacement items for Round 4. These results are typical of other Rounds. A random selection from Vale's items cannot provide adequate numbers of informative middle difficulty items as possible replacements. For both Methods, the 25 replacement items have as much information as possible at middle levels of ability, given the inadequacy of the 50-item set from which they are selected. The decline in estimated test information in Figures 19 and 20 is less for easier items because the Vale item set contains proportionally more easy items rather than hard items.

For both Methods, about 30% of the Round 4 item pool consists of items remaining from the Round 0 pool, all with estimated difficulties greater than 1.0 in absolute value. These items have been available for administration to $N = 60000$ simulees by the end
of Round 4, but have not accumulated sufficient responses to be among the 25 most used items at any Round. A different but overlapping 30% of the Round 4 item pool for each Method consists of items with fewer than 1000 (and sometimes no) responses. Most of these items have estimated difficulties greater than 1.5 in absolute value.

Discussion

Of the two methods of on-line calibration studied here, Method B would be more expensive to implement than Method A. More items must be obtained initially from which to select not only the first adaptive test item pool, but also the set of anchor items. Because the anchor items must also be seeded, either each examinee must receive more seeded items, or more examinees are required to receive seeded anchor items before an on-line calibration can occur. In any practical application of this Method, the anchor items themselves must also be replaced from time to time, requiring, at every stage, a larger number of items to be considered as candidate new items.

Although Method B is more expensive than Method A to implement, some of its properties are more desirable than Method A properties. The evidence shows that relying on ability estimates alone to maintain the scale across Rounds in on-line calibration
leads to increasingly inaccurate estimation of the parameters of the new items, which in turn, leads to increasingly inaccurate ability estimates (Figures 14 and 18). Even the approximate rescaling used here with the anchor items improves scale maintenance (Figures 15 and 18). The conditional RMSE on the true score metric of the Round 0 item pool (Figures 5 and 6) shows these effects on a metric that approximates what might actually be used for the CAT-ASVAB. Both Methods show an initial increase in RMSE after the first item pool refreshment. However, the RMSE for Method B remains about the same for subsequent Rounds, while that for Method A increases, particularly for middle true scores, across Rounds.

Both LOGIST-based Methods show scale drift over four Rounds in which about 70% of the original item pool is replaced (Figure 7). The drift is in opposite directions for the two Methods. Method A tends to underestimate slopes because estimated abilities are more spread out than those of Method B. Because of the correlation between errors of estimation, Method A underestimates difficulties for easier items and overestimates difficulties for harder items (Figures 14 and 18). Estimated abilities reflect this bias in the difficulties (Figure 12), as do estimated true
scores (Figure 5). Thus the bias is positive for moderately high true scores and negative for moderately low true scores.

Method B tends to estimate slopes better because abilities are better estimated since the items used to compute the abilities have been subjected to the approximate scaling transformation developed from the anchor items. However, this Method overestimates more slopes, and because of the correlation between errors of estimation, this Method tends to overestimate difficulties for easier items and underestimate difficulties for harder items (Figures 15 and 18). Estimated abilities reflect this bias in difficulties (Figure 13), as do estimated true scores (Figure 6). Thus the bias is negative for moderately high true scores and positive for moderately low true scores.

Based on their respective biases at the lower ranges of true scores, Method A might exclude too many minimally qualified recruits; it underestimates their scores. Method B might include too many minimally qualified recruits; it overestimates their scores. At upper true score ranges, Method A gives an overly optimistic reporting of scores, while Method B gives an overly pessimistic reporting.

The causes for this scale drift are known and seem in large part attributable to the correlation among errors of estimation.
However, these effects appear to be exaggerated by the currently accepted procedures for adaptive testing. These correlations are not the result of the particular estimation method used here (LOGIST), but are common to all item response models and associated estimation methods that assume that an item response function is a member of a family of symmetric curves. They become important in applications of IRT that rest heavily on the veracity of item level data, such as adaptive testing.

The problems begin with the inadequate calibration of the items from which the initial item pool are to be selected. By design, this is typically an unusual set of items that are very discriminating, have a wide range of difficulty, and low guessing parameters (see Table 1). Item parameter estimates for this collection of items are usually obtained by administering these items to a typical group of examinees. This sample of examinees is inadequate for good estimation of the more extreme items. Thus the estimation errors, and their correlations, are exaggerated for just those items that are important in building an adaptive test item pool.

The problems are magnified by the accepted method of building an initial adaptive test item pool by selecting items on the basis of estimated information. This guarantees that the items with the
most overestimated discriminations and misestimated difficulties will be selected. Figure 10a or Figure 11a show that the correlations between estimation errors are already present in the Round 0 item pool.

Further damage is done when items are chosen for administration in an adaptive test on the basis of estimated information. Because of the correlation between errors of estimation, the items with the most overestimated discriminations and therefore most misestimated difficulties will be chosen, insuring that the difficulties will be, on average, inappropriate for the examinee. This misestimation is already present when using the Round 0 item pool for adaptive testing, as seen in Figures 10a or 11a. Scoring an adaptive test with an algorithm that weights items with high estimated discrimination more than items with low estimated discrimination must further magnify this misestimation of abilities. Both the maximum likelihood estimator used in the on-line calibrations and the Bayesian modal estimator used in the adaptive test administration have this property.

