Electron Energy Deposition in Atomic Nitrogen

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A discrete, time-dependent energy deposition model is used to study high-energy electron-beam (100 eV to 10 MeV) deposition in atomic nitrogen. Both time-dependent and steady-state secondary electron distributions are computed. The loss function, mean energies per electron-ion pair production (\(\Delta E_{\text{W}}\)), production efficiencies, and yield functions are presented for a wide range of energies. The latest experimental and theoretical cross section are used in the model which predicts \(\Delta E_{\text{W}}\) is approximately 31 eV for a wide range of beam energies and background ionization fractions.
CONTENTS

1. INTRODUCTION ........................................................................................................... 1

2. THE SECONDARY ELECTRON DISTRIBUTION ......................................................... 3
   2.1 The Source Term S(T,t) ......................................................................................... 4
   2.2 Electron Impact Excitation Cross Sections ......................................................... 5
   2.3 Electron Ionization Cross Sections ..................................................................... 7
   2.4 Energy Loss to Plasma Electrons ....................................................................... 9

3. RESULTS ..................................................................................................................... 11

4. SUMMARY ................................................................................................................... 15

ACKNOWLEDGEMENT .................................................................................................. 15

REFERENCES .................................................................................................................. 16
ELECTRON ENERGY DEPOSITION IN ATOMIC NITROGEN

1. INTRODUCTION

The problem of electron-beam energy deposition in gaseous species is currently of interest in several areas of research, including electron beam propagation in the atmosphere, electron-beam generated lasers, and electron-beam generated discharges and their diagnostics. Detailed deposition calculations generally provide information on the secondary electron distribution, its flux, primary and secondary electron excitation rates for the internal modes of the atoms or molecules, total ionization rate, and the mean energy expended by an electron in generating an electron-ion pair, $\Delta W$. The mean energy, $\Delta W$, is often used to provide a simple description of the cumulative ionization of the gas by high-energy beam electrons, i.e., the creation of secondaries, tertiaries, etc.

In a previous paper (denoted I), we reported on the development of a new electron energy deposition code and its application to the study of deposition in atomic oxygen. While extensive work has been done on studying deposition in air and gases such as O, O$_2$, N$_2$, H$_2$, Ar, and He, among others, no such effort has been made for atomic nitrogen. Here, results are presented and discussed for electron-beam energy deposition in atomic nitrogen for beam energies between 100 eV and 10 MeV. Results are obtained using the deposition code mentioned above. In particular, secondary electron distributions are obtained by solving a time-dependent Boltzmann equation. These distribution functions relax to steady-state results from which yield spectra, production efficiencies of specific states, energy partitioning, and $\Delta W$ are computed. Loss functions are also computed and compared to Bethe's relativistic equation. Results which are specific to beam deposition in nitrogen are presented. General results are not discussed in detail since the resulting conclusions are identical to those presented in paper I. For example, examination of $\Phi(T)$ (defined in Eq. (27) in paper I) showed the continuous slowing down approximation (CSDA) to be

valid for energies greater than 1 keV. Examination of the same quantity for deposition in nitrogen gives the same result and need not be elaborated on further.

The Boltzmann equation and the specific nitrogen model are given in section 2. This includes a discussion of the electron impact excitation and ionization cross sections used in the deposition scheme. The numerical techniques implemented in the calculation were described in detail in the appendix of paper I. Results are presented and discussed in section 3. Summary remarks are reserved for section 4.
2. THE SECONDARY ELECTRON DISTRIBUTION

With the exception of specific nitrogen cross section information, much of the discussion presented in this section was also presented in section 2 of paper I.

The secondary electron distribution for a spatially homogeneous electron beam impinging upon a gas can be calculated from

\[
\frac{df(T,t)}{dt} = S(T,t) + \sum_s N_s \left\{ \sum_j \left[ \sigma_{sj}(T+V_{s_j}) v(T+V_{s_j}) f(T+V_{s_j},t) \right. \right.
\]
\[
- \sigma_{sj}(T) v(T) f(T,t) \left. \right] + \sum_i \left[ \int \left. \right| \frac{1}{2T+i_{si}} d\varepsilon \sigma_{si}(\varepsilon,\varepsilon-i_{si}-T) v(\varepsilon) f(\varepsilon,t) \right. \right.
\]
\[
+ \left. \int \frac{1}{2T+i_{si}} d\varepsilon \sigma_{si}(\varepsilon,T) v(\varepsilon) f(\varepsilon,t) - \sigma_{si}(T) v(T) f(T,t) \right\} \}
\]
\[
+ N_p(t) \frac{3}{2T} \left[ L_p(T) v(T) f(T,t) \right] ,
\]

