A Simple Model for the Initial Phase of a Water Plasma Cloud about a Large Structure in Space

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**Abstract:**

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Abstract

Large structures in the ionosphere will outgas or eject neutral water and perturb the ambient neutral environment. This water can undergo charge exchange with the ambient oxygen ions and form a water plasma cloud. Additionally, water dumps or thruster firings can create a water plasma cloud. A simple model for the evolution of a water plasma cloud about a large space structure is obtained. It is shown that if the electron density around a large space structure is substantially enhanced above the ambient density then the plasma cloud will move away from the structure. As the cloud moves away it will become unstable and will eventually break up into filaments. A true steady state will exist only if the total electron density is unperturbed from the ambient density. When the water density is taken to be consistent with Shuttle based observations the cloud is found to slowly drift away on a timescale of many tens of milliseconds. This time is consistent with the Shuttle observations.

1 Introduction

Since the dawn of Shuttle flights to the ionosphere it has been realised that the ambient ionosphere is strongly perturbed by the presence of a large body in space [Green et al., 1985]. Several experiments on board the Space Shuttle have been devoted to the measurement of the plasma parameters in the vicinity of the spacecraft [Murphy et al., 1986; Reasoner et al., 1986]. It has been found that the Shuttle or any body in space affects the ambient environment through the interaction of its associated contaminant cloud with the environment. The contaminant cloud arises from surface outgassing, leaks from life support systems, thruster firings and water dumps. The contaminant cloud can undergo several different types of interactions with the ambient environment ranging from physical interactions such as momentum transfer to chemical interactions such
as charge exchange. The result is that the self–consistent environment around a large body in space may be significantly different from the ambient environment. We choose to put the emphasis on large bodies since large bodies can have an associated contaminant cloud which makes an $O(1)$ change to the ambient environment. By an $O(1)$ change we mean that the composition of the ion density is substantially changed. For example, observations around the Shuttle[1] indicate that the observed environment differs considerably from the expected ambient environment.

It is important to study the self–consistent environment around large space vehicles such as the Shuttle for several reasons. The major reason is to understand the noise that will be observed on any sensors carried on the vehicle[1]. This will directly affect the utility of the sensors for purposes of observation. Another important reason is to build understanding of the space plasma environment and some of the processes that can occur in it. Finally study of the self–consistent environment will help in understanding such basic design issues as how to mitigate spacecraft charging in low earth orbit.

In this paper we develop a simple model for the motion of a water plasma cloud around a large structure in the ionosphere. In section 2 we derive the equations that describe the large scale plasma flow around the body. In section 3 we obtain an analytic solution for the case where there is no electrostatic modification of the motional potential. This case corresponds to a plasma cloud which does not enhance the ambient density. In section 4 we perform a multiple scale analysis and obtain the equations for the field line averaged densities and potential. For the case where the cloud density exceeds the ambient density we show that the equations of motion do not allow steady state solutions and suggest that the cloud may be unstable to the growth of $E \times B$ gradient instabilities. We then solve the model equations numerically with an Flux Conserving Transport (FCT) numerical method and show the long time behaviour of the plasma cloud. We obtain the ion residence time for a range of cloud densities. For densities typical of what is observed around the Shuttle, the residence times are consistent with the measurements. Finally in section 5 we conclude with a discussion of the significance of these results for measurements from large space vehicles such as the Shuttle. Although the equations of the water plasma cloud are generally applicable we choose
to concentrate on the dynamic behavior of an initial water plasma "puff" which could arise from an MPD thruster firing or a plasma source.

2 Derivation of the equations for the plasma cloud

Plasma clouds in the ionosphere have been studied both theoretically and experimentally for a number of years [Perkins et al., 1973; Zabusky et al.; 1973; Mitchell et al., 1985]. The major motivation has been to understand the dynamics of such clouds since they have been used for tracing the magnetic field lines and they may interfere with communications. In this work we shall follow a similar path to obtain the equations for the plasma and self-consistent electrostatic potential around a large body.

We work in a frame attached to the space structure and start by assuming that the space structure is emitting neutral water into an ambient background of oxygen ions and neutrals. The spacecraft is taken to be moving at orbital velocity with respect to the ambient environment (≈ 8 km/sec for low earth orbit). For an ambient environment whose neutral density is of the order of $10^8$ cm$^{-3}$ the mean free path of the water molecules is many kilometers. Hence for length scales of up to a few kilometers we can take the water molecules as freely expanding with their thermal velocity and write the steady state neutral water density as

$$n_{H_2O} = n_{H_2O}^0 (r_0/r)^2$$  \hspace{1cm} (1)

This assumption about the neutral density is reasonable for low water densities (water density much less than the ambient neutral density) but will have to be modified for higher densities. However since we wish to concentrate on the plasma dynamics we shall leave more detailed modeling of the neutrals for future work. In Eq. (1) we have approximated the structure as a sphere of radius $r_0$ and taken $r$ as the radial distance from the center of the sphere. This approximation will be reasonable for any body shape as long as we are several body dimensions from the body. So for example, for the Space Shuttle whose longest dimension is of the order of 40 meters can be reasonably approximated in this manner at distances of the order of 100 meters and beyond. The density $n_{H_2O}^0$ is the density of water at the surface. The assumption that the neutral species is
water is based on observations from the Space Shuttle where the dominant contaminant species was observed to be H$_2$O$^+$(Caledonia et al., 1987).

