APPLICATIONS OF DIFFERENTIAL PHASE STATISTICS TO STUDIES OF C3 AND SPREAD. (U) RANDOM APPLICATIONS INC MONTROSE CO R F PAWULA 23 MAY 87 AFOSR-TR-87-0906
Three papers have been written and two others revised and edited for publication during this year of the contract. Primary emphasis has been on the study of filtered binary processes and the development of methods for approximating probability density functions from moments. We have obtained new fundamental results in this area and have demonstrated that these methods, with origins in statistics, can be fruitfully modified and extended and applied to physical problems in signal processing.

The basic methods were shown to rigorously have their underlying foundations in the extended Fokker-Planck equation which the Principal Investigator originally developed in 1967. We are continuing to study the Fokker-Planck equation and are trying to relate the level crossings of arbitrary random processes to the Fokker-Planck coefficients with our motivations being from problems in filtered binary processes.

Extensive computations involving approximative methods and Monte Carlo simulations of filtered binary processes are being done on an on-going basis.
(a) Statement of Work

The contractor will study C\textsuperscript{3} and spread spectrum modulation techniques for operation under realistic conditions such as jamming, with particular attention to adaptive jamming. The contractor will attempt to complete the modeling of the active jammer state monitor, and will examine the effects of various monitors on error-rate and ranging performance of different types of modulations. Particular emphasis will be placed upon using the differential phase statistics to simplify results whenever possible.

For the Butterworth filtering of binary processes, the contractor will study the feasibility of getting exact solutions to the Fokker-Planck equation that have already been derived. The contractor will look at various ways of approximating the desired probability density functions from their moments, with the expectation that it may be possible to obtain a recursion relation for the moments in a fashion similar to that done by Munford.

The methods of research will be systematic, beginning with a review of the pertinent literature cited in the proposal. Attempts will be made, whenever possible to use the results of others, and to recast them if warranted in terms of the differential phase statistics. Development of the adaptive anti-jam techniques presented by the Principal Investigator will continue.

A certain amount of Monte Carlo simulation will be done for the filtered binary processes, but only to gain insight to be later used in developing mathematical methods since the primary concern is in the analytical techniques and not in the simulations themselves. Comparisons of the actual results will be given with the Monte Carlo whenever appropriate.

The mathematical analyses will be supplemented in all cases by calculations and curves to graphically depict operating characteristics for realistic parameter values. Comparisons will be made to determine worst types of jamming and other sources of error. When necessary, complex mathematical expressions will be simplified to the extent possible by series expansions and/or the development of asymptotic formulas for large values of signal-to-noise ratios. Theoretical comparisons with previous analytical work will also be made throughout.

(b) Status of Research Effort

Three papers have been written and two others revised and edited for publication during the fourth year of the contract and are listed in the next section as papers No. 8, 9, 10, 11 and 12. Papers No. 8 and 9 are the revised and edited papers and were discussed in last year's annual report.

Paper No. 10 reports on a discovery of a fundamental error in a 1934
statistics paper by Hansmann and the realization that the abortative attempts begun by Hansmann could be corrected, modified and extended. These concepts formed the basis for the work in Paper No. 12 in which new ways of approximating probability density functions from moments are presented. This is felt to be a major contribution to what has been classically known as "The Problem of Moments." The theory for this work stems from the well-known Pearson system of frequency curves, but a rigorous justification has only now been provided by Paper No. 12 by using the Principal Investigator's 1967 work on extensions of the Fokker-Planck equation. Paper No. 12 proves a beautiful theorem showing that the approximate methods are indeed exact for the beta family of probability density functions. In the paper, new approximate solutions are obtained for the classical "filter-limiter-filter" problem which use the first five even moments of the distribution. In addition, new solutions are given to the problem of second-order Butterworth filtering of the random telegraph signal and the Monte Carlo results which we published in Paper No. 8 are shown to agree extraordinarily well with the new solutions.

We are presently further refining the approximate techniques by using eighth and tenth order polynomials in place of the sixth-order polynomials used in Paper No. 12 to study the convergence properties of the approximate solutions. We intend to prepare a manuscript for publication in the IEEE Transactions on Information Theory devoted entirely to the filter-limiter-filter problem. We also plan to continue investigations of higher-order systems.

Paper No. 11 contains a generalization of a result of the Principal Investigator relating the level crossings of nonlinearly filtered binary processes to their probability density functions. The original result was derived for Markov binary process inputs with exponential time intervals and the new result shows that the time intervals can have any arbitrary interval statistics. We are currently studying a further extension of these results to arbitrary random processes by attempting to characterize the level crossings of processes in terms of their Fokker-Planck equation coefficients.

All of the areas which we are pursuing have been proved to be extremely fruitful in generating new ideas and new avenues for potential research.

(c) List of Reports and Written Publications in Technical Journals

Following is a list of papers and reports supported under this contract and the one preceding it under which the research was initiated:


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Paper No. 2 is an abridged version of Paper No. 4. Paper No. 9 is a re-titled version of

R.F. Pawula, "On the Distribution of a Butterworth Filtered Random Telegraph Signal," which was a combined version of the two internal reports


(d) List of Professional Personnel Associated with Research Effort
All of the work done under this contract was done by the Principal Investigator, Dr. Robert F. Pawula. Paper No. 1 was written with the collaboration of Mr. John H. Roberts of Plessey Electronics Company, Roke Manor, Romsey, Hampshire, England, and we regularly continue to correspond with Mr. Roberts on technical matters. Papers No. 6, 9 and 10 were written with the collaboration of Dr. S.0. Rice of the University of California at San Diego. Paper No. 10 was, in fact, Dr. Rice’s last work before his unfortunate death in November 1986. A special issue of the IEEE Transactions on Information Theory is being planned for sometime in 1988 and we are contemplating having this paper published in it.

We have been corresponding with Professor A. Munford of the Department of Mathematical Statistics and Operations Research at the University of Exeter in England and with Professor J.K. Ord at the Pennsylvania State University who is a recognized authority on Pearson-type methods in statistics.

(e) Interactions (Coupling Activities)

(i) Consultative functions to other agencies. The Principal Investigator consults on an on-going basis with Dr. Marvin K. Simon of the Jet Propulsion Laboratory of NASA on problems dealing with the mathematical aspects of communication theory and random processes. The Principal Investigator has reviewed papers for the IEEE Transactions on Communications, the IEEE Transactions on Information Theory and the (British) IEE Journal and has reviewed a proposal to AFOSR.
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