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MELBOURNE, VICTORIA
Aircraft Structures Report 432

CRITERIA FOR MATRIX DOMINATED FAILURE (U)

by

L. MOLENT, J.J. PAUL and R. JONES

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FEBRUARY 1988
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SUMMARY

This paper discusses methods for characterizing the matrix controlled strength of composite materials, the fracture of adhesives and the failure of adhesively bonded repairs. Attention is focused on the Tsai-Wu, point stress, energy density and energy release rate approaches and the relationship between these various failure theories.
1. INTRODUCTION

When designing adhesively bonded fibre composite repairs for metallic or composite structures, two of the main design requirements are:

(a) the adhesive bond must not fail

(b) the composite repair, often referred to as a patch [1], must not delaminate. These failure modes involve matrix dominated failures and require a consistent and valid failure criterion for use in the design phase of any repair.

A similar situation occurs in the compressive failure of impact damaged laminates. This failure process is often matrix dominated [2] and has received considerable attention in recent years. A review of the current status of bonded repairs and the testing and analysis methodologies for impact damaged laminates is given in [3,4] respectively.

The purpose of this present paper is to outline the main approaches currently used for the analysis of matrix dominated failures and, where possible, to explain the relationship between them.

2. FAILURE THEORIES

There are a large number of failure theories presently used to characterize the matrix controlled failure of composites and adhesives. The theories that will be discussed in the present paper are,

1. the tensor polynomial failure criteria, i.e. Tsai - Hill and Tsai - Wu,
2. the energy density failure hypothesis, and
3. the energy release rate approach.

The first two approaches differ from the latter in that they involve the stresses, and/or strains, evaluated at a characteristic distance \( r_c \) in front of the stress concentrator. The energy release rate approach does not involve a length scale, which is a material parameter, but instead requires a series of additional tests to determine the mode I, II and III energy release rates.

2.1 The Tensor Polynomial Failure Criteria

In recent years numerous forms of the polynomial failure criteria have been proposed. The most generally accepted of these can be expressed in the form

\[
F_i \sigma_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k = 1
\]

where \( F_i, F_{ij} \) and \( F_{ijk} \) are the constants which must be experimentally determined, see [5].

When a body contains a stress concentrator, such as a matrix crack, then Wu [6] stated that this failure criterion should not be evaluated at the stress concentrator but at a distance \( r_c \) from it. Indeed it was shown that \( r_c \) is a material constant which can be obtained experimentally.

In the case of the matrix dominated failure of a fibre composite laminate Hahn and Tsai [7] subsequently proposed that the failure criterion

\[
F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad (i = 2, ..., 6)
\]

1
replaces equation (1). The main difference here lies in the neglect of the contribution of the fibre stresses, i.e., \( \sigma_f \). Equation (2) is still evaluated at a distance \( r_c \) from a stress concentrator.

This theory has been successfully used by many researchers and has been applied to the problem of delamination [8] as well as to the fracture of centre-notched panels [5].

If the problem under consideration does not involve shear stresses then equation (1) reduces to

\[
F_2 \sigma_2 + F_{22} \sigma_2^2 |_{r=r_c} = 1
\]

which is similar to the point stress failure criterion [9] and which has been extensively used to characterise matrix dominated failure [10,11,12]. In references using the point stress formulation the term \( r_c \) is often denoted as \( d_0 \).

For a general problem it is necessary to determine the failure location as well as the failure load. One method for locating the point of failure initiation was proposed by Wu [5]. This approach considers both the strength vector \( F \) to the failure surface, as given by equation (1), and the stress vector \( S \) where,

\[
|S| = (k_{ij} \sigma_i \sigma_j)^{\frac{1}{2}}
\]

Failure is then said to occur at the location where

\[
S |_{r=r_c} = F |_{r=r_c}
\]

Details of this hypothesis are given in [5].

