INFLUENCE OF SCATTERING ON SEISMIC WAVES:
PHYSICAL MECHANISMS CONTRIBUTING TO ATTENUATION IN THE CRUST

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Influence of Scattering on Seismic Waves: Physical Mechanisms Contributing to Attenuation in the Crust (unclassified)

M. Nafi Toksöz, Ru-Shan Wu, Denis P. Schmitt

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The mechanisms contributing to the attenuation of earthquake ground motion in the distance range of 10 to 200 km are studied with the aid of laboratory data, coda waves and strong motion attenuation measurements in the northeastern United States and Canada and theoretical models. The relative contributions to attenuation of anelasticity of crustal rocks (constant Q), fluid flow and scattering are evaluated. Scattering is found to be strong with albedo of $B_0 = 0.9$ and scattering extinction length of about 17 km. The intrinsic attenuation in the crust can be explained by a high constant Q (500 < $Q < 2000$) and a frequency dependent mechanism most likely due to fluid effects in rocks and cracks. A fluid-flow attenuation model gives a frequency dependence ($Q \propto f^{-5}$) similar to those determined from the analysis of coda waves of regional seismograms.
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Preface

The following document is the text of a paper titled "Physical mechanisms contributing to seismic attenuation in the crust" by M. N. Toksöz, R. S. Wu and D. P. Schmitt. This paper has appeared in the Proceedings of the NATO ASI "Strong Ground Motion Seismology, M. O. Erdik and M. N. Toksöz, eds., pp. 225-247, published by Reidel in 1987. The work was supported by the U. S. Geological Survey and the Air Force Geophysics Laboratory under contract F19628-86-K-0004; the paper is being submitted as a Scientific Report for this contract.
Introduction

The amplitude of seismic waves from an earthquake source decreases with increasing distance because of geometric spreading and because of attenuation resulting from the absorption and conversion of seismic energy into heat. Just like seismic velocities, the attenuation properties vary in the crust both as a function of depth and laterally. Generally, attenuation variations are larger by about one order of magnitude than the velocity variations. In this paper, we study the physical processes contributing to seismic attenuation in the crust. The primary mechanisms we consider are the anelasticity of crustal rocks, scattering due to heterogeneities, and fluid movements within pores and cracks in the crust. These are shown schematically in Figure 1.

We consider attenuation in the distance range of 10 to 100 km. This interval is ideal for several important reasons. At distances shorter than about 10 km from the source, non-linear behavior of materials due to high strains \((e \geq 10^{-5})\) can dominate. At distances greater than 100 km, the geometric spreading effects, due to velocity-depth functions and multiple branches of travel-time curves, become site-specific and uncertain. Another important factor for favoring this distance range is that a considerable amount of new attenuation data has been obtained both from strong motion records and the analysis of seismic coda waves.

Before reviewing the attenuation data, it is important to define the terminology. The attenuation for a given wave type \((P\) or \(S)\) is defined as the inverse of the quality factor \(Q\), and related to other measures by:

\[
\frac{1}{Q} = \frac{\alpha V}{\pi f} = \frac{\delta}{\pi}
\]

where \(\alpha\) is the attenuation coefficient, \(V\) the wave velocity, \(f\) the frequency, and \(\delta\) the logarithmic
Attenuation $Q^{-1}$ or the quality factor $Q$ are dimensionless quantities. Physically, $Q^{-1}$ is equal to the ratio of energy dissipated per cycle to the total energy. For small attenuation, (i.e. $Q^{-1} \leq 0.1$), additional relationships can be established in terms of stress-strain relationships:

$$\frac{1}{Q} = \frac{M_I}{M_R} = \tan \phi \approx \phi$$

(2)

where $M_I$ and $M_R$ are the imaginary and real parts of the appropriate elastic modulus ($M = M_R + iM_I$) and $\phi$ is the phase lag of the strain behind the stress (i.e., loss tangent). The dimension of the attenuation coefficient $\alpha$ is generally given as dB/unit length or nepers per unit length. The relationship between the two is $\alpha$(dB/unit length) = 8.686 $\alpha$(nepers/unit length).

