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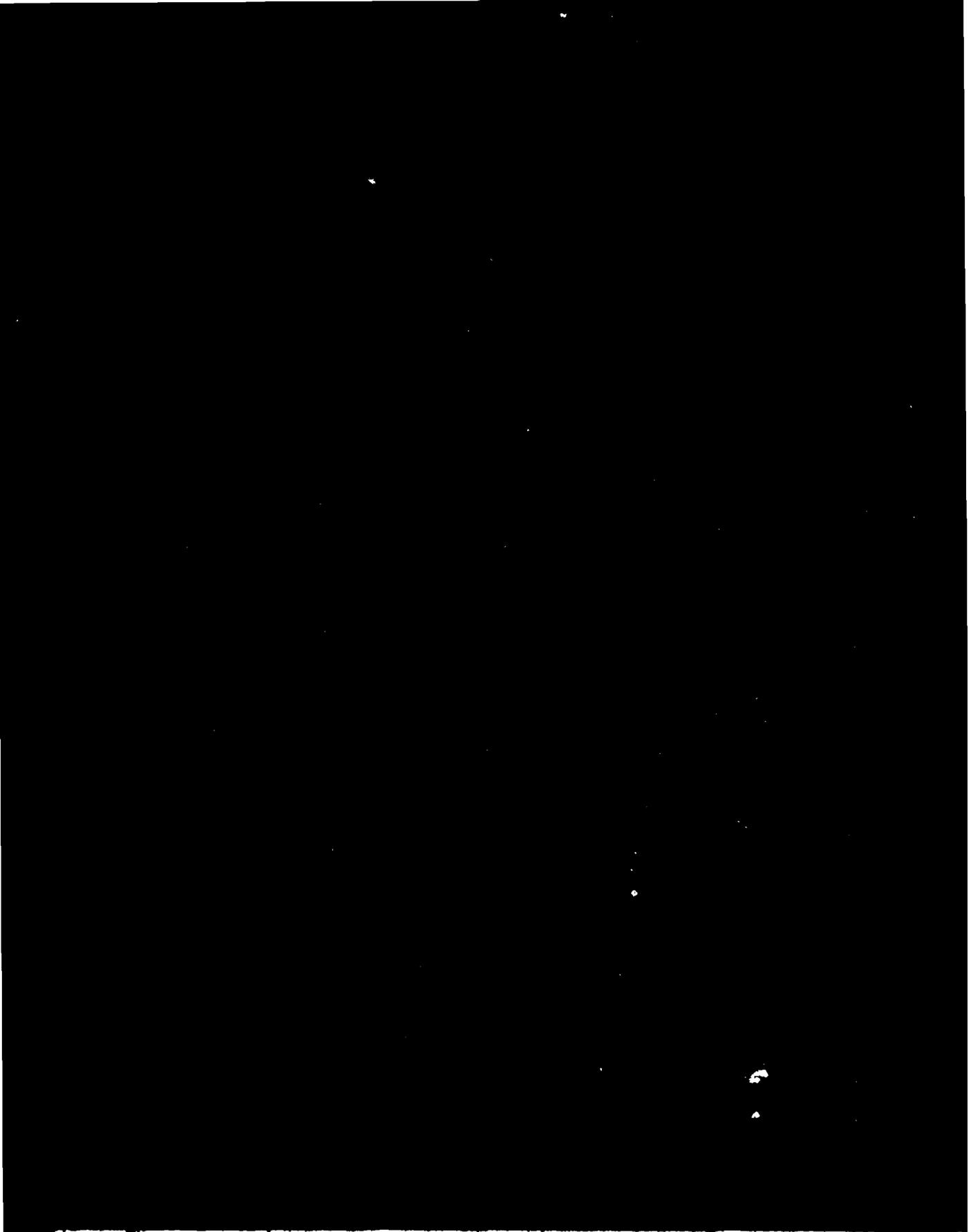
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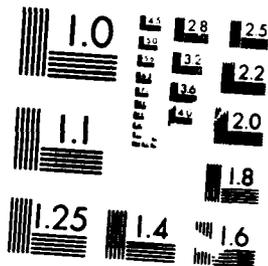
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Structures Technical Memorandum 469

APPLICATIONS OF ENERGY DENSITY  
THEORY IN CYCLIC PLASTICITY (D)

by

J. PAUL and L. MOLENT

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**APPLICATIONS OF ENERGY DENSITY  
THEORY IN CYCLIC PLASTICITY (U)**

by

J. Paul and L. Molent

**SUMMARY**

This paper discusses the application of energy density theory to a variety of problems related to fatigue life enhancement procedures, and on understanding the mechanisms which govern failure.



