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ANALYTICAL AND EXPERIMENTAL CHARACTERIZATION OF DAMAGE  
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TROY NY DEPT OF CIVIL ENGINEERING. G J DVORAK ET AL.

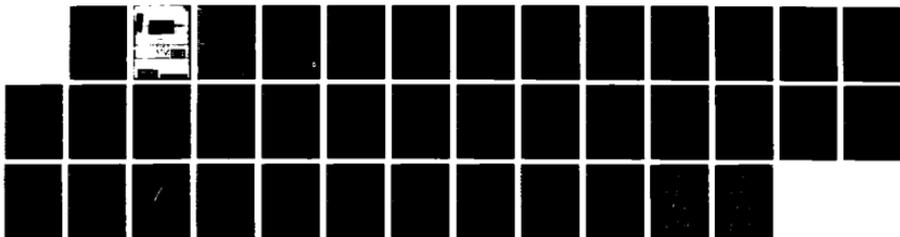
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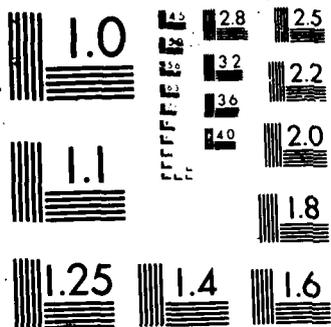
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Annual Progress Report

**AFOSR-TR. 87-1451**

ANALYTICAL AND EXPERIMENTAL  
CHARACTERIZATION OF DAMAGE PROCESSES  
IN COMPOSITE LAMINATES

by

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ABSTRACT

➤ This report presents a brief summary of results obtained in our research program on damage development in fibrous composite laminates. The following technical subjects are described: (i) Effect of fiber breaks on stiffness changes in unidirectional composites. (ii) Stress analysis of transverse cracks. (iii) Progressive transverse cracking of 0/90 laminates. (iv) Analysis of cracks approaching a boundary between two materials. (v) Fatigue of B/A<sub>2</sub> laminates.

*composite materials; ply laminates; initiation  
crack propagation; cracking (fracturing); aluminum  
Boron; Polymers* ←

## 1. INTRODUCTION

This research project is conducted as a cooperative effort of two investigators. Dr. George J. Dvorak is the Principal Investigator of the program at Rensselaer Polytechnic Institute, which is the primary contractor. Dr. Norman Laws is Principal Investigator of the part of the program subcontracted from RPI to the University of Pittsburgh.

In the second year of this research program major accomplishments were achieved in the following areas:

- Effect of fiber breaks on stiffness changes in unidirectional composites.
- Full stress analysis of transverse cracks.
- Progressive transverse cracking of 0/90 laminates.
- The image crack method for analysis of cracks approaching a boundary between two materials.
- Analysis of fatigue in B/A<sub>2</sub> laminates.

The principal results are described in the sequel.

## 2. STATUS OF THE RESEARCH

### 2.1 Effect of Fiber Breaks and Aligned Penny-Shaped Cracks on the Stiffness and Energy Release Rates in Unidirectional Composites

It is known that unidirectional graphite-epoxy composites subject to loading parallel to the fibers eventually suffer fiber breakage. The density of such fiber breaks can be quite large before final fracture occurs. At the end of each broken fiber one usually finds a penny-shaped crack (whose normal is in the fiber direction). Under continued loading, these cracks may extend. In addition there is ample evidence that in cross-ply composite laminates, one often sees fiber breaks in the  $0^\circ$  plies at the end of transverse cracks in the  $90^\circ$  plies. Another application arises in fibrous composites which can develop aligned matrix cracks on planes perpendicular to the fibers. Such cracks are confined to the matrix and do not extend through the fibers. They have been observed in  $0^\circ$  plies of cyclically loaded B-Al laminates by Dvorak and Johnson [1].

As a prelude to study of fiber breaks in composite laminates, certain critical problems were studied in unidirectional composites. In particular attention was focussed on changes in stiffness and strength due to fiber breaks accompanied by penny-shaped cracks at the ends of the broken fibers.

Various models were studied [2]. More precisely we found the self-consistent estimates, but in addition we obtained the only non-trivial Hashin-Shtrikman bound on the moduli of the cracked solid. We also derived the appropriate differential scheme model for the loss of

stiffness and showed that both the self-consistent and differential scheme results are entirely consistent with the Hashin-Shtrikman bounds. As far as we are aware this piece of work represents the first proof of the status of the self-consistent and differential scheme predictions for cracked solids in relation to the Hashin-Shtrikman bounds.

The only other work which has any bearing on the preceding results is due to Mura and Taya [3]. It is noteworthy that we were able to show explicitly that the Mura-Taya results for loss of stiffness coincide with the Hashin-Shtrikman bound.

Some typical results for loss in stiffness (of a unidirectional composite) are shown in Figs. 1 and 2. We note that the differential scheme predicts smaller stiffness loss than does the self-consistent model. Of course, both predictions are greater than the Hashin-Shtrikman bound.

We now turn to the important study of the influence of cracks on energy release rates. Here we consider two different cases. First we consider a solid containing a family of aligned penny-shaped cracks, each of radius  $a$ . The corresponding crack density is  $\alpha = Na^3$  where  $N$  is the number of cracks per unit volume. The solid is subject to macroscopically uniform loading and all cracks are open. Suppose now that each crack extends from radius  $a$  to radius  $(a + \delta a)$ , while the total number of cracks remains constant. One can then compute the crack extension force, or energy release rate per unit length,  $G_A$ , of each crack. Here the subscript  $A$  is used to signify that  $G_A$  is the energy release rate of each crack when all cracks extend simultaneously.

Next, we consider a single penny-shaped crack in the effective cracked solid. The energy release rate for this single crack is  $G_S$ .

It is important to emphasize that both  $G_A$  and  $G_S$  depend upon crack density. This is to be contrasted with the usual situation in fracture mechanics wherein a single crack extends in a material of fixed properties. The tendency for either one crack to propagate or all cracks to extend may be inferred from the ratio  $G_A/G_S$ . In fact, it is clear that if  $G_A/G_S \geq 1$  extension of all cracks is indicated rather than extension of a single crack.

It is interesting to note that the differential scheme predicts that  $G_A = G_S$ . Thus, this scheme can provide a useful border between those models which predict  $G_A$  to be greater or smaller than  $G_S$ .

Numerical results have been obtained for the T300/5208 graphite epoxy systems considered earlier. In particular the self-consistent estimates and (trivially) the differential scheme estimates are shown in Fig. 3. Since  $G_A/G_S \geq 1$ , both models predict extension of all cracks.

A further conclusion on energy release rates for a single crack is worthy of note. Consider mixed mode loading  $\bar{\sigma}_{33} \neq 0$ ,  $\bar{\sigma}_{23} \neq 0$  where the fibers are aligned in the 3-direction. Then

$$G_S = a \Lambda_{33}(\bar{\sigma}_{33})^2 + a \Lambda_{44}(\bar{\sigma}_{23})^2,$$

where  $\Lambda_{33}$  and  $\Lambda_{44}$  are the appropriate energy release rate factors. Figs. 4 and 5 contain the self-consistent and differential scheme results for these energy release rate factors for T300/5208 systems. Clearly, both models predict that, at fixed crack density,  $\Lambda_{33}$  and  $\Lambda_{44}$  decrease as the volume fraction of fibers increases. Thus  $G_S$  decreases as the volume fraction of fibers increases. Further, it is evident that, for fixed  $c_f$ ,  $G_S$  increases with crack density  $\alpha$ .

Finally, we note that the results obtained in this investigation show that progressive cracking is governed by spatial variation of fiber strength, sufficient crack energy is available for each fiber break to produce a new crack.

## 2.2 Full Stress Analysis of Transverse Cracks

The basic problem of determining the elastic field in a cross-ply laminate containing cracks has yet to be solved satisfactorily. Indeed a major effort has been devoted to this problem. Clearly the problem is extremely difficult, which is doubtless the reason for the lack of any literature on the subject. In fact the closest approach so far is due to Erdogan [4] and his co-workers. But there remain major difficulties in extending the Erdogan techniques to cope with our problem. The fundamental difficulty lies in the boundary conditions. Nevertheless we have reduced the basic problem to the solution of a singular integral equation. This integral equation has a potentially difficult kernel.

Attempts to solve this integral equation on the University of Pittsburgh DEC-10 and VAX/VMS 8600 computer systems were not successful because of time and basic machine limitations. To put the problem in a nut-shell, use of the VAX/VMS system was too expensive and the results which we had obtained were of doubtful accuracy.