Most of the data for crustal attenuation comes from coda waves (Aki and Chouet, 1975; Aki, 1980; Pulli, 1984; Singh and Herrmann, 1983; Singh, 1985; Gupta et al., 1983; Rautian and Khalturin, 1978; Roecker et al., 1982; Herrmann, 1980). These measurements generally give attenuation that decreases with frequency in the frequency range of $f = 0.5$ to 25 Hz. Some typical coda $Q$ values are:

$$Q_c(f) = 460f^{0.4}$$

(3)

for New England (Pulli, 1984);

$$Q_c(f) = 1000f^{0.2}$$

(4)

for the Central United States (Singh and Herrmann, 1983); and

$$Q_c(f) = 150f^{0.45}$$

(5)

for the Western United States (Singh and Herrmann, 1983).
The increase of $Q$ with frequency and the high values ($Q \geq 1000$) at frequencies above 10 Hz in the Eastern United States cannot be reconciled with the laboratory measurements of $Q$ in crustal rocks (see Toksöz and Johnston, 1981 for a comprehensive compilation). Most laboratory data suggest that, at least for dry rocks, $Q$ is independent of frequency (Birch and Bancroft, 1938; Peselnick and Outerbridge, 1961; Klima et al., 1964; Knopoff, 1964; Pandit and Savage, 1973; Toksöz et al., 1979; Nur and Winkler, 1980; Johnston and Toksöz, 1980; Tittman et al., 1981).

Water saturation generally decreases $Q$ values of both $P$ and $S$ waves, although the decrease is much greater for $S$-waves than for $P$-waves.

$Q$ increases with increasing confining pressure. However, the laboratory $Q$ values at pressures of 2 kilobars or more in crystalline rocks are generally less than 1000 (Klima et al., 1964; Bradley and Fort, 1966; Mason et al., 1970). It is only in the case of totally outgassed and volatile free rocks that $Q$ values of 2000 or more have been obtained (Clark et al., 1980; Tittman et al., 1974). These values have been observed in the completely dry environment of the moon (Dainty et al., 1976).

The Earth's crust is not free of water and volatiles and the high $Q$ values cannot be attributed to dehydration. The high $Q$ values still need to be explained.

Although $Q$ is independent of frequency in dry rocks, it may be frequency dependent in saturated and partially saturated rocks (Gardner et al., 1964; Winkler and Nur, 1979; Spencer, 1981; Tittman et al., 1981). The saturation may produce relaxation peaks at certain frequencies and increase and decrease of $Q$ on two sides of a peak. The question we wish to investigate is whether such relaxation phenomena and fluid motions can explain the frequency dependence of crustal $Q$ values measured from coda waves.
Attenuation measurements in the Earth using coda waves or strong motion seismograms include the contribution of scattering due to heterogeneities, fluid-flow effects in fractures and intrinsic anelasticity of crustal rocks. The laboratory measurements incorporate effects of anelasticity and fluids in pores and coating the grains. In order to compare the Earth data with the laboratory results, it is necessary to separate the effects of scattering and fluid motions in fractures. In this study, we do this in two steps. First, we calculate the scattering effects. Then we interpret the remaining attenuation and its frequency dependence in terms of constant-\(Q\) type intrinsic attenuation and fluid flow attenuation. Each of these steps are model-dependent so that the results depend on the validity of the models.
Scattering Attenuation

Scattering of elastic waves propagating in a heterogeneous medium contributes to the attenuation of these waves. Scattering attenuation is not an energy dissipation mechanism, but only an energy redistribution in space and time, therefore, it is a geometric effect. Under the single scattering approximation, the scattering attenuation cannot be separated from the intrinsic attenuation. In order to separate these two attenuation mechanisms, we need to use the multiple scattering theory. There is no general solution for the multiple scattering theory. However, several special cases have been studied (O’Doherty and Anstey, 1971; Kopnichev, 1977; Dainty and Toksöz, 1977, 1981; Richards and Menke, 1983; and Gao et al., 1983a, b). Wu (1984, 1985) formulated the multiple scattering problem in the frequency domain using radiative transfer theory. In the case of isotropic scattering with a point source in an infinite random medium, an exact solution can be obtained (Appendix A).

