MODELING MULTIPLE REPAIRABLE EQUIPMENT
AND
LOGISTIC SYSTEMS
by
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ABSTRACT

An orderly classification hierarchy is described which shows one way that many existing logistics models and logistics support analyses can be together in the context of the ILS planning function for Multiple Repairable Equipment and Logistic Systems (MREAL Systems). This hierarchy suggests nine (there may be more...) logistics subproblems which define the MREAL System planning problem. The hierarchy establishes a perspective which is intended to stimulate thought, discussion and research about REAL and MREAL Systems. Research regarding one possible model of the MREAL System which results from this hierarchy is described.

INTRODUCTION

The concept of Integrated Logistic Support (ILS) has been defined by Blanchard (Ref. 2) as "A management function... to assure a system that will not only meet performance requirements, but which can be expeditiously and economically supported... ILS assumes the integration of the various elements of support..." ILS consists of two main processes: planning and execution.

One of the most important systems to which the ILS planning process is applied is the Repairable Equipment and Logistic (REAL) System. This system consists of a population of repairable equipment deployed to meet a demand and the associated logistic support elements. When one organization owns many REAL Systems, each meeting a different demand, this collection of systems will be called a Multiple Repairable Equipment and Logistic (MREAL) System.

The complex nature of the MREAL System has impeded development of holistic models which encompass the policy decisions required for its management. Most logistics modeling work has concentrated on one or another aspect of the system, typically along functional lines. Only a few models have been developed which address this problem from a holistic viewpoint. In Hart (Ref. 13), Fabrycky and Hart (Ref. 5), and Fabrycky (Ref. 4) a model was developed which represented a single REAL system by the total number of units of repairable equipment (N), the number of repair channels (M), and the retirement age (a) of the equipment. Mean failure and repair rates were allowed to change with equipment age and a single level of maintenance was assumed. The model minimizes expected annual equivalent costs of equipment, repair facilities and shortages. The model was applied to several different prime equipment designs.

Hinger (Ref. 15) extended this model to include two levels of maintenance, and constraints on population size and retirement age. The optimization technique for both models was the sectioning search of Friedman and Savage (Ref. 8). Mackliler (Ref. 18) adapted the basic ideas of these previous two models in a Monte-Carlo simulation. While this simulation model did not include an optimization procedure, it did allow for representation of the transient stochastic behavior of the REAL System.

In 1976, Gross and Kahn (Ref. 11) formulated a model of the REAL System which involved the number of repair channels (M), and the number of spare units of repairable equipment (which equals the population (N) minus the demand), as a function of time. The model also allows demand to change with time. The objective function of the Gross-Kahn model is simpler than in the models mentioned previously. Salvage values and most operating costs are neglected, and shortages are not included in the objective function. Rather, an availability constraint is given for each time period.

The probabilistic aspect of the REAL System is modeled by Gross and Kahn using the finite source queuing approximation. They define the failure rate for each time period in terms of the failure rate from the previous period, modified by the failure rate for new units (acquired for the current period) and by the failure rate for units repaired during the last period. They assume that repairing an item improves its failure rate. Gross and Kahn require the repair rate to be identical for all units during a particular time period, but allow it to change from period to period.

While Gross and Kahn suggested a heuristic procedure for optimizing their model, Falk and Rappaport (Ref. 7) developed an optimization algorithm for the same basic model. This algorithm is based on an integer branch and bound scheme which generates subproblems that can be solved by a procedure involving an enumeration scheme (to define the feasible region) and then dynamic programming.

Fabrycky, Malmberg, Moore, and Brammer (Ref. 6) created an interactive program for the IBM personal computer using an improved version of the Fabrycky-Hart model (Ref. 5). The program allows a user to quickly obtain total REAL System expected annual equivalent cost for user-given choices of population size (N), repair channels (M), and retirement age (a). Since the behavior of a REAL System is not intuitively obvious to the casual observer, the
software has significant potential as an educational tool for managers and owners of REAL Systems. It can also be used to compare alternate equipment designs (with respect to N, M, and R), and so help illustrate the tradeoffs between design and logistics.

