LASER SENSING FOR IDENTIFICATION AND CONTROL OF DISTRIBUTED PARAMETER SYSTEMS (U) STATE UNIV OF NEW YORK AT ALBANY RESEARCH FOUNDATION D J INMAN 81 MAY 88

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This instrumentation award funded the purchase of a laser vibrometer system, mass computer data storage and data acquisition equipment. This equipment used in conjunction with existing vibration testing and control facilities provides a sophisticated low frequency velocity measurement system for use in identifying the coefficients in partial differential equation models of distributed mass structures. In addition, the vibrometer system provides straightforward and direct velocity feedback for such systems. These flexible structures characteristically have very low natural frequencies which cannot be detected by accelerometers. This system has and is being used to perform tests on models and sub-assemblies of large space structures for the purpose of evaluating existing identification and control strategies as well as to stimulate new research in the area of control, observers (estimators) and identification.
Introduction Research and testing performed with the equipment purchased under this instrumentation award during the first year of operation consists of three projects. The first was an identification of various damping mechanisms in a damped beam made of quasi isotropic composite material. The second was an identification of nonlinearities in a joint used in the ABES structures. The third was in conjunction with accelerometer measurement used to verify a combined velocity and acceleration observation method. Each project is briefly described in the following:

Damping Identification Several intense experiments using the laser vibrometer were performed to measure the response of a quasi isotropic cantilevered beam with a removable tip mass excited by an impulse at various locations. This data was collected, stored, and sent to AFOSR researchers at Brown University using BITNET. The data was analyzed in reference 1 using a spline based estimation procedure, starting with a partial differential equation model of the structure. A clear advantage over modal methods based on a finite dimensional model of the same system was observed, this is described in reference 1, which is attached.

Nonlinearity In response to a query from the Air Force Weapons Lab, a single bay of the ABES structure was tested using the laser sensing system. The goal here was to identify the nonlinearities that occurred in this structure to the presence of the joints. It was found that if the joints are properly torqued, there exists a reasonable level of amplitude and
a reasonable frequency range for which the system behaves in a linear fashion. This is
detailed in reference 2, which is also attached.

**Combined Observation**  A two signal approach to observation was proposed and
tested. This method uses an accelerometer and the laser system to simultaneously measure
both velocity and acceleration. With this information it is illustrated that the sampling
theorem can be violated and correct modal data can still be identified. This is discussed in
reference 3.

**Control Experiments**  Several control experiments are currently under way using the
laser vibrometer system. However, no results are currently available. The experiments
under way are 1) vibration control of a slewing beam using laser feedback and 2) structural
control of a truss using velocity feedback and proofmass actuators.

**References**

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2. Banks, H.T. and Inman, D.J. "An Experimental Investigation of Nonlinearities in the
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An Experimental Investigation of Nonlinearities in the Joints of the ABES Structure

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Introduction A single bay of the ABES test structure was experimentally investigated for the purpose of clarifying nonlinearities thought to be present in the joints. The joints of the ABES structure are a threaded male/female configuration with a common nut used to lock the structure’s threaded members into the joints. Joints of this type are typical to most large space structures and are generally thought to be a source of non linearity, making linear system identification of such structures impossible.

In most cases it is desirable to be able to use linear mathematical models for both identification and control of a structure. Hence the joints of the ABES structure are investigated experimentally for nonlinear behavior in hopes of finding and classifying regions of linear behavior.

Experimental Procedure In order to isolate the effects of the joint, a single joint with two members was configured for testing. This arrangement was suspended as indicated in figure 1.

Experimental Configuration for Joint Tests
An impact hammer (Kistler model number 9724A500 with charge amplifier model number 5004) was used as input to the test structure. Two accelerometers (Kistler model number 8628A5 with charge amplifier model number 5004) were used to measure the resulting acceleration. The output signals were calibrated against and monitored by noncontacting velocity measurements taken by a DANTEC Laser Vibrometer System (consisting of a Hughes Laser model number 3225H/PC, a DANTEC tracker model number 9055 No 214 and a DANTEC frequency shifter model number 9055 No 012). In all reported data, the laser velocity measurements agreed with the accelerometer measurements. A GENRAD 2515 sixteen channel analyzer was used to provide the data and calculate the appropriate transfer functions.

