SEA ICE KINEMATICS:
SPACE AND TIME SCALES

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EXECUTIVE SUMMARY

The arctic is a unique and hazardous operating environment. Sea ice, together with the cold and dark specific to the region, are the major hazards affecting Naval operations there. Because the arctic has become a principal strategic location, knowledge and prediction of sea-ice conditions and the ability to cope with them have become essential. Unfortunately, most studies to date of sea ice processes have been general in character, a result of the complexities of working in the arctic. Naval operational activities in the arctic require a detailed knowledge of the various modes of sea ice motion and their spatial and temporal variations. Thus, a critical need exists to determine the space and time scales over which modes of sea ice motion occur.

In this report, ice kinematics are described in terms of the 5 basic modes of motion: divergence (D), vorticity (ζ), deformation rate (T), and ice translation (U). Seasonal time histories of these ice kinematic parameters (IKP) were calculated using position data from drifting buoys in the arctic during May, August, and November 1979. The results were used to determine seasonal space and time scales of D, ζ, T, and translation speed variations in the arctic. An e-folding scale is used as a measure of the temporal coherency. Spatial variability is defined in terms of the degree of similarity between the magnitudes of a parameter at various locations.

Results of seasonal space and time scale analyses indicate that the divergence was the most temporally and spatially variable of the IKP for all seasons. The translation speed was the most consistent in space and time. It was found that significant variations in divergence occurred in some areas on the order of 110 km and within 4 hours. In contrast, significant variations in the translation speed occurred in some areas on the order of 705 km and over a period as great as 80 hours.

The information obtained in this analysis is appropriate for the setup of numerical simulations of ice motion. The Navy's Polar Ice Prediction System (PIPS) is the only operational system which can furnish predictive ice kinematic information throughout the arctic. To provide useful operational information, the PIPS must be run at the appropriate space and time scales. The results obtained in this study are discussed in terms of how the grid size and time step of the PIPS may be altered such that, on the average, critical ice motion characteristics will be resolved. Although it is felt that the PIPS grid size is appropriate, the time scale analysis indicates a need to decrease the time step of the model.

NOMENCLATURE

D (Divergence) - the change in the size of the parcel (without an orientation or shape change).
ζ (Vorticity) - the change in the orientation of the parcel (without a shape or size change).
T (Deformation rate) - the change in the shape of the parcel (without a size or orientation change).
U (Translation speed) - the net displacement per unit time of an ice parcel.

\( e_T \) (e-folding time) - time lag (hours) at which the autocorrelation of an ice kinematic parameter drops to e^{-0.36}.

L - the distance (km) between two locations beyond which the average similarity between IKP at the locations is approximately constant.

\( S_{\text{min}} \) - the constant, minimum average similarity associated with the distance L.
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1. INTRODUCTION

The arctic is a principal area for ice research because of its economical resources and strategic location. Since man's scientific, military, and economical interests in the arctic have grown, knowledge and prediction of sea-ice conditions and the ability to cope with them have become essential. Accurate forecasting of sea-ice behavior, for example, is indispensable to military and offshore exploration operations. As a result, much research has been aimed at understanding the processes affecting sea ice and its distribution in the arctic.

In order to prepare for specific conditions in an environment, one must determine the modes of motion in the environment. Kinematic analysis is one of the most basic methods of defining sea-ice processes. The basic modes of motion for a parcel of ice are translation, divergence (area change), vorticity (rotation rate), and deformation (related to shape changes). These ice kinematic parameters (IKP) are particularly important in an ice-infested environment due to the variations in sea-ice loading that each variable causes. Also, the IKP are related to other important physical phenomena. For example, ice convergence is associated with ridging, which can produce thick multi-year ice floes and much under-ice noise. Ice divergence is also associated with lead formation, which can greatly modify polar atmospheric heat fluxes.

Our knowledge of sea-ice processes and ice kinematics has been substantially enhanced over the last 5 or 10 years. A great amount of research in this area has already been performed in the arctic, and these studies have provided valuable insights into ice motion and related phenomena. Hibler (1974) used kinematic analysis to point out that sea ice tends to diverge under atmospheric lows during summer but converge under lows during winter. McPhee (1978) used kinematic analysis to quantify lower frequency ice divergence, rotation, and deformation rates in the Beaufort Sea. Lewis and Denner (1986) extended McPhee's work to quantify the seasonal variance of ice translation rate, divergence, rotation rate, and deformation rate. Moreover, their analysis resulted in the general description of summertime ice motion in terms of inertial oscillations plus a background mode forced by the synoptic winds.

