An Investigation of the Failure Response of Laminates Under Biaxial Stress

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Advanced fiber composites are often used in laminate form in strength critical applications. However, the ultimate strength of laminates is very poorly understood, primarily because of a lack of valid experimental data. A biaxial test specimen for laminates based on a tubular geometry was developed and used to determine the failure mechanics of two laminates. The results showed a failure process that includes matrix cracking, but this matrix cracking does not appear to directly affect fiber failure. Fiber failure in the laminate studied determines ultimate laminate strength, and can be predicted based on either a maximum fiber stress or fiber strain criterion applied on a ply level.
An Investigation of the Failure Response of Carbon/Epoxy Laminates Under Biaxial Stress

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ABSTRACT

Advanced fiber composites are often used in laminate form in strength critical applications. However the ultimate strength of laminates is very poorly understood, primarily because of a lack of valid experimental data. The Mechanics of Composites Laboratory at the University of Utah has developed a biaxial test specimen for laminates based on a tubular geometry. This specimen was used to determine the failure mechanics of two laminates. The results showed a failure process that includes matrix cracking, but this matrix cracking does not appear to directly affect fiber failure. Fiber failure in the laminates studied determines ultimate laminate strength, and can be predicted based on either a maximum fiber stress or fiber strain criterion applied on a ply level.

INTRODUCTION

Advanced composite materials are finding increased usage in applications requiring high strength and stiffness combined with light weight. Composites are usually used in laminate form, which for analysis purposes can include filament wound structures. Although the stiffness of laminates is well understood, the same cannot be said about strength properties. At the present time there is no well accepted theory for laminate failure available in the literature, and in particular no theory that has been compared with experiments over a suitable range of conditions. The problem is largely due to a lack of experimental data, which in turn reflects the experimental difficulties of performing well characterized laminate failure tests.

The lack of valid experimental data on the failure of laminates is largely due to some problems peculiar to composites, including sensitivity to stress concentration, highly directional strength, multiplicity of failure modes, and difficulty in characterizing the state of stress at failure. The Mechanics of Composites Laboratory at the University of Utah has developed a biaxial test specimen for composites that eliminates a number of the experimental problems. The specimen is based on a cylinder loaded by internal pressure and axial force. The ends of the specimen have been carefully reinforced to minimize stress concentrations, and of course the cylinder has no edges and thus no free-edge effect to complicate the data interpretation.

The present program involved tests on two separate families of laminates. The results have been detailed in two papers submitted for publication, which are given as Appendix A and B. In the following we highlight these results and the rationale for the test program.

DISCUSSION

There are a number of important issues that must be addressed in considering failure of advanced composite laminates. A major question is whether laminate failure can be related to ply failure properties. The alternative is to consider each laminate as a separate material, which is obviously undesirable. A second issue is related to the multiplicity of laminate failure modes. It is known that with typical fiber and resin systems the resin matrix can crack well before ultimate laminate failure. This cracking certainly affects the laminate for some purposes; it lowers the stiffness of the cracked plies, and increases the permeability of the laminate. The question has been posed as to whether the matrix cracks produce stress concentrations that affect the failure of the fibers. A final issue is what criteria can be used to predict failure of a laminate.
The results of this investigation are presented in the form of two papers included in Appendices. Appendix A contains the paper "Biaxial Tests of Off-Axis Quasi-Isotropic Laminates," and Appendix B contains the paper "An Examination of Failure Strength in [0/±60] Laminates under Biaxial Stress." A number of conclusions were reached in this work that significantly extend our understanding of laminate failure. A fundamental result is that under a wide range of conditions laminate failure can be based on ply properties and the idea of failure of a critical ply. This approach to laminate failure prediction requires that matrix cracking and fiber failure be considered as separate events, and that independent criteria be used to predict the two failure modes independently.

Extensive matrix cracking was observed in micro-sections of the laminates. The matrix crack spacing in the transverse plies was on the order of twice the ply thickness, with larger spacing in the angle plies. These findings are in general accord with the mechanics of matrix cracking worked out previously (Swanson, 1986; Ref 20 of Appendix A).

The fiber failure criterion that appears to correlate fiber failure is either fiber direction stress or strain. There is only a small difference between these two criteria. There appears to be no significant effect of matrix cracking on fiber failure, and no discernible effect of transverse normal and shear stresses on fiber failure.

One of the specific laminates used in this experimental program was a quasi-isotropic [0/±45/0]s layup, rotated so that the 00 fibers were at angles with respect to the principal load axes of 0, 11.25, and 22.5 degrees (the maximum misalignment possible). The primary experimental variables were the varying alignment and the ratio of applied biaxial stresses. The direct experimental results showed no decrease in laminate strength with increase in misalignment of fiber and loading direction. This result is consistent with either the maximum fiber stress or fiber strain failure criteria, and is contrary to the predictions of criteria that include transverse normal and shear stresses.

The second laminate tested was a [0/±60/0]s that was loaded so that the 00 direction coincided with the hoop direction of the cylindrical specimens. This laminate is typical of many laminates with loads primarily in the 00 and smaller loads in the lateral direction. It is also typical of a standard pressure vessel layup. The primary experimental variable was the ratio of applied biaxial stress. The results showed the usual increase in laminate strength with increasing biaxial stress ratio in accordance with the predictions of the maximum fiber stress or fiber strain criteria, but again contrary to those fiber failure criteria that include transverse normal and shear stresses. Perhaps surprisingly, the simplified theory termed netting analysis appears to give good agreement with the data.