When the material is isotropic the strength function \( F \) reduces to the modified von Mises yield criterion

\[
W_d = AdV + B
\]

where \( W_d \) is the distortional energy, \( A \) and \( B \) are constants and \( dV \) is the change in volume per unit volume i.e.,

\[
dV = (\sigma_1 + \sigma_2 + \sigma_3)/3K
\]

where \( K \) is the bulk modulus. This form of the yield criterion has been widely used for the analysis of polymers and adhesives [13,14].

### 2.2 Energy Density Theory

The concept of a critical energy density has been extensively used in the design and failure analysis of composites and adhesively bonded repairs. Applications of this theory range from

(a) fatigue life evaluation of damaged laminates [15]
(b) evaluation of the effects of combined mechanical and environmental loads on hole elongation and subsequent failure [16]
(c) estimating the residual strength of impact damaged laminates [17,18] and laminates containing edge delaminations [19]
(d) characterizing the fracture behaviour of fibre composite laminates [20,21,22].
It has also been used to design adhesively bonded repairs for cracked metallic components [23] and was used to design a boron/epoxy reinforcement to the wing pivot fitting of F-111 aircraft in service with the Royal Australian AirForce [24].

This theory also involves the use of a characteristic dimension $r_c$ and states that, for matrix dominated failures, failure initiates in the direction $\Theta_0$ which, at $r = r_c$, coincides with the local minimum of the energy in the matrix, defined as $W_e$. Failure occurs when this value reaches a critical value $W_c$. Mathematically this can be written as

$$\frac{\partial (W_e - W_c)}{\partial \Theta} = 0 \text{ at } \Theta = \Theta_0, \quad r = r_c$$

(9)

where $W_e$ is given by

$$W_e = \frac{1}{2} \sigma_{ij} \epsilon_{ij} - W_f$$

(10)

where $W_f$ is the energy in the fibres. When applying this hypothesis to the failure of adhesively bonded joints $W_f$ is considered to be zero.

As in the tensor polynomial failure criterion this approach involves:

1. the use of a characteristic dimension $r_c$
2. the neglect of the contribution due to the fibres.

Now consider the failure of a centre notched panel where the fibres are parallel to the direction of the crack. It follows from the solution to this problem [25] that the value of $r_c$ used in the energy density theory is related to that used in the tensor failure theory, which we will denote as $r'_c$, by the expression

$$r_c = r'_c (1 - \nu_{12} \sqrt{E_{22}/E_{11}})^{\frac{1}{3}}$$

(12)

This allows the test methodologies developed for the measurement of $r'_c$ to be used to evaluate $r_c$. As a first estimate it is often convenient to use $r_c = r'_c$.

The critical energy $W_c$ is a function of $dV$ and the level of local constraint, where

$$dV = \sigma_1 \left( \frac{1}{E_{11}} - \frac{\nu_{12}}{E_{11}} - \frac{\nu_{13}}{E_{12}} \right) + \sigma_2 \left( \frac{1}{E_{22}} - \frac{\nu_{12}}{E_{11}} - \frac{\nu_{23}}{E_{23}} \right) + \sigma_3 \left( \frac{1}{E_{33}} - \frac{\nu_{13}}{E_{12}} - \frac{\nu_{23}}{E_{23}} \right)$$

(13)

This dependence is shown in Table 2 where the strengths and moduli used are taken from Tsai [26] and are given in Table 1.

Table 1: COMPOSITE PROPERTIES

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>$E_{11}$</th>
<th>$E_{11}$</th>
<th>$\nu_{11}$</th>
<th>$G_{12}$</th>
<th>$E_{11}/G_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 T300/5208</td>
<td>181</td>
<td>10.3</td>
<td>0.28</td>
<td>7.17</td>
<td>1.44</td>
</tr>
<tr>
<td>2 B4/5502</td>
<td>204</td>
<td>18.5</td>
<td>0.33</td>
<td>5.59</td>
<td>3.31</td>
</tr>
<tr>
<td>3 AS/3501</td>
<td>138</td>
<td>8.96</td>
<td>0.3</td>
<td>7.1</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Note: values in Table are in GPa
Table 2: DESIGN ALLOWABLES - ORTHOTROPIC APPROACH