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## 1. INTRODUCTION

Fracture mechanics has developed in an attempt to understand the fracture of materials and to translate this understanding into structural failure prediction. Research workers in this field have searched for the fundamental quantities governing structural behaviour and thermomechanics. As a result present theories of fracture have been formulated in terms of stress, strain and energy. One approach which is extensively used is based on a critical energy density [1,2]. Advantages of energy density theory are that it may be applied to both notched and unnotched specimens [1-6], and can be used in both the elastic and the elasto-plastic regime. A detailed review of the development of energy density theory is presented in [3].

Energy density is not limited to metallic structures. It has been used to design the boron epoxy reinforcement applied to the upper surface of the Wing Pivot Fitting (WPF) of F-111C aircraft, in service with the RAAF [8], and to provide a fatigue analysis methodology for graphite epoxy structures [9].

The purpose of this report is to show how energy density theory can be used to understand the mechanisms governing failure. In a recent paper [7] it was shown that energy density theory is able to explain several phenomena associated with fatigue life enhancement techniques. In the problem considered it was demonstrated that during the loading/unloading process, the increment  $dW$  of the local energy density becomes out of phase with respect to the increment in the global work. This 'change in phase' was a necessary condition for fatigue life enhancement. However, it will be shown that depending on the subsequent load history this change in phase may be detrimental.

## 2. ENERGY DENSITY THEORY

The strain energy density criterion applies to all material regardless of its behaviour [1]. The minima and maxima of the strain energy density, denoted by  $dW/dV$ , and defined in terms of the stress  $\sigma_{ij}$  and strain  $\epsilon_{ij}$  by

$$\frac{dW}{dV} = \int_0^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}, \quad (1)$$

can be used to locate the failure sites corresponding to excessive dilatation and distortion, respectively, even when gross yielding takes place. In searching for the location of failure initiation, it suffices to obtain the local minima and/or maxima of  $dW/dV$ . This is accomplished by considering a system of local co-ordinate systems  $(x_j, y_j)$  ( $j = 1, 2, \dots, n$ ); see Figure 1. The local stationary values of  $dW/dV$  are then found by differentiating  $dW/dV$  with respect to the space variables  $r_j$  and  $\theta_j$ . Here  $r_j$  is fixed at a constant value  $r_0$  while  $dW/dV$  is assumed to vary only with  $\theta_j$ . For many practical problems failure occurs along the symmetry planes of load and specimen geometry and there is no need to compute  $dW/dV$  at every point in the continuum.

Failure under monotonic loading is thus assumed to initiate at the site where the local minimum of  $dW/dV$  is a maximum, i.e.,  $(dW/dV)_{\min}^{\max}$  and it occurs when

$$\left(\frac{dW}{dV}\right)_{\min}^{\max} = \left(\frac{dW}{dV}\right)_{\text{crit}} \quad (2)$$

If the load is applied repeatedly, the hysteresis energy density accumulated for each cycle is thought to control failure initiation. Let  $N$  designate the total number of cycles, then the fatigue life of the material element is said to be governed by the following equation.

$$\sum_{j=1}^N \left[ \left(\frac{\Delta W}{\Delta V}\right)_{\min}^{\max} \right]_j = \left(\frac{dW}{dV}\right)_{\text{crit}} \quad (3)$$

Equation (2) assumes failure to occur during the first load cycle and equation (3) after  $N$  cycles of loading.

### 3. APPLICATION TO FATIGUE ASSESSMENT

As stated in the introduction, energy density theory is able to explain several phenomena associated with fatigue life enhancement techniques. In particular it was shown that a phase difference occurs between the rate of change of the local energy density and the rate of change of the global work. In this paper it will be demonstrated that whilst this is a necessary condition for fatigue life enhancement it is not a sufficient condition. To illustrate this, let us consider the stress/strain field in front of a cold worked hole. In the problem considered in [7], it was shown that as the mandrel was removed from a cold worked hole the energy density changed phase with respect to the global work and during subsequent remote loading the energy density near the hole decreased. It is this decrease in energy density during remote loading that is thought to be responsible for the fatigue life enhancement.

It is interesting to note that this phenomenon is supported by the experimental results given in [11]. This reference presents the measured strain in front of a cold worked hole during the insertion and removal of the mandrel. The strains measured during removal of the mandrel are reproduced in Figures 2 and 3. The insertion and removal process is measured by the number of turns of the mandrel. Turn number 11 corresponds to the beginning of the removal process which has been completed at turn number 24. Here we see that both the hoop and radial strains change phase with respect to the global work, which in this case is decreasing as the mandrel is removed. This observation supports the phase change phenomenon outlined in [7].

There may be a class of problems, where the energy density goes out of phase with respect to the global work and then, during subsequent loading, increases at a greater rate than would have been expected. Such a behaviour would result in a decrease in fatigue life. Consequently a knowledge of the behaviour of the local energy density field during cyclic loading is essential in order to predict whether the loading mechanism, which produced the observed change in phase, is beneficial or detrimental to the fatigue life of the structure.