CONCLUSIONS

A consistent pattern of laminate failure behavior is emerging from our studies. Matrix cracking is observed well before ultimate laminate failure. This cracking is sensitive to in-plane transverse normal and shear stresses. Softening of the cracked plies results from this matrix cracking, with associated stress redistribution within the laminate. However in laminates usually described as "fiber dominated" this matrix cracking does not directly affect ultimate laminate failure. Rather laminate failure takes place only when fiber failure occurs. Fiber failure can be described by applying either a maximum fiber stress or a maximum fiber strain criterion on a ply basis.

The present results are in many respects unique. This is primarily due to the experimental capability that has been developed in our laboratory to study laminate failure without the complications of end and edge effects.
Appendix A

Biaxial Tests of Off-Axis Quasi-Isotropic Laminates

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to be presented at the

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Science and Engineering

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Biaxial Tests of Off-Axis Quasi-Isotropic Laminates

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ABSTRACT
An experimental investigation was carried out to determine the failure properties of quasi-isotropic AS4/3501-6 carbon/epoxy laminates under conditions where the loading axis did not coincide with the fiber axis. A tubular specimen subjected to combinations of pressure and axial tension was used to determine the response under various tension-tension biaxial stress states. The results showed no loss of strength with rotation of the laminate orientation with respect to the loading axes at angles including 22.5°, the maximum possible for the laminate used. Maximum fiber direction strains at failure were essentially independent of the rotation of the laminate as well as the biaxial stress state.

INTRODUCTION
The failure properties of advanced fiber composite laminates are of interest both from a theoretical and practical viewpoint. Since advanced composites are frequently used in strength critical applications, a knowledge of the strength properties of composite laminates is essential for rational design. Because of the number of laminates of interest, it is desirable to be able to relate laminate properties to the properties of the constituents. Surprisingly, there is little agreement in the literature about how to relate laminate strength to the strength of the constituent materials, that is to the fiber and matrix properties. Perhaps the major reason for this is that experiments on laminates have proved to be quite difficult to perform. These difficulties are largely due to complications peculiar to composites such as the free edge effect for laminates, the difference in strength between fiber and matrix, and the general sensitivity of composites to stress concentration.

It is perhaps natural that some of the early work on composite failure involved trying to extend ideas useful for isotropic materials. For example the Tsai-Hill [1] and Tsai-Wu [2] interaction formulas are a logical generalization of multiaxial stress yield criteria for isotropic materials to orthotropic materials. While the application of these criteria to lamina failure has been demonstrated [3,4], the mode of failure may be either fiber or matrix and is usually not directly predicted. For example, in an off-axis tensile coupon test of a unidirectional lamina, the failure mode for very small angles between the fibers and the applied stress direction would be expected to be that of fiber failure, while for angles on the order of 5 degrees or larger the failure mode would be expected to be matrix failure. Thus, while for example the Tsai-Wu quadratic stress criterion fits off-axis tensile coupon data, a change in failure mode from fiber to matrix is inherently involved. As will be discussed below, this change in failure mode makes a direct application of an interaction formula to laminate failure somewhat uncertain.

We have developed a biaxial test for composites that is based on internal pressure and axial tension or compression loading of a cylinder [5-7]. There have been a number of previous attempts to use this geometry for biaxial tests [8-15]. We believe that we have successfully solved the practical problems associated with stress concentrations due to pressure seals and end gripping. Thus we believe that our test results are relatively free from end effects problems. A careful analysis of our design is given in [7].

In our previous work we have conducted tests on quasi-isotropic [90/±45/0]s AS4/3501-6 carbon/epoxy laminates under a variety of biaxial stress states involving both tension-tension and tension-compression. A typical result, taken from [16], is shown in Fig. 1 where we have compared the measured laminate failure stresses with several schemes for relating laminate failure to lamina properties. A number of conclusions can be
drawn from these results. It appears to be well established in the experimental data that the apparent strength of the laminate increases with increasing biaxial tension, relative to a uniaxial applied stress. This trend is well matched by using either a fiber direction stress or a fiber direction strain criterion at the ply level. Matching the mixed tension-compression data requires that two values be used, corresponding to tension and compression failure values.

The use of a ply criterion for laminate failure requires that a differentiation be made between matrix and fiber failure, and a judgement be made about the effect of each mode of failure. For example, the use of matrix failure properties [17,18] for the tests of Fig. 1 would indicate that the off-axis plies fail by matrix cracking well before ultimate laminate failure. Although there is some uncertainty about relating matrix properties from lamina tests to laminates due to a possible insitu effect [19], the evidence presented in [18,20] indicates that micro-cracking does occur in the off-axis plies. Thus it must be assumed that this ply failure is contained by the adjacent plies, and that overall laminate failure does not occur until all plies are predicted to fail, which in this case would then involve fiber failure.

It is possible to use a stress interaction formula as a laminate failure criterion in the sense described above by requiring that the criterion be satisfied for all of the plies, under the assumption that the failure predicted for the first plies corresponds to matrix cracking. Thus ultimate laminate failure is predicted only when all plies have been predicted to fail. As shown in Fig. 1, this procedure gives very poor agreement with the experimental data, being conservative by large factors under tension-tension stress states, and non-conservative for mixed tension-compression.

It would be expected that the microcracking in the off-axis plies would affect the stress distribution in the laminate. A number of investigators have attempted to include this effect by means of nonlinear constitutive modeling [18,20,21]. A related question is whether the microcracking affects the strength of the remaining plies, as suggested in [22]. The present data would suggest that this effect is not very strong, if present at all. The delivered ply properties appear to be very close to those measured in unidirectional tensile coupons.

