BASIC Desk-Top Computer Program for the Three-Dimensional Static Configuration of an Extensible Flexible Cable in a Uniform Stream

by

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BASIC Desk-Top Computer Program for the Three-Dimensional Static Configuration of an Extensible Flexible Cable in a Uniform Stream

A generalized computer program for predicting the three-dimensional, static configurations and tensions of extensible, flexible cable systems is described. The program is written in BASIC language for use on desktop computers. The report includes development of the equations, descriptions of the program, instruction for program usage, sample problems, and a program listing for a Hewlett-Packard 9836 desktop computer.
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R  Cable drag per unit length when the cable is normal to the free-stream direction
Re  Reynolds number
s  Stretched cable length
so  Unstretched cable length
Sinc  Cable length printout interval
T  Cable tension
V  Free-stream velocity
W  Cable weight per unit length in a fluid
Wa  Cable weight per unit length in air
X  Axis of space-fixed orthogonal coordinate system positive in direction of tow (negative in free-stream direction)
Y  Axis of space-fixed orthogonal coordinate system positive to the right
Z  Axis of space-fixed orthogonal coordinate system positive in the direction of gravity
X₁, Y₁, Z₁  Intermediate coordinate system defined by a rotation of angle $\phi$ about the X-axis
x, y, z  Distances along the X, Y, and Z directions
$\beta$  Cable kite angle measured from the Z-axis to the tangent of a cable element projected onto the Y-Z plane
$\varepsilon$  Cable strain
$\nu$  Fluid kinematic viscosity
$\rho$  Fluid density
VOTATION (Continued)

σ  Cable stress
ψ  Towline angle measured from the free-stream direction (X-axis) to the tangent of a cable element
ABSTRACT

A generalized computer program for predicting the three-dimensional, static configurations and tensions of extensible, flexible cable systems is described. The program is written in BASIC language for use on desktop computers. The report includes development of the equations, description of the program, instruction for program usage, sample problems, and a program listing for a Hewlett-Packard 9836 desktop computer.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

This report describes a generalized computer program for predicting the three-dimensional, static configurations and tensions of extensible, flexible cable systems in a uniform stream. The program is written in BASIC language and is specifically intended to provide time-efficient solutions on a desktop computer. Although the program initially was written for use with Hewlett-Packard (HP) computers, it can be adapted with suitable variations in input/output format to other computers that use BASIC. To date, the program has been used on an HP-9826, an HP-9835, an HP-9836, and an HP-87. A program listing for the HP-9836 is contained in the Appendix.

The program is designed to solve initial value problems when the conditions at one end of the cable are known. Boundary value problems must be solved by an
iterative procedure to satisfy prescribed conditions at both ends of the cable. The program allows the cable to assume any configuration without regard to or prior knowledge of cable quadrant. The correct configuration is automatically established by the signs of the initial force components. Cable elasticity is entered as an option. In some cases, as for example with steel cables, elongation may be insignificant. If this is the case, program input is simplified by choosing the nonelastic option. If elongation is significant, as with many composite and synthetic lines, then the elastic option will provide greater accuracy. To use this option, cable elastic modulus and cross-sectional area must be entered as input data. The elastic modulus may be entered either as a constant or as a linear function of cable stress. Synthetic lines frequently exhibit a variable modulus proportional to stress.

The program allows intermediate bodies to be added at any desired point along the cable. In addition, certain properties may be changed along the cable as desired; for example, the cable may be part ribbon-faired and part bare or the fluid may change from water to air. Trail, kite, depth, and cable angle stop values also may be imposed. To include an intermediate body or to change a cable parameter, the cable length to that point or a stopping parameter is specified. The program then will calculate the cable configuration to the specified value and request the option desired. In this way, the computation can be continued to any desired point of termination.

This report includes derivations of the generalized equations which govern the static configuration and tension. Also, the assumed representations of the fluid forces and cable elastic properties are discussed in detail. Finally, the computer program, including the method of solution and its usage, is described.
DERIVATION OF CABLE CONFIGURATION EQUATIONS

The differential equations describing the three-dimensional static configuration and tension of a cable in a uniform stream are derived from the equilibrium of external forces acting on an element of the cable. A free-body diagram showing a segment of cable of elemental length $ds$ acted upon by hydrodynamic, hydrostatic, gravitational, and tension forces is illustrated in Fig. 1. The $(X, Y, Z)$ coordinate system shown is a right-hand, orthogonal system fixed in space with the $X$-axis positive in the direction of tow (or negative in the free-stream direction) and the $Z$-axis positive in the direction of gravity.

The equations are conveniently derived for an orthogonal coordinate system fixed to the cable. In Fig. 1, the hydrodynamic force has been resolved into components $Fds$, $Gds$, and $Hds$ where $F$ is the force component per unit length normal to the cable in the plane defined by the cable element and the free-stream direction, $G$ is the force component per unit length tangential to the cable, and $H$ is the (side) force component per unit length normal to the plane defined by the cable element and the free-stream direction. The cable-fixed coordinate system defined by the directions of $F$, $G$, and $H$ may be obtained by first rotating the spatial system by an angle $\theta$ about the $X$-axis and then rotating the resulting intermediate $(X_1, Y_1, Z_1)$ coordinate system by an angle $\phi$ about the $Y_1$-axis. In towed systems nomenclature, the angles $\theta$ and $\phi$ generally are referred to as the kite angle and the cable angle, respectively. Since the orientation of the cable changes in space, the angles $\theta$ and $\phi$ are functions of cable length $s$.

For equilibrium, the sums of the force components in each of the directions $F$, $G$, and $H$ must all be zero. As shown in Fig. 1, the sum of the force components in the direction of $F$ is
Fig. 1. Forces acting on a segment of cable of elemental length ds.
\[ Fds + (W_a - B) \cos \beta \cos \phi \, ds \]
\[ -(T + \frac{1}{2}dT) \sin(\phi + \frac{1}{2}d\phi) \cos(\frac{1}{2}d\beta) \cos \phi \]
\[ +(T + \frac{1}{2}dT) \cos(\phi + \frac{1}{2}d\phi) \sin \phi \]
\[ +(T - \frac{1}{2}dT) \sin(\phi - \frac{1}{2}d\phi) \cos(-\frac{1}{2}d\beta) \cos \phi \]
\[ -(T - \frac{1}{2}dT) \cos(\phi - \frac{1}{2}d\phi) \sin \phi = 0 \]

where

\( F \) is the hydrodynamic force component per unit length normal to a cable element in the plane defined by the cable element and the free-stream direction,

\( s \) is the stretched cable length,

\( W_a \) is the cable weight per unit length in air,

\( B \) is the cable buoyancy per unit length,

\( T \) is the cable tension,

\( \beta \) is the kite angle measured from the Z-axis to the tangent of the cable-element projection onto the Y-Z plane, and

\( \phi \) is the cable angle measured from the free-stream direction (X-axis) to the tangent of the cable element.

From trigonometry,

\[ \sin(\phi + \frac{1}{2}d\phi) = \sin \phi \cos(\frac{1}{2}d\phi) + \cos \phi \sin(\frac{1}{2}d\phi) \] (2a)
\[ \cos(\phi + \frac{1}{2}d\phi) = \cos \phi \cos(\frac{1}{2}d\phi) - \sin \phi \sin(\frac{1}{2}d\phi) \] (2b)
\[ \sin(\phi - \frac{1}{2}d\phi) = \sin \phi \cos(\frac{1}{2}d\phi) - \cos \phi \sin(\frac{1}{2}d\phi) \] (2c)
\[ \cos(\phi - \frac{1}{2}d\phi) = \cos \phi \cos(\frac{1}{2}d\phi) + \sin \phi \sin(\frac{1}{2}d\phi) \] (2d)

Also, since \( d\phi \) and \( d\beta \) approach zero,

\[ \cos(\frac{1}{2}d\phi) = 1, \] (3a)
\[ \sin(\frac{1}{2}d\phi) = \frac{1}{2}d\phi, \] (3b)
\[
\cos(\frac{\pi}{4}d\beta) = 1, \quad \text{and} \quad (3c)
\]
\[
\cos(-\frac{\pi}{4}d\beta) = 1. \quad (3d)
\]

Substituting Eqs. 2 and 3 into Eq. 1 and then expanding and combining terms gives

\[
F_{ds} + (W_a - B) \cos\beta \cos\phi ds - Td\phi = G
\]

or

\[
-T \frac{d\phi}{ds} + F + (W_a - B) \cos\beta \cos\phi = 0. \quad (4)
\]

The sum of the force components in the direction of \( G \) is

\[
G_{ds} - (W_a - B) \cos\beta \sin\phi ds
\]

\[
+ (T + \frac{1}{2}dT) \sin(\phi + \frac{1}{2}d\phi) \cos(\frac{\pi}{4}d\beta) \sin\phi
\]

\[
+ (T + \frac{1}{2}dT) \cos(\phi + \frac{1}{2}d\phi) \cos\phi
\]

\[
-(T - \frac{1}{2}dT) \sin(\phi - \frac{1}{2}d\phi) \cos(-\frac{\pi}{4}d\beta) \sin\phi
\]

\[
-(T - \frac{1}{2}dT) \cos(\phi - \frac{1}{2}d\phi) \cos\phi = 0
\]

where \( G \) is the hydrodynamic force component per unit length tangential to the cable element.

Substituting Eqs. 2 and 3 into Eq. 5 and then expanding and combining terms gives

\[
G_{ds} - (W_a - B) \cos\beta \sin\phi ds + dT = 0
\]

or

\[
\frac{dT}{ds} + G - (W_a - B) \cos\beta \sin\phi = 0. \quad (6)
\]

The sum of the force components in the direction of \( H \) is

\[
H_{ds} - (W_a - B) \sin\beta ds
\]

\[
-(T + \frac{1}{2}dT) \sin(\phi + \frac{1}{2}d\phi) \sin(\frac{\pi}{4}d\beta)
\]

\[
+(T - \frac{1}{2}dT) \sin(\phi - \frac{1}{2}d\phi) \sin(-\frac{\pi}{4}d\beta) = 0
\]
where $H$ is the hydrodynamic force component per unit length normal to the plane defined by the cable element and the free-stream direction.