The results of such kinematic analyses have been very general in character. Operational activities in the arctic require a more detailed knowledge of a given parameter and its variations. Therefore, a critical need exists to establish the time and space scales over which sea ice and sea-ice processes vary. These scales indicate the coherency of a parameter both in time at a given location and in space within a given region. The scales are a function of ice characteristics and of the response of ice to particular forcings. As the characteristics of ice and/or forcing vary, so do the associated scales. As such, one might expect to observe seasonal and regional variations in the scales. Such scales are important for military operations, establishing design criteria, aiding in the set up and/or verification of numerical simulations of ice motion, and guidance in developing future studies and monitoring systems.

Space and time scales can also be important in determining ice characteristics and the relative importance of various modes of ice motion in a region. Knowledge of space and time scales can aid in identifying the causes of regional differences and be used to delineate seasonal boundaries between regions. Moreover, regions with similar space and time scales most likely have the same ice characteristics and the same relative importance of the modes of ice motion. Findings concerning ice characteristics and ice motion mechanisms could, therefore, be extrapolated from one region to other similar regions.

In this study, ice pack kinematics are based on the motion of ice parcels. The ice kinematics are defined as the translation of a parcel of the ice pack as well as any rotation, area change, or deformation that occurs during the translation (Fig. 1). We define the following four independent components of motion as ice kinematic parameters (IKP): translation (U), vorticity (\(\bar{\zeta}\)), divergence (D), and deformation rate (T). These parameters are interpreted as follows:

\[ U \] - the net displacement per unit time of the parcel,
\[ \bar{\zeta} \] - the change in the orientation of the parcel (without a shape or size change),
Fig. 1. Mathematical and physical definitions of the four basic components of the motion of a parcel of ice. The closed integral is about the perimeter of the ice parcel (with area $A$), the velocity vector is that along the perimeter, and the unit vectors $n_i$ are the outward normal $(n_1,n_2)$ for $i = 1$, the cyclonic parallel $(-n_2,n_1)$ for $i = 2$, $(n_1,-n_2)$ for $i = 3$, and $(n_2,n_1)$ for $i = 4$. 

Translation ($\bar{u}, \bar{v}$) 

Divergence 

\[ \frac{1}{A} \int \bar{u} \cdot \hat{n}_1 \, dl \]

Vorticity (Rotation Rate) 

\[ \frac{1}{A} \int \bar{u} \cdot \hat{n}_2 \, dl \]

Deformation Rate 

\[ \frac{1}{A} \left[ \left( \int \bar{u} \cdot \hat{n}_3 \, dl \right)^2 + \left( \int \bar{u} \cdot \hat{n}_4 \, dl \right)^2 \right]^{1/2} \]
D - the change in the size of the parcel (without an orientation or shape change), and

T - the change in the shape of the parcel (without a size or orientation change).

The last three parameters are referred to as the differential kinematic parameters (DKP) and describe relative motion within the ice parcel as it translates. In addition, the speed at which the ice parcel translates is defined as \((u^2 + v^2)^{1/2}\), where \(u\) and \(v\) are the components of velocity.

The utility of the kinematic analysis is that each mode of motion has a specific physical interpretation. The divergence, for example, is related to lead and ridge formation. Translation speed is related to the ice parcel moving through the water. The shape changes caused by deformation are associated with the rearrangement of individual floes within the ice parcel.

Because each mode of ice motion affects all forms of operation in the arctic, it is necessary to determine the spatial and temporal scales over which ice motion occurs. Such scales indicate the coherency of each mode of motion both in time at a given location and in space within a given region. They are also important in determining ice characteristics and the relative importance of various modes of ice motion in a region. To date, few studies of space and time scales of sea-ice processes in the arctic have been performed. Lewis and Denner (1986) determined the seasonal time scales of the IKP in the Beaufort Sea for spring, summer, and fall 1975 and winter 1976. Their results (Fig. 2) indicated that, for each season, the divergence always had a small time scale. They found maximum time scales of the order of 12 hours for divergence, 20 hours for vorticity, and 30 hours for deformation rate. The magnitudes of the variations of the IKP were represented in terms of the standard deviation about the IKP mean. In all cases, the variations were much larger than the means and suggested that movement by any mode of motion for all seasons was never insignificant.