Abilities estimated from adaptive test administrations to a typical group are biased because of the biases in the estimates of item difficulties (Figures 12 and 13), and contain more error than
the true abilities (Figures 14 and 15). These estimates are used to determine the parameters of the new items. The new items are unscreened, and typically cover a wide range of difficulty. Only a few of these items are effective replacements for items that are eliminated (Figures 21 and 22). If no scale adjustments are made, item discriminations become underestimated, with predictable misestimation of difficulties (Figure 18). If an approximate rescaling is employed, discriminations are better estimated, but also with predictable misestimation of difficulties (also Figure 18). If items are selected for inclusion in the pool on the basis of estimated information, the correlation of errors of estimation is exaggerated once again. Over subsequent cycles of on-line calibration, these problems continue to grow.

The following suggestions for improving the process seek to retain the spirit of conventional wisdom in adaptive testing, although an unthinking application of this conventional wisdom is appropriate only when true item parameters and abilities are known. It is possible that some of these suggestions might mitigate the inevitable scale drift in on-line calibration. As always, there seems to be a choice of mitigating bias by giving up precision, or increasing test length.
Scale Drift

67

Suggestion #1

Any method of on-line calibration should begin by obtaining the best possible estimates of item parameters for the items from which the item pool will be selected. The current design could be improved by the use of a more appropriate sample of examinees. If the items have a wide range of difficulties, a sample with a wide range of abilities is required for the adequate calibration of the more extreme items. When employing a LOGIST-based approach to on-line calibration, it may be possible to correct these estimates for bias, at least approximately, using formulas appropriate for the simultaneous estimation of item and ability parameters. Formulas for this situation do not exist, but could possibly be derived using methods similar to Lord and Wingersky (1985).

Suggestion #2

Selecting the initial item pool on the basis of estimated information only exaggerates the correlation of estimation errors. A better method would stratify on estimated item difficulty alone and select randomly within each difficulty stratum. This reduces the correlation between estimation errors for item discrimination and difficulty. Alternatively, items could be eliminated using only conventional criteria such as item content or poorly behaving distractors, and all other items accepted. This method, too,
would reduce the correlation. Both of these methods might degrade the real quality of the item pool over the entire range of ability, requiring a longer adaptive test to achieve the same accuracy of estimated ability. An additional alternative available in a LOGIST-based approach would be to choose only those items that are well estimated, thus eliminating extreme items and saving middle difficulty ones. While methods exist for computing the appropriate standard errors (see Lord and Wingersky, 1985) these methods are difficult and cumbersome to implement.

Suggestion #3

The selection of items for administration in an adaptive test is typically based on estimated information or estimated discrimination. However, we know that for given levels of estimated difficulty, drawing items on estimated discrimination implies that on average estimated difficulty is not equal to the true difficulty because of the correlation of estimation errors. A better design would select items for administration on the basis of estimated difficulty alone. Even if the errors are correlated, this increases the chance that, for given levels of estimated difficulty, on average the estimated parameters are equal to their true values. This design, although mitigating the correlations among estimation errors, degrades the adaptive test over the
entire range of interest, requiring the administration of more items to achieve similar levels of accuracy. Alternatively, it may be possible to develop a confidence interval approach to estimated information functions and select items with narrow confidence bands around the estimated item information function. This latter method has the advantage of possibly maintaining the quality of the item pool in the middle of the ability range, while at the same time eliminating more extreme and more poorly estimated items. This is similar to Suggestion #2, choosing items with well-estimated parameters; here we consider items with well-estimated functions of parameters.

Suggestion #4

Adaptive test scoring algorithms should not weight items with high estimated discrimination more than items with low estimated discrimination. Both maximum likelihood and Bayesian procedures have this property. Use of these estimators will undo the efforts to use true item parameters on average. Instead, new scoring methods, such as those suggested by Lewis (1985) or Jones, Wainer, and Kaplan (1984), that take into account errors in item parameter estimates when estimating ability, should be considered.
Suggestion #5

The selection of items to be seeded for on-line calibration should be closely tied to algorithms for eliminating items in the current pool. Reasonable screening procedures based on classical item statistics, if applied to items before they are considered for seeding, should improve the maintenance of the quality of the item pool by providing more useful new items from which to select replacements. Replacement items should be selected from the candidate new items using methods that reduce the correlation among errors of estimation, as in Suggestion #2.

Consideration should be given to eliminating items with too few responses in addition to too many responses. This provides more room in the item pool for middle difficulty items, at the expense of poorer measurement properties at more extreme levels of ability. While this may appear to be antithetical to the purposes of adaptive testing, it may be cost effective to concentrate efforts on maintaining the information structure of the item pool in regions where most of the examinees are to be found.