Several combinations of input output transfer function measurements were made. The effect of impulse amplitude and bolt torque (the joint tightness) on the resulting non linearity of the frequency response was examined.

All experiments were performed in the Mechanical System Laboratory at SUNYAB, in Buffalo, New York.

**Nonlinear Behavior** The measurement of and parameter identification of structures which exhibit nonlinear behavior is complicated by several effects. First, a fundamental assumption in the input/output measurement of linear structures is that the transfer function between any two points on the structure is the same regardless of which point is chosen as the input and which point is chosen as the output. This assumption is used extensively in standard modal testing for linear structures to reduce the number of calculations needed in constructing multiple degree of freedom models of large structures.

A second nonlinear phenomenon that leads to severe error in linear measurement and identification is that of secondary resonance. The common approach to structural identification in linear systems is to measure the frequency response of the test structure. The peaks that show up in the magnitude versus frequency plots are interpreted as resulting from the resonance phenomena. Hence each peak is assumed to occur at a natural frequency of the structure and the number of peaks then indicates the number of degrees of freedom one should use in a lumped mass multiple degree of freedom model of the test structure. In fact, a single degree of freedom linear oscillator equation is curve fit to each peak. In
this way, the parameter estimation yields a modal damping ratio and an undamped
natural frequency for each peak assumed to be a single degree of freedom model.
In contrast, the force response of a single degree of freedom nonlinear system
produces an amplitude versus frequency plot with multiple frequencies. This
inherently nonlinear characteristic is referred to as secondary resonance - either
superharmonic of subharmonic depending on the frequency range of interest. These
secondary resonances cause linear parameter estimation methods to predict incorrect
parameters as well as a false model order.

To illustrate the existence of subharmonics and superharmonics in the testing
of structures consider a simple single degree of freedom model of the structure with
mild nonlinearity in its stiffness term. A well studied equation is Duffings' equation,
given by

\[ \ddot{X}(t) + 2\mu \dot{X}(t) + \omega_0^2 X(t) + \epsilon \alpha X^3(t) = F_0 \cos \omega t \]  

(1)

Here \( X(t) \) represents the displacement, \( \dot{X}(t) \) the velocity and \( \ddot{X}(t) \) the acceleration.
The quantities \( \epsilon, \mu \) and \( \alpha \) are constants, \( \mu \) indicating the damping, \( \alpha \) the strength
of the nonlinear stiffness and \( \epsilon \) a scaling factor. The driving frequency is \( \Omega \), the
magnitude of the forcing input is \( F_0 \) and \( \omega_0 \) represents the linearized systems
undamped natural frequency.

The solution of equation (1) can be obtained, using perturbation methods, for
a range of driving frequencies. First, consider the steady state response of the
system of equation (1) to input driving frequencies near three times the linear
undamped natural frequency. This is represented by \( \Omega = 3\omega_0 + \epsilon \sigma \), where \( \sigma \) is a
small parameter used to define the frequency range of interest.

The response of (1) can be shown to be of the form

\[ X(t) = a(t) \cos \left( \frac{\Omega}{3} t - \frac{1}{3} \gamma(t) \right) - \frac{F_0}{\omega^2 \cdot \omega_0^2} \cos \omega t + O(\epsilon) \]  

(2)

Here, \( O(\epsilon) \) represents all those terms of order \( \epsilon \) and higher, and the
functions \( a(t) \) and \( \gamma(t) \) vary slowly with time. The frequency content of \( X(t) \) reveals
oscillation at two frequencies rather than one as in the linear case. The
appearance of the second frequency of oscillation is called a subharmonic oscillation.
because it occurs at 1/3 of the driving frequency. A standard modal analysis of such a system would imply a two degree of freedom system with natural frequencies of \(\omega_0\) and \(\omega_0/3\), rather than a nonlinear single degree of freedom system.