Accordingly, a proposal was submitted, and accepted, by the NSF through the Pittsburgh Supercomputing Center to enable us to use the CRAY to solve the fundamental single integral equation. This work is going well and some preliminary results are available. More precisely, Figs. 6 and 7 show some stress intensity factor and energy release rate curves

for some particular graphite-epoxy systems. We remark that these results are entirely consistent with finite element calculations and with results previously obtained by Isida [5], Erdogan [4], and Mishra and Misra [6]. However, a full understanding of the physical interpretation of these preliminary results must await a thorough analysis of comprehensive numerical experiments and the associated parametric studies. These topics continue to demand a major effort in our continuing work.

### 2.3 Progressive Transverse Cracking in Composite Laminates

The process of damage development made under monotonic loading of fibrous composite laminates continues to be a major part of the research effort. Further work has been undertaken to relate progressive cracking to measurable material properties. This work is a continuation of the work reported earlier. A significant achievement has been our success in using a properly formulated shear-lag theory which properly reflects the statistics of the cracking process [7]. In complete contrast to some models which have been proposed in the literature, the model developed does not involve any adjustable parameters. But it must be noted that experimental evidence for critical energy release rates, for example, is not definitive. Indeed it is not out of place to draw attention to the pressing need for some definitive test methods for the determination of critical energy release rates.

It is also of interest to observe that some recent work by Hashin [8] bears on the problem of determining the loss of stiffness of composite laminates due to transverse cracking. Actually, Hashin obtains lower bounds for the loss stiffness. It is noteworthy that the

self-consistent model developed by the present authors, and described fully elsewhere [7], is consistent with the Hashin results. It is also significant that for the  $(0/90_3)_S$  E-glass epoxy laminates studied by Hashin, the difference between the SCM prediction and Hashin's lower bound is only slight, and certainly not sufficient to merit a graphical comparison.

But the more difficult problem of predicting transverse cracking under load is not feasible using the classical extension principles of elasticity as in Hashin's [8] work. In order to address, and indeed to solve, this problem we have further pursued the validation of our statistical shear lag analysis with experiment. The agreement between theory and all known experimental data is quite outstanding.

First we present results for the  $(0/90_3)_S$  E-glass/epoxy laminates studied by Highsmith and Reifsnider [9]. The loss of stiffness predictions are compared with experiment in Fig. 8. We remark that both the shear-lag and SCM predictions are greater than the Hashin bound, and that this bound is virtually indistinguishable from the SCM curve. Also in Fig. 9 we show the excellent agreement between theory and experiment for transverse crack density as a function of applied load. This agreement is encouraging but the  $(0/90_3)_S$  laminates are clearly not typical of graphite-epoxy laminates.

The experiment in graphite-epoxy systems are reported in a series of papers by Wang and Crossman and co-workers [10,11,12,13,14]. The work of these authors is exclusively concerned with graphite-epoxy systems and is centered on the prediction of crack density as a function of the applied (monotonic) load. Those parts of the work of these latter authors which are devoted to fatigue are not relevant to the present discussion.

We first consider the AS-3501-06 graphite-epoxy systems, for which experimental results are conveniently available in the survey article by Wang [13]. Data for  $(O_2/90)_S$ ,  $(O_2/90_2)_S$  and  $(O_2/90_3)_S$  laminates are given in Figs. 11, 12, 13 of Wang's [13] article. The material parameters for these systems are quoted by Wang [13], see also [14]. Comparison of the respective shear lag predictions with the experimental results are shown in Fig. 10. We draw the reader's attention to the fact that we have omitted Wang's [13] numerical results from Fig. 9 because it is impossible to do justice by replicating the published graphs. The important observation is that both shear lag and the Wang-Crossman theory give very good predictions. This is quite remarkable since the respective theories are based on entirely different premises.

Finally we turn to the data reported by Wang [13] for some T300/934 laminates. Again we use the basic material parameters given by Wang [13,14] in our calculations. The theoretical predictions are compared with the experimental data in Fig. 11. Again, it is encouraging to report excellent agreement.

The success of this work has prompted similar study of more general laminates and loading. In particular work on longitudinal splitting, damage due to H cracks, bending and temperature induced damage are being actively pursued. Progress to date is encouraging, especially in relation to some data reported by Muri and Gunn [15].

## 2.4 The Image-Crack Method for Analysis of Crack Approaching a Boundary Between Two Materials

It is well known that stress analysis of cracks which approach a boundary between two dissimilar solids is a difficult problem. Various solutions of this problem has been presented in the literature [4,5,6], but implementation of these solutions in damage analysis of fibrous composites is often difficult, as discussed in Section 2.2 above. Therefore, an approximate approach to this problem has been developed in the course of this research program.

The new approach is based on a normal image crack method (ICM), in which the effect of a boundary between two solids on the stress field of an approaching crack is represented by superposition of several simple crack solutions in the two solids. A complete description of the method is beyond the scope of this report, but several examples of problems that can be solved in this way are presented.

So far, the image crack method has been fully developed for anti-plane shear crack problems in homogeneous solids. Solutions have been obtained for one or more cracks perpendicular to the boundary of two finite or infinite solid layers, and for one or more crack in a layer bonded to two layers of different materials. These results have been compared with available exact solutions.

Fig. 12 shows results of a simple case of a shear crack approaching a boundary of two infinite solids. Fig. 13 shows results for a crack in a layer which is bonded to two half-planes of another solid. In each case we present a comparison of normalized stress intensity factors of the two crack tips, as functions of the distance of the crack from the

boundary, crack size, and the ratio of the shear moduli of the two solids.

Fig. 14 shows the result for a crack in a layer bonded to two different solids. No solutions of this problem have apparently appeared in the literature.

Many results of this type have been obtained, all compare very well with exact solutions to the extent that such solutions are actually available. We have also resolved the problem of many adjacent cracks in a layer bonded to two layers of finite thickness; again this problem has not been resolved by any other method.

Applications of the image crack method to other than antiplane shear problems in isotropic solids require much further work, which we hope to pursue in the future.

#### 2.5 Analysis of Fatigue Damage in B/A<sub>2</sub> Laminates

Initial work on this problem was described in our 1986 progress report [16] Section 2.1. A complete description of the analysis is now available in Chapter 9 of Ref. [17], which is enclosed with this report.

#### 2.6 Work in Progress

In addition to the results described herein, several research subjects are now in progress.

One of these projects is work on stress analysis of cracks in a anisotropic composite layer which is bonded to two adjacent anisotropic layers of any orientation. This is a general problem in damage analysis of cracked laminates. We are developing an approximate solution of this

problem which is expected to describe the essential interaction between many cracks in a composite ply of any orientation in a laminate.

Another project in progress is an experimental investigation of transverse cracking in glass-epoxy composite tubes. The tubes are of the  $(0_n/90_m)_s$  layup, about 3 in. in diameter. Testing is performed in a tension-torsion closed-loop machine. The tubes are loaded by combined axial stress and shear stress. The results include observations of crack growth, measurement of crack density, and evaluation of stiffness changes caused by cracks.

We expect to describe the results of these experiments in our next annual report.

#### ACKNOWLEDGEMENT

This work was monitored by Major George Haritos who provided encouragement and useful technical suggestions.

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FIGURES

- Figure 1 Longitudinal Young's modulus of Various T300/5208 systems:  
(a) bound ——— (b) self-consistent method - - - - -  
(c) differential scheme . . . . .
- Figure 2 Longitudinal shear modulus of Various T300/5208 systems:  
(a) bound ——— (b) self-consistent method - - - - -  
(c) differential scheme . . . . .
- Figure 3 Ratio of energy release rates for various T300/5208 systems according to self-consistent method.
- Figure 4 Energy release rate factor for various T300/5208 systems:  
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(a) self-consistent method ——— (b) differential scheme . . . . .
- Figure 6 Stress intensity factor of cross ply laminates.
- Figure 7 Values of J-integrals for laminates of different transverse ply thickness.
- Figure 8 Experimental and theoretical values for stiffness loss of  $(0/90)_S$  E-glass epoxy laminate: (i) Highsmith-Reifsnider prediction . . . . . (ii) shear lag ——— (iii) lower bound - - - - . Experimental data from reference [9].
- Figure 9 Progressive cracking in  $(0/90)_S$  E-glass epoxy laminates. Data from reference [9]. Predictions obtained from probability distortion 3 for indicated values of  $G_c$ .
- Figure 10 Theory versus experiment for progressive cracking of AS-3501-06 laminates. Data from Wang [13].
- Figure 11 Theory versus experiment for progressive cracking of T300/934 laminates. Data from Wang [13].
- Figure 12 Comparison of stress intensity factor computed by image-crack method and exact solution for a shear crack approaching a boundary of two infinite solids.
- Figure 13 Comparison of stress intensity factor computed by image-crack method and exact solution for a shear crack in a layer bonded to two half-planes of another solid.
- Figure 14 Stress intensity factor computed by image-crack method for a shear crack in a layer bonded to two different solids.

