RATIONAL CONSIDERATIONS FOR MODELLING HUMAN THERMOREGULATION DURING COLD WATER IMMERSION

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Rational Considerations for Modelling Human Thermoregulation During Cold Water Immersion

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Immeraion in cold water brings about large changes in body temperature and metabolism that add to the complexity of modelling human thermoregulation. Three specific problems peculiar to such modelling are examined; they are 1) finite-difference solution of the bioheat equation; 2) differences between predicted and measured initial conditions; and 3) prediction of convective heat loss. An optimization of the finite-difference solution of the simpler, but related, heat conduction problem is presented. A greater benefit is obtained by increasing the number of nodes rather than decreasing the integration time interval. A procedure is given for matching the predicted and measured initial values of the deep body temperature and the metabolic rate which allows a more accurate determination of set-point values for thermoregulation. To circumvent the acute sensitivity to the skin-water temperature difference using the conventional prediction of convective heat loss, use of a heat balance during steady-state of the skin temperature is outlined.
ABSTRACT

Immersion in cold water brings about large changes in body temperature and metabolism that add to the complexity of modelling human thermoregulation. Three specific problems peculiar to such modelling are examined; they are 1) finite-difference solution of the bioheat equation; 2) differences between predicted and measured initial conditions; and 3) prediction of convective heat loss. An optimization of the finite-difference solution of the simpler, but related, heat conduction problem is presented. A greater benefit is obtained by increasing the number of nodes rather than decreasing the integration time interval. A procedure is given for matching the predicted and measured initial values of the deep body temperature and the metabolic rate which allows a more accurate determination of set-point values for thermoregulation. To circumvent the acute sensitivity to the skin-water temperature difference using the conventional prediction of convective heat loss, use of a heat balance during steady-state of the skin temperature is outlined.

Keywords: heat stress; acclimatization; hypothermia; computer modeling;

environment heat transfer human factors
immersion modelling thermal response
Nude immersion in water colder than the body temperature represents an acute exposure to cold since convective heat loss is many times greater than in air. Temperature gradients become large and physiological responses are dramatic. These factors contribute to the complexity of mathematically modelling human thermoregulation for cold water immersion. Some recent attempts, though, have been made with reasonable success. These include multi-segment models that use physical representations of the human body, principles of heat conduction and feedback control mechanisms (Miller and Seagrave 1974; Montgomery 1974; Wissler 1985). In an effort to improve the predictive capability of such models, three specific problems will be addressed: 1) finite-difference solution of the bioheat equation; 2) differences between predicted and measured initial conditions, and 3) prediction of the convective heat loss.

The basic multi-segment thermoregulatory model can be separated into two distinct systems, the controlled system and the controlling system. The latter, which integrates thermoreceptor output signals from various body sites and determines the appropriate thermoregulatory response, will not be discussed (interested readers should refer to Stolwijk and Hardy 1966; Montgomery 1974; Hwang and Konz 1977; Arkin and Shitzer 1984; Wissler 1985). The controlled system considers the human body as a passive heat transfer system composed of segments representing the various components of the body (e.g., head, trunk, etc.). Each segment is further divided into compartments representing different tissue mass. Each compartment is usually assigned a unique set of physiological, thermophysical and geometric characteristics that determine its temperature through the bioheat equation (see Equation 1). Figure 1 shows the physical representation of the human body modelled by Stolwijk (1970). This model and its modification for cold water immersion (Montgomery 1974) were the basis for the present study.
Pennes (1948) outlined the bioheat equation to predict the steady-state temperature distribution of a compartment. In cold water immersion, most studies are concerned with the transient or unsteady temperature distribution. Analytic solutions of the unsteady temperature distribution (Shitzer 1985; Klinger 1985) are unwieldy for thermoregulatory modelling. To simplify the solution of the unsteady problem, finite difference methods are often applied. Although analyses have been conducted for specific conditions using this approach, no general optimization has been undertaken. The simpler, but related, heat conduction problem can be used to outline an approximate optimization scheme for using the finite-difference method of solution.