Since $d\phi$ approaches zero,

$$\sin\left(\frac{1}{2}d\phi\right) = \frac{1}{2}d\phi \quad \text{and} \quad \sin(-\frac{1}{2}d\phi) = -\frac{1}{2}d\phi.$$ (8a)

$$\sin(-\frac{1}{2}d\phi) = -\frac{1}{2}d\phi.$$ (8b)

Substituting Eqs. 2, 3 and 8 into Eq. 7 then expanding and combining terms gives

$$Hds - (W_a - B) \sin\beta ds - T\sin\phi ds - \frac{1}{2}d\theta d\phi \cos\phi = 0$$

or, since the third order term is negligible,

$$-T \sin\phi \frac{d\phi}{ds} + H - (W_a - B) \sin\beta = 0.$$ (9)

Equations 4, 6, and 9 are the three differential equations of force equilibrium in the cable-fixed reference system. The differential equations for cable displacements in the X, Y, and Z directions can be obtained from Fig. 1 by inspection. They are

$$dx = ds \cos\phi \quad \text{or} \quad \frac{dx}{ds} = \cos\phi,$$ (10)

$$dy = -ds \sin\phi \sin\beta \quad \text{or} \quad \frac{dy}{ds} = -\sin\phi \sin\beta, \text{ and}$$ (11)

$$dz = -ds \sin\phi \cos\beta \quad \text{or} \quad \frac{dz}{ds} = -\sin\phi \cos\beta.$$ (12)

where $x$, $y$, and $z$ are distances along the $X$, $Y$, and $Z$ directions, respectively.

**FLUID HYDRODYNAMIC FORCE COMPONENTS**

The fluid hydrodynamic force components $Fds$, $Cds$, and $Hds$ arise, in the cases of $F$ and $G$, from fluid drag forces and, in the case of $H$, from fluid side or lift forces.
FLUID DRAG FORCES

The fluid drag force components per unit cable length $F$ and $G$ lie in the plane defined by a cable element and the free-stream flow direction. Generally, these components are assumed to be the product of a drag per unit length that is a function only of Reynolds number and a loading function that is a function only of cable angle $\phi$. Under this convention, the normal force per unit length $F$ and the tangential force per unit length $G$ are of the following form:

$$F(Re, \phi) = -R(Re) \cdot f_n(\phi)$$  \hspace{1cm} (13)

$$G(Re, \phi) = -R(Re) \cdot f_t(\phi)$$  \hspace{1cm} (14)

where

- $R$ is the cable drag per unit length when the cable is normal to the free-stream direction ($R = \frac{1}{2} \rho C_R V^2 d$),
- $Re$ is the Reynolds number ($Re = Vd/\nu$),
- $\rho$ is the fluid density,
- $C_R$ is the cable normal drag coefficient based on frontal area expressed as a function of Reynolds number,
- $d$ is the stretched cable diameter,
- $V$ is the free-stream flow velocity,
- $\nu$ is the fluid kinematic viscosity, and
- $f_n, f_t$ are the normal and tangential hydrodynamic loading functions, respectively.

The negative (-) signs in Eqs. 13 and 14 appear since the cable drag is in the negative (-) X direction as defined in Fig. 1.
The hydrodynamic loading functions $f_{n,t}$ are commonly expressed in the form of a trigonometric series\(^1\),\(^2\) as follows:

$$f_{n,t}(\phi) = A_0 + \sum_{n=1}^\infty A_n \cos(n\phi) + B_n \sin(n\phi)$$  \hspace{1cm} (15)

where $A_0$, $A_n$, and $B_n$ are constant coefficients. Generally, cable loading is adequately expressed by inclusion of only the first five terms ($n = 2$) of Eq. 15.

**FLUID SIDE FORCES**

Fluid side forces occur as a result of cable instabilities or lateral shape asymmetries. In the case of twisted wire ropes or double-armored electro-mechanical cables, the helical twist of the exposed wire strands causes a lateral shape asymmetry which can produce significant side forces.

In a manner similar to that used with drag forces, the side force component per unit length $H$ is expressed conveniently as the product of a side force that is a function only of Reynolds number and a loading function that is a function only of cable angle $\phi$ as follows:

$$H(Re, \phi) = F_s(Re) \cdot f_s(\phi)$$  \hspace{1cm} (16)

where

- $F_s$ is the cable side force per unit length at the orientation of the cable that produces the maximum value ($F_s = \frac{1}{2} \rho C_s V^2 d$),
- $C_s$ is the cable side force coefficient based on frontal area expressed as a function of Reynolds number, and
- $f_s$ is the hydrodynamic side force loading function.
The maximum side force $F_s$ may or may not occur when the cable is normal to the free-stream. The side force loading function $f_s$ also is adequately expressed using the first five terms of the trigonometric series indicated in Eq. 15.

CABLE ELONGATION

The cables used in towing and mooring applications generally exhibit three types of elongation as follows:

1. Permanent Elongation -- This type is nonrecoverable and is associated primarily with fabrication looseness inherent in new cables, ropes, and lines before the wires or strands are seated. The amount of permanent elongation depends on the construction type and the degree of pre-stressing. Permanent elongation becomes fairly constant after repeated loadings. For wire rope, this type of elongation is typically between 0.25 and 0.33 percent of initial length depending upon maximum stress.  

2. Creep Elongation -- This type of elongation is associated with material properties and depends on both the tensile stress and the length of time the stress is applied. Creep elongation is recoverable over a period of time after the stress is reduced or removed. Metallic cables and certain forms of composite lines exhibit minimal creep within normal working stresses, whereas many synthetic lines exhibit noticeable creep.

3. Elastic Elongation -- This type of elongation is immediately recoverable and depends on the type of construction as well as the material properties. Elastic elongation is essentially proportional to stress in most metallic cables and ropes but is frequently a nonlinear function of
stress in synthetic lines - most notably nylon, polyester, and polypropylene.

Of the three types of elongation, only elastic elongation is examined in this report and included in the computer program. Permanent elongation can be accounted for by assuming a preconditioned cable. Creep elongation is difficult to treat because a stress-time history is required.

Elastic elongation curves for representative towline types are shown in Fig. 2. These curves are based on vendor data and are intended to illustrate general relationships rather than to provide specific design information. Ideally the elastic properties of each towcable should be verified individually. From standard mechanics of deformable media\(^4\), stress is related to elastic strain by the equation

\[
E_t = \frac{d\sigma}{d\varepsilon}
\]

where

- \(E_t\) is the slope of the stress-strain curve at any point and generally is referred to as the modulus of elasticity,
- \(\sigma\) is the cable stress \((\sigma \equiv T/A_c)\),
- \(T\) is the cable tension,
- \(A_c\) is the cable or strength-member cross-sectional area,
- \(\varepsilon\) is the cable strain \((\varepsilon \equiv \frac{s-s_0}{s_0})\),
- \(s\) is the stretched cable length, and
- \(s_0\) is the unstretched cable length.

The cable modulus of elasticity in Eq. 17 depends on cable construction and, therefore, should not be confused with material modulus of elasticity which generally is higher. For the nonlinear stress-strain curves of Fig. 2, the
modulus of elasticity $E_c$ can be approximated by a linear function of stress $\sigma$ as follows:

$$E_c = E_o + E_1 \sigma$$  \hspace{1cm} (18)

where

- $E_o$ is the slope of the stress-strain curve at a stress $\sigma$ equal to zero, and
- $E_1$ is a proportionality factor.

Substituting Eq. 18 into Eq. 17 and solving for strain, the result is,

$$\int \sigma \text{d} \varepsilon = \int \frac{d\sigma}{(E_o + E_1 \sigma)} = \frac{1}{A_c} \left[ \frac{dT}{(E_o + E_1 \frac{T}{A_c})} \right]$$

(19)

Equation 19 can be integrated between zero and any value of $T$ to obtain the strain or stretch at that value. The result is

$$\frac{s - s_o}{s_o} = \frac{K}{A_c} \quad \text{or} \quad s = s_o \left( 1 + \frac{K}{A_c} \right)$$ \hspace{1cm} (20a)

where

$$K = \frac{A_c}{E_1} \log_e \left( \frac{(E_o + E_1 \frac{T}{A_c})}{E_o} \right), \ E_1 \neq 0$$ \hspace{1cm} (20b)

or

$$K = \frac{T}{E_o}, \ E_1 = 0$$ \hspace{1cm} (20c)

Equation 20a,c is the classical linear stress-strain relationship. For the nonlinear stress-strain curves shown in Fig. 2, Eq. 20a,b predicted the stretched cable length to within $\pm 0.5$ percent at all stress levels.

In terms of an incremental cable segment of unstretched length $ds_o$, Eq. 20a becomes

$$ds = ds_o \left( 1 + \frac{K}{A_c} \right)$$ \hspace{1cm} (21)
Fig. 2. Elastic elongation of various towline types.
As the cable stretches under tension, both its diameter and its stretched weight per unit length decreases. For a homogeneous and isotropic material, transverse strain is related to axial strain by Poisson's ratio. However, most cables are neither homogeneous nor isotropic, and therefore the assumption is made here that the stretched cable preserves its original volume. Under this assumption, the stretched diameter $d$ and stretched weight in the fluid per unit length $W$ are related to their unstretched states $d_0$ and $W_0$, respectively, by the following relationships:

\[
d = d_0 \left[ \frac{1}{(1 + K/A_C)} \right]^{\frac{1}{2}}
\]

and,

\[
W = W_0 \left[ \frac{1}{(1 + K/A_C)} \right]
\]

where $K$ is defined in Eq. 20b,c.

Substitution of Eqs. 21, 22, and 23 into the configuration equations allows solutions of elastically elongated cables to be obtained in terms of unstretched reference parameters.

