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SOME SPECIAL CASES OF SPIN-YAW LOCK-IN

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A slightly asymmetric missile is a basically symmetric missile with a nonzero pitch moment at zero angle of attack. In flight this moment causes a trim angle that rotates with the missile and has a maximum when the spin is near resonance with the missile's natural pitch frequency. This report considers a roll moment that can be induced by this trim angle and can cause resonant lock-in spin. Simple expressions for this induced roll moment are given and the existence and stability conditions for equilibrium spin are derived. The special case of the induced roll moment caused by a radial center of mass offset is considered and a number of different types of possible equilibrium spin combinations are

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19. ABSTRACT (continued)

shown. The possibility of resonant lock-in spin in the opposite sense to the expected steady-state spin is indicated. It is further shown that there are induced roll moments for which the design steady-state spin will not occur under any launch conditions. If several stable equilibrium spins are possible, the one that occurs in flight can be determined by the orientation of the initial pitch angular velocity. Finally the form of an induced pitch moment is given and its possible effect on the angular motion is discussed.

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I. INTRODUCTION

In 1953, J. D. Nicolaides introduced the concept of spin-yaw resonance of missiles having slight configurational asymmetries.[1] In this paper, he showed the possibility of large trim angles when the spin rate happened to be near the natural pitch frequency of the missile. Later authors considered various aspects of the pitching motion of the missile when its spin was near resonance.[2-5] The combination of symmetric nonlinear aerodynamic moments and a trim moment produced by a configuration asymmetry has been studied extensively for non-resonant spin and the possible existence of subharmonic response, as well as a variety of limit motions, has been demonstrated.[6-9]

The very important question of the existence of a spin moment that forces the spin to its resonance value was first answered by Nicolaides[2] when he introduced an induced roll moment which was a function of the total angle of attack and the roll angle between a particular fin and the plane of the total angle of attack. (This "induced" roll moment is induced by the non-rotationally symmetric flow field over a finned missile at angle of attack.) Glover[10] considered the effect on spin of mass and aerodynamic asymmetries. He showed that a laterally offset c.m. location introduces an induced roll moment which is a function of two angles: (a) the total angle of attack and (b) the angle between the angle-of-attack plane and the plane containing the c.m. and axis of symmetry. This analysis has been extended to sounding rockets and re-entry vehicles.[11-12]

In this report we will consider the variety of spin lock-ins that can occur for a slightly asymmetric missile whose rolling motion is controlled by the usual linear roll moments and a generalized induced roll moment. Three different types of lock-in are shown and conditions on the induced roll moment are given. Finally, the effect of an induced pitch and yaw moment is briefly considered.

II. SLIGHTLY ASYMMETRIC MISSILE

A missile whose linear aerodynamic forces and moments have the same symmetry as those for a body of revolution will be called basically symmetric. If this basic symmetry is disturbed so that its normal force and static moment are not zero at zero angles of attack and sideslip, the missile is a "slightly asymmetric" missile.* This disturbance can be caused by small cant angles of fin surfaces and induces a trim pitch angle that rotates with the missile and is a function of the spin rate. When the spin rate is near the fast precession rate, a maximum value of the trim angle occurs and this event is called spin-pitch resonance.

Aeroballistic axes pitch and yaw with the missile but have zero roll rate. The linear aerodynamic force and moment for a slightly asymmetric missile can be written in these coordinates as:

*A slightly asymmetric missile has equal zero-spin pitch and yaw frequencies. If these frequencies differ slightly, the missile is called an "almost symmetric" missile.[13]

$$C_{\tilde{y}} + i\, C_{\tilde{z}} = -C_{N_\alpha}\, \tilde{\xi} - C_{N_o}\, e^{i(\phi+\phi_N)} \tag{2.1}$$

$$C_{\tilde{m}} + i\, C_{\tilde{n}} = \left[\phi'\, C_{M_{p\alpha}} - i\, C_{M_\alpha}\right] \tilde{\xi} - i \left(C_{M_q} + C_{M_{\dot{\alpha}}}\right) \tilde{\xi}' - i\, C_{M_o}\, e^{i(\phi+\phi_M)} \tag{2.2}$$

where

$$\tilde{\xi} = \tilde{\beta} + i\tilde{\alpha}$$

$$(\,)' = \frac{d}{ds}$$

$$s = \int_0^t (V/\ell)\, dt$$

$$\phi = \text{roll angle}$$

and where C_{N_o}, C_{M_o} are non-negative.

This force and moment can be inserted in the usual differential equations for the pitching and yawing motion:[14]

$$\tilde{\xi}'' + (H - i\sigma\phi')\, \tilde{\xi}' - (M + i\sigma\phi' T)\tilde{\xi} = -\, M_A\, e^{i(\phi+\phi_M)} \tag{2.3}$$

where

$$H = \frac{\rho S \ell}{2m} \left[C_{L_\alpha} - C_D - k_t^{-2}\left(C_{M_q} + C_{M_{\dot{\alpha}}}\right)\right]$$

$$\sigma = I_x / I_t$$

$$M = \frac{\rho S \ell^3}{2 I_t}\, C_{M_\alpha}$$

$$T = \frac{\rho S \ell}{2m} \left[C_{L_\alpha} + k_a^{-2} C_{M_{p\alpha}} \right]$$

$$M_A = - \frac{\rho S \ell^3}{2I_t} \left[C_{M_o} - i\, k_t^2 \,(1 - \sigma)\phi' C_{N_o}\, e^{i(\phi_N - \phi_M)} \right]$$

At resonance, ϕ' is of order 10^{-2} and so the C_{N_o} term in M_A can and will be neglected. For zero spin, the steady-state motion can be described by a constant trim angle:

$$\tilde{\xi} = \delta_{T0}\, e^{i\phi_M} \tag{2.4}$$

where

$$\delta_{T0} = M_A/M = - C_{M_o} \Big/ C_{M_\alpha}.$$

The independent variable in Eq. (2.3) is dimensionless time, s. For spin-yaw resonance, it is convenient to use a second dimensionless time, τ, which is defined to be

$$\tau = [-M/(1-\sigma)]^{1/2} s. \tag{2.5}$$

Eq. (2.3) now becomes

$$\ddot{\tilde{\xi}} + (\hat{H} - i\sigma\dot{\phi})\, \dot{\tilde{\xi}} + (1-\sigma - i\sigma\dot{\phi}\, \hat{T})\, \tilde{\xi} = \delta_{T0}\, (1-\sigma)\, e^{i(\phi+\phi_M)} \tag{2.6}$$

where

$$\hat{H} = H\, [-(1-\sigma)/M]^{1/2}$$

$$\hat{T} = T\, [-(1-\sigma)/M]^{1/2}$$

For constant spin, we assume the steady-state response to the aerodynamic trim has the form

$$\tilde{\xi} = \xi\, e^{i\phi} \tag{2.7}$$

where

$$\xi = \xi_T \text{ is constant.}$$

A direct substitution of Eq. (2.7) in Eq. (2.6) yields

$$\xi_T = \frac{\delta_{TO}\, e^{i\phi_M}}{1-\dot{\phi}^2 + i\dot{\phi}h} = \delta_T\, e^{i(\phi_T+\phi_M)} \tag{2.8}$$

where*

$$h = (1 - \sigma)^{-1} [\hat{H} - \sigma\hat{T}]$$

The simple form of Eq. (2.8) is due to our use of τ as the independent variable. According to Eq. (2.8), resonance occurs at $|\dot{\phi}| = 1$ and the resonance value of the trim angle magnitude is $\delta_{TR} = |h|^{-1}\delta_{TO}$. The magnitude of the trim angle grows from δ_{TO} to a maximum which is approximately δ_{TR} and then decays to zero. The phase angle varies from zero at zero spin to -90° at resonance and then to -180° for infinite spin.

III. ROLL EQUATION

The roll moment for a slightly asymmetric missile usually has two components[15] - a constant spin-producing moment caused by differential cant of the fin surfaces, δ_f, and a spin-damping moment proportional to the spin rate. If the missile is not a body of revolution, a roll moment can be induced by an angle of attack and varies with θ, the angular orientation of the angle-of-attack plane with respect to the missile.

In the usual missile-fixed axes, the complex angle of attack has the form

$$\xi = \beta + i\alpha = \delta\, e^{i\theta} \tag{3.1}$$

Since the aeroballistic axes differ from these axes by the roll angle ϕ,

$$\tilde{\xi} = \delta\, e^{i\tilde{\theta}} = \xi\, e^{i\phi} \tag{3.2}$$

$$\therefore\ \tilde{\theta} = \theta + \phi \tag{3.3}$$

*Dynamic stability near resonance requires[14] that $h < 0$.