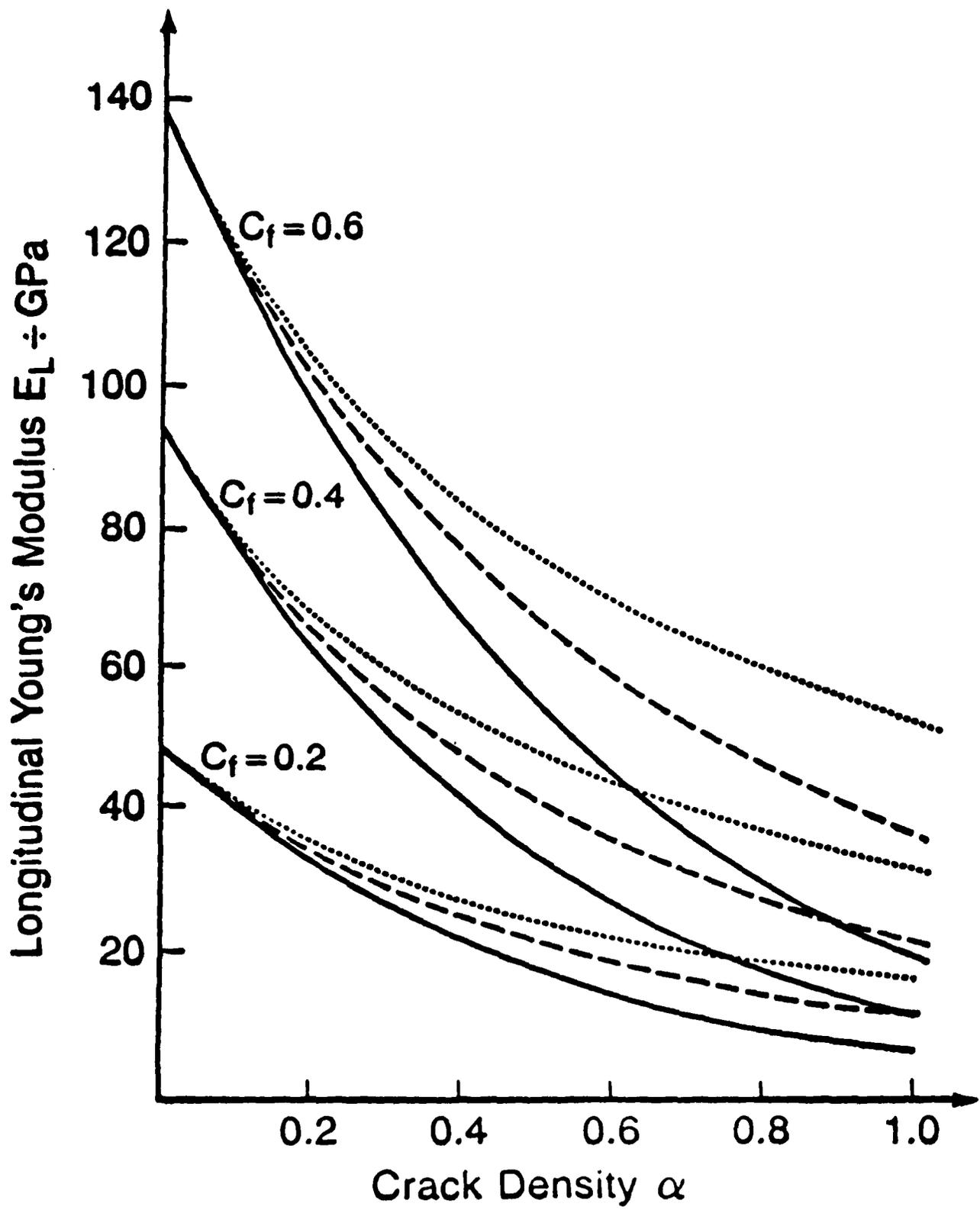


Figure 1 Longitudinal Young's modulus of Various T300/5208 systems:  
 (a) bound ——— (b) self-consistent method - - - - -  
 (c) differential scheme . . . . .

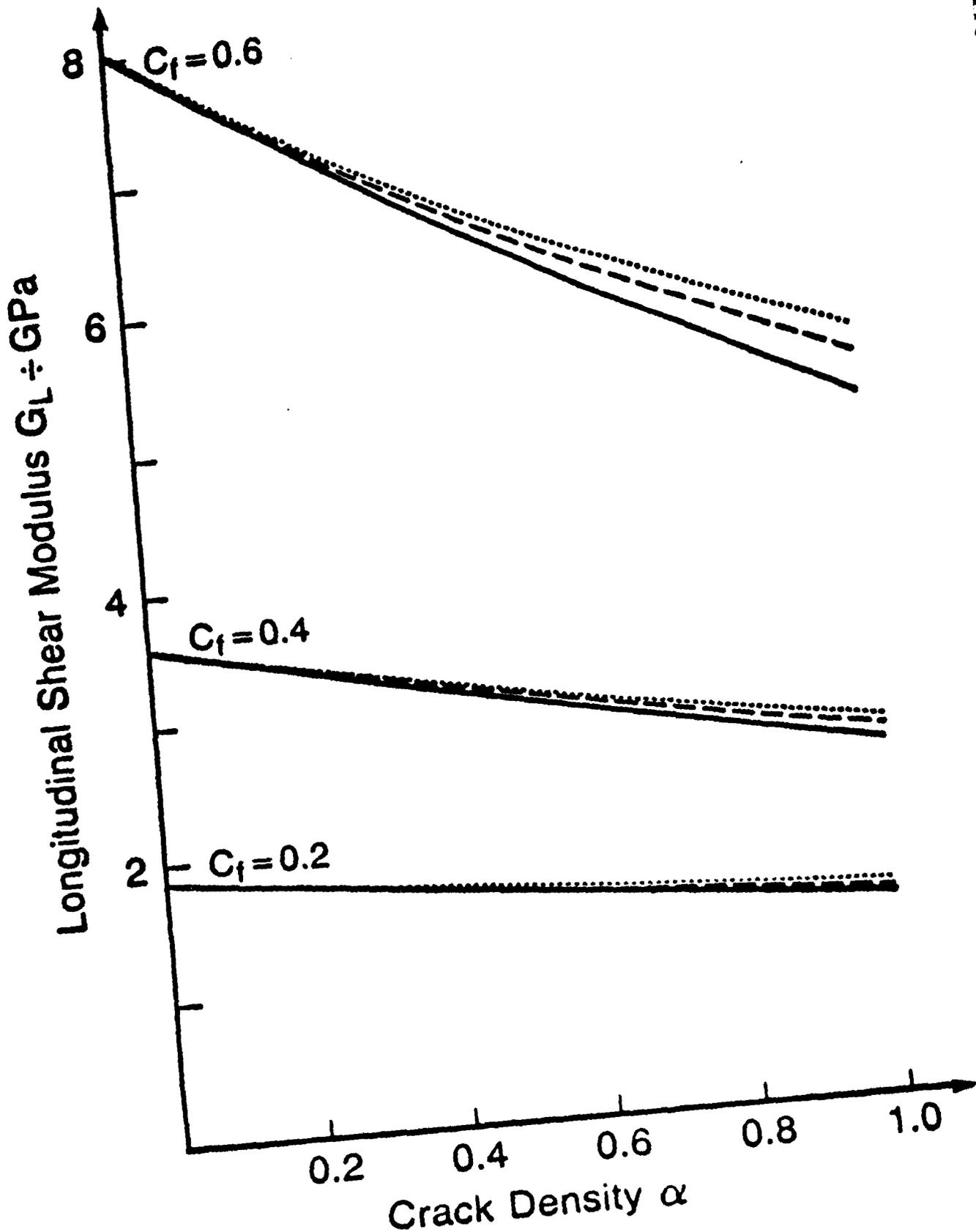


Figure 2 Longitudinal shear modulus of Various T300/5208 systems:  
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 (c) differential scheme . . . . .

