INSTABILITY OF LAMINAR SEPARATION BUBBLES: CAUSES AND EFFECTS

by

Tuncer Cebeci

September 1987

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A combination of interactive boundary layer and stability theories has been used to investigate the reasons for the instability of laminar separation bubbles on the leading edge of thin airfoils. It is shown that transition plays an important role and is likely to preclude the existence of long separation bubbles and their supposed instability.
ABSTRACT

A combination of interactive boundary layer and stability theories has been used to investigate the reasons for the instability of laminar separation bubbles on the leading edge of thin airfoils. It is shown that transition plays an important role and is likely to preclude the existence of long separation bubbles and their supposed instability.
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1.0 INTRODUCTION

The laminar boundary layer on an airfoil grows from the stagnation point with a favorable pressure gradient which causes the flow to accelerate and is then subjected to an adverse pressure gradient which can cause separation with subsequent reattachment. The resulting bubbles are common on thin airfoils where the adverse pressure gradient can be sufficiently strong to cause the flow to separate even at small angles of attack and are important because of their association with the phenomenon of stall. The nature of the phenomenon is known to depend on the Reynolds number based on the radius of the leading edge, as will be shown, the length of the separation bubble can grow to influence the location of transition and, on occasions, transition can occur within the bubble. The angle of attack is also known to be important and can cause the separated region to grow until at some angle of attack the bubble bursts and stall sets in. This sequence of events from the first appearance of separation on the upper surface to stall is complex and requires clearer understanding than is presently available.

The prediction of separation bubbles on airfoils has been studied by a number of investigators. An important contribution was made by Briley and McDonald [1] who used an interactive boundary-layer approach and solutions of the Navier-Stokes equations to examine the case of a comparatively thick airfoil where the separation region occurred around midchord and was approximately 10% chord in extent. Crimi and Reeves [2], Kwon and Pletcher [3], Cebeci and Schimke [4] and Carter and Vatsa [5] have tackled essentially the same problem with interactive boundary-layer theory. In all cases the location of transition was either assumed to correspond to laminar separation or was computed by an empirical formula. The work of Cebeci and Schimke also examined the influence of the location of transition and showed that reattachment and transition were related. Attempts to perform calculations with the experimentally reported transition location, which occurred further downstream than those considered above, revealed a tendency for the reattachment location to move rapidly downstream with the number of sweeps used in the interactive procedure. This apparent instability of the separation bubble is examined further in this report together with its relationship to the location of transition.
The present approach can be described in two parts. First we will make use of linear stability theory and the $e^n$-method to predict transition based on calculated velocity profiles. The validity of this procedure has been demonstrated by Cebeci and Egan [6] and calculations are presented to confirm that it is appropriate for the particular flows under investigation here. Secondly, we follow the approach of Cebeci et al. [7] and examine the leading-edge separation bubble on a thin airfoil as a function of angle of attack and for a Reynolds number of $10^5$. This systematic study has been arranged to allow us to examine carefully the relationship between the growth of the separated region and transition.
2.0 CALCULATION OF TRANSITION

In a recent study, Cebeci and Egan [6] performed calculations of steady flows over and downstream of bumps identical to those examined experimentally by Fage [8]. The shape of the bump was represented in the calculations by

\[
\frac{y}{h} = \begin{cases} 
1 - 12 \left( \frac{x}{B} \right)^2 - 16 \left( \frac{x}{B} \right)^3 & -\frac{B}{2} < x < 0 \\
1 - 12 \left( \frac{x}{B} \right)^2 + 16 \left( \frac{x}{B} \right)^3 & 0 < x < \frac{B}{2}
\end{cases}
\]  

(1)

where \( h \) and \( B \) denote the height and width as shown in Figure 1. The calculation method was based on that of Cebeci et al. [7] and solved the boundary-layer equations in an inverse mode with successive sweeps over the body. The edge boundary condition was written as the sum of the inviscid velocity \( u_e^0(x) \) and a perturbation velocity \( u_e(x) \), that is,

\[
\begin{align*}
\text{at} & \quad y = \delta, \\
u_e(x) &= u_e^0(x) + u_e(x)
\end{align*}
\]  

(2)

and \( u_e(x) \) was obtained from the Hilbert integral given by

\[
\begin{align*}
u_e(x) &= \frac{1}{\pi} \int_{x_a}^{x_b} \frac{d\sigma}{x - \sigma} \left( \frac{\partial u_e}{\partial e} \right) \frac{d\sigma}{x - \sigma}
\end{align*}
\]  

(3)

with the interaction region confined between \( x_a \) and \( x_b \).