The ionic water may be formed by the charge exchange reaction

$$O^+ + H_2O \rightarrow O + H_2O^+$$ (2)

where in the frame moving with the space structure the oxygen ions sweep through with an energy of up to 5 eV. At this energy the reaction rate $k_{cz}$ has been measured and is $k_{cz} = 6 \times 10^{-10}$ cm$^3$/sec [Murad and Lai, 1986a] and for $O^+$ the charge exchange collision frequency is $\nu_{cz} = n_{H_2O} k_{cz}$.

We assume for simplicity that the body is moving perpendicular to the magnetic field with velocity in a fixed frame of $V$. We define a wind velocity in the moving frame as $\vec{V}_w = -\vec{V}$ and then define a cartesian coordinate system ($x, y, z$) in the moving frame by taking the distance $z$ to be along the direction of $-\vec{V}_w$, the distance $y$ to be along the direction of $-\vec{V} \times \vec{B}$ and the distance $z$ to be along the direction of the magnetic field $\vec{B}$. We note that the negative $y$ direction is along the direction of the unshielded motional electric field that will be seen in the far field from the moving frame. The frame is illustrated in Figure 1.

We take the oxygen and water ions to be mainly moving across the magnetic field due to their large gyroradii while the electrons are taken to dominantly flow along the magnetic field due to their small gyroradii. This assumption on the ion motion is true on the large scale where the ions have a chance to complete their gyroorbits. In Caledonia et al. (1987), the ions were taken to be almost perfectly shielded from the motional electric field and hence in the moving frame the only motion available to them was parallel motion. In this work the potential is being calculated in a self-consistent manner so that the ions can move across the magnetic field. The perpendicular velocity of the oxygen ions can be obtained from the steady state momentum balance equation [Krall and Trivelpiece, 1973] and is

$$\vec{v}_{B, O^+} = (\kappa_O \cdot \vec{F}_\perp - \kappa^2 O \cdot \vec{F}_\perp \times \vec{b}) \cdot (1 - \kappa_j^2, \cdot)$$ (3)

where $\vec{b}$ is the unit vector in the magnetic field direction with $\kappa_j$, being the Hall parameter given
by

\[ \kappa_{O^+} = \Omega_{O^+}/(\nu_{O^+} + \nu_{cz}). \]  

(4)

and \( \Omega_{O^+} \) being the oxygen cyclotron frequency while \( \nu_{O^+} \) is the oxygen ion-neutral water momentum exchange collision frequency. The vector \( \vec{F}_\perp \) is given by

\[ \vec{F}_\perp = -(cT_i/eB) \nabla \ln n_{O^+} + (ce/B) \vec{E} + \nu_{O^+} \vec{v}_{H_2O} / \Omega_{O^+}. \]

(5)

In Eq. (5) \( T_i \) is the ion temperature which we take to be constant and the first term is the force due to the ion pressure, \( \vec{E} \) is the electric field seen from the moving frame and gives the electric force on the ions in the moving frame, while \( \vec{v}_{H_2O} \) in the third term is the velocity of the water and this term expresses the force on the oxygen ions due to collisions with the water neutrals. In the derivation of the force on the oxygen ions the dominant collisions for these ions has been taken to be collisions with the neutral water molecules (rate constant \( \approx 1.2 \times 10^{-9} \text{ cm}^3/\text{sec} \)). Collisions with oxygen neutrals (rate constant \( \approx 3 \times 10^{-9} \text{ cm}^3/\text{sec} \)) have been neglected for simplicity since we anticipate little differential velocity between oxygen neutrals and ions. In Eq. (3) the first term gives rise to the well known Pedersen conductivity while the second contains the \( E \times B \) drift and the Hall current. We note that the Hall parameter takes very large values for parameters typical of the ionosphere (\( \nu_{O^+} \approx 2 \times 10^{-3} \text{ sec}^{-1} \) and \( \Omega_{O^+} \approx 1.8 \times 10^2 \text{ sec}^{-1} \) so that \( \kappa_{O^+} \approx 9 \times 10^4 \)).

This suggests that the dominant motion for the oxygen ions will be the \( E \times B \) drift. Hence we can write the continuity equation for the oxygen ions as

\[ \frac{\partial n_{O^+}}{\partial t} - \frac{c}{B} \nabla \phi \times \vec{b} \cdot \nabla n_{O^+} = -n_{O^+} \nu_{cz} \]

(6)

where the electric field in the moving frame is \( \vec{E} = -\nabla \phi \) and the boundary condition on Eq. (6) is that for \( |\vec{r}| \to \infty \) the oxygen density approach the ambient density.

