One paper produced in this effort dealt with the importance of intermediate boundary conditions for approximate factorization schemes. A second paper derived stability results for spectral methods applied to initial-boundary value problems for hyperbolic systems. The paper demonstrates that one can bound certain weighted $L_2$ spatial norms of the solution in terms of norms of the boundary data. A third paper deals with domain decomposition methods in the content of spectral techniques. Stability and convergence results are obtained for one and two dimensional cases.
INSTRUCTIONS FOR PREPARATION OF REPORT DOCUMENTATION PAGE

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Block 1b. Restricted Marking: Enter the restricted marking or warning notice of the report (e.g., CNWDI, RD, NATO).

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Block 6b. Office Symbol: Enter the office symbol of the Performing Organization.

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Subject Terms: These may be descriptors, keywords, posting terms, identifiers, open-ended terms, subject headings, acronyms, code words, or any words or phrases that identify the principal subjects covered in the report, and that conform to standard terminology and are exact enough to be used as subject index terms. Each acronym or "buzz words" may be used if they are recognized by specialists in the field and have a potential for becoming accepted terms. "Laser" and "Reverse Osmosis" were once such terms.

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Responsible Individual: Give name, telephone number, and
office symbol of DoD person responsible for the accuracy of the completion of this form
I. Approximate factorization schemes are widely used to obtain efficient solutions to problems in Computational Fluid Dynamics. In many cases, they have provided a significant increase in efficiency over previously-used solution methods in particular problems. Some outstanding examples are the classical Alternating-Direction-Implicit method of Peaceman and Rachford, the Briley-McDonald Linearized Block Implicit scheme and the Beam and Warming [3] Approximate Factorization (AF) scheme for the compressible Navier-Stokes equations. In the transonic potential-flow area, some AF schemes which have significantly improved solution efficiency are the work of Ballhaus and Steger, Ballhaus et al., Holst and Jameson.

All of these schemes have the common feature that the solution procedure is broken down into a sequence of easily-implemented stages; i.e., easily-inverted matrix factors. Each of the stages usually requires boundary conditions for an "intermediate" variable (vector) which is not always a consistent approximation to the solution function desired. This feature can make satisfaction of implicit boundary conditions difficult, at best, and impossible, at worst. Dwoyer and Thames demonstrated serious boundary-condition problems associated with the class of AF schemes called "Locally One-Dimensional", even in explicit schemes.

The present paper further highlights the importance of intermediate boundary conditions by focusing on a specific example -- a boundary-induced stability restriction in Holst's AF2 scheme for the transonic full-potential equation. An analysis of the effect of the intermediate boundary condition is given by use of the usual von Neumann method and also the methods of Gustaffson, Kreiss, and Sundstrom and Osher.
II. In a recent paper, we derived stability results for spectral methods applied to initial-boundary value problems for hyperbolic systems. The paper demonstrates that one can bound certain weight $L^2$ spatial norms of the solution in terms of norms of the boundary data (homogeneous initial conditions are assumed). The bounds also contain powers of $N$, which is the degree of the approximating polynomials.

Here we show that the approximations discussed above actually converge to the exact solution, at least when this solution is smooth. We bound the error in the numerical method by a power of $N$ multiplying a term which depends only on the exact solution --- more precisely, this is the interpolation error of the initial value and boundary derivatives. For sufficiently differentiable functions, this interpolation error will decay fast enough to drive the full approximation error to zero. We have not attempted to derive the sharpest bound of this type, but merely to show that such a bound exists.

The method of proof here is similar to the one in [1,2], where basic results are first deduced for a scalar equation, and then extended to the full system. Accordingly, the paper is divided into two sections, the first dealing with the scalar case and the second with the system. By the means of Gauss-Lobatto quadrature formulas, we first bound the error at outflow for a single scalar equation. Then we use this estimate, together with the basic stability result, to bound the overall error for a system.

A paper on this subject was accepted for publication in the SIAM Journal on Numerical Analysis.
III. Recently, many numerical analysts working in the field of spectral methods are concentrating their efforts on domain decomposition methods.

The various techniques now available are in general very efficient, although often they are not supported by a theoretical background.

In this paper, we shall provide a complete theoretical analysis of the so-called patching collocation method for elliptic boundary value problems. This method was introduced by Orszag and reads as follows. Suppose that the initial domain is decomposed into rectangular regions inside of which Gauss-Lobatto points are considered. Then, the equation to be approximated is collocated at the internal nodes of each subdomain, while continuity of the polynomial solution and its normal derivative are required at the nodes of the interfaces.

Preliminary theoretical results for Legendre approximation were given. In that paper, instead of the continuity of the normal derivative, a different interface condition was imposed. The underlying idea was to also collocate the differential equation at the nodes of the interfaces, modifying the resulting expressions by additional terms, in order to ensure the continuity of the normal derivative as the polynomial degree tended to infinity. Stability and convergence results were given for the one and two dimensional cases.

Here, we shall review the results in [6] for Chebyshev approximations. The work is divided into two parts. In this first part, we shall discuss the one dimensional case, leaving to a further paper the results for the two dimensional case. Second order Neumann-Dirichlet boundary value problems are considered. We analyze two different patching conditions. The first one is the exact equivalent of that suggested. The second one is that presented. Both the schemes originate from a suitable variational formulation. We show stability and give a priori spectral type error estimates. In contrast with Legendre approximations, difficulties now arise due to the fact that integration by parts has to be handled very carefully when the
Chebyshev weight is used.

Some numerical experiments are discussed. The conclusion is that the strategy here proposed, of collocating the equation also at the interfaces (as well at the boundary nodes), gives better results than that suggested.

A paper on this subject was completed by D. Funaro.
END
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