While the above suggestions for improving on-line calibration have been couched in terms of adaptive testing, the principles behind them are also relevant to on-line calibration for other model-based testing paradigms that emphasize short computerized
tests. Any such test design, by virtue of the fact that test lengths are short and the seeding of potential additional items is restricted, may have to rely on imperfect information about ability to maintain the scale across different versions of the item pool. Thus, the accuracy of the initial item calibration, the method of selecting items for the initial pool, methods of selecting items for administration, and test scoring should all be designed to mitigate, as much as possible, the effects of the correlated errors of estimation. Careful attention should be paid to methods that insure that on average the estimated parameters are equal to the true parameters. Such methods will improve the accuracy of the estimated ability used to maintain scales across Rounds of item pool refreshment, and thereby reduce scale drift.
References


Acknowledgment

This work was supported by Contract No. N00014-83-C-0457, project designation NR 150-520, from Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research and Educational Testing Service. Reproduction in whole or in part is permitted for any purpose of the United States Government. The author wishes to acknowledge the invaluable assistance of Maxine B. Kingston in carrying out this study.
Table 1

LOGIST Parameter Estimates of Vale's Experimental WK Items (n = 258)

Taken by Armed Forces Inductees (N = 8171)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>N</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min</th>
<th>Max</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>8171</td>
<td>-0.00</td>
<td>1.06</td>
<td>-7.26</td>
<td>5.16</td>
<td>-1.22</td>
<td>-0.65</td>
<td>-0.04</td>
<td>0.63</td>
<td>1.34</td>
</tr>
<tr>
<td>a</td>
<td>258</td>
<td>1.25</td>
<td>0.47</td>
<td>0.26</td>
<td>2.50</td>
<td>0.66</td>
<td>0.88</td>
<td>1.23</td>
<td>1.54</td>
<td>1.88</td>
</tr>
<tr>
<td>b</td>
<td>258</td>
<td>-1.08</td>
<td>2.00</td>
<td>-6.10</td>
<td>3.45</td>
<td>-3.58</td>
<td>-2.42</td>
<td>-1.33</td>
<td>0.52</td>
<td>1.76</td>
</tr>
<tr>
<td>c</td>
<td>258</td>
<td>0.15</td>
<td>0.08</td>
<td>0.01</td>
<td>0.45</td>
<td>0.10</td>
<td>0.11</td>
<td>0.14</td>
<td>0.19</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Table 2

Variance of Estimated and True Abilities, Both Methods, across All Rounds of Adaptive Testing*

<table>
<thead>
<tr>
<th>Round</th>
<th>Method A</th>
<th></th>
<th>( \hat{\theta} )</th>
<th>( \theta )</th>
<th>difference</th>
<th>Method B</th>
<th></th>
<th>( \hat{\theta} )</th>
<th>( \theta )</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>( \hat{\theta} )</td>
<td>( \theta )</td>
<td>difference</td>
<td>N</td>
<td>( \hat{\theta} )</td>
<td>( \theta )</td>
<td>difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 1</td>
<td>14,979</td>
<td>1.15</td>
<td>1.05</td>
<td>.10</td>
<td>14,979</td>
<td>1.15</td>
<td>1.05</td>
<td>.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Round 0 pool)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 2</td>
<td>14,965</td>
<td>1.09</td>
<td>1.04</td>
<td>.05</td>
<td>14,968</td>
<td>1.10</td>
<td>1.04</td>
<td>.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Round 1 pool)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 3</td>
<td>14,979</td>
<td>1.16</td>
<td>1.05</td>
<td>.11</td>
<td>14,976</td>
<td>1.11</td>
<td>1.05</td>
<td>.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Round 2 pool)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 4</td>
<td>14,969</td>
<td>1.13</td>
<td>1.02</td>
<td>.11</td>
<td>14,979</td>
<td>1.07</td>
<td>1.03</td>
<td>.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Round 3 pool)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 5</td>
<td>14,965</td>
<td>1.19</td>
<td>1.03</td>
<td>.16</td>
<td>14,976</td>
<td>1.04</td>
<td>1.03</td>
<td>.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Round 4 pool)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Simulees with all-wrong or all-right response patterns are excluded from the computations of both \( \hat{\theta} \) and \( \theta \).
### Table 3

Average (across Items) Weighted RMSE for the New Items Calibrated in Each Round, for Each Method

<table>
<thead>
<tr>
<th>New items in Round</th>
<th>Method A Average RMSE</th>
<th>Untransformed Average RMSE*</th>
<th>Method B Average RMSE</th>
<th>Number of items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 0</td>
<td>0.023</td>
<td>(0.023)</td>
<td>0.023</td>
<td>100</td>
</tr>
<tr>
<td>Round 1</td>
<td>0.025</td>
<td>(0.025)</td>
<td>0.026</td>
<td>50</td>
</tr>
<tr>
<td>Round 2</td>
<td>0.027</td>
<td>(0.026)</td>
<td>0.026</td>
<td>50</td>
</tr>
<tr>
<td>Round 3</td>
<td>0.029</td>
<td>(0.024)</td>
<td>0.027</td>
<td>50</td>
</tr>
<tr>
<td>Round 4</td>
<td>0.031</td>
<td>(0.024)</td>
<td>0.025</td>
<td>50</td>
</tr>
</tbody>
</table>

*Computed prior to the application of the approximate scaling transformation for Method B; see Analysis 4 for explanation.*
Table 4
Parameters Developed for the Approximate Scaling Transformation, Based on the Anchor Items, for Method B

<table>
<thead>
<tr>
<th>Round</th>
<th>Multiplicative Constant</th>
<th>Additive Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.921</td>
<td>-.021</td>
</tr>
<tr>
<td>2</td>
<td>.897</td>
<td>-.034</td>
</tr>
<tr>
<td>3</td>
<td>.897</td>
<td>-.016</td>
</tr>
<tr>
<td>4</td>
<td>.917</td>
<td>-.027</td>
</tr>
</tbody>
</table>
Scale Drift

80

Figure 1. Global design of 'high-fidelity' simulation study for the ONR On-line Calibration Project.
Figure 2. 'True' nonparametric item response functions for some typical items.
Figure 3. Estimated and observed item response functions from simulated adaptive test responses.
Method A

Simulate adaptive test using Round 0 pool. Identify 25 items to be removed.
N = 13000
Seed 50 new items.
5 items/item.
N = 13000/item
Seed 25 anchor items.
5 items/person.
N = 3000/item.