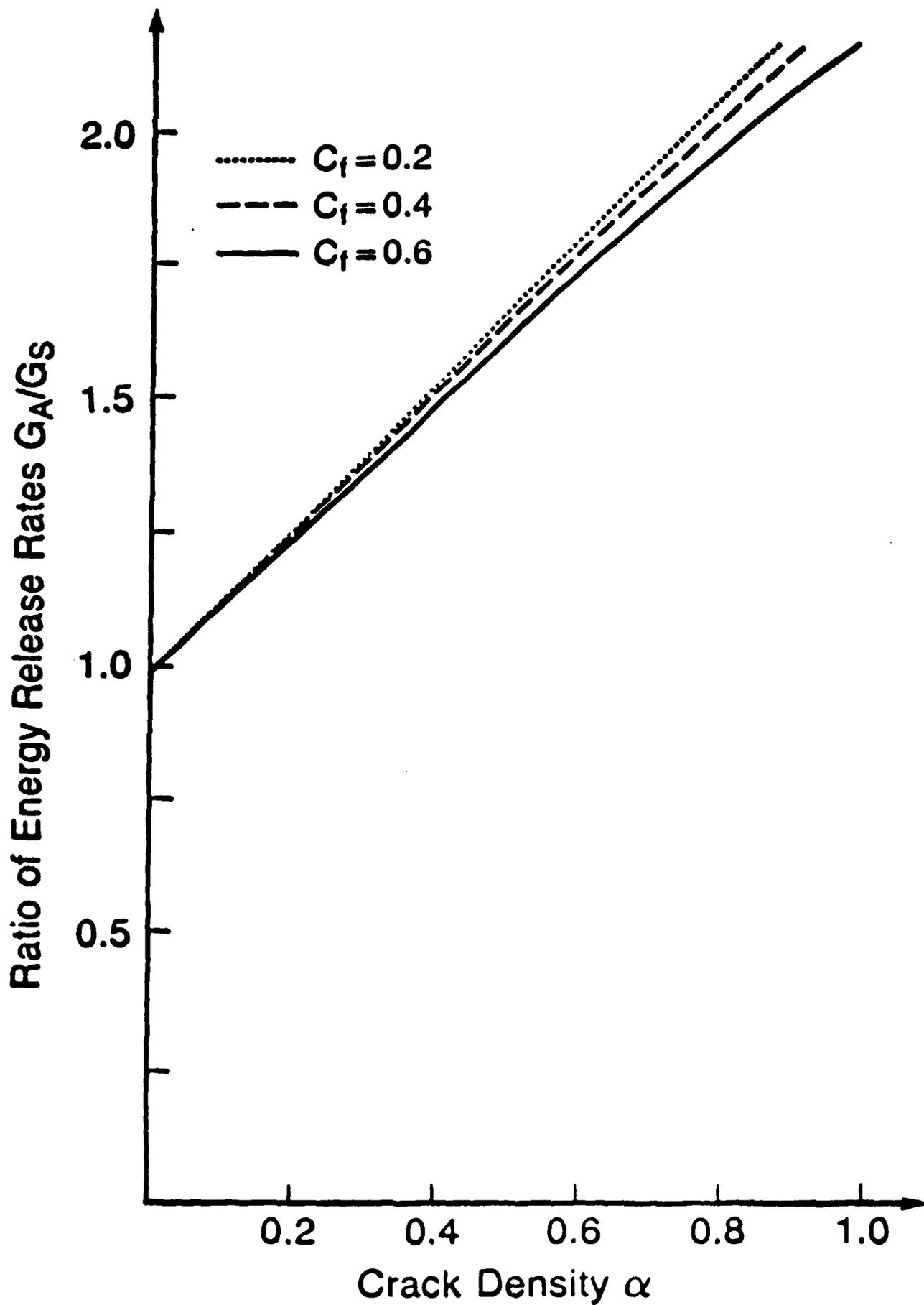


Figure 3 Ratio of energy release rates for various T300/5208 systems according to self-consistent method.

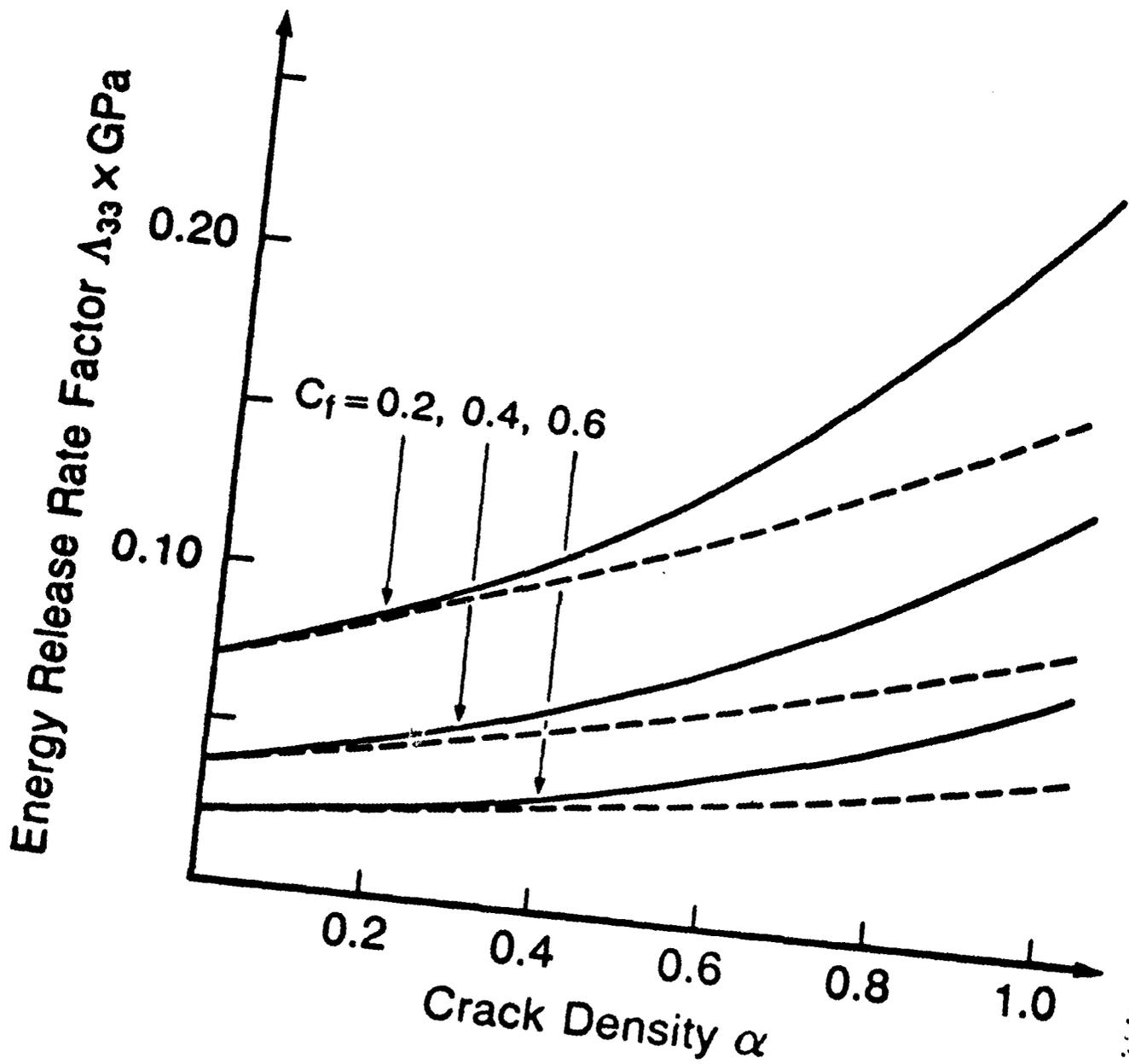


Figure 4 Energy release rate factor for various T300/5208 systems:  
 (a) self-consistent method — (b) differential  
 scheme . . . . .