where \( f(T,t) \) is the secondary electron density (cm\(^{-3}\) eV\(^{-1}\)) for electrons with kinetic energy \( T \) and speed \( v(T) \), \( S(T,t) \) the production rate (cm\(^{-3}\) sec\(^{-1}\) eV\(^{-1}\)) due to the incident electron beam, \( T_m \) the maximum secondary electron energy (discussed below), \( N_s \) the number density of species \( s \), \( N_p(t) \) the plasma electron number density, and \( L_p(T) \) the loss function of a secondary electron with energy \( T \) to the plasma electrons. Loss functions such as \( L_p(T) \) are often used to model the effects of electron-electron collisions.\(^{11,2,16,19}\) The effects of inelastic and ionizing collisions are accounted for by using detailed cross sections. For example, \( \sigma_{sj}(T) \) is the \( j \)th electron impact excitation cross section for species \( s \) where \( V_{s_j} \) is the excitation energy. For ionization channel \( i \), the total ionization cross section is \( \sigma_{si}(T) \), \( i_{si} \) the ionization potential, and \( \sigma_{si}(\varepsilon,T) \) the (differential) ionization cross section for an incident electron with energy \( \varepsilon \) producing an additional electron with energy \( T \).
In Eq. (1), electric and magnetic field effects due to the beam are assumed negligible. Elastic and superelastic collisions with the heavy particles are also neglected. Furthermore, in this paper, we consider the case of a single component background gas (atomic nitrogen) and assume that both the gas density \( N_s = N_0 \) and source term are time independent. The nitrogen atoms initially reside in the \( 4S^0 \) (ground) state. The model accounts for energy loss to the \( 2p^0 \) and \( 2p^0 \) metastable states, all \( n = 3 \) states, 1 specific Rydberg series (\( n \geq 4 \)) state, and the \( 3P \ N^+ \) state. Within the context of these assumptions and this model, the various terms appearing in Eq. (1) are discussed in detail below. Analytic forms for the relevant excitation and ionization cross sections are presented in sections 2.2 and 2.3. A detailed discussion of these forms, their reduction to well-known theoretical results, and their relative accuracy in comparison to existing measurements and calculations is given elsewhere.\(^{20}\)

2.1 The Source Term \( S(T, t) \)

The emphasis in this paper is on high-energy electron-beam deposition in nitrogen. Beam electrons are assumed to make at most one collision and then leave the area of interest. The source term for the generation of the secondary electrons is, therefore,

\[
S(T, t) = S(T) = N_0 \ N_b(T_b) \ v(T_b) \ \sum_i \ \sigma_i(T_b,T) , \tag{2}
\]

where \( N_b(T_b) \) is the number density of beam electrons whose energy is \( T_b \), \( \sigma_i(T_b,T) \) the differential ionization cross section for producing a secondary electron with energy \( T \) (restricted to energies \( \leq (T_b - I_i)/2 \)), and the time-independence is made explicit. The exact form of the differential cross section is discussed in section 2.3.

For the case of energetic electrons which do not leave the area of interest, but are completely stopped by the medium, the source term may be given by

\[
S(T, t) = S(T) = N_1 \ \delta(T - T_b) , \tag{3}
\]

where \( N_1 \) is the number of completely stopped electrons per cm\(^3\) per sec. This case is not treated in this paper.
2.2 Electron Impact Excitation Cross Sections

