THE EFFECT OF COLLISION ANGLE ON MACH REFLECTION (U)
MATERIALS RESEARCH LABS ASCOT VALE (AUSTRALIA)
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THE EFFECT OF COLLISION ANGLE ON MACH REFLECTION

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ABSTRACT

Following the successful application of the HULL code to the modelling of shock waves colliding with a smooth planar surface, as described in a previous report, the influence of collision angle on reflection type was investigated. The numerical investigations revealed examples of both regular and Mach reflection. From these an estimate of the transition collision angle for a non-decaying air shock wave of $M_1 = 1.472$ (overpressure of 137.9 kPa) was made. This angle was found to be approximately $41.5^\circ$. 

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1. INTRODUCTION

Blast damage to targets from detonating explosives may be enhanced significantly by the coalescence of an incident shock wave and a wave that has reflected from a nearby surface, normally the ground. This superposition of shock waves may form a Mach stem which not only has a greater overpressure and impulse than the incident wave, but also produces considerable turbulence in its wake. The formation of this Mach stem is a function of both the velocity (or pressure) of the incident shock and the angle at which reflection occurs. Thus, for a given shock wave velocity, it should be possible to determine an angle of reflection at which a transition from regular to Mach reflection, or vice versa, occurs.

Many experiments studying such shock interactions and transitions have been conducted in shock tubes. This is because shock tubes allow fairly complex phenomena to be viewed in close detail and with a minimum of danger. Accurate computer modelling of these same events would expedite the overall research programme.

The HULL code as used at Materials Research Laboratories, has already been successful at modelling the formation of a Mach stem from the collision of a shock front with a smooth wedge [1]. Here, it is reported that the HULL code can simulate the transition from regular reflection to Mach reflection. The formation of the Mach stem as a function of collision angle, or alternatively, the angle of reflection, is also investigated.

2. THE INITIAL CONDITIONS

The simulation was set up as displayed in Figure 1. Here, a non-decaying shock wave is shown travelling along a smooth-walled, air filled shock tube just prior to impacting a smooth ramp. The reflection process was
studied by changing the angle of elevation of the wedge. The horizontal tube was modelled in cartesian co-ordinates with the top and bottom boundaries perfectly reflective, and the right boundary transmissive. From the left boundary the non-decaying shock wave was input.

The standard symbols [2] used to represent the physical quantities of interest, both in front of, and behind, the shock wave were $T$ for absolute temperature, $P$ for pressure, $\rho$ for density and $u$ for shock front velocity. $\theta_w$ is the wedge angle, and $\alpha$, the collision angle, its complement.

![Diagram](https://via.placeholder.com/150)

**FIGURE 1** Problem description. All symbols follow the standard notation [2].

The equation of state due to Doan and Nickel [3] was employed to model the air. This semi-empirical equation is valid up to a temperature of approximately 17000 K, with an error usually much less than 2%. The temperature limit on this equation of state is far in excess of the 600 K which may be expected theoretically for most problems of interest. Thus the equation is suitable for this simulation.

The initial conditions for the shock front are provided in Table 1. These represent a shock wave of $M_i = 1.472$ moving through still air at ambient temperature and pressure. The shock values were derived through the well-known Rankine-Hugoniot relations [4], and were determined in a previous paper [1]. The overpressure developed by this incident shock wave is 137.9 kPa, so taken to correspond to a nominal 20 psi.

The computational grid, although not the same for each problem, usually consisted of between 10,000 and 30,000 cells, depending on the wedge angle. The cell dimensions were selected so that the wedge plane could pass exactly through two diagonally opposite cell vertices. This produced the smoothest possible wedge surface when the SHORE option in HULL was selected. Typically the cell dimensions were of the order of 2 mm.
## TABLE 1

Physical Quantities of the System

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNIT</th>
<th>AMBIENT</th>
<th>SHOCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>kg/m$^3$</td>
<td>1.225</td>
<td>2.222</td>
</tr>
<tr>
<td>Specific Internal Energy</td>
<td>J/kg</td>
<td>$2.044 \times 10^5$</td>
<td>$2.692 \times 10^5$</td>
</tr>
<tr>
<td>Pressure</td>
<td>kPa</td>
<td>101.3</td>
<td>239.2</td>
</tr>
<tr>
<td>Shock Velocity</td>
<td>m/s</td>
<td>-</td>
<td>$5.009 \times 10^2$</td>
</tr>
<tr>
<td>Particle Velocity</td>
<td>m/s</td>
<td>0</td>
<td>$2.247 \times 10^2$</td>
</tr>
</tbody>
</table>

### 3. RESULTS

For each collision angle modelled, both the density and the pressure contour plots were analysed. From these, the collision angles of $57^\circ$ and $35^\circ$ were selected as typical examples of Mach and regular reflection, respectively.