Figure 2 shows the distribution of seismic wave energy with distance calculated by the theory. In the figure, the energy density is normalized by the extinction length $L_e$, which is the reciprocal of the extinction coefficient $\eta_e$:

$$ L_e = \frac{1}{\eta_e} $$

$$ \eta_e = \eta_a + \eta_s $$

(6)

where $\eta_a$ is the energy absorption coefficient due to anelasticity of the medium and $\eta_s$ is the scattering coefficient which is defined as the total scattered power by a unit volume of random medium per unit incident power flux density. Note that $\eta_a$ is related to the attenuation coefficient given in equation (1) by $\eta_a = 2\alpha$. In Figure 2 the curve shapes change depending on the seismic
albedo $B_0$ of the medium, which is defined as:

$$B_0 = \frac{\eta_s}{\eta_e} = \frac{\eta_s}{\eta_s + \eta_a}$$  \hspace{1cm} (7)

For the case of large albedo ($B_0 > 0.5$), i.e. when the medium is strongly heterogeneous, and scattering is significant, the curves are of arch shape. The maxima of the curves depend on the extinction coefficient ($\eta_e = \eta_s + \eta_a$). Therefore it is possible to obtain $B_0$ and $\eta_e$ from the energy density-distance curves, and thus separate the scattering effect from the intrinsic attenuation.

The theory has been applied to local earthquakes in Hindu Kush region (Wu, 1984; Wu and Aki, 1985) with the conclusion that the scattering attenuation in that region is not the dominant factor ($B_0 \leq 0.5$). In this study, we look at the attenuation data in the eastern United States where anelastic attenuation may be low.

Figures 3a and 3b are the strong motion data (pseudo velocity) in Northeastern America for the case of $f = 5$ Hz and 1 Hz respectively (with 5% damping). The solid lines in the figures are the best fits to the data. If we assume that the received strong motions are composed of both the direct arrivals and the scattered waves, then we can compare curves given in Figure 2 with the data to obtain the seismic albedo $B_0$ and the intrinsic quality factor $Q_a$. In Figure 4, the PSV data are corrected for the geometric spreading ($1/R$ for body waves) and then squared to compare with the theoretical predictions. The best theoretical curves are also drawn in the figure. We can see that in the first 100 km the fit between theory and data is generally good except for a few points which are very close to the source. For greater distances, the data gradually deviate from the theory and become flatter. This may be due to the dominance of $L_g$ waves at great distances. The discrepancy of data and theory at very close distances is probably due to the non-linear effects. From these
Table 1: Medium parameters at \( f = 1 \) and \( f = 5 \) Hz based on multiple scattering theory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( f = 1 ) Hz</th>
<th>( f = 5 ) Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_e )</td>
<td>15 km</td>
<td>15 km</td>
</tr>
<tr>
<td>( B_0 )</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>( \eta_s )</td>
<td>0.06/km</td>
<td>0.06/km</td>
</tr>
<tr>
<td>( L_s (= 1/\eta_s) )</td>
<td>16.7 km</td>
<td>16.7 km</td>
</tr>
<tr>
<td>( \eta_a )</td>
<td>0.0067 km</td>
<td>0.0067 km</td>
</tr>
<tr>
<td>( L_a (= 1/\eta_a) )</td>
<td>150 km</td>
<td>150 km</td>
</tr>
<tr>
<td>( Q_s (= kL_s) )</td>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>( Q_a (= kL_a) )</td>
<td>270</td>
<td>1350</td>
</tr>
</tbody>
</table>

Comparisons of data with theory, we obtain the average seismic albedo \( B_0 = 0.9 \) and the extinction length \( L_a = 15 \) km for both the 1 Hz and 5 Hz waves. The medium parameters based on these values are listed in Table 1. In Figure 5 we plot the theoretical curves of PSV-distance relation for different seismic albedo \( B_0 \) when the extinction length is fixed at 15 km. A smaller albedo means a smaller intrinsic \( Q \) and therefore has a steep decrease of amplitude with distance. Figure 6 shows different curves of different albedos when the intrinsic \( Q \) is fixed at \( Q_a = 1350 \). We can see that the strong scattering will make the apparent attenuation much bigger than the intrinsic attenuation when the distance is larger than the absorption extinction length, \( L_a \). However, the amplitude change is not exponential for small distances.