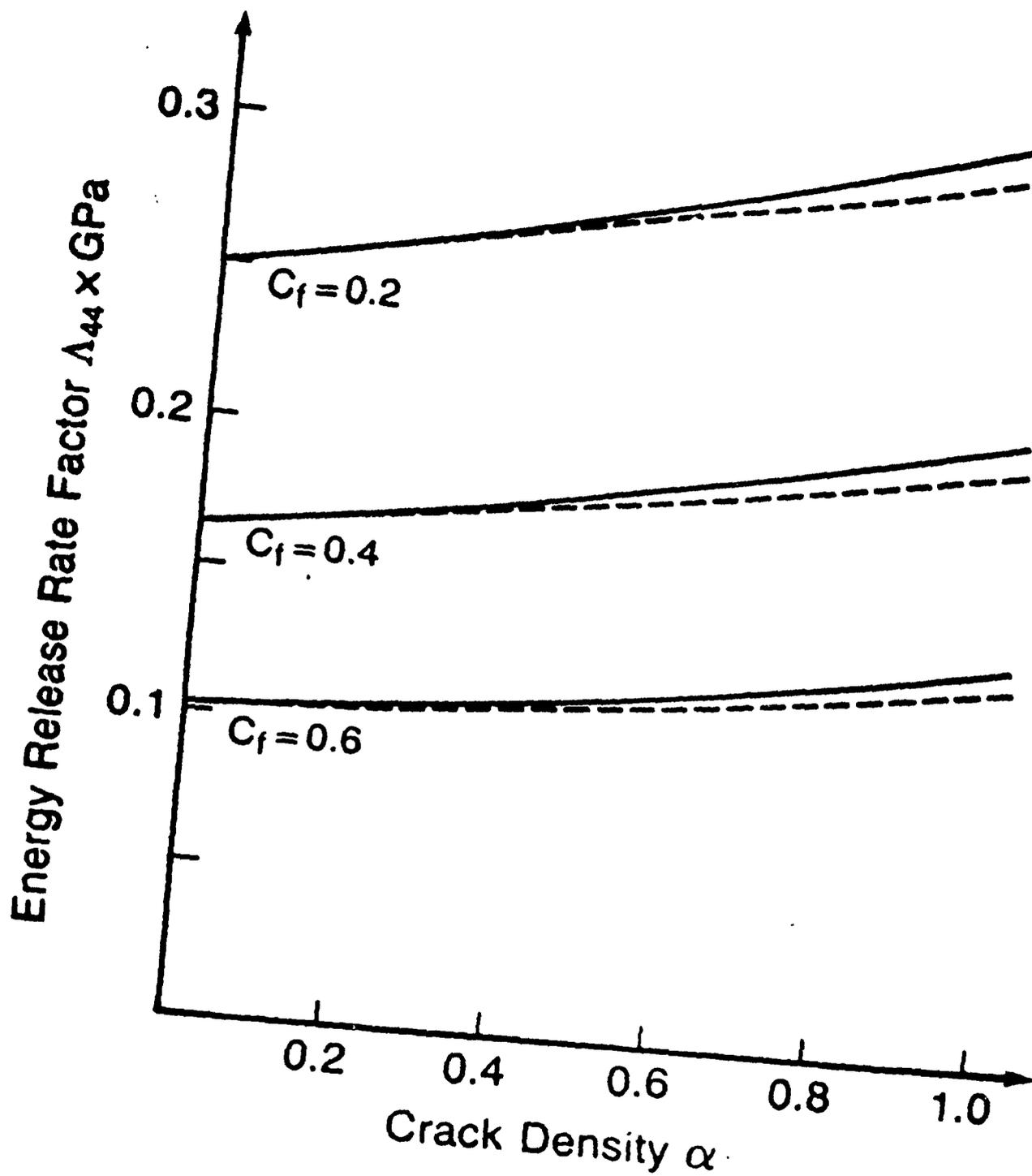


Figure 5 Energy release rate factor for various T300/5208 systems:  
 (a) self-consistent method — (b) differential  
 scheme . . . . .

# STRESS INTENSITY FACTOR of $[0/90/0]$ Laminates first set of materials

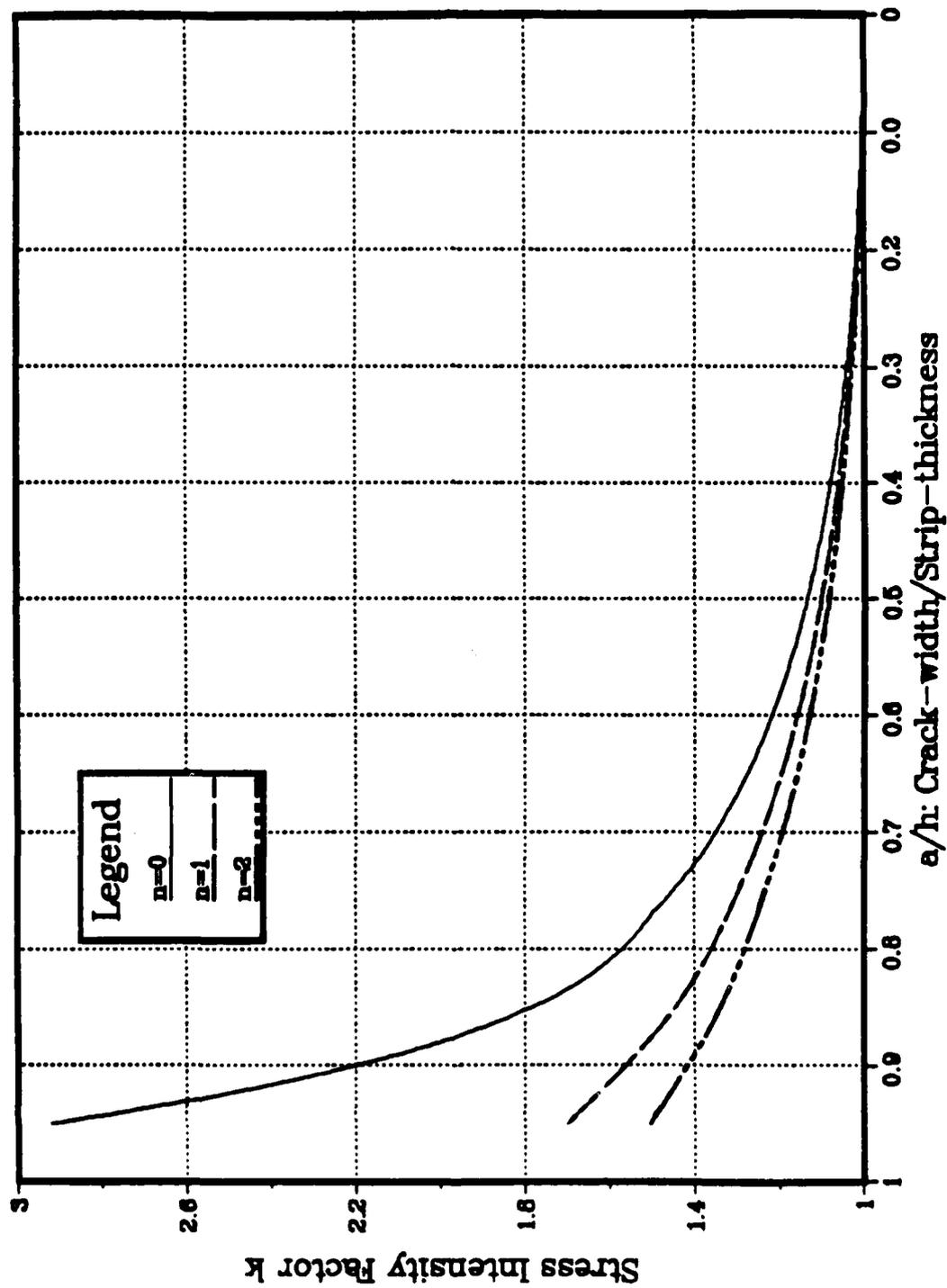


Figure 6 Stress intensity factor of cross ply laminates.