Compute $b_0$'s.
Fix $b_0$'s, calibrate new items.
Select 25 replacement items.

Simulate adaptive test. Identify items. Seed new items.

Compute $b_0$'s.
Fix $b_0$'s, calibrate new items.
Select replacements.

Simulate adaptive test. Identify items. Seed anchor items.

Compute $b_0$'s.
Fix $b_0$'s, calibrate new items.
Select replacements.

Simulate adaptive test. Identify items. Seed new items.

Compute $b_0$'s.
Fix $b_0$'s, calibrate new items.
Select replacements.

Simulate adaptive test. Identify items. Seed anchor items.

Compute $b_0$'s.
Fix $b_0$'s, calibrate new items.
Select replacements.

Simulate adaptive test. Identify items. Seed new items.

Figure 4. Detailed design of simulation study of two LOGIST-based methods of on-line calibration.
Figure 5. For all Rounds of Method A, conditional RMSE (left) and conditional bias (right) of estimated true score on the 100-item reference test composed of Round 0 items. Dashed vertical lines mark quintiles of the distribution of true scores for the final Round.
Figure 6. For all Rounds of Method B, conditional RMSE (left) and conditional bias (right) of estimated true score on the 100-item reference test composed of Round 0 items. Dashed vertical lines mark quintiles of the distribution of true scores for the final Round.
Figure 7. After all Rounds for both Methods, conditional RMSE (left) and conditional bias (right) of estimated true score on the 100-item reference test composed of Round 0 items. Dashed vertical lines mark quintiles of the distribution of true scores for the final Round.
Figure 8. Typical item response functions estimated by LOGIST (dotted lines) and the corresponding true item response functions (solid lines). Both estimated and true IRF's are on the scale of the Round 0 pool.
Figure 9.  Four hundred random draws from a bivariate normal distribution (left), conditional bias function (middle), and conditional error function (right). See text.
Figure 10a. Snapshot for Method A, Round 1. See text.
Figure 10b. Snapshot for Method A, Round 2. See text.
Figure 10c. Snapshot for Method A, Round 3. See text.
residual slopes for starting pool

residual difficulties for starting pool

approximate bias in estimated abilities

approximate error in estimated abilities

residual slopes for new items

residual difficulties for new items

Figure 10d. Snapshot for Method A, Round 4. See text.
Figure 10e. Snapshot for Method A, Round 5. See text.
Figure 11a. Snapshot for Method B, Round 1. See text.
Figure 11b. Snapshot for Method B, Round 2. See text.
Figure 11c. Snapshot for Method B, Round 3. See text.
Figure 11d. Snapshot for Method B, Round 4. See text.
residual slopes for starting pool

residual difficulties for starting pool

approximate bias in estimated abilities

approximate error in estimated abilities

Figure 11e. Snapshot for Method B, Round 5. See text.
Figure 12. For all Rounds of Method A, the residuals for the slopes and the difficulties for items in the starting pool and the approximate bias in the estimated ability. See text.
Figure 13. For all Rounds of Method B, the residuals for the slopes and the difficulties for items in the starting pool and the approximate bias in the estimated ability. See text.
Figure 14. For all Rounds of Method A, the approximate error function for the estimated abilities and the residuals for the slopes and difficulties for the new items. See text.
<table>
<thead>
<tr>
<th>Approximate Error in Estimated Abilities</th>
<th>Residual Slopes</th>
<th>Residual Difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 15. For all Rounds of Method B, the approximate error function for the estimated abilities and the residuals for the slopes and difficulties for the new items. See text.
Figure 16. Method A, item 244, true IRF and estimated IRF's from each Round.
Figure 17. Method B, item 244, true IRF and estimated IRF's from each Round.
Figure 18a. The Round 1 residuals for the slopes and difficulties for new items for Method A (top), Method B before scaling (middle), and Method B after scaling (bottom). See text.
Figure 18b. The Round 2 residuals for the slopes and difficulties for new items for Method A (top), Method B before scaling (middle), and Method B after scaling (bottom). See text.
Figure 18c. The Round 3 residuals for the slopes and difficulties for new items for Method A (top), Method B before scaling (middle), and Method B after scaling (bottom). See text.
Figure 18d. The Round 4 residuals for the slopes and difficulties for new items for Method A (top), Method B before scaling (middle), and Method B after scaling (bottom). See text.
Figure 19. Method A estimated test information functions for the 100-item pools at each Round of simulations.
Figure 20. Method B estimated test information functions for the 100-item pools at each Round of simulations.
Figure 21. Method A estimated test information functions for the target, the 50 new items and the 25 replacement items, Round 4.
Figure 22. Method B estimated test information functions for the target, the 50 new items and the 25 replacement items, Round 4.
## Scale Drift