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>TENSION</th>
<th>COMPRESSION</th>
<th>SHEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{xx}$</td>
<td>$\sigma_{yy}$</td>
<td>$\sigma_{zz}$</td>
</tr>
<tr>
<td></td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>800</td>
<td>27.4</td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>1860</td>
<td>41.4</td>
</tr>
<tr>
<td>3</td>
<td>61.7</td>
<td>1336</td>
<td>35.2</td>
</tr>
</tbody>
</table>

where: $W = W \times E$, $d\tilde{V} = dV \times E$

Note: $\nu_{12}$ is taken as 0.3

The failure envelope predicted for AS/3501, using both the energy density and Tsai - Hill theories, is shown in Figure 1. Both theories predict essentially the same behaviour. Figure 2 shows the critical available energy density for the three composite materials considered in Table 1.

As mentioned in section 2.1, the Tsai - Hill theory is based on a particular generalization of the modified von Mises criterion for isotropic material. An alternative approach, for isotropic materials, is to add the energy due to a change in volume, denoted by $W_v$, to both sides of equation (7) and then noting that

$$W_v = \frac{Ed\tilde{V}^2}{6(1 - 2\nu)}$$

and that the total energy density $W = W_d + W_v$. We obtain an alternative form of the modified von Mises yield criterion, viz:

$$W = AdV + B + Ed\tilde{V}^2/6(1 - 2\nu) = F(dV)$$

For any given material the function $F(dV)$ should be experimentally determined. This equation has been used in [24] to design bonded repairs to damaged metallic components.

If we replace $F(dV)$ by $W_v$ we see that both equations (1) and (15) represent possible alternative extensions of the modified von Mises yield criterion.

The concept of a critical energy density forms the basis for another approach to the design of adhesively bonded joints, see [13]. In this approach the true stress-strain curve is replaced by a simpler curve with a constant post yield slope. This idealized curve is chosen to have the same energy density as the true stress-strain curve. It is also assumed that, for a symmetric joint, the only stresses in the adhesive are shear stresses. These approximations allow a closed form analytical solution to be obtained. In this approach the shear strain in the adhesive is chosen as the critical design variable. However, because of the choice of the idealized stress-strain curve there is a one to one relationship between shear strain and energy density.
2.3 Energy Release Rate Approaches

For mode I self similar crack growth of a through crack in an isotropic body in the absence of body forces Atluri [27] showed that the energy release rate $G$ can be written as,

$$G = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} [W n_i - t_i u_{i,1}]ds$$

(16)

where $n_i$ is the component of the unit normal to the path in the $x_i$ direction, $t_i$ is the traction tensor defined as $t_i = n_i \sigma_{ij}$, $W$ is the strain energy, $u_i$ is the displacement tensor and $\Gamma_\varepsilon$ is a vanishing small path surrounding the crack tip.

It has also been shown [27] that for a general problem this integral will not equal the value of $G$ obtained from the movement of load points method, i.e.,

$$G \neq \frac{P^3}{2B} \frac{\partial C}{\partial \alpha^2}$$

(17)

where $P$ is the applied load, $C = \delta / P$, $\alpha$ is the movement of the load points, $a$ is the crack length and $B$ is the thickness of the structure.

This formulation was extended to composite materials by Jones [28]. In this case $G$ is given by the same expression as in (16) with $W$ replaced by the energy density for a composite material, i.e.,

$$W = \frac{1}{2} t_{ijkl} C_{ijkl} - \beta_{ij} \epsilon_{ij} (T - T_0) - \phi_{ij} \epsilon_{ij} (M - M_0) + C_1 (T, M)$$

(18)

where $M$ is the mass flux of moisture per unit volume, $\beta_{ij}$ and $\phi_{ij}$ are the coefficients related to the thermal and moisture expansion coefficients of the body respectively, $T_0$ and $M_0$ are the reference temperature and moisture content respectively, $C_{ijkl}$ is the stiffness tensor and $C_1$ is a function of the temperature and moisture.

This formulation assumes that the terms in the integrand which are functions of temperature and moisture alone integrate to zero. As shown in [29] the existence of body forces, which for matrix dominated problems arise due to closure [30], again result in compliance measurements giving inappropriate measurements of $G$.