Electron impact excitations of nitrogen are either optically (dipole) allowed or optically forbidden. The cross section for optically allowed transitions from the ground state to an excited state \( j \) is given by,

\[
\sigma_j = A_j \frac{4n_a^2 R^2 f_j}{E^2} \left( \frac{E}{W_j} - 1 \right) \left\{ \ln \left[ \frac{4 C_j E}{W_j} (1 - \beta^2)^{-1} \right] - \beta^2 \right\}, \tag{4}
\]

where \( n_a^2 \) is the atomic unit cross section (0.88 \( \times \) \( 10^{-16} \) cm\(^2\)), \( R \) the hydrogen atom ionization potential (13.60 eV), \( W_j \) the transition energy, \( f_j \) the optical oscillator strength, \( A_j \) and \( C_j \) adjustable parameters, and \( \beta = v/c \) where \( v \) is the electron velocity. The energy \( E \) is \( mv^2/2 \) where \( m \) is the electron rest mass. For energies less than \( 10^4 \) eV, \( E \) is approximately equal to the electron kinetic energy, \( T \); relativistic effects begin to play a role above that value. All excitation (and ionization) cross sections are assumed equal to 0.0 for energies less than the excitation (or ionization) thresholds. Equation (4) accounts for the threshold behavior in a manner first proposed by Drabin \(^{21} \) and includes the energy dependence expected according to the relativistic Bethe formula based on the Born approximation. \(^{22,23} \) It is reasonable to use this form for energies up to \( 10^9 \) eV. Above that energy, coupling to the radiation field cannot be neglected.

The parameters used in Eq. (4) for optically allowed transitions are given in Table 1. The oscillator strengths were taken from Wiese et al. \(^{24} \)

In atomic nitrogen, transitions to forbidden states are categorized as those involving 1) the metastable ground states or 2) higher-lying Rydberg states. Generally, these transitions proceed because of existing electric quadrupole moments, magnetic moments, or electron exchange effects. Cross sections for electron impact excitation of such transitions do not display the characteristic \( E^{-1} \ln E \) behavior at high energies, but instead decrease more rapidly. \(^{25} \) To describe such transitions, we adopt a form used extensively by Green and coworkers, \(^{9,26-30} \)
where \( a, b, c, \) and \( A_j \) are parameters and the other quantities are defined above. Equation (5), with the proper choice of parameters, reduces to well-known theoretical results.  

Table 2 gives the parameters for collisional excitation of the \( ^2D_0 \) and \( ^2P_0 \) metastable states. Two sets of parameters were used to model the metastable state cross sections. These give good agreement with existing experimental\(^{31} \) and theoretical\(^{32-34} \) results. Our preference for using two sets of parameters is that the desired \( E^{-3} \) high energy behavior\(^{20,21,32} \) be maintained.  

Electron impact excitation cross sections for transitions to high-lying excited states are obtained using the general procedure developed by Green and Dutta.\(^{27} \) Setting \( b = 1 \) and rewriting Eq. (5) to correspond to their form, the cross section is given by,

\[
\sigma_j = c_j f_j \frac{4na_o^2 R^2}{W_j^2} \left[ 1 - \frac{W_j}{E} \right]^c \left( \frac{W_j}{E} \right)^a .
\]  

The effective oscillator strength, \( f_j \), is defined as

\[
f_j = \frac{f^*}{(n - \rho)^3} ,
\]  

where \( \rho \) and \( f^* \) are assumed constant within a Rydberg series and \( \rho \) is determined from

\[
W_j = I_j - \frac{R}{(n - \rho)^2}.
\]  

In principle, the value of \( f^* \) is obtained by equating \( f_j \) with the known oscillator strength of an optically allowed transition in the series. As an alternative, the parameters \( c_j, a, c, \) and \( \rho \) may be chosen according to the criteria proposed by Jusick et al.\(^{35} \) Dalgarno and Lejeune\(^{2} \) used this method for oxygen, letting \( f^* \) vary only according to
which ionization continuum the series belonged to. Adopting their approach, the parameters for all \( n = 3 \) states included in this calculation are presented in Table 3. For the \( ^3P \) continua, \( \varepsilon^* = 3.475 \). We have not included states which ionize to the \(^1D\) or \(^1S\) continua.

Energy loss to Rydberg states with \( n \geq 4 \) and belonging to a series containing one of the allowed transitions given in Table 1 are also modeled by Eq. (6). The effective \( f_j \) is obtained by solving Eq. (8) for \( p \) (\( \bar{V}_j \) is the energy of the \( n = 4 \) state) and integrating Eq. (7) from \( n = 4 \) to \( -\infty \). \( \bar{V}_j \) (the average of \( V_j \) and \( \bar{I}_j \)) is used in Eq. (6) in place of \( V_j \). Parameters are given in Table 4.

