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THE PREDICTION OF SATELLITE LIFETIMES

by

D.G. King-Hele
Doreen M.C. Walker

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SUMMARY

Accurate prediction of the lifetimes of satellites decaying naturally under the action of air drag is one of the most difficult problems of orbital dynamics. Lifetime estimation is also a matter of practical importance when a satellite perceived as 'dangerous' is about to decay. The time when the satellite finally plunges into the lower atmosphere is affected by at least eight identifiable types of variation in upper-atmosphere density, by odd zonal harmonics in the gravitational field, and by possible changes in the mode of rotation of the satellite.

For use in the RAE Table of Satellites, we have since 1958 made (and revised) lifetime estimates for all satellites, rockets and other major components (but not fragments). More than 10000 such predictions have been made, nearly all being calculated from the current observed decay rate. This Report summarizes the methods used in the past and gives the results of new orbital theory developed to take better account of the effects of atmospheric oblateness and odd zonal harmonics in the geopotential. The Report also describes new approximate methods to allow for the effect of the solar-cycle and day-to-night variations in air density, and includes discussion of other sources of error. There are separate sections, giving techniques for use when no observed values of decay rate are available. Numerical-integration methods are not covered, but we give the graphical-analytical methods devised for two common types of high-eccentricity orbit - geosynchronous transfer orbits and Molniya-type orbits.

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1 INTRODUCTION

Since 1958 the likely lifetimes of all spacecraft and rockets in orbit have been estimated and updated at RAE for use in the *RAE Table of Earth Satellites*¹. For orbits of low eccentricity, the methods² have been based on the current observed decay rate and orbital theory³ derived on the assumption that the air density has no time variations; but a correction has been made for the semi-annual variation in air density. Recently⁴, the theory has been extended to derive corrections for the oscillations in perigee height (of order 10 km) caused by the odd zonal harmonics in the geopotential and by atmospheric oblateness. The present Report incorporates the corrections for these two effects and also derives approximate correction factors for the effects of long-term solar activity and the day-to-night variation in density, giving a total of five corrections to the basic lifetime estimate. The effects of short-term variations in air density are still unmodelled, however, because they are unpredictable, and they may be expected to produce errors of up to 10% in the prediction of the remaining lifetime of the satellite, unless solar activity remains steady.

For high-eccentricity orbits, lunisolar perturbations need to be taken into account. Although numerical integration is usually needed for these orbits, two analytical methods have been developed to estimate lifetimes approximately, for low-inclination transfer orbits⁵ and for Molniya-type orbits⁶. For completeness, the necessary graphs for using these methods have also been included here, with some small improvement on the original formulations.

More accurate predictions for high-eccentricity orbits can be made by numerical integration of the equations of motion, although this integration often has to continue for more than 25 years and is therefore very demanding in computer time. For these satellites the zero-drag PROD computer program⁷ is used for making the first lifetime estimate, which is often quite accurate when perigee rises to a great height and then clearly descends to below 100 km, but is less satisfactory if the perigee tends to oscillate at heights of 150-250 km. As decay approaches, we use the program PTDEC⁸, which includes drag via an atmospheric model but is much more expensive in computer time. The use of these programs is described elsewhere⁹, and they are not discussed here.

It is of course possible to predict the lifetime of a low-eccentricity orbit by numerical integration instead of the graphical methods presented here. But the program for numerical integration must incorporate an atmospheric model that covers as many variations as the graphical method if the results are to be

as accurate as those of the graphical method. (Unfortunately PTEC does not include semi-annual or day-to-night effects, or variations in solar activity, and, even so, cannot in practice be run for long enough⁹.)

It would also be possible to computerize the graphs given here: but the calculations for lifetime estimation are trivial with the graphical method, once the decay rate is known. The 'difficult' parts of the task are deriving a reliable decay rate, and arranging to be able to lay hands quickly on several sets of orbital elements for any of the 6000 satellites in orbit. The methods presented here allow lifetimes to be predicted without computer capability.

The arrangement of the Report is as follows: after a digression in section 2 on the 'threat' of satellite decay, section 3 gives the basic method for low-eccentricity orbits, in terms of the currently-observed decay rate; then sections 4 to 7 detail the corrections required, which are summarized in section 8, where other sources of error are also discussed. Section 9 gives the alternative method to be used when values of the current decay rate are not available. Section 10 outlines the methods for the transfer and Molniya-type orbits.

2 SATELLITE DECAY AS A 'THREAT'

There is a widespread perception, among the public and particularly the media, that the decay of a large spacecraft poses a threat to human well-being. Consequently, as with Skylab 1 in 1979, there can be a voracious demand for forecasts of the impact point or, more realistically, of the track over the Earth on the final revolutions in orbit: an arcane art suddenly becomes of public interest.

The 'danger' to people on the ground from pieces of satellites falling out of the sky is in reality minimal by comparison with the other hazards of life. There are about two satellites per week from which pieces survive the heating during atmospheric entry and tumble to the ground, and the impact points are scattered almost randomly world-wide. By contrast there are more than 100 incidents per week when aircraft, or parts of aircraft, fall out of the sky, and these impacts tend to be concentrated near centres of population. Also some of the aircraft are very large and some are filled with inflammable fuel, whereas the satellite fragments are inert lumps (see chapter 2 of Ref 10 for illustrations). So far as is known, no one has yet been injured by a piece of a decaying satellite, and the probability of anyone being killed in the next 10 years is small, whereas more than 50 people per year on the ground are killed by falling aircraft and more than 50000 per year by traffic accidents. So the

danger of satellites to people on the ground is truly negligible. But there are no limits to human folly, and if the media choose to label a particular satellite as 'dangerous', it will be perceived as dangerous.

Debris from satellites falling through the lower atmosphere does pose a greater threat to aircraft in flight in that region, because their vulnerable area is so large. This danger is still very small by normal aircraft safety standards, but it may be prudent to divert aircraft known to be flying in the vicinity at the time when a large satellite is expected to decay.

If the decaying satellite has a nuclear reactor as a power source, the scattering of radioactive particles in the atmosphere would have some long-term ill-effects. However, the satellites are small and the ill-effects are likely to be much less than those caused by tests of atomic bombs in the atmosphere, by accidents in nuclear power stations and by the small amounts of radioactivity that are released under normal operation from nuclear installations.

3 BASIC METHOD FOR LIFETIME ESTIMATION

3.1 Introduction

There are two possible approaches to lifetime estimation. One is to use the observed decay rate of the satellite (with an approximate value of eccentricity) to assess its future life. The other is to calculate the lifetime from the current orbital elements and knowledge of the size, shape and mass of the satellite and the current air density. The first of these methods is much to be preferred, because the second method requires knowledge of the mass and dimensions of the satellite, and these are not accurately known for about 90% of the satellites launched. Also the values of eccentricity available may not be accurate enough to give a good value of perigee height; and even when the perigee height is accurately known, the current density at perigee is tedious to calculate, because it depends on at least 7 parameters; and even then will only be accurate to about $\pm 10\%$. In this section the first method is covered, but an outline of the second method is given in section 9, for use when the observed decay rate is not available.

3.2 Practical procedure

The most widely used data for satellite orbits are NORAD 2-line elements issued regularly by the NASA Goddard Space Flight Center. In these elements the mean motion n is given in revolutions per day, and so it is convenient to express the lifetime in terms of \dot{n} , the rate of change of n expressed in

revolutions/day². In practice, values of \dot{n} should always be obtained by taking the difference between two values of n , and dividing by the time interval between them, which needs to be long enough to ensure a change of at least 0.001 rev/day in n . The value of \dot{n} given in the 2-line element should not be used (except as a last resort), because it is essentially an 'instantaneous' value.

The basic lifetime L^* is written in the form

$$L^* = Q/\dot{n}, \quad (1)$$

where Q is a function of n , the eccentricity e , the atmospheric density scale height[†] H (which depends on the height ν and solar activity), and the gradient H , that is dH/dy , which we denote by μ . Values of H and μ for low and high solar activity are shown in Fig 1. The values are from the *COSPAR International Reference Atmosphere 1972*¹¹ and apply when the solar radiation energy at 10.7 cm wavelength, $F_{10.7}$, has values of 80 or $150 \times 10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$ (The value of $F_{10.7}$ is usually not much less than 80 at solar minimum, but may greatly exceed 150 at a strong solar maximum)

The variation of Q with e for high and low solar activity and for perigee heights of 200 or 600 km is shown in Fig 2. For $e < 0.2$ these graphs are based on equations (5) and (6) of Ref 2, and for $e > 0.2$ on Ref 12: there is a slight discontinuity at $e = 0.2$.

Although solar activity does not appear to have much effect in Fig 2, appearances are deceptive because lifetimes most often have to be calculated for near-circular orbits, with $e < 0.01$, and this is the region where solar activity has most effect. More detailed diagrams are needed for accurate calculation.

The first of these, Fig 3, shows how Q varies with e for $\nu \leq e \leq 0.03$ for various values of n , covering perigee heights between 150 and 600 km. For perigee heights near 600 km, the change from low to high solar activity increases Q by 30% for $e = 0$, but the change is much less when $e = 0.03$ (about 5%).

If the orbit is nearly circular, it is generally better to work with the graph of Q versus n for $0 \leq e \leq 0.01$, given as Fig 4

[†] H is a measure of the rate of decrease of density ρ with height, being given by $H = -\rho/(d\rho/dy)$.

Fig 4 terminates at $n = 16.4$ rev/day and does not cover the very lowest circular orbits. For a circular orbit the lifetime is given by

$$L^* = \frac{3H_8 n \eta}{2an} \quad (2)$$

from equations (6.79) and (4.94) of Ref 3. In this equation H_8 is the value of H at height H_8 below perigee, n is given by equation (4.88) of Ref 3 and a is usual the semi major axis (here merely the radius of the orbit). If the height is 150 km, which corresponds to $n = 16.5$ rev/day, $H_8 \approx 13$ km for high solar activity and $H_8 \approx 12$ km for low solar activity, from Fig 1. Also, if the lifetime is assumed to end when the height decreases to 120 km, $(a_c - a_L) = 30$ km in (4.88), and $\beta \approx 0.1 \text{ km}^{-1}$, so that $\eta \approx 0.95$. Equation (2) then gives $Q = 0.047$ for high solar activity and $Q = 0.043$ for low solar activity. These values are indicated by circles in Fig 4.

Solar activity has most effect on Q for near-circular orbits at heights of 300-500 km, as is obvious from the wide range of values of h at these heights in Fig 1. For example, with $e = 0$ and $n = 15.4$ (corresponding to 450 km height), Fig 4 gives $Q = 0.21$ for high solar activity and $Q = 0.16$ for low.

When $0.03 \leq e \leq 0.2$, a more accurate value of Q can be obtained by plotting Q/e against e for various values of n , as shown in Fig 5. For given e and n , the variation of Q with solar activity never exceeds 10%; and the variation of Q with n , for given e , is also nearly always less than 10%. With care, Fig 12.4 can be read with an accuracy of 1%, which is slightly better than the theory on which it is based. Although Q is insensitive to solar activity when $e > 0.03$, the lifetime is of course shorter when solar activity is high, because the greater atmospheric density leads to a higher value of \dot{h} , and hence a lower value of Q/\dot{h} .

Figs 3 to 5 are the same as the corresponding graphs in Ref 2. For $e > 0.2$, the theory has recently been revised¹², and Fig 6 gives the variation of Q with e on the assumption that the lifetime is reduced by a factor $(1 - 0.1 \mu)$ when μ is taken into account. This is close to the result obtained in Ref 12. In using Fig 6 it should be remembered that when $e > 0.4$, quite large lun solar perturbations in perigee height may arise. However, if the amplitude of the lunisolar perturbation in perigee height is less than 100 km, and if the lifetime extends to several cycles of perturbation, L^* may be approximately estimated by

calculating the average value of \dot{n} over a cycle, $\bar{\dot{n}}$ say[†], and taking $L^* = Q/\bar{\dot{n}}$. If the lunisolar perturbations are large and not easily recognizable as periodic, numerical integration is usually needed⁹, though analytical methods are still sometimes feasible (see section 10).

These are the basic techniques, from Ref 2 with some additions. We now look at methods for taking account of several awkward variations that sometimes spoil the accuracy of the basic method.

4 EFFECT OF OSCILLATIONS IN PERIGEE HEIGHT CAUSED BY ODD ZONAL HARMONICS AND ATMOSPHERIC OBLATENESS

Theory for these two effects, which both produce variations in perigee height of up to 10 km, has been developed in Ref 4. If L^* is the basic lifetime, as derived in section 3 from spherical-atmosphere theory, the true lifetime L may be expressed as

$$L = L^* F(\alpha z) F(\alpha_0), \quad (3)$$

where $F(\alpha z)$ is the factor to take account of the effect of odd zonal harmonics, and $F(\alpha_0)$ is the factor that allows for atmospheric oblateness. Expressions for both these factors are derived in Ref 4.

The value of \dot{n} in equation (1) is the observed current or (in terms of the theory) the *initial* value of \dot{n} , and should strictly be written as \dot{n}_0 . For simplicity, the suffix 0 has been omitted in section 3, but clarity now demands its inclusion.

The gravitational attraction of the odd zonal harmonics in the geopotential has the effect of producing a regular variation of the perigee distance r_p with the argument of perigee ω (the variation is virtually sinusoidal if $e > 0.01$). The amplitude B of this oscillation, as given by the odd zonal harmonics of Ref 13, is shown in Fig 7. The theory of Ref 4 requires that $B/H < 0.3$ and shows that the value of $F(\alpha z)$ depends on the parameter

$$b = \frac{B}{H} \left(1 - \frac{1}{2z_0} \right), \quad \left(z_0 > \frac{1}{2} \right) \quad (4)$$

where z_0 is the initial (*ie* current) value of $z = ae/H$. In practice, a typical value of a/H is 125, so that $z \approx 1$ for $e = 0.008$ and $z \approx 10$ for $e = 0.08$. The

[†] Averaging $1/\dot{n}$ is not recommended, because the observed value of \dot{n} can be very small (or even slightly negative as a result of solar radiation pressure) when perigee height is at its maximum.

value of b is zero for $z_0 \leq \frac{1}{2}$. The theory of Ref 4 is not valid over a small range of inclinations near 63.4° , where B becomes very large. Since B/H must be less than 0.3, the maximum permissible value of B depends on the value of H : generally we assume $H > 33$ km, as happens if the perigee height exceeds 180 km; and this requires that $B < 10$ km. If H is larger, correspondingly larger values of B are permissible.

Fig 8 shows how $F(oz)$ varies with ω_0 when $b = 0.2$ for various values of $\omega_L - \omega_0$, where ω_L is the value of ω at the end of the satellite's life. The thick line shows the value, F_∞ say, when ω goes through many revolutions. The thinner lines show the value, F_f say, when ω completes only a fraction f of a revolution, so that $|\omega_L - \omega_0| = 2\pi f$. If ω goes through $(N + f)$ revolutions, $F(oz)$ may be taken as $(NF_\infty + fF_f)/(N + f)$, or may be calculated from equation (61) of Ref 4.

It is also shown in Ref 4 that the atmospheric oblateness factor $F(a_0)$ depends on c'_0 and λ : here λ depends only on z_0 and is shown in Fig 9, and c'_0 is the initial value of $c' = cI_2/I_0$, where c depends on the ellipticity of the atmosphere, ϵ , and is given by

$$c = \frac{\epsilon}{2H} r_{p0} \sin^2 i, \quad (5)$$

with i as the orbital inclination. Also I_n denotes the Bessel function of the first kind and imaginary argument, of order n and argument z , and the variation of I_2/I_0 with z is shown in Fig 10.