As an illustration of this philosophy let us consider the local stress strain behaviour in the stiffener runout of the F-111 Wing Pivot Fitting (WPF) during a Cold Proof Load Test (CPLT). As a result of cracks in the integral stiffeners under the upper surface of the WPF, the Aeronautical Research Laboratories (ARL) was

tasked to design a suitable reinforcement scheme. Reference [8] describes the design concept for a boron/epoxy doubler bonded to the upper surface of the WPF over the critical area.

As a result of this design study a detailed two dimensional finite element model of the unreinforced WPF was developed, (see [8] for details). A view of the critical section is shown in Figure 4.

This paper examines the behaviour of the local energy density, in the vicinity of the critical region, during the CPLT. An elastic-plastic analysis was conducted in which the model was subjected to incremental loading to a maximum load corresponding to 7.3 g and a minimum load corresponding to -2.4 g, (see [8] for details). At each load increment the energy density was calculated at the nodal points shown in Figure 5.

The results of this analysis are shown in Figures 6, 7, 8 and 9. It can be seen that, in each case, on unloading the energy density changes phase with respect to the global load. However, unlike the cold worked hole problem, the second load case, which corresponds to loading from 0 to -2.4 g, gives rise to an increase in energy density. The magnitude of this increase becomes significantly larger towards the re-entrant corner (node 267, see Figure 5). The energy density at node 267 is such that a small flaw at this location may cause the critical energy density to be exceeded. This explains the failures occurring in the CPLT.

As in the fatigue life enhancement procedure discussed in [7] the phase change in energy density is due to prior plasticity. However, unlike cold working the effect of this phase change is detrimental. This clearly shows that the phase change phenomenon report in [7] is a necessary but not a sufficient condition for fatigue life enhancement.

In fracture related problems, the observed phase change of the local energy density with respect to the global work means that current J integral methods and other extension of linear elastic fracture mechanics, which are based on global behaviour (i.e., load point displacements, etc), are invalid. Only in the cases where the global behaviour is closely related to the local crack tip behaviour can J be considered valid. Energy integrals which are more closely related to the behaviour of the material at the crack tip are discussed by Atluri [10].

#### 4. IMPLICATIONS FOR REPAIR

From Figures 6, 7, 8 and 9 it can be seen that the major contribution to the accumulated energy density field is due to the load sequence, 0 g : 7.3 g : 0 g. In order to lower the residual energy density obtained on unloading it is necessary to rework the local geometry of the stiffener runout. Repairing, by means of the bonded doubler outlined in [8], without reworking of the stiffener runout will merely lower the energy density that is added to the already very high residual energy density. The optimum solution is to both rework the stiffener runout and reinforce the WPF. A preload applied to the wing during the doubler application, may also be used to lower the residual energy density. It is recommended that the stiffener runout be reworked in conjunction with doubler application; see [8] for details.

## 5. CONCLUSION

This paper has shown how energy density theory can be used to understand a variety of problems associated with cyclic plasticity. In particular it has demonstrated that the 'phase change' phenomena discussed in [7] is not a sufficient condition for fatigue life enhancement.

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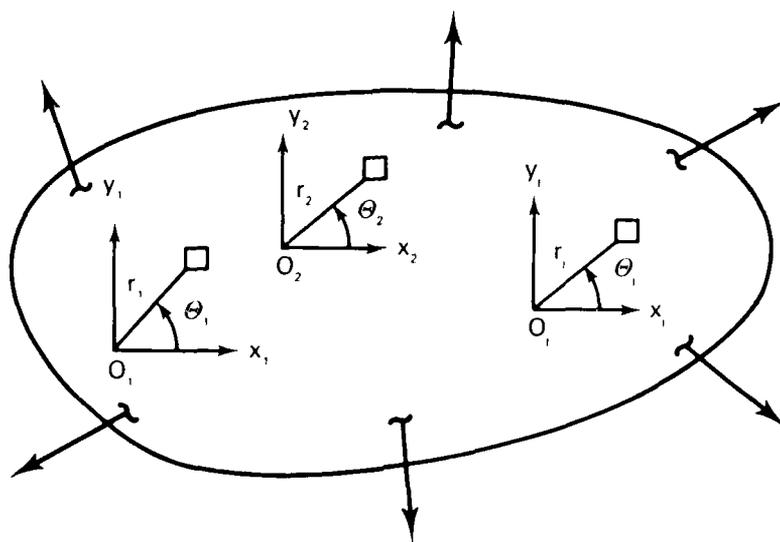


