SOLAR OSCILLATIONS AND CONVective FLOWS AS PROBES
OF STRUCTure IN THE SUBPHOTOSPHERE

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Application of inverse theory to the observation of high-degree five-minute solar oscillations has led to the detection of horizontal flows below the solar surface that are a combination of solar rotation and giant convection cells. The distinctive displacements in the centroids of the ridges evident in the power diagrams of the oscillations from one observing day to the next arise from different patterns of giant cells being rotated into view. Such observation of frequency splittings for the high-degree oscillation modes, combined with refinements in the inversion of the data using optimal averaging and spectral expansions, has shown that helioseismology should permit detailed mapping of velocity and thermal structures below the solar surface. Extensive theoretical studies of fully compressible magnetoconvection have shown that flows are indeed able to concentrate magnetic fields into concentrated flux sheets that are substantially evacuated of gas. The magnetic buoyancy instabilities have been extensively studied.
A. OVERVIEW OF RESEARCH GOALS

The research supported by this contract has been directed toward the study of the nonlinear dynamics of turbulent convection below the surface of the Sun, and the ability of such convection to produce strong magnetic fields through dynamo action. The coupling of convection with magnetic fields influences much of what is observed in the solar atmosphere, and so too most of the variations seen in the solar wind. Probing of the underlying convection is essential for understanding solar variability.

The dynamics of the convection zone has been studied under this contract by a combination of observations and theory. The five-minute oscillations of the Sun have been used to probe velocity and thermal structures below the solar surface. This has been possible because the presence of giant convection cells, as predicted by theory, leads to changes in the apparent frequencies of the acoustic waves responsible for the oscillations. Such frequency shifts with time have been measured from full-disk Doppler observations of the Sun carried out from Sac Peak. Application of inverse theory to such observed frequency splittings of the five-minute oscillations has allowed us to begin to map the velocity fields with depth below the solar surface. Such work in helioseismology has been complemented by nonlinear models of solar convection which take into account the compressibility of the medium and permit the flows to penetrate into the stably-stratified atmosphere. The theoretical models have also studied the interaction of convection with magnetic fields, dealing directly for the first time with the possible evacuation of the plasma within the magnetic flux sheets that can be formed by the
convection. The theoretical studies have proved to be very useful in also interpreting the observations with high spatial resolution of both granular and mesogranular flows on the solar surface as observed from both Sac Peak and from the Spacelab 2 SOUP instrument.

B. RESEARCH ACCOMPLISHMENTS

The research work supported in substantial part by this contract has resulted in 30 published (or in press) papers in refereed scientific journals or proceedings. The cover pages with abstracts and full citations are attached as Papers A to DD. A further 16 published abstracts of talks at national conferences have also been cited in the quarterly reports, and there have been over 28 other seminars given describing aspects of the research. Thus the contract has supported research that has been highly visible and thoroughly documented in the open scientific literature.

Group 1. (Papers A to E): Major reviews and summary discussions of helioseismology. The subject of using the five-minute oscillations to probe the interior structure and dynamics of the Sun has become an area of intense research interest and enthusiasm in solar physics. Helioseismology has now also been embraced by a wide group of physicists as a topic that should be strongly encouraged because of its substantial promise in permitting study of the inside of a star. This has permitted the Global Oscillations Network Group (GONG) to be initiated as a national project centered at the National Solar Observatory, involving both Tucson and Sac
Peak. With partial assistance from this contract, we have been able to consider helioseismology from a broad perspective and to help explain its subtleties in a number of prominent review articles within this group of papers. In particular, Papers C and D were color cover articles in *Science* and in *Scientific American*, and both elicited considerable interest and discussion from scientists and from people involved with national policy.

**Group 2. (Papers F to M):** Search for solar giant cells and study of rotation with depth. Application of inverse theory to our observations of high-degree five-minute oscillations has permitted us to detect horizontal flows below the solar surface that are a combination of solar rotation and giant convective cells. The distinctive displacements in the centroids of the ridges evident in the power diagrams of the oscillations from one observing day to the next arise from different patterns of giant cells being rotated into view. Our observation of such shifts in the frequencies of the oscillations, combined with our refinements of inversion of the data by optimal-averaging and spectral expansions, is serving to develop helioseismology as a detailed approach for studying the dynamics of the solar convection zone.