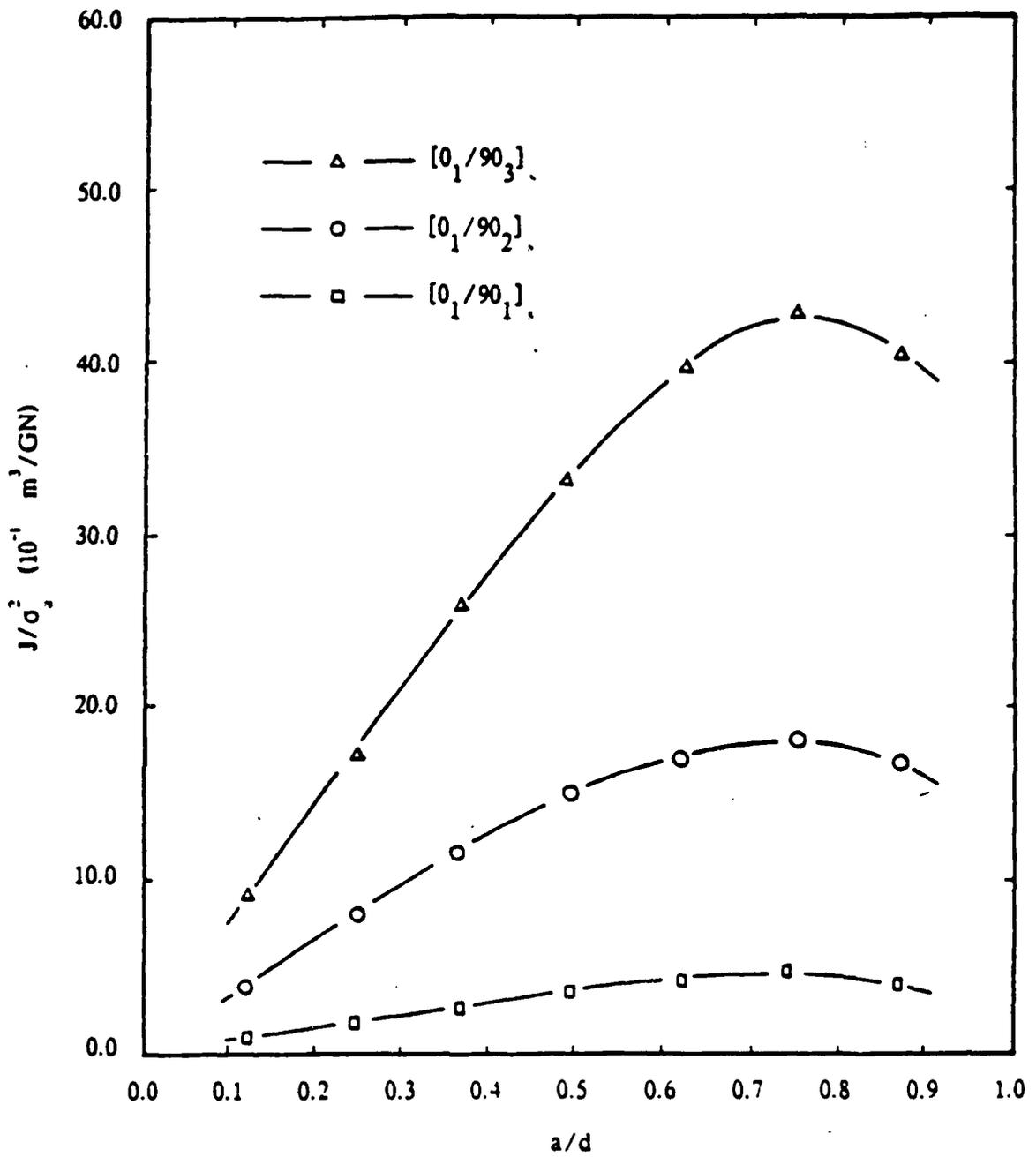


Figure 7 Values of J-integrals for laminates of different transverse ply thickness.

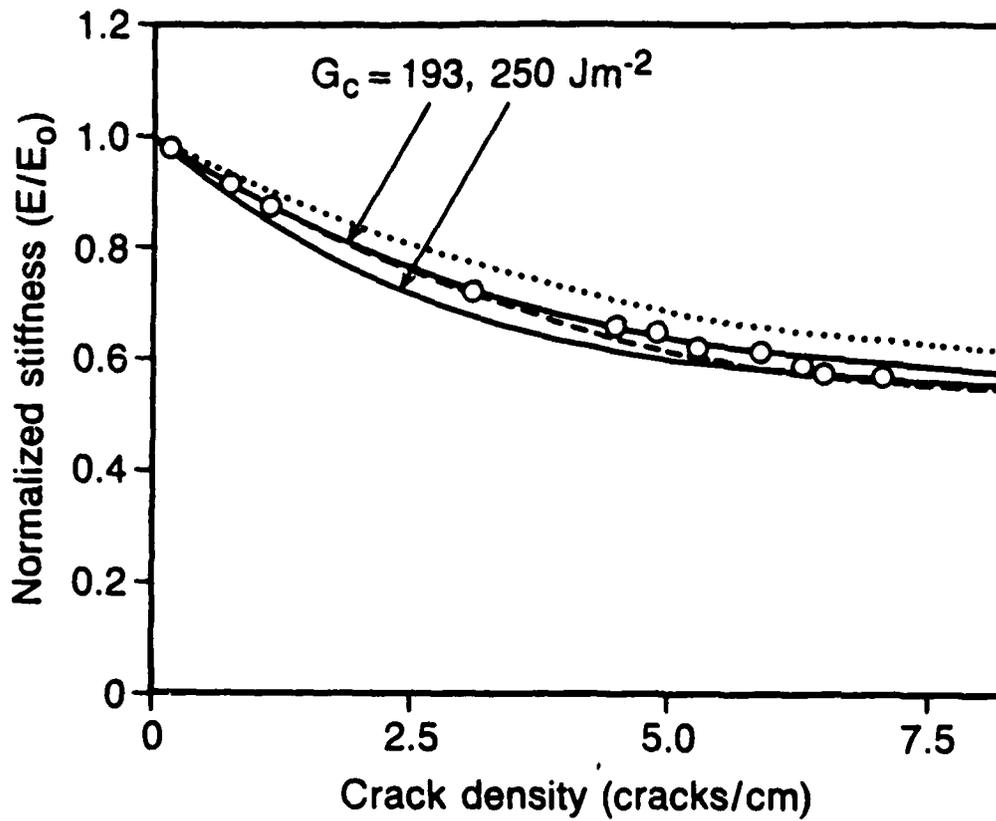


Figure 8 Experimental and theoretical values for stiffness loss of  $(0/90)_3$  E-glass epoxy laminate: (i) Highsmith-Reifsnider prediction . . . . . (ii) shear lag ——— (iii) lower bound - - - - . Experimental data from reference [9].

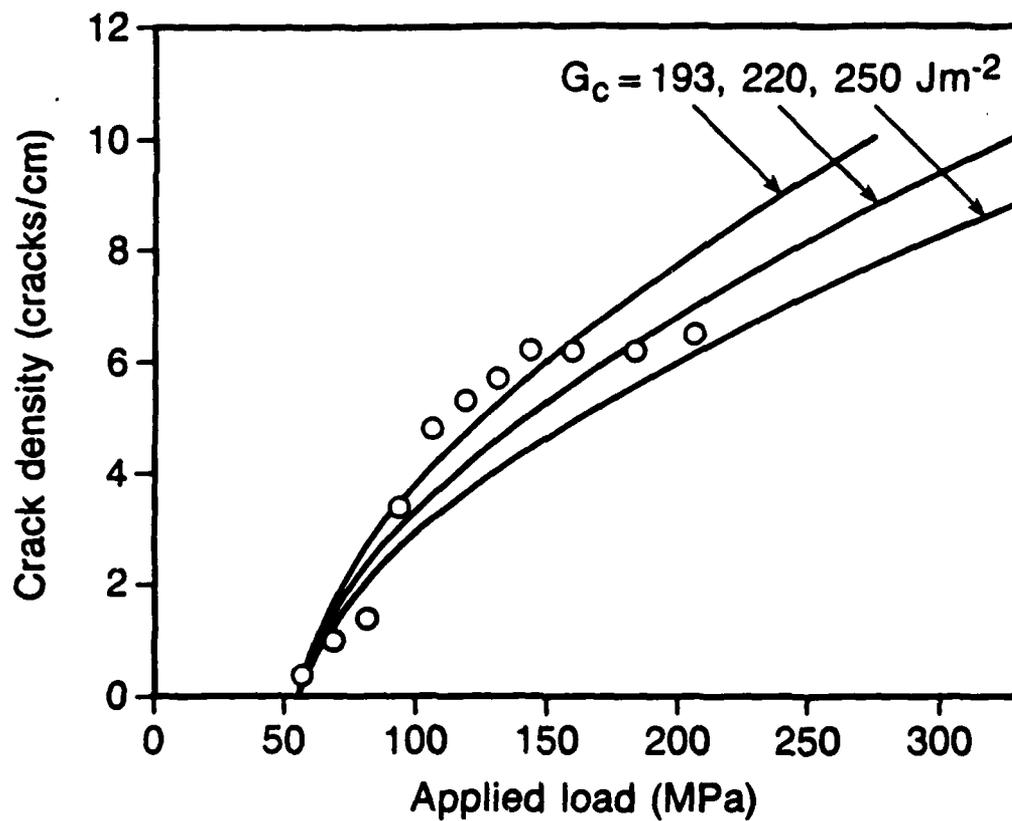


Figure 9 Progressive cracking in  $(0/90)_3$  E-glass epoxy laminates. Data from reference [9]. Predictions obtained from probability distortion 3 for indicated values of  $G_c$ .