The electrons will mainly flow along the magnetic field but will also have an \( E \times B \) drift across the field. In addition electrons will be lost as a result of dissociative recombination of the water ion. This occurs with a rate \( k_{\text{recom}} \) which we shall specify later. Hence we can write the electron continuity equation as

\[ \frac{\partial n_e}{\partial t} - \frac{c}{B} \nabla \phi \times \vec{b} \cdot \nabla n_e + \nabla \cdot \Gamma_e = -n_e n_{H_2O} k_{\text{recom}} \]

(7)
where $\Gamma_{\parallel}$ is the parallel electron flux given by

$$\Gamma_{\parallel} = \frac{e n_e \nabla \psi_e}{m_e v_e} \quad (8)$$

and the potential $\psi_e$ is defined as

$$\psi_e = \phi - \frac{T_e}{e} \ln \left( \frac{n_e}{n_{ambient}} \right) \quad (9)$$

In Eq. (9), $\phi$ is the self-consistent potential which is modified by the electron pressure. The parallel current is related to the parallel electric field through the resistivity ($\eta_e$)

$$j_\parallel = -e \Gamma_{\parallel} = \frac{1}{\eta_e} \left( -\nabla \psi_e \right) \quad (10)$$

where the resistivity is given by

$$\eta_e = \frac{m_e v_e}{n_e e^2} \quad (11)$$

In Eq. (8) the collision frequency $\nu_e$ is the electron momentum exchange frequency due to collisions with the neutrals or the ions. The physical content of this equation is that the electrons flow along the field lines and are swept across it by the self-consistent potential in the plasma cloud. We note that for any steady state to exist to this equation it is necessary that there be a convection of electrons across the field lines. This implies that the potential contours must be open. This is because closed potential contours will cause the plasma to rotate [Daly and Whalen, 1979] but will not lead to any net loss. This implies that the symmetry breaking part of the electrostatic potential will be crucial in determining the motion of electrons. This part of the potential will arise from the fact that the system is moving and hence sees a motional potential.

Once we have the equations for the oxygen ions and the electrons the equation for the water ions follows from the requirement that the plasma remain quasineutral,

$$n_{H_2O^+} = n_e - n_{O^-} \quad (12)$$

As an alternative to using Eqs. (6), (7), and (12) we could use Eqs. (6) and (12) coupled with the equation for the water ions

$$\frac{\partial n_{H_2O^+}}{\partial t} - \frac{e}{B} \nabla \cdot \nabla n_{H_2O^+} = n_{O^-} \nu_{ee} - n_e n_{H_2O} \cdot k_{rem} \quad (13)$$
Finally we need an equation to determine the self-consistent potential. This equation comes from the requirement of quasineutrality for all times. This means that in addition to Eq. (12) we require that we have charge conservation ($\nabla \cdot \vec{j} = 0$ where $\vec{j}$ is the current flowing in the plasma). We take the perpendicular current to be given by the perpendicular ion flows (Eq. (3) and the analogous equation for the perpendicular water velocity) and the parallel current to be given by the electron flows (Eq. (10)) and obtain the equation [Drake and Huba, 1986]

$$\frac{c}{B} \nabla \cdot \left( \left( \frac{n_{H_2O^+} + n_{O^+}}{\kappa_{H_2O^+}} \right) \nabla \phi \right) + \nabla \cdot \left( D_{H_2O^+} n_{H_2O^+} + D_{O^+} n_{O^+} \right)$$

$$+ \left[ \hat{e} \times \vec{v}_{H_2O} \cdot \nabla (n_{O^+} \frac{\nu_{O^+,H_2O}}{\Omega_{O^+}}) + \hat{e} \times \vec{v}_O \cdot \nabla (n_{H_2O^+} \frac{\nu_{H_2O^+,O}}{\Omega_{H_2O^+}}) \right]$$

$$+ \nabla \parallel \left( \frac{1}{\epsilon \eta_e} \nabla \parallel \psi_e \right) = 0 \quad (14)$$

In this equation for charge conservation the perpendicular ion diffusion coefficient is

$$D_{\perp, i} = \frac{1}{\kappa_i eB} \quad (15)$$

where the Hall parameter for the water ions is

$$\kappa_{H_2O^+} = \Omega_{H_2O^+}/(\nu_{H_2O^+,O} - \nu_{e}) \quad (16)$$

We note that in this expression all the $\vec{E} \times \vec{B}$ drifts have cancelled out since they do not give rise to any net current. The first term on the RHS of Eq. (14) arises from the Pedersen current due to the electric field, the second term due to the diffusion of ions down the density gradients, the third due to the Hall current while the fourth is the electron flow to balance the ions. This equation must be solved subject to the boundary condition that in the far field the electric field in the moving frame is the motional electric field. This requires that for $|\vec{r}| \to \infty$ we have $\nabla \perp \phi = -\vec{V} \times \vec{B}/c$. We can interpret the first two terms in the potential equation (Eq. (14) as giving the shielding from the plasma cloud while the last term is the charge neutralization by the parallel flow of electrons from far away in the ionosphere.
3 Analytic Solutions of the Cloud Equations

The set of equations for the potential and densities cannot in general be solved analytically. However, it is possible to obtain a special solution and draw some general conclusions from the structure of the equations.