With respect to space scales, Colony and Thorndike (1980) calculated the coherence of summer ice speed in the Beaufort Sea. Considering the spatial variability of summer ice movement, they found that synoptic ice motion on the scale of \(-100\) km was highly coherent (squared coherence of \(-0.9\)). The coherency of inertial ice motion was of the order of 0.65.

The above investigations into the space and time scales of ice motion in the arctic were confined to the Beaufort Sea. To date, the only study of space and time scales using data from throughout the arctic was done by Thorndike (1986). He used 1979 drifter data for the central arctic basin, and the results are given in Fig. 3 and Table 1. One sees that the temporal correlation falls to about 0.7 after one day, 0.4 after two days, and decreases slowly at longer lags (Fig. 3). The spatial autocorrelation of velocity (Table 1) shows a relatively high coherency out to 200 km.

In the present study, position data from buoys drifting on ice were used to calculate seasonal time histories of IKP for regions covering a large portion of the arctic. This was done using May, August, and November buoy position data from 1979. The study areas covered by the data range in size from \(178 \times 10^3\) km\(^2\) in fall to \(272 \times 10^3\) km\(^2\) in the spring. The IKP time histories were then used to calculate seasonal space and time scales for the speed (rather than velocity), divergence, vorticity, and deformation rate in the corresponding study regions.

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>(B_\parallel)</th>
<th>(B_\perp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>100</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>200</td>
<td>0.91</td>
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</tr>
<tr>
<td>400</td>
<td>0.68</td>
<td>0.51</td>
</tr>
<tr>
<td>800</td>
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<tr>
<td>1200</td>
<td>0.19</td>
<td>-0.09</td>
</tr>
<tr>
<td>1600</td>
<td>0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>2000</td>
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</tr>
<tr>
<td>2400</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1. Spatial correlation functions for sea-ice velocity (from Thorndike, 1986). \(B_\parallel(r)\) is the correlation between the components of velocity parallel to the line joining two points separated by a distance \(r\). \(B_\perp(r)\) is the correlation between the components of velocity perpendicular to that line.
Fig. 2. A schematic representing magnitude variations versus e-folding times of the various kinematic parameters for spring, summer, and fall 1975 and winter 1976 in the Beaufort Sea (from Lewis and Denner, 1986).
Fig. 3. Temporal autocorrelation for 1979 buoys (from Thorndike, 1986).
2. DATA

As part of the United States' contribution to the First GARP Global Experiment, an array of automatic data buoys was deployed in the arctic early in 1979. The data transmitted by the buoys were received by the TIROS-N and NOAA-A satellites, retransmitted to receiving stations on earth, and relayed to Service Argos in Toulouse, France. Screening of the data by Service Argos indicated that over two-thirds of the measured buoy positions fell within 300 m of their true locations. Therefore, in the present analysis, position measurements were assumed to have a root mean square (rms) position error of 500 m. The data used in this study include the time histories of the positions of those drifting buoys. Position data during May, August, and November 1979 were chosen to represent spring, summer, and fall conditions, respectively. The study areas were delineated only by those buoys which transmitted data during the entire months of May, August, and November. (Unfortunately, the position data for the winter of 1979 were too sparse and too widely separated to adequately compose a study region for that season.)

The approximate areas of the regions delimited by the buoys during spring, summer, and fall are denoted by the shaded areas in Fig. 4. The coordinate system has its origin at the North Pole, its x axis coincident with the Greenwich meridian, and its y axis coincident with the 90°E meridian. Time histories of the u and v velocity components were calculated for each buoy comprising the study area. This was done to test for anomalously large velocity or acceleration values, indicators of erroneous position data. Spurious data points were eliminated from the position data. In addition, buoys whose data records were composed of more than 1/3 faulty data were not used. A spline fit was then performed on the position data to obtain fixes at three hour intervals during May, August, and November 1979. These data were then used in the kinematic calculations.
Fig. 4. Approximate study areas covered by the data for a) May, b) August, and c) November of 1979. The areas are delimited using buoy positions on the first day of the month. Contours are of atmospheric surface pressure at 1200 GMT.
3. TECHNICAL APPROACH

Ice Kinematics

Within each study area, the population of buoys was large enough to form a number of regions, each containing at least four buoys. These areas ranged in size from $15.8 \times 10^3$ km$^2$ to $61.5 \times 10^3$ km$^2$. The spring, summer, and fall study areas were partitioned into 18, 15, and 11 regions (or ice parcels), respectively. The 3 hour position data of each buoy were used to calculate the time histories of the IKP. The model used to calculate the IKP was developed following Molinari and Kirwan (1975) and Okubo and Ebbesmeyer (1976).