The complete roll moment including the angle-of-attack-induced component can be written as

$$C_\ell = C_{\ell_\delta} \delta_f + C_{\ell_p} \phi' + C_{\ell_\theta}(\theta,\delta) \tag{3.4}$$

The Y-axis in the missile-fixed coordinates is usually taken to be in a plane of mirror symmetry of the basically symmetric missile. If this is the case, the induced roll moment coefficient is on odd function[16] of θ.

$$C_{\ell_\theta}(\theta,\delta) = -C_{\ell_\theta}(-\theta,\delta) \tag{3.5}$$

It therefore can be expanded as a Fourier sine series in θ. A rotationally symmetric missile with n similar fins has a symmetry angle of $2\pi/n$ and thus the induced roll moment should have this fundamental wavelength.

$$C_{\ell_\theta} = \sum_{k=1}^{\infty} a_k \sin nk\theta \tag{3.6}$$

where

$$a_k = a_k(\delta) \text{ and } a_k(0) = 0.$$

In terms of the complex angle of attack

$$\sin nk\theta = \frac{\xi^{nk} - \bar{\xi}^{nk}}{2i\delta^{nk}} \tag{3.7}$$

If we make the mathematically attractive assumption[16] that the roll moment is an analytic function of α and β, the $a_k(\delta)$ function can be expressed as a special power series in δ.

$$a_k = \delta^{nk} \sum_{\ell=0} b_{k\ell}\, \delta^{\ell} \tag{3.8}$$

The first term in this double series expression for the induced roll moment coefficient is

$$b_{10}\ \delta^n \sin n\theta = ((b_{10})/2i)\ (\xi^n - \bar{\xi}^n). \tag{3.9}$$

For all the calculations of this report, we will approximate the induced roll moment coefficient by this term. As a further convenience, we will select the orientation of the Y-axis in the plane of mirror symmetry so that b_{10} is positive. The resulting roll equation for this roll moment is:[14]

$$\phi'' + K_p\,[\phi' - \phi'_s - i\,K_\theta\,(\xi^n - \bar{\xi}^n)] = 0 \tag{3.10}$$

where
$$K_p = -\frac{\rho S \ell^3}{2\,I_x}\left[C_{\ell_p} + k_a^{\,2}\,C_D\right]$$

$$\phi'_s = K_\delta/K_p$$

$$K_\delta = \frac{\rho S \ell^3}{2\,I_x}\,\delta_f\,C_{\ell_\delta}$$

$$K_\theta = (b_{10}/2)\left[C_{\ell_p} + k_a^{\,2}\,C_D\right]^{-1}$$

For most finned missiles, K_p is positive. In the absence of an induced roll moment, Eq. (3.10) predicts a stable steady-state spin of ϕ'_s. This design steady-state spin is set by the designer through the fin differential deflection angle, δ_f.

The independent variable in Eq. (3.10) can be easily changed from s to τ and ξ can be scaled by its value at resonance.

$$\ddot{\phi} + \hat{K}_p\,[\dot{\phi} - \dot{\phi}_s - i\,G\,(\zeta^n - \bar{\zeta}^n)] = 0 \tag{3.11}$$

where
$$\hat{K}_p = K_p\,[-(1-\sigma)/M]^{1/2}$$

$$G = K_\theta\,[-(1-\sigma)/M]^{1/2}\,\delta_{TR}^{\,n}$$

$$\zeta = \xi\,\delta_{TR}^{-1}$$

The differential equation for the scaled complex angle of attack, ζ, can be obtained from Eqs. (2.6-2.7)

$$\ddot{\zeta} + [\hat{H} + i(2-\sigma)\dot{\phi}]\ \dot{\zeta} + (1-\sigma)[1-\dot{\phi}^2 + i\dot{\phi}h + i\ddot{\phi}]\ \zeta = (1-\sigma)|h|e^{i\phi_M} \tag{3.12}$$

Lock-in occurs when Eqs. (3.11-3.12) have a constant steady-state equilibrium solution

$$\dot{\phi}_e - \dot{\phi}_s = i\ G\ (\zeta_e^n - \bar{\zeta}_e^n) \tag{3.13}$$

$$\zeta_e = \frac{|h|\ e^{i\phi_M}}{1-\dot{\phi}_e^2 + i\dot{\phi}_e h} \tag{3.14}$$

ζ_e has a maximum amplitude of unity. Thus, if G is small compared to $\dot{\phi}_s$, the induced roll moment has very little effect on equilibrium spin, and the usual steady-state spin occurs. If, however, this is not the case, the induced roll moment can have a large contribution for $\dot{\phi}_e \cong \pm 1$ and the roll rate can be locked in at resonant spin ($\dot{\phi}^2 = 1$). At resonance, θ is $\phi_M - (\pi/2)$. As we shall see, lock-in can occur both for the normal case of resonant spin in the same sense as the expected steady-state spin and for the reverse case of resonant spin with the opposite sense.

IV. LOCK-IN STABILITY

A number of solutions to Eqs. (3.13-3.14) can exist. These may be found graphically by plotting the curve

$$y = f_1\ (\dot{\phi}_e) = i\ G\ (\zeta^n - \bar{\zeta}^n) \tag{4.1}$$

and finding its intersection with the line*

$$y = f_2\ (\dot{\phi}_e) = \dot{\phi}_e - \dot{\phi}_s \tag{4.2}$$

*A VAX 11/780 program has been written by J.W. Bradley, Launch and Flight Division, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, to find all such intersections.

Some of these equilibrium points, however, may be unstable and have no engineering significance.

We will assume a small perturbation of an equilibrium solution.

$$\dot{\phi} = \dot{\phi}_e + \eta_1 \qquad (4.3)$$

$$\zeta = \zeta_e + \eta_2 + i\ \eta_3 \qquad (4.4)$$

where

$$\eta_j = \eta_j(\tau).$$

Substitution of Eqs. (4.3-4.4) in Eqs. (3.11-3.12) yields a fifth-order differential system in the η_j's. Next, we assume a coupled exponential solution for these perturbation functions:

$$\eta_j = \eta_{j0}\ e^{\lambda\tau} \qquad (4.5)$$

where the η_{j0}'s are constants.

Direct substitution gives three equations for the three η_{j0}'s.

$$\sum_{j=1}^{3} a_{jk}\ \eta_{j0} = 0; \qquad k = 1,2,3 \qquad (4.6)$$

where the a_{jk}'s are given in TABLE 1.

The determinant of the system (4.6) must be zero for a non-trivial solution. This condition reduces to a fifth-order polynomial equation in λ. J.W. Bradley's VAX program inspects the coefficients of this polynomial and uses the Routh-Hurwitz criterion to determine the presence of negative real parts in any of the five roots. If all of the roots have negative real parts, the equilibrium is stable.

It should be noted that the location of equilibrium points is determined by the parameters n, $\dot{\phi}_s$, G, ϕ_M, and h. The stability of these equilibrium points requires values of $\hat{H}$, $\hat{K}_p$, and σ in addition to these basic five parameters.

TABLE 1. Stability Matrix Elements, a_{jk}.

$$a_{11} = \lambda + \hat{K}_p$$

$$a_{21} = -in \hat{K}_p G (\zeta_e^{n-1} - \bar{\zeta}^{n-1})$$

$$a_{31} = n \hat{K}_p G (\zeta_e^{n-1} + \bar{\zeta}_e^{n-1})$$

$$a_{21} = -R \{\zeta_e [(2\dot{\phi}_e - ih)(1-\sigma) - i\lambda]\}$$

$$a_{22} = \lambda^2 + \hat{H} \lambda + (1-\sigma)(1-\dot{\phi}_e^2)$$

$$a_{23} = -\dot{\phi}_e [(2-\sigma) \lambda + (1-\sigma) h]$$

$$a_{31} = -I \{\zeta_e [(2\dot{\phi}_e - ih)(1-\sigma) - i \lambda]\}$$

$$a_{32} = \dot{\phi}_e [(2-\sigma) \lambda + (1-\sigma) h]$$

$$a_{33} = \lambda^2 + \hat{H} \lambda + (1-\sigma)(1-\dot{\phi}_e^2)$$

V. CENTER OF MASS OFFSET

When n is three or greater, a missile has trigonal or greater rotational symmetry and its linear force and moment coefficients have the same symmetry as a body of revolution.[16] Digonal rotational symmetry ($n = 2$) would occur, for example, when a four-fin missile has pairs of fins with unequal areas. The theory for almost symmetric missiles would apply to the motion of digonal missiles. Although $n = 1$ really implies no rotational symmetry, symmetric missiles can have induced roll moments with this value of n if their centers of mass have a radial offset.

We will assume the center of mass to be radially offset by a distance of $\ell\hat{r}_c$ in the plane $\phi = 0$. The normal force can then exert a roll moment with a lever arm of $\ell\hat{r}_c \sin\theta$.

$$M_{x\theta} = F_N \, \ell\hat{r}_c \sin\theta$$

$$= -\rho S \ell V^2 \left(\frac{i}{4}\right) \hat{r}_c \, C_{N_\alpha} (\xi - \bar{\xi}) \tag{5.1}$$

If Eq. (5.1) is compared with Eqs. (3.4 and 3.9), we see that the radially offset center of mass produces an induced roll moment with $n = 1$, $b_{10} = \hat{r}_c \, C_{N_\alpha}$.