Fig 11 shows the variation of $F(a_0)$ with ω_0 when $c'_0 = 0.2$, for various values of $\omega_L - \omega_0$. The thick line shows the results for $\omega_L - \omega_0 = \pm N\pi$, where N is an integer. The thinner lines show the results when ω does not complete a half-revolution, for $\lambda = 1$ (corresponding to $e_0 > 0.2$): the broken line gives results for $\lambda = 0.8$ (corresponding to $z_0 \approx 8$, or $e_0 \approx 0.06$). If ω goes through $(N + f)$ half revolutions, $F(a_0)$ can be calculated from the expression given two paragraphs ago for $F(oz)$, or from equation (65) of Ref 4.

The values of b and c'_0 used for illustration in Figs 8 and 11, although large, are within the possible practical range for high-inclination orbits[†], and the Figures show that changes of up to about 25% in lifetime may be produced by either the odd zonal harmonics or atmospheric oblateness. Both correction factors may be scaled linearly when b and c'_0 differ from 0.2.

[†] However, c'_0 can only be as large as 0.2 if $c = 0.3$ and $z_0 > 5$.

If $\omega_L - \omega_0$ is small enough, $F(\omega)$ and $F(a_0)$ differ from 1 by less than 10%, and are then not worth evaluating; this happens when $\omega_L - \omega_0 < 40^\circ$ for $F(\omega)$ or when $\omega_L - \omega_0 < 19^\circ$ for $F(a_0)$.

5 EFFECT OF THE SEMI-ANNUAL VARIATION IN AIR DENSITY

For satellites with lifetimes between about 1 month and about 18 months, the effect of the semi-annual variation in air density is often the greatest source of error in estimating L . Unfortunately the semi-annual variation changes appreciably in form from year to year and cannot at present be accurately predicted. The only practical procedure is to take a standard form for the variation, and to hope that the variation which is in progress at the time of the prediction is not too seriously non-standard. For this purpose we take the variation shown in Fig 12, which is an average over several years¹⁴ and applies at a height of about 250 km. Most satellites with lifetimes of 1-18 months have perigee heights between 200 and 300 km, and the amplitude of the semi-annual oscillation does not vary much with height within this height range, so the use of Fig 12 is not unreasonable. However, there is no guarantee of its accuracy in future years. During some years, errors up to 10% in L may occur, particularly for lifetimes of between 2 and 6 months; during other years the variation may be near-standard, and errors from this source would be much smaller. It should also be noted that the semi-annual variation appears to be greater in the northern hemisphere than the southern¹⁵, and this will be relevant if the satellite's perigee is expected to remain in the same hemisphere until decay.

Again we define L^* as the lifetime in the absence of perturbations, that is, with density independent of time, and denote the 'semi-annual correction factor' by $F(sa)$, so that the 'true' lifetime L is given by

$$L = L^*F(sa) \quad (5)$$

Then, if ρ_0 denotes the current (initial) value of the 'standard variation' of density given in Fig 12, and if $\bar{\rho}$ denotes the mean value of ρ between the initial date and the decay date, equation (6) of Ref 4 shows that

$$F(sa) = \rho_0/\bar{\rho} \quad (7)$$

Fig 13 shows the values of $F(sa)$ for lifetimes up to 160 days and a succession of current dates through the year, on the assumption that the semi-annual variation has the form shown in Fig 12. Usually $F(sa)$ can be read

directly from Fig 13, but an iterative procedure may be needed if $F(sa)$ departs greatly from 1.0. To take an extreme example, if the basic lifetime is calculated from \dot{n} at 10 April, and $L^* = 80$ days, then Fig 3 gives $F(sa) = 1.25$, so that $L = 1.25L^* = 100$ days. The appropriate value of $F(sa)$ for a 100-day lifetime from 10 April is $F(sa) = 1.31$, giving as a 'second iteration' $L = 1.31L^* = 105$ days.

An alternative plotting of $F(sa)$ against the time of year for a specified lifetime is given as Fig 14 which is needed for lifetimes greater than 160 days. The curve for '1 year' applies equally well for 2, 3, 4, ... years.

6 EFFECT OF THE SOLAR-CYCLE VARIATION IN AIR DENSITY

It is quite useful to look on the 11-year solar-cycle variation in air density as a slow-motion version of the semi-annual oscillation. Pursuing this rather loose analogy, we see that the basic lifetime L^* should be multiplied by a factor $F(sc)$, which is given by

$$F(sc) = \rho_{po} / \bar{\rho}_p \quad (8)$$

where ρ_{po} is the current (initial) perigee density, and $\bar{\rho}_p$ the mean value of density at the same height during the remaining lifetime L .

Unfortunately, solar activity is much more irregular than the semi-annual variation, and neither the timing nor the amplitude of a future cycle has yet been reliably predicted (except by luck), though there are several confident predictions¹⁶⁻¹⁸ that the next maximum will be in 1990 or 1991, with a maximum sunspot number of 110 ± 10 . Fig 15 shows the sunspot index from 1610 to 1986, based largely on Refs 19 and 20. The irregularity is obvious, and the possibility that solar activity might return to the very low levels prevalent in the 17th century cannot be ruled out. If this happened, it would invalidate lifetime predictions made on the assumption that the mean solar activity in future cycles will be similar to that in the current cycle or recent past cycles. The lifetimes would then be much longer than predicted.

Until the solar cycle can be reliably predicted, it is only possible to adopt empirical and imprecise models. If the satellite is expected to remain in orbit for several solar cycles, and has already survived one, the obvious procedure is to use the increase in n δn say, during the time interval δt between one solar minimum and the next (or between one maximum and the next), and to take $L = Q\delta t/\delta n$. This method should give a good approximation if L exceeds

about three cycles and if subsequent cycles are of similar strength to the cycle used in calculating δn and δt . If the subsequent levels of activity are higher than in the cycle used, the real lifetime will be shorter than that predicted; and *vice-versa*.

It is important to remember that decay is much more likely to occur in a year when solar activity is high. Indeed solar maximum can be regarded as 'harvest time' for long-lived satellites: that grim but sun-tanned reaper known as 'solar Max' drags them down in hundreds. Nearly all the long-lived satellites that escape the cull will last until at least the next solar maximum. If the predicted lifetime is calculated as between 10 and 25 years, we need to adjust the prediction to take account of the work of solar Max. For example, a satellite launched at solar minimum is unlikely to have a lifetime of 10 or 20 years, because that would probably imply decay near solar minimum, which is unlikely. The lifetime is much more likely to be near 5, 15 or 25 years, and one of these values should be chosen as the best round-number estimate for lifetime, in preference to 10 or 20 years.

If a satellite is going to decay within the current solar cycle, the problem is even more difficult, because the prediction of solar activity, and hence of air density, is so unreliable. Only a very rough-and-ready method can be offered, as follows. In attempting to model the solar activity index $F_{10.7}$ over a solar cycle, a sinusoidal approximation may be best; but the density has a much stronger variation than $F_{10.7}$ (by a factor of up to 30 at 500 km height). If ρ_m is the density at perigee height at solar minimum, and ρ_M the value at the same height at solar maximum, the best simple analytical representation of the density at this fixed height seems to be

$$\rho_p = \rho_m + (\rho_M - \rho_m) \sin^4 (\pi t/P), \quad (9)$$

where P is the period of the solar cycle and t is the time measured from solar minimum. In practice the date of 'solar minimum' is ill-defined, and t should be measured from a date half-way between successive solar maxima: the important point is to try to ensure that the maximum density ρ_M occurs at the right time, so that $t = \frac{1}{2}P$ is the time of solar maximum. Equation (9) can only be called a desperate expedient rather than a realistic model. In reality, two successive maxima rarely have the same maximum density ρ_M ; the variation is asymmetrical, the rise being steeper than the decline; the period P usually varies between one cycle and the next; and the zero-date for t is subject to

error (for rising solar activity) because the date of the next maximum may be wrongly forecast.

If, after all these cautions, we still adopt equation (9) for ρ_p , the value of $\bar{\rho}_p$, found by integrating ρ_p between time t_0 and time $t_0 + L$, is

$$\bar{\rho}_p = \frac{1}{8}(5\rho_m + 3\rho_M) - \frac{P(\rho_M - \rho_m)}{4\pi L} \left[\sin \frac{2\pi(t_0 + L)}{P} - \sin \frac{2\pi t_0}{P} - \frac{1}{8} \left\{ \sin \frac{4\pi(t_0 + L)}{P} - \sin \frac{4\pi t_0}{P} \right\} \right]. \quad (10)$$

In equation (10) the term in curly brackets is in practice often negligible, though it needs to be retained for consistency (to ensure that $\rho_p \rightarrow \rho_{po}$ as $L \rightarrow 0$).

From equations (8), (9) and (10), the explicit equation for $F(sc)$ is

$$F(sc) = \frac{1 + (\rho_M/\rho_m - 1) \sin^4(\pi t_0/P)}{\frac{1}{8}(5 + 3\rho_M/\rho_m) - \frac{P(\rho_M - \rho_m)}{4\pi L} \left[\sin \frac{2\pi(t_0 + L)}{P} - \sin \frac{2\pi t_0}{P} - \frac{1}{8} \left\{ \sin \frac{4\pi(t_0 + L)}{P} - \sin \frac{4\pi t_0}{P} \right\} \right]}. \quad \dots (11)$$

When ρ_M/ρ_m is large, iteration for L may be necessary in applying equation (11). Suppose, for example, that $\rho_M/\rho_m = 4$ (appropriate for a height of about 350 km), and that a lifetime estimate made at solar maximum ($t_0 = \frac{1}{2}P$) gives $L^* = 2$ years. Obviously the lifetime will be longer than 2 years, and if we guess $L/P = \frac{1}{2}$ for simplicity, equation (11) gives $F(sc) = 1.30$ or $L = 2.60$ years. If $P = 11$ years, this gives $L/P = 0.236$, whence $F(sc) = 1.27$ from (11) and $L = 2.54$ years. A third iteration gives $L = 2.51$ years.

Fig 16 shows the variation of ρ_M/ρ_m with height for an 'average' solar maximum ($F_{10.7} \approx 150$, giving $T_m \approx 1000$ K in *CURA 1972*), and also for a high solar maximum ($F_{10.7} \approx 250$, $T_m \approx 1300$ K) similar to that of 1981. At solar minimum, T_m is taken as 700 K for the purposes of Fig 16. Most satellites with lifetimes greater than half a solar cycle have perigee heights greater than 350 km, so Fig 16 shows that in practice ρ_M/ρ_m usually exceeds 4.

Fig 17 shows how $F(sc)$ varies with t_0/P when $\rho_M/\rho_m = 4$, for selected values of lifetime L , and Fig 18 is a similar diagram for $\rho_M/\rho_m = 8$. As

expected, the values of $F(sc)$ are largest when the lifetime estimate is made at or a little after sunspot maximum ($\tau_0 = \frac{1}{2}P$), and smallest when it is made at or a little after sunspot minimum ($\tau_0 = 0$). In Fig 18, $F(sc)$ can be as large as 3 or as small as 0.2.

Values of up to 20 are possible for the density ratio, as Fig 16 shows, when perigee is near 500 km. So the variation of $F(sc)$ with ρ_M/ρ_m for values up to 20 is shown in Fig 19, for $L/P = 1, 2, 3 \dots$ and various values of τ_0/P .

Since $F(sc)$ is so strongly dependent on the density ratio, the lifetime and the initial point, it is not always easy to interpolate in Figs 17 to 19, and it is best to calculate $F(sc)$ directly from equation (11). The term in square brackets in equation (11) is zero if L/P is an integer, so that

$$F(sc) = \frac{8}{5 + 3(\rho_M/\rho_m)} \left\{ 1 + \left(\frac{\rho_M}{\rho_m} - 1 \right) \sin^4 \left(\pi \tau_0/P \right) \right\} \quad (L/P = 1, 2, 3 \dots) \dots \dots (12)$$

This form may also be used as an approximation whenever $L/P > 10$, with error less than $1.6(\rho_M/\rho_m - 1)\%$.

In Figs 17 and 18 the value of $F(sc)$ often departs from 1.0 by a large factor: so lifetime estimates can be seriously in error if solar activity is ignored or wrongly predicted. The error is greatest for lifetime estimates made at solar minimum: a good example²¹ is Skylab 1, launched in 1973, for which NASA predicted in 1974 a total lifetime of 10 years, whereas UK predictions taking account of solar activity (correctly) gave 6 years.

7 EFFECT OF THE DAY-TO-NIGHT VARIATION IN AIR DENSITY

7.1 The variation of density

At heights above 200 km the air density is highest at about 14 h local time and lowest at about 04 h local time¹¹. The variation depends on latitude as well as longitude, however, and is best regarded as a 'daytime bulge' in the contours of density, having its maximum near the equator (or, more accurately, near the sub-solar latitude) and lagging behind the sub-solar longitude by about 30° . Consequently the perigee of a satellite of high inclination, as it slowly travels round the orbit, sometimes avoids the daytime bulge by 'slipping past' at high latitude, and sometimes goes through the centre of the bulge. The track depends on the right ascension of the node and the time of year. The result is that the day-to-night variation in perigee density experienced by the orbit,

although nearly constant in period, varies in amplitude as the right ascension of the node steadily precesses.

The density at perigee depends primarily on the geocentric bulge-perigee angle ψ_p as specified in Ref 22. For orbits of high inclination, the variation of ψ_p differs from cycle to cycle (as seen in Figs 5 to 8 of Ref 14, for example), and a regular variation of ψ_p can only be assumed if $i < 35^\circ$. However, this simplest of variations does also serve as a fair approximation, on average, for satellites of high inclination, if they experience enough day-to-night cycles, although the average amplitude is then generally about 0.6 of the maximum possible, because the perigee rarely samples the full maximum and more often crosses the bulge on its lower slopes. So a correction factor based on the assumption that the variation is regular will be useful for satellites of low inclination and for long-lived satellites of higher inclination.

The most realistic simple approximation for the variation of density ρ with angular distance ϕ from the centre of the daytime bulge appears (from section 3.2 of Ref 22) to be

$$\rho = \rho_m + (\rho_M - \rho_m) \cos^4 \frac{1}{2} \phi,$$

where ρ_M and ρ_m are now the maximum and minimum densities in the *day-to-night* variation. Applying the above equation for perigee (with $\rho = \rho_p$ and $\phi = \phi_p$), and assuming ϕ_p varies linearly with time t measured from the minimum-density point[†], we have

$$\rho_p = \rho_m + (\rho_M - \rho_m) \sin^4 (\pi t/P), \quad (13)$$

where the period P is now that of the day-to-night cycle rather than the solar cycle. Equation (13) is the same as equation (9); consequently $\bar{\rho}_p$ is given by equation (10), and $F(dn)$ by equation (11).

The period P of the day-to-night cycle may be written²³ as

$$P = \frac{360}{4.982(R/a)^{3.5} (5\cos^2 i - 2\cos i - 1) - 0.986} \text{ days}, \quad (14)$$

[†] Formally, this implies that $\phi_p = \pi(1 - 2t/P)$. In reality, ϕ_p reverts to positive values when $0.5 < t/P < 1.0$, but its sign is of no consequence in evaluating $\cos^4(\frac{1}{2}\phi_p)$.

if $(1 - e^2)$ is taken as 1. Values of P given by equation (14) are plotted against 1 for $R/a = 0.8$ and 0.9 in Fig 20. This diagram is difficult to read when the period exceeds 1 year and values of $\dot{\phi}_p$ are given as an alternative. If P is in years and $\dot{\phi}_p$ in degrees/day, $P = 0.986/\dot{\phi}_p$.

The appropriate amplitude for the ratio of maximum to minimum density, ρ_M/ρ_m , in the day-to-night variation indicated in Fig 21, where the values of density are taken from *CIRA 1972* $T_\infty = 900$ K for ρ_M and $T_\infty = 700$ K for ρ_m for 'low solar activity', and with $T_\infty = 1300$ K for ρ_M and $T_\infty = 1000$ K for ρ_m for 'high solar activity'.