2.3 Electron Ionization Cross Sections

The total ionization cross section is given, analogous to Eq. (4), by

\[
\sigma_i = A_i \frac{4\pi a_0^2 R^2}{E^2} \left( \frac{E}{I_i} - 1 \right) \left\{ \ln \left( \frac{4 C_i E}{I_i} (1 - \beta^2)^{-1} \right) - \beta^2 \right\}, \quad (9)
\]

Equation (9) reduces identically to the Drawin formula\(^\text{21}\) for low energies, displays the desired \( E^{-1} \ln E \) behavior at high energies, and shows the expected rise for relativistic energies.

The differential ionization cross section, \( \sigma_i(T, \varepsilon) \), is given by,\(^\text{13,20}\)

\[
\frac{\sigma(T)}{\tan^{-1} \left( \frac{T - I_i}{2b(T)} \right)} \frac{b(T)}{b(T)^2 + \varepsilon^2}, \quad (10)
\]

for the low energy regime, defined here as \( T < I_i + 10 \text{ eV} \). The index \( i \) has been dropped for convenience. When \( T > I_i + 10 \text{ eV} \) then

\[
\sigma(T, \varepsilon) = \frac{\sigma(T)}{p(T)} \frac{b(T)}{g(T, \varepsilon)} \quad (11)
\]

where
\[
g(T, \varepsilon) = \frac{1}{(T+mc^2)^2} - \frac{(2Tmc^2 + m^2c^4)}{(T+mc^2)^2} \left( \frac{b(T)}{b(T) + \varepsilon} \right) \left( \frac{b(T) + T - \varepsilon - I}{b(T) + \varepsilon} \right)
\]

\[
+ \frac{1}{b(T)^2 + (T-I)^2} + \frac{1}{b(T)^2 + \varepsilon^2},
\]

(12)

and

\[
p(T) = \tan^{-1} \left[ \frac{T-I}{b(T)} \right] - \frac{b(T)}{T+2b(T)-I} \frac{(2Tmc^2 + m^2c^4)}{(T+mc^2)^2} \ln \left[ \frac{b(T)+T-I}{b(T)} \right]
\]

\[
+ \frac{b(T)(T-I)}{2(T+mc^2)^2}.
\]

(13)

In Eqs. (10 - 13), \( b(T) \) is an energy-dependent parameter and \( mc^2 \) the electron rest mass energy (.511 MeV). As above, \( T \) is the kinetic energy of the incident electron (primary) and \( \varepsilon \) the outgoing (secondary) electron. Equation (13) ensures that when the secondary electron is defined as the least energetic of the two, then

\[
\sigma_i(T) = \int_{0}^{(T-I_i)/2} \sigma_i(T, \varepsilon) \, d\varepsilon.
\]

(14)

The parameters \( A_i \) and \( C_i \) have been chosen so that Eq. (9) provides a compromise fit to the recent calculations of McGuire\(^{36}\) and experimental measurements of Brook et al.\(^{37}\) Additional theoretical and experimental results are discussed in detail elsewhere.\(^{20}\) Since transitions to \( N(1D) \) and \( N^+(1S) \) do not occur in photoionization and, therefore, are expected to have small collision cross sections, they are neglected in this study. The parameter \( b_i(T) \) is assumed to have the form

\[
b_i(T) = \begin{cases} 
\frac{b_{io}}{T} & T \leq \exp(k_i) \\
\frac{b_{io}k_i}{\ln T} & T \geq \exp(k_i)
\end{cases}
\]

(15)
where $b_{io}$ and $k_{i}$ are constants. The ionization parameters are given in Table 5. From the oxygen work, we found that $b_{io}$ is approximately equal to the ionization potential. The functional form of $b_{i}(T)$ at higher energies and the value of $k_{i}$ are chosen so that the loss function, $L(T)$, (discussed later below) is in agreement with Bethe's relativistic formula.  