### Appendix 1

Parameter estimates for items not included in Figures because 'true' parameters could not be developed from the tabled values of the true item response functions

<table>
<thead>
<tr>
<th>Figure</th>
<th>Row</th>
<th>Column</th>
<th>( \hat{a} )</th>
<th>( \hat{b} )</th>
<th>( \hat{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10a</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2,3</td>
<td>.31</td>
<td>-11.07</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.54</td>
<td>-4.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.99</td>
<td>-3.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.32</td>
<td>-8.85</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.20</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.21</td>
<td>-9.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.90</td>
<td>-2.79</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.68</td>
<td>-3.86</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.54</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.13</td>
<td>-14.88</td>
<td>.20</td>
</tr>
<tr>
<td>10b</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2,3</td>
<td>.76</td>
<td>-5.22</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-3.00</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.50</td>
<td>-5.88</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
<td>-2.83</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.57</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.28</td>
<td>-10.88</td>
<td>.20</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>( \hat{a} )</td>
<td>( \hat{b} )</td>
<td>( \hat{c} )</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>10c 1</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td>2 2,3</td>
<td>2</td>
<td>2,3</td>
<td>.89</td>
<td>-3.26</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.05</td>
<td>-66.42</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.60</td>
<td>-3.34</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.66</td>
<td>-3.92</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.21</td>
<td>2.83</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.50</td>
<td>-5.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.34</td>
<td>3.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.08</td>
<td>-3.35</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.57</td>
<td>-3.55</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-3.87</td>
<td>.20</td>
</tr>
<tr>
<td>10d 1</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td>2 2,3</td>
<td>2</td>
<td>2,3</td>
<td>.44</td>
<td>-5.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.40</td>
<td>-6.89</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.04</td>
<td>-3.14</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.42</td>
<td>-3.37</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.51</td>
<td>-5.32</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.11</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.61</td>
<td>-4.21</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.02</td>
<td>3.02</td>
<td>.19</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>--------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>10e</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td>11a</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2,3</td>
<td>.34</td>
<td>-10.21</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.59</td>
<td>-3.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.07</td>
<td>-2.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.35</td>
<td>-8.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
<td>-2.97</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.23</td>
<td>-9.06</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.98</td>
<td>-2.59</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-3.57</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.44</td>
<td>-6.05</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.14</td>
<td>-13.72</td>
<td>.20</td>
</tr>
<tr>
<td>11b</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2,3</td>
<td>.43</td>
<td>-7.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.74</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.47</td>
<td>-5.41</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.21</td>
<td>-2.43</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.48</td>
<td>-5.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.36</td>
<td>-8.18</td>
<td>.20</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>--------</td>
<td>----</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>11c</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2,3</td>
<td>.31</td>
<td>-7.31</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.25</td>
<td>-12.34</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.88</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.95</td>
<td>3.11</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.34</td>
<td>-8.73</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.87</td>
<td>-3.44</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.22</td>
<td>-2.38</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.79</td>
<td>-3.32</td>
<td>.20</td>
</tr>
<tr>
<td>11d</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2,3</td>
<td>.32</td>
<td>-8.18</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.39</td>
<td>-6.32</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.64</td>
<td>-3.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.48</td>
<td>-7.81</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.29</td>
<td>-9.24</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>-2.60</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.33</td>
<td>-7.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>3.21</td>
<td>.20</td>
</tr>
<tr>
<td>11e</td>
<td>1</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>( a )</td>
<td>( b )</td>
<td>( c )</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>12</td>
<td>1,2</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>3,4,5</td>
<td>1,2</td>
<td></td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td>13</td>
<td>1,2</td>
<td>1,2</td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td>3,4,5</td>
<td>1,2</td>
<td></td>
<td>.87</td>
<td>-3.94</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.70</td>
<td>3.92</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.03</td>
<td>-3.45</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.88</td>
<td>-3.72</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.11</td>
<td>10.18</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>2,3</td>
<td>.31</td>
<td>-11.07</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.54</td>
<td>-4.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.99</td>
<td>-3.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.32</td>
<td>-8.85</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.20</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.21</td>
<td>-9.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.90</td>
<td>-2.79</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.68</td>
<td>-3.86</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.54</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.13</td>
<td>-14.88</td>
<td>.20</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
<td>--------</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>14 (cont'd.)</td>
<td>2</td>
<td>2,3</td>
<td>.76</td>
<td>-5.22</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>1.01</td>
<td></td>
<td>-3.00</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.65</td>
<td></td>
<td>-2.78</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.50</td>
<td></td>
<td>-5.88</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.05</td>
<td></td>
<td>-2.83</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.41</td>
<td></td>
<td>-6.57</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.28</td>
<td></td>
<td>-10.88</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2,3</td>
<td></td>
<td>.89</td>
<td>-3.26</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>.05</td>
<td></td>
<td>-66.42</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.60</td>
<td></td>
<td>-3.34</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.66</td>
<td></td>
<td>-3.92</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.21</td>
<td></td>
<td>2.83</td>
<td>.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.50</td>
<td></td>
<td>-5.46</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.34</td>
<td></td>
<td>3.46</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td></td>
<td>-3.35</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.57</td>
<td></td>
<td>-3.55</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.65</td>
<td></td>
<td>-3.87</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2,3</td>
<td></td>
<td>.44</td>
<td>-5.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>.40</td>
<td></td>
<td>-6.89</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.04</td>
<td></td>
<td>-3.14</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.42</td>
<td></td>
<td>-3.37</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.51</td>
<td></td>
<td>-5.32</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td></td>
<td>-3.11</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.61</td>
<td></td>
<td>-4.21</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.02</td>
<td></td>
<td>3.02</td>
<td>.19</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>2,3</td>
<td>.34</td>
<td>-10.21</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>.59</td>
<td></td>
<td>-3.82</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.07</td>
<td></td>
<td>-2.94</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.35</td>
<td></td>
<td>-8.17</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.19</td>
<td></td>
<td>-2.97</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.23</td>
<td></td>
<td>-9.06</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.98</td>
<td></td>
<td>-2.59</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.73</td>
<td></td>
<td>-3.57</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.44</td>
<td></td>
<td>-6.05</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.14</td>
<td></td>
<td>-13.72</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td>Figure (cont'd.)</td>
<td>Row</td>
<td>Column</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>------------------</td>
<td>-----</td>
<td>--------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>2,3</td>
<td>.43</td>
<td>-7.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.74</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.47</td>
<td>-5.41</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.21</td>
<td>-2.43</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.48</td>
<td>-5.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.36</td>
<td>-8.18</td>
<td>.20</td>
</tr>
<tr>
<td>3</td>
<td>2,3</td>
<td></td>
<td>.31</td>
<td>-7.31</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.25</td>
<td>-12.34</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.88</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.95</td>
<td>3.11</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.34</td>
<td>-8.73</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.87</td>
<td>-3.44</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.22</td>
<td>-2.38</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.79</td>
<td>-3.32</td>
<td>.20</td>
</tr>
<tr>
<td>4</td>
<td>2,3</td>
<td></td>
<td>.32</td>
<td>-8.18</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.39</td>
<td>-6.32</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.64</td>
<td>-3.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.48</td>
<td>-7.81</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.29</td>
<td>-9.24</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>-2.60</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.33</td>
<td>-7.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>3.21</td>
<td>.20</td>
</tr>
<tr>
<td>18a</td>
<td>1</td>
<td>1,2</td>
<td>.31</td>
<td>-11.07</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.54</td>
<td>-4.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.99</td>
<td>-3.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.32</td>
<td>-8.85</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.20</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.21</td>
<td>-9.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.90</td>
<td>-2.79</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.68</td>
<td>-3.86</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.54</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.13</td>
<td>-14.88</td>
<td>.20</td>
</tr>
</tbody>
</table>
## Scale Drift

