Stress Analysis in Elastic Solids with Many Cracks

To develop a new method of analysis of many cracks problems in elastic solids that is sufficiently simple and applicable to both two- and three dimensional configurations, and to apply it to a number of practically important problems involving multiple cracking.

Such method has been developed and its accuracy was verified by checking the results against the solutions available in the literature. The new method has been applied to solving a number of problems.
Stress Analysis in Elastic Solids with Many Cracks

Final Report

Principal Investigator: Mark Kachanov

December 4, 1987

U.S. Army Research Office

Grant DAAG29-84-K-0184

Tufts University

Approved for Public Release,
Distribution Unlimited
The objective of the work done under the Grant was:

- to develop a new method of analysis of many cracks problems in elastic solids that is sufficiently simple and applicable to both two- and three-dimensional configurations, and

- to apply it to a number of practically important problems involving multiple cracking.

Such method has been developed and its accuracy was verified by checking the results against the solutions available in the literature. The new method has been applied to solving the following problems.

1. Stress analysis of the representative crack configurations, investigations of the competing effects of stress shielding and stress amplification under various loading conditions. In particular, "star-like" crack arrays (often occurring as a result of an impact) have been analysed.

2. Finding the full stress field in a solid with many cracks by relatively simple means (by assuming the crack opening displacement in the form of a polynomially distorted ellipse, with the polynomial coefficients matching the SIFs at the crack tips).

3. Several problems of three-dimensional interactions of penny-shaped cracks under the mode I conditions have been analysed. The effects produced by large differences in the sizes of interacting cracks (they are relevant to the three-dimensional crack-microcracks problems) were addressed.

4. The developed method allows one to calculate the effective elastic properties of cracked solids (including the anisotropic ones) for any concrete crack array geometry. This problem has been addressed and, in particular, fluctuations of the results from one realization of the crack statistics to another and an agreement of these results with the ones predicted by various approximate models (like the self-consistent scheme) were analyzed.
5. Interaction of a crack propagating in a brittle material with a field of microcracks (damage) in the vicinity of a crack tip. In particular, variation of the results from one realization of the microcrack statistics to another realization of the same statistics was analysed. Also, the adequacy of modelling of the microcrack field by an elastic material with reduced stiffness (a modelling frequently used in literature) was examined.

The main results (reported in detail in the publications listed below) can be briefly summarized as follows.

1. The accuracy of the new method was found to be very good up to quite close distances between cracks (substantially smaller than the crack sizes). The accuracy is even better in the three-dimensional configurations. In the case of weak crack interactions, the method is asymptotically exact. It has been also found, as a by-product, that many of the solutions reported in the handbooks of stress intensity factors are incorrect, since they do not agree with our results in the case of weak interactions.

2. The effects of stress shielding and stress amplification in the arrays of interacting cracks have been analysed. The shielding effects were found to be generally dominant under the predominantly mode I loading conditions on cracks; the opposite is true for the predominantly mode II conditions.

3. A closed form solution for the effective elastic properties of a solid with any particular arrangement of cracks has been obtained; it is accurate up to quite high crack densities. It was found, by using the Monte-Carlo method, that fluctuations of the results from one realization of the crack statistics to another such realization may be quite noticeable (up to 15-20%).

4. Results for several sample three-dimensional problems and their comparison with the ones for the analogous two-dimensional problems indicate that the interaction effects are generally weaker in the three-dimensional configurations. It was also found that, in the cases when a penny shaped crack interacts with a much smaller one (the simplest three-dimensional crack-microcrack problem) the impact of the interaction on the larger crack is highly localized along the edge of the latter.
5. For the problem of crack-microcrack interactions that is relevant for the mechanics of brittle fracture propagation in materials like ceramics, concrete and geomaterials, the following fundamental fact was discovered: there is no direct correlation between the extent of "damage" (understood as the extent of progression towards the final failure) and the deterioration of the effective elastic properties. This seems to be intuitively clear since the fracture related properties (like SIFs) are determined by the local fluctuations of the crack array geometry to which the volume average quantities, like effective elastic properties, are not very sensitive; note that many models of the damage zone that has been proposed in the literature are in contradiction with this simple fact.

Two microcrack field statistics (fully random and the one with microcrack orientations normal to the maximal tensile stress) have been investigated. It was found that there is no definite effect of either crack tip shielding or amplification: this effect was highly variable from one realization of the microcrack statistics to another realization of the same statistics. It was also found that the effect of the microcrack field in the "wake" region is statistically insignificant.

**Graduate Students**

Three graduate students have defended the M.S. Theses:


J. Zarzour, "Some three-Dimensional Crack Interaction Problems" (1986)

E. Montagut, " Star-Like Crack Arrays" (1987)

Two other graduate students (J. Laures and P. Clarke) continue to do the M.S. work. One student (E. Montagut) continues to work on the Ph.D. project. Some of the mentioned students were co-sponsored by AFOSR.
Publications


Three more papers are under preparation.
Presentations of the Work

The work done under the Grant has been presented at the following professional meetings:


International Conference on Fracture, Delhi, India, 1984.

Symposium on Mechanics of Damage and Fatigue (IUTAM) Tel Aviv, Israel, 1985.

International Conference on Fracture of Concrete, Lausanne, Switzerland, 1985.


International Conference on Fragmentation, Form and Flow in Fractured Media, Neve-Ilan, Israel, 1986 (invited lecture).


Winter Annual Meeting of ASME, Boston, 1987 (to be presented).

Also, lectures on the subject were given at the following organizations:

National Bureau of Standards (at the Building Materials Division, 1982, at the Center for Materials Science, 1985, another presentation is planned at the Building Materials Division); Rensselaer Polytechnic Institute (1985); University of Illinois at Chicago Circle (1985); Yale University (1985); Royal Shell Co. Research Center (Netherlands) (1985); Institute of Reactor Materials, W. Germany (1986); United Technologies Research Center (1986), GTE Laboratories (1986), ALCOA Technical Center (1986).

Cooperation with the Shock Physics Group at SRI International has been started.
Discussion and Suggestions for Further Research

The advantages of the developed method are that it is simple, works in both 2-D and 3-D and applies to a variety of problems related to multiple cracking.

The emphasis in the further research should be made, in our opinion, on the three-dimensional problems involving strong interactions of many cracks (note that such problems are traditionally considered as very difficult). Extension of the analysis to anisotropic materials (having in mind applications to composites) and the problems of crack-microcrack array in 3-D should be mentioned in particular. Another set of problems to be addressed includes generalization of the method for the non-circular cracks and for the inhomogeneities other than cracks (holes and cavities).
DATE
3/11/88
FILM
DITIC