In practice, satellites with perigee heights greater than about 500 km usually have lifetimes which extend over many cycles of the day-to-night variation, and it is often simplest then to calculate n as an average over one cycle and not to use $F(dn)$ at all. If the $F(dn)$ method is used mainly for perigee heights lower than 500 km, Fig 21 shows that ρ_M/ρ_m is unlikely to exceed 5, even for low-inclination orbits. Values of ρ_M/ρ_m between 2 and 4 are those most likely to arise in practice.

By deriving equation (13), we have tacitly implied that the density at perigee will be a crucial parameter. This is likely to be true if the eccentricity is high enough, but not for a circular orbit. Thus we need to divide orbits on the basis of their eccentricity, and $e = 0.03$ proves a convenient dividing line.

7.2 Orbits with $e > 0.03$

It is shown in the Appendix that, for orbits with $e > 0.03$ moving in an atmosphere having the simple sinusoidal variation of ρ with ϕ specified in Ref 22, the factor $F(dn)$ that takes account of the day-to-night variation in air density may be written as

$$F(dn) = \rho_{po}/\bar{\rho}_p \quad (15)$$

where, as before, ρ_{po} is the initial (current) value of ρ_p and $\bar{\rho}_p$ is the mean value of density at the initial perigee height. Equation (15) will be taken as the 'standard' $F(dn)$, and it is of course the same as equation (8) for $F(sc)$. Equation (15) does have limitations, however: its accuracy would be dubious if the lifetime was less than one cycle of ϕ_p , because the model is then probably inadequate. Also it applies strictly only for orbits of inclination less than 35° , or for higher-inclination orbits over several cycles of ϕ_p ; and of course e must exceed 0.03.

We assume that equation (15) still holds when ρ is represented by a more accurate model, such as equation (13). Then, within the provisos of the previous paragraph, the variation of $F(sc)$ with t_o/P in Fig 17 applies equally well to $F(dn)$, although the broken curves may be unreliable because they are for values of L/P less than 1.

In using Fig 17 to obtain $F(dn)$, the appropriate period P is of course that of the day-to-night cycle, and t_o/P represents the fraction of that cycle which has been completed at the date when the lifetime estimate is made. Thus if $t_o = 0$, perigee is at the minimum-density point; if $t_o = \frac{1}{2}P$, perigee is moving towards the maximum density and is half-way there; if $t_o = \frac{1}{4}P$, perigee is at the maximum-density point; if $t_o = \frac{3}{4}P$, perigee is half-way on its journey from maximum to minimum density.

If $\rho_M/\rho_m < 4$, as is likely in practice, the values of $F(dn)$ obtained from Fig 17 can be scaled down by use of Fig 19. A more accurate procedure is to calculate $F(dn)$ from equation (12), if $L/P = 1, 2, 3 \dots$. If $\rho_M/\rho_m < 2$, a linear variation of $F(dn)$ with ρ_M/ρ_m may be assumed. Thus if $F(dn, 2)$ denotes the value for $\rho_M/\rho_m = 2$, the value of $F(dn)$ for any other ρ_M/ρ_m less than 2 may be taken as

$$F(dn) = 1 + \left\{ \frac{\rho_M}{\rho_m} - 1 \right\} \left\{ F(dn, 2) - 1 \right\} \quad (1 \leq \rho_M/\rho_m \leq 2) \quad (16)$$

7.3 Orbits with $e < 0.03$ ($z < 3$)

Values of $F(dn)$ for $e < 0.03$ are derived in the Appendix, where it is shown that the standard value of $F(dn)$ should be scaled towards i by a factor $I_1(z_o)/I_0(z_o)$, which decreases from 0.81 at $z_o = 3$ to 0 at $z_o = 0$ (see Table A1). Thus

$$\left\{ F(dn) \right\}_{z_o < 3} = 1 + \frac{I_1(z_o)}{I_0(z_o)} \left\{ F(dn) - 1 \right\} \quad (17)$$

If $\rho_M/\rho_m < 3$, as is likely in practice for most high-inclination orbits, Fig 19 shows that $0.6 < F(dn) < 1.7$. The factor $\left\{ F(dn) \right\}_{z_o < 3}$ may therefore still be significant down to quite low values of e : to keep it within 10% of 1.0 requires $z < 0.3$, that is, an eccentricity below about 0.003.

7.4 Review of results

It should be emphasized that the day-to-night variation is a very difficult effect to treat, and the results have many limitations and approximations.

Strictly, the results for $F(dn)$ are realistic only for $i < 35^\circ$ or for satellites of higher inclination with lifetimes extending over several day-to-night cycles. For the latter the maximum density is rarely sampled and a 'scaling-down factor' of about 0.6 is recommended to find the effective value $(\rho_M/\rho_m)_{eff}$ for use in Fig 17 or 19. Thus

$$(\rho_M/\rho_m)_{eff} = 1 + 0.6 \left\{ (\rho_M/\rho_m) - 1 \right\}, \quad (18)$$

where ρ_M/ρ_m on the right-hand side is found from Fig 21. After this correction has been made (when necessary), the values of $F(dn)$ can be read from Fig 17 or 19, and then modified using equation (17) if $z_0 < 3$ ($e_0 < 0.03$).

Finally, it should be noted that if $L/P = 1, 2, 3 \dots$, or if $L > 10P$, the value of $F(dn)$ may be found from equation (12), with the reduced value $(\rho_M/\rho_m)_{eff}$ if $i > 45^\circ$.

8 REVIEW OF THE SOURCES OF VARIATION IN LIFETIME

8.1 Summary of procedure for applying the five correction factors

From the results of sections 4 to 7, the full procedure for predicting lifetime is, first, to calculate the basic lifetime L^* using Figs 3 to 6, and then to apply up to five correction factors, the true lifetime L being expressed as

$$L = L^* F(oz) F(ao) F(sa) F(sc) F(dn). \quad (19)$$

The odd zonal correction factor, $F(oz)$, is given by equation (61) of Ref 4, and here by Fig 8, with Figs 1 and 7 as auxiliaries. The second correction factor, $F(ao)$, takes account of atmospheric oblateness, being given by equation (65) of Ref 4, and being shown here in Fig 11, with Figs 9 and 10 as auxiliaries. The third correction factor, $F(sa)$, takes account of the semi-annual variation in air density and is shown in Figs 13 and 14. The solar-cycle factor, $F(sc)$, is defined by equations (8) to (11) and plotted in Figs 17 to 19, with Fig 16 as an auxiliary. The final (and least satisfactory) correction factor, $F(dn)$, provides an indication of the effects of the day-to-night variation in air density. Standard values of $F(dn)$, which are the same as for $F(sc)$, are given in Figs 17 and 19, though now with Figs 20 and 21 as auxiliaries: strictly, the standard $F(dn)$ applies only when (a) perigee goes through at least one day-to-night cycle, (b) the inclination is less than 35° , and (c) the eccentricity

is greater than 0.03. The correction is smaller for orbits of high inclination and for eccentricities less than 0.03, as explained in section 7.

It is rarely that the five factors are significant simultaneously, but all can have important effects on the lifetime. If we take any change greater than 10% to be significant, $F(a_0)$ may be of importance for lifetimes greater than 7 weeks, and $F(\omega_z)$ and $F(s_a)$ for lifetimes greater than 1 month. All these three factors generally have maximum effects of about 30%[†]. The effect of $F(s_c)$ can be significant for lifetimes greater than 1 year and is very large for satellites with perigee heights near 500 km having lifetimes of 5-20 years: Fig 18 shows that the lifetime estimate may have to be reduced by a factor of up to 5 if made at solar minimum, or increased by a factor of up to 3 if made at solar maximum. The effect of $F(d_n)$ is small if the perigee height is below 250 km, but can become large (>50%) for satellites having lifetimes of several years and perigee heights above 250 km.

8.2 Other effects

Although five important variations are covered in the previous paragraph, the effects of short-term density variations due mainly to solar activity have not been brought into the scheme of calculation because they are as yet unpredictable. A useful (though arbitrary) division of these is into four categories:

(i) Sudden changes in density occurring within a few hours, attributable to transient outbursts of solar activity and closely correlated with geomagnetic storms. For lifetimes greater than 1 week, the times of such disturbances should be avoided when calculating \dot{n} . If the satellite's lifetime is only a few days, it is wise to look at the predicted values of the geomagnetic index and to make allowance for any unexpected disturbances. These sudden changes linked with geomagnetic storms are usually most prevalent in the year or two after solar maximum.

(ii) Monthly changes. The Sun rotates relative to the Earth once every 27.3 days, and sunspot groups that persist on the Sun's face for several rotations tend to produce a 27-day recurrence in upper-atmosphere density variations. If such recurrences have arisen in preceding months, it is best to evaluate \dot{n} over about 27 days, if the lifetime exceeds about 6 months. For shorter lifetimes, \dot{n} should if possible be evaluated at a

[†] The effect of $F(\omega_z)$ can be larger at inclinations close to 63.4° , which are excluded from the theory.

level of solar activity that is likely to be the average over the remaining lifetime - a procedure that calls for some inspiration (or guess-work)

(iii) Irregular short-term variations. When all allowances have been made for the effects of solar activity, day-to-night variations, etc, there usually remains an irregular variation in density, and hence \dot{n} , having an amplitude of about $\pm 10\%$ and a timescale ranging between a day and a week; and this produces errors of up to 10% in lifetime estimation because of the variability in \dot{n} .

(iv) The possibility of large changes in solar activity. In the two or three years before and after a strong solar maximum, solar activity may undergo quite large and unexpected variations which can cause larger errors in lifetime prediction, particularly for lifetimes between about 6 months and 2 years. These errors are less likely to arise in the 2 years before and after solar minimum.

When the satellite is within a few revolutions of decay, two further sources of variation are liable to cause errors in lifetime estimation.

The first, which applies to satellites of all shapes and sizes, arises because the drag coefficient does not remain constant as assumed in the theory; instead there is a decrease in the drag coefficient as the perigee descends to heights where free-molecule flow no longer prevails. At heights of 150-400 km the drag coefficient is often taken²⁴ to be about 2.2, but the value begins to decrease when perigee descends below 130 km and the drag coefficient is reduced to about half its free-molecule value²⁵ by the time perigee has descended to 90 km. At this stage there has already been some extension of the lifetime, and the subsequent time in orbit will be double what the theory predicts. The net effect varies between one satellite and another, but a rough rule of thumb is to add one revolution to the lifetime for predictions made in the last 24 hours of the satellite's life. This very rough rule is most appropriate for quite large satellites with mass/area of order 100 kg/m^2 and having orbits of low eccentricity. The effect is smaller if

- (a) the satellite is small (because it remains in free-molecule flow down to a lower altitude),
- (b) the mass/area is lower (because the remaining life is then shorter), or
- (c) the orbit is of high eccentricity (because perigee may already be below 110 km some days before decay).

Fig 26 of Ref 2 covers this effect graphically.

The second source of error applies to all non-spherical satellites and arises from changes in attitude, which alter the drag or create lift. The simplest variation is a change in the mode of rotation of the satellite, produced by aerodynamic forces at perigee: at worst, a tumble can be converted to arrow-like flight, thus considerably reducing drag and extending the lifetime; this is unusual, but changes of $\pm 20\%$ in drag probably occur quite often, especially for satellites of complex shape (such as those with solar panels). A more drastic variation in the mass/area occurs if part of a satellite is torn off by the effects of aerodynamic heating and forces at perigee: the remaining structure then in effect becomes a new satellite, needing a new lifetime estimate, based on subsequent orbital data, which will not exist unless the satellite survives for several more revolutions. A third problem, even more intractable, arises when the attitude of the satellite as it approaches perigee is such that it generates upward lift, tends to 'skip off' the lower atmosphere and survives for a few more revolutions. (If the lift is negative, it may 'dive in' earlier than predicted.) It is easy enough to think of other scenarios disastrous to mathematical prediction. For example, aerodynamic heating at perigee may be sufficient to provoke an explosion in residual propellants remaining in a rocket.

9 LIFETIME IN TERMS OF ρ AND δ

9.1 Introduction

The basic method of section 3 requires a value for the currently-observed decay rate, \dot{n} . Sometimes this will not be available, but it may still be possible to calculate the satellite's lifetime if the mass and dimensions, and the perigee height, are quite accurately known. In the calculations a spherical atmosphere will be assumed: thus it is L^* that will be found, and the corrections $F(\alpha)$, etc can then be applied as in section 8.

The lifetime will be expressed in terms of the density at perigee, ρ_p , and the area/mass parameter, δ , defined by

$$\delta = FSC_D/m \quad (20)$$

Here m is the mass of the satellite, S its mean cross-sectional area perpendicular to the direction of motion, C_D is the drag coefficient, for which a value of 2.2 is recommended²⁴ for perigee heights of 150-400 km, and F is a factor to allow for atmospheric rotation, which is usually between 0.9 and 1.1 and is shown in Fig 2.5 of Ref 3.

9.2 Orbits with $e < 0.2$

With a spherically-symmetrical atmosphere, equation (4.22) of Ref 3, with $a = a_0$, shows that we may write the change Δa in a during one revolution as

$$\Delta a = -2\pi a^2 \nu_p \delta \exp(-z) \left\{ I_0 + 2eI_1 + O(e^2) \right\}, \quad (e < 0.2) \quad (21)$$

after dropping the suffix 0 from a , ν_p and z . Since $n^2 a^3 = \text{constant}$ by Kepler's third law, $\Delta n = -3n\Delta a / (2a)$; and, as Δt is equal to the orbital period T , we have

$$\dot{n} = \frac{\Delta n}{\Delta T} = -\frac{3n\Delta a}{2a\Delta t} = -\frac{3\Delta a}{2aT^2} \quad (22)$$

if n is expressed in revolutions per day (rather than rad/day), so that $n = 1/T$. From equations (1), (22) and (21), we have

$$L^* = \frac{T}{3\pi a \delta} X(e, H) \quad (23)$$

$$\text{where } X(e, H) = \frac{QT \exp z}{I_0(z) + 2eI_1(z) + O(e^2)} \quad (24)^{\dagger}$$

for $e < 0.2$. Values of X are plotted against e for two values of H/a in Fig 22. If a , e , T , δ , ν_p and H are known, equation (23) gives the lifetime for $e < 0.2$. Approximate values of ν_p for two levels of solar activity are available from Fig 23, but a more detailed calculation using CIRA 1972 is advisable. The same advice applies for H , for which Fig 1 gives approximate values.

It is often useful to eliminate T from equation (23) by writing $T = 2\pi(a^3/\mu')^{1/2}$, where μ' is the gravitational constant ($398600 \text{ km}^3 \text{ s}^{-2}$ for the Earth). Equation (23) then becomes

$$L^* = \frac{2}{3\pi} \frac{\delta}{\nu_p} \left(\frac{a}{\mu'} \right)^{1/2} X(e, H) \quad (25)$$

[†] The function $X(e, H)$ depends on e and H/a only, because, in the notation of equation (4.56) of Ref 3, $QT = eF(e)$.

When $e = 0$, the lifetime can be obtained from equation (4.89) of Ref 3, which, on dropping the suffix and writing $T = 2\pi(a^3/\mu')^{1/2}$, gives

$$L^* = \frac{H}{(\mu'a)^{1/2} \rho \delta} \quad (26)$$

For satellite heights of 200-1000 km, $(\mu'a)^{1/2}$ varies by less than 3% from its value at 600 km height, which is $4.55 \times 10^{15} \text{ m}^2 \text{ day}^{-1}$. So equation (26) gives the lifetime in a circular orbit as

$$L^* \approx \frac{0.22H}{10^{15} \rho \delta} \quad \text{days} \quad (27)$$

where H is in metres, ρ in kg/m^3 , and δ in m^2/kg . If we further take $C_D = 2.2$ and $F = 1$, so that $\delta = 2.2$ (S/m), equation (27) reduces to

$$L^* \approx \left(\frac{\text{m}}{\text{S}}\right) \frac{H}{10^{16} \rho} \quad \text{days} \quad (28)$$

When $\rho = 10^{-12} \text{ kg/m}^3$ (1 ng/m^3), which from Fig 23 is the value at about 550 km height when solar activity is high, and $H = 70 \text{ km}$ (from Fig 1), equation (28) gives $L^* \approx 700$ days if $\text{m/S} = 100 \text{ kg/m}^2$, a typical value. At the same height with low solar activity, the appropriate ρ would be about $8 \times 10^{-14} \text{ kg/m}^3$, from Fig 23, and H would be about 56 km, from Fig 1: so $L^* \approx 70 \text{ m/S}$, or 20 years if $\text{m/S} = 100 \text{ kg/m}^2$. This value of L^* is unrealistic because low solar activity has (in the past 250 years) never persisted for 20 years; but the calculation does show that lifetimes in circular orbits can be up to 10 times longer at solar minimum than at solar maximum for heights near 500 km.