FIG. 1: LOCAL CO-ORDINATE SYSTEMS IN A CONTINUUM

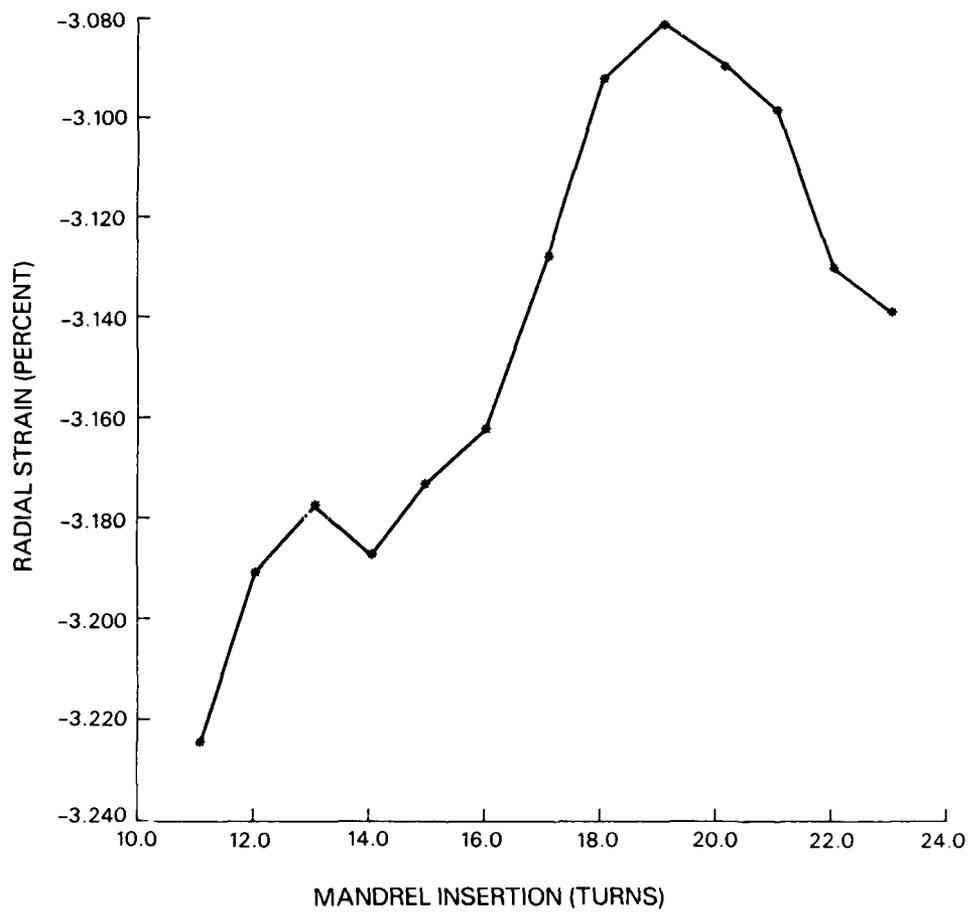


FIG. 2: RADIAL STRAIN DURING MANDREL REMOVAL AT 0.5MM FROM HOLE

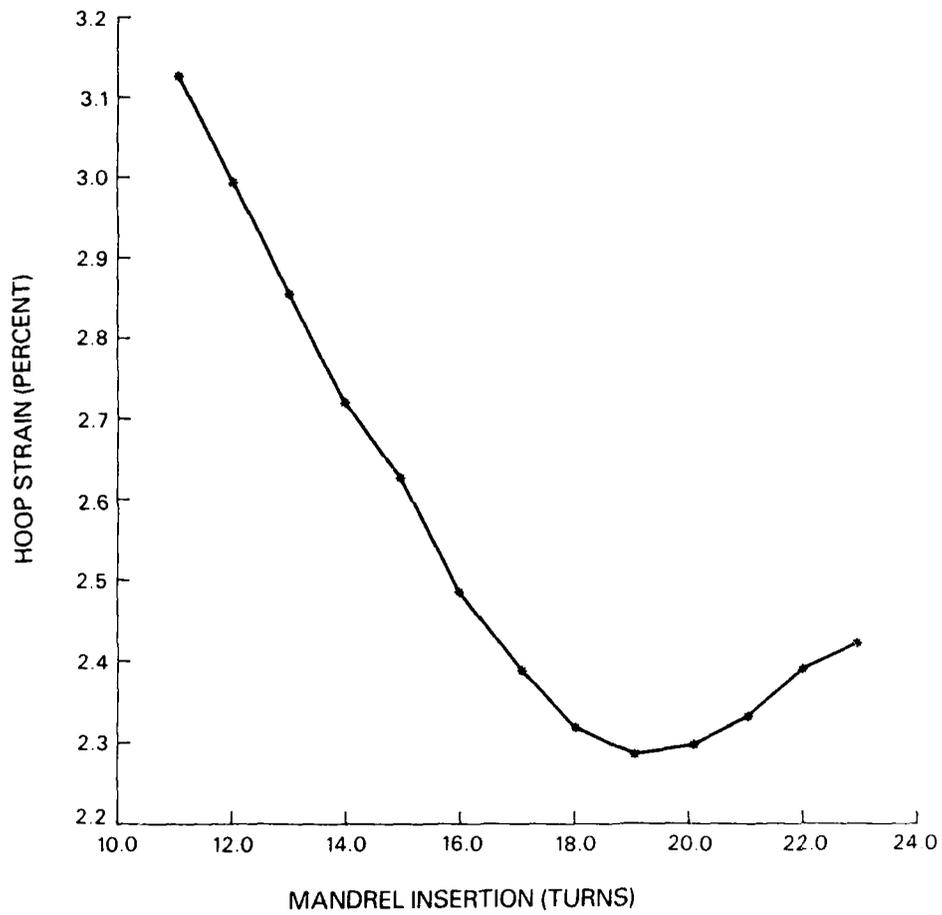


FIG. 3: HOOP STRAIN DURING MANDREL REMOVAL AT 0.5MM FROM HOLE