The offset center of mass can also produce a trim pitch moment since the drag force now has the linear arm $\ell\hat{r}_c$.

$$(C_{\tilde{m}} + i \, C_{\tilde{n}})_0 = -i \, \hat{r}_c \, C_D \, e^{i\phi} \tag{5.2}$$

$$\therefore \; \phi_M = 0, \qquad C_{M_0} = \hat{r}_c \, C_D \tag{5.3}$$

VI. INDUCED PITCH MOMENT

If an induced roll moment dependent on θ is present, the transverse moment expansion can also have a term dependent on θ.

$$(C_{\tilde{m}} + i \, C_{\tilde{n}})_\theta = -i \, C_{M_\theta} (\theta, \delta) \; \xi \, e^{i\phi} \tag{6.1}$$

For an n-gonal rotationally symmetric missile, the simplest expression for C_{M_θ} is:[16]

$$C_{M_\theta} = a_0 \, \delta^{n-2} \, e^{-n\theta i} \tag{6.2}$$

For $n = 1$, C_{M_θ} is $a_0 \, \xi^{-1}$ and the induced pitch moment is the trim pitch moment of Eq. (5.2). For $n > 1$, Eq. (3.12) becomes a nonlinear differential equation.

$$\ddot{\zeta} + [\hat{H} + i\ (2-\sigma)\ \dot{\phi}]\ \dot{\zeta} + (1-\sigma)\ [1-\dot{\phi}^2 + m_\theta + i\ \dot{\phi}\ h + i\ \ddot{\phi}]\ \zeta$$

$$= (1-\sigma)\ |h|\ e^{i\phi_m} \tag{6.3}$$

where $m_\theta = m_\theta\ (\theta,\ |\zeta|\ \delta_{TR}) = C_{M_\theta} / C_{M_\alpha}$.

The steady-state trim equation for specified spin becomes a nonlinear equation in ζ_e and θ_e

$$\zeta_e = \frac{|h|\ e^{i\phi_M}}{1 - \dot{\phi}^2 + m_\theta + i\ \dot{\phi}\ h} \tag{6.4}$$

For m_θ small compared with unity, it has very little effect on the real part of the denominator but can have a significant effect on the imaginary part. Thus, the resonance value of $|\zeta_e|$ can exceed unity. A much more important effect of m_θ can be to make all solutions for ζ_e unstable. If that is the case, ζ can vary through very large values. This possibility has been denoted as catastrophic yaw by Nicolaides.[2]

VII. DISCUSSION

According to Eqs. (3.13-3.14), the induced roll moment has maximum amplitudes near resonance. For even values of n, and $n\phi_M = \pm\pi/2, \pm 3\pi/2, \ldots$, $|f_1|$ has an absolute maximum of $2|G|$ at $\dot{\phi}_e = \pm 1$. For odd values of n, the same absolute maximum occurs at $n\phi_M = \pm 0, \pm\pi, \ldots$. Thus, resonant lock-in is only possible for $|\dot{\phi}_s| < 2G$. In actuality, for specific n, occurrence of lock-in depends on the specific values of $(h, \dot{\phi}_s, G, \phi_M)$ while the stability of the lock-in depends on $(\hat{H}, \hat{K}_p, \sigma)$. Throughout this section, $h = \hat{H} = \hat{K}_p = \sigma = 0.1$ and we will only consider different values of $\dot{\phi}_s$, G, ϕ_M .

In order to consider lock-in in more detail, we will limit the remainder of this discussion to the case of n = 1. For this case of an offset center of mass and $\phi_M = \pi$, f_1 has a minimum of $-2G$ at $\dot{\phi}_e = -1$, and a maximum of $2G$ at $\dot{\phi}_e = 1$. A simple analysis further shows that for $\phi_M = \pi/2$, f_1 varies from a minimum near -G at $\dot{\phi}_e = -1 - h$ to a maximum near G at $\dot{\phi}_e = -1 + h$ and similarly varies from a maximum near G at $\dot{\phi}_e = 1 - h$ to a minimum near -G at $\dot{\phi}_e = 1 + h$. Finally, $f_1(\theta) = -f_1(\theta+\pi)$. In Figures 1 and 2, $f_1(\dot{\phi}_e)$ is plotted for $\phi_M = \pi/2, 3\pi/2$, respectively, for G = 5. The dotted lines in these figures are $f_2(\dot{\phi}_e)$ for $\dot{\phi}_s = 3$. The parameters for these figures are the first two entries in TABLE 2.

TABLE 2. Illustrated Examples of Various Lock-In Cases.

Case	$\dot{\phi}_s$	G	ϕ_M	Stable $\dot{\phi}_e$'s	Type
1.	3.0	5.0	90°	1.01, 2.68	SN
2.	3.0	5.0	270°	-.98, 3.11	SR
3.	0.5	1.0	180°	-1.03, .52, 1.08	SNR
4.	0.5	0.5	90°	.69, .97	SN
5.	0.5	2.0	270°	-.98, .10	SR
6.	0.5	1.0	90°	.99	N
7.	0.5	5.0	270°	-.99	R

In both figures there are five equilibrium points. In Figure 1, the slightly modified steady-state spin, $\dot{\phi}_e = 2.68$, and the normal resonance, $\dot{\phi}_e = 1.01$, are the only stable equilibrium points. This is the normal lock-in model considered by engineers. If spin starts near zero, designers hope it accelerates through resonance fast enough to avoid capture and reaches steady-state spin with only a slight stimulus to its pitching motion caused by passage through resonance.

In Figure 2, the two stable equilibrium points are the steady-state spin of 3.11 and the reverse resonance spin of -.98. This resonant lock-in spin in the opposite sense to the design spin is an unexpected result. We will denote normal resonant lock-in spin by N, steady-state spin by S, and reverse resonant lock-in spin by R. Then the two stable equilibrium spins of Figure 1 could be identified by SN and the two stable equilibrium spins of Figure 2 by SR.

If we now consider $\dot{\phi}_s$ less than unity, even more remarkable possibilities appear. Five examples are given in TABLE 2. Case 3 in TABLE 2 is particularly interesting; the corresponding equilibrium spin determination is shown in Figure 3. Since $\phi_M = 180°$, f_1 has only one maximum and one minimum. There are, however, three stable equilibrium spins and this case is denoted by SNR. The next two entries in the Table are examples of types SN and SR for $|\dot{\phi}_s| < 1$ and are quite similar to the first two entries. The final two entries represent new types, N and R, for which no stable steady-state spin exists. Thus the induced roll moment can completely overpower the design steady-state spin. (Cases 4 to 7 are shown in Figures 7 - 10.)

For the cases of two or three stable equilibrium spins, the equilibrium that occurs in flight is determined by initial conditions. Equations (3.11-3.12) form a fifth-order differential system. The necessary five initial conditions are the initial spin rate $\dot{\phi}_0$, the initial complex angle ζ_0, and the initial complex angular velocity. For simplicity, we will let $\dot{\phi}_0 = \zeta_0 = 0$ and consider only the magnitude and orientation of $\dot{\zeta}_0$.

$$\dot{\zeta}_0 = |\dot{\zeta}_0| \, e^{i\theta^*} \tag{7.1}$$

For case 1, which was a type SN with $\dot{\phi}_s = 3$, $|\dot{\zeta}_0|$ was set at .1 and θ^* was varied. As can be seen from Figure 4, steady-state spin occurs for $\theta^* = 180°$ and normal lock-in spin for $\theta^* = 0$. For case 2, which was type SR with $\dot{\phi}_s = 3$, $|\dot{\zeta}_0|$ was set at 1, steady-state spin occurred for $\theta^* = 90°$ and reverse lock-in spin for $\theta^* = 270°$ (Fig. 5). Finally, for case 3 which was type SNR with $\dot{\phi}_s = .5$, $|\dot{\zeta}_0|$ was 3, steady-state spin occurred for $\theta^* = 0$, normal lock-in spin for $\theta^* = 90°$ and reverse lock-in spin for $\theta^* = 270°$ (Fig. 6). Therefore the determination of which equilibrium spin occurs in flight can be made by the orientation of the initial angular velocity.

VIII. SUMMARY

1. A roll moment can be induced by the missile's pitching and yawing motion. Simple expressions for this roll moment have been given for aerodynamically symmetric missiles and for missiles with mass asymmetries.

2. These induced roll moments can cause the rolling motion of slightly asymmetric missiles to have a variety of steady-state values. Examples of normal resonant lock-in spins in the direction of spin and reverse resonant lock-in spin in the opposite direction are shown.

3. For multiple stable equilibrium spins, the orientation of the initial pitch angular velocity can determine which one occurs in a given flight.