The above interactive boundary-layer procedure with \( u_e^0(x) = 1 \) was used to compute the boundary-layer characteristics including the velocity profiles and wall shear stress parameter \( f_w^" \) defined by

\[
f_w^" = \frac{\tau_w}{\rho \frac{u_x}{u_0}} \sqrt{\frac{u_x}{\frac{u_0}{v}}}
\]

for the conditions investigated by Fage. A sample of the results, in terms of \( f_w^" \), are included in Figure 1 for a Reynolds number of \( 4.375 \times 10^5 \) per foot and for three bump heights. Here the Reynolds number per foot is defined in terms of the measured freestream velocity \( u_{1c} \) at the position of the centerline of the bump but for the undistorted surface. The figure shows that the wall shear parameter decreases immediately prior to the bump, rises rapidly with the favorable pressure gradient imposed by the upstream surface of the
bump, reaches a maximum and decays rapidly to a minimum value before stabilizing as the influence of the bump diminishes. The influence of the bump height is to increase the magnitude of the maxima and minima of the $f_w''$ distribution with corresponding increase in its gradient.

Figure 1. Variation of wall shear parameter $f_w''$ in the bump flows of Fage for constant Reynolds number.
Figure 1 and Table 1 also show measured and calculated locations of transition with the latter obtained from the $e^n$-method and the calculated velocity profiles. This method stems from the work of Smith and Gamberoni [9] and Van Ingen [10] and is based on linear stability theory. It assumes that transition starts when a small disturbance is introduced at a critical Reynolds number and is amplified by a factor of $e^n$. For given velocity profiles, the Orr-Sommerfeld equation is solved and stability properties are examined. The amplification rates $(-\alpha_i)$ are computed as a function of x for a range of discrete values of the frequency $\omega$ and transition is assumed to occur when $e^{-\int_0^x \alpha dx}$ reaches a value equal to $e^n$ where $n$ is around 9. In the present case the profiles were available from the interactive boundary-layer calculations and the same version of the Box scheme was used to solve the stability equation with a continuation method to obtain the eigenvalues in regions of rapidly changing $f_w$ and in regions of separated flow.

The agreement between measured and calculated transition locations is shown on Figure 1 and Table 1 to be within experimental uncertainty and similar results were reported by Cebeci and Egan for the much wider range of configurations and Reynolds numbers investigated by Fage. It is clear that the location of transition moves upstream with increasing bump height and that the length of the separated region increases. These two characteristics are also to be found in the $f_w$ distributions associated with the leading edge region of thin airfoils as discussed below.

Figure 2 shows the $f_w$ distribution and the corresponding external velocity distribution for the leading edge of a thin airfoil. The result is similar to

<table>
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<th>$u_{1c}$ (ft/sec)</th>
<th>$L_{measured}$</th>
<th>$L_{calc}$</th>
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<tr>
<td>61.5</td>
<td>3.75</td>
<td>3.30</td>
</tr>
<tr>
<td>70.0</td>
<td>2.92</td>
<td>2.95</td>
</tr>
<tr>
<td>92.4</td>
<td>2.08</td>
<td>2.48</td>
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previous distributions reported by Cebeci et al. but is here presented for a Reynolds number of $10^5$ which ensures that transition occurs downstream of the separation bubble. The form of the $f_w$-curve resembles those of Figure 1, particularly downstream of the beginning of the favorable pressure gradient and will be even more similar for a wider bump. It is to be expected that the $e^n$-method will apply equally to the thin airfoil and that the transition location can also be determined reliably by the $e^n$-method in this case.