<table>
<thead>
<tr>
<th>Figure</th>
<th>Row</th>
<th>Column</th>
<th>$^a$</th>
<th>$^b$</th>
<th>$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18a (cont'd.)</td>
<td>2</td>
<td>1,2</td>
<td>.31</td>
<td>-11.07</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.54</td>
<td>-4.13</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.99</td>
<td>-3.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.32</td>
<td>-8.85</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.20</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.21</td>
<td>-9.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.90</td>
<td>-2.79</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.54</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.13</td>
<td>-14.88</td>
<td>.20</td>
</tr>
<tr>
<td>3</td>
<td>1,2</td>
<td>.34</td>
<td>-10.21</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.59</td>
<td>-3.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.07</td>
<td>-2.94</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.35</td>
<td>-8.17</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
<td>-2.97</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.23</td>
<td>-9.06</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.98</td>
<td>-2.59</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-3.57</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.44</td>
<td>-6.05</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.14</td>
<td>-13.72</td>
<td>.20</td>
</tr>
<tr>
<td>18b</td>
<td>1</td>
<td>1,2</td>
<td>.76</td>
<td>-5.22</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-3.00</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.50</td>
<td>-5.88</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
<td>-2.83</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.41</td>
<td>-6.57</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.28</td>
<td>-10.88</td>
<td>.20</td>
</tr>
<tr>
<td>2</td>
<td>1,2</td>
<td>.39</td>
<td>-8.28</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.91</td>
<td>-3.02</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.92</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.42</td>
<td>-6.00</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.08</td>
<td>-2.67</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.43</td>
<td>-6.45</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.32</td>
<td>-9.09</td>
<td>.20</td>
</tr>
<tr>
<td>Figure</td>
<td>Row</td>
<td>Column</td>
<td>( \hat{a} )</td>
<td>( \hat{b} )</td>
<td>( \hat{c} )</td>
</tr>
<tr>
<td>----------</td>
<td>-----</td>
<td>--------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>18b (cont'd.)</td>
<td>3</td>
<td>1,2</td>
<td>.43</td>
<td>-7.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.74</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.73</td>
<td>-2.65</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.47</td>
<td>-5.41</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.21</td>
<td>-2.43</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.48</td>
<td>-5.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.36</td>
<td>-8.18</td>
<td>.20</td>
</tr>
<tr>
<td>18c</td>
<td>1</td>
<td>1,2</td>
<td>.89</td>
<td>-3.26</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.05</td>
<td>-66.42</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.60</td>
<td>-3.34</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.66</td>
<td>-3.92</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.21</td>
<td>2.83</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.50</td>
<td>-5.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.34</td>
<td>3.46</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.08</td>
<td>-3.35</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.57</td>
<td>-3.55</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-3.87</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1,2</td>
<td>.28</td>
<td>-8.14</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.22</td>
<td>-13.74</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.58</td>
<td>-3.20</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.91</td>
<td>-3.08</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.85</td>
<td>3.49</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.30</td>
<td>-9.72</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.78</td>
<td>-3.82</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-2.64</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.71</td>
<td>-3.69</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1,2</td>
<td>.31</td>
<td>-7.31</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.25</td>
<td>-12.34</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.65</td>
<td>-2.88</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.01</td>
<td>-2.78</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.95</td>
<td>3.11</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.34</td>
<td>-8.73</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.87</td>
<td>-3.44</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.22</td>
<td>-2.38</td>
<td>.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.79</td>
<td>-3.32</td>
<td>.20</td>
</tr>
</tbody>
</table>
## Scale Drift