Likewise, examination of the solution of the steady state response of (1) around the driving frequency \(3\omega = \omega_0 + \epsilon\omega\) yields a solution of the form

\[
X(t) = a(t) \cos [3nt + \gamma(t)] + \frac{E_0}{n^2 \cdot \omega_0^2} \cos nt + O(\epsilon)
\]

(3)

In this case a superharmonic appears in the solution. An identification of the frequency content of this signal would contain the natural frequency and the superharmonic frequency and again yield an erroneous linear result.

The following experimental results represent a first attempt to clarify the linear range with both frequency and amplitude of excitation for the joints in the ABES structure.

**Results** As mentioned above, one clear sign of the presence of nonlinearities is the violation of the condition for linear transfer function that the frequency content of the transfer function with point A as input and point B is output should be the same as the transfer function with point B as input and point A is output. The top plot of Figure 2 illustrates the magnitude of the transfer function with A as input and B as output. The top plot of Figure 3 is the magnitude of the transfer function with B as input and A as output. The frequency content in the lower frequency range of 0 to 900 HZ appear the same. However, in the frequency range above 1000 HZ the two spectra are quite different indicating that the joint is nonlinear (joint torqued to 2 ft. lbs.) in the higher range with excitation frequencies.

The next set of tests examine the effect of input amplitude on the joint transfer function near a primary resonance (about 255 HZ) with the joint torqued to 2 ft. lbs. The amplitude of the impulse to the joint increases starting with figure 4 and moving to figure 6. Note that at some (not measured) level of input amplitude super and sub harmonics occur. This is clearest in figure 6. The existence of super and sub harmonics is of course a clear indicator of nonlinear behavior.
Figure 7 illustrates the same frequency range and fundamental harmonic as figures 4, 5 and 6. However in this case, the bolts in the joints are only tightened to 1 ft. lb. of torque instead of 2 ft. lb. Again, this configuration produces a variety of super and sub harmonics indicating nonlinear behavior.

The third set of experiments tests the joint linearity for high levels of joint torque (35 ft. lbs.). Figure 8 illustrates a low amplitude hammer hit and figure 9 illustrates a high amplitude hammer hit. The resonant frequencies for the high torque joint remain the same indicating the for high torques the ABES structure behaves in a linear fashion.

**Summary** The joints for the ABES structure are definitely capable of illustrating nonlinear behavior. The nonlinearities in the joint are a function of three factors 1) input amplitude 2) torque or joint tightness and 3) frequency range. If it is desired to model the ABES structure with a linear model it is necessary to use evenly torqued joints of about 35 ft. lbs. and to realize that at some level (high amplitude) of disturbance excitation the linear model will fail for high frequency excitations.

It is the conclusion of this study that the ABES structure can be effectively modelled with a linear model for lower amplitude disturbance provided all the joints are properly torqued.
**Figure 2:** Frequency Response with input at point A and output measured at point B

**Figure 3:** Frequency Response with input at point B and output measured at point A
Figure 4: Frequency Response with low amplitude input and joint torqued to 2 ft. lbs.

Figure 5: Frequency Response with medium amplitude input and joint torqued to 2 ft. lbs.
Figure 6: Frequency Response with heavy amplitude input and joint torqued to 2 ft. lbs. Sub and super harmonics appear.

Figure 7: Frequency Response with medium amplitude input and joint torqued to 1 ft. lb. Sub and super harmonics appear.
PARAMETER IDENTIFICATION TECHNIQUES FOR THE ESTIMATION OF DAMPING IN FLEXIBLE STRUCTURE EXPERIMENTS

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Abstract

We report on our use of spline based inverse procedures to estimate damping coefficients in distributed parameter systems for flexible structures. Damping models involving viscous (air) damping and Kelvin-Voigt damping in an Euler-Bernoulli framework are used to analyze data from vibration experiments with composite material beams.

I. Introduction

In this presentation we report on continuing efforts that we have pursued for the past year. These efforts involve a combination of experimental investigations with the simultaneous development of the mathematical and computational aspects of a theoretical methodology to support the experiments.