If the density of the cloud is very low, then we expect that the motional potential will be only weakly shielded by the cloud. In this case, we approximate the electric field everywhere by the boundary condition on Eq. (14) and write \( \nabla \cdot \phi = -\nabla \times B/c \) for all space. If we require that \( \phi = \phi(r_\perp) \) (i.e., we do not allow any parallel electric fields) then in the absence of recombination, we have from Eq. (7) in steady state that

\[
-\nabla \cdot \nabla \cdot n_e = 0,
\]

which has only two possible solutions, either that \( n_e = n_{\text{ambient}} \) everywhere or that \( n_e \) varies in only one direction in space which is physically unreasonable. The oxygen equation is

\[
-\nabla \cdot \nabla n_{O^+} = -n_{O^+} V . \mathbf{e}_z,
\]

which has the solution

\[
n_{O^+} = n_{\text{ambient}} \exp \left( -\frac{n_0^2 k_\perp}{V} \frac{r_\perp^2}{\sqrt{y^2 + z^2}^2 + \left( \frac{ x - x_0}{\sqrt{y^2 + z^2}^2 + \left( \frac{ x - x_0}{\sqrt{y^2 + z^2}^2 + \left( \frac{ x - x_0}{\sqrt{y^2 + z^2}^2 + 2 x_0^2} \right)^2} \right)} \right) \right).
\]

This shows the product of the water column density traversed by a \( O^+ \) ion and the charge-exchange cross section. Hence, the electron density is constant everywhere while the oxygen density is depleted and partially replaced by the water density. The contours of constant water column density relative to the ambient electron density are shown in Figure 2 for a type II bow shock, showing a depletion of water which is emitting water from a central region of 100 meters in radius. We choose to look at these large length scales since there is evidence from PDIP measurements around the Shuttle of water ions out to a kilometer from the Shuttle of Murphy. private communication, 1987. We see that most of the depletion for the oxygen density occurs on the fast side of the structure and for this case somewhat less than half of the oxygen ions are converted to water ions. Hence, for this case the water plasma cloud is actually a depletion relative to the ambient electron density.
4 Derivation and Numerical Solution of the Averaged Cloud Equations

Since the plasma flow along the magnetic field is extremely rapid compared to the perpendicular flow we can simplify the equations describing the plasma by doing a formal multiple scale analysis. Physically, we expect the cloud to take a cigar shape along the magnetic field. The equations for the field line averaged density and field line averaged potential can be obtained by averaging the density and potential equations where the averaging operator is given by

\[ F = \frac{1}{l_B} \int \frac{dl}{B} \int \frac{dl}{B} \]

Hence from this point, \( N_e, N_{H_2O^+}, N_{O^+}, \Phi \) refer to the field line averaged electron density, water ion density etc. The field line average operator annihilates the parallel terms in the electron density and potential equations and gives for the density equations (from Eqs. (6) and (7))

\[ \frac{\partial N_{O^+}}{\partial t} - \frac{c}{B} \nabla \Phi \times \vec{b} \cdot \nabla N_{O^+} = -N_{O^+} \nu_{cz} \]  

(20)

and

\[ \frac{\partial N_e}{\partial t} - \frac{c}{B} \nabla \Phi \times \vec{b} \cdot \nabla n_e = -N_e N_{H_2O^+} k_{recom} \]

(21)

while the potential equation (Eq. (14)) is

\[ \frac{c}{B} \nabla \cdot \left( \left( \frac{N_{H_2O^+}}{\kappa_{H_2O^+}} + \frac{N_{O^+}}{\kappa_{O^+}} \right) \nabla \Phi \right) + \nabla \cdot \left( D_{\perp H_2O^+} N_{H_2O^+} + D_{\perp, O^+} N_{O^+} \right) + \left[ \vec{b} \times \vec{v}_{H_2O} \cdot \nabla \left( N_{O^+} \frac{\nu_{O^+ + H_2O}}{\Omega_{O^+}} \right) + \vec{b} \times \vec{v}_{O} \cdot \nabla \left( N_{H_2O^+} \frac{\nu_{H_2O^+ + O}}{\Omega_{H_2O^+}} \right) \right] = 0. \]  

(22)

In obtaining these equations we have used the multiple scale assumption to say that the densities and potential have a strong dependence on the perpendicular direction and a weak dependence on the parallel direction. This is a reasonable assumption for the plasma quantities but cannot be applied to the neutrals. Therefore in Equations (20), (21), and (22) we have evaluated the neutral water density in the plane \( z = 0 \). This assumption enables us to obtain a closed set of equations for the field line averaged quantities and still contains the appropriate physics.
This set of equations describes the dynamic evolution of a water plasma cloud as seen from a moving frame and with the potential calculated self-consistently. We shall use these equations to study the evolution a water plasma cloud where the initial density may be larger than the ambient electron density. As pointed out in Caledonia et al. [1987] such an enhanced density cannot arise self-consistently solely by chemical processes such as charge exchange. Therefore in those cases where the water plasma density exceeds the ambient density we assume that the water plasma has been injected at the initial time due to some mechanical mechanism such as the firing of a plasma source.