For circular orbits, $X(e,H) = 1.5 H/a$, as comparison of equations (25) and (26) shows: the lifetime is longer when H is larger, because the increase of density as time goes on is less rapid when H is larger. For $e > 0.01$, however, Fig 22 shows that $X(e,H)$ is larger when H is smaller. This is because the density above perigee decreases more strongly when H is smaller, thus giving rise to a smaller \dot{n} for given ρ_p and e . When z is large, $X(e,H)$ is approximately proportional to $(a/H)^{1/2}$ for given e . These variations with H are of course relatively small: for given e , it is mainly $\rho \delta$ that controls the lifetime in equation (25).

9.3 Orbits with $e > 0.2$

The results so far have been for $e < 0.2$. When $e > 0.2$, equation (21) has to be replaced by the high-eccentricity version, equation (4.107) of Ref 3, which gives

$$\Delta a = -a^2 \rho_p \delta \left(\frac{2\pi}{z} \right)^{\frac{1}{2}} \frac{(1+e)^{3/2}}{(1-e)^{1/2}} \left\{ 1 + 0(0.003) \right\} \quad (e > 0.2) \quad (29)$$

Then L^* is given by equation (23) with

$$X(e, H) = \frac{QT^{\frac{1}{2}} z (1-e)^{\frac{1}{2}}}{(1+e)^{3/2}} \quad (e > 0.2) \quad (30)$$

Writing $QT = eF(e)$ in the notation of equation (4.133) of Ref 3, and $z(1-e) = r_p e/H$, we have

$$X(e, H) = \left(\frac{e}{1+e} \right)^{3/2} F(e) \left(\frac{2\pi r_p}{H} \right)^{\frac{1}{2}} \quad (e > 0.2) \quad (31)$$

Using the values of $F(e)$ given in Fig 4.10 of Ref 3, and taking $H/r_p = 0.01$, we find $X(e, H)$ increases from 1.6 at $e = 0.2$ to 4.4 at $e = 0.4$ and 13.2 at $e = 0.8$.

9.4 Results for average solar activity

Fig 24 is a graph of satellite lifetime as a function of perigee height and e , the values of perigee density ρ_p and scale height H being taken from *CIRA 1972* with $T_\infty = 900$ K, which is approximately the average over a solar cycle. The quantity plotted in Fig 24 is $L^*(S/m)$; so, if $m/S = 100 \text{ kg/m}^2$, the lifetime in years is 100 times the value shown. In drawing Fig 24 (which is from Ref 2), it was assumed that $C_D = 2.2$ and $F = 0.91$. If the lifetime obtained from Fig 24 is less than 10 years, the solar activity is unlikely to be 'average', and the mean solar activity over the predicted lifetime ought to be used. For such short lifetimes the best technique is to calculate L^* with the current perigee density, from equation (23) or (27), and then to correct for solar activity by the method already given in section 6. The other corrections to L^* can be applied whenever they are appropriate and appreciable

10 HIGH-ECCENTRICITY ORBITS WITH LUNISOLAR PERTURBATIONS10.1 Introduction

The methods of sections 3 to 9 are valid for eccentricities greater than 0.2, if lunisolar perturbations in perigee height are small enough. When $e > 0.4$, however, lunisolar perturbations usually have a significant effect and need to be computed. If the lunisolar perturbations produce a variation which is recognizably periodic, the lifetime can be estimated by using a value of \dot{n} averaged over a cycle; if the perturbation in perigee height is large (>50 km) and not recognizably periodic, it is usually necessary to resort to numerical integration. However, it is worth trying to devise analytical methods for some of the heavily-populated types of high-eccentricity orbit, and two such methods will now be described, for low-inclination transfer orbits to geosynchronous height and for Molniya-type orbits.

10.2 Low-inclination transfer orbits ($i \approx 25^\circ$, $e_0 \approx 0.73$)

Most of the many geosynchronous satellites now in orbit have made their way to the geosynchronous height of 36000 km via a transfer orbit with perigee height near 200 km and apogee height 36000 km. Such transfer orbits have eccentricities close to 0.73, and most have orbital inclinations between 20° and 30° †. A discarded rocket usually remains in the transfer orbit and, as there are now more than a hundred of these, an approximate analytical method for determining their lifetimes has been developed⁵.

In the absence of lunisolar perturbations the lifetime L^* is given by equations (25) and (31), and, on inserting the numerical values of a and e for a transfer orbit, and taking $C_D = 2.2$ and $F = 0.925$, we have

$$L^* = \frac{23.6}{10^9 \rho_p \sqrt{H}} \left(\frac{m}{100S} \right) \text{ years}, \quad (32)$$

with m in kg, S in m^2 , ρ_p in kg/m^3 and H in km. The variation of L^* with perigee height for a standard value of $m/S = 100 \text{ kg/m}^2$ is shown in Fig 25. If m/S differs from 100 kg/m^2 , the lifetime should of course be multiplied by $m/100S$.

The average period of the lunisolar perturbation in perigee height for a transfer orbit is about 9 months, though the amplitude and period of the

† The exceptions ($i \approx 10^\circ$) are satellites launched by Ariane.

perturbation vary appreciably from cycle to cycle. If the satellite survives several cycles of the lunisolar perturbation, we can calculate the perturbation approximately, determine the mean perigee height and take that as the 'perigee height' in Fig 25. It can be shown⁵ that if λ is the solar longitude and $i = 25^\circ$, the arithmetic mean perigee height \bar{y}_p is given by

$$\bar{y}_p = y_{p0} + 33 \cos\{2(\Omega_0 + \omega_0 - \lambda_0)\} + 26 \cos(\Omega_0 + 2\omega_0) - 21 \cos\{2(\Omega_0 + \omega_0)\} \text{ km} \quad \dots (33)$$

where suffix 0 denotes initial values as usual, and oscillations of amplitude less than 10 km have been ignored (because the only orbits available for these satellites generally have an accuracy in perigee height not better than 10 km). If we write the last three terms in equation (33) as Δy_{p1} , Δy_{p2} and Δy_{p3} , their variations are as shown in Fig 26.

However, \bar{y}_p is not the right value to use in equation (32): we need the value of y_p that correctly gives the mean *density* at perigee over a cycle of the oscillation. This height, y^1 say, is always somewhat less than \bar{y}_p and we may write

$$y^1 = \bar{y}_p - \Delta y_{p4} \quad (34)$$

where Δy_{p4} is as given in Fig 27. The lifetime can now be read from Fig 25, taking the perigee height as y^1 .

If the inclination differs from 25° , the three numerical values in equation (33) change. If we call them A_1 , A_2 and A_3 , we find that A_1 decreases from 43 km at $i = 10^\circ$ to 28 km at $i = 30^\circ$; in contrast, A_2 increases from 8 km at $i = 10^\circ$ to 33 km at $i = 30^\circ$; and A_3 increases from 15 km at $i = 10^\circ$ to 26 km at $i = 30^\circ$. The calculation of y_p should be modified accordingly if $i \neq 25^\circ$.

The method is valid if the lifetime extends to several cycles of the perturbation, that is, if L exceeds about 3 years. This implies $y^1 > 150$ km from Fig 25, if $m/S = 100 \text{ kg/m}^2$. The results obtained with the method should be valid to within a factor of 1.5, subject to the usual proviso about solar activity.

10.3 Molniya-type orbits ($i_0 = 65^\circ$ or 63° , $e_0 \approx 0.74$)

The launches of Molniya communications satellites with 12-hour orbital periods and initial inclinations near 65° began in 1965, and 31 such satellites were launched, with initial eccentricities near 0.74, accompanied by 31 separated rockets in similar but slightly different orbits. Then in 1973 there was a change of inclination to 63° , which has been retained for all the 82 launches in the years 1974-1986.

The perigee height of a Molniya increases from about 500 km initially to more than 1000 km, and then descends some years later. For the Molnias at 65° inclination, an approximate analytical solution can readily be devised⁶ and the results are recorded here in case launches at 65° inclination are restarted. For the 63° Molnias, the lifetimes are much longer and less amenable to analytical evaluation, although an approximate indication can be given.

For the 65° Molnias, numerical integration with the aid of the PROD computer program⁷ shows that the inclination nearly always stays between 64° and 66° , and i may be taken as 65° with an error in $\sin i$ of less than 0.7%. The changes in eccentricity due to lunisolar perturbations²⁶ can then be expressed algebraically. If only the most important terms are included, it can be shown⁶ that the perigee height is driven down to 100 km, and the lifetime ends, when

$$\frac{0.566 + 20c' - 0.31 \cos \Omega_0}{1 - 0.36 \cos \Omega_0} = -\cos 2\omega \pm 0(0.04) \quad (35)$$

where c' is an empirical parameter to cater for the departure of the initial eccentricity from 0.74: it is found that $-0.005 < c' < 0.007$. If we exclude the first launch as non-standard, 29 of the remaining 30 Molniya launches at 65° inclination had $\omega_0 = 285^\circ$ (with one at 284°). Gracefully accepting this devastating demonstration of launching accuracy, we may take $\omega_0 = 285^\circ$ as standard and construct a diagram, Fig 28, showing the permitted theoretical lifetime domains as a function of Ω_0 . The actual decay dates of 29 of the 30 satellites are also shown: all are either inside or close to the theoretical domains. According to the theory, the lifetime of a 65° Molniya will not exceed 7 years. The Molniya satellites are of three types and are numbered accordingly in Fig 28: thus Molniya 1-12 is the 12th launch of type 1. In Fig 28, Molniya 1-23 has been used as an example to show the variation of lifetime with Ω_0 given by PROD.

The theory ignores drag, which is usually important only in the last month of the life. But the drag at that time can sometimes extend the life by a few months, because the drag reduces e and therefore reduces the rate at which the perigee height is being driven down by lunisolar perturbations.

For the Molnyas with initial inclinations of 63° , numerical integration shows that the inclination may subsequently vary between 62.8° and 65.1° , so that ω varies in a complex manner, usually increasing when $i < 63.4^\circ$ and decreasing when $i > 63.4^\circ$. It is possible for ω to become trapped near 270° for several years, so that $-\cos 2\omega$ remains near 1.0 and may then always be greater than the left-hand side of equation (35), implying an indefinitely long lifetime.

The results for 63° Molnyas are therefore much less definite and may be summarized as follows⁶:

- (i) Early decay, after 2 to 4 years in orbit, is possible if $135^\circ < \Omega_0 < 225^\circ$; but it is unlikely (and has not yet happened, except for a rocket)
- (ii) If $90^\circ < \Omega_0 < 240^\circ$, there is a good chance (>50%) of decay between 9 and 14 years after launch.
- (iii) If $-120^\circ < \Omega_0 < 90^\circ$, decay is likely between 12 and 20 years after launch, unless ω librates about 270° . If libration does occur (and it is likely only when $-60^\circ < \Omega_0 < 60^\circ$, no upper limit can be specified for the lifetime.

Fig 29 shows these lifetime domains graphically, with all available actual lifetimes and others predicted by numerical integration with PROD. (The actual lifetimes are all less than 13 years, because the 63° Molnyas did not start until 1973.) In view of the gross approximations made, it is surprising that all the points are within the domains indicated in Fig 29; indeed it seems that the domains might be made smaller.

There are other special high-eccentricity orbits but they are (a) not so heavily populated as the transfer and Molnyas-type orbits, and (b) mostly not so amenable to treatment by approximate methods, because the inclination often varies rather widely.

⁶ Note that the upper limit for Ω_0 has been revised, from 270° in Ref 6 to 240° here, in the light of experience.

11 CONCLUSIONS

Anyone who has struggled through to the end of this Report will emerge with some idea of the difficulties and complexities of predicting the decay date of a satellite. Accurate lifetime prediction is one of the most intractable problems of orbital mechanics.

It cannot be said too often that there is no glory to be gained from lifetime prediction. If you do succeed in making an accurate prediction, most people think it must be easy and the rest will say it is just luck. If your prediction is wrong, you can expect sneers from the ignorant and the semi-learned.

We favour the graphical methods presented here, because they are simple and of adequate accuracy and comprehensiveness. Others may prefer more sophisticated methods, such as numerical integration; but their predictions are unlikely to be more accurate, because the errors arise chiefly from the future variations in air density, which are uncertain.

Appendix

ORBITAL LIFETIME IN AN ATMOSPHERE WITH DAY-TO-NIGHT DENSITY VARIATION

We assume, as in Ref 22, that the density ρ at distance r from the Earth's centre is given in terms of the bulge-perigee angle ϕ by

$$\rho = \rho'(1 + F \cos \phi) \exp\left\{-\frac{(r - r')}{H}\right\}, \quad (\text{A.1})^\dagger$$

where F is a constant, equal to $(\rho_M - \rho_m)/(\rho_M + \rho_m)$ in the notation of section 7; ϕ is as defined in Ref 22; and the constant ρ' is the density at distance r' from the Earth's centre for $\phi = 90^\circ$, where the value of r' is open to choice (it would normally be taken as the initial perigee distance r_{po} , but this is not compulsory).

If $T (=1/n)$ is the orbital period, we have $\dot{T} = 3\Delta a/(2a)$, where Δa is the change in a during one revolution, and hence, from equation (35) of Ref 22,

$$\dot{T} = -3\left(\frac{\pi}{2z}\right)^{\frac{1}{2}} a \delta \rho_p \left\{ 1 + 2e + \frac{1}{8z} - \frac{F \cos \phi_p}{2z} + 0\left(e^2, \frac{0.1}{z^2}, \frac{F}{z^2}, \frac{F^2}{2z}\right) \right\} \quad (\text{A.2})$$

for Phase 1, that is, for $3H/a < e < 0.2$. Here δ is the area/mass parameter defined in section 9. In view of the other uncertainties in the analysis, we shall neglect terms less than 0.1 in equation (A.2). Since the highest value of F attained¹¹ is 0.7, we lose little by assuming that $F < 0.6$, so that $F \cos \phi_p/(2z) < 0.1$. Also $1/(8z) < 0.05$. Thus equation (A.2) reduces to

$$\dot{T} = -3(\pi/2z)^{\frac{1}{2}} a \delta \rho_p \left\{ 1 + 2e + 0(e^2, 0.1/z, \frac{1}{2}F/z) \right\}. \quad (\text{A.3})$$

Following the method of Ref 4, we write down the corresponding equation for the unperturbed orbit (with $F = 0$), denoting values on that orbit by suffix u :

$$\dot{T}_u = -3(\pi/2z_u)^{\frac{1}{2}} a_u \delta \rho_{pu} \left\{ 1 + 2e_u + 0(e^2, 0.1/z) \right\}. \quad (\text{A.4})$$

We may drop the suffix u on z , a and e , because the maximum change in $(r_{po} - r_p)/H$ due to the day-to-night variation is, from equation (72) of Ref 22, of order $\frac{1}{6}F \cos \phi_p$, which is less than 0.1 for $F < 0.6$; and it then follows from equation (15) of Ref 4 that z_u , a_u and e_u are not significantly different from z , a and e . Also, from equation (A.1) with $F = 0$,

[†] The symbols r_o and ρ_o in Ref 22 have been replaced by r' and ρ' , to avoid confusion over the suffix o .

$$\rho_{pu} = \rho_{pou} \exp\left\{\frac{(r_{po} - r_p)}{H}\right\}. \quad (A.5)$$

Thus equation (A.4) becomes

$$\dot{T}_u = -3\left(\frac{\pi}{2z}\right)^{\frac{1}{2}} \rho_{pou} \left\{1 + 2e + 0(e^2, 0.1/z)\right\} \exp\left\{\frac{(r_{po} - r_p)}{H}\right\}. \quad (A.6)$$

We require that $\dot{T} = T_u$ initially⁴: comparing (A.3) and (A.6), we see that this requires ρ_{pou} to be defined as equal to ρ_{po} . As Ref 4 shows, $F(dn)$ is given by the mean value of \dot{T}_u/\dot{T} during the remaining life of the satellite. Thus, dividing (A.6) by (A.3) and noting that ρ_{po} is constant, we have

$$F(dn) = \text{mean}\left(\frac{\dot{T}_u}{\dot{T}}\right) = \frac{\rho_{po}}{\text{mean}\left[\rho_p \exp\left\{\frac{(r_p - r_{po})}{H}\right\}\right]}. \quad (A.7)$$

The denominator here is the mean value of the perigee density at the initial perigee height (but of course at the local value of ϕ_p), and represents the quantity denoted by $\bar{\rho}_p$ in equations (8) and (10). Thus $F(dn)$ is as given in equation (15). Since equation (15) is the same as equation (8), and the solar-cycle variation of density, given by equation (9), has the same form as the day-to-night variation, given by equation (13), the results obtained for $F(sc)$ apply also for $F(dn)$, if $0.03 < e < 0.2$.

