Contract N00014-84-K-0583

HEATING EFFICIENCY OF BEAT WAVE EXCITATION IN A DENSITY GRADIENT

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PPG-1130 February 1982

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LOS ANGELES
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IN A DENSITY GRADIENT

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February 1988

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A simple model is presented which yields analytical expressions for the heating efficiency of beat-wave excitation in a plasma with a linear density profile. The effect of self-consistent Landau damping by tail electrons is included without recourse to WKB approximations.
Presently there is renewed interest in beat-wave excitation\(^1\) of Langmuir waves to generate fast electrons, two possible applications being advanced particle accelerators\(^5\) and modification of the earth's ionosphere. Although the description of this process in uniform plasmas is relatively well understood, the presence of a density gradient introduces several complications which have been handled in a WKB sense in the pioneer work by Rosenbluth and Liu\(^2\) and in computer simulations by Cohen, et al\(^4\). Because in certain experiments the role of density gradients may not be negligible or in fact may be useful in achieving a desired result, it is convenient to obtain analytical expressions which explicitly exhibit the dependence of the heating efficiency on the density scale length \(L\). In this brief note we present a relatively simple model which yields such a result and incorporates the self-consistent effect of Landau damping by fast electrons without recourse to WKB approximations. It is hoped that this result may be useful in estimating the performance of future experiments and that the model may provide a basis for more elaborate calculations of analogous phenomena in nonuniform plasmas.

The one-dimensional model consists of separating the roles of warm background electrons having a spatially varying density \(n_0(z)\), and fast tail electrons whose density \(n_t\) is considered to be uniform (because of their longer collision mean free path) over the region where the relevant wave-particle interactions occur. The tail density is assumed small \((n_t/n_0 \ll 1)\) so that the principal contribution of the fast particles enters through Landau damping, while the warm background electrons determine the propagation features of the driven Langmuir wave. The background electrons are treated as a warm fluid with thermal velocity \(\tau\) and the tail electrons are described kinetically assuming an unperturbed tail distribution.
\[ f_{ot} = \frac{n_t}{(2\pi v t^2)^{1/2}} \exp[-v^2/2v_t^2] \]  

Beat excitation is envisioned to arise from the beat ponderomotive force acting on the background electrons and caused by two transparent electromagnetic waves having frequencies and wave numbers \( \omega_j, k_j \); \( j = 1,2 \). These waves propagate along the nonuniform direction \( z \) and can individually move in the direction of decreasing density \( (k_j > 0) \) or towards increasing density \( (k_j < 0) \). The self-consistent beat excited longitudinal electric field has a harmonic time dependence \( E(z) \exp(-i\omega t) \) with \( \omega = \omega_2 - \omega_1 \), and is determined from Poisson's equation

\[ \frac{3}{\hbar^2} [E(z) - E_0(z)] = -4\pi e [\tilde{n}(z) + \tilde{n}_t(z)] , \]  

in which the effective pump source \( E_0(z) \) for beat excitation is

\[ E_0 = -\left( \frac{e}{mc} \right) \left( \frac{\omega}{\omega_1 \omega_2} \right) E_{j1} E_{j2} , \]  

where \( E_{j0} \) is the complex amplitude of the \( j \)th electromagnetic wave, \( e \) and \( m \) are the charge and mass of an electron, and \( c \) is the speed of light. In Eq. (3) the frequencies of the electromagnetic waves are assumed high enough that their phase velocities are essentially the speed of light. The perturbed charge densities (oscillating at frequency \( \omega \)) for the background \( \tilde{n}_o \) and tail \( \tilde{n}_t \) are calculated from the fluid and Vlasov equations, respectively.

Introducing the Fourier transform of the driven field

\[ E(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \tilde{E}(k) e^{ikz} , \]
and assuming a linear density profile with scale length \( L \), i.e.,
\[
l - \omega_p^2(z)/\omega^2 = z/L,
\]
where \( \omega_p(z) \) is the local electron plasma frequency, transforms Eq. (2) into a differential equation in \( k \)-space
\[
\frac{i}{L} \frac{d}{dk} \tilde{E}(k) + \left[ \frac{d}{dk} g(k) - \frac{3k^2}{k_D^2} \right] \tilde{E}(k) = 2\pi k_0 \delta(k - k_b),
\]
(5)

with \( k_b = k_2 - k_1 \) the beat wave number, \( k_D = \omega/\nu \), and
\[
g(k) = \sqrt{\frac{n}{2}} \left( \frac{\pi \nu}{n_0} \right) \left( \frac{\nu}{\nu_L} \right) k_D \, \text{sgn}(k) \exp\left[ - \frac{\omega^2}{2k^2 \nu_L \gamma} \right],
\]
(6)

where \( \text{sgn}(k) = 1 \) for \( k > 0 \), \(-1\) for \( k < 0 \).

Equation (5) can be integrated to yield the spectrum
\[
\tilde{E}(k) = -2\pi \epsilon_0 L \delta(k - k_b) \exp\left[ F(k) - F(k_b) \right],
\]
(7)

\[
F(k) = -i \frac{k^3 L}{k_D^2} - g(k) L,
\]
(8)

where \( \delta(k) \) is the Heaviside step function.