**Group 3. (Papers N to O):** Studies of active regions and flares. These two papers have dealt with regions possessing strong magnetic fields. The first, involving coordinated SMM satellite and ground-based observations, revealed that active regions possess a broad spectrum of rapid brightening events,
suggesting that magnetic field reconnection may be proceeding almost continuously and stochastically. The second paper is unusual in describing helioseismologic observations that were carried out during several days that were interrupted by a major solar flare. This permitted the first evaluation of the flare site as a possible source of acoustic mode excitation.

Group 4. (Papers P to V): Theory of compressible non-linear convection and magnetoconvection. Through the development of compressible theoretical models for convection, coupled with the use of supercomputers, we have studied the interaction of penetrative convection with magnetic fields. In particular, we have examined how convection can penetrate downward into the stable stratification below the convection zone, finding that lateral swaying of the convective plumes can lead to the excitation of gravity waves there. That region is thought to be able to support magnetic dynamo action, and our studies have provided estimates of the ability of convection to supply mechanical energy into that region. We have also considered the behavior of compressible magnetoconvection, finding that the magnetic fields are concentrated into sheets by the convective motions. These sheets can be substantially evacuated of gas to achieve a balance between the gas pressure outside and the strong magnetic pressure within the sheet. Remarkably enough, the sheets can appear almost as stagnant regions of fluid surrounded by sheaths of fast downflow, not unlike the behavior near magnetic pores on the Sun. These are the first dynamical calculations to consistently treat compressibility and field evacuation in magnetoconvection.
Group 5. (Papers W to BB): Magnetic buoyancy instability. These six papers study the nature of both linear and nonlinear instability, and its consequences, on magnetic field configurations through effects of magnetic buoyancy. This research has revealed that a wide range of magnetic field stratifications can be unstable, for there are about three varieties of instabilities that can disrupt the fields. This has significant implications upon whether it is possible to store strong magnetic fields at the base of the convection zone without them being disrupted by magnetic buoyancy instabilities. These studies have been supported by extensive nonlinear numerical simulations of the manner in which emerging magnetic flux loops twist and fragment.

Group 6. (Papers CC to DD): Nonlinear dynamical systems. These two papers report on the very rich behavior observed in doubly-diffusive convection that can lead to periodic, quasi-periodic and chaotic motion. The first paper has been of considerable importance in revealing that partial differential equations can allow the remarkable transitions previously observed with much simpler mathematical systems. This has bearing on the forms of time dependence that should be achieved with magnetoconvection as one example. The second paper has found that double-diffusive systems can admit travelling waves for the convection, and has studied their relative stability to standing waves normally studied in convective systems.
OVERVIEW OF SOLAR SEISMOLOGY: OSCILLATIONS AS PROBES OF INTERNAL STRUCTURE AND DYNAMICS IN THE SUN

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ABSTRACT

The physical nature of solar oscillations is reviewed. The nomenclature of the subject and the techniques used to interpret the oscillations are discussed. Many of the acoustic and gravity waves that can be observed in the atmosphere of the Sun are actually resonant or standing modes of the interior; precise measurements of the frequencies of such modes allow deductions of the internal structure and dynamics of this star. The scientific objectives of such studies of solar seismic disturbances, or of solar seismology, will be outlined. The reasons for why it would be very beneficial to carry out further observations of solar oscillations both from ground-based networks and from space will be discussed.