Results given in Table 1 give a good fit to the data with a consistent set of parameters at \( f = 1 \) Hz and \( f = 5 \) Hz. They suggest a frequency dependent anelastic \( Q \) with \( Q_a = 270 \) at 1 Hz and \( Q_a = 1350 \) at 5 Hz. As it was discussed in the first section while reviewing the laboratory data, such
variation of $Q$ with frequency cannot be explained without a relaxation mechanism. In the crust, the fluids may provide such a mechanism.
Effects of Fluids on Attenuation

A fracture medium can be viewed as a fully saturated porous material of low porosity and relatively high permeability. Following Biot (1956a, b; 1962), such a finite porosity rock is modeled as a statistically isotropic material composed of a solid elastic matrix permeated by a network of interconnected pores saturated by a compressible viscous liquid. The liquid phase is then continuous and the wavelength of the signal is considered to be large compared to the characteristic pore dimension. As it allows the analysis of the propagation of a total wavefield, this model has been the basis of numerous studies in various fields. However, it was not until recently that Plona (1980) and Plona and Johnson (1980) experimentally demonstrated the validity of the theory. It predicts the existence of three types of body waves: a compressional wave of the first kind ($P_1$), which displays high velocity and quasi elastic properties; a compressional wave of the second kind ($P_2$), associated with low velocity and quasi viscous characteristics, and a shear wave. All three body waves are dispersive and dissipative: their velocities and attenuations are frequency dependent. In this study, we will focus on the traditional $P$ (i.e., $P_1$) and $S$ waves.

Energy dissipation due to fluid flow is related to the relative motion of the two phases which are coupled through inertial and viscous forces. These are characterized by a viscous ($b(\omega)$) and a mass ($\rho_{22}(\omega)$) coupling coefficients which can be expressed as functions of the imaginary and real parts (respectively) of the spectral signature of the material which is itself function of the pore shape and the pore geometry (Auriault et al., 1985; Schmitt, 1985). These forces are of the same order of
magnitude for a so-called critical frequency $f_{ci}$ given by:

$$f_{ci} = \frac{b(o)}{2\pi \rho^2(0)} = \frac{\nu \phi}{2\pi \kappa \rho_f \theta}$$  \hspace{1cm} (8)

where $\nu$ is the dynamic viscosity of the fluid, $\rho_f$ is the fluid density, $\theta$ is a coefficient characteristic of the pore shape and pore geometry and $\kappa$ is the intrinsic permeability of the porous material.

Below the critical frequency, i.e., in the low frequency range, the viscous forces are dominant and the fluid flow follows Poiseuille's law. The attenuation ($Q^{-1}$) of $P$ and $S$ waves is then proportional to frequency. In the high frequency range, i.e., above the critical frequency, the viscosity effect takes place in a very thin boundary layer close to the pore wall and the inertial forces become dominant. The attenuation of both $P$ and $S$ waves is then proportional to the inverse of the square root of the frequency. The attenuation is maximum at the critical frequency for both the $P$ and the $S$ waves. The theoretical prediction of the behavior of the attenuations above the critical frequency approximates quite well the observed frequency dependence (i.e., $Q$ is proportional to $f^{0.4}$ in the Northeast and to $f^{0.45}$ in the West). It implies that the critical frequency is $f_{ci} \leq 1$ Hz.

In addition to the fluid flow attenuation, the $P$ and $S$ waves are attenuated due to Coulomb friction between grains of the rock. This attenuation is independent of frequency (Walsh, 1966). In a porous material, elastic constants can be expressed as functions of the bulk moduli of the constitutive grains $K_s$, the skeleton $K_b$ and the fluid $K_f$, the shear modulus of the skeleton $\mu_b$ and the porosity $\phi$ (see Appendix B). The constant $Q$ of the solid can be introduced through equation $B - 4$.

To calculate attenuation due to fluid flow, we take a crustal crystalline rock model saturated with water. The physical parameters of the medium are given in Appendix B. The viscosity used
for the fluid is 0.2 centipoise. This value corresponds to water viscosity at a temperature of 100°C.