Brammer (Ref. 3) developed a model for determining the period by period maintenance requirements for a REAL System. The model estimates the number of repair channels, the repair channel staffing and the spare parts demand for the system in each period using a Markovian analysis approach.

An examination of these and other models along with the modeling needs of the logistics community, reveals a number of shortcomings: (1) existing models make numerous assumptions which may not hold for real REAL Systems, (2) probabilistic system behavior is modeled approximately, (3) the transient behavior of a system can be modeled, but only at considerable cost and usually not in a form which lends itself to optimization, and (4) the integrated logistic characteristics of equipment designs are addressed by the models as an after-thought or not at all. It is also clear that many of the existing models were developed within the scope of a single functional area. According to Stein (Ref. 18), this "functional parochialism" is responsible for considerable "functional parochialism" may have also discouraged the development of policy optimization models which deal with entire REAL or MREAL Systems.

CLASSIFICATION HIERARCHY

These realizations lead to the consideration of the MREAL System in its entirety and to an attempt to develop a representation of the associated logistic policy problem. The result was a classification hierarchy which suggests an overall structure to the MREAL System. It appears to encompass, as subproblems, much of the existing modeling work and many of the functional areas of logistics. While this hierarchy is not the last word, it is hoped it will lead to better, more useful models by (1) encouraging the model creator to think about the elements of the MREAL System not covered by his/her model, (2) making the model user and the model creator more aware of the interactions between the elements of MREAL systems, and (3) showing the user how the model contributes to the overall ILS planning process for the MREAL system.

Figure 1 shows the classification hierarchy for the MREAL System. Models which deal with the optimal allocation of resources for collections of REAL Systems comprise the highest level of the hierarchy. The key issue is that different REAL Systems satisfy demands which differ in importance. A good MREAL System model helps manage resource allocations between REAL Systems, in light of these differences.

The second level of the hierarchy consists of the REAL Systems, each representing a different demand for a major end item of repairable equipment. Each demand is determined by the mission of the organization and environment in which the equipment functions.

A good REAL System model integrates all elements of the system which affect the performance of the REAL System and cannot be modeled separately. Such a model is an important tool for the allocation of resources within a REAL System, i.e. between resources spent on spare end items, maintenance channels, physical design, operator training, etc.

The third level of the hierarchy consists of the major subproblems of the REAL System planning problem. Nine subproblems have been identified:

1. Mechanic Training Problem (MTP): determine the type of training for mechanics, as it pertains to the effect of training on time to repair; determine the number of mechanics to assign to each repair channel at each maintenance level for the REAL System.

2. Optimal Level of Repair/Level of Repair Analysis Problem (OLRP): determine the maintenance level at which each type of equipment failure will be repaired.

3. Machine Design Problem (MDP): choose the design, design specifications, or procurement source which yields the best REAL System (or MREAL System) performance.

4. Maintenance Configuration Problem (MCP): determine the number of levels of maintenance (organizational, field, depot, etc.); determine the number of channels and tooling for each level; determine the queuing/service discipline for each channel.

5. Spare Machine Problem (SMP): determine the number of units of prime mission equipment (including spares) to be procured over time to meet the demand.


7. Replacement Policy Problem (RPP): determine when to retire equipment; define catastrophic damage for the equipment; determine how to handle catastrophic damage.

8. Inspection and Testing Policy (ITP): determine frequencies and types of inspection and testing.

9. Operator Training Problem (OTP): determine the level of training for operators of prime equipment; determine the envelope of allowable and recommended equipment operations.

The fourth level of the hierarchy contains subelements from several of the subproblems of the REAL System. These subelements are either problems whose solutions depend upon the policies established for the "parent" element, or they are significant problems which must be solved in order to establish the policies for the "parent" element. The subelements

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TWENTIETH ANNUAL SYMPOSIUM - 1985
FIGURE 1: Classification Hierarchy for Multiple REAL Systems
The process of creating the hierarchy lead to an attempt to create a holistic model of an MREAL System. Four elements and subelements of each REAL System were chosen for inclusion in a model named MREALI: the Spare Machine Problem (SMP), the Retirement Age Problem (RAP), the Levels and Channels Problem (LCP), and the Machine Design Problem (MDP). The model is intended to give simultaneous guidance regarding the following questions:

1. How much capital should be invested in each REAL System?
2. How many units of prime equipment should be procured in each REAL System in order to meet the demand?
3. How many repair channels should serve each REAL System?
4. At what age should equipment be retired in each REAL System?
5. From what source should the equipment be procured for each REAL System? Alternatively, what MTBF and MTTR should be designed for the prime equipment of each REAL System?