The quasi-isotropic laminates discussed above are often referred to as "fiber dominated", in that fibers are placed in the load directions and would reasonably be expected to carry the applied stresses. For this situation, our data show that laminate ultimate failure can be described by limit values of fiber direction stress or strain in the critical plies. The question is still open, however, as to the range of conditions over which this behavior would be expected to hold. To give further insight on this question, we have conducted a series of experiments in which quasi-isotropic laminates were loaded at an angle with respect to fiber directions. In this situation it is not obvious whether the laminate is still "fiber dominated" or whether the matrix properties would play a more important role, as appears to be the case in angle ply laminates. In previous work by Zhou and Sun [23], it was found that off-axis loading of quasi-isotropic laminate coupons significantly reduced the strength relative to on-axis loading. However this reduction was ascribed to edge effects, leaving open the question of the laminate response if edge effects were not present.

In the following, we give the results of experiments on tubular specimens composed of quasi-isotropic laminates, in which the angle between the fibers and the specimen (and load) axis was treated as a variable.

EXPERIMENTAL

The tubular specimen used in this investigation is shown in a schematic view in Fig. 2. The reinforcement is added to the ends of the tube in an effort to reduce stress concentrations associated with the pressure seals and gripping. A description of the reinforcing materials and the results of a finite element analysis of the specimen has been given previously [[6,7]. The specimen is nominally 96 mm (3.8 in) inside diameter, is lined with a thin rubber bladder, and is loaded by combinations of internal pressure and
axial load. An aluminum plug of either of two configurations is placed inside the specimen to reduce fluid (hydraulic oil) volume. In the solid plug configuration, the seals are located in the one-piece plug so that about 95% of the axial load is carried by the plug. In the split plug configuration, the axial load is transmitted directly into the specimen, so that the usual 0.5 ratio of axial to hoop laminate stress results if no additional load is applied. Strains are measured by means of strain gages located on the exterior of the specimen.

The specimens were hand layed-up on a cylindrical mandrel by Hercules Aerospace, and were made of AS4/3501-6 carbon/epoxy laminate in a [90/±45/0], configuration. However the "90" direction was actually located at angles of 0, 11.25, and 22.5 degrees with respect to the cylinder hoop direction, so that the laminate angles with respect to the major load axis were actually [78.75/33.75/-11.25/-56.25]s and [67.5/22.5/-67.5/-22.5]s for the off-axis laminates. The tensile coupon properties of the fiber lot used in this investigation are given in Table 1.

RESULTS

The specimens were subjected to tests at low load levels to determine linear elastic properties, and then tested to ultimate failure at various combinations of axial and hoop stress. The specimens were first loaded in axial tension without the pressure plug installed, to minimize any possible effects of seal friction on the results. The rubber bladder and solid plug were then installed, and the specimen loaded with internal pressure with only a small axial stress. The resulting stress-strain plots from these two tests were then used to determine the laminate elastic constants. The results are given in Table 2 for the three laminate configurations. It can be seen that the laminate elastic response is indeed quasi-isotropic within the variability of the data, in that the properties are independent of the orientation of the laminate with respect to the loading axis. The stiffness data are also consistent with the lamina stiffness values and the usual application of classical lamination theory.

The stress-strain results up to failure load levels are shown in Figs 3-6. We have plotted the biaxial stress-strain results in the form of the orthotropic stress-strain law given as follows:

\[
\sigma_\theta \text{ vs } \theta \quad \sigma_\theta \epsilon_\theta + \sigma_z \epsilon_z \\
\sigma_z \text{ vs } z \quad \tau_{\theta z} \epsilon_{\theta z} + \sigma_{zz} \epsilon_z
\]

Thus the results should follow a line of unit slope if the material were linearly elastic. In fact the results do follow this relationship quite closely. As shown in Fig. 3-5, there seems to be no systematic difference in the stress-strain response between the different layup configurations. Although we have shown only one example of the axial stress-strain response, the same result was also obtained in that the axial response was not significantly different between the three layups. We have used the elastic properties determined for each individual specimen in plotting these results. The same conclusions would have been reached if average properties had been used, although specimen to specimen variations in observed stiffness would have shown up more strongly in the stress-strain plots.

The stresses and strains at failure are given in Table 3 and plotted in Figs. 7 and 8. The hoop strains at failure are plotted in Fig. 7 as a function of the ratio of applied laminate axial stress to hoop stress, for all three laminate configurations. The solid line is the tensile coupon strain at failure for the fiber lot used in the present investigation. It can be seen that no significant difference exists in the measured failure strains for the three laminate configurations. Further, the results are very close to the tensile coupon strains. Thus there seems to be little loss in strain capability associated with the laminations relative to coupon values.
The laminate stresses at failure are shown in Fig. 8. The trends in the data are similar to those seen previously, in that increasing axial stress is seen to strengthen the laminate. It is possible that there are systematic differences between the stress values for the three laminate orientations, but the differences are not large and are masked by data scatter.

DISCUSSION

The major point of the present investigation is to establish the failure properties of advanced fiber composite laminates, and in particular carbon/epoxy laminates. A fundamental question to be answered is whether laminate failure can in fact be related to independently measured ply properties, or whether each laminate must be treated as an individual material and characterized separately. Because of the multiplicity of laminates of possible interest, there is obviously great motivation for using lamina properties. Once it has been established that lamina properties can be utilized, the next question is obviously just how to relate these properties to laminate failure.