This set of field line averaged equations for a plasma cloud around an space structure still cannot be solved analytically but it can be shown that the only steady state solution allowed is the one previously discussed where the electron density is uniform everywhere. The proof of this statement is given in the appendix and follows a similar proof in work by Dungey [1958]. Since in the far field the electron density must match the ambient density we conclude that plasma clouds whose density exceeds the ambient density cannot be steady state structures. Therefore the enhanced densities seen around the Shuttle [Pickett et al., 1985] must be transient phenomena. The fact that a plasma cloud whose density exceeds the ambient will continuously evolve is also suggested by the following physical arguments: such a plasma cloud will have a density distribution which will probably peak towards the center of the cloud. The moving cloud will have Pedersen currents in it which act to modify the imposed motional field. These Pedersen currents are density dependent since they arise from the bulk motion of ions due to the potential gradients. Therefore we expect and Eq. (22) confirms this, that the highest density regions of the cloud will modify the motional potential most effectively. Hence the low density cloud edges will see a self consistent $E \times B$ drift different than the higher density regions. This leads to a velocity shear through the cloud which causes it to continuously distort. Therefore no steady state will exist. The timescale on which this distortion will take place will be a function of density. It can be estimated on the basis of the arguments given by Perkins et al. [1973] if we assume that the dominant effect of the cloud is to shield the potential. If the cloud has density $n_c \gg n_{\text{ambient}}$ then the electric field in the cloud is
\[ E_y \simeq n_{\text{ambient}}/n_e E_0 \text{ where } \vec{E}_0 = \vec{V} \times \vec{B}/c = E_0 \vec{e}_y. \text{ From } \nabla \times \vec{E} = 0 \text{ we obtain} \]

\[ E_z \simeq -y(n_{\text{ambient}}/n_e^2)(dn_e/dx)E_0. \]

The cloud velocity is \[ \vec{V}_c = c\vec{E} \times \vec{B}/B^2 \] which gives

\[ v_x \simeq -V(n_{\text{ambient}}/n_e) \]

and

\[ v_y \simeq -V(y n_{\text{ambient}}/n_e^2)(dn_e/dx). \]

We see that in the moving frame the cloud will drift backwards with a velocity which depends on the density and will drift sideways with a velocity which depends both on the density and on how sharp the density gradient is in the direction of the neutral wind. From these estimates we can see that distortion will occur on a timescale of \[ L_n/(V(n_{\text{ambient}}/n_e)) \] where \( L_n \) is the density length scale. For the case of the Space Shuttle if we take the density length scale to be the Shuttle length (\( \simeq 50 \) meters) and take a water ion cloud of \( 10^7 \) cm\(^{-3} \) in an ambient density of \( 10^5 \) cm\(^{-3} \) then for a velocity of 8 km per second the distortion time is 0.6 seconds.

We have solved the averaged cloud equations (Eqs. (20), (21), and (22)) for a 5km by 5km square region using a uniform 101 by 101 cartesian mesh. The center of the square was fixed on the structure. With \( n_{\text{H}_2\text{O}^+}(x, y) \) and \( n_{\text{O}^+}(x, y) \) given at some time we calculate the potential from Eq. (22) and use that result to obtain the flow field velocities. Then that information is used in Eqs. (20) and (21) to advance the densities by one timestep.

Successive Point Over Relaxation (SPOR) was used to solve the elliptic current balance equation. This scheme introduces a relaxation parameter into the equation and which then allows an iterative solution of the problem [Roach, 1976]. The derivatives were approximated by second order finite differences and iterations were carried out until the maximum error was less than \( 10^{-5} \) between two successive iterations. Neumann boundary conditions were applied in both the x and y directions. We required the potential to match the following boundary conditions in the far field

\[ \nabla \phi = -\frac{\vec{V} \times \vec{B}}{c} \]
in the $\mathbf{V} \times \mathbf{B}$ direction and

$$\nabla \perp \phi = 0$$

in the direction of motion. The potential of the central structure was taken to be 4 Volts which is consistent with measurements taken from the Shuttle.