Initial conditions usually assume a deep core temperature ($T_c$) and a metabolic rate (MR) based on "standard" man in a state of thermal neutrality (i.e., zero heat storage). This makes subsequent comparison between predicted and measured values of these variables during immersion ambiguous since a person's pre-immersion values do not necessarily match "standard" man's, especially if the person is lean or fat. A procedure to match the initial values of $T_c$ and MR under a condition of thermal neutrality is outlined.

Conventional methods of predicting the convective heat loss are sensitive to the assumptions of body shape and water motion, and especially to the skin-water temperature difference when the skin temperature ($T_{sk}$) is near or at its steady-state value. An alternative approach which relies on the heat balance of the skin compartment is developed.

**Finite-Difference Method**

The bioheat equation expressed in cylindrical co-ordinates is

$$\rho c \frac{d}{dt} = \frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + (\rho c v)_{b1} \cdot (T_{b1} - T) = a$$

(1)
where \( r \) is the radial distance from the center, \( k \) is the thermal conductivity, \( T \) is the temperature, \( \rho \) is the density, \( c \) is the specific heat, \( m \) is the metabolic rate per unit volume, \( t \) is the time, and the subscript \( \text{bl} \) refers to the blood. Refinements such as separate heat exchange with the arterial and venous blood, tangential and angular temperature gradients, and counter-current heat exchange (Kusnetz 1979; Arkin and Shitzer 1984; Wissler 1985) are not considered here. To test the accuracy of the finite-difference solution of Equation 1, predictions are either compared to experimentally-determined values (Chan et al. 1973) or to exact solutions that are made possible by the selection of specific initial and boundary conditions. One such set to predict temperatures in living tissue was used by Gordon and Roemer (1975) and they found that errors were reduced by decreasing the integration time interval from 10 to 1 min, by increasing the number of nodes from 4 to 10, and by decreasing the nodal spacing for large temperature differences. For tissue characteristics and conditions different from the ones considered by Gordon and Roemer, a more general set of guidelines is desirable to optimize the numerical integration of Equation 1.

An approximate optimization scheme can be outlined by considering the simpler heat conduction problem with tissue-blood heat conduction and tissue metabolism removed so that an exact solution of the average tissue temperature (\( T_{\text{avg}} \)) can be found. \( T_{\text{avg}} \) serves as a good reference point for comparisons between the exact and finite-difference solutions of Equation 1 since the change in \( T_{\text{avg}} \) is directly related to the tissue heat storage which is the quantity that determines net heat exchange.

Consider the problem defined in Figure 2 where the initial tissue temperature has a uniform value \( a \) and the surface temperature (\( T_s \)) is fixed. The exact solution of \( T_{\text{avg}} \), normalized for dimensionless analysis, is given by (Carslaw and Jaeger 1959; Crank 1975)
\[ \frac{T_{\text{avg}} - T_s}{T_s} = 1 - \sum_{j=1}^{\infty} \left( \frac{2}{\alpha_j R} \right)^2 \exp \left( -K\alpha_j^2 t \right) \]  \hspace{1cm} (2)

where \( \alpha_j \) is the positive root of \( J_0 (\alpha_j R) \), \( J_0 \) is the Bessel function of the first kind of zero order, and \( K \) equals \( k/\rho c \). Values of Equation 2 are plotted against the dimensionless quantity \( Kt/R^2 \) in Figure 3.