The critical frequency defined by equation 8 is a function of the pore geometry, the permeability and the porosity. To obtain a critical frequency close to 1 Hz, using cylindrical ducts in two perpendicular directions, typical values of permeability and porosity are: 50 darcies, 0.5%; 100 darcies, 1%; and 200 darcies, 2%. These sets of parameters give a critical frequency of 1.21 Hz. If instead of ducts we use fractures of equivalent permeability, we obtain fracture apertures of 20 to 800 μm. With increasing porosity, this leads to fracture densities of 0.6, 1.2, and 2.5 per meter. Although the fracture widths and densities are not unrealistic for the shallow part of the crystalline crust where drillings core and borehole studies have been done, it is necessary to do more detailed modelling to evaluate the effects of interconnected fractures systems.

Figures 7a, b display the velocity dispersion and attenuation for $P$ and $S$ waves between 0.1 and 100 Hz due to fluid flow. For both waves, the velocity dispersion is small. The maxima of attenuation decreases with decreasing porosity because of smaller volume of fluid. The attenuation of the $S$ wave is much greater than that of the $P$ wave. For shear waves $Q$ values at successive maxima are equal to 700, 1400, and 2800. At 5 Hz, we obtain shear $Q$ values of 1100, 2000 and 4000, in the range of $Q$ values given in Table 1.

Adding the attenuation in the solid as constant $Q$ raises the total attenuation values. Figures 8a, b display the results obtained with addition of constant quality factors of 500. For the $S$ wave, the attenuation maxima now obtained corresponds exactly to the sum of the inverse of both quality factors (this rule does not hold for the $P$ waves because of the presence of the $P_2$ wave). For shear waves, the $Q$ values at 1.2 Hz for the three models are 290, 370 and 420, respectively. These fall
in the range of $Q$ values given in Table 1. For a higher $Q$, any $Q=2000$, shear $Q$ values will more nearly approximate those given above. If we accept anelastic $Q$ values given in Table 1 ($Q = 270$ at 1 Hz and $Q = 1350$ at 5 Hz), then we need to combine the fluid flow mechanism with a constant $Q$ value of 500 to 2000 in order to explain the attenuation. The upper limit of 2000 is obtained from the minimum plausible solid attenuation (Clark et al., 1980).

These simple calculations show that a combination of attenuation due to solid friction (constant $Q$) and fluid flow can explain the attenuation values after removing the scattering effects. The importance of fluid flow contribution is that it can explain the frequency dependence of observed intrinsic $Q$ values of $S$ waves.
Discussion and Conclusions

In this paper, we proposed three mechanisms to explain the attenuation of earthquake ground motion in the distance range of 10 to 100 km. These include multiple scattering due to heterogeneities in the crust, Coulomb friction in rocks and viscous dissipation due to fluid motions in cracks. In order to determine the relative importance of these mechanisms, we considered the $Q$ measurements made in the laboratory, determined by the decay of coda waves of local seismograms and the amplitude decay of strong motion records in northeastern United States and Canada.

Laboratory data suggest that in rocks where there is no fluid flow $Q$ is constant over a wide range of frequency. Fluid motion in pores and cracks introduces a frequency dependent $Q$. Frequency dependence is strongly controlled by a critical frequency which is a function of crack or pore geometry, porosity, permeability and fluid viscosity. Below the critical frequency $Q$ decreases with frequency and above the critical frequency, $Q$ increases with frequency. This increase is proportional to the square root of frequency.

The increase of $Q$ with frequency and proportionality constant ($Q \propto f^{0.5}$) is very close to values determined for the crust from the coda wave analysis. Since coda decay provides a measure of the intrinsic attenuation in the crust, it is reasonable to assume that, in addition to constant $Q$, fluid flow plays an important role in attenuation in the crust. For the Northeast, we find the intrinsic constant $Q$ to be high ($500 \leq Q \leq 2000$). The fluid flow effects on attenuation are as large or larger than that of the intrinsic attenuation in the frequency range of 1 to 10 Hz.