The only formal limitation that has accrued during the analysis is that e should be 'Phase 1', approximately $0.03 < e < 0.2$. However, there is another limitation: the model for density, (A.1), will not always be adequate if the satellite decays before ϕ_p completes one revolution, because there may be a great decrease in r_p , so that the values of F and H ought to be considerably reduced. This caution is not needed if $F < 0.2$, as happens if the perigee height is less than 200 km.

The result (A.7) obtained for Phase 1 applies *a fortiori* for $e > 0.2$ because the terms in $1/z$ become quite negligible, and the terms in e and e^2 in equation (A.2) can be grouped⁴ as $(1+e)^{3/2}(1-e)^{-1/2}$ without requiring the assumption that e is small.

For $e < 0.03$, the analysis takes a slightly different form. From equation (31) of Ref 22, we have

$$\dot{r} = -3\pi a \delta \rho' \left[I_0 + I_1 F \cos \phi_p + O(e, eF) \right] \exp\left\{ (r' - a)/H \right\} . \quad \text{..... (A.8)}$$

From equation (A.1),

$$\rho_{po} = \rho'(1 + F \cos \phi_{po}) \exp\left\{ (r' - r_{po})/H \right\} . \quad \text{(A.9)}$$

Dividing equation (A.8) by equation (A.9) gives

$$\dot{r} = -3\pi a \delta \rho_{po} \left\{ \frac{I_0 + I_1 F \cos \phi_p + O(e, Fe)}{1 + F \cos \phi_{po}} \right\} \exp\left\{ \frac{r_{po} - r_p}{H} - z \right\} . \quad \text{(A.10)}$$

On the unperturbed orbit, equation (A.10) takes the form

$$\dot{r}_u = -3\pi a \delta \rho_{pou} \left\{ I_0 + O(e) \right\} \exp\left\{ \frac{r_{po} - r_p}{H} - z \right\} , \quad \text{(A.11)}$$

where we have again assumed that z and a do not differ significantly from their unperturbed values, relying for this assumption on equation (130) of Ref 22, which is similar in its main terms to equation (72) of Ref 22. (There may be problems with this assumption as $z \rightarrow 0$; but then the effect of the day-night variation tends to zero.)

Dividing equation (A.11) by equation (A.10), we have

$$\frac{\dot{r}_u}{\dot{r}} = \left(\frac{\rho_{pou}}{\rho_{po}} \right) \frac{1 + F \cos \phi_{po}}{1 + (I_1/I_0) F \cos \phi_p} \left\{ 1 + O(e, Fe) \right\} . \quad \text{(A.12)}$$

The value of ρ_{pou} is defined by the requirement that $\dot{r} = \dot{r}_u$ initially. Thus

$$\frac{\dot{r}_u}{\dot{r}} = \frac{1 + (I_{10}/I_{00}) F \cos \phi_{po}}{1 + (I_1/I_0) F \cos \phi_p} \left\{ 1 + O(e, Fe) \right\} , \quad \text{(A.13)}$$

where I_{no} is the initial value of I_n .

To obtain $F(dn)$ for $e < 0.03$, we integrate \dot{r}_u/\dot{r} over the remaining lifetime L , as in Ref 4, so that

$$F(dn) = \frac{1}{L} \int_0^L \left[1 - (I_1/I_0) F \cos \phi_p + (I_{10}/I_{00}) F \cos \phi_{p0} \right] dt, \quad (A.14)$$

if we assume that F is small enough for terms in F^2 to be ignored. In attempting to integrate equation (A.14), the obvious first approximation is to assume that I_1/I_0 remains constant at its initial value. (The decrease of I_1/I_0 with time, shown in Fig 30 for $z_0 = 3$, assuming $z/z_0 = (1 - t/L)^{1/2}$, is very slow until near the end of the life.) Then assuming that ϕ_p varies linearly with time, with $\dot{\phi}_p = 2\pi/P$, equation (A.14) gives

$$F(dn) = 1 + \frac{I_{10}}{I_{00}} F \left[\cos \phi_{p0} - \frac{P}{2\pi L} \left\{ \sin \left(\phi_{p0} + \frac{2\pi t}{P} \right) - \sin \phi_{p0} \right\} \right]. \quad (A.15)$$

The corresponding equation for $0.03 < e < 0.2$ is obtained on dividing equation (A.6) by equation (A.3), to give

$$\frac{\dot{r}_u}{\dot{r}} = (\rho_{p0}/\rho_p) \exp \left\{ (r_{p0} - r_p)/H \right\}, \quad (A.16)$$

since $\rho_{pou} = \rho_{po}$ in Phase 1, as already noted after equation (A.6). From equation (A.1),

$$\frac{\rho_{po}}{\rho_p} = \frac{1 + F \cos \phi_{p0}}{1 + F \cos \phi_p} \exp \left\{ (r_p - r_{p0})/H \right\}. \quad (A.17)$$

From equations (A.16) and (A.17),

$$\frac{\dot{r}_u}{\dot{r}} = \frac{1 + F \cos \phi_{p0}}{1 + F \cos \phi_p}. \quad (A.18)$$

Comparison with equation (A.13) shows that the result for $0.03 < e < 0.2$ is the same as for $e < 0.03$, except that I_1/I_0 is replaced by 1. Thus if F^2 terms are ignored, $F(dn)$ for $0.03 < e < 0.2$ is given by equation (A.15) with $I_{10}/I_{11} = 1$.

Consequently, when $z_0 < 3$, the 'strength' of $F(dn)$, i.e. its departure from 1.0, needs to be scaled down by a factor $I_1(z_0)/I_0(z_0)$ and is given by equation (17). Values of I_1/I_0 appear in Table A.1 at the end of this Appendix.

In integrating equation (A.14) for $z_0 < 3$, we have assumed that I_1/I_0 is constant. This is an acceptable approximation if z_0 is near 3, but overestimates the 'strength' of $F(dn)$, particularly for $z_0 < 1$. An opposite extreme is to assume that I_1/I_0 decreases linearly to zero, and Fig 30 shows that the truth lies between these approximations. Taking

$$\frac{I_1}{I_0} = \frac{I_{10}}{I_{00}} \left(1 - \frac{z}{L}\right)$$

in equation (A.14), we find

$$F(dn) = 1 + \frac{I_{10}}{I_{00}} F \left[\cos \phi_{po} + \frac{P}{2rL} \sin \phi_{po} + \frac{P^2}{4r^2 L^2} \left\{ \cos \left(\phi_{po} + \frac{2\pi L}{P} \right) - \cos \phi_{po} \right\} \right] \dots (A.19)$$

Two of the terms within square brackets in equation (A.15) are the same as in equation (A.19), and the third differs from the remaining terms in equation (A.19) by less than 0.1 if $L/P > 1$ and $F < 0.3$. Thus the two approximations give effectively the same result.

In deriving the results for $e < 0.03$, we have ignored the term $\left\{ (F I_0/I_1) \cos \phi_p \right\}^2$, which integrates to give

$$\frac{1}{2} \left(\frac{F I_{00}}{I_{10}} \right)^2 \left\{ 1 + 0 \left(\frac{P}{2rL} \right) \right\}.$$

This term is less than 0.1 if $F < 0.55$ for $z_0 = 3$, or if $F < 0.64$ for $z_0 = 2$, so its neglect is in close accord with our previous assumption that $F < 0.6$.

Table A.1

Values of I_1/I_0 for $0 \leq z \leq 3$

z	0	0.5	1.0	1.5	2.0	2.5	3.0
I_1/I_0	0	0.242	0.446	0.596	0.698	0.765	0.810

REFERENCES

- | <u>No.</u> | <u>Author</u> | <u>Title, etc</u> |
|------------|--|--|
| 1 | D.G. King-Hele
D.M.C. Walker
J.A. Pilkington
A.N. Winterbottom
H. Hiller
G.E. Perry | <i>The RAE table of Earth satellites 1957-1986.</i>
London, Macmillan Press (1987) |
| 2 | D.G. King-Hele | Methods for predicting satellite orbital lifetimes.
<i>J. Brit. Interplan. Soc.</i> , <u>31</u> , 181-196 (1978)
RAE Technical Report 77111 (1977) |
| 3 | D.G. King-Hele | <i>Satellite orbits in an atmosphere: theory and applications.</i>
Glasgow, Blackie (1987) |
| 4 | D.G. King-Hele
D.M.C. Walker | The contraction of satellite orbits under the
influence of air drag. Part VIII: Orbital lifetime in
an oblate atmosphere, when perigee distance is
perturbed by odd zonal harmonics in the geopotential.
RAE Technical Report 86069 (1986) |
| 5 | D.G. King-Hele | Lifetime prediction for satellites in low-inclination
transfer orbits.
<i>J. Brit. Interplan. Soc.</i> , <u>35</u> , 339-344 (1982)
RAE Technical Report 81119 (1981) |
| 6 | D.G. King-Hele | The orbital lifetimes of Molniya satellites.
<i>J. Brit. Interplan. Soc.</i> , <u>28</u> , 783-796 (1975)
RAE Technical Report 75052 (1975) |
| 7 | G.E. Cook | PROD, a computer program for predicting the development
of drag-free satellite orbits. Part 1: Theory.
RAE Technical Report 71007 (1971)
[<i>Celestial Mechanics</i> , <u>1</u> , 301-314 (1973)] |
| 8 | G.E. Cook
H.D. Palmer | The elliptic orbit integration program POINT.
RAE Technical Report 76129 (1976) |
| 9 | D.M.C. Walker
D.G. King-Hele | Lifetime predictions for the RAE table of satellites.
ESA SP-246, pp 29-37 (1986) |
| 10 | D.G. King-Hele | <i>Observing Earth satellites.</i>
Macmillan, London (1983) |

REFERENCES (continued)

- | No. | Author | Title, etc |
|-----|---|---|
| 11 | - | COSPAR International Reference Atmosphere 1972
(CIRA 1972).
Berlin, Akademie-Verlag (1972) |
| 12 | D.G. King-Hele
D.M.C. Walker | The contraction of satellite orbits under the influence
of air drag. Part VII: Orbits of high eccentricity
with scale height dependent on altitude.
RAE Technical Report 86013 (1986) |
| 13 | D.G. King-Hele
C.J. Brookes
G.E. Cook | Odd zonal harmonics in the geopotential, from analysis
of 28 satellite orbits.
<i>Geophys. J. Roy. Astronom. Soc.</i> , <u>64</u> , 3-30 (1981)
RAE Technical Report 80023 (1980) |
| 14 | D.M.C. Walker | Variations of air density from January 1972 to
April 1975 at heights near 200 km.
<i>Planet. Space Sci.</i> , <u>26</u> , 291-309 (1978)
RAE Technical Report 77078 (1977) |
| 15 | K. Maeda
A.E. Hedin
H.G. Mayr | Hemispheric asymmetries of the thermospheric semi-
annual oscillation.
<i>J. Geophys. Res.</i> , <u>91</u> , 4461-4470 (1986) |
| 16 | K.h. Schatten
A.E. Hedin | A dynamo theory prediction for solar cycle 22: sunspot
number, radio flux, exospheric temperature, and total
density at 400 km.
<i>Geophys. Res. Lett.</i> , <u>11</u> , 87-876 (1984) |
| 17 | A.K. Paul | Analysis and prediction of sunspot numbers.
<i>Geophys. Res. Lett.</i> , <u>12</u> , 833-834 (1985) |
| 18 | R.N. Bracewell | Simulating the sunspot cycle.
<i>Nature</i> , <u>323</u> , 516-519 (1986) |
| 19 | M. Waldemeier | <i>Sunspot activity in the years 1610-1960.</i>
Zurich, Schulthess (1961) |
| 20 | J.A. Eddy
P.A. Gilman
D.E. Trotter | Solar rotation during the Maunder minimum.
<i>Solar Physics</i> , <u>46</u> , 3-14 (1976) |

REFERENCES (concluded)

- | <u>No.</u> | <u>Author</u> | <u>Title, etc</u> |
|------------|---------------------------------|--|
| 21 | A. Wilson | Skylab - the last hours.
<i>Spaceflight</i> , <u>22</u> , 36-7 (1980) |
| 22 | G.E. Cook
D.G. King-Hele | The contraction of satellite orbits under the influence of air drag. Part V: With day-to-night variation in air density.
<i>Phil. Trans. Roy. Soc. A</i> , <u>259</u> , 33-67 (1965)
RAE Technical Report 64029 (1964) |
| 23 | D.G. King-Hele
D.M.C. Walker | Upper-atmosphere density during the years 1957 to 1961, determined from satellite orbits.
<i>Space Research II</i> , pp 918-957
Amsterdam, North-Holland Publ. Co. (1961) |
| 24 | G.E. Cook | Satellite drag coefficients.
<i>Planet. Space Sci.</i> , <u>13</u> , 929-946 (1965) |
| 25 | K.S. Hadjimichalis | A study of sphere drag in the transition from continuum to free-molecular flow.
University of Oxford, Department of Engineering Science, Report 1073/73 (1973) |
| 26 | G.E. Cook | Lunisolar perturbations of the orbit of an Earth satellite.
<i>Geophys. J. Roy. Astronom. Soc.</i> , <u>6</u> , 271-291 (1962)
RAE Technical Note GW 582 (1961) |

Fig 1

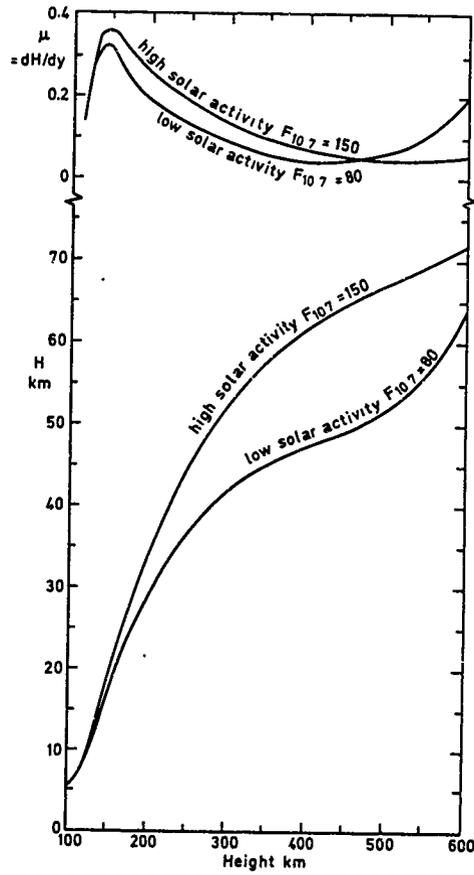


Fig 1 The variation with height y of density scale height H and its gradient $\mu = dH/dy$ as given by CIRA 1972, for high and low solar activity