The explicit finite-difference solution of Equation 1 using a three-point approximation is given by (Ozisik 1985); for the innermost nodal point \( n=0 \)

\[ T_0 (t + \Delta t) = T_0 (t) + \frac{K\Delta t}{q} (T_1 (t) - T_0 (t)) \] \hspace{1cm} (3a)

and for \( 1 \leq n \leq N-1 \)

\[ T_n (t + \Delta t) = T_n (t) + \left( \frac{K\Delta t}{q_{n+1}^2 - q_n^2} \right) \left( \begin{array}{c} r_{n+1} - r_n \\ r_n - r_{n-1} \end{array} \right) \left( \begin{array}{c} T_{n+1} (t) - T_n (t) \\ T_n (t) - T_{n-1} (t) \end{array} \right) \] \hspace{1cm} (3b)

where \( \Delta t \) is the integration time interval, \( r \) and \( q \) are radii defined in Figure 2, and the subscript refers to the nodal point. The outermost nodal point \( n=N \) temperature is fixed at \( T_s \). \( T_{\text{avg}} \) is determined by weighting the temperature of each node according to the thermal capacity of tissue around the node defined by the boundaries at \( q \). Values of normalized \( T_{\text{avg}} \) for 3, 4, 6 and 10 nodes spaced apart by equal volumes are shown in Figure 3. These values were obtained by choosing \( \Delta t = 10^{-4} R^2/K \) which allowed ten integration intervals prior to the earliest values shown.

Four factors contribute to the size of error using the finite-difference solution: they are 1) value of \( \Delta t \); 2) number of nodes; 3) time of prediction; and 4) nodal spacing. Decreasing \( \Delta t \) further than its present value led to improvements of less...
than 0.1%. Therefore, the size of error indicated in Figure 3 is only minimally attributed to the presence of Δt. Increasing Δt worsens the prediction but the extent depends upon the number of nodes used. As shown in Table 1, the prediction is increasingly sensitive to Δt as the number of nodes increase.

An infinite number of nodes will model the heat conduction problem exactly, therefore, increasing the number will logically decrease the error as shown in Figure 3. The minimum number of nodes required by the present finite-difference method is three, yet after a long period of time the error using three nodes becomes negligible. Therefore, the optimal choice of the number of nodes depends upon the acceptable size of error at a given time. This can be best illustrated by a practical example. Consider a cylindrical model compartments of radius R = 2 inch (5 cm) and a typical tissue conductivity K = 0.00018 inch^2/s (0.001 cm^2/s). If the error must not exceed 10% after 1 min (i.e., at Kt/R^2 = 0.0024), then ten nodes can be used as seen in Figure 3. If, however, this constraint is relaxed so that the same level of error must not be exceeded after 10 min, then four nodes can be used. Of course, this example is predicated on the use of Δt = 10^{-4} R^2/K, hence some adjustment may be necessary if Δt is increased. Otherwise Figure 3 is a reliable guide to approximate the optimal number of nodes for any combination of tissue conductivity and cylindrical dimension.

The size of error using a geometry that separates nodes by equal volumes (EV), as in the present study, is considerably smaller than that generated by using equally spaced nodes (EN). This is demonstrated in Table 1 where the percent error is listed for both ways of separating nodes. This result is consistent with the finding of Gordon and Roemer (1975) that the error is reduced by decreasing the nodal spacing where temperature gradients are highest.
These guidelines are generally applicable for the heat conduction problem outlined in Figure 2. The extent to which these guidelines would change with the inclusion of tissue-blood heat conduction and tissue metabolism is not known. We suggest, therefore, that the above guidelines serve as an approximate starting point for optimizing the finite-difference solution of the unsteady bioheat equation.

**Initial \( T_c \) and MR values**

Initial conditions usually assume a state of thermal neutrality which is the starting point for many experimental studies. The conventional procedure for determining the model's initial temperature distribution is to assume the physiological, thermophysical and geometrical characteristics of "standard" man and simulate an exposure to air within the zone of thermal neutrality (Stolwijk 1970). The thermoregulatory model will then generate a steady-state temperature distribution with zero heat storage in each compartment. However, when comparing model predictions to actual measurements, the initial values of \( T_c \) and MR between the model and the subject may not be matched. Unless these values are matched, it would be difficult to ascertain to what extent differences between these values during the subsequent immersion period are attributed to the model. In addition, initial differences in \( T_c \) and MR between the model and the subject impose an arbitrary thermoregulatory response at the outset of the exposure. Simply applying a subject's anthropometric characteristics to the model using the conventional procedure will not assure that initial values of \( T_c \) and MR between model and subject are matched. The following describes a procedure for matching these values under a condition of thermal neutrality, similar to the method used by Miller and Seagrave (1974) in another application.