For the convective equations we used the two-dimensional flux correction method (FCT) of Zalesak [1979]. The high order scheme was a leapfrog–trapezoidal [Roach, 1976] with fluxes calculated with the flux formulae developed by Zalesak [1984] while a donor–cell scheme [Roach, 1976] was used for the lower scheme to complete the FCT algorithm. Symmetric boundary conditions were applied on the density distributions and the flux limiter was applied on every iteration. No assumptions were made about the symmetry of the solutions so that the computations were carried out through the whole mesh. The oxygen density approached the ambient ion density (taken to be $2 \times 10^5 \text{ cm}^{-3}$) at the edges of the square. The water ion density was assumed to have an initial gaussian profile falling off on a length scale of 500 meters.

\[
n_{H_2O^+} = n_{H_2O^+}(\vec{z} = 0) \exp \left[-\left(\frac{r - r_s}{R_L}\right)^2\right]
\]

where $|r_s| = 50$ meters and $R_L = 500$ meters. This simulates a large scale ionic water injection. The initial water ion profile for a typical simulation is shown in Figure 3 where the central density for this case was $n_{H_2O^+}(\vec{z} = 0) = 10^6 \text{ cm}^{-3}$. In this figure the x–direction corresponds to the negative wind velocity direction. That is the neutral wind is blowing from positive x to negative x in this figure. The negative y direction is the direction of the motional electric field.

In Figures 4–6 we show the initial distribution of potential for a density profile as given in Figure 3 and for three initial ion water densities. In Figure 4 the initial water density was $n_{H_2O^+}(\vec{z} = 0) = 10^4 \text{ cm}^{-3}$. In this figure we see the shielding that arises when the total ion density exceeds the ambient oxygen ion density and the cloud behaves as a classical dielectric. In Figure 5 the central water density was taken to be $n_{H_2O^+}(\vec{z} = 0) = 10^6 \text{ cm}^{-3}$. Here we see exactly the opposite tendency from Figure 4. The electric field inside the cloud increases rather than decreases. This occurs because the density gradient terms in the current balance equation are large enough to overcome the natural polarization which occurs due to the imposed motional electric field. The
electric field increases since a large mobility driven electric flux is necessary to balance the density gradient driven ion flux. This tendency is confirmed in Figure 6 where we give the potential distribution for \( n_{H_2O+}(z = 0) = 10^8 \text{ cm}^{-3} \). The potential contours here can be well modeled as due to a uniform motional field and a positive charge on the positive y side of the cloud and a negative charge on the negative y side of the cloud. This suggests that the picture of the plasma cloud around large objects in the ionosphere as being a dielectric shield [Katz et al., 1984] is too simplistic a model. The important effect of the plasma density gradients must also be taken into account.

A typical simulation is shown in Figures 7–13 for the initial water ion puff given in Figure 3 and with an initial density of \( n_{H_2O+}(z = 0) = 10^8 \text{ cm}^{-3} \). Since an enhanced density of this magnitude would have to be caused by some means such as a plasma source which would probably emit warm electrons we took the recombination rate for the water as \( k_{\text{recom}} = 3 \times 10^{-8} \text{ cm}^3/\text{secs} \) [Murad and Lai, 1986b] which corresponds to an electron energy of 1 eV. The neutral water is assumed to have the form given in Eq. (1). In Figure 7 we show the density contours at 0.1 seconds after the start of the simulation. This corresponds to 16 gyroperiods which is enough time so that a substantial \( \vec{E} \times \vec{B} \) drift can occur. The water ion cloud is seen to be drifting backwards and to be undergoing some distortion on the backside of the cloud. The central region of the cloud is seen to be breaking into two regions with very steep gradients on the backside of the central region. The fact that most of the distortion is concentrated on the backside of the cloud can be understood by noting that the high density core of the cloud will drift more slowly than the rear fringes of the cloud so that the back edge of the cloud will collide with the central core so leading to the observed distortion. Also around the source ionic water is still being formed due to charge exchange and then immediately drifting backward so that a very steep density gradient in the ionic water is observed near the source. The distortion on the backside of the cloud may be the development of the \( \vec{E} \times \vec{B} \) instability. In barium cloud studies where the equations [Perkins et al., 1973] for the barium cloud are very similar to the ones used here the cloud was observed to break up into filaments both experimentally and numerically [Zabusky et al., 1973]. For barium clouds it is believed that the filaments may be associated with the formation of \( \vec{E} \times \vec{B} \) instabilities [Drake and Huba,
1986]. This leads us to suggest that the filaments may be due to a similar reason. This important topic of possible instabilities is the subject of ongoing research and will be reported in a future publication. This may be another reason why no steady state may exist. This also suggests that electrostatic noise may be inextricably linked with the large water densities observed around large space objects. In Figure 8 we show the associated oxygen density contours at the same time. This region delimits the area where new ionic water is being created from the area where ionic water exists because it is came from the initial conditions. Comparison of this figure with the last figure suggests that part of the breakup of the central core may be due to the creation of ionic water there in an asymmetric manner. In this figure we also see the very sharp density gradients in the oxygen density along the edges of the oxygen hole. In 0.1 seconds the region of oxygen depletion extends back behind the structure a distance of approximately 0.7 km. This suggests that the center of the water cloud is moving with a drift velocity of approximately 7 km per second. This is less than the velocity an individual ion would drift backwards with if it saw only the motional electric field and occurs because the ions in the cloud are being affected by the self-consistent electric field and not the motional electric field. In Figure 9 we show the potential contours at 0.1 seconds. The fact that the cloud has moved relative to the structure is clearly seen in this figure since the region where the potential contours is distorted has moved backwards. For Figure 10-13 we show the time development of the water ion cloud at 0.15, 0.2, 0.25 and 0.3 seconds. The cloud continues to develop a tadpole-like shape in the direction of cloud motion and the central breakup and gradients become more pronounced. The instability on the backside of the cloud has entered a nonlinear state where distinct fingers have formed on the backside of the cloud. This has also been seen in barium cloud releases in the ionosphere. The cloud distribution over this period of time suggests that measurements of the cloud density from the structure will see a density that is both time dependent and spatially anisotropic. This suggests that density measurements from the Space Shuttle must be interpreted with care.