The objective function of MREALI contains expected total annual equivalent costs of: (1) procurement and operation of the equipment, (2) procurement and operation of the repair facilities, and (3) shortages. In order to have a fair comparison between procurement and operating costs for the equipment, the procurement cost is annualized over the equipment life (R) using the time value of money and an estimate of salvage value at retirement. The first portion of the MREALI objective function thus accounts for the annualized procurement cost and operating cost for each unit of repairable equipment in each REAL System.

The procurement cost of a repair channel in each REAL System is similarly annualized. It is assumed, however, that the retirement age of a repair channel is not a policy issue, but is set by outside considerations. The operating costs of the repair channels are fixed on a per channel-year basis, but may be different for each REAL System. Total repair channel costs do not change as the repair facilities are more or less loaded. (This may be an important assumption to be relaxed in future models.)

The expected total shortage cost for each REAL System is computed by estimating the mean number of shortage-year experiences and multiplying by the cost of a shortage-year. MREALI assumes that the equipment in each REAL System has exponential failure and repair characteristics which may change with "age." The stochastic behavior of such a population can be represented by a two-stage cyclic queue with as many classes of customers as there are "age" groups. MREALI designates such a class for each year-group over the life of the equipment in each REAL System.
Gross and Ince (Ref. 9) have shown how the stochastic behavior of a two-stage cyclic queue can be modeled under the first-come, first-served queuing discipline assumed by MREAL. Unfortunately, as the number of "age" groups increases, the computational resources required to solve the model increase at an even faster rate. Gross and Ince show that the classical finite source queueing model can approximate the solution to the two-stage cyclic queue under certain circumstances. MREAL uses one of these approximations (rare averaging) to estimate the distribution on shortage-years, from which a mean is computed.

There are several links between the REAL Systems modeled by MREAL. One such link involves the respective values given the costs of shortage-years. The model automatically allocates more resources to those REAL Systems with higher shortage-year costs. Another link involves budget limitations on the organization's procurement and operating expenditures. Several constraints are used to represent these limitations.

The first of these constraints limits the annualized procurement and operating costs for both the equipment and the repair channels, for all of the REAL Systems, to a single amount. While this constraint is computationally simple, it lacks considerable intuitive appeal because it deals with annualized, not budgeted, amounts, and because it combines procurement and operating costs.

A second type of constraint represents separate limitations on procurement and operating costs. The limitation on procurement cost deals only with expenditures for the replacement of retiring equipment.

Two other types of constraints are formulated. One is written for each REAL System and allows the specification of a maximum allowable equipment unavailability. Another specifies a maximum allowable probability for the event of exceeding a specified number of shortages. These additional, alternate constraints are described to demonstrate the flexibility of this approach to modeling MREAL Systems.

Three kinds of notation appear in the formulation of MREAL: (1) decision variables, (2) system parameters, and (3) system performance measures. Decision variables represent system operating policies and are capitalized in the notation. System parameters are costs and other factors which cannot be directly controlled by system owners/managers. System parameters appear in lower case. System performance measures represent the cost and availability quantities which result from the system parameters and policies. Performance measures appear in the mixed case. See Table I for the definitions of decision variables, system parameters and system performance measures.