TR 87030

Fig 2

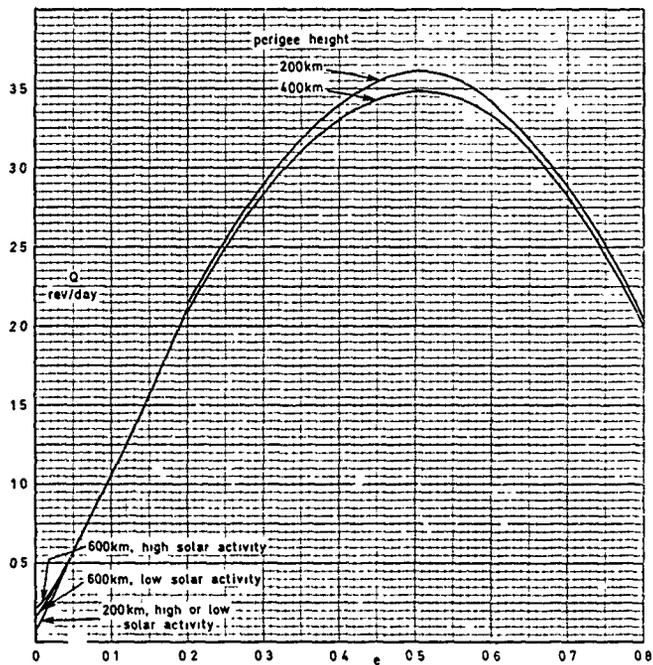


Fig 2 Variation of Q with e for $0 \leq e \leq 0.8$ and various perigee heights

TR 87010

Fig 3

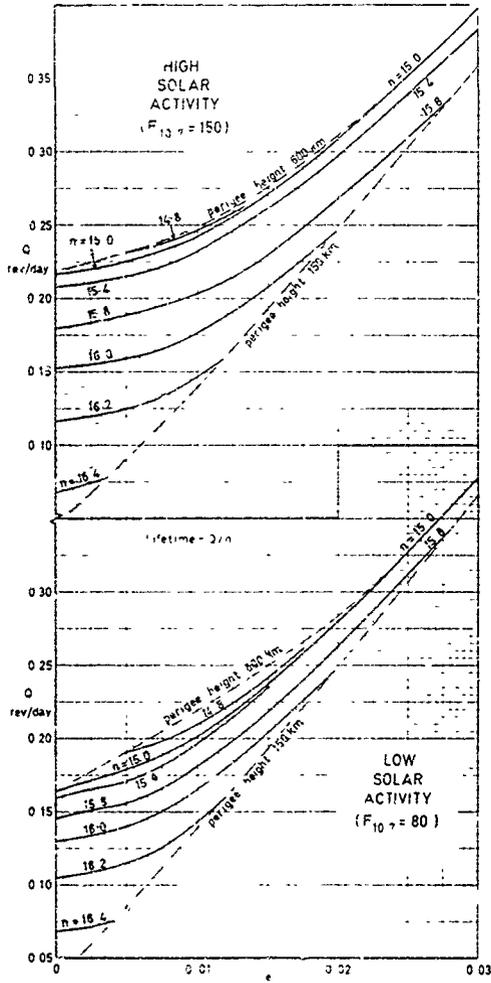


Fig 3 Variation of Q with e for $e < 0.03$, with μ from Fig 1

Fig 4

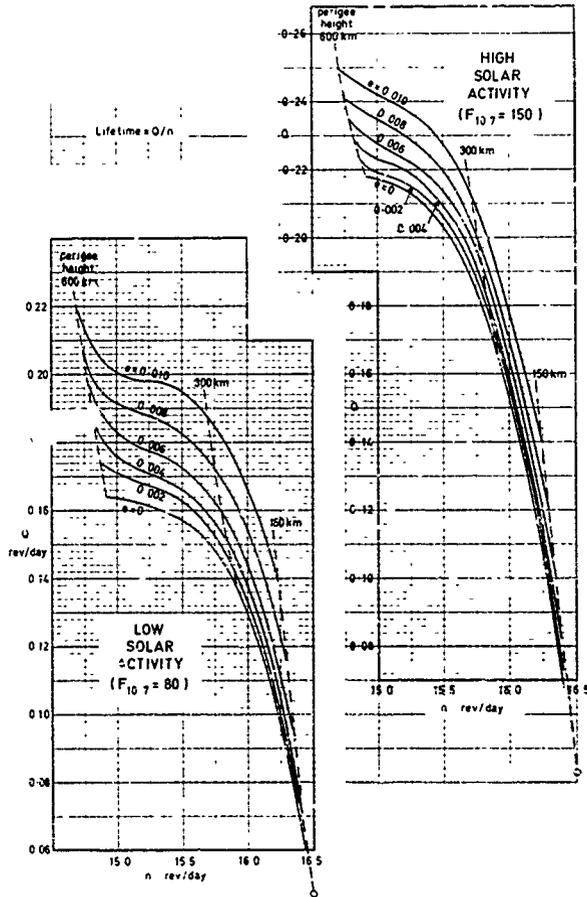
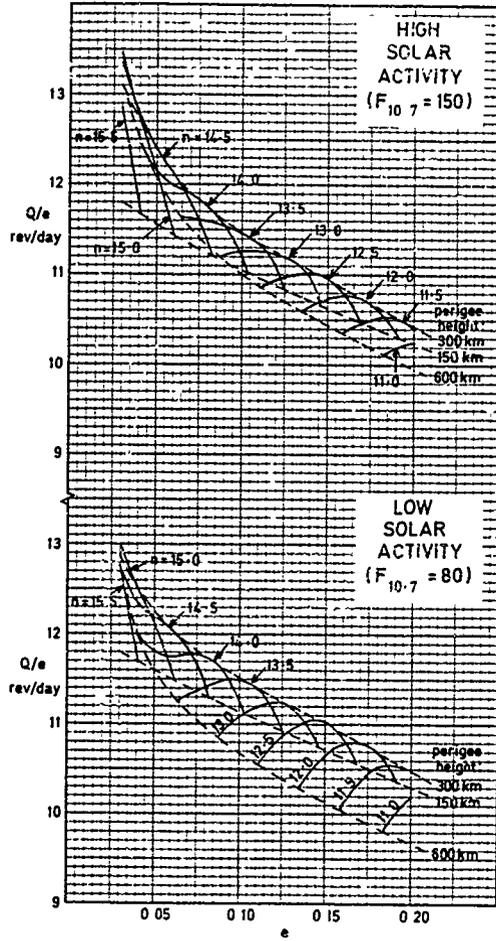


Fig 4 Variation of Q with n for $e < 0.01$, with μ from Fig 1

Fig 5



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Fig 5 Variation of Q/e with e for $0.03 < e < 0.2$, with μ from Fig 1

Fig 6

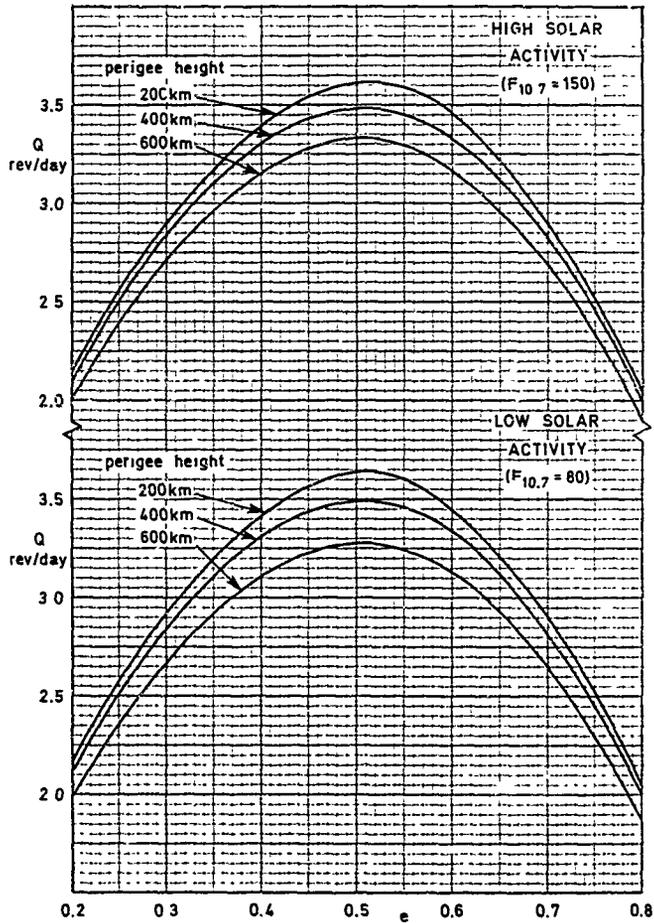


Fig 6 Variation of Q with e for $0.2 < e < 0.8$, with μ from Fig 1

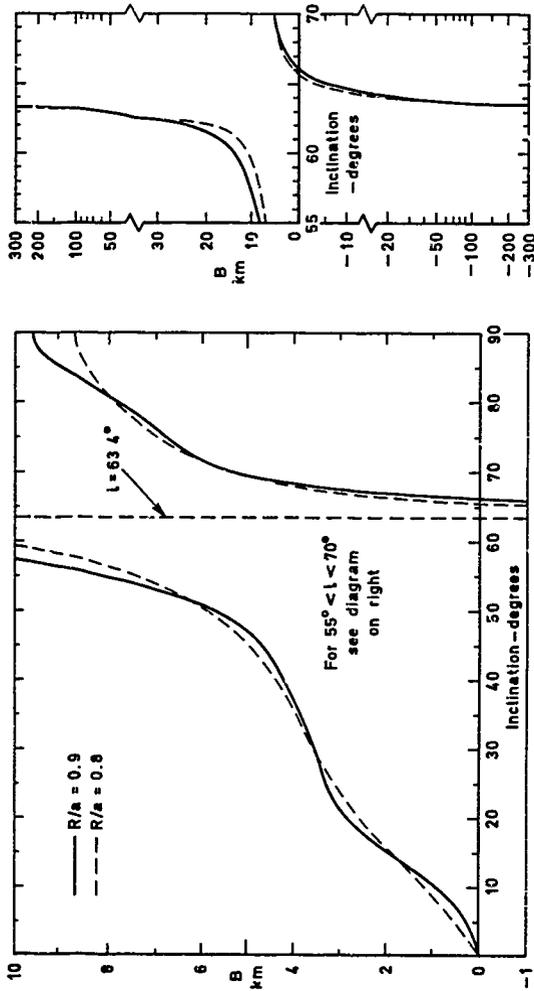


Fig 7

Fig 7 Variation of B with inclination for $R/a = 0.9$ and 0.8 , from Ref 13

Fig 8

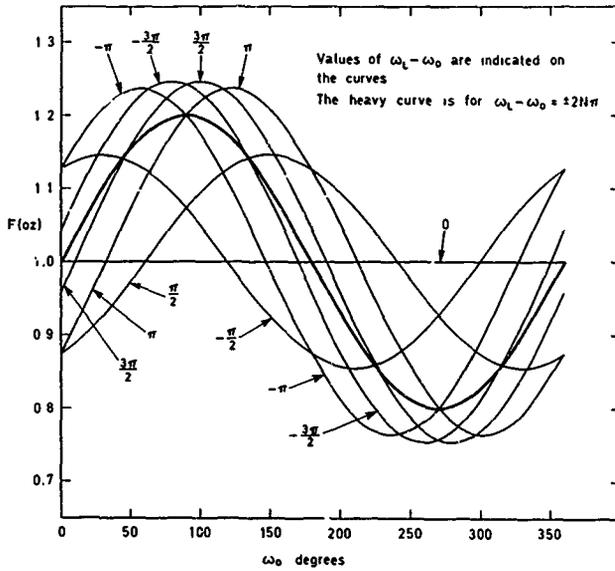


Fig 8 Variation of $F(oz)$ with ω_0 for $b = 0.2$ and various values of $\omega_L - \omega_0$

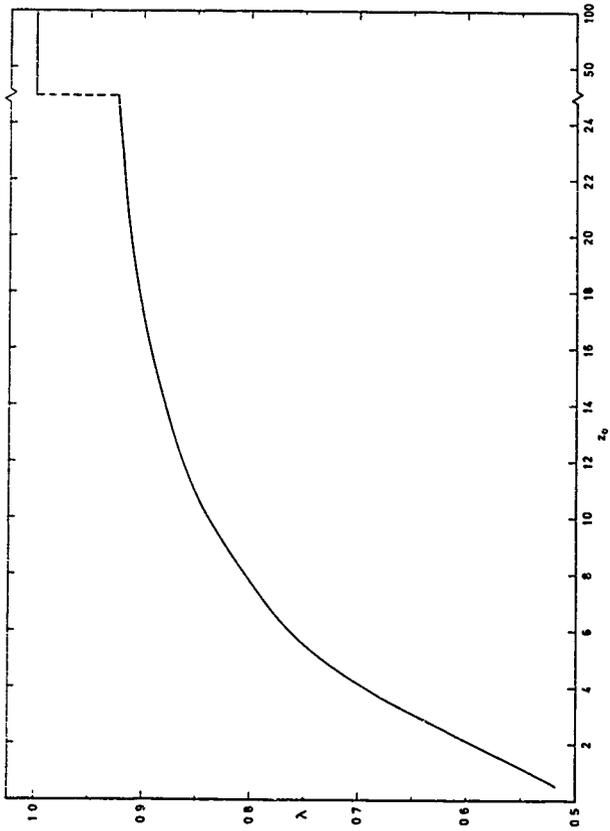


Fig 9

Fig 9 Variation of λ with z_0

Fig 10

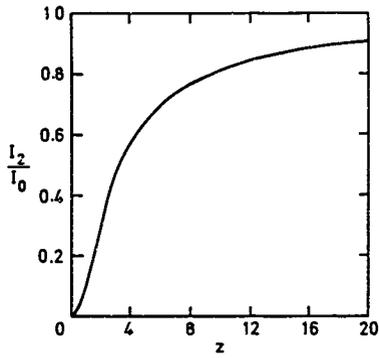


Fig 10 Variation of I_z/I_0 with z

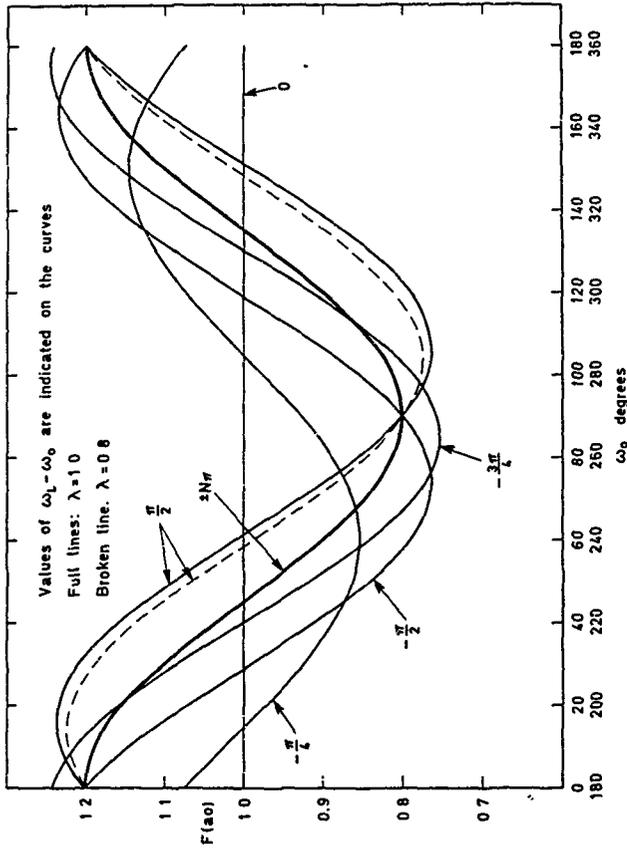


Fig 11

Fig 11 Variation of $F(\omega)$ with ω_0 for $c_0 = 0.2$ and various values of $\omega_L - \omega_0$