Examples of time series of the ice divergence and vorticity during spring, summer, and fall 1979 are shown in Figs. 5-7. The ice parcel from which these calculations were made was delineated by 5 buoys located in the central Beaufort Sea in the spring and summer and 4 buoys in the fall. The kinematic results provide an interesting contrast to those of Lewis and Denner (1986) (also shown in Figs. 5-7). Lewis and Denner calculated the time histories of the DKP for an ice parcel delineated by manned camps in the central Beaufort Sea during 1975. The area encompassed by the manned camps was approximately $8.3 \times 10^3$ km$^2$ during each season. Calculations done in the present analysis show that, during fall 1979, all modes of ice motion were very energetic. The oscillations of the ice kinematics were much higher in frequency and larger in amplitude during the fall than during the spring or summer of 1979. Amplitude variations of the ice kinematics during summer appear to be slightly larger than those during spring for all modes of motion. The frequency of oscillation of the ice kinematics appears to be similar during spring and summer. In contrast, the oscillations of the ice kinematics during early summer 1975 became higher in frequency and larger in amplitude (Fig. 6). Motion at the inertial frequency was very energetic. The kinematics for the fall of 1975 had longer period and smaller amplitude oscillations (Fig. 7).

The most intriguing aspect of the comparison between the two years is that the arctic fall conditions during 1979 apparently assumed characteristics similar to the summer of 1975. The causes for the differences between the two years is discussed in detail by Lewis et al. (1988).

Time Scales

To consider scales of temporal variations, the autocorrelations of the IKP were calculated using the spring, summer, and fall time histories of the IKP. One should recall that the IKP, when used in conjunction with space and time scales, includes the speed of the ice parcel instead of the velocity. The time scale of the variations in the IKP was defined as that time lag at which the autocorrelation dropped to $e^{-1}=0.36$ (e-folding scale). This allowed us to define a seasonal e-folding time, $e_T$, for each IKP. The $e_T$ indicates the average time at which one could expect significant variations in the IKP at a given location and season. The e-folding time scales can be thought of in terms of persistence, with longer e-folding times implying a slower rate of change in the variable. If the calculations did not show a correlation dropping below $e^{-1}$ for up to three days time lag (at 3 hour intervals), 80 hours was used as the $e_T$. In addition, each $e_T$ was associated with the average position of the centroid of the corresponding cluster of buoys. From this information, two-dimensional contour maps of seasonal $e_T$'s for each IKP were produced.

Time varying oscillations can be considered in terms of the magnitude of the variations and how rapidly these variations occur. The $e_T$'s are a measure of the time scales on which these variations occur. In order to intercompare the $e_T$'s, it is necessary to ensure that the magnitudes of the variations are comparable as well (i.e., of the same size). Intercomparisons of IKP time scales have little meaning when the magnitudes of the IKP variations are significantly different from one region to the next. There is the possibility that the magnitudes of the variations of ice kinematics in one region are insignificant in comparison with kinematics from other regions. In such a case, a comparison of regional e-folding scales or two-dimensional contour maps have questionable meanings. To confirm that the calculated e-folding scales were true indications of sea ice motion with similar magnitudes of temporal oscillations, the variances of the IKP of all the ice parcels were calculated and compared. It was found that the time-varying oscillations of the IKP for each region had similar variance magnitudes (Table 2). Thus, based on the results in Table 2, the time scales for every cluster can be readily intercompared.
Space Scales
The spatial autocorrelation of a parameter is a measure of how well the pattern of variations of the parameter is correlated with distance. However, the autocorrelation gives no indication of the magnitude of the changes of the parameter with distance. Because we wanted to quantify the variations in the magnitude of the IKP, we, therefore, did not use spatial autocorrelations. Instead, we considered spatial similarities of the IKP.