In three dimensions the integral on the right-hand side of equation (16) no longer equals $G$ and is referred to as $T^*$, see [27]. Approaches based on energy release rate considerations have been widely used in characterising matrix failure [31,32,8,30]. However, as mentioned above, it must be emphasised that compliance measurements do not always characterize near-tip behaviour. This is particularly true in the case of combined mechanical and environmental loading, see [28,33]. In this case the equation governing the coupling between mechanical deformation and thermal energy is given by,

$$-\kappa_{ij} + \rho q \frac{\partial}{\partial T} = \rho C_T T - (\epsilon_{ij} \frac{\partial C_{ijkl}}{\partial T} - \frac{\partial}{\partial T} (\beta_{ij} (T - T_0)) - \frac{\partial}{\partial T} \phi_{ij} (M - M_0)) \epsilon_{ij}$$

(19)

where $\kappa_{ij}$ is the heat flux tensor, $q$ is the heat absorbed when 1 gram of moisture is absorbed by 1 gram of material and $\rho$ is the density, see [28]. Experimental confirmation of this formulation is given by Wong [34] for the special case when the
material is isotropic. If the material is cyclically stressed at a frequency such that
adiabatic conditions hold then

\[ p\dot{M} - \rho C_v \dot{T} = - (\varepsilon_{ij} \frac{\partial C_{ijkl}}{\partial T} - \frac{\partial}{\partial T}(\delta_{ij}(T - T_0)) - \frac{\partial}{\partial T} \sigma_{ij}(M - M_0))T^{ij} \]  

(20)

Let us now consider a bar subjected to uniaxial strain,

\[ \varepsilon_{22} = \Delta \cos \omega t \ \epsilon = 0 \text{ if } i, j \neq 2 \]  

(21)

substituting into (20) gives,

\[ p\dot{M} - \rho C_v \dot{T} = (\beta_{22} \Delta \cos \omega t + \frac{1}{2} \frac{\partial E_{22}}{\partial T} \omega^2 \Delta \sin^2 \omega t)T \]  

(22)

and integrating with respect to \( t \) we obtain:

\[ p\Delta M - \rho C_v \Delta T = (\beta_{22} \Delta \sin \omega t - \frac{1}{4} \frac{\partial E_{22}}{\partial T} \Delta \omega^2 (1 - \cos 2\omega t))T_0 \]  

(23)

This shows that although the load is applied at a single frequency \( \omega \) the temperature and moisture fields have a response at both \( \omega \) and \( 2\omega \) and that the amplitude of the response at frequency \( 2\omega \) is proportional to the square of the local strain. Hence for a problem involving environmental effects the load point behaviour, which only reflects the behaviour at the applied frequency \( \omega \), cannot provide information on the component of the near-tip field which is responding at a frequency of \( 2\omega \). Furthermore the near-tip component has an amplitude proportional to \( \Delta \varepsilon^2 \) and can thus be expected to be very large. Near the tip the large strain field, theoretically infinite, means that adiabatic conditions are never realized so that the fatigue behaviour will depend on test frequency. Indeed this has been confirmed by a series of recent laboratory tests [35].

The existence of a response in the temperature field at frequencies \( \omega \) and \( 2\omega \) has been confirmed experimentally in [36] using thermal emission techniques.

It must be stressed that energy release rate approaches are best used when growth is co-planar and self-similar. In other circumstances they must be used with caution [28,37]. An alternative approach, which is related to the energy release rate method, was suggested by Watanabe [38]. This method uses the first term in the expression for \( T^* \), which we will define as \( W_e \), to characterize the damage, viz:

\[ W_e = \lim_{r \to 0} \oint_{\Gamma_r} Wdy \]  

(24)

This formulation has not, as yet, been widely used but when applied to the failure of a damaged fastener hole under both thermal and mechanical loads appeared to correlate with the observed failure load, see [28]. The parameter \( W_e \) coincides with \( T^* \) if the delamination has a blunt tip. Further work is required before this can be considered a valid fracture parameter.