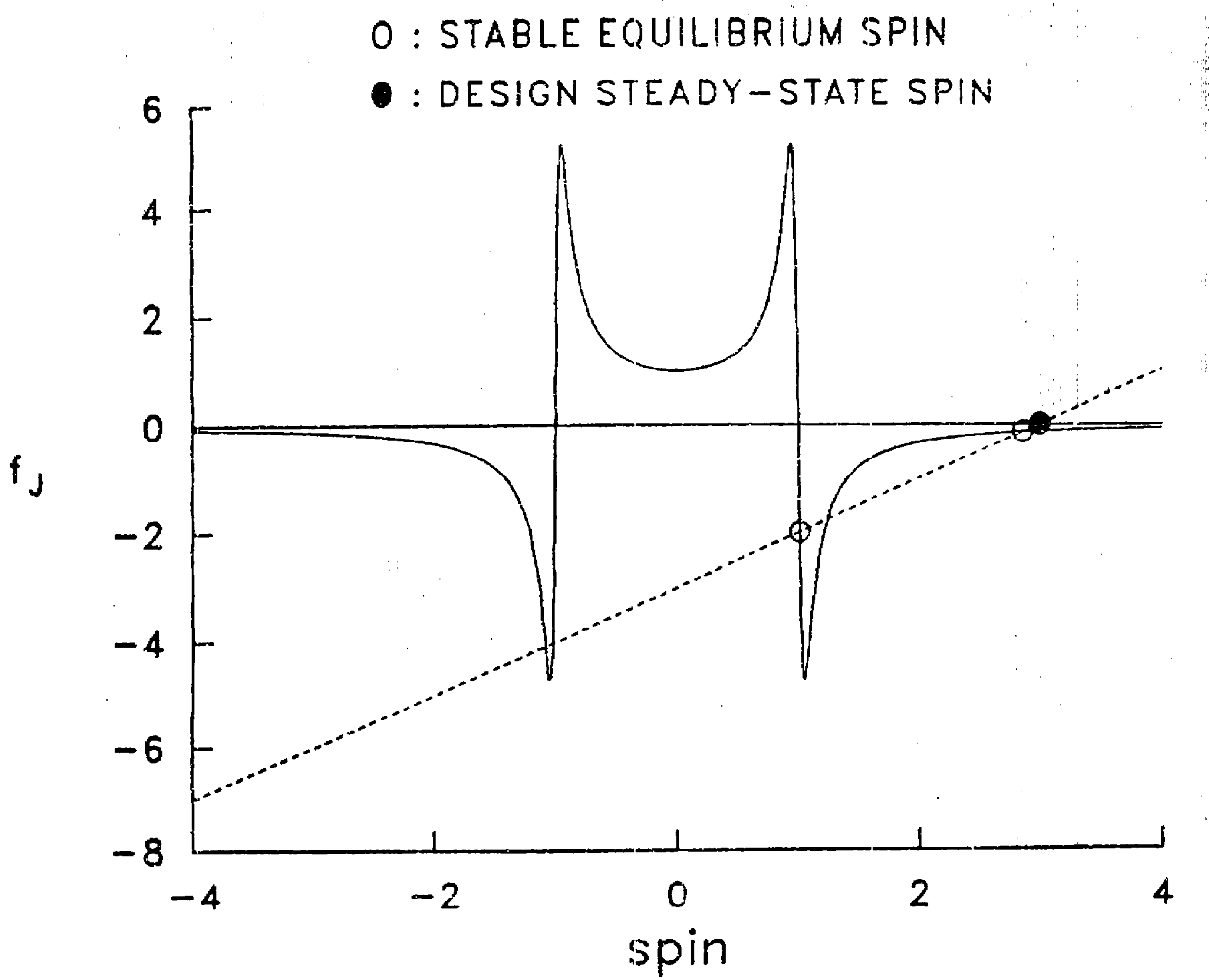

Figure 1. f_j versus $\dot{\phi}$ for case 1. Design steady-state spin and normal lock-in.

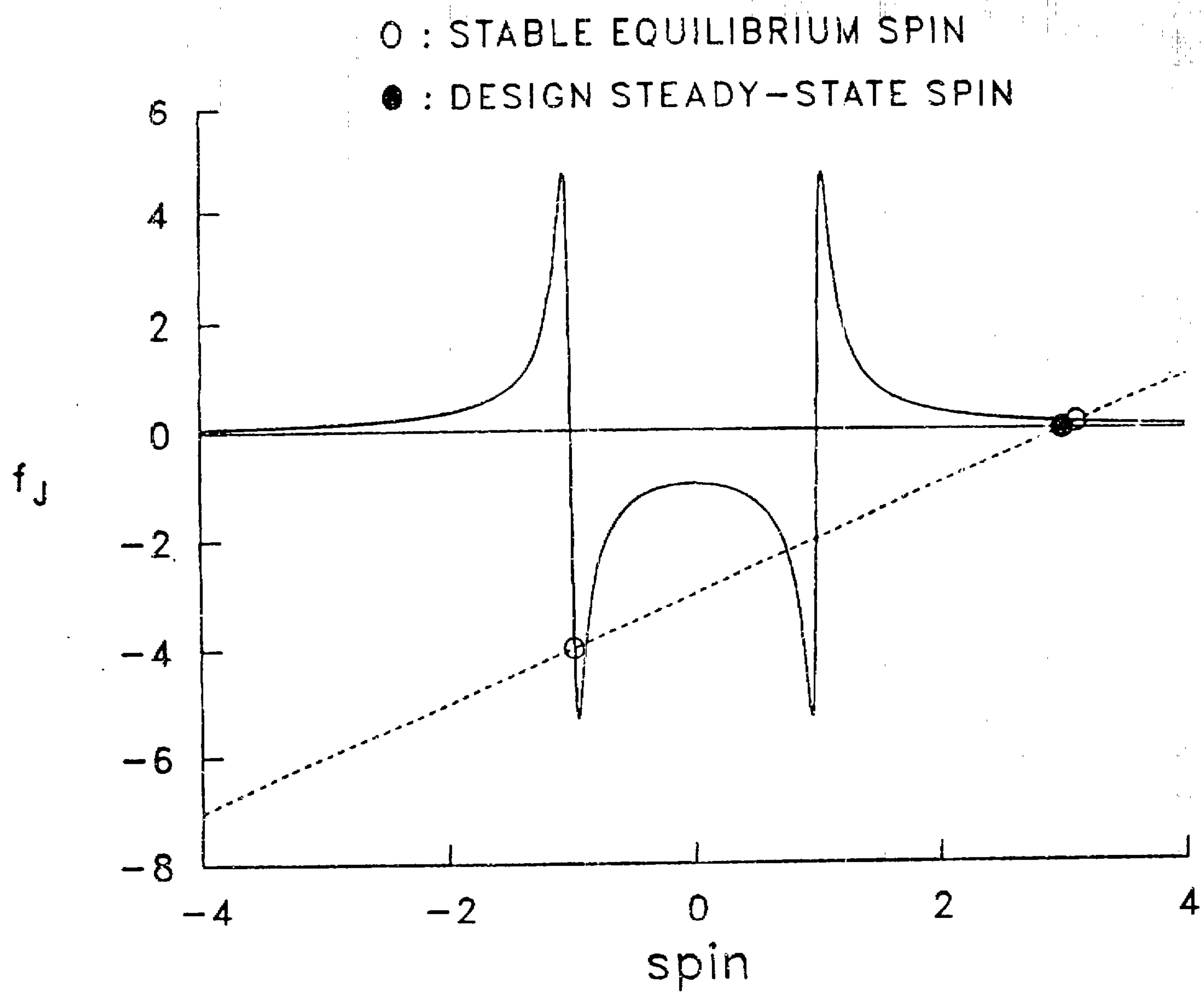

Figure 2. f_j versus $\dot{\phi}$ for case 2. Design steady-state spin and reverse lock-in.