Figure 2. Variation of wall shear parameter $f_w$ and external velocity $U_e$ near the leading-edge of a thin airfoil.
3.0 SEPARATION BUBBLES ON THIN AIRFOILS

Following the approach of Cebeci et al. [7], we consider a thin ellipse in nose-centered coordinates

\[
\frac{(x - a)^2}{a^2} + \frac{y^2}{b^2} = 1
\]

(4)

at an angle of attack of \(\alpha\) and in a uniform stream of velocity \(u_\infty\). Attention is directed to the nose region where the ellipse is approximated locally by a "nose-fitting" parabola, and an expression is derived for the external velocity distribution \(\bar{u}_e\),

\[
\frac{\bar{u}_e}{u_\infty(1 + t)} = \frac{\xi + \xi_0}{\sqrt{1 + \xi^2}}
\]

(5)

Here \(t\) denotes the thickness ratio \(b/a\), \(\xi_0\) corresponds to a reduced angle of attack, \(\alpha/t\), and the parameter \(\xi\) is a dimensionless distance from the nose related to the \(x\)- and \(y\)-coordinates of the ellipse by \(x + a = 1/2 \at^2\xi^2\), \(y = \at^2\xi\). The parameter \(\xi\) is related to the surface distance \(s\) by

\[
s = \at^2 \int_0^{\xi} \sqrt{(1 + \xi^2)}^{1/2} d\xi
\]

(6)

The investigation of Cebeci et al. [7] made use of the external velocity distribution given by Eq. (5) and showed that the laminar boundary layer near the leading edge was well behaved and unseparated if \(\xi_0 < \xi_s = 1.16\), although there was significant adverse pressure gradient. At higher values of \(\xi_0\), however, separation occurred with an associated singularity and required the use of an interactive theory to link the viscous and inviscid flows. With this theory, solutions were obtained for separation bubbles at \(R(\Xi 2 u_\infty a/v) = 2 \times 10^6\) and for \(t = 0.1\) but reattachment occurred in a very limited range of the reduced angle of attack. For \(\xi_0 > 1.218\), calculations broke down shortly after the flow reversal in the boundary layer and the subsequent studies of Stewartson, Smith and Kaups [11] led them to suggest that a dramatic switch to another separated form of motion can occur.

Similar calculations have been performed for a Reynolds number of \(10^5\), for which we can be sure that the transition occurs in the region downstream of
the separated flow, and are presented in Figures 3 and 4 with increase of the reduced angle of attack $\xi_0$ implying an increase in the strength of the adverse pressure gradient. Consistent with the observations of Cebeci et al. [7], the region of separated flow was found to increase in extent with $\xi_0$ and at $\xi_0 = 1.296$ revealed a tendency to expand slowly with each sweep (Fig. 4) with the tendency becoming bigger at $\xi_0 = 1.298$ (Fig. 5). This instability of the bubble may have a counterpart in the bump flows of Section 2 since, with a very severe adverse pressure gradient corresponding to a larger bump than those considered by Fage, it may be expected that laminar flow will undergo transition before reattachment. This possibility can be tested with the help of the stability theory described in the previous section.

![Figure 3. Variation of $f''_w$ and transition location (*) with $\xi$ for various reduced angles of attack $\xi_0$ for $R = 105.$](image-url)
Figure 4. Variation of $f''_w$ and transition location (*) with number of sweeps for $\xi_0 = 1.296$, $R = 105$.

Figure 5. Variation of $f''_w$ with number of sweeps for $\xi_0 = 1.298$ and $R = 105$. 
The application of the e^n-method to the leading-edge flows of Figure 3 led to transition locations identified in the figure. They exhibit the same trend noted in connection with Figure 1 in that transition moves forward with increasing reduced angle $\xi_0$ which is analogous to bump height $h$. It is of particular note that, as $\xi_0$ tends to the value of 1.296 for which instability has been observed, the transition location moves towards and inside the separation bubble at $\xi_0 = 1.292$. With further increase in the reduced angle of attack, the transition location moves further inside the separation bubble. At $\xi_0 = 1.296$ (see Fig. 4), the transition location moves upstream with each sweep. These results imply that the real flow will become turbulent and have a shorter recirculation region which is consistent with experiments. It also suggests that there is little merit in expending effort to calculate the large laminar separation bubbles which would be obtained with larger reduced angles. This observation is likely to be independent of the use of interactive boundary layer or Navier-Stokes procedures.
4.0 REFERENCES


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