SOLAR INVERSE THEORY

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Helioseismological inversion, as with the inversion of any other data, is divided into three phases. The first is the solution of the so-called forward problem: namely, the calculation of the eigenfrequencies of a theoretical equilibrium state. The second is an attempt to understand the results, either empirically by determining how those frequencies vary as chosen parameters defining the equilibrium model are varied, or analytically from asymptotic expansions in limiting cases of high order or degree. A familiarity with at least the qualitative dependence of the eigenfrequencies on various properties of the solar model is necessary not only for personal enlightenment but also for arming oneself to interpret the rather more abstract third phase. That phase is to pose and solve an inverse problem, which seeks to find a plausible equilibrium model of the Sun whose eigenfrequencies are consistent with observation.

The three phases are briefly discussed in this review, and the third, which is not yet widely used in helioseismology, is illustrated with some selected inversions of artificial solar data.
Seismology of the Sun

Jørgen Christensen-Dalsgaard, Douglas Gough, Juri Toomre

Summary. Oscillations of the sun make it possible to probe the inside of a star. The frequencies of the oscillations have already provided measures of the sound speed and the rate of rotation throughout much of the solar interior. These quantities are important for understanding the dynamics of the magnetic cycle and have a bearing on testing general relativity by planetary precession. The oscillation frequencies yield a helium abundance that is consistent with cosmology, but they reinforce the severity of the neutrino problem. They should soon provide an important standard by which to calibrate the theory of stellar evolution.

Helioseismology

Acoustic waves within the sun are visible as oscillations on the solar surface. Their pattern and period hold clues to structure, composition and dynamics in the sun's interior

by John W. Leibacher, Robert W. Noyes, Juri Toomre and Roger K. Ulrich

Oscillations of the sun's surface are the result of sound waves resonating in its interior; here four of the 10 million resonances that occur in the sun are modeled by computer. Surface regions that are approaching the observer are colored blue, regions that are receding are colored red. In actual observation such surface displacements are evident as Doppler shifts in the wavelengths of light absorbed by the moving gases and as variations in brightness. The spatial pattern and the period of a surface oscillation enable investigators to deduce the three-dimensional structure of the resonance and to infer properties of the solar interior. For each oscillation the degree (ℓ) and the azimuthal order (m) are given. Degree describes the pattern in terms of its total number of nodal curves along which the surface of the sphere is motionless, seen here as bands of gray between the zones of color; azimuthal order is an indication of the number of nodes that intersect the solar equator.
PROPERTIES OF SOLAR OSCILLATIONS

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ABSTRACT. Many of the oscillations that can be observed in the atmosphere of the Sun are resonant acoustic or gravity modes of the interior. Accurate measurement of the frequencies of these $p$ and $g$ modes permits deductions about the internal structure and dynamics of this star. Some of the methods of interpretation, involving a close interplay between observation and theory, can be carried over to the study of more distant stars.


ATTENTION TO MEASURE THE SOLAR SUBSURFACE VELOCITY

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ABSTRACT — Five-minute oscillation modes are advected by horizontal velocities below the solar surface, and thus can be used as probes of rotation and large-scale convective flows. Results of inverse theory applied to observations of high-degree modes carried out on six separate days reveal variations in horizontal velocities with depth from day to day that may be the result of giant convection cells, though noise in the data makes this interpretation somewhat tentative.
ROTATIONAL INVERSION FROM GLOBAL SOLAR OSCILLATIONS

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We investigate the degree to which various sets of solar oscillations can resolve the solar internal rotation. Genuine observations were simulated by the following procedure: First an artificial angular velocity was invented by one of us, and from it the rotational splitting of a set of normal modes was calculated; to that was added some random noise. The result was treated as artificial data by the other author, acting as an observer, who attempted to recover the rotation law by using the Backus-Gilbert optimal averaging procedure. The observer knew neither the original rotation law nor the amount of noise that had been added. Finally his conclusion was compared with the actual artificial angular velocity.

SENSITIVITY OF INFERRED SUBPHOTOSPHERIC VELOCITY FIELD TO
MODE SELECTION, ANALYSIS TECHNIQUE AND NOISE

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ABSTRACT

The horizontal velocity immediately below the photosphere can be inferred from observations of high-degree solar oscillations by an optimal-averaging inversion technique. We investigate the sensitivity of the results to various details of both the inversion and the determination of the frequencies. The results are shown to be quite stable to the choice of most parameters, suggesting that this procedure produces reliable estimates of the subsurface velocity.
THE EFFECTS OF A NEARLY 100% DUTY CYCLE ON OBSERVATIONS OF SOLAR OSCILLATIONS

PAPER I.