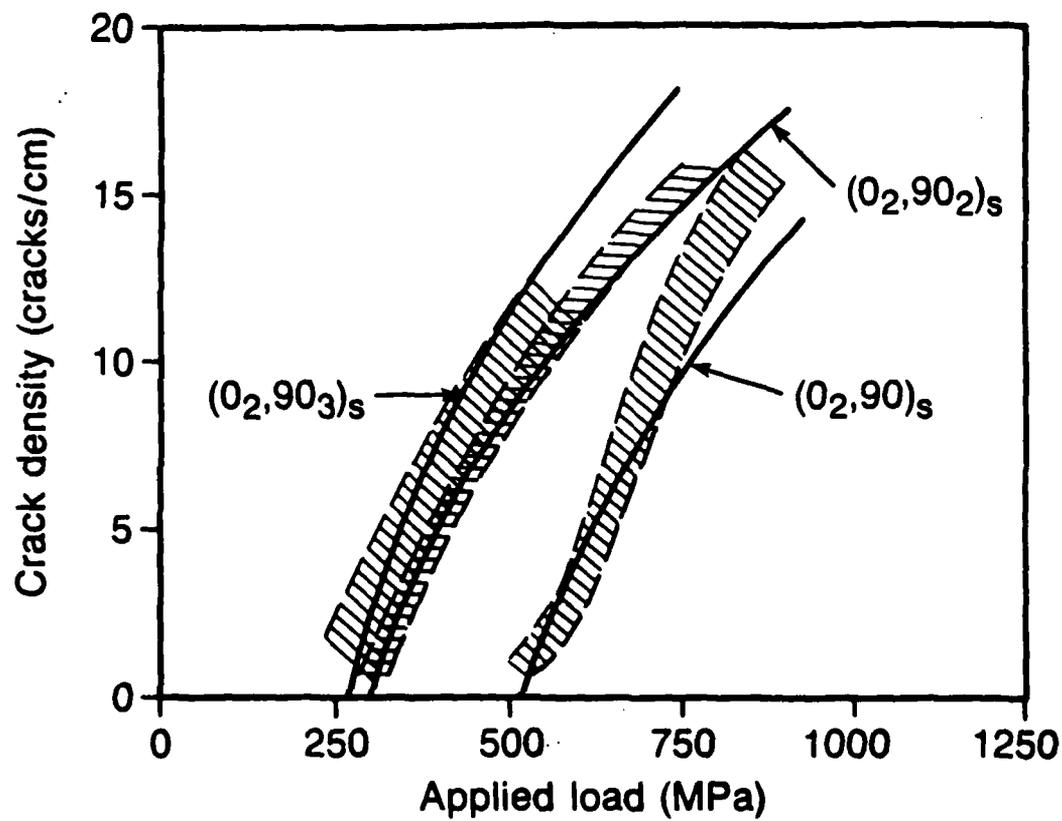


Figure 10 Theory versus experiment for progressive cracking of AS-3501-06 laminates. Data from Wang [13].

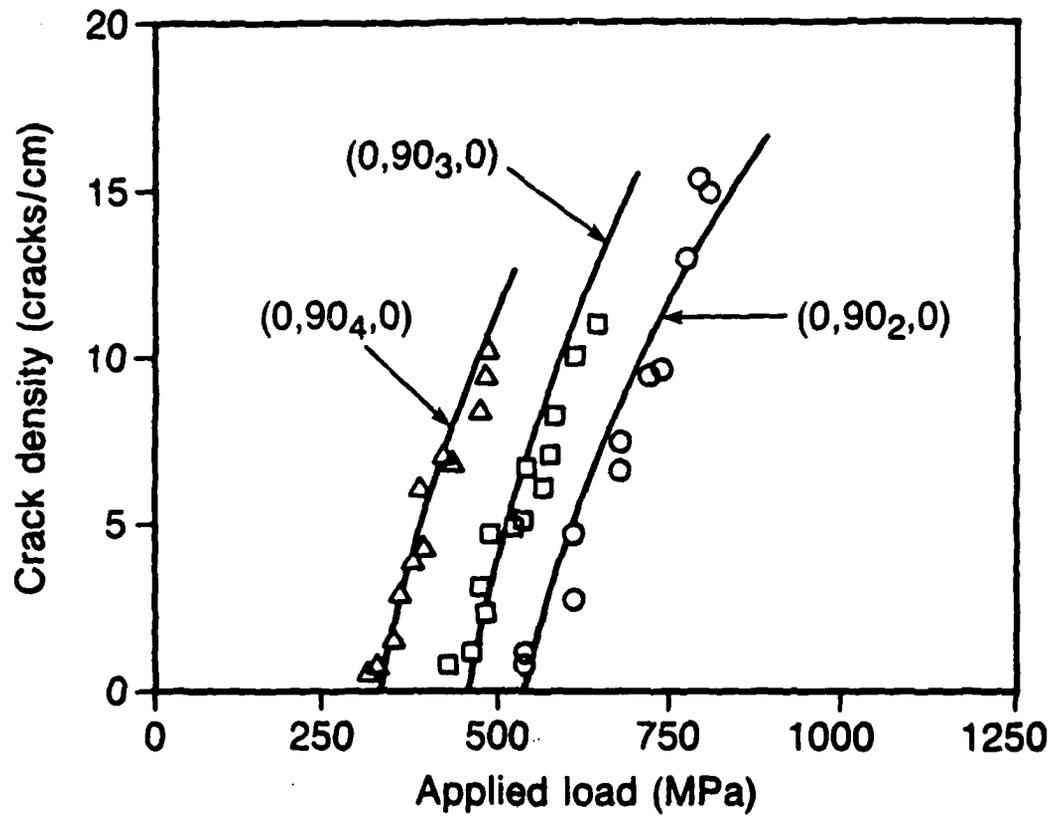


Figure 11 Theory versus experiment for progressive cracking of T300/934 laminates. Data from Wang [13].

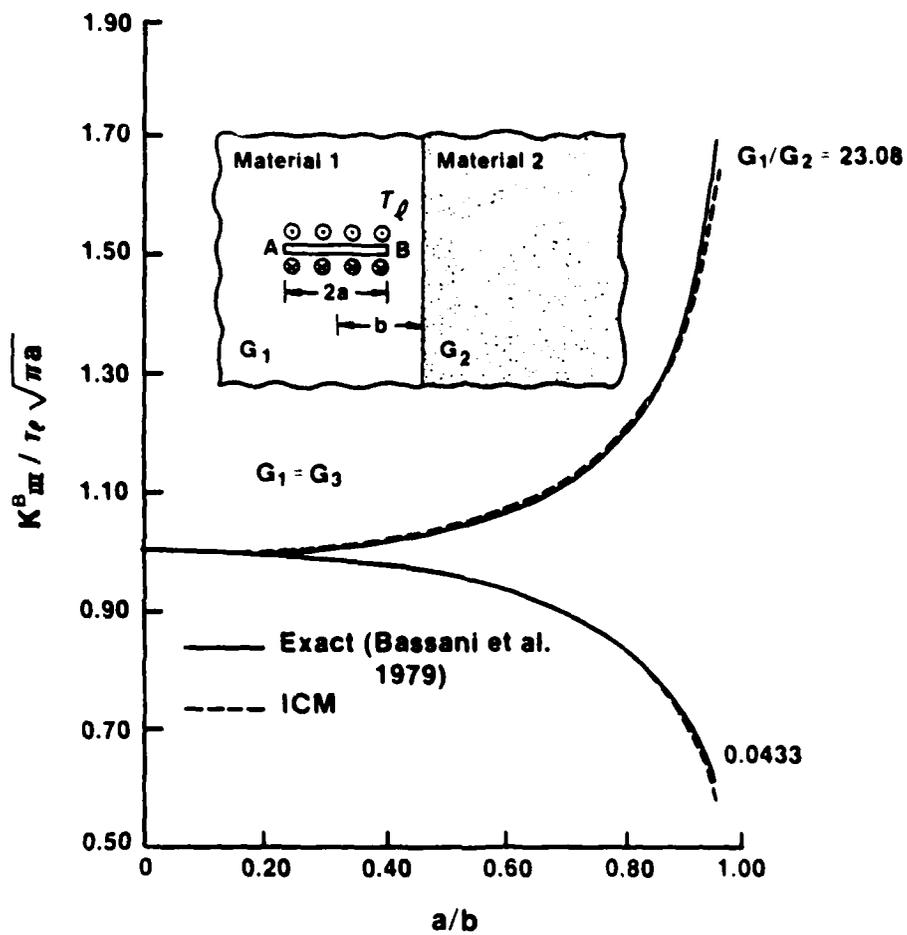


Figure 12 Comparison of stress intensity factor computed by image-crack method and exact solution for a shear crack approaching a boundary of two infinite solids.