The time that ions spend in the vicinity of the structure is critical to determining the range of chemical reactions that they can undergo. This question is of substantial interest since there is evidence that chemical reactions can occur over surfaces in space Green et al. 1985. In Table 1
Table 1: Residence time against central ion water density

<table>
<thead>
<tr>
<th>$n_{H_2O^+}(z = 0)$ (cm$^{-3}$)</th>
<th>$r$ (milliseconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4$</td>
<td>44</td>
</tr>
<tr>
<td>$10^5$</td>
<td>19</td>
</tr>
<tr>
<td>$10^6$</td>
<td>8</td>
</tr>
<tr>
<td>$10^7$</td>
<td>5</td>
</tr>
</tbody>
</table>

we give an estimate of the residence time of a water ion in the vicinity of the Space Shuttle as a function of the initial central water ion density. The residence time ($r$) was defined as $r = L/v_D$ where $L$ is the length of the Shuttle taken as 50 meters and $v_D$ is the cloud drift velocity determined from the simulation. We see that the residence time of the ions is in the millisecond to tens of milliseconds range and furthermore the higher density cases lose ions substantially faster than the low density cases. This suggests that measurement of the decay rate of a puff of plasma will not give a unique answer for the ion residence time [Sasaki et al., 1985]. Rather the ion residence time is a function of the density as we would expect for a loss process which is not linear. For the low density case the residence time is consistent with measurements from the Shuttle [Caledonia et al., 1987]. In Caledonia et al. [1987] it was shown that a signal of $n_{H_2O^+}/n_{O^+}$ of 0.1 was consistent with a loss time for the water ions of 40 milliseconds. We also note that these times are consistent with results from Spacelab 2 where enhanced ionic densities were not seen and substantial shielding of the motional potential was not observed [J. Raitt, private communication, 1986].

5 Conclusions

We have formulated a simple model for the plasma cloud around a large structure in the ionosphere. One such structure is the Space Shuttle and another may be a system like the Space Station. The model has been applied to an ionic water puff and the dynamic behavior of the cloud followed.

A number of important conclusions can be drawn. First is that the cloud possesses no steady state if the electron density is enhanced above the ambient. The second is that the effect of density
gradients in the cloud may actually enhance the electric field that the cloud sees so that the picture of the cloud always acting as a shield against the motional potential is too simplistic. The third is the cloud may be subject to electrostatic instabilities which will grow and saturate with distortion of the cloud structure. This will mean that electrostatic noise and turbulence may always accompany these clouds. Finally we conclude that the cloud structure will be highly anisotropic which suggests that measurements of the cloud density from the structure must be treated with care.

Future work will concentrate on elucidation of the electrostatic instabilities associated with the cloud and on resolution of the ion residence times with the observed chemistry around the Shuttle.

These conclusions indicate that sensors on board systems like the Space Shuttle or a Space Station will have to cope with a background which is spatially anisotropic, temporally varying and also electrostatically noisy. These issues will also be studied in future work.

6 Acknowledgements

The authors would like to acknowledge useful discussions with Dr. S. Zalesak in the course of this work. This work was supported by the Air Force Geophysics Laboratory under contract number F19628-86-K-0018.

A Analytic Proof that Cloud Equations have no Steady State

We consider Eqs. (20), (21), and (22) with recombination in the electron equation ignored for simplicity. We define two potentials by

$$\psi_{O^+} = \frac{n_{O^+}}{\kappa_{O^+}} - \frac{B}{c} D_{O^+}, \ln n_{O^+}, \tag{23}$$

and

$$\psi_{H_2O^+} = \frac{n_{H_2O^+}}{\kappa_{H_2O^+}} - \frac{B}{c} D_{H_2O^+}, \ln n_{H_2O^+}. \tag{24}$$

We assume that the density equations for the oxygen and water possess a steady state and that the plasma cloud is moving with some constant velocity \( \vec{V} \). With this assumption we can write

$$\frac{\partial n_{O^+}}{\partial t} = \vec{V} \cdot \nabla n_{O^+} \cdot n_{O^+}, \nu_{r}. \tag{27}$$
and

\[
\frac{\partial n_{H_2O^+}}{\partial t} = -\vec{V}_e \cdot \nabla n_{H_2O^+} + n_{O^+}\nu_{ez}.
\]  