Consider a model consisting of \( i \) number of segments each consisting of \( J \) number of compartments. The heat storage of the \( j \)th compartment of the \( i \)th segment is given by Stolwijk (1970).
\[ HF_{ij} = Q_{ij} - E_{ij} - BC_{ij} + TD_{ij-1} - TD_{ij} \]  
where \( Q \) is the metabolic rate, \( E \) is the evaporative heat loss, \( BC \) is the heat conducted to the central blood, and \( TD \) is the heat conducted to the adjacent outer compartment. The latter two terms are explicitly given by,

\[ BC_{ij} = c_{blj} \cdot BF_{ij} \cdot (T_{ij} - T_{bl}) \]  
and

\[ TD_{ij} = TC_{ij} \cdot (T_{ij} - T_{ij+1}) \]

where \( c_{bl} \) is the volumetric heat capacity of the blood, \( BF \) is the blood flow rate, and \( TC \) is the thermal conductance (defined by the coefficients in Equation 3). For the innermost compartment,

\[ HF_{i,1} = Q_{i,1} - E_{i,1} - BC_{i,1} - TD_{i,1} \]  
where \( E \) represents the respiratory heat loss for the head and trunk segments only, otherwise its value is zero. For the outermost compartments,

\[ HF_{i,J} = Q_{i,J} - E_{i,J} - BC_{i,J} + TD_{i,J-1} - H_{i} \cdot (T_{i,J} - T_{amb}) \]  
where \( H \) is the combined radiative and convective heat transfer coefficient in air, and \( T_{amb} \) is the ambient temperature. For the central blood,

\[ HF_{bl} = \sum_{i=1}^{I} \sum_{j=1}^{J} BC_{ij} \]  

Under a condition of thermal neutrality, the heat storage of each compartment, including the central blood, is set equal to zero. An iterative procedure is used to solve the thermal neutral temperature distribution. By equating the heat storage of each compartment to zero and choosing a starting value of \( T_{bl} \), the system of heat storage equations reduces to \( I \) independent sets of \( J \) equations each that can be represented in matrix notation. For example, the matrix for a segment comprised of four compartments \(( J = 4)\) as (illustrated in Figure 1) is given by
For all compartments except that representing the muscle, \( Q \) is given by the compartment's basal metabolic rate. For the muscle compartment,

\[
Q_{i,2} = (IMR - BMR) \cdot WORKM_i + QB_{i,2}
\]

where IMR and BMR are the subject's initial and basal metabolic rates and \( WORKM_i \) is the fractional contribution of the \( i \)th segment to the difference between IMR and BMR (Stolwijk 1970). The Gauss Elimination Method (Kreyszig 1967) can be used to solve for the temperatures \( T_{i,1}, T_{i,2}, T_{i,3} \) and \( T_{i,4} \). The iteration continues until values of \( T_{bl} \) and \( T_{amb} \) are found such that the trunk core temperature of the model equals the subject's deep body temperature (usually the rectal) and the heat storage of the blood equals zero. The temperature distribution obtained for "standard" man by this procedure agrees to that obtained using the conventional procedure, but for conditions other than "standard", this procedure has the advantage of matching model and subject initial values of \( T_c \) and MR.

As an example of the differences that can be expected between individuals of different morphology, consider the thermal neutral temperature distributions obtained under the same basal and environmental conditions for a lean and a fat individual.
shown in Figure 4. Because of the higher body insulation of the fat individual, internal temperatures are higher, yet skin temperatures are close to those of the lean individual. In other words, the fat individual has a higher temperature difference between the inner and outer compartments than the lean individual under the same thermal neutral condition. Therefore, the model would assign different thermoregulatory set-points for these individuals.