The spatial similarity is defined as the degree of similarity (1.0 being identical) between the values of an IKP at two locations. Let $P_1$ and $P_2$ be defined as values of an IKP at positions 1 and 2, respectively. We define the spatial similarity, $S$, as the average for all observations of the value

$$S = \frac{\text{Min}(|P_1|, |P_2|)}{\text{Max}(|P_1|, |P_2|)}$$

when $P_1$ and $P_2$ have the same sign. Thus, $S$ is a measure of the average change in $P$ (in terms of a ratio) with distance. The space scale of variations is defined as that distance at which the spatial similarity for an IKP dropped to a value of 0.6.

Because space scales are a function of ice characteristics and forcing, seasonal and regional variations of these scales were expected. Therefore, we attempted to determine regions with IKP's that had similar curves of $S$ vs distance. However, the results suggested that distinct regions of spatial coherence within the arctic could not be defined. We, therefore, considered the entire arctic as one region in performing the seasonal space scale analysis of the IKP.

<table>
<thead>
<tr>
<th>VARIANCE</th>
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<tr>
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<td>SPRING</td>
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<td>SUMMER</td>
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<td>FALL</td>
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<tr>
<td>-----------</td>
</tr>
<tr>
<td>D</td>
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<tr>
<td>$\zeta$</td>
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<tr>
<td>T</td>
</tr>
<tr>
<td>U</td>
</tr>
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</table>

Table 2. Ranges of magnitude variations (in terms of variances about the means) of the kinematic parameters for all ice parcels present during spring, summer, and fall 1979. Units are $10^{-15}/s^2$ for $D$, $T$, and $\zeta$ and $10^{-4}$ $m^2/s^2$ for $U$. 


Fig. 6. Time histories of divergence and vorticity in the Beaufort Sea from Lewis and Denner (1986) (left) and from this study (right) for August.
Fig. 7. Time histories of divergence and vorticity in the Beaufort Sea from Lewis and Denner (1986) (left) and from this study (right) for November.
4. RESULTS

Time Scales

Divergence appears to have had a large temporal variability during all seasons, with the smallest $e_T$'s being ~2 hours for each season (Table 3). During the spring, divergence underwent its widest range in temporal consistency, from 2.3 to 26.8 hours. Within the study areas, the coherency in time of the divergence appears to have generally decreased slightly from spring to fall. For all seasons, low $e_T$'s of divergence were prevalent over the central portions of each study area (Figs. 8a-8c). Towards the vicinity of the North Pole and the central arctic, the divergence was more consistent in time.

Vorticity had a distinctly larger temporal coherency for all seasons than did the divergence. Vorticity time scales had a range of 8.0 to 80.0 hours from May to November. The smallest $e_T$ for vorticity (8.0 hours) occurred during spring and the largest (80.0 hours) during summer (Table 3). The temporal consistency of the vorticity within the study areas gradually increased from spring to fall. Areas of larger $e_T$'s of vorticity (temporally consistent) varied from one season to the next (Figs. 9a-9c).

Temporal variations in the deformation from May to November yielded a range of $e_T$'s from 2.7 to 32.8 hours (Table 3). The smallest $e_T$ occurred during fall and the largest during spring. From spring to summer, the temporal coherency of the deformation generally decreased nearly two-fold. There was then a slight increase from summer to fall. Lower $e_T$'s of deformation occurred in the southern Beaufort Sea and the vicinity of the North Pole during spring and summer (Figs. 10a-10b). However, this trend was not very pronounced. During the fall the largest $e_T$'s of deformation occurred in the vicinity of 78°N, 158°W at the south-western edge of the study area (Fig. 10c).

From spring to fall, $e_T$'s for the ice parcel translation speed ranged from 13.0 to 80.0 hours (Table 3). Near the North Pole, the temporal coherency of speed decreased markedly from spring to summer and then increased slightly from summer to fall (Figs. 11a-11c). The spatial structure of the speed was similar to that of the deformation in fall and summer. During the fall, the largest $e_T$'s occurred at the south-western edge of the study area, while the lowest values were present in the vicinity of the North Pole during the summer (Fig. 11b-11c). However, during spring the largest $e_T$'s of speed were found in the central arctic and near the North Pole (Fig. 11a).

Space Scales

Spatial similarities of the IKP were determined as functions of distance. The length scale for each IKP is defined as that distance at which the spatial similarity fell to 0.6. These scales for each IKP and season are given in Table 4.