Our previous work, although limited to only a single carbon/epoxy system, has suggested that at least it is possible to relate lamina properties to laminate failure. However, to do this a clear distinction must be made about the mode of failure. Matrix cracking can correspond to total failure of a lamina, while in many cases matrix cracking in a laminate is contained by the other plies. Thus it is important to differentiate between contained ply cracking and ultimate ply failure. In our previous work on quasi-isotropic laminates it appeared that fiber failure was closely correlated with ultimate laminate failure. It is obviously important to know when matrix degradation can lead to, or strongly affect, ultimate laminate failure.

The present investigation is an attempt to broaden the experimental evidence on laminate failure to include loading not coincident with fiber directions. Obviously this has practical significance in terms of design, as it may in some cases be difficult to establish load directions. However, it also has theoretical significance concerning failure criteria. The evidence shown in Figs. 7 and 8 indicates that there is no loss of strength associated with rotation of the laminate with respect to the loading axes, over a range of biaxial tension-tension stress states. The rotation included a value of 22.5°, which is the maximum possible for this laminate. The measured maximum fiber direction strains at failure appear to be independent of the laminate rotation, as well as independent of the ratio of applied laminate stress as had also been seen for on-axis loading.

The present data provide additional information on the choice of failure criterion for ply failure under different applied stress conditions. Obviously interaction formulas such as the Tsai-Wu quadratic illustrated do not at all match the measured response, being off by up to a factor of 4. Even the trends are apparently incorrect; the interaction formulas predict weakening with increased biaxial stress, while the data shows a strengthening. Additionally, the interaction formulas show a significant weakening with rotation of the laminate that is not supported by the data. A possibility exists that the use of an interaction formula would be improved by a better calculation of the ply stresses. The use of linear theory at present undoubtedly overpredicts the transverse stresses in the critical ply, relative to a calculation that includes progressive ply softening due to microcracking and perhaps other mechanisms. However, it would be difficult to accurately include these softening effects, due to uncertainties in the calculation as well as the sensitivity of the interaction formulas to transverse stress.

As discussed in the Introduction, our previous work has shown that either maximum fiber direction ply stress or fiber direction strain can be effectively used as a fiber failure criterion in laminates. Because of the low in-plane minor Poisson's ratio, the two criteria give essentially the same prediction. The present work adds additional corroboration to this, although not in complete detail. As shown in Figs. 1 and 8, the general trends of laminate strengthening with increased biaxial stress are predicted by either of
these criteria. However, in detail an increase in failure stress with rotation of the laminate direction would be predicted. As shown in Fig. 8, while no decrease is seen, the evidence for an increase in strength is not obvious. It is possible that there is indeed a small increase, but the normal data scatter makes it difficult to say either way.

The present data agree with that obtained previously on the AS4/3501-6 system in that measured fiber direction strains in laminates agree closely with the ultimate strain measured in tensile coupons. While a tremendous simplification for dealing with this fiber and resin system, this is probably not a universal result. Our recent work on other carbon/epoxy systems [24] suggests that this translation of properties from tensile coupon to laminate may depend on the properties of the matrix.

SUMMARY AND CONCLUSIONS
An experimental study has been carried out to determine the strength of AS4/3501-6 carbon/epoxy laminates in a [0/±45/90]s configuration, in which the laminate configuration has been rotated by angles of 0, 11.25, and 22.50 with respect to the biaxial stress loading axes. A tubular test specimen loaded by combinations of internal pressure and axial load was employed. The results showed that little difference could be observed in both the elastic response and the failure properties with respect to the laminate rotation. The present results agreed well with previous work, in that it was possible to correlate laminate failure with lamina failure properties, using either a maximum fiber direction stress or strain criterion. Further, the failure values measured in tensile coupon tests could be used directly for the laminate failure prediction. In detail, while no strength loss was observed with laminate rotation, the increase in strength predicted by either the maximum fiber stress or strain criteria was masked by data scatter and probably at best is smaller than predicted.

ACKNOWLEDGMENTS
This work was supported by the Air Force Office of Scientific Research under Award No. AFOSR 86-0115. Specimens for this program were furnished by Hercules Aerospace, Magna, Utah. This support is gratefully acknowledged.

REFERENCES


Figure 1. Failure stresses in AS4/3501-6 quasi-isotropic cylinders under axial stress.

Figure 2. Schematic of the four inch tubular specimen with end grips and internal pressure plug.

Figure 3. Comparison of hoop stress with measured strains and stiffness of an aligned quasi-isotropic carbon/epoxy laminate, tested at a stress ratio cy/cy of 0.500.

Figure 4. Comparison of hoop stress with measured strains and stiffness of a 35° degree rotated quasi-isotropic carbon/epoxy laminate, tested at a stress ratio cy/cy of 0.500.
Figure 5. Comparison of hoop stress with measured strains and stiffness of a 22.5 degree rotated quasi-isotropic carbon/epoxy laminate, tested at a stress ratio $\sigma_2/\sigma_1$ of 0.500.

Figure 6. Comparison of axial stress with measured strains and stiffness of an aligned quasi-isotropic carbon/epoxy laminate, tested at a stress ratio $\sigma_2/\sigma_1$ of 0.500.

Figure 7. Effect of stress ratio on fiber strain at failure for quasi-isotropic carbon/epoxy laminate.

Figure 8. Comparison of stresses at failure for off-axis-loading of quasi-isotropic carbon/epoxy laminates with LPT predicted maximum fiber strain and Test We failure stresses.
Table 1.