**Convective Heat Loss**

The conventional prediction of convective heat loss (C) in water is highly sensitive to the skin-water temperature difference, especially as $T_{sk}$ nears the temperature of the water ($T_w$):

$$C = h_c (T_{sk} - T_w)$$  \hspace{1cm} (12)

where $h_c$ is the convective heat transfer coefficient. In nude immersion, the steady-state skin temperature ($T_{sk,s}$) is usually 0.5 to 1.5°C above $T_w$. Small variations in $T_{sk}$ about the value of $T_{sk,s}$ can therefore cause large variations in the predicted value of $C$. Such variations in $C$ critically affect the heat storage of the skin compartment which in turn can perpetuate a fluctuation in the predicted value of $T_{sk}$. Unless the integration time interval is sufficiently small, these fluctuations may be unacceptably large and instability may result. This problem is exacerbated by the assumptions of body shape and water motion that determine $h_c$ (Sekins and Ecosy 1982) which partly explains the wide disparity among values reported in the literature (Boutetier et al. 1977). To circumvent this difficulty of a potentially fluctuating value of $C$, an alternative method of predicting this value is proposed, as follows.

It has been shown experimentally that $T_{sk}$ of a nude subject falls exponentially during immersion in cold water (Strong et al. 1984). Therefore, the rate of change of $T_{sk}$ should be proportional to the difference between $T_{sk,s}$ and its current value, expressed in differential form as
\[ T_{sk} = \lambda \cdot (T_{sk} - T_{sk}) \]  
where \( \lambda \) is a proportionality coefficient. The change in \( T_{sk} \) over a time interval in which \( \lambda \) is fixed is given by

\[ \Delta T_{sk} = (T_{sk} - T_{sk}) \cdot (1 - \exp(-\lambda \cdot \Delta t)) \]

where \( T_{sk} \) is the skin temperature at the start of the time interval. At the same time, the rate of change of \( T_{sk} \) must, by definition, equal the skin compartment's heat storage \( (HF_{sk}) \) divided by its heat capacity \( (C_{sk}) \). By equating this quantity to Equation 13, the value of \( \lambda \) is defined which for small values of \( \Delta t \) can be approximated by:

\[ \lambda = \frac{HF_{sk}}{C_{sk}} \cdot (T_{sk} - T_{sk}) \]

During the unsteady state of falling \( T_{sk} \), the value of \( HF_{sk} \) is determined through Equation 8 with the modification that the last term is replaced by the expression for \( C \) given by Equation 12. Although the sensitivity discussed above with predicting \( C \) is present during this period, this sensitivity diminishes as \( T_{sk} \) falls. Once \( T_{sk} \) nears \( T_{sk} \) (a difference of 0.005°C is considered negligible), the value of \( C \) can be predicted by assuming zero heat storage of the skin compartment (i.e., \( HF_{sk} = 0 \)) so that

\[ C_i = Q_i - BC_{i, J} + TD_{i, J-1} \]

In this way, the value of \( C \) during steady-state of the skin temperature is not sensitive to the skin-water temperature difference.

Throughout this development, a value of \( T_{sk} \) has been implicitly assumed. Although the value of \( T_{sk} \) cannot be wholly predicted in advance, a good estimate can be made because of the narrow range of values found experimentally for nude immersion in water. In addition, the values of \( BC_{i, J} \) and \( TD_{i, J-1} \) are not sensitive to the choice of \( T_{sk} \), so little error can be expected using Equation 16 during steady-state of the skin temperature.
Conclusions

Three specific problems with modelling the controlled system of human thermoregulation for cold water immersion have been examined: 1) the finite-difference solution of the bioheat equation; 2) differences between predicted and measured values of Tc and MR; and 3) the prediction of C. An approximate optimization scheme for using the finite-difference solution of the bioheat problem was outlined by considering the simpler, but related, general heat conduction problem. In concurrence with other studies, it was found that the error was reduced by decreasing the integration time interval, by increasing the number of nodes, and by decreasing the nodal spacing for large temperature differences. However, by a judicious choice of these factors, the finite-difference procedure can be optimized. For example, a greater benefit is obtained by doubling the number of nodes rather than halving the integration time interval.