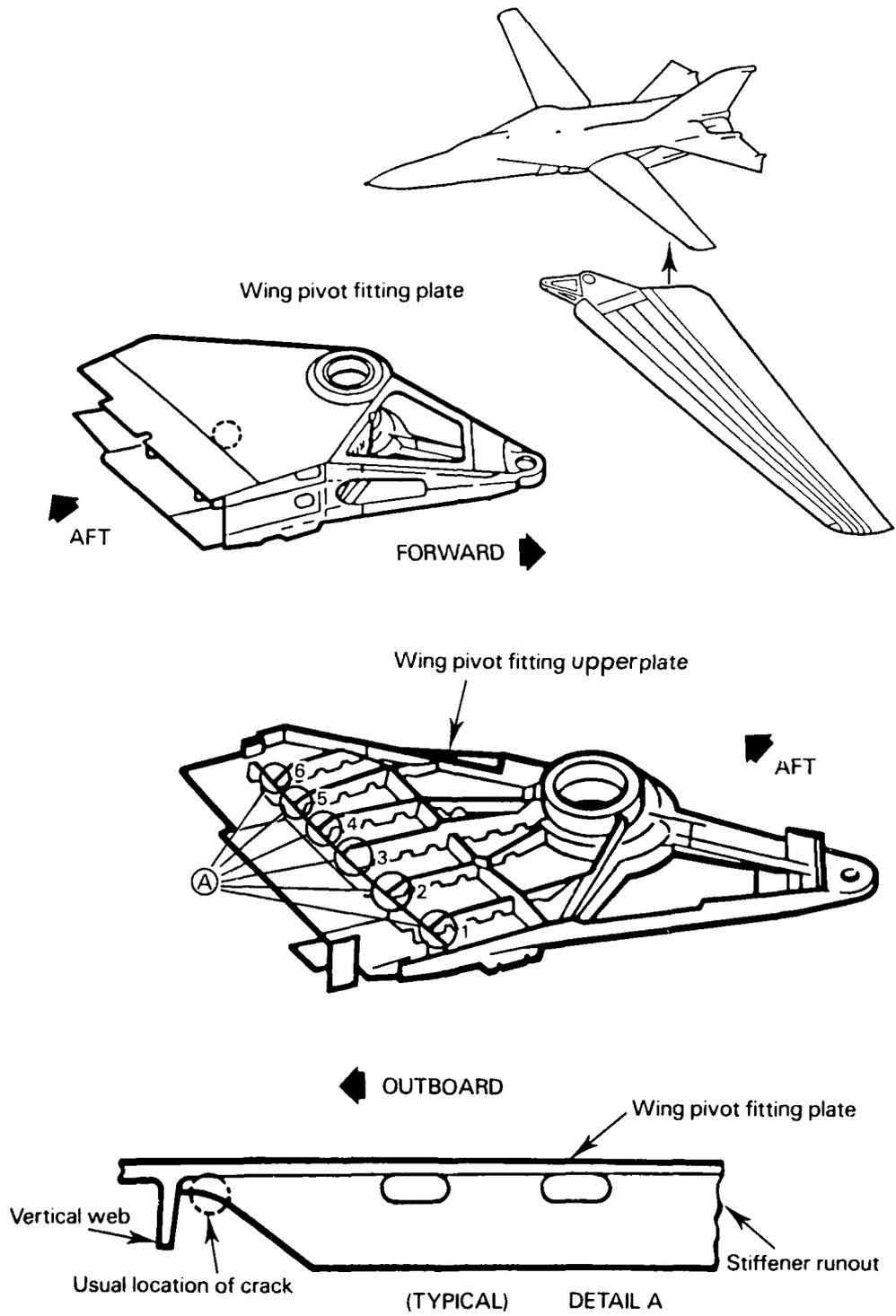


FIG. 4: F-111 WING PIVOT FITTING - LOCATION OF CRITICAL REGION OF STIFFENER RUNOUT SECTION

Stiffener runout region

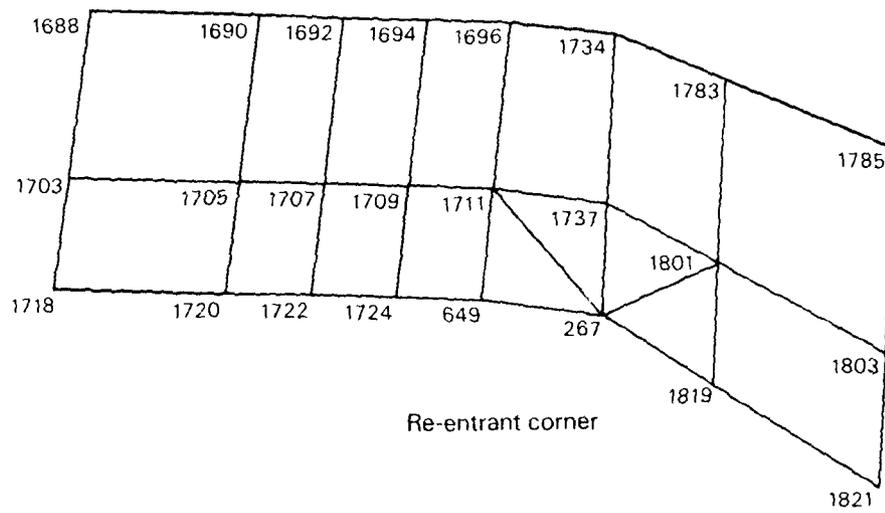


FIG. 5: CRITICAL REGION, FINITE ELEMENT MESH SHOWING NODE NUMBERS.