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ABSTRACT: Power spectra of window functions with duty cycles between 80% and 99% and with randomly spaced gaps are computed and their effect on observations of solar oscillations are discussed. It is found that for all the cases considered, observations of solar oscillations would not be severely impacted as long as the gap structure is random rather than periodic.


THE EFFECTS OF IMAGE MOTION ON THE $x$-$v$ DIAGRAM

PAPER J.

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ABSTRACT: A simple two-dimensional $(x,t)$ model of the solar oscillatory velocity field is subjected to a form of differential image motion. This image motion is meant to approximately model the effect of the Earth's atmosphere on observations of high degree solar oscillations. The distorted velocity field is analyzed to provide the apparent frequencies of the modes. Comparison of the results with the frequencies obtained from the undistorted case shows that the image motion can produce a discrepancy of as much as 12 µHz.

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INFLUENCE OF SPATIAL FILTERING ON POSSIBLE ANISOTROPIES IN SOLAR OSCILLATIONS

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ABSTRACT. We have used full disk Doppler observations of solar oscillations to compare the amplitudes of sectoral modes propagating along the equator with those of similar modes propagating along a great circle aligned with the poles. We find that the amplitudes are generally not equal for the two classes of modes, but the results are sensitive to analysis procedures attempting to isolate the different modes of oscillation. Spatial filtering of the data using spherical harmonics suggests that greater amplitudes are associated with "polar" sectoral modes than with "equatorial" sectoral modes.


SOLAR EQUATORIAL ROTATION RATE INFERRED FROM INVERSION OF FREQUENCY SPLITTING OF HIGH-DEGREE MODES

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ABSTRACT — The equatorial rotation rate has been inferred as a function of depth through the outer 16 Mm of the Sun from observations of high-degree five-minute oscillations. An optimal averaging inversion procedure due to Backus & Gilbert (1970) has been applied to frequency splittings measured from power spectra obtained using Doppler data spanning three and five consecutive days. The resulting rotation curves have proven to be much more stable than the curves obtained from data sets of single days. The results imply that the solar rotation rate increases with depth by 0.023 μHz reaching a maximum at about 2 Mm below the surface, then decreases by 0.037 μHz down to 16 Mm.
EFFECTS OF MODE BEATING ON DETERMINATION OF SOLAR OSCILLATION FREQUENCIES

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ABSTRACT. Numerical simulations have been used to assess effects of mode beating, or interference between unresolved modes of solar oscillation, upon the determination of their frequencies. Synthetic data sets of oscillations are constructed for modes with spherical harmonic degrees $\ell$ ranging from 50 to 400. The isolation of sectoral modes is sought by using a narrow rectangular observing window aligned with the equator and averaging the data in longitude before carrying out the Fourier transforms to determine power spectra. The simulations suggest mode beating can produce differences of about 15 $\mu$Hz between sectoral mode frequencies measured from ridge centroids in the power spectra and the actual input frequencies. Variations in mode amplitudes and phases from one realization to another in these simulations can lead to differences of about 10 $\mu$Hz when the measured frequencies are compared. These errors are a significant source of noise for inversion of helioseismic data.
FREQUENT ULTRAVIOLET BRIGHTENINGS OBSERVED IN A SOLAR ACTIVE REGION WITH SOLAR MAXIMUM MISSION