PROGRAM DESCRIPTION

The computer program consists of a main input section and six subroutines.
MAIN INPUT SECTION

The input section provides a format for entering fluid density, towing speed (or flow speed), initial body force components in the three principal directions X, Y, and Z, cable characteristics, cable length printout interval, and various spatial conditions at which the integration is to stop. The initial cable angle $\phi$ and kite angle $\theta$ are automatically determined within the program by the magnitudes and directions of the body force components. This method of entering body force allows the integration to proceed without regard to or prior knowledge of the cable angle quadrant. The program also will compute cable configurations, with a small starting error, when the initial input force is zero. This feature is useful in towed array problems where usually no body is attached to the outboard end of the array.

SUBROUTINE TANGLE

Subroutine "Tangle" computes initial tension $T$, cable angle $\phi$, and kite angle $\theta$ using the input force components. Starting values for these variables are required to initiate the program solution.

SUBROUTINE CRITANG

Subroutine "CritAng" computes the cable angle $\phi$ and kite angle $\theta$ when the starting input force is zero. Although this subroutine is not required for the program to obtain a solution, it does improve the accuracy at the outboard end of the cable and does reduce computation time considerably when the starting force is zero.
SUBROUTINE INTEGRATOR

Subroutine "Integrator" solves the differential equations of force equilibrium, Eqs. 4, 6, and 9, using a self-starting, modified Euler's method. A large integration step \( \Delta s \) is used in the program to minimize solution time. Initially, the step size is set equal to the specified printout interval \( S_{inc} \) or to a length increment of 50 ft, whichever is less. At each integration interval, the calculation is iterated about the changes in cable angle \( \Delta \phi \) and kite angle \( \Delta \beta \) until succeeding iteration values of \( \Delta \phi \) and \( \Delta \beta \) are within \( \pm 0.0002 \) rad (\( \pm 0.01 \) deg). If, however, the computation produces angle changes greater than 0.09 rad (5 deg) or the computation does not converge within 10 iteration loops, then the step size is reduced, and the iteration is repeated for the smaller value. By this technique, the integration step size is efficiently adjusted to provide a good degree of accuracy and a short computation time. Once the variables \( T \), \( \phi \), and \( \beta \) are determined, the cable displacement variables \( x \), \( y \), and \( z \) (Eqs. 10, 11, and 12) are computed using the average calculated values of \( \phi \) and \( \beta \) between successive integration steps. A flow chart of the integration process is shown in Fig. 3.

In general, the trigonometric series used to specify the cable hydrodynamic loading functions is composed of both sin and cos terms and provides the correct loading value only in the cable angle range \( \phi \) from zero to 90 deg. Therefore, if a computation falls outside this range, a corrected value must be used. This correction is made within the program by assuming that the normal loading function for all types of cables and the side loading function for faired cables are symmetrical about \( \phi = \pm 90 \) deg and complementary about \( \phi = 0 \) deg, that the tangential loading function for all types of cables is symmetrical about \( \phi = 0 \) deg and complementary about \( \phi = \pm 90 \) deg, and that the side loading function for
Fig. 3. Flow chart of integration technique.
stranded (twisted) cables is complementary about both $\phi = 0\, \text{deg}$ and $\phi = \pm 90\, \text{deg}$. These relationships are summarized in Table 1.

The integration is terminated at the maximum specified value of cable length $S_{\text{max}}$ or at a specified stop value corresponding to trail distance $x$, kite distance $y$, depth $z$, or cable angle $\phi$. After each integration step, the program compares the computed value to the specified stop value. If a specified stop value has been exceeded, the program performs a simple iteration to converge on the specified value. The iteration works by halving the cable length segment which brackets the point until the desired stop value is approached to within a certain limiting criterion. The convergence criterion is 0.01 ft for distance variables and 0.0005 rad (0.03 deg) for the angular variable.

Table 1 - Relationships of the hydrodynamic loading functions in various quadrants of cable angle $\phi$.

| Loading Function | Cable Angle Range (deg) | | |
|------------------|-------------------------| | |
|                  | $\phi$                  | $-180\, \text{TO} -90$ | $-90\, \text{TO} 0$ | $0\, \text{TO} +90$ | $+90\, \text{TO} +180$ |
|                  | $\phi_1$ - $180$ minus $|\phi|$ | $|\phi|$ | $\phi$ | $180$ minus $\phi$ |
| Normal           | $f_n(\phi)$ - $-f_n(\phi_1)$ | $-f_n(\phi_1)$ | $+f_n(\phi_1)$ | $+f_n(\phi_1)$ |
| Tangential       | $f_t(\phi)$ - $-f_t(\phi_1)$ | $+f_t(\phi_1)$ | $+f_t(\phi_1)$ | $-f_t(\phi_1)$ |
| Side (stranded)  | $f_s(\phi)$ - $+f_s(\phi_1)$ | $-f_s(\phi_1)$ | $+f_s(\phi_1)$ | $-f_s(\phi_1)$ |
| Side (faired)    | $f_s(\phi)$ - $-f_s(\phi_1)$ | $-f_s(\phi_1)$ | $+f_s(\phi_1)$ | $+f_s(\phi_1)$ |

SUBROUTINES PODE, RIBBON, AND OTHER LOADING

Subroutines "Pode" and "Ribbon" provide hydrodynamic loading functions for bare and ribbon-faired cables, respectively. These subroutines are included for convenience since these types of cables are used frequently. Subroutine "Other
loading" allows the user to specify the coefficients of the trigonometric series corresponding to any desired loading. Pode and ribbon-faired loading are discussed in the next paragraphs.

Pode Normal and Tangential Hydrodynamic Loading

Pode hydrodynamic loading is commonly used to predict the performance of unfaired (bare) cable systems. In terms of the previously discussed trigonometric series (refer to section on FLUID DRAG FORCES), the normal and tangential loading functions \( f_n \) and \( f_t \) are, respectively,

\[
\begin{align*}
  f_n &= \sin^2 \phi = 0.5 - 0.5 \cos(2\phi) \\
  f_t &= f
\end{align*}
\]

where

\( f \) is the Pode frictional parameter.

Ribbon Normal and Tangential Hydrodynamic Loading

Normal and tangential hydrodynamic loading functions have been obtained from at-sea data for a 0.78-in. (19.8-mm) diameter double-armored electro-mechanical cable. The ribbon used was 0.015-in. (0.381-mm)-thick, 0.78-in. (19.8-mm)-wide polyurethane. Ribbon spacing was 0.78 in. (19.8 mm) between centers. The ribbon length (equal to one-half the loop length) was six times the cable diameter or 4.68 in. (237.7 mm). The normal and tangential loading functions \( f_n \) and \( f_t \) obtained for this ribbon faired cable are, respectively,

\[
\begin{align*}
  f_n &= 0.4986 - 0.2499 \cos \phi + 0.2527 \sin \phi - 0.2487 \cos(2\phi) \\
  f_t &= -0.2255 + 0.3417 \cos \phi + 0.2255 \sin \phi - 0.0811 \sin(2\phi).
\end{align*}
\]
These loading functions correspond to a normal drag coefficient,

$$C_R = 5.7467 - 0.93 \log_{10} \text{Re}; \quad 50,000 < \text{Re} < 120,000$$

(28)

where Re is Reynolds number based on diameter. Hydrodynamic loading may vary with different ribbon geometries, and therefore the above equations should be used with caution.

**Bare and Ribbon-Faired Side Force Hydrodynamic Loading**

The side force on 1 x 19 and 6 x 19 twisted wire ropes has been measured in a wind tunnel. The data, reduced to side loading function form, are shown in Fig. 4. These data represent average values over a Reynolds number range from 32,600 to 43,400 for the 1 x 19 rope and from 16,400 to 83,700 for the 6 x 19 rope.

![Fig. 4. Hydrodynamic side loading function for two twisted wire ropes.](image_url)
The side force coefficients $C_s$, averaged over the same Reynolds number ranges, are 0.135 and 0.232 for the $1 \times 19$ and the $6 \times 19$ ropes, respectively. Average values are given here since the data were scattered to the extent that a reasonable Reynolds number dependency could not be established. A least-squares fit to the side loading data resulted in the following trigonometric series for the side force loading function $f_s$:

$$f_s = 1.4792 + 0.0841 \cos \theta - 3.0425 \sin \theta - 1.5633 \cos (2\theta) + 1.5213 \sin (2\theta).$$  \hspace{1cm} (29)

Although this loading function was determined on the basis of only two cable constructions, the general form probably is consistent with other stranded cables as well, and therefore it has been entered into the program as a general loading function for both bare and ribbon-faired cables. The side force coefficient $C_s$, however, is strongly influenced by the number of wire strands exposed to the flow, by the twist or helical angle, and possibly by the ribbon geometry.

STORAGE AND TIME REQUIREMENTS

The fully working program without comment statements requires approximately 10,000 bytes of storage on an HP-9836; with comment statements, approximately 13,000 bytes of storage are required. The length of time required to compute a cable configuration depends primarily on the change in cable angle and the tension. On an average, an HP-9836 requires approximately 5 to 10 sec. However, for cases in which the initial input tension is near zero, computing time may increase to 20 sec or more. Computation time also may be greater on other computers. For example, an HP-9835 is approximately three times slower than an HP-9836; an HP-87 is approximately ten times slower.
ACCURACY

The large integration step size and relatively loose convergence criteria used in the program do not appear to significantly affect accuracy. Comparisons made with other cable programs\textsuperscript{11,12} indicate differences of less than 0.05 percent in the output variables for the same input conditions.

PROGRAM USAGE

Data are entered into the program by a series of command statements which appear on the CRT screen. The command statements are listed in Table 2. The program prints the data variables immediately after they are entered to maintain a running check of the values.

COMMENTS CONCERNING DATA INPUT

The following comments refer to command statements listed in Table 2.