A detailed review of current energy release rate test methodologies is given in [2], whilst certain problem areas arising when using energy release rate approaches are highlighted in [37].
It is interesting to note that the point stress and average stress criteria can also be used to obtain estimates of the energy release rate using either double cantilever or centre-notched test specimens, see [10,11]. The values of \( G_c \) obtained in this way tend to be higher than those obtained from compliance measurements, see [2,10].

It is well known that for metals the critical stress intensity factor \( K_c \) is strongly dependent on the level of local restraint. Indeed recent tests at the University of Melbourne [39] have shown that \( K_c \) for an 8mm thick 2024 T6 aluminium alloy is approximately 82 MPa\( \sqrt{m} \) whilst its plane strain value is approximately 30 MPa\( \sqrt{m} \). This corresponds to an eight fold increase in \( G_c \). However, this effect is often overlooked for composites. Similarly in a structure the level of restraint is variable and that experienced by an edge delamination may be quite different to that experienced by a delamination surrounding a fastener or a joint. It is clear that a methodology is required to quantify this level of restraint. One such methodology based on the local change of surface and volume energies is presented in [40].

In the case of a crack lying entirely within an adhesive layer the energy release rate \( G \) can be related to the energy density \( W \) by,

\[
G = \frac{2(1-\nu)}{(1-2\nu)} r_c W
\]

However, as seen in section 2.2, the critical value of \( W \) may be dependent on \( dV \). This infers that \( G_c \) may be a function of \( dV \). In the case of a delamination in a composite it is well known that \( G_{2c} \) is much larger than \( G_{1c} \), see [2]. It is therefore tempting to conjecture that,

\[
G_c = G_{1c} \frac{dV}{dV_{1|\infty}} + (1 - \frac{dV}{dV_{1|\infty}}) G_{2c}
\]

where \( dV \) is the value of \( dV \) at \( r = r_c \) which occurs when \( G_c = G_{1c} \). However, this expression lacks a physical basis and its usefulness in describing mode III failure is questionable. To date the majority of methods used to determine \( G_c \) for mixed mode failure are empirical and require a knowledge of the individual mode I, II and III energy release rates as well as their respective failure energies.

3. MATERIAL NONLINEARITIES

In the previous sections it has been assumed that, up to failure, the material is behaving elastically. However a recent investigation indicated that, prior to failure, there was significant dissipated energy [41]. To confirm this a series of tests was performed on a 56-ply XAS-914C laminate with a ply configuration of \([\pm 45/02]_T\). Six specimens with dimensions approximately 315 by 6.0 by 4.8 mm were prepared and loaded in an 500 kN Instron testing machine at a loading rate of 600 kN/sec. This rate of loading was chosen in an attempt to achieve adiabatic conditions.

For each specimen the temperature of a centrally located point on the surface of the specimen was measured using an infrared detector. A typical non-dimensionalised temperature response is shown in Figure 3. In each case failure of the specimen did not occur near the point at which the temperature was measured. Details of the test methodology are given in [42].

From Figure 3 it is seen that the measured temperature response is similar to that for metals [43]. There is an initial cooling during the linear regime, which for
metals is described by Kelvin's law, followed by extensive heating. The amount of heating is a direct measure of the dissipated energy, and as such supports the results given in [41].

As a result of this investigation it is clear that further work is necessary in order to understand the role of dissipated energy in the failure process.

4. CONCLUSION

This paper has outlined several methods currently used for analysis of the matrix dominated failure in composite materials and adhesively bonded joints. Particular attention has been given to the relationship between these approaches.

Attention has also been focused on the role of dissipated energy in the failure process. This is a topic which is not, as yet, fully understood and with the advent of more ductile matrix materials requires greater attention.
REFERENCES


FIGURE 1: FAILURE ENVELOPE FOR AS/3501
FIGURE 2: PLOT OF CRITICAL AVAILABLE ENERGY DENSITY
FIGURE 3: TEMPERATURE RESPONSE OF A POINT ON A GRAPHITE EPOXY SPECIMEN LOADED ADIABATICALLY
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