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ABSTRACT

Observations in the ultraviolet of sites of enhanced intensity within an active region on the Sun reveal frequent and rapid brightenings in Si iv and O iv line emission. These transition region lines were observed with 0.08 s sampling in time using the Ultraviolet Spectrometer and Polarimeter (UVSP) instrument on the Solar Maximum Mission (SMM) satellite. The observations suggest that intermittent heating events of modest amplitude are occurring at many sites within an active region. By selecting the brightest site at any given time within an active region and then sampling its behavior in detail within a 120 s interval, we found that about two-thirds of the samples showed variations of the Si iv line intensity. The brightenings typically lasted about 40-60 s, though some were as brief as 20 s. Intensity increases of about 20%-100% were commonly observed. It appears that the UV emission may be coming mostly from small unresolved elements within our spatial sampling window of 3' x 3'. Some modulations in intensity are found to correlate with the jitter in the satellite pointing: this allows us to make estimates of the possible size and contrast of these bright elements. The relatively weak brightenings reported here occur much more frequently than those of larger amplitude detected with OSO 8; the energetic events classified as flares are much rarer yet. Our results suggest that heating due to magnetic field reconnection within an active region is proceeding almost stochastically; events involving only a modest release of energy occur the most frequently.

Subject headings: Sun: activity — Sun: spectra — ultraviolet: general — ultraviolet: spectra


RESPONSE OF THE SOLAR FIVE-MINUTE OSCILLATIONS TO A MAJOR FLARE

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ABSTRACT — Solar five-minute oscillations of intermediate-degree l were observed both before and after a very strong white-light flare. Intensity images of the full Sun taken on two sides of the Fe I λ 5576 spectral line were recorded on film, digitized with 8'' spatial resolution, and then converted into Doppler velocities. The data were projected onto both equatorial and polar sectoral modes and Fourier transformed in time. Comparing the resulting power spectra, we find a substantial increase in power in the p_s ridge of the equatorial modes on the day after the flare; such an increase may be a consequence of the solar flare. When data from all the ridges are considered, there is an average increase in power of only a few percent the day after the flare. This overall increase is probably not significant due to uncertainties from effects of the beating of unresolved modes.
Penetrative cellular convection in a stratified atmosphere

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Summary. Penetrative convection has been investigated within a simple compressible model consisting of three layers of differing stratification prior to the onset of convection. The middle one is a convectively unstable polytrope bounded above and below by two stably stratified polytropes. Nonlinear anelastic modal equations are used to describe these convective motions. Penetrative boundary conditions are imposed at the top and bottom of the computational domain. One- and two-mode steady solutions with hexagonal planforms have been studied for Rayleigh numbers up to about $10^5$ times critical, and for a range of Prandtl numbers, horizontal wavenumbers and stratifications. These show that: 1) Penetration into the lower stable layer by downward-directed plumes is substantially larger in a stratified medium than in a Boussinesq fluid, and produces an extended region of adiabatic stratification. 2) Overshooting into the upper stable layer can be sharply diminished by a reversal of the buoyancy occurring in the upper part of the unstable zone if the stratification is strong enough. These effects are all related to large pressure perturbations that arise in such convective flows. The strong asymmetry between upward and downward penetration in compressible media has major implications for the mixing of stable regions above and below stellar convection zones.

Key words: stellar structure – convection zones – astrophysical fluid dynamics
STRONG DOWNWARD PLUMES RESULTING FROM COMPRESSIBILITY IN NONLINEAR CONVECTION AND THEIR COUPLING TO GRAVITY WAVES

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ABSTRACT

Two-dimensional numerical simulations are used to model fully compressible nonlinear convection spanning multiple scale heights within a stellar envelope. The resulting cellular flows display prominent downward-directed plumes surrounded by broader regions of upflow. Such asymmetry arises because pressure fluctuations accentuate buoyancy driving in the concentrated plumes and can lead to buoyancy braking in the surrounding ascending flows. When such convection is allowed to overshoot into regions of stable stratification above and below the unstable zone, then the dynamics of penetration is dominated by the downward plumes. This leads to the excitation of a broad spectrum of internal gravity waves in the lower stable zone, and these waves in turn can produce a significant modulation in the amplitude of the convection through their feedback upon the plumes.

