Title: Interactive Synthesis of Computer Programs

Abstract:
Research concentrated on three topics:
1. Deductive synthesis of data flow networks,
2. Binary-search algorithms,
3. Theory of plans.

In the first area, a method of deductive synthesis of deterministic networks was developed. In the second area, general binary-search schema were constructed and specialized for particular applications. Finally, a variant of situational logic in which plans are explicit objects was introduced into the program-synthesis research.
TECHNICAL SUMMARY

Our research concentrated on the following three topics. For each topic a copy of a published paper is enclosed.

- Deductive Synthesis of Dataflow Networks


The synthesis of concurrent programs is much more complicated than the synthesis of sequential programs. In general, a concurrent program does not have a single input value and a single output value, but receives several inputs and sends several outputs during its execution. If we consider sequences of input and output values, then we can specify a concurrent program by giving a relation between the sequence of input values and the sequence of output values. This specification method is natural especially for networks of deterministic processes that communicate asynchronously by sending messages over buffered channels. Deterministic data flow networks fall into this category.

We have developed a method for the deductive synthesis of deterministic dataflow networks, which are specified by a relation between sequences of input values and sequences of output values.

Our synthesis method consists of two stages. The first stage, the deductive-synthesis stage, starts from a specification of the network. Using the deductive-tableau techniques of Manna and Waldinger, a system of recursive equations is synthesized. This system can be regarded as an applicative program that satisfies the specification for the network, but it does not directly represent any structure or parallelism of a network. In the second stage, the system of recursive equations is transformed into a dataflow network.
• Binary-Search Algorithms


Some of the most efficient numerical algorithms rely on a binary-search strategy; according to this strategy, the interval in which the desired output is sought is divided roughly in half at each iteration. This technique is so useful that some authors (e.g., Dershowitz and Manna, Smith) have proposed that a general binary-search paradigm or schema be built into program synthesis systems and then specialized as required for particular applications.

It is certainly valuable to store such schemata if they are of general application and difficult to discover. This approach, however, leaves open the question of how schemata are discovered in the first place. We have found that the concept of binary search appears quite naturally and easily in the derivations of some numerical programs. The concept arises as the result of a single resolution step, between a goal and itself, using our deductive-synthesis techniques.

The programs we have produced in this way (e.g., real-number quotient and square root, integer quotient and square root, and array searching) are quite simple and reasonably efficient, but are bizarre in appearance and different from what we would have constructed by informal means. For example, we have developed by our synthesis techniques the following real-number square-root program $\text{sqrt}(r, \varepsilon)$:

$$
\text{sqrt}(r, \varepsilon) = \begin{cases} 
\text{if } \max(r, 1) < \varepsilon \\
\text{then } 0 \\
\text{else if } \left[\text{sqrt}(r, 2\varepsilon) + \varepsilon\right]^2 < r \\
\text{then } \text{sqrt}(r, 2\varepsilon) + \varepsilon \\
\text{else } \text{sqrt}(r, 2\varepsilon).
\end{cases}
$$

The program tests if the error tolerance $\varepsilon$ is sufficiently large; if so, 0 is a close enough approximation. Otherwise, the program finds recursively an approximation within $2\varepsilon$ less than the exact square root of $r$. It then tries to refine this estimate, increasing it by $\varepsilon$ if the exact square root is large enough and leaving it the same otherwise.

This program was surprising to us in that it doubles a number rather than halving it as the classical binary-search program does. Nevertheless, if the repeated occurrences of the recursive call $\text{sqrt}(r, 2\varepsilon)$ are combined by common-subexpression elimination, this program is as efficient as the familiar one and somewhat simpler.

• A Theory of Plans


Problems in commonsense and robot planning were approached by methods adapted from our program-synthesis research; planning is regarded as an application of automated deduction. To support this approach, we introduced a variant of situational logic, called plan theory, in which plans are explicit objects. A machine-oriented deductive-tableau inference system is adapted to
plan theory. Equations and equivalences of the theory are built into a unification algorithm for the
system. Frame axioms are built into the resolution rule.

Special attention was paid to the derivation of conditional and recursive plans. Inductive
proofs of theorems for even the simplest planning problems, such as clearing a block, have been
found to require challenging generalizations.

SCIENTIFIC COLLABORATION

The research was conducted in full collaboration with Dr. Richard Waldinger (Artificial Intelli-
gence Center, SRI International), Eric Muller (programmer), and the following Ph.D. students of
the Computer Science Department at Stanford University: Yoni Malachi (graduated 1986), Bengt
Jonsson, Tomas Feder, Jon Traugott, Alex Bronstein, David Kashtan, and Marianne Baudinet.

Invited lectures and seminars, on the research reported here, were given in the following
countries: Australia, China, England, India, Israel, Italy, Japan, Thailand, and in the USA.