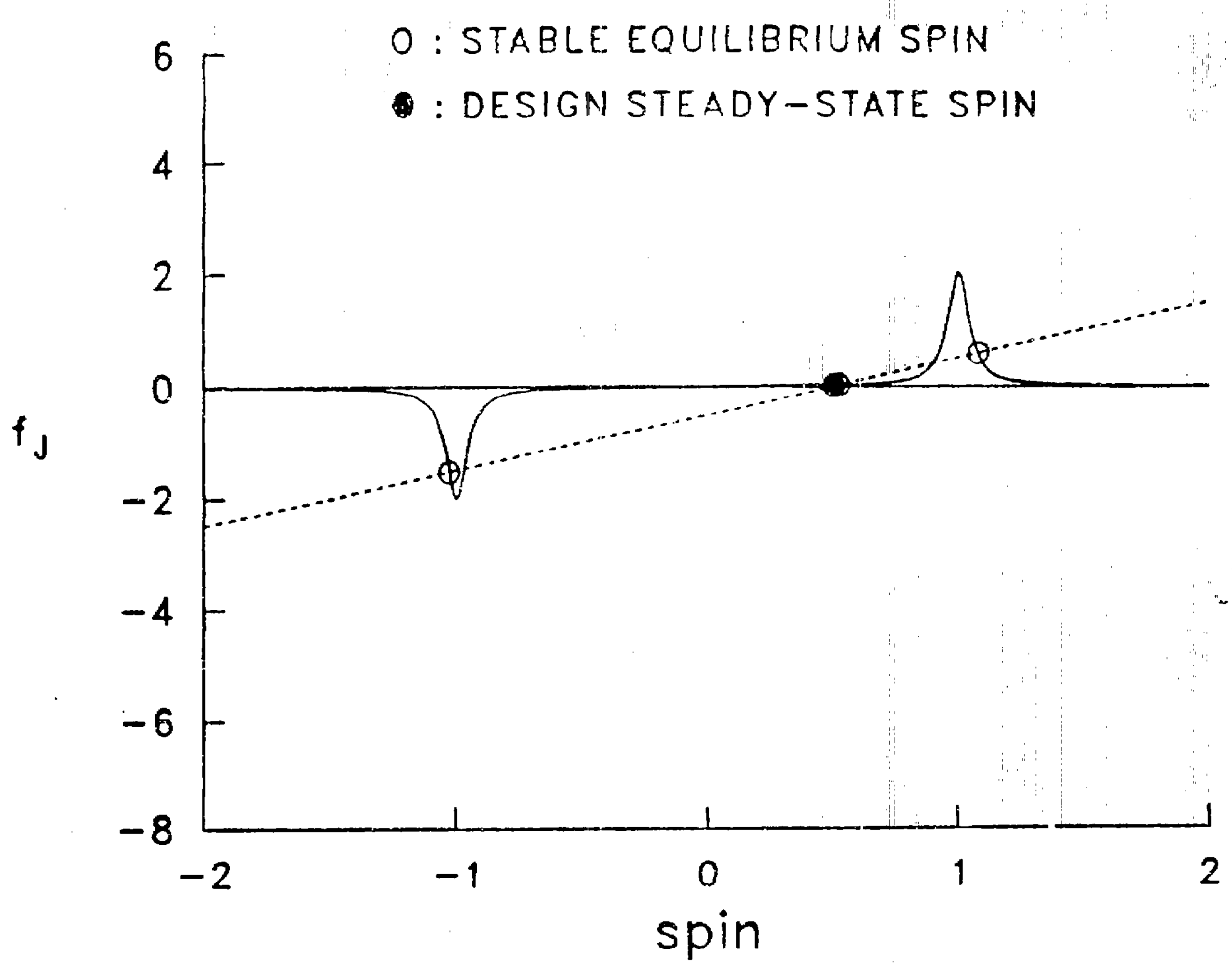

Figure 3. f_j versus $\dot{\phi}$ for case 3. Design steady-state spin, normal lock-in, and reverse lock-in.

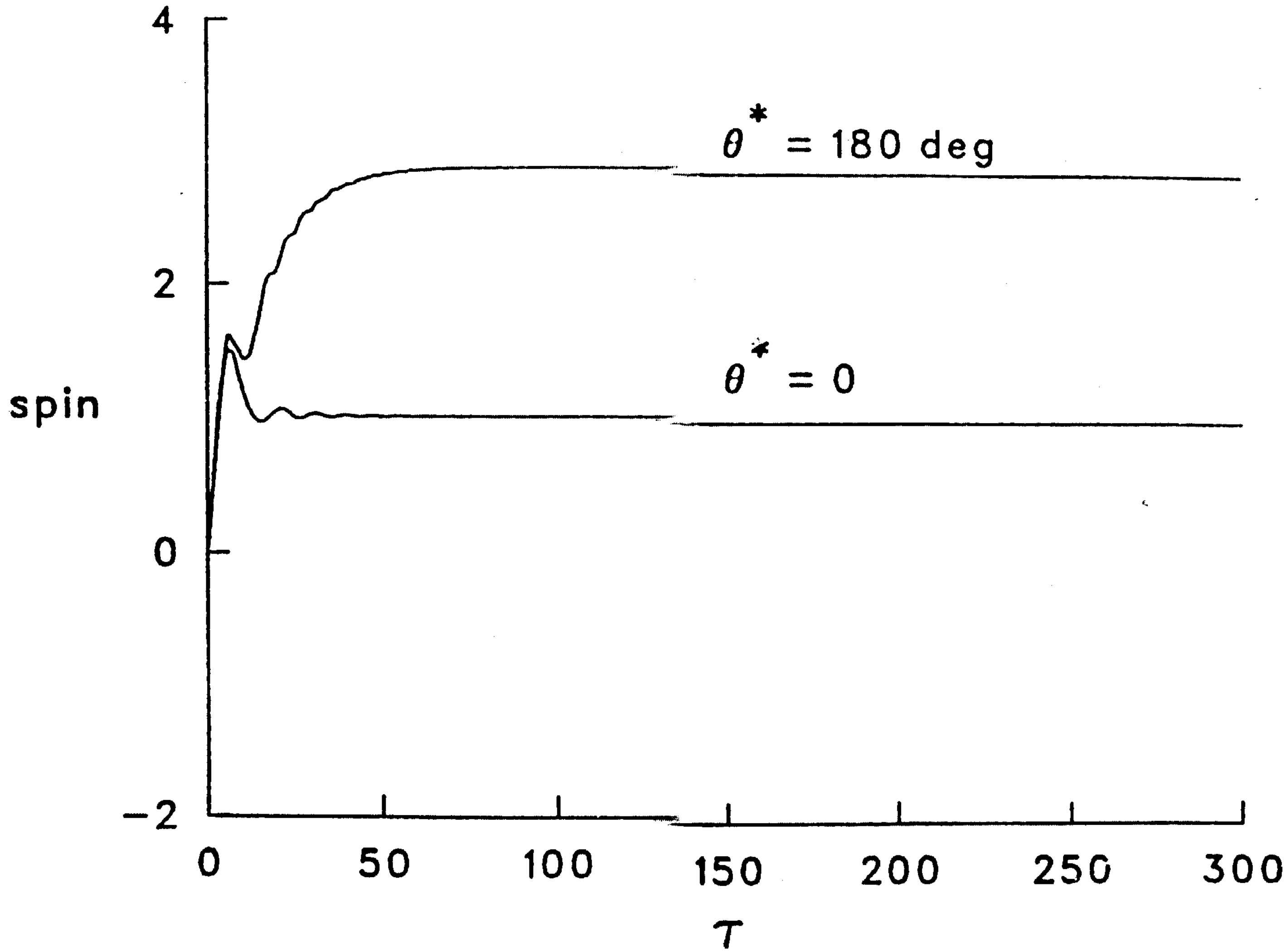

Figure 4. $\dot{\phi}$ versus τ for case 1 with $|\dot{\zeta}| = 0.1$. Design steady-state spin occurs for $\theta^* = 180°$ while normal lock-in occurs for $\theta^* = 0°$.

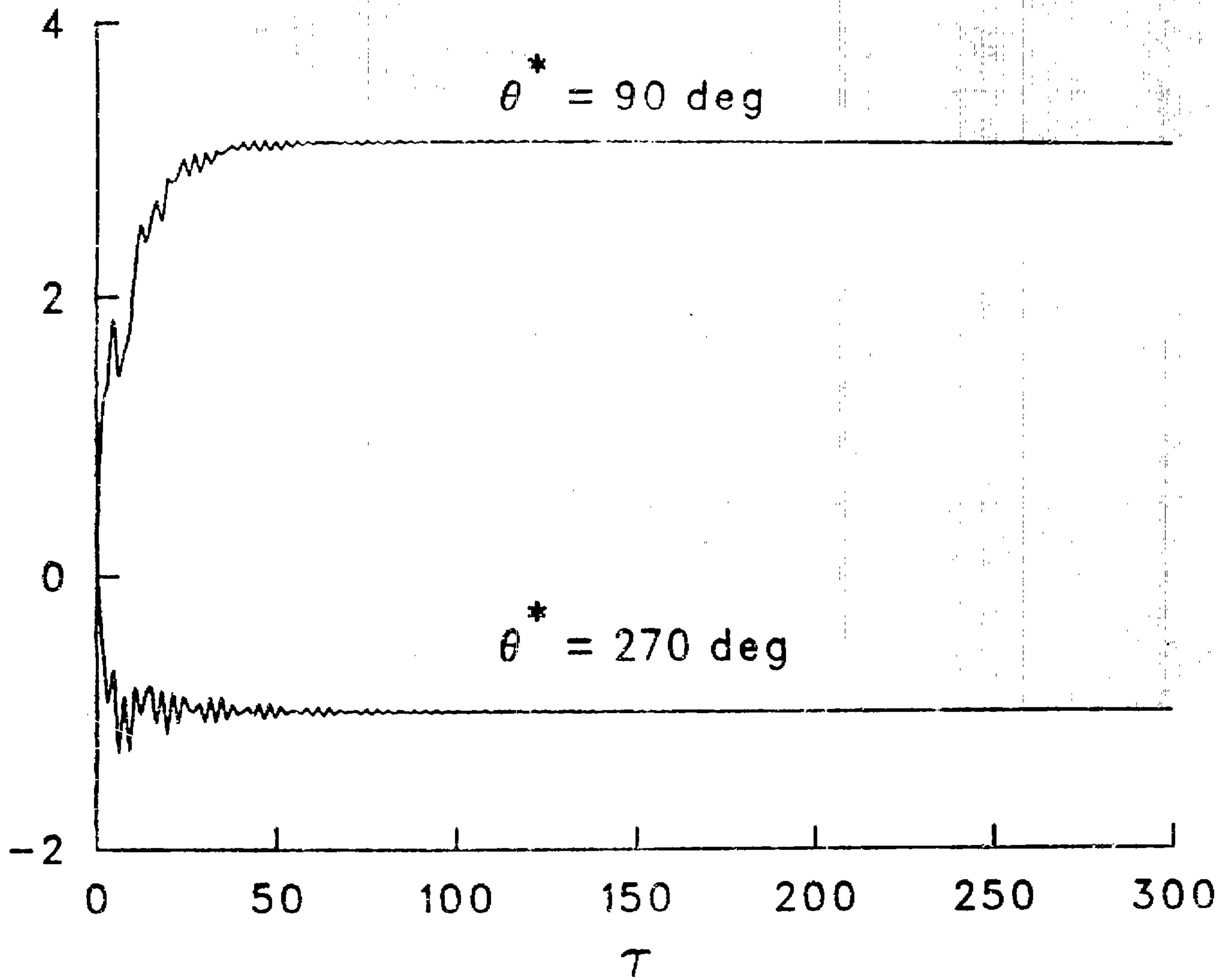

Figure 5. $\dot{\phi}$ versus τ for case 2 with $|\dot{\zeta}| = 1.0$. Design steady-state spin occurs for $\theta^* = 90°$ while reverse lock-in occurs for $\theta^* = 270°$.