1. Command statement number 1 selects the printout medium.

2. The title statement number 2 is dimensioned to allow a title length of up to 80 characters.

3. Statement numbers 5 through 9 request the type of hydrodynamic loading desired. If "Pode" loading is chosen, the statement PODE LOADING will be printed and statement number 6, requesting the Pode frictional parameter, will appear next on the CRT. After the frictional parameter is entered, the program will skip to command statement number 10. Similarly if "Ribbon" loading is chosen, the statement RIBBON LOADING will be printed. If neither "Pode" nor "Ribbon" loading is entered, the coefficients of a generalized loading function will be requested (command statements 7, 8, and 9). Five values for each loading function $f_n$, 

22
Table 2. Program command statements.

<table>
<thead>
<tr>
<th>Step</th>
<th>Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>FOR HARDCOPY, ENTER: 1 [For CRT, press CONT]</td>
</tr>
<tr>
<td>02</td>
<td>ENTER: TITLE</td>
</tr>
<tr>
<td>03</td>
<td>ENTER SPEED: V (Knots)</td>
</tr>
<tr>
<td>04</td>
<td>ENTER FLUID MASS DENSITY: Rho (Lb*Sec^2/Ft^4)</td>
</tr>
<tr>
<td>05</td>
<td>ENTER CABLE LOADING: Pode=1; Ribbon=2; Other=3</td>
</tr>
<tr>
<td>06</td>
<td>ENTER PODE FRICTIONAL PARAMETER: f</td>
</tr>
<tr>
<td>07</td>
<td>ENTER NORMAL LOADING FUNC, fs: A0,A1,B1,A2,B2</td>
</tr>
<tr>
<td>08</td>
<td>ENTER TANGENTIAL LOADING FUNC, ft: A0,A1,B1,A2,B2</td>
</tr>
<tr>
<td>09</td>
<td>ENTER SIDE LOADING FUNC, fs: A0,A1,B1,A2,B2</td>
</tr>
<tr>
<td>10</td>
<td>ENTER CABLE NORMAL DRAG COEFF: Cr</td>
</tr>
<tr>
<td>11</td>
<td>ENTER CABLE SIDE FORCE COEFF: Cs</td>
</tr>
<tr>
<td>12</td>
<td>ENTER CABLE THICKNESS: d (In.)</td>
</tr>
<tr>
<td>13</td>
<td>ENTER CABLE WEIGHT IN FLUID: W (Lb/Ft)</td>
</tr>
<tr>
<td>14</td>
<td>ENTER CABLE ELASTICITY: Non-Elastic=1; Elastic=2</td>
</tr>
<tr>
<td>15</td>
<td>ENTER MODULUS OF ELAS: E=E0+EI*T/Ac (Lb/In.^2)</td>
</tr>
<tr>
<td>16</td>
<td>ENTER CABLE CROSS-SECTIONAL AREA: Ac (In.^2)</td>
</tr>
<tr>
<td>17</td>
<td>ENTER BODY FORCE COMPONENTS: Fx,Fy,Fz (Lb)</td>
</tr>
<tr>
<td>18</td>
<td>ENTER MAXIMUM CABLE LENGTH: Smax (Ft)</td>
</tr>
<tr>
<td>19</td>
<td>ENTER CABLE LENGTH PRINTOUT INTERVAL: Sinc (Ft)</td>
</tr>
<tr>
<td>20</td>
<td>ENTER LIMIT: Xstop (Ft) [If none, press CONT]</td>
</tr>
<tr>
<td>21</td>
<td>ENTER LIMIT: Ystop (Ft) [If none, press CONT]</td>
</tr>
<tr>
<td>22</td>
<td>ENTER LIMIT: Zstop (Ft) [If none, press CONT]</td>
</tr>
<tr>
<td>23</td>
<td>ENTER LIMIT: Phistop (Deg) [If none, press CONT]</td>
</tr>
<tr>
<td>24</td>
<td>ADD BODY, ENTER: 1; ADD CABLE, ENTER: 2 [NEW CASE, press CONT]</td>
</tr>
</tbody>
</table>
\( f_t, \text{ and } f_s \text{ are required. In each case, the values to be entered represent the coefficients of the following trigonometric series:} \\
\[ f_{n,t,s} = A_0 + A_1 \cos \phi + B_1 \sin \phi + A_2 \cos(2\phi) + B_2 \sin(2\phi). \]

After the coefficients of all three loading functions have been entered, the functions with their assigned coefficient values will be printed in the output.

4. Command statement number 14 provides the option of selecting either a nonelastic cable or an elastic cable. If the nonelastic option is chosen, the program skips the command statements (numbers 15 and 16) which relate to the elastic properties of the cable. If the elastic option is chosen, the ensuing command statements (numbers 15 and 16) require input of cable (not material) modulus of elasticity and strength-member cross-sectional area. Two values representing the coefficients of the equation

\[ \varepsilon_0 + E_1 \left( \frac{\sigma_c}{A_c} \right), \]

where \( \varepsilon \) is cable stress, must be input for the modulus of elasticity. For a linear stress-strain curve, the first coefficient \( E_0 \) is the slope of the curve and the second coefficient \( E_1 \) is set equal to zero. For a nonlinear stress-strain curve, the coefficient \( E_0 \) represents the slope of the stress-strain curve at zero stress and the coefficient \( E_1 \) represents a factor proportional to stress. The sign of \( E_1 \) must be positive. This method of representing modulus of elasticity is useful for synthetic lines which frequently exhibit nonlinear stress-strain relationships. Modulus of elasticity is discussed further in the section entitled CABLE ELONGATION.
5. Body force is entered into the program (command statement number 17) in the form of components in the three principal directions $X$, $Y$, and $Z$ where $F_x$ is positive in the direction of tow (negative in the free-stream direction), $F_y$ is positive to starboard, and $F_z$ is positive in the direction of gravity. Entering body forces in this manner avoids confusion with respect to cable angle values and quadrants. Body drag $F_x$ should be entered as a negative (-) number; a positive (+) value indicates thrust. The initial input force can be either at the fixed end of the cable (which implies a positive value for $F_x$) or at the towed body or free end of the cable. Care should be exercised, however, in starting a computation at a fixed end; in this case, if the starting cable angle is near the critical angle, the resulting cable shape is exceptionally sensitive to the force component values.

6. If external body force components are input into the program as the initial condition, specified cable length $S_{\text{max}}$ (command statement number 18) should always be positive (+). If a negative cable length is specified, the initial input forces represent internal cable forces rather than external forces. The cable length printout interval $S_{\text{inc}}$ (command statement number 19) can be specified as either positive or negative; the proper sign is selected by the program.

7. Four stopping conditions other than $S_{\text{max}}$ are included in the program (command statement numbers 20 through 23). The conditions $X_{\text{stop}}$, $Y_{\text{stop}}$, and $Z_{\text{stop}}$ will terminate the calculation whenever trail ($X$-direction), kite ($Y$-direction), or depth ($Z$-direction) reach the specified value as measured from the starting point at the initial body. The condition $\Phi_{\text{stop}}$ will terminate the computation when the specified cable angle $\phi$
value is reached. If no stopping condition is specified, the calculations will continue until Smax is reached.

8. Once the computation is completed to the specified stopping point, command statement number 24 will appear on the CRT screen. If the option ADD BODY is chosen, command statement numbers 17 through 23 will reappear on the CRT in sequence. If the option ADD CABLE is selected, command statement numbers 4 through 16 and 18 through 23 will appear on the CRT. The ADD CABLE option allows the cable properties as well as the fluid density (as, for example, from water-to-air) to change. If the option NEW CASE is chosen, the program will print the final cable force components and return to command statement number 1 to begin a new cable configuration.

9. When an HP-9836, an HP-9835, or an HP-9826 is used, most of the input values will be retained unless they are specifically changed or the computer program is stopped. The exceptions are the stopping parameters (command statement numbers 20 through 23). These values must be re-entered each time the command statement appears on the CRT. With an HP-87, however, all values must be re-entered each time a command statement appears on the CRT.

SAMPLE PROBLEMS

The following three sample problems illustrate program usage:

Problem 1 - Submarine Towing a Submerged Buoy

Determine the length of double-armored steel cable necessary to tow a subsurface buoy 300 ft above the deck of a submarine at 10 kn. Ribbon fairing is
attached to the cable for the first 300 ft of length starting at the buoy to reduce cable strumming. Printout of cable variables is desired at 50-ft intervals along the cable length. The cable will be assumed to be nonelastic. The characteristics of the fluid, cable, and buoy are listed in Table 3; program output is listed in Table 4.

Problem 2 - Ship Towing a Paravane

Determine the length of faired towline necessary to tow a paravane 500 ft to the side of a ship at 25 kn. The towline strength member is parallel-strand fiberglass in a epoxy matrix. The ship towpoint is 15 ft above the water surface. Printout of cable variables is desired at 50-ft intervals along the cable length. The characteristics of the fluid, towline, and paravane are listed in Table 5; final program output is listed in Table 6. Note that, since the towline passes through the air-water interface but the total side distance to the ship is specified, the problem is boundary valued and must be iterated to find the side distance at which the towline pierces the water surface.

Problem 3 - Helicopter Towing Two Depressors

A helicopter is required to tow two depressors spaced 50-ft apart along a braided nylon towline at a speed of 20 kn. Determine the length of towline necessary to place the lower depressor at a depth of 75 ft if the helicopter is flying at an altitude of 100 ft. The towline is unfaired. Both depressors produce identical towing forces. Printout of cable variables is desired only at end points of cable segments. The characteristics of the fluids, towline, and depressors are listed in Table 7; program output is listed in Table 8.
Table 3. Physical characteristics of sample problem 1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, $V$ (kn)</td>
<td>10.0</td>
</tr>
<tr>
<td>Fluid Density, $\rho$ (lb-sec$^2$/ft$^4$)</td>
<td>1.9905</td>
</tr>
</tbody>
</table>

**Cable**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $d$ (in.)</td>
<td>0.35</td>
</tr>
<tr>
<td>Weight in Fluid, $W$ (lb/ft)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**Ribbon Loading**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Drag Coefficient, $C_r$</td>
<td>1.25</td>
</tr>
<tr>
<td>Side Force Coefficient, $C_s$</td>
<td>0.10</td>
</tr>
<tr>
<td>Cable Length, $S_{max}$ (ft)</td>
<td>300.0</td>
</tr>
</tbody>
</table>

**Bare (Pode) Loading**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pode Frictional Parameter, $f$</td>
<td>0.01</td>
</tr>
<tr>
<td>Normal Drag Coefficient, $C_r$</td>
<td>1.90</td>
</tr>
<tr>
<td>Side Force Coefficient, $C_s$</td>
<td>0.10</td>
</tr>
<tr>
<td>Total Depth, $Z_{stop}$ (ft)</td>
<td>300.0</td>
</tr>
</tbody>
</table>

**Buoy**

<table>
<thead>
<tr>
<th>Force, $F$ (lb)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag Force, $F_x$</td>
<td>-500.0</td>
</tr>
<tr>
<td>Side Force, $F_y$</td>
<td>0.0</td>
</tr>
<tr>
<td>Down Force, $F_z$</td>
<td>-2000.0</td>
</tr>
</tbody>
</table>
Table 4. Program output for sample problem 1.