In Figure 2 a pressure profile for a collision angle of $35^\circ$ is shown. The contour pattern is typical of that found for regular reflection.

In Figure 3 the pressure contour plot for a collision angle of $57^\circ$ is shown. In this case, Mach stem formation is evident and the pattern displayed is very similar to that which would be expected for Mach reflection.

For the regular reflection case, as shown in Figure 4, the density contours behind the incident wave tend to intersect the ramp surface almost perpendicularly, as is found experimentally [5]. At the convolution of the incident and reflected wave the plot provides a spread of contours emanating from the contact point. This is not supported experimentally. This spread is thought to be partly due to the grid size employed in the simulation being too large, with the consequent smoothing of density values from one cell to its neighbour. This could also mean that the ramp has not been modelled smoothly enough. Further refinement of the grid size is, however, considered impractical. It may also be that the HULL code has an inherent difficulty in modelling the contact point well.
FIGURE 2  Pressure contour plot for 55° wedge.
FIGURE 3  Pressure contour plot for 33° wedge.
FIGURE 4  Density contour plot for 55° wedge.
FIGURE 5  Density contour plot for $33^\circ$ wedge.
The Mach reflection case shown in Figure 5 does not display their inconsistency at the contact point. This suggests that perhaps the code is better suited to the study of Mach reflection. In fact, the curvature of the density contours (not all shown here) close behind the Mach stem, and near the surface of the wedge and the general form of the plot, appear to support this contention. Although the precise location of a slipstream is not evident from this diagram the distinctive bow in the isopycnics does imply the existence of a vortex as is found experimentally [6].

Comparing isopycnics is of more value than comparing isobars as density contours can be compared directly with interferometric results obtained from experiments. One such comparison has been made by using data supplied by Professor Takayama [7]. The comparison is shown in figure 6, where a shock wave of $M_1 = 1.49$ is demonstrated colliding with a wedge set at $30^\circ$, i.e. initial conditions very close to those for figure 5. There are some similarities between figures 5 and 6. The distinctive sharp outline of the Mach stem is very clear in both figures. Although the discontinuity of the isopycnics behind the Mach stem is not shown in Figure 5, the general shape is in good agreement with Figure 6. The computational approach is thus producing results that well simulate the available experimental data.

The transition from regular reflection to Mach reflection can be found by plotting the peak pressure against the collision angle. The peak pressure was obtained from the Station data of the HULL code. These data provide the time histories of various parameters, such as pressure at a given point in the computational space. For each simulation a number of stations were located, as near as possible to the wedge surface, in order to detect the pressure profile. Thus a range of pressure values were obtained for each collision angle. The average values are listed in Table 2 and plotted in Figure 7. Generally the percentage error in measuring the peak pressure was fairly constant. The main reasons for any variance were due to the stations not being exactly on the surface of the wedge and the final value being smoothed over a number of neighbouring cells. These problems were found to be of greatest importance for the high collision angle Mach reflection regime (see Table 2). Here the Mach stem is short and the measurement of pressure at any distance removed from the wedge surface would ensure a decrease in pressure detected.

When the values of Table 2 are plotted (see Figure 7) a discontinuity is found. The left hand side, or low collision angle region, corresponds to the regular reflection regime, while the right hand side, or high collision region, corresponds to the Mach reflection regime.

Figure 7 suggests that the peak pressure is fairly constant for the regular reflection area, until the collision angle approaches the transition region. This is typical of the behaviour found for pressure variation within the regular reflection region [8].

An estimate of the peak pressure expected for a collision angle of zero degrees, i.e. normal reflection, can be found by employing the approximation of an ideal gas [4]. Then,
\[ P_r = \frac{(\gamma^2 \pm 2(\gamma - 1))(2\gamma^2 - (\gamma - 1))}{(\gamma^2 - 1)M_1^2 + 2(\gamma + 1)} \]

where \( \gamma \) is the ratio of specific heats and \( P_r \) is the normally reflected pressure. \( P_r \) is found to be in basic agreement with the value of the peak pressure obtained from Figure 7 at a collision angle of 0°. This agreement should be considered in light of the fact that an ideal gas has been assumed for this calculation.