MULTIPLE REAL SYSTEMS MODEL (MREAL)

Minimize: $Z = \sum_{i=1}^{n} [C_i(p) + C_i(f) + C_i(s)]$

subject to: $\sum_{i=1}^{n} [C_i(p) + C_i(f) + C_i(s)] < b_{pr} + b_{op}$

$\sum_{j=1}^{n} C_{ij} S_{ij} < b_{caj}$ for all $i$

**Alternate formulation:**

$\sum_{i=1}^{n} C_i(p) < b_{pr}$

$\sum_{i=1}^{n} C_i(op) < b_{op}$

**Alternative formulation:**

$\sum_{i=1}^{n} C_{ij} S_{ij} < b_{caj}$ for all $i$

Equipment Population Cost $- C_i(p)$

$C_i(p) = N_i [(A|P, l_0, R_i) S_{ij} + I_i f_i S_{ij}]$

Repair Facility Cost $- C_i(f)$

$C_i(f) = \sum_{i=1}^{n} (A|P, l_0, R_i)[(C_i - c_{ij}) S_{ij} + I_i S_{ij}]$

Shortage Cost $- C_i(s)$

$C_i(s) = \sum_{i=1}^{n} [C_i(k) q_{kj}]_{k=1}^{N_i} (N_i - k) q_{kj}$

where:

$$q_k = \begin{cases} 
\sum_{j=1}^{N_i} \left[ \frac{N_i!}{(N_i-k)!k!} \right]^{k} S_{ij} & \text{if } k < M_i \\
\sum_{j=1}^{M_i} \left[ \frac{N_i!}{(N_i-k)!k!} \right]^{k} S_{ij} & \text{if } k \geq M_i 
\end{cases}$$

Annual Replacement Capital Cost $- C_i(pr)$

$C_i(pr) = (N_i/R_i) \sum_{j=1}^{n} [P_{ij} S_{ij}]$

The annual replacement capital cost represents only the steady state annual requirement for new equipment (replacing retiring equipment). It does not include initial procurement capital for the REAL System.
Table 1. MREAL Model - Symbols and Definitions

**Decision Variables**

1. \( N_i \) - The size of the population of repairable equipment in the \( i \)th REAL system.
2. \( n_i \) - The number of repair channels operated in the \( i \)th REAL system.
3. \( R_i \) - Retirement age of the repairable equipment in the \( i \)th REAL system.
4. \( S_{ij} \) - Takes the value 1 only if design \( j \) is selected in the \( i \)th REAL system.

**System Parameters**

1. \( d_i \) - The demand to be met by the equipment in the \( i \)th REAL system.
2. \( N \) - The number of REAL systems considered.
3. \( q_i \) - The number of designs (sources) considered in the \( i \)th REAL system.
4. \( p_{ij} \) - Unit procurement cost of design \( j \) in the \( i \)th REAL system.
5. \( f_{ij}(R_i) \) - Unit salvage value, a function of \( R_i \).
6. \( o_{ij} \) - Annual unit operating cost of design \( j \) in the \( i \)th REAL system.
7. \( c_i \) - The cost of one unit-year of shortage in the \( i \)th REAL system.
8. \( rp_i \) - Acquisition cost of a repair facility in the \( i \)th REAL system.
9. \( rf_i \) - Salvage value of a repair facility in the \( i \)th REAL system.
10. \( ro_i \) - Annual operating cost of a repair facility in the \( i \)th REAL system.
11. \( rr_i \) - The retirement age of repair facilities in the \( i \)th REAL system.
12. \( g_{ij}(t) \) - The MTBF age profile for design \( j \) in the \( i \)th REAL system.
13. \( h_{ij}(t) \) - The MTTR age profile for design \( j \) in the \( i \)th REAL system.

**System Performance Measures**

1. \( C_i(p) \) - The equivalent annual procurement and operating costs for all the repairable equipment in the \( i \)th REAL system.
2. \( C_i(f) \) - The equivalent annual procurement and operating costs for the repair facilities operated in support of the \( i \)th REAL system.
3. \( C_i(s) \) - The average annual total cost of shortages in the \( i \)th REAL system.
4. \( C_i(pr) \) - Annual requirement for replacement capital in the \( i \)th REAL system.
5. \( C_i(op) \) - Annual requirement for operating capital in the \( i \)th REAL system.
6. \( C_{unai} \) - Inherent system unavailability for design \( j \) in the \( i \)th REAL system.
7. \( C_{unaij}(sh_i) \) - Catastrophic unavailability: the probability that the system experiences more than \( sh_i \) shortages.
8. \( \lambda_{ij}(R_i) \) - Mean breakdown rate for design \( j \) in the \( i \)th REAL system: a function of \( R_i \).
9. \( \mu_{ij}(R_i) \) - Mean repair rate for design \( j \) in the \( i \)th REAL system: a function of \( R_i \).