SUBMARINE TOWING A SUBMERGED BUOY USING A STEEL CIRCULAR CABLE

V=10.00 Knots
Rho=1.9905 Lb • Sec^-2/ Ft^-4

RIBBON LOADING

\[
\begin{align*}
\mathbf{fn} &= 0.4996 - 0.2499 \cos(\Phi) + 0.2527 \sin(\Phi) - 0.2487 \cos(2\Phi) + 0.0000 \sin(2\Phi), \\
\mathbf{ft} &= -0.2255 + 0.3417 \cos(\Phi) + 0.2255 \sin(\Phi) + 0.0000 \sin(2\Phi) - 0.0811 \sin(2\Phi), \\
\mathbf{fs} &= 1.4792 + 0.0041 \cos(\Phi) - 3.0425 \sin(\Phi) - 1.5633 \cos(2\Phi) + 1.5213 \sin(2\Phi).
\end{align*}
\]

\[
\begin{align*}
C_r &= 1.250, \\
C_s &= 1.00, \\
d &= 0.350 \text{ in}, \\
W &= 0.150 \text{ Lb/ Ft}.
\end{align*}
\]

BODY FORCE COMPONENTS

\[
\begin{align*}
F_x &= -500.00 \text{ Lb}, \\
F_y &= 0.00 \text{ Lb}, \\
F_z &= -2000.00 \text{ Lb}.
\end{align*}
\]

<table>
<thead>
<tr>
<th>Sref(Ft)</th>
<th>Sstr(Ft)</th>
<th>X(Ft)</th>
<th>Y(Ft)</th>
<th>Z(Ft)</th>
<th>Phi(Deg)</th>
<th>Beta(Deg)</th>
<th>T(Lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>76.0</td>
<td>180.0</td>
<td>2061.6</td>
</tr>
<tr>
<td>50.0</td>
<td>50.0</td>
<td>17.1</td>
<td>4.4</td>
<td>46.8</td>
<td>63.9</td>
<td>181.0</td>
<td>2081.7</td>
</tr>
<tr>
<td>100.0</td>
<td>100.0</td>
<td>43.1</td>
<td>1.6</td>
<td>89.5</td>
<td>53.9</td>
<td>182.3</td>
<td>2112.7</td>
</tr>
<tr>
<td>150.0</td>
<td>150.0</td>
<td>79.4</td>
<td>3.6</td>
<td>127.6</td>
<td>45.8</td>
<td>183.7</td>
<td>2151.6</td>
</tr>
<tr>
<td>200.0</td>
<td>200.0</td>
<td>112.2</td>
<td>6.3</td>
<td>161.3</td>
<td>39.4</td>
<td>185.2</td>
<td>2197.1</td>
</tr>
<tr>
<td>250.0</td>
<td>250.0</td>
<td>152.5</td>
<td>9.3</td>
<td>191.0</td>
<td>34.7</td>
<td>186.5</td>
<td>2247.2</td>
</tr>
<tr>
<td>300.0</td>
<td>300.0</td>
<td>194.7</td>
<td>12.6</td>
<td>217.4</td>
<td>30.0</td>
<td>187.8</td>
<td>2300.5</td>
</tr>
<tr>
<td>350.0</td>
<td>350.0</td>
<td>238.9</td>
<td>16.0</td>
<td>240.4</td>
<td>25.5</td>
<td>189.0</td>
<td>2304.9</td>
</tr>
<tr>
<td>400.0</td>
<td>400.0</td>
<td>284.7</td>
<td>19.3</td>
<td>268.4</td>
<td>22.2</td>
<td>190.0</td>
<td>2309.9</td>
</tr>
<tr>
<td>450.0</td>
<td>450.0</td>
<td>331.4</td>
<td>22.6</td>
<td>277.9</td>
<td>19.5</td>
<td>191.1</td>
<td>2315.0</td>
</tr>
<tr>
<td>500.0</td>
<td>500.0</td>
<td>378.8</td>
<td>25.7</td>
<td>293.4</td>
<td>17.4</td>
<td>192.0</td>
<td>2320.5</td>
</tr>
<tr>
<td>525.0</td>
<td>525.0</td>
<td>400.8</td>
<td>27.2</td>
<td>300.0</td>
<td>16.6</td>
<td>192.4</td>
<td>2322.2</td>
</tr>
</tbody>
</table>

Rho= 1.9905 Lb • Sec^-2/ Ft^-4

PODE LOADING

\[
\begin{align*}
\mathbf{fn} &= 0.5000 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) - 0.5000 \cos(2\Phi) + 0.0000 \sin(2\Phi), \\
\mathbf{ft} &= 0.1000 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) + 0.0000 \sin(2\Phi) + 0.0000 \sin(2\Phi), \\
\mathbf{fs} &= 1.4792 + 0.0041 \cos(\Phi) - 3.0425 \sin(\Phi) - 1.5633 \cos(2\Phi) + 1.5213 \sin(2\Phi).
\end{align*}
\]

\[
\begin{align*}
C_r &= 1.900, \\
C_s &= 1.100, \\
d &= 0.350 \text{ in}, \\
W &= 0.150 \text{ Lb/ Ft}.
\end{align*}
\]

<table>
<thead>
<tr>
<th>Sref(Ft)</th>
<th>Sstr(Ft)</th>
<th>X(Ft)</th>
<th>Y(Ft)</th>
<th>Z(Ft)</th>
<th>Phi(Deg)</th>
<th>Beta(Deg)</th>
<th>T(Lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300.0</td>
<td>300.0</td>
<td>194.7</td>
<td>12.6</td>
<td>217.4</td>
<td>30.0</td>
<td>187.8</td>
<td>2330.5</td>
</tr>
<tr>
<td>350.0</td>
<td>350.0</td>
<td>238.9</td>
<td>16.0</td>
<td>240.4</td>
<td>25.5</td>
<td>189.0</td>
<td>2304.9</td>
</tr>
<tr>
<td>400.0</td>
<td>400.0</td>
<td>284.7</td>
<td>19.3</td>
<td>268.4</td>
<td>22.2</td>
<td>190.0</td>
<td>2309.9</td>
</tr>
<tr>
<td>450.0</td>
<td>450.0</td>
<td>331.4</td>
<td>22.6</td>
<td>277.9</td>
<td>19.5</td>
<td>191.1</td>
<td>2315.0</td>
</tr>
<tr>
<td>500.0</td>
<td>500.0</td>
<td>378.8</td>
<td>25.7</td>
<td>293.4</td>
<td>17.4</td>
<td>192.0</td>
<td>2320.5</td>
</tr>
<tr>
<td>525.0</td>
<td>525.0</td>
<td>400.8</td>
<td>27.2</td>
<td>300.0</td>
<td>16.6</td>
<td>192.4</td>
<td>2322.2</td>
</tr>
</tbody>
</table>

FINAL FORCE COMPONENTS

\[
\begin{align*}
F_x &= -2226.63 \text{ Lb}, \\
F_y &= -142.75 \text{ Lb}, \\
F_z &= -647.27 \text{ Lb}.
\end{align*}
\]
Table 5. Physical characteristics of sample problem 2.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, $V$ (kn)</td>
<td>25.0</td>
</tr>
<tr>
<td>Water Density, $\rho_w$ (lb-sec$^2$/ft$^4$)</td>
<td>1.9905</td>
</tr>
<tr>
<td>Air Density, $\rho_a$ (lb-sec$^2$/ft$^4$)</td>
<td>0.0024</td>
</tr>
<tr>
<td>Towline</td>
<td></td>
</tr>
<tr>
<td>Normal Drag Coefficient, $C_r$</td>
<td>0.15</td>
</tr>
<tr>
<td>Side Force Coefficient, $C_s$</td>
<td>0.0</td>
</tr>
<tr>
<td>Thickness, $d$ (in.)</td>
<td>0.5</td>
</tr>
<tr>
<td>Weight in Water, $W_w$ d (lb/ft)</td>
<td>0.2</td>
</tr>
<tr>
<td>Weight in Air, $W_a$ (lb/ft)</td>
<td>0.55</td>
</tr>
<tr>
<td>Modulus of Elasticity, $E$ (lb/in.$^2$)</td>
<td>9.0 x 10$^6$</td>
</tr>
<tr>
<td>Structural Cross-Sectional Area, $A_c$ (in.$^2$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Normal Loading Function, $f_n$</td>
<td>-4.102 + 4.330 cos$\phi$ + 4.878 sin$\phi$ - 0.228 cos2$\phi$ - 2.165 sin2$\phi$</td>
</tr>
<tr>
<td>Tangential Loading Function, $f_t$</td>
<td>0.078 + 0.383 cos$\phi$ - 0.078 sin$\phi$</td>
</tr>
<tr>
<td>Side Loading Function, $f_s$</td>
<td>1.000 sin$\phi$</td>
</tr>
<tr>
<td>Athwartship Distance, $Y_{stop}$ (ft)</td>
<td>500.0</td>
</tr>
<tr>
<td>Tow Height, $Z_{stop} - Depth$ (ft)</td>
<td>15.0</td>
</tr>
<tr>
<td>Paravane</td>
<td></td>
</tr>
<tr>
<td>Drag Force, $F_x$ (lb)</td>
<td>-1000.0</td>
</tr>
<tr>
<td>Side Force, $F_y$ (lb)</td>
<td>5000.0</td>
</tr>
<tr>
<td>Down Force, $F_z$ (lb)</td>
<td>1000.0</td>
</tr>
</tbody>
</table>
Table 6. Program output for sample problem 2.