AS4/3501-6 Lamina Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11,\text{elastic}}$</td>
<td>18.4 Msi</td>
<td>(126.87 MPa)</td>
</tr>
<tr>
<td>$E_{11,\text{secant}}$</td>
<td>20.65 Msi</td>
<td>(142.38 MPa)</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>1.6 Msi</td>
<td>(11.03 MPa)</td>
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<tr>
<td>$G_{12}$</td>
<td>0.8 Msi</td>
<td>(5.52 MPa)</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>.28</td>
<td></td>
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<tr>
<td>$\varepsilon_{1f}$</td>
<td>1.384 %</td>
<td></td>
</tr>
<tr>
<td>$X_T$</td>
<td>288 ksi</td>
<td>(1986 MPa)</td>
</tr>
<tr>
<td>$Y_T$</td>
<td>6.95 ksi</td>
<td>(47.9 MPa)</td>
</tr>
<tr>
<td>$S$</td>
<td>13.88 ksi</td>
<td>(95.7 MPa)</td>
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* at failure

Table 2.

Average measured engineering constants of tubular quasi-isotropic carbon/epoxy laminates

<table>
<thead>
<tr>
<th>Orientation</th>
<th>$E_{zz}$ (Msi)</th>
<th>$E_{\theta\theta}$ (Msi)</th>
<th>$v_{zz}$</th>
<th>$v_{\theta\theta}$</th>
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<tbody>
<tr>
<td>Aligned</td>
<td>7.983 (55.0)</td>
<td>7.139 (49.2)</td>
<td>.344</td>
<td>.302</td>
</tr>
<tr>
<td>CV* [ % ]</td>
<td>[3.6]</td>
<td>[6.5]</td>
<td>[7.3]</td>
<td>[7.8]</td>
</tr>
<tr>
<td>11.25 deg.</td>
<td>7.662 (52.8)</td>
<td>7.391 (51.0)</td>
<td>.326</td>
<td>.315</td>
</tr>
<tr>
<td>CV* [ % ]</td>
<td>[5.1]</td>
<td>[3.8]</td>
<td>[9.4]</td>
<td>[10.8]</td>
</tr>
<tr>
<td>22.5 deg.</td>
<td>7.633 (52.6)</td>
<td>7.361 (50.8)</td>
<td>.293</td>
<td>.283</td>
</tr>
<tr>
<td>CV* [ % ]</td>
<td>[3.2]</td>
<td>[5.4]</td>
<td>[9.7]</td>
<td>[10.7]</td>
</tr>
<tr>
<td>Overall</td>
<td>7.739 (53.4)</td>
<td>7.312 (50.4)</td>
<td>.319</td>
<td>.300</td>
</tr>
<tr>
<td>CV* [ % ]</td>
<td>[4.5]</td>
<td>[5.4]</td>
<td>[11.0]</td>
<td>[11.0]</td>
</tr>
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</table>

* Coefficient of Variation
### Table 3.

Measured strength properties of tubular quasi-isotropic carbon/epoxy specimens

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Fiber Strain at Failure</th>
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<tr>
<td></td>
<td>%</td>
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<tr>
<td>LTCU-86-1-#2</td>
<td>1.366</td>
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<tr>
<td>LTCU-86-1-#3</td>
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</tr>
<tr>
<td>LTCU-86-7-#3</td>
<td>1.264</td>
</tr>
<tr>
<td>LTCU-86-8-#2</td>
<td>1.290(^a)</td>
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<tr>
<td>LTCU-86-8-#3</td>
<td>1.376(^a)</td>
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<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Stresses at Failure</th>
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<tr>
<td></td>
<td>Ksi (MPa)</td>
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<tr>
<td>LTCU-86-1-#2</td>
<td>62.2 (428.7)</td>
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</tr>
</tbody>
</table>

\(^a\) Extrapolated values
Appendix B

An Examination of Failure Strength in [0/±60] Laminates under Biaxial Stress

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An Examination of Failure Strength in \([0/\pm 60]\) Laminates under Biaxial Stress

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ABSTRACT

Reliable criteria for predicting failure of fiber composite laminates are necessary for rational analysis and design. The lack of experimental data, particularly under multiaxial stress conditions, has hampered the selection of these criteria. A study of failure in tubular specimens composed of AS4/3501-6 carbon/epoxy laminates was carried out, using loadings of internal pressure and axial force to vary the states of stress. The results showed that laminate failure could be based on predicted fiber failure in a critical ply. The maximum fiber direction strain was found to correlate the failure stresses within the variability of the data. This finding is consistent with previous results on other laminates.

INTRODUCTION

The question of the strength of fiber composite laminates under mechanical load is somewhat controversial, despite the importance in practical design. There have been a number of previous studies of laminate failure under multiaxial stress conditions [1-15], although it has been difficult to establish overall trends. Fiber composite laminates fail in a somewhat complicated manner. In some laminates that are often termed "fiber dominated" ultimate failure is controlled by fiber failure. This ultimate failure is often preceeded by matrix failure, which occurs by the matrix developing cracks between the fibers. This matrix cracking is particularly susceptible in plies with a high transverse stress. While the matrix cracking undoubtedly softens the laminate somewhat, the overall structural integrity of the laminate appears in many cases to be preserved by the plies with more favorable orientations, that are thus loaded primarily in the fiber direction.