The scattering analysis of strong motion records at 1 Hz and 5 Hz in the distance range of 10-200 km gives a large albedo ($B_0=0.9$), implying strong scattering. In addition to albedo the
only property of the scatterers that can be determined is the scattering extinction length. We obtain an extinction length of about 17 km for $f=1$ Hz and 5 Hz. The scatterers could be geologic fractures such as individual plutons, rock type changes, shear zones, dikes, sills or, most likely, a combination of all these.

It is important to state that the above discussions are based on a limited amount of data and theoretical models that make simplifying assumptions. It is necessary to analyze additional near-field data and to improve the models in order to draw firm conclusions.
Acknowledgements

We would like to thank Dr. Anton M. Dainty for critical discussions and his valuable suggestions. This work was supported by the United States Geological Survey under contract number 14-08-0001-G1092 and by the Advanced Research Project Agency of the Department of Defense and monitored by the Air Force Geophysical Laboratory under contract number F19628-86-K-0004. The views expressed in this report, however, are solely those of the authors and do not necessarily represent the views of the United States Geological Survey, the Advanced Research Projects Agency, the Air Force Geophysical Laboratory, or the United States Government.
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APPENDIX A

The elastic wave energy received by an isotropic point receiver in a random heterogeneous medium can be represented by the average energy density $E(r)$ at that point. The energy density in radiative transfer theory is defined as (see Wu, 1985):

$$E(r) = \frac{1}{C} \int_{4\pi} I(r, \hat{n}) d\Omega$$  \hspace{1cm} (A - 1)

where $C$ is the wave velocity, and $I(r, \hat{n})$ is the specific intensity or directional intensity. It gives the power flowing within a unit solid angle in the direction $\hat{n}$ ($\hat{n}$ is the unit vector) received by a unit area perpendicular to $\hat{n}$, in a unit frequency band. The specific intensity is defined for a frequency $\omega$, which is omitted in the notation. Since the $P$ wave energy is much smaller than the $S$ wave energy for earthquakes, we consider here $I(r, \hat{n})$ as only the $S$ wave energy by neglecting the mode converted energy from $P$ waves. We assume here also that the wave energy described by $I(r, \hat{n})$ is depolarized, i.e. the energy is equally partitioned between the two orthogonal components of $S$ waves. This agrees generally with the observations.

From the radiative transfer equations we can obtain the equation for the transfer energy density:

$$E(r) = E_i e^{-\eta t} + \int_V \left[ \eta E(r_1) + \epsilon(r_1, \hat{n}) \right] G_0(r - r_1) dV_1$$  \hspace{1cm} (A - 2)

where $E_i$ is the incident field and,

$$\epsilon(r_1, \hat{n}) = \frac{4\pi}{C} W(r_1, \hat{n})$$  \hspace{1cm} (A - 3)

is the source energy density function, where $W$ is the source power spectral density, and

$$G_0(r_1 - r) = \frac{e^{-\eta t R}}{4\pi R^3} = \frac{e^{-\eta t |r - r_1|}}{4\pi |r - r_1|^3}$$  \hspace{1cm} (A - 4)
In (A-2), suppose the incident field $E_{in} = 0$ and the isotropic point source is located at $r = 0$, radiating total power $P_0$. Then,

$$
\varepsilon(r) = \frac{P_0}{C} \delta(r) = E_0 \delta(r) \tag{A-5}
$$

and Equation (A-2) becomes:

$$
E(r) = E_0 \frac{e^{-\eta r}}{4\pi r^2} + \int \frac{\eta \epsilon E(r_1)}{4\pi} \frac{e^{-\eta(\epsilon \|r-r_1\|)}}{4\pi \|r-r_1\|^2} dV_1 \tag{A-6}
$$