Fig 12

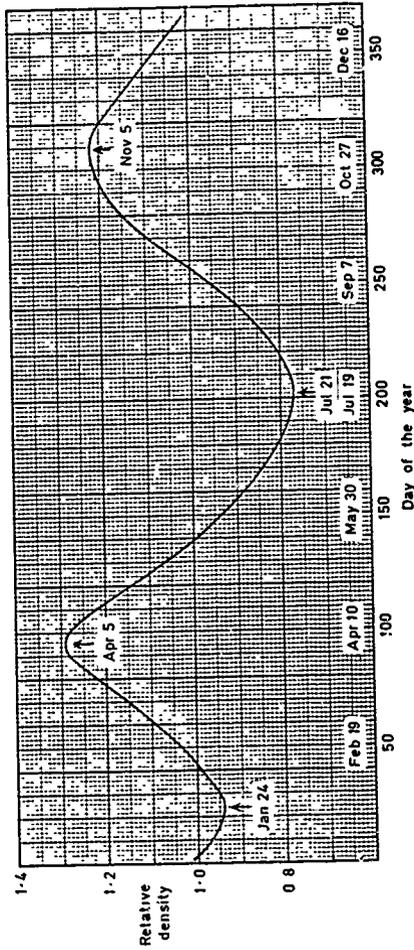


Fig 12 Semi-annual variation of air density for heights of 200-250km, from Ref 14

Fig 13

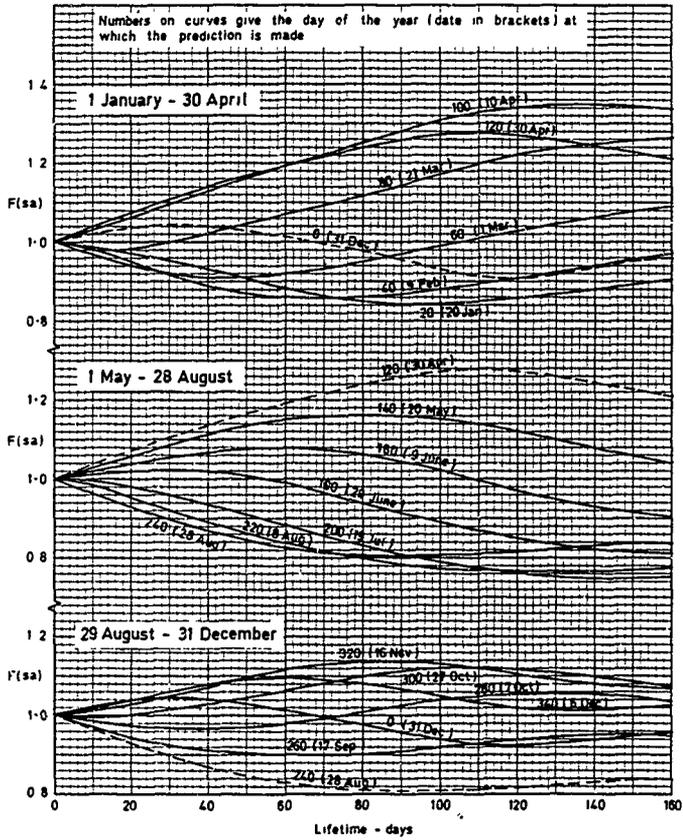


Fig 13 Variation of semi-annual correction factor $F(sa)$ with lifetime

Fig 14

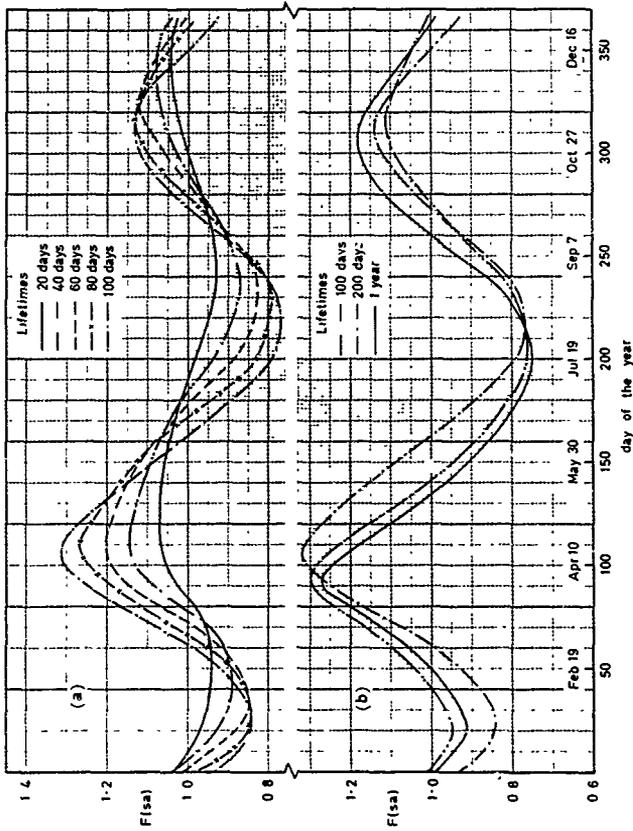


Fig 14. Variation of semi annual correction factor $F(sa)$ during the year for satellites with lifetimes of (a) 20-100 days and (b) 100 days-1 year

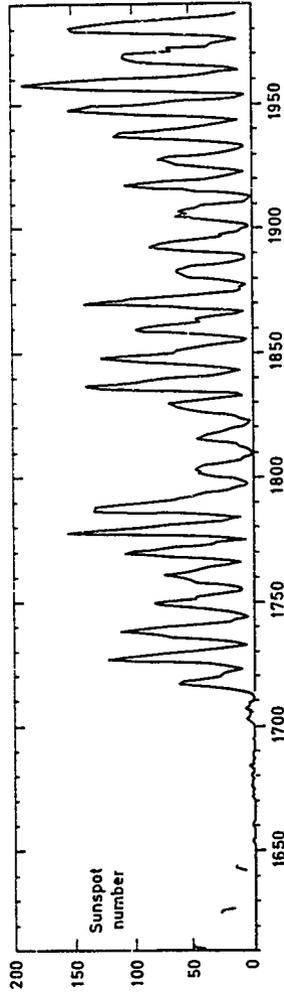


Fig 15

Fig 15 Yearly sunspot numbers, from 1610 to 1986

Fig 16

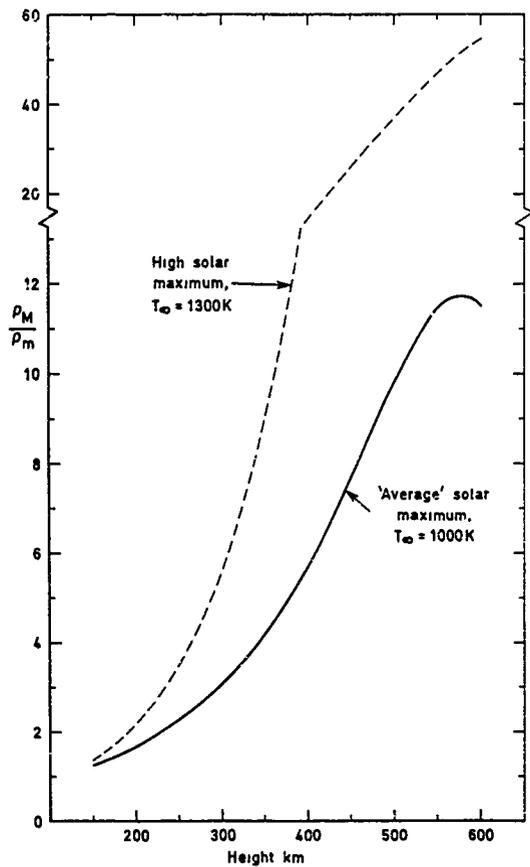


Fig 16 Ratio of maximum to minimum air density over a solar cycle of average or high intensity

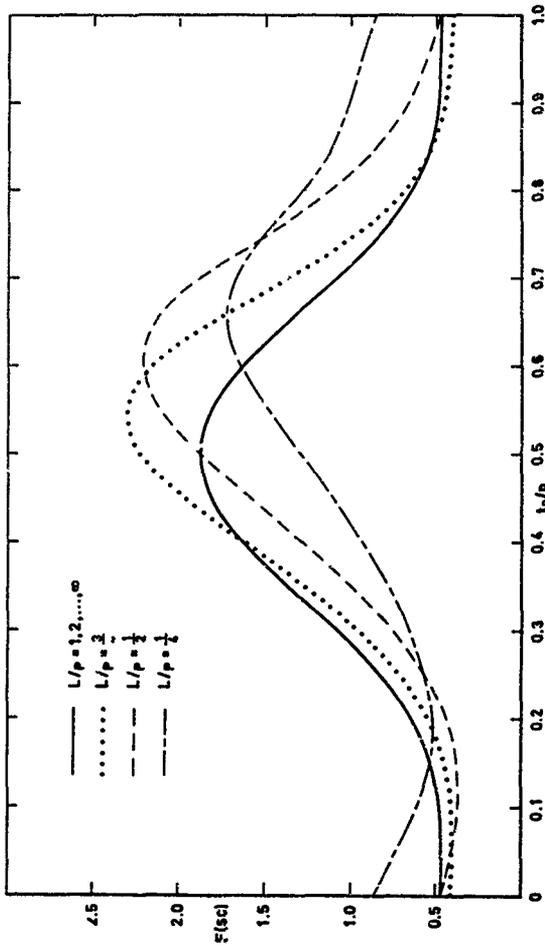


Fig 17

Fig 17 Variation of $F(isc)$ with t_0/p for $\rho_M/\rho_m = 4$

Fig 18

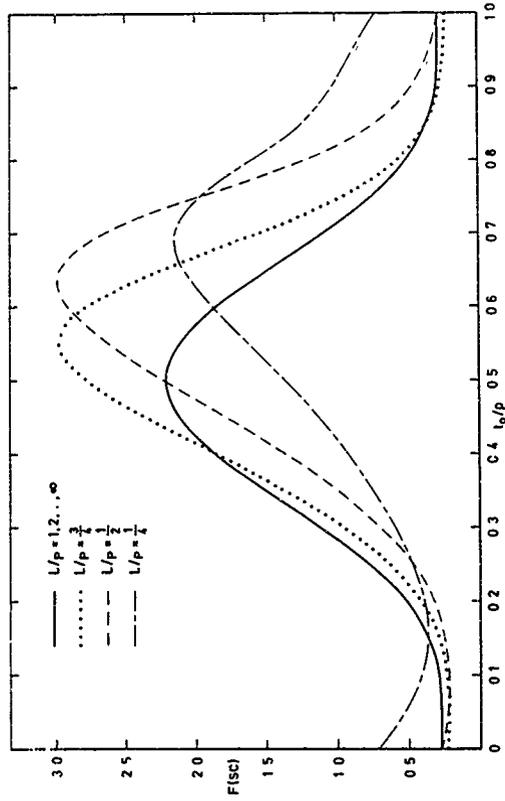


Fig 18 Variation of F (sec) with t_0/P for $\rho_M/\rho_m = 8$

Fig 19

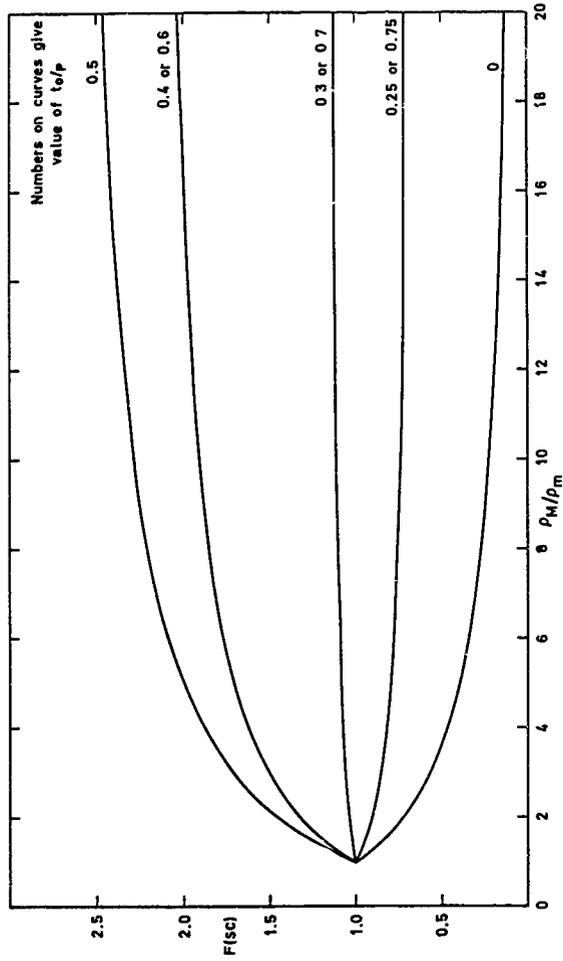


Fig 19 Variation of $F(sc)$ with ρ_M/ρ_m for $L/P = 1, 2, \dots, \infty$ and various t_0/P

Fig 20

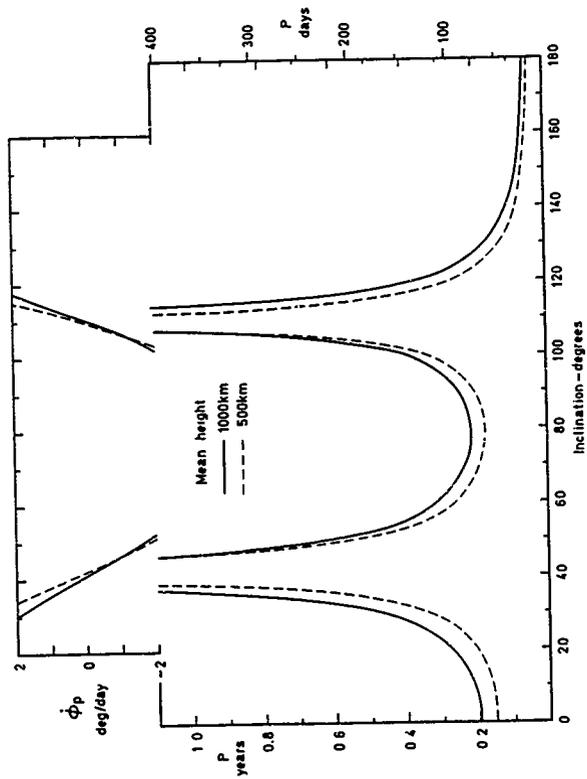


Fig 20 The average period P of the day-to-night movement of perigee, for orbits with mean height 500 or 1000 km

Fig 21

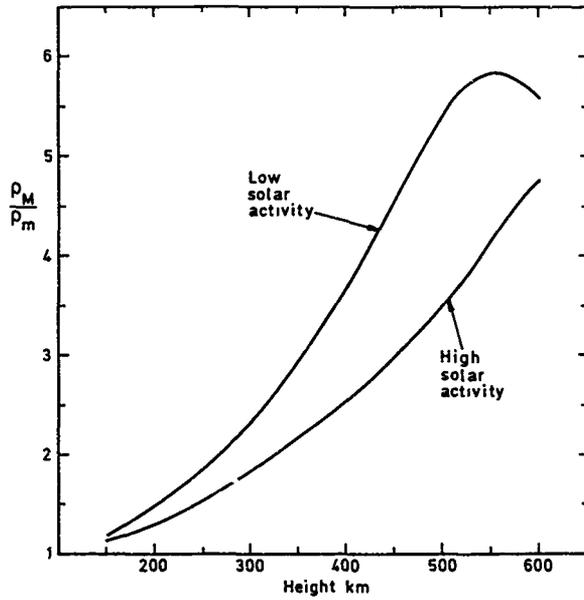


Fig 21 Ratio of maximum daytime to minimum night-time density for high and low solar activity