As the plies can be considered "building blocks" for laminates, it is desirable to base considerations of laminate failure on the properties of the plies. The alternative would be to treat each laminate as a distinct material, which is obviously undesirable because of the number of possible laminates of interest. Thus it would seem desirable to characterize the failure properties of the individual ply under the multiaxial stress states of interest. However it has proved difficult to follow this procedure in practice, as the failure modes of a lamina are different from those of a laminate. That is, contained matrix cracking can occur in a laminate, while this cracking corresponds to ultimate failure in a lamina. Since the strength of matrix and fiber can differ by a factor of 50, it is thus necessary to distinguish
fiber and matrix failure modes. This point has been made previously by Hashin [16] and Hahn, Erikson, and Tsai [17].

It thus appears that it is desirable to be able to predict both when a ply will fail and in what failure mode. As ply stresses are in general multiaxial even when the overall laminate loading is uniaxial, multiaxial stresses are an additional part of the problem. There are a number of possible criteria that could be used to predict ply failure under multiaxial stresses [16,18]. However a number of these have been developed for materials that show a less significant difference in directional strengths, so that it is not important to differentiate between failure modes. Theories of this type have less usefulness for fiber composite materials.

Perhaps a fundamental difficulty in assessing failure theories is the lack of experimental data. This lack of data reflects the difficulty of performing multiaxial stress tests on well characterized specimens. The Mechanics of Composites Laboratory at the University of Utah has developed a biaxial specimen based on internal pressure and axial force loading of a cylindrical tube. The specimen appears useful for tests on laminates. The data from these laminate tests can serve to indirectly establish lamina failure criteria. It must be assumed that the fiber failure mode corresponds to ultimate laminate failure. Results have been presented previously for AS4/3501-6 carbon/epoxy laminates in a [0/±45/90]s quasi-isotropic configuration [19-21]. The results have corroborated the comments made above about separating matrix and fiber failure criteria. The results show that fiber failure is accurately represented by either a maximum fiber stress or a maximum fiber direction strain criterion. Criteria that use quadratic polynomials in the ply stress components were significantly in error, by as much as a factor of four, for predicting ultimate laminate failure.

The present paper presents further biaxial stress results on a [0/±60/0]s laminate. This laminate is of importance for several reasons, including the fact that it is typical of many laminates designed for loads primarily in the 0 direction. Under certain stress states it can have significantly more inplane shear than the [0/±45/90] laminates examined previously, and thus provides a more stringent test of possible failure criteria. Finally, because the laminate uses only two independent fiber angles it can be analyzed by means of netting theory. As will be discussed further subsequently, netting analysis is a simplified theory that is often employed in pressure vessel design. Varying the ratio of applied biaxial stresses in the present experiments will permit the accuracy of netting theory to be established over a wide range of conditions.

In the following we briefly review the experimental techniques employed. The results are then examined in terms of ply failure theories.
EXPERIMENTAL

The specimen used for the biaxial tests is a thin walled cylinder with reinforced ends, loaded with internal pressure and axial force. A schematic is shown in Fig.1. The analysis of the specimen has been described in detail in [19,22].

The material used for the present tests is AS4/3501-6 carbon/epoxy prepreg in a [90/±30/0]_s layup. Here 90 corresponds to the hoop direction, which is the direction of the highest loading in the cylindrical configuration. Conventional notation for this laminate would be to have the highest laminate stress in the 0° direction. We have used both notations here to describe the same layup, but no confusion should arise if the reader notes that only one layup was used. Note that the laminate is not quasi-isotropic because of the extra 90 ply. The specimens were made by Hercules Aerospace, Magna, Utah. The fiber lot acceptance tensile test values are given in Table 1 for the fiber lot used in this program. The fiber ultimate strains are slightly lower than obtained in previous test programs with a different fiber lot [19-21].

The tests were all performed with proportional loading of the internal pressure and axial load. This mode is obtained by using load feedback in a servo-controlled test machine with the pressure signal as the load command. Very precise control of the ratio of stresses can be obtained in this manner. Strains were measured with strain gage rosettes placed on the specimen axial midplane and located around the circumference.

RESULTS

The cylinders were first subjected to low axial and pressure loads, applied separately, to obtain the laminate stiffness coefficients. The results of these tests are given in Table 2. It can be seen that the results are reasonably consistent with each other and also with previous data for the lamina elastic constants.

The specimens were then loaded to failure at a prescribed ratio of axial to hoop laminate stress. Typical stress-strain curves are shown in Figs 2 and 3. These figures are plotted so that if the laminate stress-strain response were linear, the data would follow a line of unit slope. In fact the data follow this line quite closely, indicating linear response. Although some scatter was observed among the various specimens, no significant trends other than shown in these two figures were observed.

The measured laminate failure strains and stresses are shown in Figs. 4 and 5 and listed in Table 3. We have plotted the average hoop strain at failure as a function of the ratio of applied laminate stresses in Fig. 4, along with the average uniaxial coupon failure strain. It can be seen that no significant change in failure strain is seen at the different ratios.
of applied stresses. This is a significant finding that is in accord with the results obtained previously with quasi-isotropic laminates. Further, the average of the laminate results is not significantly different than the strain obtained in uniaxial coupon tests, although it does seem to be slightly lower.

The laminate failure stress in the hoop direction is seen in Fig. 5 to increase with increased axial stress, over the range of values obtained. Again, this is a result also seen previously with quasi-isotropic laminates.

The laminate failure stresses can most easily be interpreted by comparison with predictions of various failure criteria. This comparison is carried out below.