The length scales given in Table 4 provide interesting information on the IKP during spring, summer, and fall 1979. One sees that the divergence had the largest spatial variability of all the kinematic parameters considered, with space scale values ranging from 100 to 200 km. The lowest similarity occurred during summer, with significant variations having a scale of ~100 km. From these results, as well as from the results of the time scale analyses, we see that the divergence is the most spatially and temporally incoherent of the IKP.

In contrast, results in Table 4 indicate that speed was the most spatially consistent of the IKP, with space scale values ranging from 650 to 700 km. During summer, the spatial similarity in speed was at its greatest value. The variabilities during the spring and fall were nearly equal and slightly larger than that of summer.

The deformation rate also had high spatial coherency, particularly during spring. The range of space scale values was from 650 km in spring to 400 km in fall. A gradual increase in

<table>
<thead>
<tr>
<th></th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
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<tbody>
<tr>
<td>D</td>
<td>2.3-26.8</td>
<td>2.6-15.2</td>
<td>1.7-10.1</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>8.0-38.5</td>
<td>9.2-80.0</td>
<td>20.46.1</td>
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<tr>
<td>T</td>
<td>10.2-32.8</td>
<td>4.9-15.9</td>
<td>2.7-24.3</td>
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<tr>
<td>U</td>
<td>14.9-80.0</td>
<td>13.0-31.5</td>
<td>21.3-27.4</td>
</tr>
</tbody>
</table>

Table 3. Range of e-folding times (in hours) for each ice kinematic parameter during each season. An e-folding time of 80 hours implies that the temporal autocorrelation never fell below $e^{-1}$. 


The magnitudes of the $S_{\text{min}}$ values shown in Table 5 suggest a simple explanation for this characteristic in the similarity plots (A. Thorndike, personal communication). Suppose we have a variable $X$ which is uniformly distributed over the interval $(a,b)$. It can be shown that the mean similarity of sets of independent samples of $X$ from the interval $(a,b)$ has a value of 0.5. The similarities of all the IKP also tend toward a value of 0.5, but at different distances for each variable. Thus, a simple interpretation of this phenomena is that, at distances $> L$, the spatial variations of the IKP can be considered nearly uniformly distributed. Geophysically, this implies that $L$ is beyond the range at which one can detect, in the mean, a distinct trend in the spatial distribution of the IKP. As expected from the space scale results, ice pack divergence has the smallest $L$ scale, being ~475 km. The trend for larger $L$ scales for vorticity, then deformation, and then speed is followed, with the ice speed having an $L$ scale of ~825 km.

<table>
<thead>
<tr>
<th>Spring</th>
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<tr>
<td>D 195</td>
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<td>160</td>
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</tr>
<tr>
<td>U 660</td>
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</table>

Table 4. Length scales (in km) for the ice kinematic parameters for each season.

The vorticity had small length scales in comparison with those of speed and deformation rate. Values ranged from $\approx$300 km in summer to 350 km in spring. There was an increase in spatial variability from spring to summer and then a gradual decrease from summer to fall.

An interesting point can be noted about the plots of similarity as a function of distance (Figs. 12-14). Beyond a given distance $L$, the average spatial similarity for each IKP appears to be nearly constant. Thus, for a given IKP, the minimum average similarity between the IKP at two locations is reached when the locations have a separation of at least $L$. The value $L$ may be defined as the distance between two locations beyond which the average similarity between IKP at the locations is approximately constant and at a minimum.

We define $S_{\text{min}}$ as the constant, minimum average similarity associated with the distance $L$. One sees that, for a given IKP, the $S_{\text{min}}$'s and the $L$'s are nearly equivalent during spring, summer, and fall (Figs. 12-14). For example, during spring, summer, and fall, the $S_{\text{min}}$ for vorticity is approximately 0.47, and the value of $L$ is about 575 km. Table 5 gives values of $S_{\text{min}}$ and $L$ for each IKP for each season.