Our long term goals include the understanding of damping mechanisms in complex distributed structures constructed from composite materials. Our quest has obvious motivation from and potential consequences for the design of control (active and passive) systems for large flexible space structures.

The initial efforts on which we report here entail the testing and development of spline based approximation techniques for inverse problems arising in attempts to model and quantify damping in composite material beams. Substantial previous mathematical efforts along with numerical tests on simulated data can be found in the literature (see [BCR], [BR] for a sample of some of these results and references to other work).

We briefly outline the underlying mathematical ideas, describe the experimental equipment employed, and then report on two representative experiments and our findings using our techniques with the data from these experiments.

II. Mathematical Foundations

The basic mathematical model that we have considered in connection with the efforts on which we report in this note consists of the equations for a cantilevered Euler-Bernoulli equation with tip mass, damping, and an applied transverse force. Specifically we take as our model the system for transverse vibrations of a long slender (length, $l$) cantilevered beam:

$$\rho \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} + c_D \frac{\partial u}{\partial t} + \gamma \frac{\partial u}{\partial x} = f(x,t), \quad 0 < x < l, \quad t > 0,$$

$$u(0,t) = 0, \quad t > 0$$

(2.1)

$$EI \frac{\partial^2 u}{\partial x^2} (t,t) + c_D \frac{\partial u}{\partial t} (t,t) = 0, \quad t > 0$$

(2.2)

$$EI \frac{\partial^4 u}{\partial x^4} (t,t) + c_D \frac{\partial^2 u}{\partial x^2} (t,t) = \gamma \frac{\partial u}{\partial x} (t,t), \quad t > 0$$

(2.3)

$$u(0,x) = \phi(x), \quad \frac{\partial u}{\partial x} (0,x) = \phi(x), \quad 0 < x < l.$$ (2.4)
of pultruded quasi-isotropic composite used by DOD and NASA in studies of potential materials for the proposed Space Station [WM]. The beams rectangular cross sections measured on the order of 6 mm by 100 mm provide a lower cost, higher specific strength alternative to extruded aluminum alloy structures.

The experiments consisted of recording the vibrations after a free release from an initial bent configuration or after an impulse was imparted to a resting beam using a Kistler Instrument Corporation force hammer, instrumented with a load cell to provide measurements of the input force magnitude. Kistler accelerometers (Model 8628 piezo beam) were attached to the free tip of the beams. The data acquisition was performed with a GENRAD 2515 analyzer.

Time response data was collected and sent via BITNET to Brown University for use in the above described algorithms. A copy of data was also used as input to a standard modal analysis package [SDRC] residing in the GENRAD as well as an Eigenystem Realization Algorithm residing in a VAX785.

The modal analysis results were then interpreted in terms of the partial differential equation model given by equations (2.1) - (2.5) above. The measured natural frequencies were used as data in a simple average to produce initial estimates for E. The damping parameters were calculated by using an appropriate number of modal damping ratios. This procedure, detailed below, represents a relatively standard engineering approach to estimating the parameters in a vibration test of a distributed mass structure. These parameters were then used in a numerical simulation of the response and compared with the response predicted by the spline based methods presented here. Note, that this modal analysis procedure, however, requires that closed form solutions of the frequency equations be available. This is a shortcoming not shared with the spline based procedures.

IV. Results

We present our findings and some preliminary conclusions on two representative examples. Further discussions and results from other experiments will be given in a paper currently in preparation.

Example FR1.TM86

In this experiment we used a composite material beam of length 1m, width .102m, thickness .006m with collocated tip mass and accelerometer. The linear mass density was \( \rho = 0.021546 \) slug/ft and the tip mass was \( m = 0.07182 \) slug. The beam was cantilevered at \( x = 0 \); it was displaced from equilibrium (with this initial displacement measured for use as initial data). At time \( t = 0 \), the beam was released and the free oscillations were then observed; 2048 time observations at \( \Delta t = 0.007813 \) seconds intervals were recorded. Thus time domain data over the period \([0,16] \) seconds were available. Upon taking the FFT of this data, we found that only one mode was observable in the oscillations; this was at 1.875 Hz. For our model we used (2.1) - (2.5) with \( E1 \) calculated with experimental modal information to be 126.7083 lbft².