2.4 **Energy Loss to Plasma Electrons**

Perkins has derived the rate of energy loss to plasma electrons by a test electron. These results have been utilized by Schunk and Hayes to obtain expressions for the energy loss for nonrelativistic energies. For relativistic energies, the energy loss is given by Tsytovich. In Eq. (1), these results are used to calculate the loss function, $L_{p}(T) = - (1/N_{p})(dT/dx)$, where

$$- \frac{dT}{dx} = \frac{\omega_{p}^{2} e^{2}}{\gamma_{o} e^{2} \omega_{p}} \log \frac{mv^{3}}{\gamma_{o} e^{2} \omega_{p}} , \quad \text{for } kT_{e} \ll T \ll 14.6 \text{ eV} ,$$

$$- \frac{dT}{dx} = \frac{\omega_{p}^{2} e^{2}}{2v^{2}} \left\{ \log \frac{mv_{T}^{2}}{I_{e}} + 1 - \left[ 2(1-\beta^{2})^{1/2} + \beta^{2} \right] \ln 2 \right. $$

$$+ \frac{1}{8} \left[ 1 - (1-\beta^{2})^{1/2} \right]^{2} \right\} , \quad T \geq 14.6 \text{ eV} ,$$

and $\omega_{p}$ is the plasma frequency ($= [4\pi N_{p} e^{2}/m]^{1/2}$), $\ln \gamma_{o}$ is Euler's constant (0.577), $T_{e}$ the electron temperature, and $I_{e}$ is an average excitation energy of the plasma electrons given in terms of the dielectric function of the electron gas, $\varepsilon_{e}(\omega)$, as

$$\ln I_{e} = \frac{2}{\pi \omega_{p}^{2}} \int_{0}^{\infty} \omega \ \text{Im} \left[ - \varepsilon_{e}(\omega)^{-1} \right] \ln \omega \ d\omega .$$
In the limit of nonrelativistic energies, small damping$^{36}$ ($I_e = 5 \omega_p$), and $N_p \ll 1.6 \times 10^{24}$ cm$^{-3}$, Eq. (17) reduces to the high-energy, nonrelativistic equation given by Perkins.$^{38}$ The cutoff energy, 14.6 eV, is chosen to ensure continuity between Eqs. (16 & 17).

In Eq. (1), the ionization fraction, $N_p/N_0$, is entered as a parameter. For nonrelativistic energies, Dalgarno and Lejeune$^2$ analyzed the sensitivity of their oxygen deposition model to changes in the fractional ionization. Similar results are presented in the next section.
3. RESULTS

The discrete energy deposition scheme described above and in paper I is utilized for beam electrons with energies ranging from 100 eV to 10 MeV. For all cases, the beam flux was fixed at $1.99 \times 10^{18}$ cm$^{-2}$ sec$^{-1}$ and the background nitrogen density was fixed at $2.46 \times 10^{19}$ cm$^{-3}$. Unless otherwise noted, the fractional ionization was approximately zero ($4.1 \times 10^{-20}$). The quantities defined below are also defined in paper I. Results for deposition in nitrogen are discussed in this section.

The total inelastic cross section, $\sigma^T(T)$, contains contributions from electron impact excitation, $\sigma^T_e(T)$, and ionization, $\sigma^T_i(T)$, and is given by,

$$\sigma^T(T) = \sigma^T_e(T) + \sigma^T_i(T)$$

Using the information presented in sections 2.2 and 2.3, these cross sections are shown in Fig. 1 for energies up to 10 MeV.

The loss function accounts for energy loss to excitation, $L_e(T)$, and ionization, $L_i(T)$, as well as energy carried away by secondary electrons, $L_s(T)$, and may be written as

$$L(T) = L_e(T) + L_i(T) + L_s(T)$$

$$(T-I_i)/2 \int_{0}^{(T-I_i)/2} \varepsilon \, \sigma_i(T,\varepsilon) \, d\varepsilon$$

Figure 2 shows $L(T)$ and its components for energies up to 10 MeV. Most of the energy goes into producing secondary electrons, while loss to excitation is significant only below the ionization threshold. Figure 2 also shows a comparison between $L(T)$ and Bethe's relativistic loss function.\textsuperscript{18}
\[ L_B(T) = \frac{2\pi r_e^2 mc^2}{\beta^2} Z \left\{ \log \left[ \frac{T^2 (\gamma + 1)}{I_o^2} \right] + \frac{1}{\gamma^2} - \frac{2\gamma - 1}{\gamma^2} \log 2 \right. \\
+ \left. \frac{1}{8} \left( \frac{\gamma - 1}{\gamma} \right)^2 \right\}. \]  

Eq. (21)

In Eq. (21), \( Z = 7 \) for nitrogen, \( r_e \) is the classical electron radius, \( I_o \) a mean excitation energy (\( I_o = 85.0 \) for nitrogen\(^{40}\)), and \( \gamma = (1 - \beta^2)^{-1/2} \). For energies greater than 1 keV, \( L(T) \) and \( L_B(T) \) are in close agreement. The energy loss of an electron traversing a material is lessened because of polarization of the medium.\(^{40-42}\) This density effect is small for the energies in the present studies.