122

<table>
<thead>
<tr>
<th>Figure</th>
<th>Row</th>
<th>Column</th>
<th>(\hat{a})</th>
<th>(\hat{b})</th>
<th>(\hat{c})</th>
</tr>
</thead>
<tbody>
<tr>
<td>18d</td>
<td>1</td>
<td>1,2</td>
<td>0.44</td>
<td>-5.94</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.40</td>
<td>-6.89</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.04</td>
<td>-3.14</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.42</td>
<td>-3.37</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.51</td>
<td>-5.32</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>-3.11</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.61</td>
<td>-4.21</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.02</td>
<td>3.02</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1,2</td>
<td>0.29</td>
<td>-8.83</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.36</td>
<td>-6.86</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.59</td>
<td>-4.26</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.44</td>
<td>-8.48</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.26</td>
<td>-10.05</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.14</td>
<td>-2.81</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.31</td>
<td>-7.74</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.14</td>
<td>3.53</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1,2</td>
<td>0.32</td>
<td>-8.18</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.39</td>
<td>-6.32</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.64</td>
<td>-3.94</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.48</td>
<td>-7.81</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.29</td>
<td>-9.24</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>-2.60</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.33</td>
<td>-7.13</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.24</td>
<td>3.21</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Educational Testing Service/Stocking

Dr. Terry Ackerman  
American College Testing Programs  
P.O. Box 168  
Iowa City, IA 52243

Dr. Robert Ahlers  
Code N711  
Human Factors Laboratory  
Naval Training Systems Center  
Orlando, FL 32813

Dr. James Algina  
1403 Norman Hall  
University of Florida  
Gainesville, FL 32605

Dr. Erling B. Andersen  
Department of Statistics  
Studiestraede 6  
1455 Copenhagen  
DENMARK

Dr. Eva L. Baker  
UCLA Center for the Study of Evaluation  
145 Moore Hall  
University of California  
Los Angeles, CA 90024

Dr. Isaac Bejar  
Mail Stop: 10-R  
Educational Testing Service  
Rosedale Road  
Princeton, NJ 08541

Dr. Menucha Birenbaum  
School of Education  
Tel Aviv University  
Ramat Aviv 69978  
ISRAEL

Dr. Arthur S. Blaiwes  
Code N712  
Naval Training Systems Center  
Orlando, FL 32813-7100

Dr. Bruce Bloxom  
Defense Manpower Data Center  
550 Camino El Estero,  
Suite 200  
Monterey, CA 93943-3231

Dr. R. Darrell Beck  
University of Chicago  
NORC  
6030 South Ellis  
Chicago, IL 60637

Cdt. Arnold Bohrer  
Sectie Psychologisch Onderzoek  
Rekruterings-En Selectiecentrum  
Kwartier Koning Astrid  
Bruijnstraat  
1120 Brussels, BELGIUM

Dr. Robert Breaux  
Code 78  
Naval Training Systems Center  
Orlando, FL 32813-7100

Dr. Robert Brennan  
American College Testing Programs  
P.O. Box 168  
Iowa City, IA 52243

Dr. James Carlson  
American College Testing Program  
P.O. Box 168  
Iowa City, IA 52243

Dr. John B. Carroll  
409 Elliott Rd., North  
Chapel Hill, NC 27514

Dr. Robert M. Carroll  
Chief of Naval Operations  
OP-01B2  
Washington, DC 20350

Dr. Raymond E. Christal  
UES LAMP Science Advisor  
AFHRL/MQEL  
Brooks AFB, TX 78235

Dr. Norman Cliff  
Department of Psychology  
Univ. of So. California  
Los Angeles, CA 90089-1061
Director, 
Manpower Support and 
Readiness Program 
Center for Naval Analysis 
2000 North Beauregard Street 
Alexandria, VA 22311

Dr. Stanley Collyer 
Office of Naval Technology 
Code 222 
800 N. Quincy Street 
Arlington, VA 22217-5000

Dr. Hans F. Crombag 
Faculty of Law 
University of Limburg 
P.O. Box 616 
Maastricht 
The NETHERLANDS 6200 MD

Dr. Timothy Davey 
Educational Testing Service 
Princeton, NJ 08541

Dr. C. M. Dayton 
Department of Measurement 
Statistics & Evaluation 
College of Education 
University of Maryland 
College Park, MD 20742