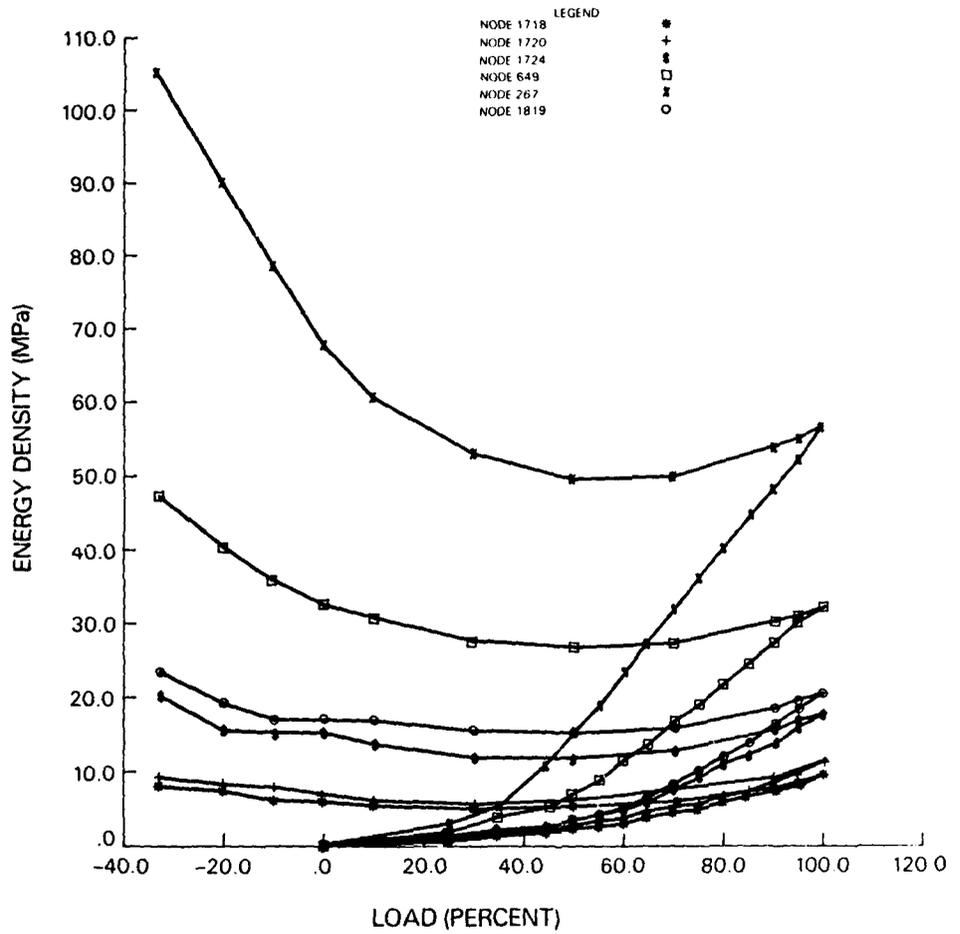


FIG. 6: PLOT OF ENERGY DENSITY VERSUS LOAD FOR NODES ALONG BOUNDARY OF THE STIFFENER

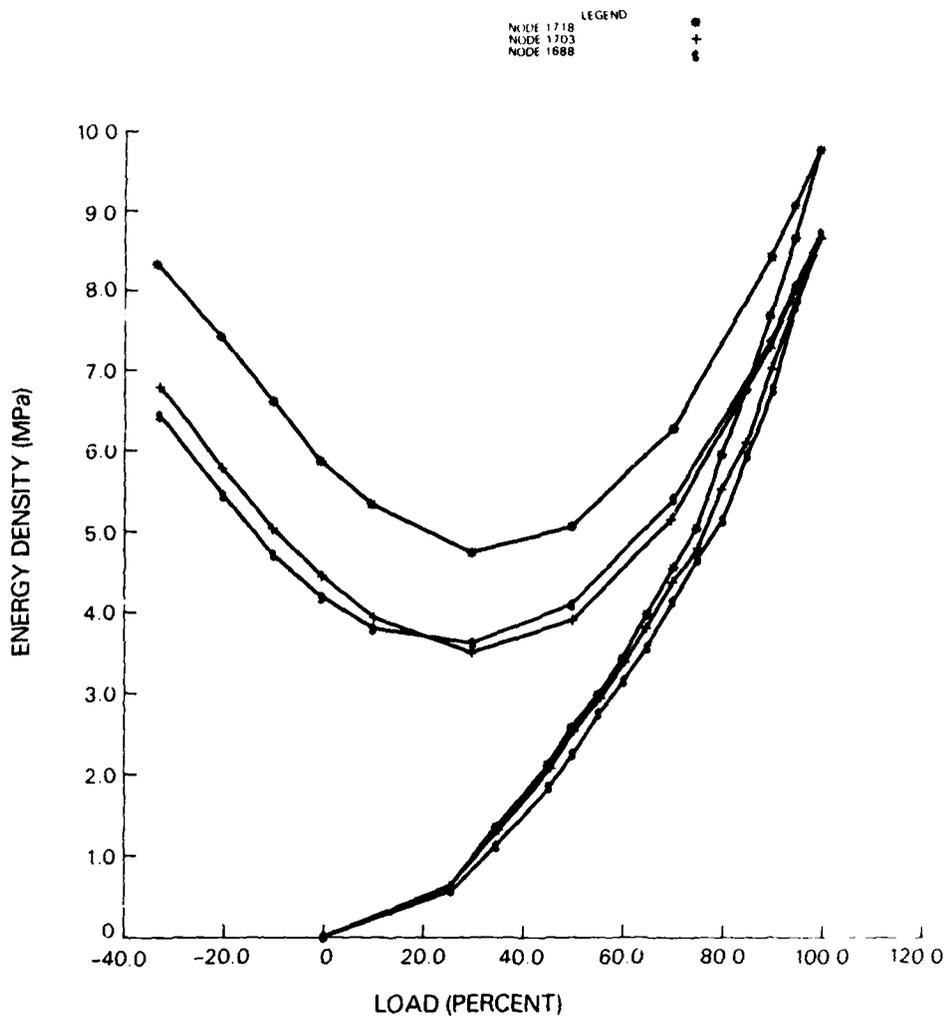