Average excitation (\( \bar{Q}_e \)), ionization (\( \bar{I} \)), and secondary (\( \bar{\varepsilon} \)) energies (per event) are defined as

\[ \bar{Q}_e = \frac{L_e(T)}{\sigma_e(T)}, \]  
\[ \bar{I} = \frac{L_i(T)}{\sigma_i(T)}, \]  
\[ \bar{\varepsilon} = \frac{L_s(T)}{\sigma_s(T)}. \]

and shown in Fig. 3. There is very little change in \( \bar{I} \) and \( \bar{Q}_e \) for energies greater than -100 eV (asymptotically, \( \bar{I} = 14.5 \) eV and \( \bar{Q}_e = 11.9 \) eV). The average energy of the secondary electron can be expressed as a function of the incident primary energy, with an accuracy of better than 4%, by

\[ \bar{\varepsilon} = 7.93 \left( \ln T - 2.88 \right), \quad 100 \text{ eV} \leq T \leq 3 \text{ keV}, \]  
\[ \bar{\varepsilon} = 2.75 \left( \ln T + 6.78 \right), \quad 3 \text{ keV} \leq T \leq 10 \text{ MeV}. \]

These averages show that most of the energy in a typical collision goes into producing secondaries.
The CSDA is valid for energies greater than approximately 1 keV. Within this approximation, total excited state and ion populations after degradation of the primary may be calculated using the ratios $\sigma_e^T(T)/L(T)$ and $\sigma_i^T(T)/L(T)$. These quantities are shown in Fig. 4.

The time-dependent Boltzmann equation was solved using numerical methods discussed in the appendix of paper I. In general, the distribution function, $f(T,t)$, relaxes to a steady-state solution, $f(T)$. For a 10 MeV beam, this relaxation is shown in Fig. 5. The characteristic relaxation time is energy-dependent. Intermediate energies ($\sim 100$ eV) relax first, followed by the lower part of the spectrum ($\sim 10$ eV, but greater than the lowest excitation energy), and, finally, the high energies ($\geq 1$ keV). Similar observations were made by Bretagne et al. for Ar and in paper I for oxygen. Figure 6 shows steady-state distribution functions for various beam energies.

Yield spectra, $Y(T)$, $[= N_o \sigma(T)v(T)f(T)]$ are shown in Fig. 8. These are proportional to similar quantities analyzed by Green and coworkers. For completely stopped source electrons (from 50 eV to 10 keV), Green et al. find that the yield functions fall off with energy as $T^{-1.58}$. Our oxygen results show that this energy dependence holds for beam sources as well. For nitrogen, the energy behavior of the yield spectra are also independent of source energy. This result was also found by Garvey et al. for $H_2$ and beam energies up to 10 MeV.

As mentioned previously, the energy necessary to produce an electron-ion pair, $\Delta W$, is particularly useful for simplifying the description of ionization in a gas. For beam sources, it is given by

$$\Delta W = \frac{N_o N_b(T_b) v(T_b) L(T_b)}{\int S(T) \, dT + \sum N_o \int \sigma_i(T) v(T) f(T,t) \, dT}$$

where the first term in the denominator gives the rate for producing electron-ion pairs directly by beam ionization and the second term is the production rate for all generations of secondaries. Equation (29)
shows that $\Delta W$ is, in general, a time-dependent function, through its
dependence on $f(T,t)$. The steady-state values of $\Delta W$ for beam energies
ranging from 100 eV to 10 MeV are shown in Table 6. These values are
nearly constant (to within 2%) over the entire energy range. The near
constancy of $\Delta W$ at energies $\geq$ 100 eV is well-known.10,34,44 Fano44
attributes this result to the fact that the ratios of the excitation and
ionization cross sections are insensitive to energy. Garvey et al.10
use the independence of the yield spectra on source energy to give an
analytic demonstration of this effect.