(26)

We substitute Eqs. (25) and (26) into Eq. (20) and the equivalent equation for the water ions and use the definitions of the potentials in Eqs. (23) and (24) to obtain

\[
-\vec{V}_e \cdot \nabla n_{O^+} = \frac{e}{B} (\nabla \psi_{O^+} \times \vec{b}) \cdot \nabla n_{O^+}
\]  

(27)

and

\[
-\vec{V}_e \cdot \nabla n_{H_2O^+} = \frac{e}{B} (\nabla \psi_{H_2O^+} \times \vec{b}) \cdot \nabla n_{H_2O^+}.
\]  

(28)

These equations have the general solution

\[
\nabla \psi_{O^+} = -\vec{H} + \frac{1}{n_{O^+}} \nabla \psi_{O^+}
\]  

(29)

and

\[
\nabla \psi_{H_2O^+} = -\vec{G} + \frac{1}{n_{H_2O^+}} \nabla \psi_{H_2O^+}
\]  

(30)

where \(\vec{H}\) and \(\vec{G}\) are constant vectors related to \(\vec{V}_e\) and \(\nabla \psi_{H_2O^+}\) and \(\nabla \psi_{O^+}\) are arbitrary functions of the water ion density and oxygen ion density respectively. If we substitute these expressions for the potentials in Eq. (22) we obtain

\[
\nabla^2 (\psi_{O^+} + \psi_{H_2O^+}) = \vec{M} \cdot \nabla n_{O^+} + \vec{N} \cdot \nabla n_{H_2O^+}
\]  

(31)

where the vectors \(\vec{M}\) and \(\vec{N}\) are independent of the densities. We integrate Eq. (31)over a volume \(V\) bounded by a surface of constant electron density to obtain

\[
\int_V \nabla^2 (\psi_{O^+} + \psi_{H_2O^+}) dV = 0
\]  

(32)

Now if we multiply Eq. (31) by \(\psi_{O^+} + \psi_{H_2O^+}\) integrate over the same volume and use Eq. (32) and Green's theorem we obtain

\[
\int_V (\nabla \psi_{O^+} \cdot \nabla \psi_{H_2O^+})^2 dV = 0
\]  

(33)

This is only possible if

\[
(\nabla \psi_{O^+} \cdot \nabla \psi_{H_2O^+}) = 0
\]  

(34)
This indicates that the electron density equation must be of the form

\[ \vec{V}_e \cdot \nabla n_e = 0. \] (35)

The only nonconstant solution to this equation is for the electron density to vary in only one direction in space which is physically unreasonable. Therefore we can conclude that if the electron density has closed contours then it has no steady state and if there is a steady state then the electron density must be the ambient density everywhere.
References


Figure 1. Coordinate System
Figure 2. Steady State Water Ion Density Contours for a Uniform Electron Density.
Figure 3. Initial Water Ion Density Contours for Numerical Simulations. This corresponds to an initial puff of water plasma.
Figure 4. Initial Potential Contours for a Central Water Ion Density of $10^3$ cm$^{-3}$ where Water Plasma has the contours of Figure 3.
Figure 5. Initial Potential Contours for a Central Water Ion Density of $10^6$ cm$^{-3}$ Where Water Plasma Has the Contours of Figure 3
Figure 6: Initial Potential Contours for a Central Water Ion Density of 10^8 cm^-3 Where Water Plasma Has
the Contours of Figure 3
Figure 7. Water Ion Density Contours at 0.1 sec for an Initial Central Water Ion Density of $10^6$ cm$^{-3}$
PLASMA CLOUD
O+ NORMALIZED DENSITY DISTRIBUTION

INC= 0.500E-01

Time = 0.1sec

Figure 3. Oxygen Ion Density in Spacecraft Wakes for a Water-Ion Density of 10^6 cm^-3.

Note: The ions are not drawn to scale. The figure illustrates the wake density enhancement.
Figure 4. Potential Contours at 0.1 sec for an Initial Central Water Ion Density of $10^6$ cm$^{-3}$. Potential at the origin is taken to be 4 V.
PLASMA CLOUD
H2O+ NORMALIZED DENSITY DISTRIBUTION

INC= 0.500E-01

Time = 0.15sec

Figure 10: Water Ice Density Contours at 0.15 sec for an Initial Central Water Ion Density of 10^6 cm^-3.
Figure 11. Water Ion Density Contours at a time of 0.2 seconds with Water Ion Density of $10^6$ cm$^{-3}$.
PLASMA CLOUD
H2O+ NORMALIZED DENSITY DISTRIBUTION

INC= 0.500E-01

Time = 0.3 sec

Figure 13. Water Ion Density Contours at 0.3 sec for an Initial Central Water Ion Density of 10^6 cm^-3
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