FIG. 7: PLOT OF ENERGY DENSITY VERSUS LOAD FOR NODES THROUGH THE DEPTH OF THE STIFFENER

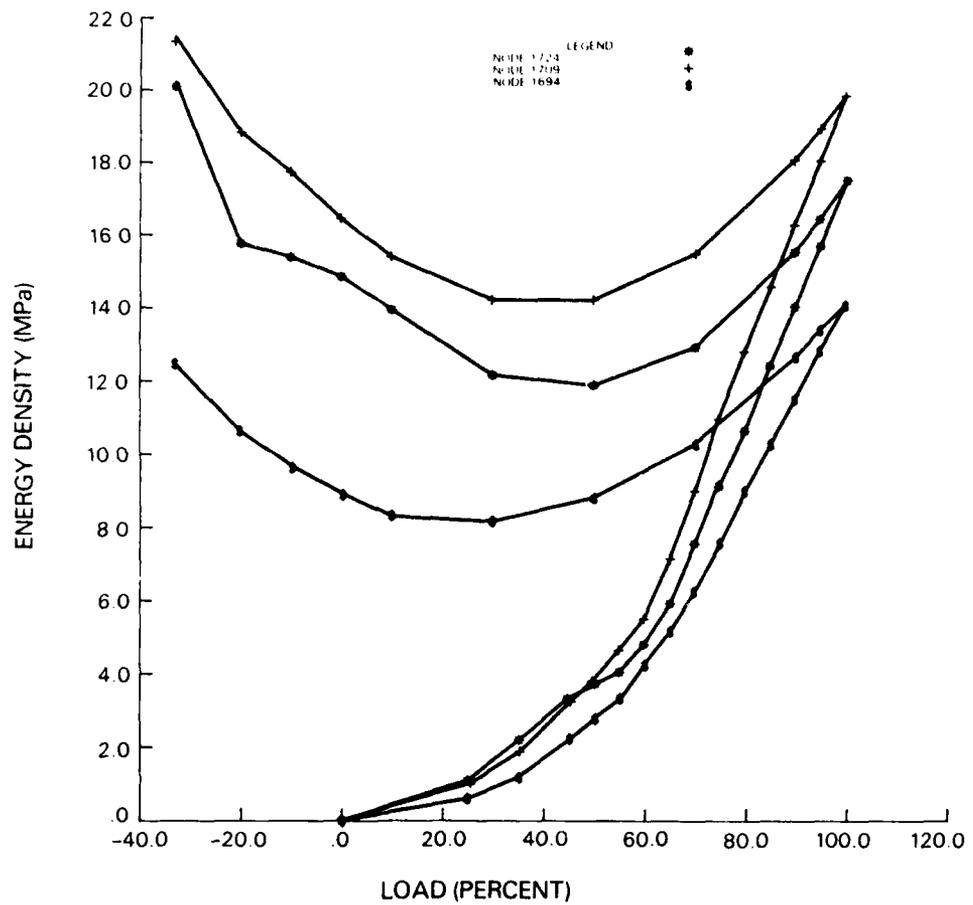


FIG. 8: PLOT OF ENERGY DENSITY VERSUS % LOAD FOR NODES THROUGH THE DEPTH OF THE STIFFENER