**COMPARISON WITH FAILURE THEORIES**

The estimation of the strength of a laminate under general stress conditions is vital to rational design and usage of composites. This process is greatly facilitated by having a reliable failure criterion that will correlate basic ply properties with the ultimate failure of the laminate. We have thus compared a number of failure criteria with the present data. The criteria that will be displayed are the maximum fiber direction strain (or stress, which is almost identical), netting analysis, the Tsai-Wu stress quadratic polynomial [23], and a similar polynomial given by Hashin [16]. Expressions for the later two criteria are given below, as

Tsai-Wu: \[ f_1 \sigma_1 + f_{11} \sigma_1^2 + f_2 \sigma_2 + f_{22} \sigma_2^2 + f_{66} \tau_{12}^2 = 1 \]  
Hashin: \[ f_1 \sigma_1 + f_{11} \sigma_1^2 + f_{66} \tau_{12}^2 = 1 \]

Again, it should be emphasized that it is intended to use these criteria for the prediction of ultimate laminate failure, and not the initiation of matrix cracking unless that corresponds to ultimate failure. Accordingly, it will be required that the Tsai-Wu criterion be satisfied for all of the plies, on the basis that the first ply failure will correspond only to matrix cracking. The Hashin criterion and the maximum fiber strain (or stress) criterion already pertain only to laminate failure, so the first ply to satisfy these expressions indicates laminate failure.

Netting analysis is based on neglecting any contribution from the matrix and determining the fiber direction stresses from equilibrium alone. Thus it can only be applied to laminates with two independent angles, such as studied at present. If the ratio of overall laminate axial to hoop stress is defined as R, then netting analysis predicts that failure will occur in the hoop direction for \( R < 0.75 \) (for this laminate) with the laminate hoop stress given by
Netting: \[ \sigma \theta = \frac{X_t}{t} \frac{t_\theta}{(1 - R\tan^2\alpha)} \]

where \(X_t\) is fiber direction tensile strength, \(t_\theta\) is fiber thickness in the hoop direction, \(t_t\) is total laminate thickness, and \(\alpha\) is the helical angle of 30\(^\circ\).

Several complications arise in applying these criteria. One of these is establishing the necessary constants to be used, which are based on material property tests. The values that were used are listed in Table 4 along with references to the sources of the data. An additional problem is the somewhat nonlinear behavior of the carbon/epoxy stress-strain response. Although the laminate stress-strain relationship appears to be quite linear as seen in the measured stress-strain response presented above, in detail the behavior is more complex. The stress-strain response of unidirectional coupons stressed in the fiber direction stiffens with strain. The present material had a final secant modulus approximately 10% higher than the initial modulus. However the shear modulus softens with strain, as shown in [24] and elsewhere. Further, it has been inferred by a number of investigators that transverse cracking will lead to softening of the plies that are stressed in the transverse direction [25,26], due to microcracking. Thus there is a redistribution of the laminate stresses at higher strains. Presumably these effects could all be included in a nonlinear response model, but to avoid these complexities at present an adjusted value of fiber direction failure stress has been used. This adjusted value is just the tensile coupon failure strain multiplied by the initial modulus. This adjusted value will be used in the Tsai-Wu and Hashin criteria that involve laminate stresses that are to be predicted by linear classical lamination theory.

A plot of the various failure criteria is shown in Fig. 6, compared with the experimental data. It can be seen that the only criterion that does not fit the data at all is the Tsai-Wu quadratic. On the other hand, the Hashin criterion which is also a quadratic gives a reasonable fit. However at a stress ratio of zero, the Hashin criterion predicts failure of the helical plies, which does not agree with the experimental findings. It appears that perhaps the best fit is obtained with the maximum fiber strain criterion, although the practical difference is in fact not very significant between these criteria with the exception of the Tsai-Wu quadratic. It can be seen that netting analysis shows the correct trend of increasing strength with increasing stress ratio, over the range of conditions tested. This is surprising in view of the simplifications of that theory.

DISCUSSION
The primary focus of this paper is to establish rules for predicting the failure of fiber composite laminates. It would appear that the principal difficulty in accomplishing this previously has been the lack of experimental data. Thus the present data appears to be useful, and significantly extends our previous work which has been limited to quasi-isotropic laminates only.

The basic assumption tentatively used to examine various failure theories is that the criteria should be based on ply stresses, and applied to the individual plies of the laminate. This of course requires that a further assumption relating ply failure to laminate failure be made. The assumption at present is that fiber failure within a ply would correspond to ultimate laminate failure. Thus the criteria examined were presumed to apply to fiber failure. The comparison of the selected failure theories with the data seen in Fig. 6 shows that the maximum fiber direction strain criterion fits the trends of the data very well. This finding has been also seen in our previous work on quasi-isotropic laminates as well, including tests in which the loading directions did not coincide with the fiber directions [27]. Thus the maximum strain criterion is emerging as a practical tool for predicting laminate failure. It should be noted that the maximum fiber direction ply stress criterion is for all practical purposes the same as the strain criterion due to the typically low value of minor Poisson's ratio for the carbon/epoxy materials.

It is surprising that netting analysis also tends to predict the same trends of the data with changes in applied stress ratio. If some adjustment in the value of fiber strength is allowed, this criterion is as accurate as the fiber strain criterion for the laminate studied. Thus netting analysis would appear to be a reasonable criterion, particularly for preliminary design because of the ease of use, when the ply stresses can be determined from equilibrium. It should be noted that the present data involves changes in the ratio of helical to hoop ply stresses (as calculated by netting theory) from 0 to 0.87. In the design of pressure vessels there has been some concern that the ratio of stresses also influenced the outcome of a netting analysis prediction of strength. The present results appear to indicate that this is not the case. They also illustrate the advantage of the present experimental setup, where the applied stress ratio can be changed without changing the laminate layup.