1Also at E.T.S. Ingenieros de Caminos, Universidad Polytecnica de Barcelona, Barcelona, Spain.
ON THE DETERMINATION OF THE LIFETIME OF VERTICAL VELOCITY PATTERNS IN MESOGRAINULATION AND SUPERGRANULATION

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ABSTRACT

Observational studies of the vertical velocities of mesogranulation and supergranulation provide conflicting results for the lifetimes of these patterns when analyzed by two different methods. Visual inspection of the velocity images suggests that mesogranulation has a lifetime in excess of 2 hours, while cross-correlation methods imply a lifetime of only about 40 min. For supergranulation, the correlation technique yields a lifetime of 2.7 hours, far short of the 24 hours found by many other studies considering network structures or horizontal velocities. These discrepancies may be due to temporal changes in the apparent velocity zero, by differential distortions in the images due to large-scale flows, or by evolution in the flows controlled by the magnetic fields.

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**PAPER S.**

**TWO-DIMENSIONAL COMPRESSIBLE CONVECTION EXTENDING OVER MULTIPLE SCALE HEIGHTS**

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**ABSTRACT**

Two-dimensional simulations are used to study fully compressible thermal convection spanning multiple density scale heights typical of a stellar envelope. The fluid is assumed to be a perfect gas with constant thermal conductivity and dynamic viscosity. The unstably stratified layer is thus represented by a polytropic index typically taken to be $n = 1$ (with $n = 3/2$ equivalent to an adiabatic stratification). The mean density ratio (bottom to top of the layer) ranges from about 1 to 21, with Rayleigh numbers up to about 1000 times critical. These highly nonlinear flows are studied with a two-dimensional numerical scheme based on a modified two-step Lax-Wendroff method. The convective motions in these simulations span the full height of the unstable layer, with no tendency to form a succession of rolls in the vertical as has been assumed in mixing length treatments of convection. Further, the flows remain subsonic because the center of the cell shifts toward the bottom of the layer as the density stratification is strengthened. The flows then display prominent downward-directed plumes surrounded by broader regions of upflow. The flow asymmetry leads to a kinetic energy flux which is directed downward, whereas the enthalpy or convective flux is upward. Such asymmetry comes about because pressure fluctuations accentuate buoyancy driving in the downward plumes and can lead to buoyancy braking in the surrounding ascending flows. Compressional work is significant in the overall energetics.

**Subject headings:** convection — hydrodynamics — stars: atmospheres

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**PAPER T.**

**NONLINEAR COMPRESSIBLE CONVECTION PENETRATING INTO STABLE LAYERS AND PRODUCING INTERNAL GRAVITY WAVES**

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**ABSTRACT**

Penetrative convection spanning multiple scale heights is studied within a simple stellar envelope consisting of three layers: a convectively unstable middle layer bounded above and below by stably stratified polytropes. Two-dimensional numerical simulations are used to investigate the fully compressible nonlinear motions that ensue. The cellular flows display prominent downward-directed plumes surrounded by broader regions of upflow. Such asymmetry arises because pressure fluctuations accentuate buoyancy driving in the concentrated plumes and can even lead to weak buoyancy braking in the surrounding ascending flows. As the plumes plunge downward into a region of stable stratification, they serve to excite a broad spectrum of internal gravity waves there. The induced waves are not passive, for they feed back upon the plumes by deflecting them sideways, thereby modulating the amplitude of the convection in time even in the unstable layer. The penetrative motions that billow upward into the upper stable zone are distinctly weaker, and they cascade back downward toward the unstable zone over a broad horizontal scale. The strong excitation of gravity waves by the convection has implications for gradual mixing deep within a star.