The sign of \( k_b \) determines the direction in which the directly driven Langmuir wave propagates. For \( k_b > 0 \), propagation occurs in the direction of decreasing density and results in a single wave whose phase velocity decreases as \( z \) increases (in the WKB sense as \( \sqrt{3} \, \nu \, (z/L)^{-1/2} \)). For \( k_b < 0 \) the spatial pattern consists of two waves. The directly driven wave propagates towards increasing density and upon reaching the cut-off point \( (z = 0 \text{ where } \omega_p = \omega) \) exhibits partial (because of Landau damping) reflection and thus generates a second wave which behaves analogously to the one directly driven for \( k_b > 0 \). These features imply that for \( k_b > 0 \), tail electrons with velocity \( \nu > 0 \) are
accelerated and this results in an increase of the heat flux $\delta Q_+$ as $z \to \infty$.

However, for $k_b < 0$, tail modification occurs for both $v < 0$ and $v > 0$ particles, and causes heat flux modifications in both directions. The self-consistent modifications in the time-averaged tail distribution function, accurate to second-order in the amplitude of the longitudinal electric field, are

$$\langle \delta f^+_L (z = \pm \infty, v) \rangle = (\frac{e}{2m})^2 \frac{3}{v} \frac{\partial}{\partial v} \left| \bar{E} (k = \frac{\omega}{v}) \right|^2 \frac{3}{v} \frac{\partial}{\partial v} f^+_0 (v) , \quad (9)$$

where the label $\pm$ refers to particles moving in the positive or negative $z$ direction.

In determining the efficiency of energy transfer to the plasma the quantities of interest are the asymptotic heat fluxes

$$\delta Q_\pm = \pm \int_0^\infty dv \frac{mv^3}{2} \langle \delta f^+_L \rangle , \quad (10)$$

which using Eq. (9) and integrating by parts can be put in the form

$$\delta Q_\pm = \pm \frac{e^2}{4m} \int_0^\infty dv \left| \bar{E} (k = \frac{\omega}{v}) \right|^2 \frac{3}{v} \frac{\partial}{\partial v} f^+_0 (v) . \quad (11)$$

From Eq. (6) it is seen that $g(k = \omega/v)$ is proportional to $f_0 (v)$ and hence $|\bar{E} (k = \omega/v)|^2$ depends exponentially on $f_0 (v)$. This implies that the integral in Eq. (11) can be evaluated analytically without approximations.

Defining the heating efficiency $\eta_\pm$ as the ratio of the enhanced tail heat flux to the Poynting flux $P_j = c |E_j|^2 / 8\pi$ of one (say $j = 1$) of the electromagnetic waves, it is found that for direct excitation with $k_b > 0$
where \( \alpha = (\pi/2)^{1/2} \left( \frac{n_t}{n_0} \right) \left( \frac{\bar{v}}{v_L} \right) (k_D L) \) measures the combined effect of tail density and scale length, and \( \xi = \omega/\left( \sqrt{2} k_D v_L \right) \). In Eq. (12) the background density \( n_0 \) is evaluated at \( z = 0 \).

In the limit of large \( \alpha \) (i.e., long scale length)

\[
\eta_+ = 2\pi \left( \frac{\omega}{\omega_j} \right)^5 (k_0jL) \left( \frac{P_2}{n_0mc^2} \right) \equiv n_0 \quad ,
\]

where \( k_0j = \omega_j/c \) since \( \omega_1 = \omega_2 >> \omega \). It should be noted that in this limit the heating efficiency is independent of tail parameters. In the opposite limit, i.e., \( \alpha \ll 1 \), corresponding to small tail densities and/or short scale lengths,

\[
\eta_+ = n_0 \alpha (1 - e^{-\xi^2}) \quad ,
\]

showing a scaling proportional to \( n_t \) and \( L^2 \).

For excitation towards increasing density, i.e., \( k_b < 0 \), the contributions from the two heat fluxes at \( z = \pm \) must be included in evaluating the efficiency, which yields

\[
\eta_- \equiv \frac{\delta Q_+ - \delta Q_-}{P_1} = n_0 \left[ 1 - \exp[-2\alpha(1 + e^{-\xi^2})] \right] \quad ,
\]

and for large \( \alpha \) reduces to \( \eta_- = n_0 \), as in the \( k_b > 0 \) case. In the small \( \alpha \) limit Eq. (15) becomes
\[ n_- = \eta_0 \alpha (1 + e^{-\xi^2}) \quad \text{(16)} \]

For transparent electromagnetic waves propagating in the same direction (i.e., \( k_1 \cdot k_2 > 0 \)) it is likely that in many applications \( \xi \gg 1 \). In this special case it is found that \( \delta_\xi \rightarrow 0 \), and thus

\[ n_+ = n_- = \eta_0 (1 - \exp(-2\alpha)) \quad \text{(17)} \]

The physics behind Eq. (17) is that particle acceleration results from the transit-time of \( v > 0 \) particles through the spatially varying envelope (and not the phase) of the beat excited wave.

In summary, a simple model has been developed that yields a compact analytic expression

\[ n_\pm = \eta_0 \{1 - \exp\{-2\alpha(1 \mp e^{-\xi^2})\}\} \quad \text{(18)} \]

for the heating efficiency of beat-wave excitation in the direction of decreasing (+) and increasing (-) density in a nonuniform plasma. This result includes nonuniform wave propagation and self-consistent Landau damping without recourse to WKB approximations.

The authors acknowledge useful discussions with Dr. W. Mori and Mr. S. Srivastava. This work is sponsored by the Office of Naval Research.
References


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