For a given state, the production efficiency, $P_j(T_b)$, is defined as
the number of excitations of that state per electron-ion pair created,
i.e.,

$$P_j(T_b) = \frac{\int \sigma_j(T) v(T) f(T,t) \, dT}{\int \frac{N_o N_b(T_b) v(T_b) L(T_b)}{\Delta W}} .$$

For a 10 MeV beam, the six largest production efficiencies are given in
Table 7. These production efficiencies are nearly constant for beam
energies of 100 eV to 10 MeV. The production efficiency for the single
ionization continuum is unity.

The results discussed above depend, in part, on the value of the
assumed ionization fraction.2 The sensitivity of these results has been
investigated. In particular, Fig. 8 shows that the distribution
function, $f(T)$, is insensitive to changes in the background ionization
for energies greater than $\sim$ 20 eV, but highly sensitive for lower
energies. Table 8 shows $\Delta W$ and the percentage of deposition energy lost
to the background electrons for various ionization fractions. As
expected, the energy loss increases with increasing numbers of
background electrons, however, $\Delta W$ remains nearly constant until the
fraction approaches $10^{-2}$.
4. SUMMARY

A discrete model has been used to study energy deposition by relativistic electron beams in atomic nitrogen. For beam energies between 100 eV and 10 MeV, the energy required to produce an electron-ion pair is approximately 31 eV. For a 10 MeV beam, ΔW is also constant for background ionization fractions between 0.0 and 10⁻³. Production efficiencies remain nearly constant over most of the energy range. The time-dependent model is used to observe the relaxation of the secondary electron distribution to a steady state solution. These are the first results for deposition in atomic nitrogen.

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35. A.T. Jusick, C.E. Watson, L.R. Peterson, and A.E.S. Green, J. Geophys. Res. 72, 3943 (1967).


44. U. Fano, Phys. Rev. 70, 44 (1946).
Table 1

Electron Impact Excitation Cross Section Parameters for Optically Allowed Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>$v_j$</th>
<th>$A_j$</th>
<th>$C_j$</th>
<th>$f_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4p^4s$</td>
<td>10.33</td>
<td>1.0</td>
<td>0.3125</td>
<td>0.130</td>
</tr>
<tr>
<td>$4p^2s2p^4$</td>
<td>10.92</td>
<td>1.0</td>
<td>0.3125</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Table 2

Electron Impact Excitation Cross Section Parameters for Metastable-State Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>$v_j$</th>
<th>$A_j$</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>≤ 40.0 eV</th>
<th>≥ 40.0 eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2p^0 2p^3$</td>
<td>2.38</td>
<td>0.025</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>≤ 40.0 eV</td>
<td>≥ 40.0 eV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>227.72</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2p^0 2p^3$</td>
<td>3.57</td>
<td>0.0175</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>≤ 40.0 eV</td>
<td>≥ 40.0 eV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>356.04</td>
<td>3</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
### Table 3

Electron Impact Excitation Cross Section Parameters for Rydberg-State Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>$v_j$</th>
<th>$c_j$</th>
<th>$f_j$</th>
<th>$a$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2\text{P}$ 3p</td>
<td>10.68</td>
<td>0.40</td>
<td>0.150</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^2\text{S}^o$ 3p</td>
<td>11.60</td>
<td>0.30</td>
<td>0.191</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^4\text{D}^o$ 3p</td>
<td>11.75</td>
<td>0.10</td>
<td>0.130</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$^4\text{P}^o$ 3p</td>
<td>11.84</td>
<td>1.50</td>
<td>0.130</td>
<td>0.7</td>
<td>2.5</td>
</tr>
<tr>
<td>$^4\text{S}^o$ 3p</td>
<td>11.99</td>
<td>0.07</td>
<td>0.150</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>$^2\text{D}^o$ 3p</td>
<td>12.00</td>
<td>0.14</td>
<td>0.140</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^2\text{F}^o$ 3p</td>
<td>12.12</td>
<td>0.40</td>
<td>0.150</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^2\text{D}$ 3s'</td>
<td>12.35</td>
<td>--</td>
<td>--</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^2\text{P}$ 3d</td>
<td>12.97</td>
<td>0.40</td>
<td>0.150</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^4\text{F}$ 3d</td>
<td>12.98</td>
<td>0.10</td>
<td>0.130</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$^2\text{F}$ 3d</td>
<td>12.99</td>
<td>0.14</td>
<td>0.140</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3 (continued)