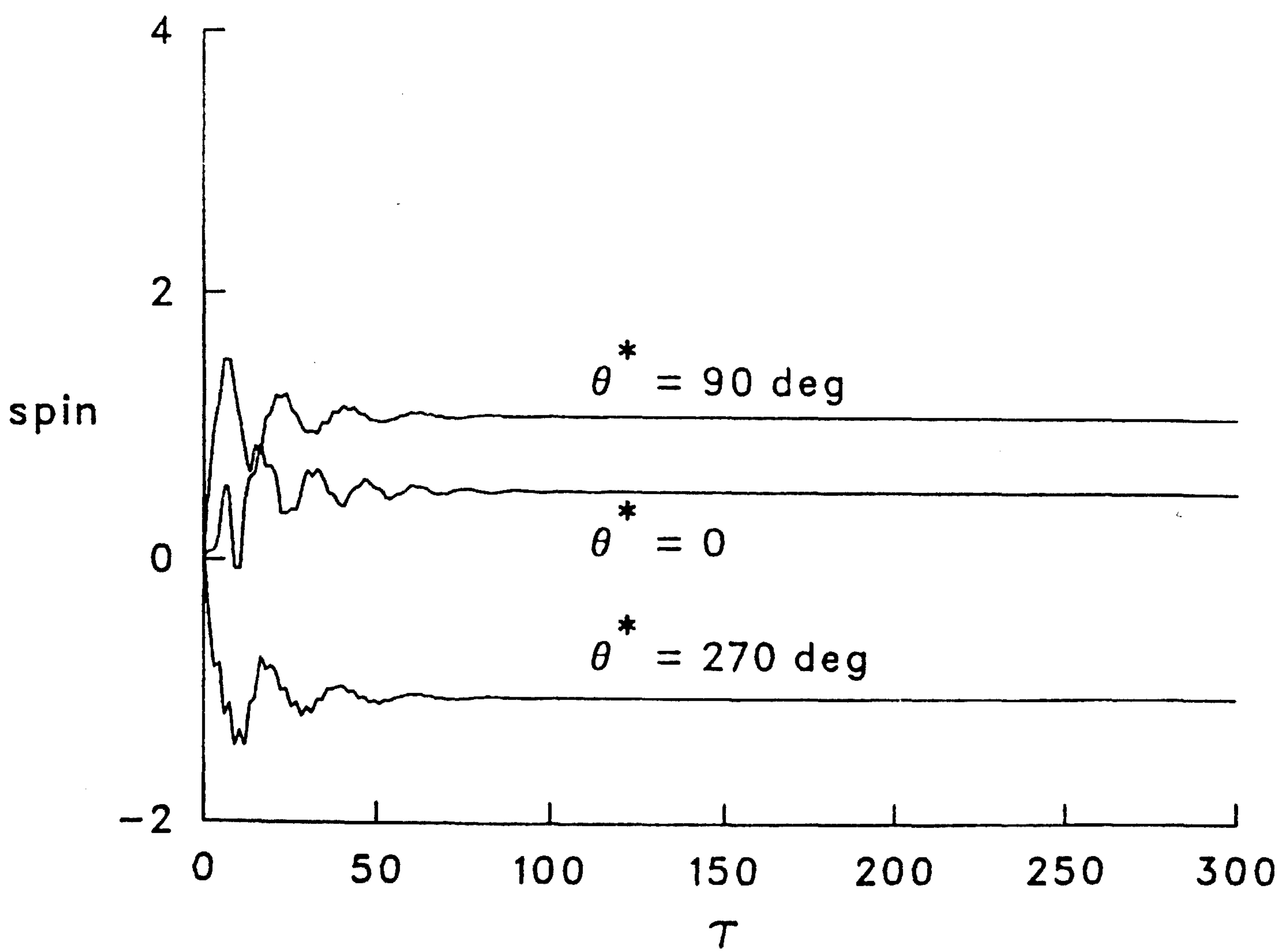

Figure 6. $\dot{\phi}$ versus τ for case 3 with $|\dot{\zeta}| = 3$. Design steady-state spin occurs for $\theta^* = 0$, normal lock-in for $\theta^* = 90°$, and reverse lock-in for $\theta^* = 270°$.

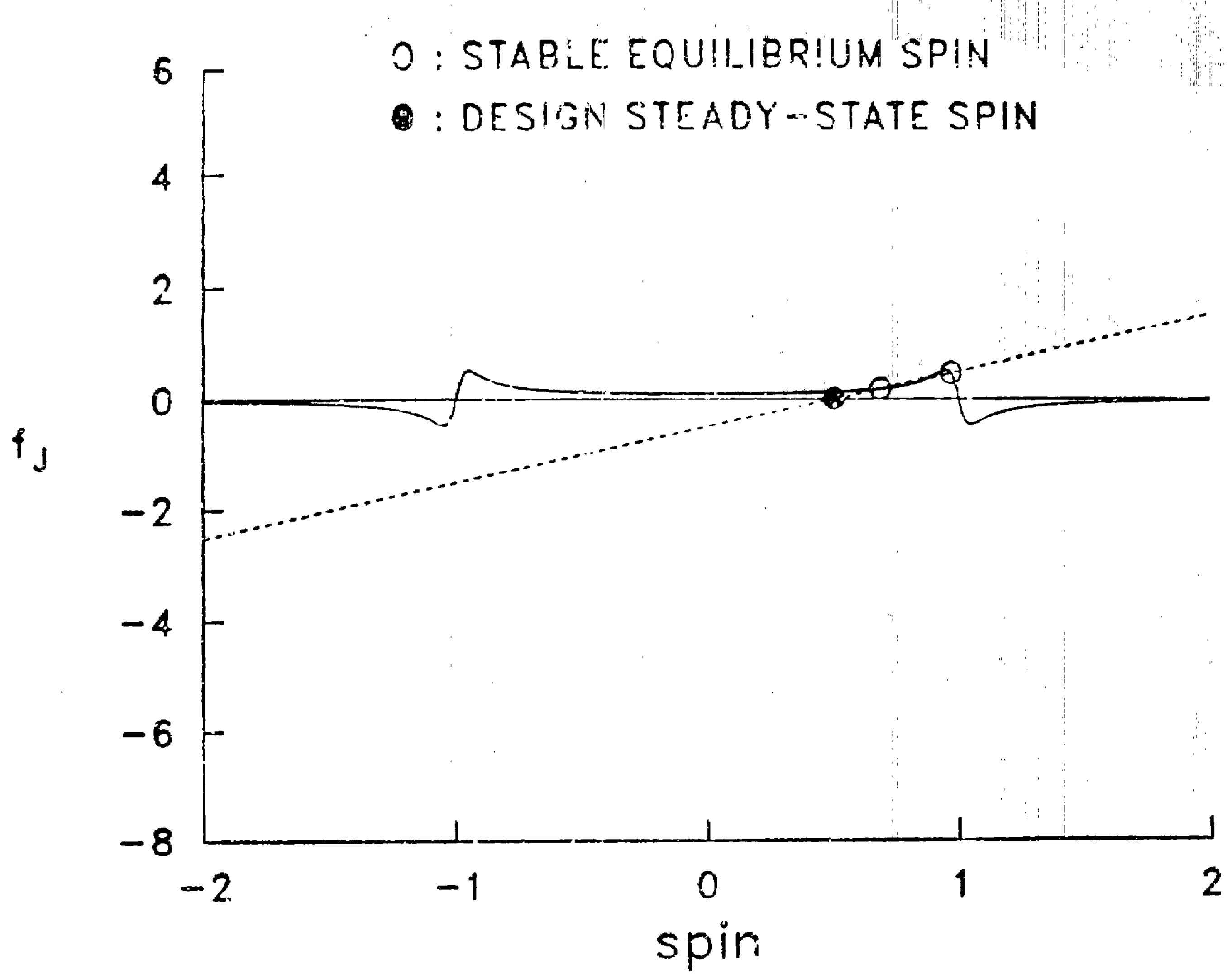

Figure 7. f_j versus $\dot{\phi}$ for case 4. Design steady-state spin and normal lock-in.