**SHIP TOWING A PARAVANE USING A FAIRED FIBERGLASS TOWLINE**

<table>
<thead>
<tr>
<th>V</th>
<th>25.00 Knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rho</td>
<td>1.9305 Lb·Sec²/Ft⁴</td>
</tr>
<tr>
<td>fn</td>
<td>4.1020+4.3300 COS(Phi)+4.8780 SIN(Phi)-0.2280 COS(2<em>Phi)-2.1650 SIN(2</em>Phi)</td>
</tr>
<tr>
<td>ft</td>
<td>0.0780+0.3680 COS(Phi)-0.0780 SIN(Phi)+0.0000 COS(2<em>Phi)+0.0000 SIN(2</em>Phi)</td>
</tr>
<tr>
<td>fs</td>
<td>0.0000+0.0000 COS(Phi)+1.0000 SIN(Phi)+0.0000 COS(2<em>Phi)+0.0000 SIN(2</em>Phi)</td>
</tr>
<tr>
<td>Cr</td>
<td>0.150</td>
</tr>
<tr>
<td>Cs</td>
<td>0.000</td>
</tr>
<tr>
<td>d</td>
<td>0.500 In.</td>
</tr>
<tr>
<td>W</td>
<td>2.000 lb/Ft</td>
</tr>
<tr>
<td>E</td>
<td>9.000 × 10⁶ Lb/In.²</td>
</tr>
<tr>
<td>Ac</td>
<td>0.100 In.²</td>
</tr>
</tbody>
</table>

**BODY FORCE COMPONENTS**

<table>
<thead>
<tr>
<th>Sref(Ft)</th>
<th>Sstr(Ft)</th>
<th>X(Ft)</th>
<th>Y(Ft)</th>
<th>Z(Ft)</th>
<th>Phi(Deg)</th>
<th>Beta(Deg)</th>
<th>T(Lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>79.9</td>
<td>78.7</td>
<td>5198.2</td>
</tr>
<tr>
<td>50.0</td>
<td>50.3</td>
<td>12.0</td>
<td>-47.9</td>
<td>-9.6</td>
<td>73.6</td>
<td>78.6</td>
<td>5349.9</td>
</tr>
<tr>
<td>100.0</td>
<td>100.6</td>
<td>28.1</td>
<td>-94.6</td>
<td>-19.1</td>
<td>69.0</td>
<td>78.5</td>
<td>5322.3</td>
</tr>
<tr>
<td>150.0</td>
<td>150.9</td>
<td>47.8</td>
<td>-139.9</td>
<td>-29.4</td>
<td>65.0</td>
<td>78.4</td>
<td>5410.9</td>
</tr>
<tr>
<td>200.0</td>
<td>201.2</td>
<td>70.4</td>
<td>-183.9</td>
<td>-37.5</td>
<td>61.5</td>
<td>78.2</td>
<td>5513.2</td>
</tr>
<tr>
<td>250.0</td>
<td>251.5</td>
<td>95.6</td>
<td>-225.5</td>
<td>-46.4</td>
<td>58.5</td>
<td>70.1</td>
<td>5627.3</td>
</tr>
<tr>
<td>300.0</td>
<td>301.8</td>
<td>122.9</td>
<td>-267.9</td>
<td>-55.7</td>
<td>55.9</td>
<td>78.0</td>
<td>5751.2</td>
</tr>
<tr>
<td>350.0</td>
<td>352.1</td>
<td>151.9</td>
<td>-308.1</td>
<td>-65.8</td>
<td>53.6</td>
<td>77.9</td>
<td>5883.7</td>
</tr>
<tr>
<td>400.0</td>
<td>402.5</td>
<td>182.6</td>
<td>-347.2</td>
<td>-72.2</td>
<td>51.6</td>
<td>77.8</td>
<td>6023.4</td>
</tr>
<tr>
<td>450.0</td>
<td>452.8</td>
<td>214.3</td>
<td>-385.3</td>
<td>-80.5</td>
<td>49.9</td>
<td>77.7</td>
<td>6169.5</td>
</tr>
<tr>
<td>500.0</td>
<td>503.2</td>
<td>247.9</td>
<td>-424.4</td>
<td>-88.7</td>
<td>48.3</td>
<td>77.5</td>
<td>6321.1</td>
</tr>
<tr>
<td>515.5</td>
<td>516.8</td>
<td>257.7</td>
<td>-433.8</td>
<td>-91.2</td>
<td>47.9</td>
<td>77.5</td>
<td>6389.2</td>
</tr>
</tbody>
</table>

| Rho | 0.0024 Lb·Sec²/Ft⁴ |
| fn | 4.1020+4.3300 COS(Phi)+4.8780 SIN(Phi)-0.2280 COS(2*Phi)-2.1650 SIN(2*Phi) |
| ft | 0.0780+0.3680 COS(Phi)-0.0780 SIN(Phi)+0.0000 COS(2*Phi)+0.0000 SIN(2*Phi) |
| fs | 0.0000+0.0000 COS(Phi)+1.0000 SIN(Phi)+0.0000 COS(2*Phi)+0.0000 SIN(2*Phi) |
| Cr | 0.150 |
| Cs | 0.000 |
| d | 0.500 In. |
| W | 2.000 lb/Ft |
| E | 9.000 × 10⁶ Lb/In.² |
| Ac | 0.100 In.² |

**FINAL FORCE COMPONENTS**

| Fx | -42174.22 Lb |
| Fy | 4610.46 Lb |
| Fz | 1072.54 Lb |
Table 7. Physical characteristics of sample problem 3.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, V (kn)</td>
<td>20.0</td>
</tr>
<tr>
<td>Water Density, ρ_w (lb·sec^2/ft^4)</td>
<td>1.9905</td>
</tr>
<tr>
<td>Air Density, ρ_a (lb·sec^2/ft^4)</td>
<td>0.0024</td>
</tr>
<tr>
<td>Towline (Pode Loading)</td>
<td></td>
</tr>
<tr>
<td>Pode Frictional Parameter, f</td>
<td>0.007</td>
</tr>
<tr>
<td>Normal Drag Coefficient, C_r</td>
<td>1.80</td>
</tr>
<tr>
<td>Side Force Coefficient, C_s</td>
<td>0.0</td>
</tr>
<tr>
<td>Diameter, d (in.)</td>
<td>0.625</td>
</tr>
<tr>
<td>Weight in Water, W_w d(lb/ft)</td>
<td>0.000</td>
</tr>
<tr>
<td>Weight in Air, W_a (lb/ft)</td>
<td>0.130</td>
</tr>
<tr>
<td>Modulus of Elasticity, E (lb/in.²)</td>
<td>23,367.0 + 14.4 (T/A_c)</td>
</tr>
<tr>
<td>Cross-Sectional Area, A_c (in.²)</td>
<td>0.307</td>
</tr>
<tr>
<td>Distance Between Depressors, Smax1 (ft)</td>
<td>50.0</td>
</tr>
<tr>
<td>Depth to Lower Depressor, Zstop1 (ft)</td>
<td>75.0</td>
</tr>
<tr>
<td>Total Vertical Separation, Zstop2 (ft)</td>
<td>175.0</td>
</tr>
<tr>
<td>Paravane</td>
<td></td>
</tr>
<tr>
<td>Drag Force, F_x (lb)</td>
<td>-300.0</td>
</tr>
<tr>
<td>Side Force, F_y (lb)</td>
<td>0.0</td>
</tr>
<tr>
<td>Down Force, F_z (lb)</td>
<td>1500.0</td>
</tr>
</tbody>
</table>
Table 8. Program output for sample problem 3.

HELICOPTER TOWING TWO DEPRESSORS USING A BRAIDED NYLON TOWLINE

\[ \begin{align*}
V &= 20.00 \text{ Knots} \\
\text{Rho} &= 1.9905 \text{ Lb} \cdot \text{Sec}^{-2} / \text{Ft}^4 \\
\text{PODE LOADING} \\
\text{fn} &= 0.5000 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) - 0.5000 \cos(2\Phi) + 0.0000 \sin(2\Phi) \\
\text{ft} &= 0.0070 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) + 0.0000 \cos(2\Phi) + 0.0000 \sin(2\Phi) \\
\text{fs} &= 1.4792 + 0.0041 \cos(\Phi) - 3.0425 \sin(\Phi) - 1.5633 \cos(2\Phi) + 1.5213 \sin(2\Phi) \\
\end{align*} \]

\[
\begin{align*}
\text{Cr} &= 1.800 \\
\text{Cs} &= 0.000 \\
\text{d} &= 0.625 \text{ In.} \\
\text{W} &= 1.000 \text{ Lb/Ft} \\
\text{E} &= (23.3670 + 0.0144 \cdot T/Ag) \cdot 10^{-3} \text{ Lb/In.}^2 \\
\text{Ac} &= 0.307 \text{ In.}^2 \\
\end{align*} \]

\text{BODY FORCE COMPONENTS}

\[
\begin{align*}
\text{Fx} &= -300.00 \text{ Lb} \\
\text{Fy} &= 0.00 \text{ Lb} \\
\text{Fz} &= 1500.00 \text{ Lb} \\
\text{Sref} (\text{Ft}) &\quad \text{Sstr} (\text{Ft}) \quad \text{X} (\text{Ft}) \quad \text{Y} (\text{Ft}) \quad \text{Z} (\text{Ft}) \quad \Phi (\text{Deg}) \quad \Beta (\text{Deg}) \quad T (\text{Lbs}) \\
0.0 &\quad 0.0 &\quad 0.0 &\quad 0.0 &\quad 0.0 &\quad 78.7 &\quad 0.0 &\quad 1529.7 \\
50.0 &\quad 54.9 &\quad 44.4 &\quad 0.0 &\quad -28.1 &\quad 14.7 &\quad 0.0 &\quad 1568.7 \\
\end{align*} \]