The Hashin and Tsai-Wu stress quadratic polynomials are superficially similar, except that the Hashin expression has removed the transverse normal stress terms on the basis that they correspond to matrix failure only. The difference resulting from this change is striking. As shown in Fig. 6 the Tsai-Wu expression is significantly at variance with the data, while the Hashin expression gives an excellent fit. As mentioned before, the Hashin polynomial does predict failure in the wrong ply at low stress ratios. At higher stress ratios the importance of the shear stress is diminished and the criterion reduces to that of
maximum fiber direction stress in the hoop plies. It appears that the Hashin polynomial 
over-emphasizes the shear stress contribution to fiber failure. This is perhaps 
understandable in the sense that the matrix would fail first under high shear stresses, thus 
relieving the stress on the fibers. This possibility was also suggested by Hashin [16], as 
well as Hahn, Erikson, and Tsai [17]. It may be that the shear stresses, and perhaps the 
normal stresses, should be in the fiber failure criterion but at reduced levels. The present 
data are probably not precise and extensive enough to establish this result.

A point made earlier is that the calculation of laminate stresses with linear theory 
limits the accuracy possible in predicting laminate failure. While the nonlinearity in the 
present and previously studied carbon/epoxy laminates does not appear to be large in 
comparison with say metal plasticity, it does have complicating effects. For example, the 
stresses in the fiber directions are likely to be larger than those predicted by linear theory, 
by a maximum on the order of 10%. The transverse and shear stresses are likely to be 
lower than given by linear theory, perhaps by a much higher percentage due to the 
softening involved. Failure theories that involve these transverse and shear stresses would 
thus seem to be subject to larger uncertainties due to the difficulty of establishing these 
stresses accurately. Further refinements of nonlinear laminate analysis would appear to be 
helpful here. On the other hand, use of a fiber strain criterion would avoid the necessity of 
having an accurate knowledge of these stresses.

SUMMARY AND CONCLUSIONS

Tests were carried out on a [0/±60/0]_5 laminate in tubular form. Multiaxial stress 
conditions were imposed by combinations of internal pressure and axial load. The 
measured stress-strain response was in reasonable accord with classical lamination theory. 
The failure stresses and strains could be well correlated by means of predicted ply failure 
using a maximum fiber direction strain (or stress) criterion. The Hashin polynomial 
predicted failure in the wrong plies at low values of axial to hoop laminate stress, but 
agreed well with the data at higher stress ratios. Netting analysis gave a surprisingly good 
correlation with the measured data. The Tsai-Wu quadratic stress polynomial did not agree 
well with the data, being conservative by large factors even though used on a "last ply 
failure" basis.

ACKNOWLEDGMENTS

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Award No. 86-0115. The specimens for the program were contributed by Hercules 
Aerospace, Magna, Utah. This support is gratefully acknowledged.
REFERENCES


Table 1


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<th>Mean Value</th>
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<td>Tensile Strength, $X_T$</td>
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<td>Fiber Strain</td>
<td>1.384 %</td>
<td>4.6 %</td>
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Table 2

Measured stiffness coefficients of tubular [90/30/-30/90/-30/30/90] carbon/epoxy specimens.

<table>
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<th>Specimen Number</th>
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Table 3

Measured strength properties of tubular [90/30/-30/90/-30/30/90] carbon/epoxy specimens

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<th>Stresses at Failure</th>
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<td>$\sigma_\theta$</td>
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Table 4
Material Properties used in Failure Analysis of AS4/3501-6

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<td>$X_T$ Modified for use in Linear Analysis</td>
<td>1758 Mpa (255 ksi)</td>
<td>See Text</td>
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<td>48 Mpa (6.95 ksi)</td>
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<td>Transverse Normal Strength in Compression, $Y_C$</td>
<td>168 Mpa (-24.4 ksi)</td>
<td>[28]</td>
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<td>In-Plane Shear Strength, $S$</td>
<td>96 Mpa (13.88 ksi)</td>
<td>[24,28]</td>
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<tr>
<td>Fiber Direction Failure Strain</td>
<td>0.0138</td>
<td>See Text</td>
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Figure 1  Schematic of the four inch tubular specimen with end grips and internal pressure plug.
Figure 2 Comparison of hoop stress to measured strain and stiffness for a carbon/epoxy laminate with a stacking sequence of [90/30/-30/90/-30/30/90], tested at a stress ratio $\sigma_2/\sigma_0$ of 0.683.
Figure 3 Comparison of axial stress to measured strain and stiffness of a carbon/epoxy laminate with a stacking sequence of [90/30/-30/90/-30/30/90], tested at a stress ratio $\sigma_z/\sigma_0$ of 0.683.
Figure 4 Comparison of fiber strain at failure to stress ratio $\sigma_2/\sigma_0$ for a carbon/epoxy tubular laminate with a stacking sequence of [90/30/-30/90/-30/30/90].
Figure 5 Comparison of hoop and axial stresses at failure for a carbon/epoxy laminate with a stacking sequence of [90/30/-30/90/-30/30/90].
Figure 6 Comparison of stresses at failure for a carbon/epoxy laminate with a stacking sequence of [90/30/-30/90/-30/30/90] to maximum fiber strain, netting analysis, Tsai-Wu stress polynomial, and Hashin failure criteria.
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