<table>
<thead>
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<th>$S_{\text{min}}$</th>
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<td>0.59</td>
<td></td>
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<table>
<thead>
<tr>
<th>L (km)</th>
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<tbody>
<tr>
<td>D 478</td>
</tr>
<tr>
<td>$\zeta$ 560</td>
</tr>
<tr>
<td>T 625</td>
</tr>
<tr>
<td>U 810</td>
</tr>
</tbody>
</table>

Table 5. Values of $S_{\text{min}}$ and $L$ for each IKP during each season.
Fig. 8a. Contour map of the e-folding time scales of divergence across the study area during May 1979.
Fig. 8b. Contour map of the e-folding time scales of divergence across the study area during August 1979.
Fig. 8c. Contour map of the e-folding time scales of divergence across the study area during November 1979.
Fig. 9a. Contour map of the e-folding time scales of vorticity across the study area during May 1979.
Fig. 9b. Contour map of the e-folding time scales of vorticity across the study area during August 1979.
Fig. 9c. Contour map of the e-folding time scales of vorticity across the study area during November 1979.
Fig. 10a. Contour map of the e-folding time scales of deformation across the study area during May 1979.
Fig. 10b. Contour map of the e-folding time scales of deformation across the study area during August 1979.
Fig. 10c. Contour map of the e-folding time scales of deformation across the study area during November 1979.
Fig. 11a. Contour map of the e-folding time scales of speed across the study area during May 1979.
Fig. 11b. Contour map of the e-folding time scales of speed across the study area during August 1979.
Fig. 11c. Contour map of the e-folding time scales of speed across the study area during November 1979.
Fig. 12. Spatial similarities of a) divergence, b) vorticity, c) deformation, and d) speed as functions of distance for spring 1979.
Fig. 13. Spatial similarities of a) divergence, b) vorticity, c) deformation, and d) speed as functions of distance for summer 1979.
Fig. 14. Spatial similarities of a) divergence, b) vorticity, c) deformation, and d) speed as functions of distance for fall 1979.
5. SUMMARY AND CONCLUSIONS

Ice kinematics can be described in terms of the 5 basic modes of motion: divergence (D), vorticity (\( \zeta \)), deformation rate (T), and ice translation (U). In this study, seasonal time histories of these ice kinematic parameters (IKP) were calculated using position data from drifting buoys in the arctic during May, August, and November 1979. The results were used to determine seasonal space and time scales of D, \( \zeta \), T, and translation speed variations in the arctic. An e-folding scale was used as a measure of the temporal coherency. Spatial variability was defined in terms of the degree of similarity between the magnitudes of a parameter at two locations.

The results of the seasonal space and time scale analyses are given in Figs. 8-11 and Tables 3 and 4. In general, the divergence was the most temporally and spatially variable of the IKP during spring, summer, and fall. In contrast, the translation speed showed the highest degree of temporal and spatial coherency during all seasons.

Sea-ice kinematics can be determined using observed ice motion or numerical models. To date, the Navy's Polar Ice Prediction System (PIPS) is the only operational system which can provide predictive ice kinematic information throughout the arctic. To provide useful operational information, this system must be run on the appropriate spatial and temporal scales. The results obtained in this study can be used to configure the model to run at the appropriate space and time scales such that, on the average, major variations in ice characteristics will not be missed.

The size of the ice parcels considered in this study (15.8 to 61.5 \( \times 10^3 \) km\(^2\)) is of the same order of magnitude as the grid size used in the PIPS model (16.1 \( \times 10^3 \) km\(^2\)). Therefore, direct comparisons of the kinematic results from this study can be made with the IKP determined by the PIPS. We first begin with a discussion of the time scale results. The time scale calculations indicate that significant variations of some ice kinematics can occur on the order of 2 hours. The minimum time scales of divergence and deformation were of the order of the sampling interval of the drifter position data (3 hours). Thus, there is the possibility that the average minimum time scales for these parameters may actually be lower than those calculated in this study.

The implication of the time scales is that the pack ice dynamics has an energetic, short time-frame component. And since divergence is involved (see Table 3), that energetic component is not a negligible factor. Divergence of the ice pack deals with the opening and closing of leads and the production of ice ridges (and thicker ice). Thus, divergence is directly linked to the tensile and compressive strength of sea ice, a very important parameter. If one were to only consider the external forcing of arctic pack ice by the atmosphere, he would expect considerably longer time scales for the ice kinematics. The rotation of the earth introduces an additional consideration, the Coriolis effect. From this, one would expect a time scale of about five hours in the arctic, especially under free-drift conditions. But not all of the kinematic parameters always have such a small minimum time scale as the ice divergence. Obviously, there is some other factor to be considered.