To predict a subject's initial temperature distribution under a condition of thermal neutrality, zero heat storage was assumed for each model compartment and the resulting system of linear heat balance equations were solved simultaneously. An iterative procedure was adopted so that the solution yielded a value of trunk core temperature equal to the subject's deep core (rectal) temperature. Since set-point values for thermoregulation are based on the thermal neutral temperature distribution, this procedure of matching the model and subject initial temperatures avoids any arbitrary thermoregulation at the outset of the exposure.

To circumvent the uncertainties associated with predicting the convective heat loss during steady-state of the skin temperature, a method was developed to assure a smooth exponential fall in Tsk to a steady-state value. Once this value was attained, zero heat storage of the skin compartment was assumed so that C could be predicted through a heat balance of the skin compartment. In this way, the
predicted value of C is not sensitive to the small skin-water temperature difference that occurs with nude immersion in water. This is potentially more important for exposures less severe than cold water immersion since the relative error using incorrect set-points is greater when actual temperatures are close to neutral conditions.

Although the problems addressed in this paper arose out of a need for greater accuracy when modelling the human thermoregulatory response for immersion in cold water, the procedures for handling these problems may be applicable in other circumstances, especially where finite-difference methods are used and matching of initial conditions are important.
The views, opinions, and/or findings contained in this report are those of the authors and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other official documentation.

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REFERENCES


Table 1: Percent error at $Kt/R^2 = 0.01$ for nodes spaced apart by equal volumes (EV) and by equal radial displacement (EN).

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>$\Delta t = 0.0001 , R^2/K$</th>
<th>$\Delta t = 0.001 , R^2/K$</th>
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<tr>
<td></td>
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<td>EN</td>
</tr>
<tr>
<td>3</td>
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<td>117.2</td>
</tr>
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<td>4</td>
<td>17.7</td>
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<tr>
<td>6</td>
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<td>25.1</td>
</tr>
<tr>
<td>10</td>
<td>1.9</td>
<td>7.4</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Figure 1. Schematic of the human body (one side shown) modelled by Stolwijk (1970). Each body segment is composed of four concentric annular compartments, the head modelled as a sphere and the others as cylinders. Length of the cylinders are given in inches. Radial dimensions are determined by the mass of the individual modelled.

Figure 2. Schematic of the heat conduction problem.

Figure 3. Normalized values of the average tissue temperature plotted against the dimensionless quantity \( Kt/R^2 \). The solid line represents the exact solution of the heat conduction problem outlined in Figure 2 and the dashed lines represent the finite-difference solution. Numbers above the dashed lines indicate the number of nodes used. The integration time interval was \( R^2/K \times 10^{-4} \).

Figure 4. Thermal neutral temperature distributions for the Stolwijk (1970) model assuming basal conditions (BMR = 14.5 BTU/ft²/h (45.7 w/m²)) and a simulated exposure to a 86°F (30°C) environment with a 20 ft/min (10 cm/s) air movement. The open bars indicate the temperature distribution of a lean person (height = 68 in (173 cm), weight = 140 lb (63.5 kg), body fat = 7.3%) and the hashed bars indicate the temperature distribution of a fat person (height = 68 in (173 cm), weight = 200 lb (90.7 kg), body fat = 23.8%). Compartments of each segment are core, muscle, fat and skin ordered from left to right.
Fig. 1
\[ T = a \quad 0 < r < R, \quad t = 0 \]
\[ T = T_s \quad r = R, \quad t \geq 0 \]
\[ k, \rho, c \quad \text{constant} \]