**Subject headings:** convection — hydrodynamics — stars: interiors
MAGNETIC FIELDS INTERACTING WITH NONLINEAR COMPRESSIBLE CONVECTION

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ABSTRACT

Two-dimensional numerical simulations are used to study fully compressible convection in the presence of an imposed magnetic field. Highly nonlinear flows are considered that span multiple density scale heights. The convection tends to sweep the initially uniform vertical magnetic field into concentrated flux sheets with significant magnetic pressures. These flux sheets are partially evacuated, and effects of buoyancy and Lorentz forces there can serve to suppress motions. The flux sheets can be surrounded by a sheath of descending flow. If the imposed magnetic field is sufficiently strong, the convection displays a variety of oscillations. The unstably stratified fluid layer has an initial density ratio (bottom to top of layer) of 11. Surveys of solutions at fixed Rayleigh number sample Chandrasekhar numbers from 1 to 1000 and magnetic Prandtl numbers from 1/16 to 1. These nonlinear simulations utilize a two-dimensional numerical scheme based on a modified two-step Lax-Wendroff method.

Subject headings: convection – hydrodynamics – stars: interiors
Penetration and Mixing Below a Convection Zone

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Abstract

Two-dimensional numerical simulations are used to investigate how fully compressible nonlinear convection penetrates into and mixes a stably stratified zone beneath a stellar convection zone. The dynamics of penetration is dominated by downward plumes which can extend far into the stable material and which lead to the excitation of a broad spectrum of internal gravity waves in the lower stable zone. The depth of penetration is controlled by a balance between the kinetic energy carried into the stable layer by the plumes to and the buoyancy braking they experience there. The motions mix the chemical stratification in the penetrative layer in a few convective overturning times. The model fluid is taken to be a perfect gas with a constant dynamic viscosity. The computational domain is divided into regions of stable and unstable thermal stratifications by varying the thermal conductivity with depth. The mean density ratio (bottom to top of the layer) is initially 6 across the unstable zone and 114 across the entire computational domain, with Rayleigh numbers in the unstable layer taken to be about 100 times critical.
Magnetic Buoyancy Instabilities for a Static Plane Layer

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Recent calculations suggest that the bulk of the solar magnetic field may be stored in a thin convectively stable region situated between the convection zone proper and the radiative zone. Determining the stability properties of such a field is therefore important with implications for both the generation and escape of magnetic flux. The magneto-Boussinesq equations are used to perform a linear stability analysis of a static plane layer. Several new instability mechanisms are revealed showing previous ideas concerning magnetic buoyancy instabilities to be over simplified. The most important result is that instability may occur even for fields which increase with height. Detailed results are presented for 2 and 3 dimensional motions for a weakly stratified magnetic field together with a simple calculation for the 2 dimensional instability of a strongly varying field.
Magnetic Buoyancy Instabilities Incorporating Rotation

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Recent calculations suggest that the bulk of the solar toroidal field may be stored in a thin, convectively stable region situated between the convection zone proper and the radiative zone. Determining the stability properties of such a field is therefore important with implications for both the generation and escape of magnetic flux. The plane layer, linear stability analysis of Hughes (1985) is extended to incorporate the effects of uniform rotation. Detailed studies are made of interchange, or "axisymmetric" modes and of undular, or wavelike, motions, considering modes of both low and high frequency. The force due to rotation acts to constrain the fluid motions, a feature which is strongly stabilizing for direct modes, but can, in certain circumstances, be destabilizing for oscillatory motions.

For the interchange modes we show that the instability discussed at length by Hughes (1985), driven by fields increasing with height, is still present and indeed may be enhanced by rotational effects. We also study the more conventional instabilities, discussing the transformation between direct and oscillatory modes and considering in detail some peculiar properties of the oscillatory instabilities.

The more relevant instabilities in an astrophysical context are likely to be undular modes. Previous studies of low frequency modes driven by top heavy field gradients are extended to consider modes of various frequencies for a wide range of parameter values. Of particular interest is the occurrence of two distinct modes of instability for bottom heavy field gradients. We also exhibit some of the peculiar stability boundaries which can result when none of the competing influences in the problem is dominant.

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Finite-Amplitude Solutions for Interchange Instabilities Driven by Magnetic Buoyancy

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Magnetic buoyancy instabilities may be a significant cause of flux loss from the solar interior. This paper extends the linear theory for such instabilities, which has been widely studied, into the weakly non-linear regime to obtain solutions valid for small but finite-amplitude motions. We consider both static and rotating equilibrium states but, for simplicity, focus attention only on the two-dimensional interchange modes. The method used in obtaining the solutions is due to Coullet and Spiegel (1983)—a summary of the method, together with its advantages, is contained in Section 2.