One might immediately suspect that the short-term variability is a result of measurement noise. However, upon closer inspection this is seen to be unlikely. First, the methodology in calculating differential motion is one in which the bias of the position error is estimated and then removed (Kirwan and Chang, 1979). Secondly, in almost all cases only the divergence of a cluster for a given month had a short e-folding time while the vorticity and deformation had time scales that were up to 8-12 times longer. Finally, low pass filtering the position data (to remove position errors) had little effect on the time scale results. Data were low passed filtered using a 10.5 hr half-power point filter that passed 95% of the energy at 12 hr. In general, the speed, vorticity, and deformation time scales of the filtered data were similar to those of the unfiltered data (within 10%). The time scales of the divergence using the filtered position data increased from 2-3 hr to 5-6 hr, still 5-7 times smaller than the scales of the other parameters. These factors indicate that position error is not the cause of the short time scales of ice pack divergence.

It is quite possible that the internal stresses of sea ice introduce the additional variability, and this would naturally be seen primarily by the ice divergence. Internal ice pack stresses may be one of the weakest areas...
of our knowledge of the geophysics of pack ice. If this factor is the cause of the short time scales of ice pack divergence, it would seem reasonable to deal with it on the same time scale in our modeling. Aside from the internal stresses, ice divergence/convergence is of great importance for the operational needs of the Navy (surfacing, firing, etc.). These factors would seem to warrant running the PIPS with a shorter time step, of the order of 1 to 2 hours.

As for space scales, the minimum length scales of divergence and vorticity (110 and 280 km, respectively) are of the order of the distance between the ice parcels considered in this study (Table 4). The actual space scales for divergence and vorticity may, therefore, be somewhat less than those determined in this analysis. Figs. 12a and 14a suggest that the space scale for divergence may be less than 100 km. We again are faced with the question as to what factor results in such short scales for ice pack divergence? Whatever that factor is, it does not seem to have the same affect on the other ice kinematic parameters. In this case, we expect the atmospheric forcing to be the primary determinant for the space scales of pack ice. The variability of the Coriolis effect might shorten the space scale somewhat, but we would expect only some secondary affects. The compressive and tensile forces of the pack ice again seem to be the likely cause of the shorter space scales for ice divergence. These forces are a function of the spatial orientation of the crystals of the pack ice floes as well as the orientation of the forcing on the individual floes. Although the orientation of the compressive or tensile forces can be determined by an ice model, it is not now possible to determine mean crystalline orientation within a grid cell. As a result, the adjustment of the PIPS grid to a smaller size based on the space scale results (<100 km) does not appear to be warranted.

The space scale analysis allows one to have confidence in IKP contour maps produced from PIPS model output. The grid spacing of the PIPS model (127 km) is of the order of the smaller space scales of the IKP. Thus, contour maps produced from the PIPS model output could be expected, on the average, to resolve nearly all the significant spatial variations in the IKP. As for IKP determined only from drifter data, contour maps can be trusted only when the centroids of the drifter clusters are separated by distances not much larger than the IKP space scales. Contouring data with spatial gaps in cluster centroid positions greater than IKP scales could result in overlooking significant variations in the IKP. The space scale results of this study (Table 4) indicate the following guidelines in contouring IKP from drifter cluster data: cluster centroid spacings of 500 to 1000 km for U and T; 400 to 500 km for \( \zeta \); and no more than 200 km for D, preferably less. In addition, the minimum space scale results (~100 km) indicate that the areal extent of drifter clusters should not be much larger than 10-20 x 10^3 km^2. Otherwise, significant variations in IKP (divergence, in particular) may not be resolved.

In final summary, the results of this work indicate the following:

1) Ice divergence is the most temporally and spatially incoherent of the ice kinematic parameters.
2) Ice translation speed is the most temporally and spatially coherent of the ice kinematic parameters.
3) The short space and time scales of ice pack divergence are not reflected in the other ice kinematic parameters.
4) Because of the operational and mechanical importance of ice divergence, it is recommended that the PIPS time step be reduced to the order of 1 to 2 hours.
5) The space scale analysis indicates somewhat smaller scales than the PIPS grid size. However, it is felt that going to a smaller PIPS grid size would not result in more accurate ice motion predictions at the present state of the model technology.
6) Contour maps produced from the PIPS model output can be expected, on the average, to resolve nearly all the significant spatial variations of the ice kinematic parameters.

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