\text{BODY FORCE COMPONENTS}

\[
\begin{align*}
\text{Fx} &= -300.00 \text{ Lb} \\
\text{Fy} &= 0.00 \text{ Lb} \\
\text{Fz} &= 1500.00 \text{ Lb} \\
\text{Sref} (\text{Ft}) &\quad \text{Sstr} (\text{Ft}) \quad \text{X} (\text{Ft}) \quad \text{Y} (\text{Ft}) \quad \text{Z} (\text{Ft}) \quad \Phi (\text{Deg}) \quad \Beta (\text{Deg}) \quad T (\text{Lbs}) \\
186.6 &\quad 209.1 &\quad 189.3 &\quad 0.0 &\quad -75.0 &\quad 8.4 &\quad 0.0 &\quad 2527.5 \\
\end{align*} \]

\text{Rho} = 0.0024 \text{ Lb} \cdot \text{Sec}^{-2} / \text{Ft}^4

\text{PODE LOADING}

\[
\begin{align*}
\text{fn} &= 0.5000 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) - 0.5000 \cos(2\Phi) + 0.0000 \sin(2\Phi) \\
\text{ft} &= 0.0070 + 0.0000 \cos(\Phi) + 0.0000 \sin(\Phi) + 0.0000 \cos(2\Phi) + 0.0000 \sin(2\Phi) \\
\text{fs} &= 1.4792 + 0.0041 \cos(\Phi) - 3.0425 \sin(\Phi) - 1.5633 \cos(2\Phi) + 1.5213 \sin(2\Phi) \\
\end{align*} \]

\[
\begin{align*}
\text{Cr} &= 1.800 \\
\text{Cs} &= 0.000 \\
\text{d} &= 0.625 \text{ In.} \\
\text{W} &= 1.000 \text{ Lb/Ft} \\
\text{E} &= (23.3670 + 0.0144 \cdot T/Ag) \cdot 10^{-3} \text{ Lb/In.}^2 \\
\text{Ac} &= 0.307 \text{ In.}^2 \\
\end{align*} \]

\text{Sref} (\text{Ft}) &\quad \text{Sstr} (\text{Ft}) \quad \text{X} (\text{Ft}) \quad \text{Y} (\text{Ft}) \quad \text{Z} (\text{Ft}) \quad \Phi (\text{Deg}) \quad \Beta (\text{Deg}) \quad T (\text{Lbs}) \\
186.6 &\quad 209.1 &\quad 189.3 &\quad 0.0 &\quad -75.0 &\quad 8.4 &\quad 0.0 &\quad 2735.6 \\
\end{align*} \]

\text{FINAL FORCE COMPONENTS}

\[
\begin{align*}
\text{Fx} &= -2707.38 \text{ Lb} \\
\text{Fy} &= 2.00 \text{ Lb} \\
\text{Fz} &= 469.51 \text{ Lb} \\
\end{align*} \]
ACKNOWLEDGMENTS

The help of several members of the Towed Systems Branch who contributed criticisms and comments is gratefully acknowledged. Discussions with John Johnston, William Stewart, and Dr. Paul Rispin were particularly helpful.
APPENDIX

HP-9836 PROGRAM LISTING
Program CAOL36 calculates the three-dimensional static configuration of an extensible, flexible cable in a uniform stream.

Main input section

```
120 RAD
130 DIM A(80)
140 M1: Output=0
150 Continue=0
160 S=0
170 Ss=0
180 X=0
190 Y=0
200 Z=0
210 T=0
220 FX=0
230 FY=0
240 FZ=0
250 INPUT "FOR HARDCOPY, ENTER: 1 [For CRT, press CONT]",Output
260 IF Output=0 THEN PRINTER IS I
270 IF Output=1 THEN PRINTER IS 7M1
280 INPUT "ENTER TITLE",A#
290 PRINT "------------------------------------------------------------------------"
300 PRINT A#
310 PRINT "------------------------------------------------------------------------"
320 PRINT
330 INPUT "ENTER SPEED: V (Knots)".V
340 PRINT USING II"V".V,"Knots"
350 I: IMAGE 2A,DD.2D,X,5A
360 M2: PRINT
370 INPUT "ENTER FLUID MASS DENSITY: Rho (Lb*Sec^2/Ft^4)".Rho
380 PRINT USING 12"Rho-.Rho."Lb*Sec"2/Ft"4"
390 IMAGE 4A,Z.40,X.13A
400 INPUT "ENTER CABLE LOADING: Pode-1; Ribbon-2; Other-3",Loading
410 IF Loading=1 THEN GOSUB Pode
420 IF Loading=2 THEN GOSUB Ribbon
430 IF Loading=3 THEN GOSUB Other loading
440 PRINT
450 PRINT USING 131"fn-.Fa0,Fa1,"COS(Phi)",Fb1,"SIN(Phi)",Fa2,"COS(2*Phi)",
460 "Fb2,"SIN(2*Phi)"
470 PRINT USING 131"ft-.Ga0,Ga1,"COS(Phi)",Gb1,"SIN(Phi)",Ga2,"COS(2*Phi)",
480 "Gb2,"SIN(2*Phi)"
490 PRINT USING 131"fs-.Ha0,Ha1,"COS(Phi)",Hb1,"SIN(Phi)",Ha2,"COS(2*Phi)",
500 "Hb2,"SIN(2*Phi)"
510 13: IMAGE 3A,MZ.4D,2(SZ.4D,X,9A),2(SZ.4D,X,10A)
520 PRINT
530 INPUT "ENTER CABLE NORMAL DRAG COEFF: Cr",Cr
540 PRINT USING 141"Cr-.Cr"
550 I4: IMAGE 4X,3A,DD.3D
560 PRINT USING 151"Cs-*.Cs"
570 IMAGE 4X,3A,MD.30
```

36
560 INPUT "ENTER CABLE THICKNESS: d (In.)", D, d
570 PRINT USING I6; "d", D, "In."
580 I6: IMAGE 4X, 2A, 3D, X, 3A
590 INPUT "ENTER CABLE WEIGHT IN FLUID: W (Lb/Ft)", W
600 PRINT USING 17T; "W", W, "Lb/Ft"
610 17: IMAGE 4X, ZA, MD0, 3D, X, SA
620 INPUT "ENTER CABLE ELASTICITY: Non-Elastic=1; Elastic=2", Elas
630 IF Elas<>2 THEN M3
640 INPUT "ENTER MODULUS OF ELAS: E=E0+EI*T/Ac (Lb/In.^2)", E0, E1
650 IF E1=0 THEN PRINT USING 18; "E", E0/1000000, "*10^6 Lb/In.^2"
660 18: IMAGE 4X, 2A, 3D, X, 14A
670 IF E1<>0 THEN PRINT USING 19; "E", (E0/1000, E1/1000, "*T/Ac") *10^3 Lb/In.^2
680 19: IMAGE 4X, 3A, 3D, 4D, SSD, 4D, 21A
690 INPUT "ENTER CABLE CROSS-SECTIONAL AREA: Ac (In.^2)", Ac
700 PRINT USING I10; "Ac", Ac, "In.^2"
710 I10: IMAGE 4X, 3A, DD, 3D, X, 5A
720 M3: IF Continue=2 THEN MS
730 M4: INPUT "ENTER BODY FORCE COMPONENTS: Fx,Fy,Fz (Lb)", Fx, Fy, Fz
740 PRINT
750 PRINT "BODY FORCE COMPONENTS"
760 PRINT USING III; "Fx", Fx, "Lb"
770 PRINT USING III; "Fy", Fy, "Lb"
780 PRINT USING III; "Fz", Fz, "Lb"
790 III: IMAGE 4X, 3A, MSD, 2D, X, 2A
800 MS: INPUT "ENTER MAXIMUM CABLE LENGTH: Smax (Ft)", Smax
810 INPUT "ENTER CABLE LENGTH PRINTOUT INTERVAL: Sinc (Ft)", Sinc
820 Cstop=0
830 Xstop=10^3
840 Ystop=10^3
850 Zstop=10^3
860 Phidstop=10^3
870 INPUT "ENTER LIMIT: Xstop (Ft) [If none, press CONT]", Xstop
880 INPUT "ENTER LIMIT: Ystop (Ft) [If none, press CONT]", Ystop
890 INPUT "ENTER LIMIT: Zstop (Ft) [If none, press CONT]", Zstop
900 INPUT "ENTER LIMIT: Phistop (Deg) [If none, press CONT]", Phistop
910 Phistop=Phidstop/57.2958
920 PRINT
930 PRINT "\Sref(Ft) Str(Ft) X(Ft) Y(Ft) Z(Ft) PhL(Deg) Beta(Deg) T(Lbs)"
940 IF Continue=2 THEN GOSUB Tangle
950 GOSUB Integrator
960 Continue=0
970 INPUT "ADD BODY, ENTER: 1; ADD CABLE, ENTER: 2 [NEW CASE, press CONT]"
980 IF Continue=1 THEN M4
990 IF Continue=2 THEN M2
1000 PRINT
1010 PRINT "FINAL FORCE COMPONENTS"
1020 PRINT USING III; "Fx", Fx, "Lb"
1030 PRINT USING III; "Fy", Fy, "Lb"
1040 PRINT USING III; "Fz", Fz, "Lb"
1050 IF Output=1 THEN PRINT CHR$(12)
1060 GOTO M1
Subroutine "Tangle" calculates starting cable angles and tension from input force components.

\[ F_x = F_x + F_{x_0} \]
\[ F_y = F_y + F_{y_0} \]
\[ F_z = F_z + F_{z_0} \]

\[ T_{zy} = \sqrt{(F_z^2 + F_y^2)} \]
\[ T = \sqrt{(T_{zy}^2 + F_x^2)} \]