<table>
<thead>
<tr>
<th>State</th>
<th>$v_j$</th>
<th>$c_j$</th>
<th>$f_j$</th>
<th>a</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^4P$ 3d</td>
<td>12.99</td>
<td>1.50</td>
<td>0.130</td>
<td>0.7</td>
<td>2.5</td>
</tr>
<tr>
<td>$^4D$ 3d</td>
<td>13.01</td>
<td>0.10</td>
<td>0.130</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$^2D$ 3d</td>
<td>13.03</td>
<td>0.14</td>
<td>0.140</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$^2D^o$ 3p'</td>
<td>13.70</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^2P^o$ 3p'</td>
<td>13.72</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^2P^o$ 3p'</td>
<td>13.92</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^2S$ 3s''</td>
<td>14.41</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^2G$ 3d'</td>
<td>14.89</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$^2S$ 3d'</td>
<td>14.94</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 4

Electron Impact Excitation Cross Section Parameters
for Rydberg Series \((n \geq 4)\) Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>(\tilde{v}_j)</th>
<th>(C_j)</th>
<th>(f_j)</th>
<th>(a)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4P_{ns})</td>
<td>13.70</td>
<td>1.50</td>
<td>0.109</td>
<td>0.7</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 5

Electron Impact Ionization Cross Section Parameters

<table>
<thead>
<tr>
<th>Final State</th>
<th>(A_i)</th>
<th>(C_i)</th>
<th>(b_{io})</th>
<th>(k_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N^+(3P))</td>
<td>2.20</td>
<td>0.25</td>
<td>15.0</td>
<td>7.40</td>
</tr>
</tbody>
</table>

Table 6

\(\Delta W\) Versus Electron Beam Energy, \(T_b\)

<table>
<thead>
<tr>
<th>(T_b) (eV)</th>
<th>(10^2)</th>
<th>(10^3)</th>
<th>(10^4)</th>
<th>(10^5)</th>
<th>(10^6)</th>
<th>(10^7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta W) (eV)</td>
<td>30.9</td>
<td>30.8</td>
<td>31.0</td>
<td>31.4</td>
<td>31.4</td>
<td>30.8</td>
</tr>
</tbody>
</table>
Table 7
Production Efficiencies for a 10 MeV Beam

<table>
<thead>
<tr>
<th>State</th>
<th>P_j</th>
<th>State</th>
<th>P_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2D_0$ 2p$^3$</td>
<td>1.55</td>
<td>$^4P$ 3p</td>
<td>0.13</td>
</tr>
<tr>
<td>$^2P_0$ 2p$^3$</td>
<td>0.35</td>
<td>$^4P$ 3d</td>
<td>0.11</td>
</tr>
<tr>
<td>$^4P$ 3s</td>
<td>0.32</td>
<td>$N^+ ({}^3P)$</td>
<td>1.0</td>
</tr>
<tr>
<td>$^4P$ 2s2p$^4$</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8
$\Delta W$ and Energy Loss Versus Ionization Fraction

<table>
<thead>
<tr>
<th>Fraction</th>
<th>0.0</th>
<th>$10^{-6}$</th>
<th>$10^{-5}$</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta W$ (eV)</td>
<td>30.8</td>
<td>30.8</td>
<td>30.8</td>
<td>30.8</td>
<td>30.9</td>
<td>31.8</td>
</tr>
<tr>
<td>Energy Loss (X)</td>
<td>0.0</td>
<td>3.5</td>
<td>4.0</td>
<td>5.9</td>
<td>11.9</td>
<td>22.0</td>
</tr>
</tbody>
</table>
Fig. 1. Total excitation (long dashes), total ionization (short dashes), and total inelastic (solid line) cross sections.
Fig. 2. Total loss function (solid line) partitioned into excitation (lower solid line), ionization (long dashes), and secondary electron (short dashes) contributions and compared to Bethe's formula, Eq. (21) (dash-dotted line).
Fig. 3. Average excitation, ionization, and secondary energies defined according to Eqs. (22 - 24).
Fig. 4. Ratio of the total excitation and ionization cross sections to the total loss function.
Fig. 5. Relaxation of $f(T,t)$ to steady-state for $T = 10^1 - 10^6$ eV.
Fig. 6. Steady-state distribution function, f(T), for beam energies of 100 eV to 10 MeV.
Fig. 7. Yield spectra for beam energies of 100 eV to 10 MeV.
Fig. 8. f(T) for fractional ionizations of $0 - 10^{-2}$. 
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