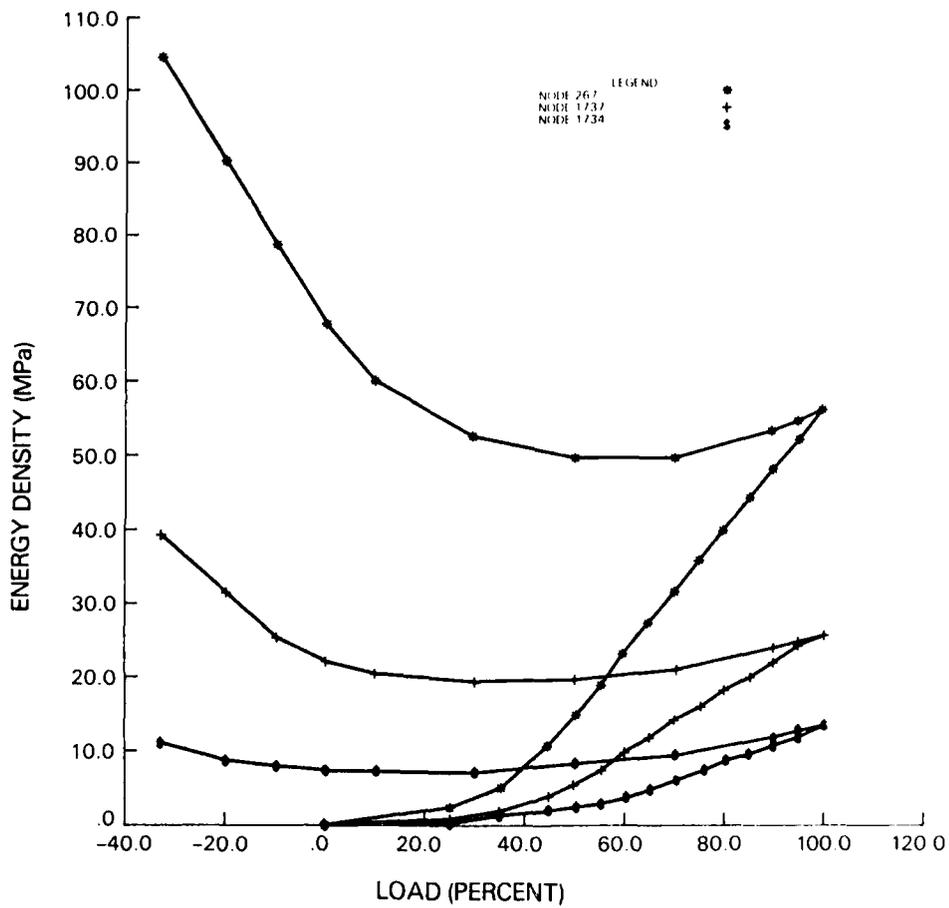


FIG. 9: PLOT OF ENERGY DENSITY VERSUS LOAD FOR NODES THROUGH THE DEPTH OF THE STIFFENER

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