IF \( T_{zy} = 0 \) THEN \( \beta = 0 \)
IF \( (F_z = 0) \) AND \( (F_y > 0) \) THEN \( \beta = \pi*F_y/(2*\text{ABS}(F_y)) \)
IF \( F_z > 0 \) THEN \( \beta = \text{ATN}(F_y/F_z) \)
IF \( (F_z = 0) \) AND \( (F_y < 0) \) THEN \( \beta = \pi \)
IF \( (F_z < 0) \) AND \( (F_y = 0) \) THEN \( \beta = \text{ATN}(F_y/F_z) + \pi*F_y/\text{ABS}(F_y) \)
IF \( F_x = 0 \) AND \( (T_{zy} = 0) \) THEN \( \phi = \pi/2 \)
IF \( F_x < 0 \) THEN \( \phi = \text{ATN}(-T_{zy}/F_x) \)
IF \( F_x > 0 \) THEN \( \phi = \text{ATN}(-T_{zy}/F_x) + \pi \)
IF \( T = 0 \) THEN GOSUB Critang
IF \( T < 1 \) THEN \( T = 1 \)
RETURN

Subroutine "Critang" calculates critical angles to begin the integration when the starting tension is zero.

\[ \beta = 0 \]
\[ \phi = 0 \]

IF \( W = 0 \) THEN P3
\[ R = 118723*Rho*Cr+Dia*V*V \]
\[ F_s = 118723*Rho*Dia*V*V \]
\[ \Phi_{inc} = 0.09 \]
\[ \Phi = \text{SIN}(\Phi_{inc}) \]
\[ C_p = \text{COS}(\Phi_{inc}) \]
\[ \Phi_{inc} = \Phi_{inc}/2 \]
IF \( |W| > |H| \) THEN P2
\[ \Phi = \Phi_{inc}/2 \]
GOTO P1
IF \( W > 0 \) THEN \( \beta = \text{ASN}(H/W) \)
IF \( W < 0 \) AND \( (H > 0) \) THEN \( \beta = -\text{ASN}(H/W) - \pi \)
IF \( (W < 0) \) AND \( (H < 0) \) THEN \( \eta = -\text{ASN}(H/W) + \pi \)
\[ F = R*(F_{a_0} - \Phi_{inc} + 2*C_{2p} + F_{b_2}*S_{2p}) \]
\[ \text{Sum} = F + W*\text{COS}(\Phi_{inc}) + \text{COS}(\eta) \]
IF \( |\text{Sum}| < 0.0001 \) THEN P3
IF \( \text{Sum} = 0 \) THEN \( \Phi_{inc} = \Phi_{inc}/2 \)
\[ \Phi = \Phi_{inc}/\text{ABS}(\text{Sum}) \]
GOTO P1
P3: RETURN
Subroutine "Integrator" solves the differential equations of cable force equilibrium using a self-starting, modified Euler's method.

```plaintext
1620 Subroutine "Integrator"
1630 !
1640 Integratr: 1
1650 !
1660 !
1670 !
1680 Sinc=ABS(Sinc)
1690 IF Smax@0 THEN Sinc=Sinc
1700 Dies=Dies
1710 W=W
1720 N1: Phid=Phi*57.2958
1730 Beta=Beta*57.2958
1740 PRINT USING 112;S,5s,X,Y,Z,Phid,Beta,T
1750 112:IMAGE S(MSD,D,X),2X,2(M3D,D,4X),70,D
1760 IF (Cstop!1) OR (ABS(Smax-S)<.01) THEN N9
1770 SI=INT((ABS(S)+.01)/ABS(Sinc)) Sinc= Sinc
1780 N2: Dels=S-S
1790 IF ABS(Dels)>S0 THEN Dels=S0+Dels/ABS(Dels)
1800 IF ABS(S+Dels)>ABS(Smax) THEN Dels=Smax-S
1810 N3: Dels=Dels
1820 I=I+1
1830 Delp=0
1840 Delp=0
1850 Delb=0
1860 N4: J=J+1
1870 Delp=Delp
1880 Delb=Delb
1890 Betav=Beta+Delb/2
1900 Phiv=Phiv+Delb/2
1910 Tau=T+Delt/2
1920 IF Eles<>2 THEN N5
1930 E=E0+El+Tav/Ae
1940 IF E<>0 THEN B=1+Tav/(Ae+E)
1950 IF E<>0 THEN B=1+LOG(E/E0)/E1
1960 Dies=Dies+SQR(I/B)
1970 W=W*(I/B)
1980 Dels=Dels+8
1990 N5: R=.118725*Rho*Cr+Dies+V+V
2000 Fs=.118723*Rho*Ca+Dies+V+V
2010 Sp=SIN(Phiv)
2020 IF ABS(Sp)<.0001 THEN Sp=.0001
2030 Cmp=COS(Phiv)
2040 S2p=SIN(2*Phiv)
2050 Sb=SIN(Betav)
2060 Cmp=COS(Betav)
2070 Phiv=ABS(ASN(SIN(Phiv)))
2080 Spl=SIN(Phiv)
2090 Cmp=COS(Phiv)
2100 S2p=SIN(2*Phiv)
2110 C2p=COS(2*Phiv)
2120 Fl=R*(Fa0+Fa1*Cp1+Fb1*Sp1+Fa2*C2p1+Fb2*S2p1)
2130 G1=R*(Ga0+Ga1*Cp1+Gb1*Sp1+Ga2*C2p1+Gb2*S2p1)
2140 H1=R*(Ha0+Ha1*Cp1+Hb1*Sp1+Ha2*C2p1+Hb2*S2p1)
2150 IF Sp<>0 THEN F=F1
2160 IF Sp<0 THEN F=-F1
2170 IF Cp<>0 THEN G=G1
2180 IF Cp<0 THEN G=-G1
2190 IF (Loading<2) AND (S2p<=0) THEN H=H1
2200 IF (Loading<2) AND (S2p<0) THEN H=-H1
```
2210  IF (Loading>2) AND (Sp>0) THEN H=H1
2220  IF (Loading>2) AND (Sp<0) THEN H=H1
2230  Delt=(-G+Ws*Sp*Cb)*Dels
2240  Delb=(H-Ws*Sb)*Dels/(Tav*Sp)
2250  Delp=(F+Ws*Cp*Cb)*Dels/Tav
2260  Delsl=Dels
2270  IF I>10 THEN Dels=Delsl/2
2280  IF ABS(Delt)>0.09 THEN Dels=ABS(.07/Delt)*Delsl
2290  IF ABS(Delp)>0.09 THEN Dels=ABS(.07/Delp)*Delsl
2300  IF Dels<Delb THEN N3
2310  IF (ABS(Delt-Delp))>.0002 OR (ABS(Delt-Delp))>.0002 THEN N4
2320  S+S+Dels
2330  Ss=Ss+Dels
2340  X=X+Dels*Cp
2350  Y=Y-Dels*Sp*Sb
2360  Z=Z-Dels*Sp*Cb
2370  T=T+Delt
2380  Phi=Phi+Delt
2390  Beta=Beta+Delb
2400  IF Cstop=1 THEN N7
2410  IF ABS(X)>ABS(Xstop) THEN N6
2420  IF ABS(Y)>ABS(Ystop) THEN N6
2430  IF ABS(Z)>ABS(Zstop) THEN N6
2440  IF Phi>Phistop THEN N6
2450  IF (ABS(Smax-S))>0.1 AND (ABS(S1-S))>0.1 THEN N2
2460  GOTO N1
2470  N6: Cstop=1
2480  N7: IF ABS(Abs(X))>ABS(Xstop))<.01 THEN N1
2490  IF ABS(Abs(Y))>ABS(Ystop))<.01 THEN N1
2500  IF ABS(Abs(Z))>ABS(Zstop))<.01 THEN N1
2510  IF ABS(Phi-Phistop)<.0005 THEN N1
2520  Dels=ABS(Dels)
2530  IF ABS(X)>ABS(Xstop) THEN N8
2540  IF ABS(Y)>ABS(Ystop) THEN N8
2550  IF ABS(Z)>ABS(Zstop) THEN N8
2560  IF Phi<Phistop THEN N8
2570  Dels=Dels/2
2580  IF S<0 THEN Dels=+Dels
2590  GOTO N3
2600  N8: Dels=-Dels/2
2610  IF S<0 THEN Dels=-Dels
2620  GOTO N3
2630  N9: Fx=-T*COS(Phi)
2640  Fy=T*SIN(Phi)*SIN(Beta)
2650  Fz=T*SIN(Phi)*COS(Beta)
2660  RETURN
Subroutine "Pods" provides coefficients for Pod or bare cable loading functions.

```
2720 |---------------------------
2730 | Pods:
2740 | Subroutine *Pods* provides coefficients for Pod or bare cable loading functions.
2750 |
2760 | Fe0,.5
2770 | Fe1=0
2780 | Fb1=0
2790 | Fe2=-.5
2800 | Fb2=0
2810 | Gb1=0
2820 | Gb2=0
2830 | Ha0=1.4792
2840 | Ha1=.0841
2850 | Hb1=3.8425
2860 | Ha2=.15633
2870 | PRINT
2880 | PRINT "PODE LOADING"
2890 | INPUT "ENTER PODE FRICTIONAL PARAMETER: f", &e0
2900 | RETURN
2910 |-----------------------------
2920 | Ribbon:
2930 | Subroutine "Ribbon" provides coefficients for ribbon-faired cable loading functions.
2940 |
2950 | Fa0=.4986
2960 | Fa1=.2499
2970 | Fb1=.2527
2980 | Fa2=.2487
2990 | Fb2=0
3000 | Gb1=.3417
3010 | Gb2=0
3020 | Ha0=.2355
3030 | Ha1=3.255
3040 | Ha2=.15633
3050 | PRINT
3060 | PRINT "RIBBON LOADING"
3070 | RETURN
3080 |-----------------------------
3090 | Other-loading:
3100 | Subroutine "Other-loading" allows input of generalized cable loading functions.
3110 | INPUT "ENTER NORMAL LOADING FUNC, fn: A0,A1,B1,A2,B2", Fa0, Fa1, Fb1, Fa2, Fb2
3120 | INPUT "ENTER TANGENTIAL LOADING FUNC, ft: A0,A1,B1,A2,B2", Ga0, Ga1, Gb1, Gb2
3130 | INPUT "ENTER SIDE LOADING FUNC, fs: A0,A1,B1,A2,B2", Ha0, Ha1, Hb1, Ha2, Hb2
3140 | RETURN
3150 | END
```
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