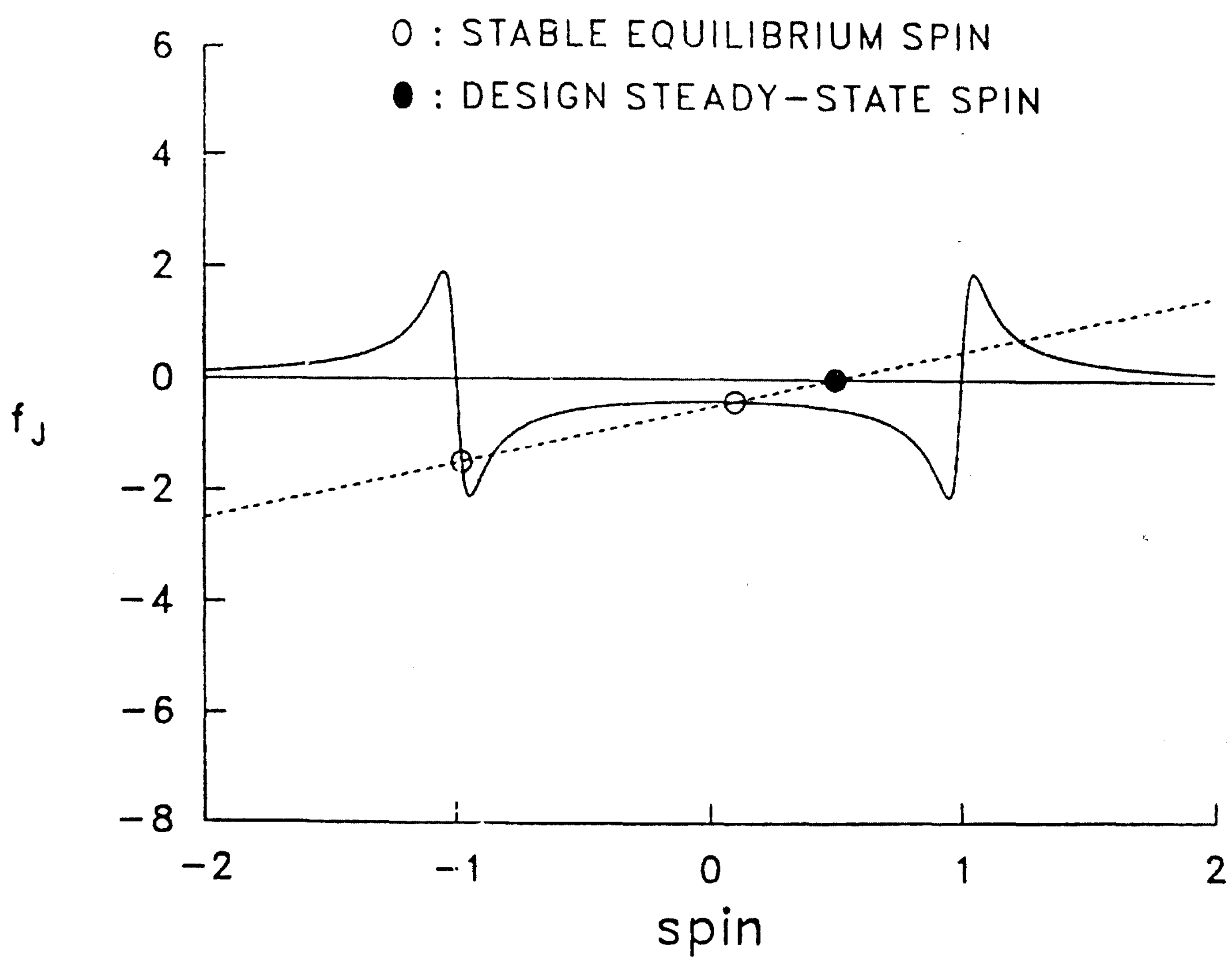

Figure 8. f_j versus $\dot{\phi}$ for case 5. Design steady-state spin and reverse lock-in.

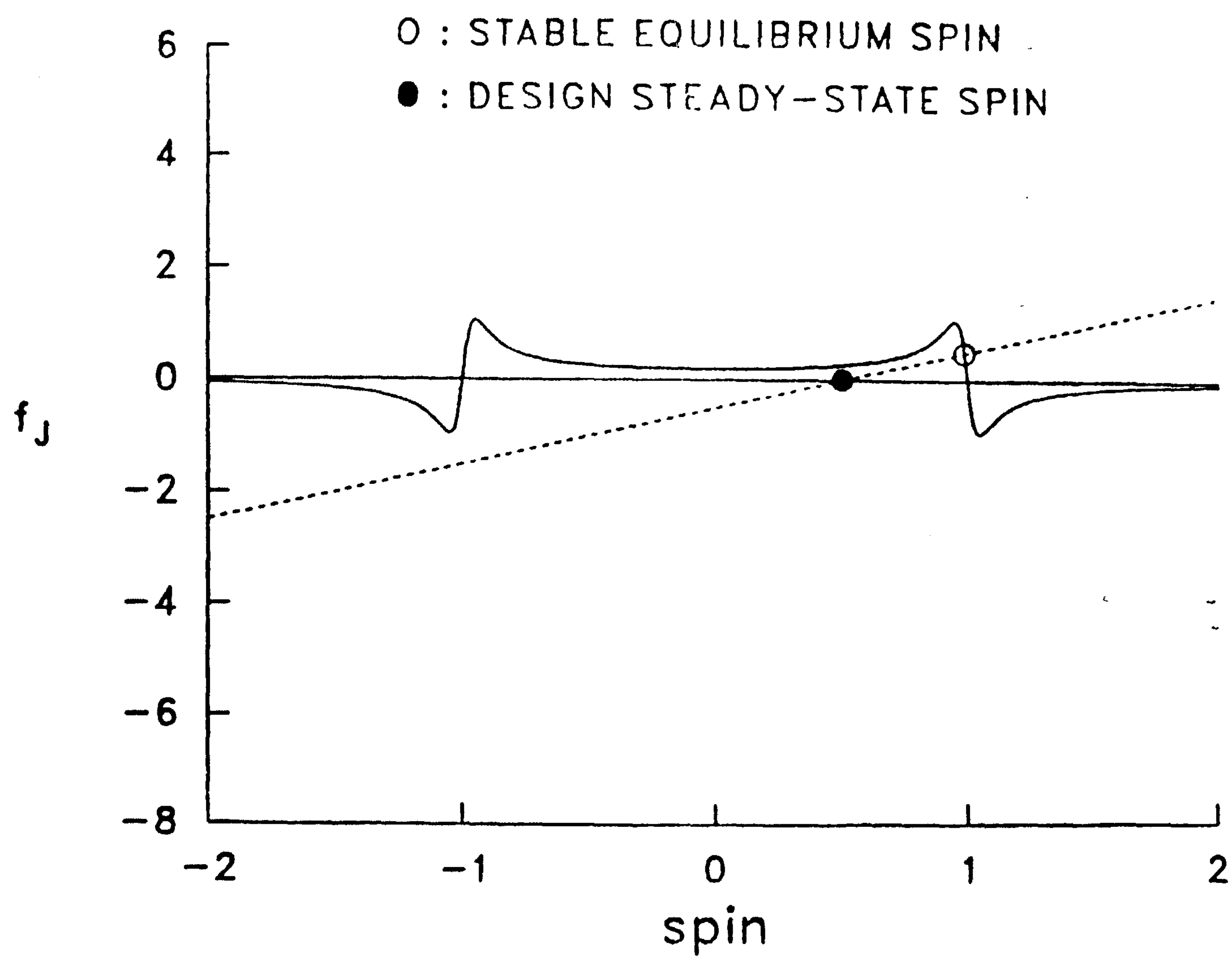

Figure 9. f_j versus $\dot{\phi}$ for case 6. No design steady-state spin occurs but normal lock-in does occur.