Fig 22

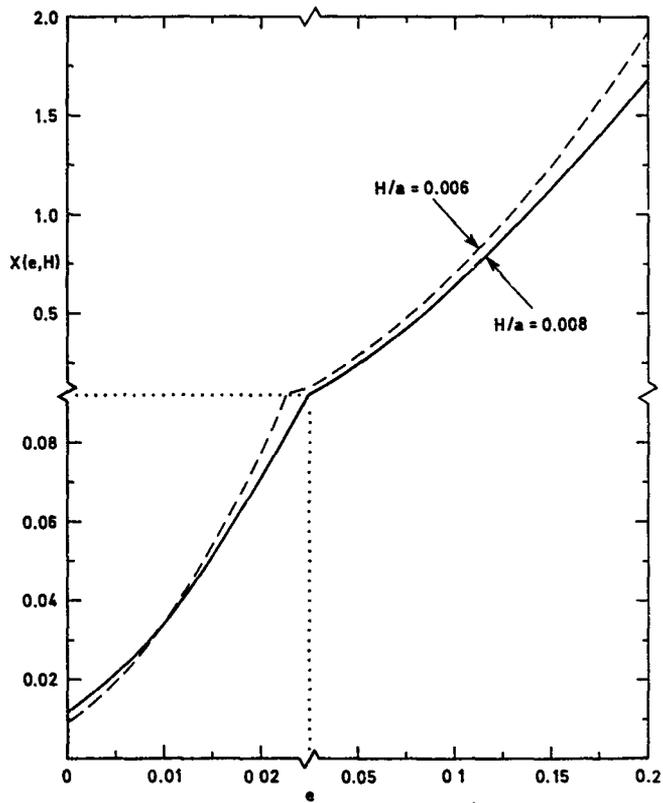


Fig 22 Variation of $X(e,H)$ with e

Fig 23

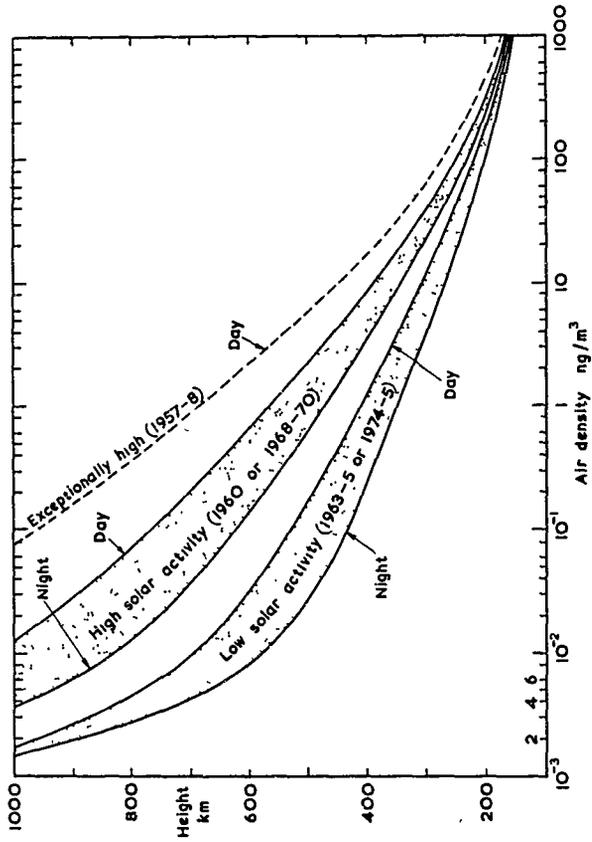


Fig 23 Variation of air density with height from 150 to 1000 km for high and low solar activity. Based on CIRA 1972

Fig 24

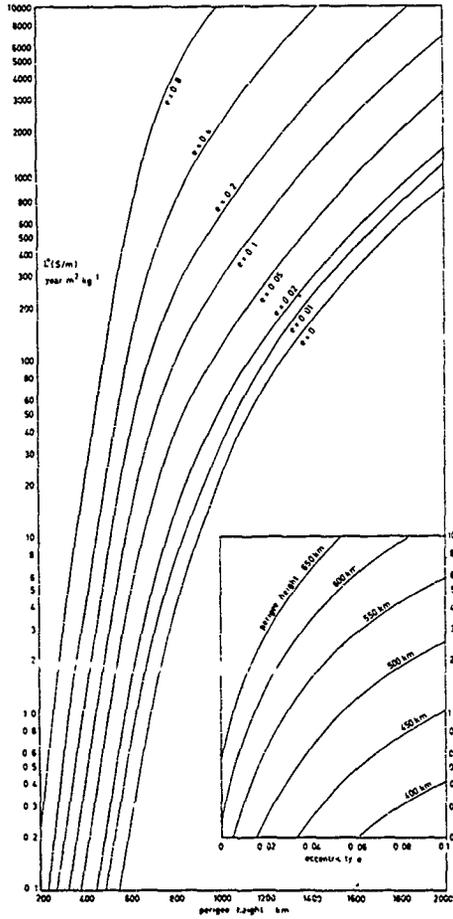


Fig 24 Lifetimes of long-lived satellites, based on mean density over an average solar cycle

Fig 25

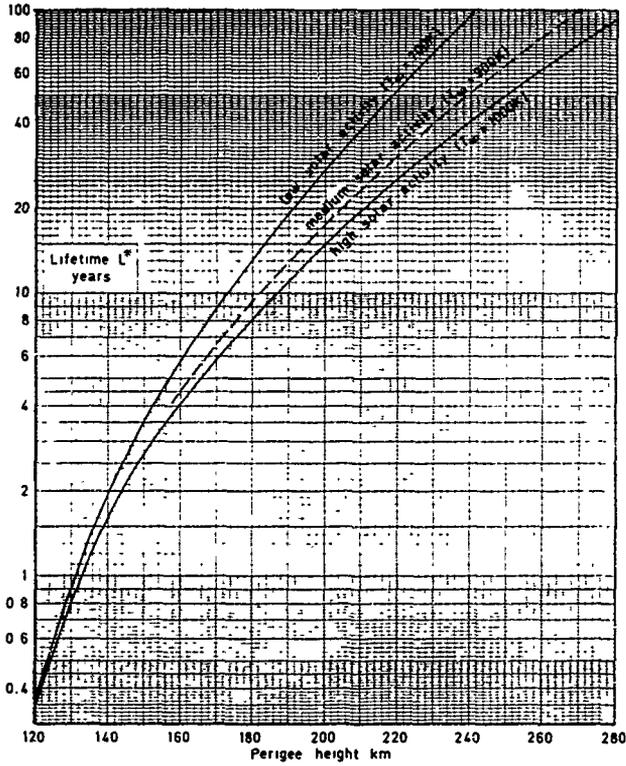


Fig 25 Lifetime L^* of satellite of mass/area $m/S = 100 \text{ kg/m}^2$ in transfer orbit of eccentricity 0.73. If $m/S \neq 100$, multiply L^* by $m/100S$. Lunisolar perturbations ignored

Fig 26

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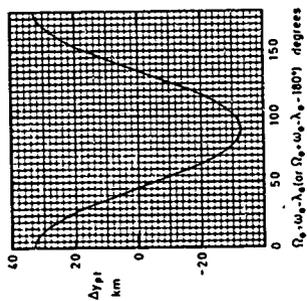
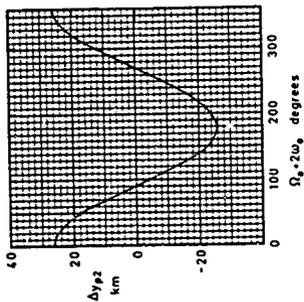
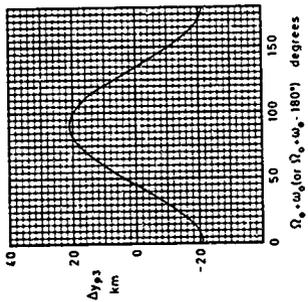


Fig 26 Graphs for finding ΔY_{p1} , ΔY_{p2} and ΔY_{p3}

Fig 27

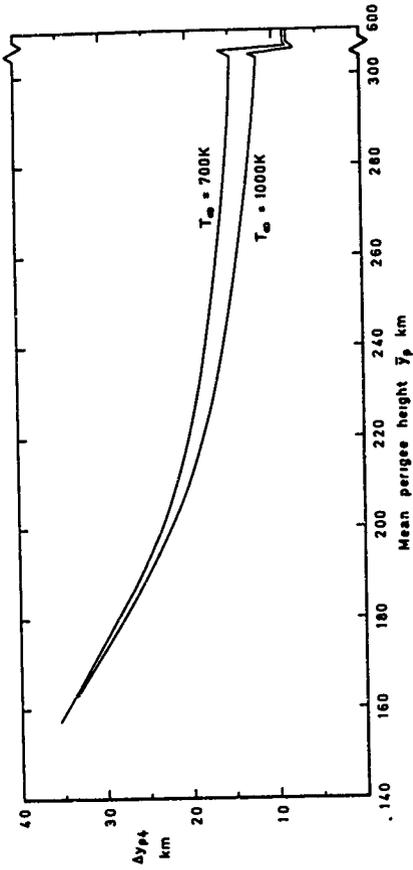


Fig 27 Variation of Δy_{p4} with \bar{y}_p

Fig 23

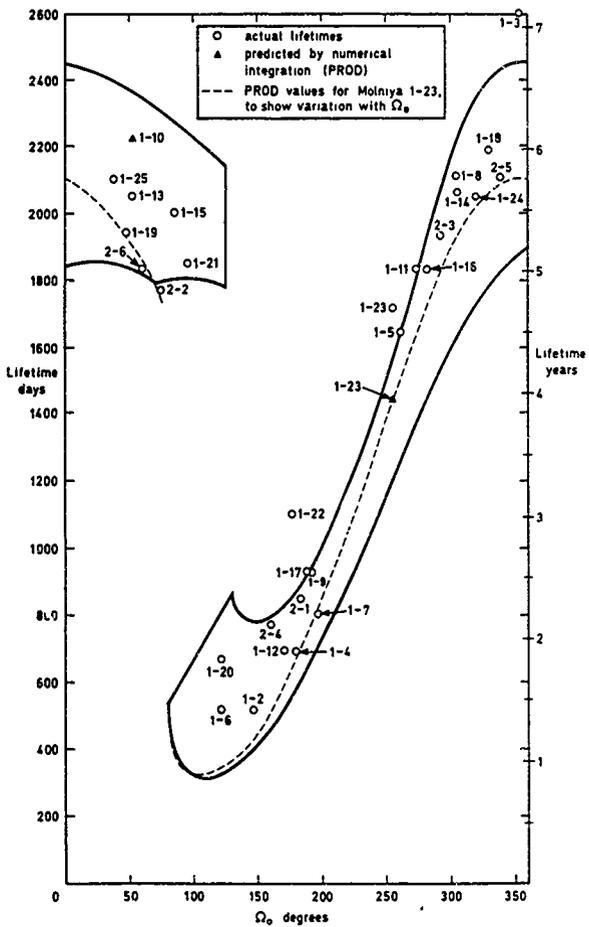


Fig 28 The permitted theoretical lifetime domains for 65° Molniyas, with actual lifetimes and those predicted by PROD

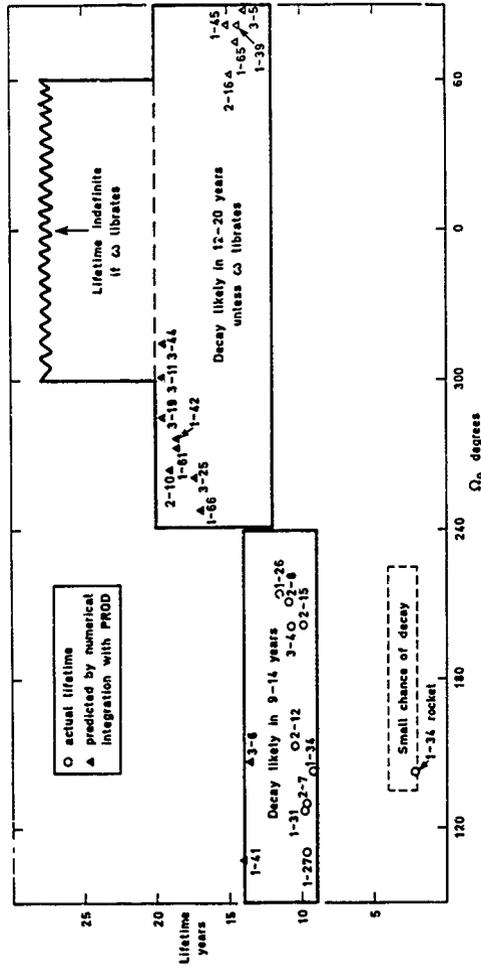


Fig 28

Fig 29 Approximate lifetime domains for 63° Molniyas, with actual lifetimes and those predicted by PROD

Fig 30

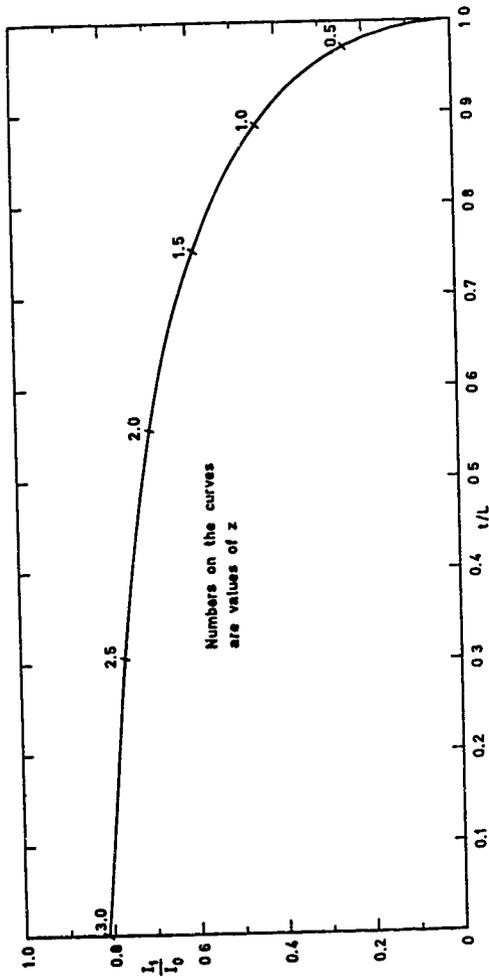


Fig 30 Variation of I_1/I_0 with time, for $z_0 = 3$

REPORT DOCUMENTATION PAGE

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16. Descriptors (Keywords) (Descriptors marked * are selected from TEST) Satellite lifetimes. Satellite orbits. Zonal harmonics. Orbital theory. Upper-atmosphere density. Solar activity. Lunisolar perturbations.					
17. Abstract Accurate prediction of the lifetimes of satellites decaying naturally under the action of air drag is one of the most difficult problems of orbital dynamics. Lifetime estimation is also a matter of practical importance when a satellite perceived as 'dangerous' is about to decay. The time when the satellite finally plunges into the lower atmosphere is affected by at least eight identifiable types of variation in upper-atmosphere density, by odd zonal harmonics in the gravitational field, and by possible changes in the mode of rotation of the satellite. For use in the RAE Table of Satellites we have since 1958 made lifetime estimates for all satellites, rockets and other major components. More than 10000 such predictions have been made, nearly all being calculated from the current observed decay rate. This Report summarizes the methods used in the past and gives the results of new orbital theory developed to take better account of the effects of atmospheric oblateness and odd zonal harmonics in the geopotential. The Report also describes new approximate methods to allow for the effect of the solar-cycle and day-to-night variations in air density, and includes discussion of other sources of error.					