In our initial parameter estimation efforts, we tried to fit the data over \([0,2] \) seconds with a model featuring only viscous damping (i.e. \( c1 = 0 \)). With our spline inverse procedures we found \( \gamma = 0.0315 \) slug/(ft sec) and obtained the model response depicted in Figure 1.

![Figure 1](image1)

We also used modal techniques to try to recover a value for \( \gamma \) in (2.1) - (2.5). That is, we used standard modal techniques to estimate \( \gamma \) the % critical damping on the mode excited, as \( \gamma = 0.269 \). Since one can readily argue that \( \gamma = 2\sqrt{E1\rho}/3 \), this approach yields the calculated value \( \gamma_{cal} = 0.0826 \). Using this value in the model, one does not obtain a very good fit to the data. This is shown in Figure 2.

![Figure 2](image2)
However, these initial findings are somewhat misleading. Even the spline inverse procedures don't yield a good fit of the model (2.1) - (2.3) with only viscous damping if we consider the time domain response over a long period of time. The best results we could obtain are depicted in Figure 3 where we note also we optimize over mass (this was done to adjust the m value since the "tip mass" was actually a small tip body centered at x = .963m or 3.2cm from the tip end). The "optimal" value of γ was \( \bar{\gamma} = 0.015174. \)

Several important features of our findings here are worth noting. (i) This is a "1 mode - 1 damping parameter" model fit in either case. Our findings reveal that such a situation does not automatically lead to success. If the wrong damping model is hypothesized, even good quantitative methods won't yield satisfactory results. (ii) It is important to use the data over a long time interval whether we are using FFT data or time domain data.

Example IR3.1NTM1186

Again a composite material beam of length 1 m was used. The width of this beam was .104m, the thickness .00524m. No tip mass was attached but as usual a tip accelerometer was available for data collection. We observed the response of the beam after it was hit with an impulse hammer (soft tip) at \( x = 0.7m. \) Three dominant modes were observed in the response, at 3.7Hz, 23.3Hz and 61 Hz, respectively. The beam parameters were \( \rho = 0.2144 \) slug/ft and \( EI = 107.103 \) lb-ft² (calculated approximately using modal information). We first optimized on \( EI \) to obtain a frequency match between our model and the FFT of the data. A value of \( \bar{EI} = 109.2367 \) lb-ft² was obtained. Modelling of the input (f in equation (2.1)) is in general, nontrivial since it is not strictly an impulse. We used the input signal (from a transducer in the hammer) to model the shape, height, and duration of the input to the beam. We then used our spline inverse procedures to estimate the two damping parameters \( \gamma \) and \( c_D \), obtaining values \( \gamma = 0.004235 \) slug/(ft see) and \( c_D = 0.005385 \) slug ft/sec. The corresponding time domain response for the model and the FFT are shown in Figures 5 and 6. As is obvious, we obtained a very good fit to the data (with 3 modes excited) using only two damping mechanisms in (2.1) - (2.5).
If we try to estimate $\gamma$ and $c_{i1}$ using modal techniques (via % critical damping and principles relating $\gamma$ to $\gamma$ and $c_{i1}$ similar to that noted in the discussion of the previous experiment), we obtain values $\gamma_{\text{est}} = .2982$ slug/(ft sec) and $c_{i1,\text{est}} = .4511$ slug ft/sec. If we use these in the model equations (2.1) - (2.5), we find that the response is damped out much too quickly and does not resemble the experimental data.

A final comment on our findings in this example has important implications for use of modal methods in attempts to understand damping mechanisms in distributed parameter structures. While we found that the viscous damping ($\gamma$) was extremely important in the overall modeling of the low frequency mode and the Kelvin-Voigt damping ($c_{i1}$) was most important in modeling correctly the two high frequency modes, the damping does not decouple on the modes. We believe this poses serious difficulties for any attempts to understand damping mechanisms using only modal techniques.

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References


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