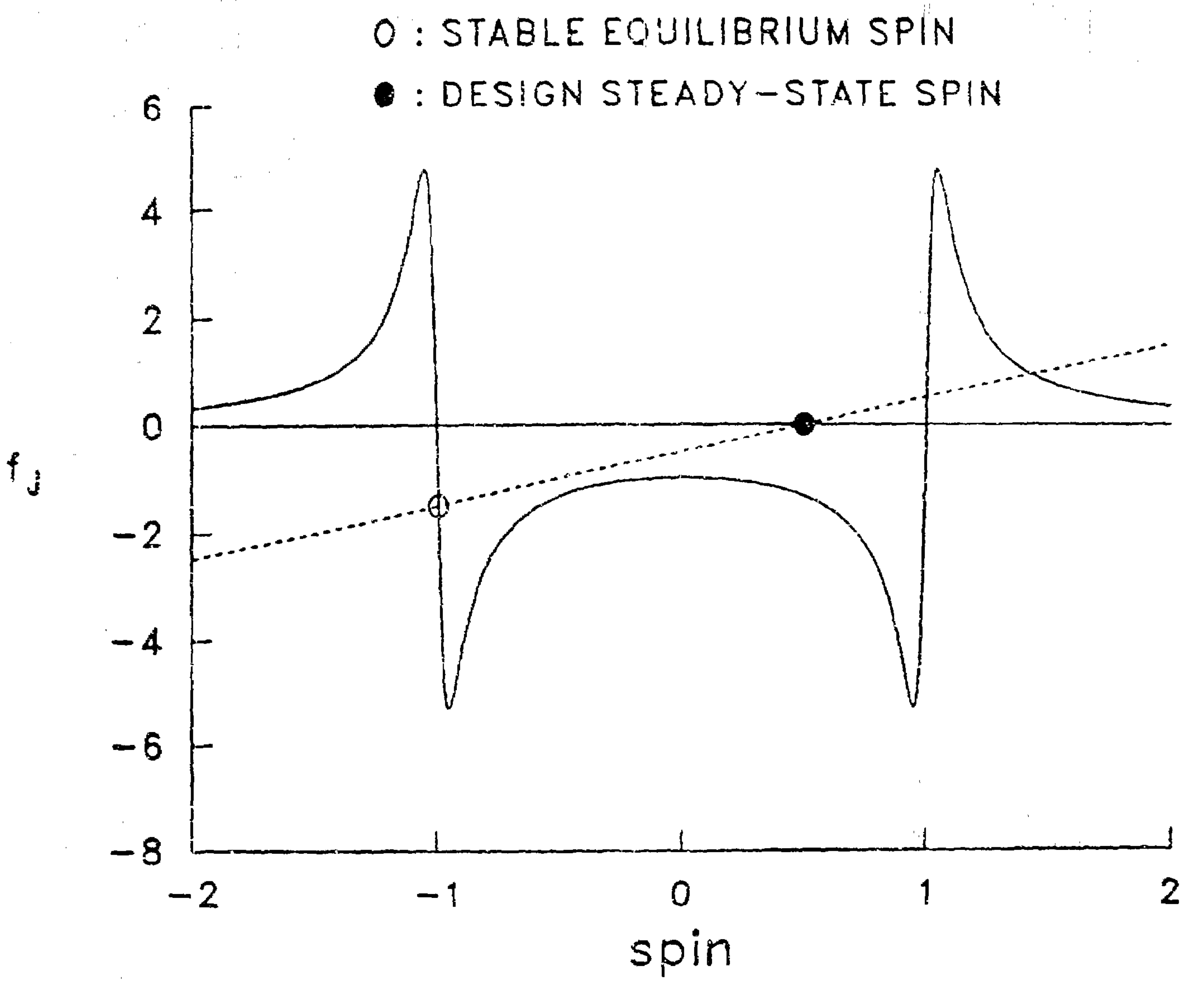

Figure 10. f_j versus $\dot{\phi}$ for case 7. No design steady-state spin occurs but reverse lock-in does occur.

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LIST OF SYMBOLS

C_D	Drag force/$(1/2)\rho V^2 S$, the drag force coefficient
C_ℓ	$M_X/(1/2)\rho V^2 S\ell$ where M_X is the axial aerodynamic force
C_{ℓ_p}	Spin-damping moment coefficient [Eq. (3.4)]
C_{ℓ_δ}	Spin-producing moment coefficient due to canted fins [Eq. (3.4)]
C_{ℓ_θ}	Spin moment coefficient induced by the angle of attack [Eq. (3.4)]
C_{L_α}	$C_{N_\alpha} - C_D$, the lift force coefficient
$C_{\tilde{m}}$, $C_{\tilde{n}}$	$(M_{\tilde{Y}}, M_{\tilde{Z}})/(1/2)\rho V^2 S\ell$ where $M_{\tilde{Y}}$, $M_{\tilde{Z}}$ are the transverse aerodynamic moments in the aeroballistic system
$C_{M_{p\alpha}}$	Magnus moment coefficient [Eq. (2.2)]
$C_{M_q} + C_{M_{\dot{\alpha}}}$	Damping moment coefficient sum [Eq. (2.2)]
C_{M_0}	Asymmetry moment coefficient [Eq. (2.2)]
C_{M_α}	Static moment coefficient [Eq. (2.2)]
C_{M_θ}	Induced pitch moment coefficient [Eq. (6.1)]
C_{N_0}	Asymmetry force coefficient [Eq. (2.1)]
C_{N_α}	Normal force coefficient [Eq. (2.1)]
$C_{\tilde{y}}$, $C_{\tilde{z}}$	$(F_{\tilde{y}}, F_{\tilde{z}})/(1/2)\rho V^2 S$ where $F_{\tilde{y}}$, $F_{\tilde{z}}$ are the transverse aerodynamic forces in the aeroballistic system
f_1	$i\,G\,(\zeta^n - \bar{\zeta}^n)$
f_2	$\dot{\phi}_e - \dot{\phi}_s$
F_N	Normal force due to radially offset center of mass
G	$\hat{K}_\theta\, \delta^n_{TR}$
h	$(\hat{H} - \sigma\hat{T})/(1 - \sigma)$

LIST OF SYMBOLS (continued)

H	$\frac{\rho S \ell}{2m}\left[C_{L_\alpha} - C_D - k_t^{-2}\left(C_{M_q} + C_{M_{\dot\alpha}}\right)\right]$
I_x	Axial moment of inertia
I_t	Transverse moment of inertia
k_a^2	$I_x/m\ell^2$
k_t^2	$I_t/m\ell^2$
K_p	$-\frac{\rho S \ell^3}{2\ I_x}\left[C_{\ell_p} + k_a^2\ C_D\right]$, [Eq. (3.10)]
K_δ	$\frac{\rho S \ell^3}{2\ I_x}\ \delta_f\ C_{\ell_\delta}$, [Eq. (3.10)]
K_θ	$(b_{10}/2)\left[C_{\ell_p} + k_a^2\ C_D\right]^{-1}$, [Eq. (3.10)]
ℓ	Reference length (diameter)
m	Mass
m_θ	$C_{M_\theta}/C_{M_\alpha}$, [Eq. (6.3)]
M	$\frac{\rho S \ell^3}{2\ I_t}\ C_{M_\alpha}$
M_A	$-\frac{\rho S \ell^3}{2\ I_t}\ C_{M_o}$
$M_{x\theta}$	Roll moment due to radially offset center of mass [Eq. (5.1)]
n	Symmetry number; the symmetry angle is $2\pi/n$ radians

N	Type designator: normal resonant lock-in
$\hat{r}_c$	Radial offset of the center of mass (calibers)
R	Type designator: reverse resonant lock-in
s	$\int_0^t (V/\ell)\, dt$
S	(1) Reference area $\pi \ell^2/4$ (2) Type designator: steady-state spin
t	Time
T	$\frac{\rho S \ell}{2m} \left[C_{L_\alpha} + k_a^{-2} C_{M_{p_\alpha}} \right]$
V	Magnitude of the velocity
α, β	Angles of attack and sideslip in the missile-fixed system
$\tilde{\alpha}, \tilde{\beta}$	Angles of attack and sideslip in the aeroballistic system
δ	Absolute value of ξ and $\tilde{\xi}$
δ_f	Differential fin cant angle
δ_t	Absolute value of ξ_T
δ_{TR}	$\delta_{To}/\lvert h \rvert$
δ_{To}	$M_A/M = -C_{M_o}/C_{M_\alpha}$
ζ	ξ/δ_{TR}
$\dot{\zeta}_0$	Complex angular velocity at $\tau = 0$.
θ	Orientation angle of ξ [Eq. (3.1)]
$\tilde{\theta}$	Orientation angle of $\tilde{\xi}$ [Eq. (3.2)]

θ^*	Orientation angle of $\dot{\zeta}_0$
λ	Constant $\dot{\eta}_j/\eta_j$ [Eq. (4.5)]
ξ	$\beta + i\alpha = \delta e^{i\theta}$, complex angle of attack in the missile-fixed system
$\tilde{\xi}$	$\tilde{\beta} + i\tilde{\alpha} = \delta e^{i\tilde{\theta}}$, complex angle of attack in the aeroballistic system
ξ_T	Constant trim angle value of ξ [Eq. (2.8)]
ρ	Air density
σ	I_x/I_t
τ	$[-M/(1-\sigma)]^{1/2}$ s
ϕ	Roll angle
ϕ_M	Asymmetry moment orientation angle [Eq. (2.2)]
ϕ_N	Asymmetry force orientation angle [Eq. (2.1)]
ϕ'_s	K_δ/K_p: steady-state spin (rad/cal)
ϕ_T	$\mathrm{Tan}^{-1}[-\dot{\hat{\phi}}\, h/(1-\dot{\hat{\phi}}^2)]$ [Eq. (2.8)]

Superscripts:

$(\bar{\ })$	Complex conjugate
$(\hat{\ })$	$[-(1-\sigma)/M]^{1/2}$ ( )
$(\dot{\ })$	d( )/dτ
( )'	d( )/ds

Subscripts:

$(\ )_e$	Equilibrium value
$(\ )_s$	Steady-state value

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