**TABLE 2**

Peak Pressure for various collision angles and for \( M_1 = 1.472 \)

<table>
<thead>
<tr>
<th>RAMP ANGLE ( \theta' )</th>
<th>COLLISION ANGLE ( \alpha' )</th>
<th>PEAK PRESSURE ( \text{kPa} )</th>
<th>TYPE OF REFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>80</td>
<td>( 250 \pm 20 )</td>
<td>Mach</td>
</tr>
<tr>
<td>20.2</td>
<td>69.8</td>
<td>( 330 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>26.9</td>
<td>63.1</td>
<td>( 350 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>30.8</td>
<td>59.2</td>
<td>( 380 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>57</td>
<td>( 400 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>40.5</td>
<td>49.5</td>
<td>( 450 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>44</td>
<td>( 480 \pm 20 )</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>35</td>
<td>( 440 \pm 30 )</td>
<td>Regular</td>
</tr>
<tr>
<td>65</td>
<td>25</td>
<td>( 450 \pm 30 )</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>10</td>
<td>( 450 \pm 40 )</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 7  Peak Pressure vs Collision Angle as defined from HULL output.
In the Mach reflection region the pressure gradually increases as the transition angle is approached. This increase in peak pressure is due to the steadily increasing value of the Mach number of the Mach stem. This is explained in the following argument:

To a first approximation the Mach number of the produced Mach stem \( M_m \) can be defined as [4].

\[
\frac{M_m}{M_1} = \frac{\sin \alpha}{\sin \alpha}
\]  

(2)

where \( \alpha \) is the collision angle.

As the collision angle decreases from 90°, \( M_m \) must increase. Since the pressure behind the Mach stem is a function of \( M_m \) as given in equation 1, the peak pressure must rise as the transition point is reached.

In the Mach reflection regime, as the incident shock moves over the wedge, the Mach stem increases in height. If the rate of increase is assumed to be constant, a growth angle for the locus of the triple point can be determined. When this growth angle is plotted against collision angle and extrapolated back to zero growth angle, i.e. no Mach stem, the transition angle may be estimated.

From the available pressure and density contour plots the position of the triple point was estimated. The average values for the Mach stem growth angle were then calculated together with standard deviations. These values are given in Table 3. The relatively large error bars associated with the growth angles were due to the fact that the real position of the triple point was difficult to determine consistently. This was particularly the case when the collision angle was large, as then the growth rate of the Mach stem was also large.
TABLE 3

Mach Stem Growth Angle as a Function of Collision Angle
for $M_1 = 1.472$

<table>
<thead>
<tr>
<th>RAMP ANGLE $\theta^\circ$</th>
<th>COLLISION ANGLE $\alpha^\circ$</th>
<th>MACH STEM GROWTH ANGLE (X$^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>80</td>
<td>13.1 ± 0.8</td>
</tr>
<tr>
<td>14</td>
<td>76</td>
<td>12.1 ± 0.6</td>
</tr>
<tr>
<td>20.2</td>
<td>69.8</td>
<td>9.4 ± 1.0</td>
</tr>
<tr>
<td>26.9</td>
<td>63.1</td>
<td>8.2 ± 0.8</td>
</tr>
<tr>
<td>30.8</td>
<td>59.2</td>
<td>6.6 ± 0.4</td>
</tr>
<tr>
<td>33</td>
<td>57</td>
<td>5.8 ± 0.4</td>
</tr>
<tr>
<td>40.5</td>
<td>49.5</td>
<td>2.2 ± 0.3</td>
</tr>
</tbody>
</table>

Using the values of Table 3, the Mach stem growth angle was plotted against the collision angle (Figure 8). This revealed a transition angle from regular reflection to Mach reflection of approx. $41.5^\circ ± 0.5$. This is in general agreement with the empirically determined figure of approximately $42.7^\circ$ for a Mach number of 1.472 [4].
FIGURE 8. Mach stem growth angle vs. Collision angle as measured from the HULL output.
4. CONCLUSION

It has been demonstrated that the Materials Research Laboratories version of the HULL code can simulate the transition from regular reflection to Mach reflection with some confidence. It provides valid answers as to where the transition from regular reflection to Mach reflection occurs. It also indicates what the pressures associated with that transition might be, in addition to supplying useful time histories of various shock wave characteristics.

HULL does, however, seem to have a few shortcomings. It appears to have difficulty describing the internal structure behind the Mach stem for Mach reflection, and defining the contact point for regular reflection. These problems will be investigated in later versions of HULL.
5. REFERENCES


The effect of collision angle on mach reflection

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Shock waves
Mach reflection

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