Assuming $E_0 = 1$, the solution can be written as:

$$
E(r) = \frac{\eta \epsilon P_d}{4\pi r} \exp\left(-\eta d_0 r\right) + \frac{\eta \epsilon}{4\pi} \int_0^\infty f(s, B_0) \exp\left(-\eta r s\right) ds \tag{A-7}
$$

where

$$
P_d = \frac{2d_0^2(1-d_0)}{B_0(B_0^2 + B_0 - 1)} \tag{A-8}
$$

and $d_0$ is the diffuse multiplier determined by:

$$
\frac{B_0}{2d_0} \ln \left\{ \frac{1+d_0}{1-d_0} \right\} = 1; \tag{A-9}
$$

and

$$
f(s, B_0) = \left\{ \left[ 1 - \frac{B_0}{s} \tanh^{-1} \left( \frac{1}{s} \right) \right]^2 + \left( \frac{\pi B_0}{2 \frac{1}{s}} \right)^2 \right\}^{-1} \tag{A-10}
$$

The first term in Equation (A-7) is the diffuse term $E_d$ and the second term is the coherent term $E_c$.

Note that the diffuse multiplier $d_0$ is always less than 1. When distance $r$ is large, especially for large $B_0$, the diffuse term becomes dominant, and $E(r)$ will be approximately an exponential decay with an apparent attenuation coefficient $d_0 \eta$, which is less than the extinction coefficient $\eta$. 
The degree of reduction depends on the albedo $B_0$. Figure 1 shows the energy density distribution with distances for different albedo values.
APPENDIX B

Expressions of the elastic coefficients

The elastic coefficients $A$ and $N$ are equivalent to Lamé's coefficients. $\tilde{R}$ is a measure of the fluid pressure needed to move a given fluid volume into the porous aggregate, the total volume being constant. $T$ is related to the fluid and solid volume variations. These coefficients can be easily expressed as functions of the bulk moduli of the solid $K_s$, the skeleton $K_b$ and the fluid $K_f$, the shear modulus of the skeleton $\mu_b$ and the porosity $\tilde{\phi}$. Following Plona and Johnson (1980), one has:

$$
\begin{align*}
A &= \frac{(1 - \tilde{\phi})(1 - \tilde{\phi} - \frac{K_b}{K_s})K_s + \tilde{\phi}K_s K_b}{1 - \tilde{\phi} - \frac{K_b}{K_s} + \tilde{\phi} K_s K_f} - \frac{2}{3}N \\
T &= \frac{(1 - \tilde{\phi} - \frac{K_b}{K_s})\tilde{\phi} K_s}{1 - \tilde{\phi} - \frac{K_b}{K_s} + \tilde{\phi} K_s K_f} \\
\tilde{R} &= \frac{\tilde{\phi}^3 K_s}{1 - \tilde{\phi} - \frac{K_b}{K_s} + \tilde{\phi} K_s K_f} \\
N &= \mu_b
\end{align*}
$$

(B - 1)

In the above expression, it is assumed that the porosity does not vary with the pore pressure (Brown and Korringa, 1975; Dunn, 1985).

Denoting $\alpha_m$ and $\beta_m$, the compressional and shear wave velocities of the dry rock, one can write:

$$
\begin{align*}
K_b &= (1 - \tilde{\phi})\rho_s(\alpha_m^2 - 4\beta_m^2/3) \\
N &= (1 - \tilde{\phi})\rho_s\beta_m^2
\end{align*}
$$

(B - 2)

and for the fluid

$$
K_f = \rho_f\alpha_m^2
$$

(B - 3)
If one assumes an anelastic attenuation for the P and S waves in the skeleton characterized by quality factors $Q_{\alpha_m}$ and $Q_{\beta_m}$ and a frequency dependence $e^{i\omega t}$, it implies a velocity dispersion of the form, (e.g. Aki and Richards, 1980):

$$c(\omega) = \frac{c(\omega_0)}{\left(1 - \frac{1}{\pi Q} \log\left(\frac{\omega}{\omega_0}\right)\right)\left(1 - \frac{i}{2Q}\right)}$$  \hspace{1cm} (B - 4)

where

- $\omega_0$ is a reference angular frequency
- $c(\omega)$ is the body wave velocity ($\alpha_m$ or $\beta_m$) at angular frequency $\omega$,
- $Q$ is the corresponding quality factor ($Q_{\alpha_m}$ or $Q_{\beta_m}$).

In these conditions, $\alpha_m$ and $\beta_m$ become complex and frequency dependent as well as $K_1$ and the coefficients $A$, $N$, $T$ and $R$.