Annual Operating Capital Requirement - \( C_i(op) \)

\[
C_i(op) = \left( N_i \sum_{j=1}^{q_i} \left[ o_{ij} g_{ij} \right] + N_i ro_i \right)
\]

Computation of REAL System Unavailability

\[
Sunaij = \frac{d_i}{\sum_{k=0}^{N_i} \left( N_i - d_i + k \right) q_k / d_1}
\]

Computation of Catastrophic Unavailability

\[
C_{unaij}(sh_i) = \frac{d_i}{\sum_{k=sh_i}^{N_i} \left( N_i - d_i + k \right) \left( N_i - d_i + k \right) / d_1}
\]

*where \( q_k \) is as defined in the Shortage Cost section.*

This performance measure represents the fraction of equipment demand which, on the average, will not be satisfied because of shortages. It is the ratio of the expected number of shortages to the demand.
where $q_k$ is as defined in the Shortage Cost section. This performance measure is the steady state probability that the number of shortages will ever be $q_k$ or greater.

PROPOSED OPTIMIZATION PROCEDURE

MREAL I is a model whose objective function and feasible region are very non-linear. Three of the decision variables take integer values, but the fourth, representing design, does not have even that amount of mathematical structure. Consequently, most of the standard integer programming techniques are not suitable for MREAL.

A number of multi-variate non-linear search techniques have been examined for suitability with respect to MREAL. These have included both gradient based and gradient free methods. The advantage of these search techniques is that many of them do not require the objective function to be differentiable or continuous. Only a computable objective function is required.

The optimization technique under development begins with a point in the feasible region, if such exists. It then finds a direction in which improvement occurs and moves in that direction until improvement ceases. The process then repeats until no further improvement can be obtained. The procedure contains some protection against getting stuck at a local, but not global, minimum. However, unless a search procedure is exhaustive, it can only guarantee its results to be global under certain restrictive conditions, which have not been shown to hold for MREAL.

In order to locate a direction of improvement, an approximation to the gradient is computed. From this information an integer direction is obtained and a unidimensional Fibonacci search is performed. This search includes not only points which lie on the integer direction, but also certain points which lie appropriately near the integer direction. The unidimensional search ceases when the value of the objective function begins to increase. A gradient parton procedure is periodically performed to prevent the process from getting hung up moving through a valley. A barrier function is used to prevent the search procedure from leaving the feasible region.

CONCLUSIONS

A classification hierarchy has been described which helps expose the weaknesses of existing REAL and MREAL Systems models. The hierarchy serves as a point of departure for discussion about the need for models of REAL and MREAL Systems. It may also serve as a graphic "dictionary" of terms and concepts regarding these models. One possible model of an MREAL System, based on the ideas contained in the hierarchy, has been formulated and described, along with a proposed optimization technique.

Considering the magnitude of expenditures for REAL and MREAL Systems, and the lack of comprehensive, holistic planning models for these systems, it is imperative that more work be done in this area. The logistics community has a unique opportunity in this work to define the collection of policies which determine the performance of REAL and MREAL Systems, to determine how the tools of the mathematical modeler can be used to aid this decision making, and to build and implement such models.

BIBLIOGRAPHY


BIOGRAPHY

Thomas P. Moore is a doctoral candidate in Industrial Engineering and Operations Research at Virginia Tech, Blacksburg, Virginia. He received the M.S. in Operations Research from Stanford University in 1975 and the B.A. in Math/Physics from Northeastern University in 1974. Mr. Moore is the first recipient of the Annual Dissertation Proposal Award from the Society of Logistics Engineers (1984). He served 4 years in the United States Army, 3 of which were spent teaching at the Army Logistics Management Center. Mr. Moore is currently a Captain, Corps of Engineers, U.S. Army Reserve.

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