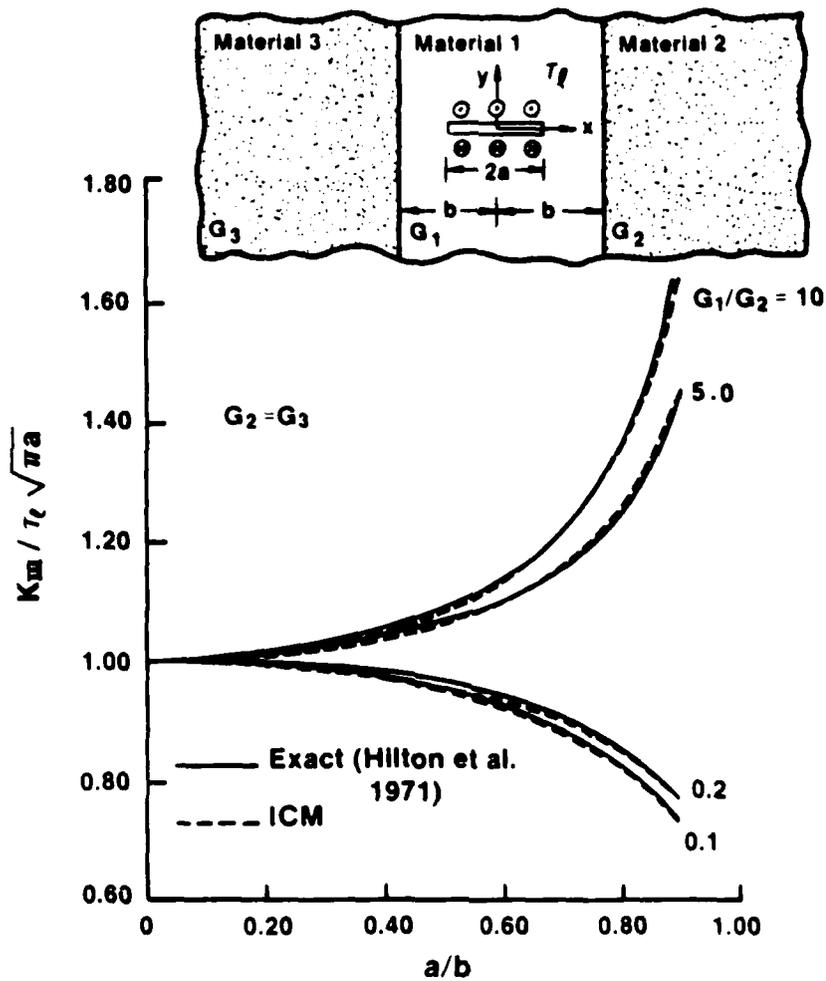


Figure 13 Comparison of stress intensity factor computed by image-crack method and exact solution for a shear crack in a layer bonded to two half-planes of another solid.

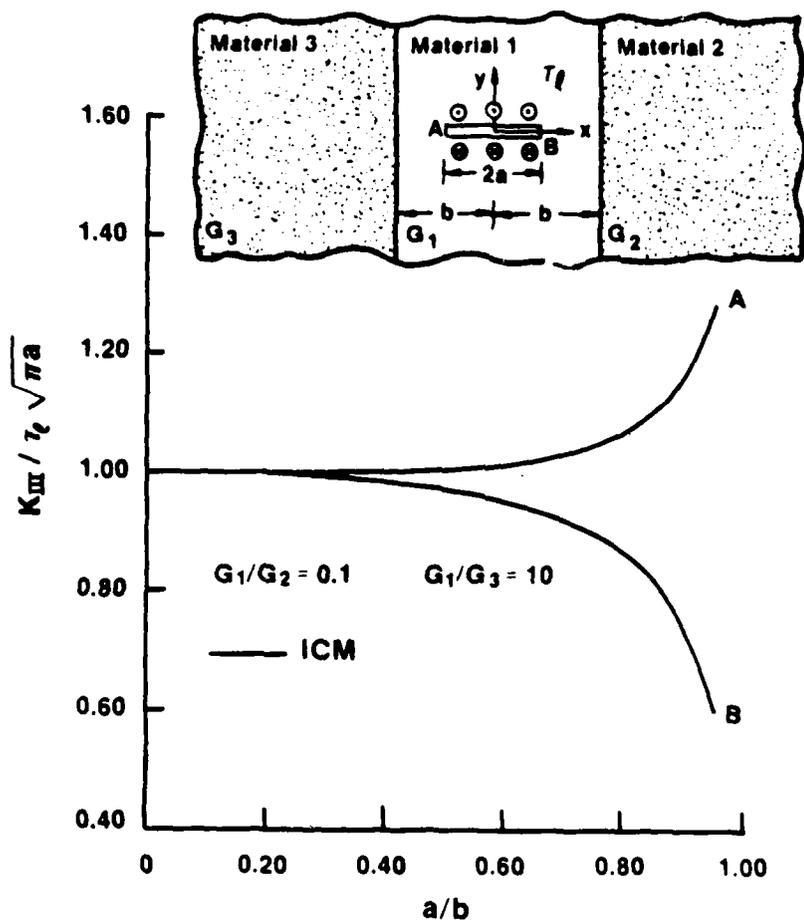


Figure 14 Stress intensity factor computed by image-crack method for a shear crack in a layer bonded to two different solids.

3. LIST OF PUBLICATIONS

1. Dvorak, C.J. and Laws, N., "Analysis of Progressive Matrix Cracking in Composite Laminates, II: First Ply Failure," J. Composite Materials 21 (1987) 309.
2. Laws, N., "A Note on Penny-Shaped Cracks in Transversely Isotropic Materials," Mechanics of Materials 4 (1985) 209.
3. Laws, N. and Dvorak, G.J., "The Effect of Fiber Breaks and Aligned Penny-Shaped Cracks on the Stiffness and Energy Release Rates in Unidirectional Composites," Int. J. Solids Structures (1987), to appear.
4. Laws, N. and Dvorak, G.J., "Analysis of Progressive Matrix Cracking in Composite Laminates, III: Stiffness Reduction and Crack Growth under Monotonic Loading," forthcoming.
5. Dvorak, G.J. and Laws, N., "Analysis of First Ply Failure in Composite Laminates," Engineering Fracture Mechanics 25 (1986) 763.
6. C.J. Wung, "Strain Space Analysis of Plasticity, Fracture, and Fatigue of Fibrous Composites," Ph.D. Dissertation, University of Utah, March 1987.

4. LIST OF PRESENTATIONS

G.J. Dvorak (September 1, 1985 - March 31, 1987)

Eleventh Annual Mechanics of Composites Review, Dayton, Ohio,  
October 22-24, 1985 (invited lecture).

Midwestern Mechanics Seminar:

University of Notre Dame, October 29, 1985.  
Illinois Institute of Technology, October 30, 1985.  
University of Illinois at Urbana-Champaign, October 31, 1985.  
Purdue University, November 1, 1985.

Midwestern Mechanics Seminar:

University of Michigan, April 9, 1986.  
University of Wisconsin, April 10, 1986.  
University of Minnesota, April 11, 1986.  
Michigan State, April 29, 1986.

Colloquium, Northwestern University, May 2, 1986. "Analysis of  
Fatigue Cracking of Fibrous Metal Matrix Laminates."

General Electric Company, Seminar, May 14, 1986.

ASME Winter Annual Meeting, invited lecturer, Anaheim, CA, December  
7-12, 1986.

ONR Workshop on Composite Materials - Interface Science, Leesburg,  
Virginia, March 11, 1987.

N. Laws (Sept. 1, 1985 - August 31, 1986)

Rensselaer Polytechnic Institute, February 1986

Texas A & M, March 1986

University of Houston, March 1986

University College, Dublin (Engineering Dept.), June 1986

University College, Dublin (Mathematical Physics), June 1986

University College, Cork, June 1986

5. LIST OF PROFESSIONAL PERSONNEL

Dr. G.J. Dvorak - Principal Investigator  
Dr. C.J. Wung - Research Associate  
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Mr. E. Martine - Graduate Student

Dr. N. Laws - Principal Investigator  
Mr. J.B. Wang, Graduate Student  
Mr. W. Li, Graduate Student

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