Dr. Ralph J. DeAyala 
Measurement, Statistics, 
and Evaluation 
Benjamin Bldg., Rm. 4112 
University of Maryland 
College Park, MD 20742

Dr. Dattprasad Divgi 
Center for Naval Analysis 
4401 Ford Avenue 
P.O. Box 16268 
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong 
Bell Communications Research 
6 Corporate Place 
PYA-1K226 
Piscataway, NJ 08854

Dr. Fritz Drasgow 
University of Illinois 
Department of Psychology 
603 E. Daniel St. 
Champaign, IL 61820

Defense Technical 
Information Center 
Cameron Station, Bldg 5 
Alexandria, VA 22314 
Attn: TC 
(12 Copies)

Dr. Stephen Dunbar 
2248 Lindquist Center 
for Measurement 
University of Iowa 
Iowa City, IA 52242

Dr. James A. Earles 
Air Force Human Resources Lab 
Brooks AFB, TX 78235

Dr. Kent Eaton 
Army Research Institute 
5001 Eisenhower Avenue 
Alexandria, VA 22333

Dr. John M. Eddins 
University of Illinois 
252 Engineering Research 
Laboratory 
103 South Mathews Street 
Urbana, IL 61801

Dr. Susan Embretson 
University of Kansas 
Psychology Department 
426 Fraser 
Lawrence, KS 66045

Dr. George Englehard, Jr. 
Division of Educational Studies 
Emory University 
210 Fishburne Bldg. 
Atlanta, GA 30322

Dr. Benjamin A. Fairbank 
Performance Metrics, Inc. 
5825 Callaghan 
Suite 225 
San Antonio, TX 78228
Dr. P-A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

Dr. Gerhard Fischer
Liebigasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1801 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research & Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric & Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service, 21-T
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut für Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY
Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. George B. Macready
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08541

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMPEP/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
16-T
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Barbara Means
SRI International
333 Ravenswood Avenue
Menlo Park, CA 94025

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Norman, OK 73071

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031
Dr. James B. Olsen  
WICAT Systems  
1875 South State Street  
Orem, UT 84058

Office of Naval Research,  
Code 1142CS  
800 N. Quincy Street  
Arlington, VA 22217-5000  
(6 copies)

Dr. Mark D. Reckase  
ACT  
P. O. Box 158  
Iowa City, IA 52243

Dr. Malcolm Ree  
AFHRL/MA  
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt  
HumRRO  
1100 South Washington Street  
Alexandria, VA 22314

Dr. Carl Ross  
CNET-PDCD  
Building 90  
Great Lakes NTC, IL 60088

Dr. J. Ryan  
Department of Education  
University of South Carolina  
Columbia, SC 29208

Dr. Fumiko Samejima  
Department of Psychology  
University of Tennessee  
310B Austin Peay Bldg.  
Knoxville, TN 37916-0900

Mr. Drew Sands  
NPRDC Code 62  
San Diego, CA 92152-6800

Wayne M. Patience  
American Council on Education  
GED Testing Service, Suite 20  
One Dupont Circle, NW  
Washington, DC 20036

Dr. James Paulson  
Department of Psychology  
Portland State University  
P.O. Box 751  
Portland, OR 97207

Dr. Jesse Oransky  
Institute for Defense Analyses  
1801 N. Beauregard St.  
Alexandria, VA 22311

Dr. Randolph Park  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Wayne M. Patience  
American Council on Education  
GED Testing Service, Suite 20  
One Dupont Circle, NW  
Washington, DC 20036

Dr. James Paulson  
Department of Psychology  
Portland State University  
P.O. Box 751  
Portland, OR 97207

Dr. Judith Orasanu  
Basic Research Office  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Dr. Carl Ross  
CNET-PDCD  
Building 90  
Great Lakes NTC, IL 60088

Dr. J. Ryan  
Department of Education  
University of South Carolina  
Columbia, SC 29208

Dr. Fumiko Samejima  
Department of Psychology  
University of Tennessee  
310B Austin Peay Bldg.  
Knoxville, TN 37916-0900

Mr. Drew Sands  
NPRDC Code 62  
San Diego, CA 92152-6800

Lowell Schoer  
Psychological & Quantitative Foundations  
College of Education  
University of Iowa  
Iowa City, IA 52242

Dr. Mary Schratz  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Dan Segall  
Navy Personnel R&D Center  
San Diego, CA 92152
Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
HumRRO
12908 Argyle Circle
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
NRC MH-176
2101 Constitution Ave.
Washington, DC 20418

Dr. Martin F. Wiskoff
Defense Manpower Data Center
550 Camino El Estero
Suite 200
Monterey, CA 93943-3231

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800
Dr. Kentaro Yamamoto  
03-T  
Educational Testing Service  
Rosedale Road  
Princeton, NJ 08541  

Dr. Wendy Yen  
CTB/McGraw Hill  
Del Monte Research Park  
Monterey, CA 93940  

Dr. Joseph L. Young  
National Science Foundation  
Room 320  
1800 G Street, N.W.  
Washington, DC 20550  

Mr. Anthony R. Zara  
National Council of State Boards of Nursing, Inc.  
625 North Michigan Avenue  
Suite 1544  
Chicago, IL 60611  

Dr. Peter Stoloff  
Center for Naval Analysis  
4401 Ford Avenue  
P.O. Box 16268  
Alexandria, VA 22302-0268