For the static equilibrium the bifurcation structure is similar to that of thermo-haline convection and magnetoconvection and is fairly straightforward to analyse. The inclusion of rotation yields more complicated non-linear behaviour with the appearance of more varied unfoldings as well as some new bifurcations. One of the key results to emerge is that for both static and rotating basic states the oscillatory modes destabilised by "bottom-heavy" field gradients (magnetic field increasing with height) are often unstable to finite-amplitude disturbances although stable to infinitesimal perturbations. A detailed discussion of all the finite-amplitude behaviour is given in Sections 3 and 4.

KEY WORDS. Magnetic buoyancy instability, bifurcation, weakly non-linear regime, equilibrium states.
A NEW LOOK AT THE INSTABILITY OF A STRATIFIED HORIZONTAL MAGNETIC FIELD

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ABSTRACT

Although the undular instabilities of a stratified horizontal magnetic field have been studied in a number of contexts we believe that the physical mechanism responsible for the instability has not been satisfactorily explained. In this paper we present a new explanation of why these instabilities occur, considering in detail the quite different cases of two-dimensional and three-dimensional motions.

MEAN ADVECTION EFFECTS IN TURBULENCE

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Abstract

The connection between the effective convection velocities for scalar and vector fields due to the action of turbulence is discussed. An explicit relation between the two is calculated for the important special case of two-dimensional flows and fields and it is shown that both velocities are zero for homogeneous turbulence. The calculation leads to new insights on the rôle of symmetry in determining the advection velocity for the case of a vector field.
MAGNETIC FIELDS IN THE OVERSHOOT ZONE: THE GREAT ESCAPE

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ABSTRACT

In order that magnetic flux be confined within the solar interior for times comparable to the solar cycle period it has been suggested that the bulk of the solar toroidal field is stored in the convectively stable overshoot region situated beneath the convection zone proper. Such a magnetic field though is still buoyant and is therefore subject to Rayleigh-Taylor type instabilities. In this paper we consider the model problem of an isolated region of magnetic field embedded in a convectively stable atmosphere. The fully nonlinear evolution of the two-dimensional interchange modes is studied, thereby shedding some light on one of the processes responsible for the escape of flux from the solar interior.


Transitions to chaos in two-dimensional double-diffusive convection

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The partial differential equations governing two-dimensional thermosolutal convection in a Boussinesq fluid with free boundary conditions have been solved numerically in a regime where oscillatory solutions can be found. A systematic study of the transition from nonlinear periodic oscillations to temporal chaos has revealed sequences of period-doubling bifurcations. Overstability occurs if the ratio of the solutal to the thermal diffusivity \( r \sim 1 \) and the solutal Rayleigh number \( R_s \) is sufficiently large. Solutions have been obtained for two representative values of \( r \).

For \( r = 0.316 \), \( R_s = 10^4 \), symmetrical oscillations undergo a bifurcation to asymmetry, followed by a cascade of period doubling bifurcations leading to aperiodicity. As the thermal Rayleigh number \( R_T \) is increased, the bifurcation sequence is repeated in reverse, restoring simple periodic solutions. As \( R_T \) is further increased more period-doubling cascades followed by chaos can be identified. Within the chaotic regions there are narrow periodic windows and multiple branches of oscillatory solutions coexist. Eventually the oscillatory branch ends and only steady solutions can be found. The development of chaos has been investigated for \( r = 0.1 \) by varying \( R_T \) for several different values of \( R_s \). When \( R_s \) is sufficiently small there are periodic solutions whose period becomes infinite at the end of the oscillatory branch. As \( R_T \) is increased chaos appears in the neighbourhood of these heteroclinic orbits. At higher values of \( R_T \) chaos is found for a broader range in \( R_T \). A truncated fifth-order model suggests that the appearance of chaos is associated with heteroclinic bifurcations.
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