The parameters chosen for the formation are $\alpha_m = 5500$ m/s; $\beta_m = 3300$ m/s, $K_1 = 4.5 \times 10^{10}$ Pa, and $\rho_s = 2700$ kg/m$^3$. When introduced, the quality factor is identical for both body waves of the skeleton and is equal to 500.
Figure Captions

Figure 1. Schematic illustration of earthquake strong ground motion attenuation mechanism discussed in this paper. a) Rock anelasticity refers to frequency independent $Q$ associated with relative motions and frictional losses across grains and dislocations. b) Scattering is due to structural and geologic heterogeneities in the crust. c) Fluid flow incorporates fluid motions in pores and fractures induced by P and S waves.

Figure 2. Normalized energy distribution curves corrected for spherical spreading, $4\pi r^2 E(r)$ as a function of normalized distance $D_e = r/L_e$ where $L_e$ is the extinction length defined by Equation 6 in the text.

Figures 3a,b. Ground velocity (PSV) at 5 Hz (3a) and 1 Hz (2a) as a function of distance for events in northeastern United States and Eastern Canada. Values normalized to a common magnitude. Data are from compilation of Risk Engineering, Inc., under EPRI sponsorship. The solid line in each case is a “best” fit to data.

- △ 11-1-82, New Brunswick, $M - b = 5.5$, ECTN data
- ○ 19-1-82, New Hampshire, $M_b = 4.8$, strong motion data and ECTN
- □ 31-3-82, New Brunswick, $M_b = 4.8$, strong motion data and ECTN
- ◊ 6-5-82, New Brunswick, $M_b = 4.0$, strong motion data
- △ 16-6-82, New Brunswick, $M_b = 4.6$, strong motion data and ECTN
- ○ 7-10-83, Adirondacks, New York, $M_b = 5.6$, ECTN
- □ 11-10-83, Ottawa, Canada, $M_b = 4.1$, ECTN

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Figure 4a,b. Match between the multiple scattering model \((B_o = 0.9 \text{ and } L_e = 15 \text{ km})\) and the observed ground motion data as a function of radial (epicentral) distance \(R\), at frequencies 5 Hz (4a) and 1 Hz (4b). PSV curves are the "best" fit curves of Figures 3a,b. \((PSV \cdot R/10)\) and \((PSV \cdot R/10)^2\) are calculated from PSV curves. Note the goodness of fit between the model and data curves in the distance range of \(R = 10\) to 100 km where model approximations are valid.

Figure 5. Sensitivity of theoretical curves (Power versus radial distance) at \(f = 5\) Hz, to different model parameters. The model that fit the data best is shown as a "heavy" line.

Figure 6. Sensitivity of theoretical curves to albedo \((B_o)\) values at \(f = 5\) Hz as a function of distance.

Fixed parameters are \(L_e = 15\) km, \(Q_a = 1350\). The model that fit the data best is shown as a solid line.

Figure 7. Velocity and attenuation \((Q^{-1})\) of P and S waves, as a function of frequency, due to fluid flow. The three models are for different porosity \((\phi)\) and permeability \((k)\) values of fractured rock. A: \(\phi = 0.5\%\), \(k = 50\) darcies; B: \(\phi = 1\%\), \(k = 100\) darcies; C: \(\phi = 2\%\), \(k = 200\) darcies. The rock anelasticity is assumed to be zero. Note that velocity dispersion is small, but changes in attenuation are significant.

Figure 8. Velocity and attenuation of P and S waves due to fluid flow and rock anelasticity \((Q_0 = 500)\). All other parameters are the same as those of Figure 7.
ATTENUATION MECHANISM

a - Rock Anelasticity

b - Multiple Scattering

source

station

c - Fluid Flow

Pressure driven flow

Shear driven flow

Fractures

Figure 1
Figure 4a

$B_o=0.9$
$Le=15\text{km}$
Figure 4b
Figure 5
Figure 6

- $f = 5 \text{Hz}$
- $Q_a = 1350$
- $B_0 = 0$
- $B_0 = 0.5$
- $B_0 = 0.7$
- $B_0 = 0.8$
- $B_0 = 0.9$
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