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ENTRY AND EXIT

by

Robert Wilson

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1. Introduction

Analyses of industrial competition have attained a new vigor with the application of game-theoretic methods. The process of competition is represented in models that reflect genuine struggles for entry, market power, and continuing survival. Dynamics and informational effects are captured explicitly, although so far only in simplified formulations. Recognition of the importance and intricate complexities of competitive processes began a half-century ago in the work of Joan Robinson [1934], and the first game-theoretic formulations were developed by Shubik [1959] a quarter-century later, but the flowering of this approach began in the 1980s with the recognition that informational asymmetries are crucial ingredients to bring the flavor of struggle to the ensuing dynamic processes. Models that admit both dynamics and private information formulate competition as essentially a bargaining process in which credible communication is limited to costly actions.

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Each firm's claim to survival is signalled by its willingness to offer lower prices longer than others. Firms' struggles for the advantages of monopoly power bring benefits to consumers by dissipating a substantial part of the subsequent profits in the battle to obtain them. This is not the entire story of competition, of course, since also important are, for example, races for cost advantages, product development and differentiation, as well as the imposition of search and switching costs on customers to sustain monopoly pricing; nevertheless, it brings theories that describe more realistically the Darwinian aspect of competition.

This chapter presents a few of the models developed recently to study competitive processes that affect a firm's entry into a market, and the decision to exit. The focus is on firms' strategies to gain or protect monopoly power. We omit the ordinary sort of daily battles for market share; the intent is to study battles for survival. That is, we study competition as economic warfare.

The analyses use the methods of game theory, since the key feature is the strategic behavior of the participants as they jockey for position in a game of economic life or death. Some of the theory is adapted from evolutionary biology, and indeed would change little if one were studying two animals competing for prey or a mate. In the economic context, strength derives from superior resources, lower costs, etc. Often the overall effect of warfare is simply the discovery of which contestant is stronger, as revealed by the eventual capitulation of the weaker one. Thus, in several of the models described here the essential

ingredient is some initial uncertainty as to which one is the stronger, and the ensuing battle is primarily a demonstration of which one has the greater power to sustain combat. There is no possibility that a battle can be avoided by persuasion: the only credible evidence that one is stronger is the demonstration itself.

We begin with an examination in §1 of models of 'predatory' behavior aimed at deterring entry, and this theme is continued in §2 where we examine in more detail wars of attrition intended to drive out competitors. §3 returns to the theme of entry deterrence via limit pricing strategies. §4 considers the maintenance of monopoly prices by the threat of price wars. In §5 we examine firms' incentives to obtain powers of commitment via the timing of capacity additions. Finally, §6 establishes the role of reputational effects in collusive situations with incomplete information. Concluding remarks are presented in §7 and bibliographic references in §8.

#### 1. Predation

We begin with an examination of the long-standing controversy about the power and motives of large firms to drive out smaller competitors by driving prices below costs. This controversy is associated with the so-called 'deep pocket' hypothesis and the 'demonstration effect'. The deep pocket hypothesis refers generally to the proposition that a firm with superior resources can sustain a pitched battle longer, but more particularly it invokes the idea that multi-product conglomerates selling in several markets might be able to subsidize price cuts in one

market by drawing on profits from other markets. This idea may seem far-fetched, since surely a firm tries to maximize profits in every market, but in another form it is more plausible: a firm able to retain earnings from other markets can avoid using capital markets in times of stress, and its other earning can ameliorate shareholders' distress. The deep pocket hypothesis obtains its strongest form, however, when it is combined with the demonstration effect; namely, the conglomerate's motive to respond aggressively to competition in any one market so as to deter entry in other markets. The notion here is that a stern response in one market is taken as a signal by potential entrants in other markets as to what they might expect if they were to enter. To the extent the demonstration effect is real, it multiplies the conglomerate's incentive to fight: large losses in the entered market can be justified as insignificant compared to the losses incurred if failure to respond now were to initiate a later flood of entries into other markets.

The classic example of predatory behavior is Rockefeller's purported conduct in building the dominant position of the old Standard Oil in the latter part of the 19th century. More recent examples, subject to debate, from the late 1960s and early 1970s include IBM's response to the plug-compatible equipment manufacturers, and Maxwell House's response to Folger's attempt to enter the East-coast markets for coffee.

What one means by multiple markets depend on the context. Generally, markets are distinct to the degree that the products are not perfect substitutes. This may mean that the products are technically different, but it can also mean that they are sold in geographically

separated locales, or that they are targeted to different groups of customers. We do not dwell on such distinctions here but rather take the market segmentation as a datum. Similarly, we do not enter into a detailed analysis of the actual policies that constitute the response to entry (pricing, advertising, and other marketing devices); rather we mostly assume that there is a clear-cut distinction between predatory (or aggressive) and acquiescing behavior and that this distinction is manifested in the profits that the firms obtain.

The central difficulty in any analysis of predatory behavior is to illuminate the motives of the incumbent firm and the entrant. Can it possibly be that the actions costly for the incumbent in the short run are part of the optimal strategy in the long run? If so then the incumbent must be investing in something early that will pay off later. That something can be either tangible, such as a monopoly position in the market if it drives out the entrant, or intangible, such as a reputation for being tough that will deter others contemplating entry. For the entrant, a lot hinges on the credibility attached to the incumbent's threat to engage in battle. Is it irrational, a bluff, or is it truly in the incumbent's best interest?

These questions do not have easy answers, and speculation can not substitute for the particulars of the case, and especially for the personalities peculiar to the leadership of the firms. Our discussion, therefore, focuses on developing sketchy models that capture a few salient aspects to see what they predict a rational, profit-maximizing participant would do.

### 1.1 Complete Information

To begin we consider two examples involving an incumbent firm, already established in a market, and a potential entrant who is considering entering. Whether or not entry occurs, a sequence of events ensues with financial consequences, but here we encapsulate all the follow-on events in a static description that uses present values to represent subsequent income streams. The entrant's choices are simply to enter or not. Following entry the incumbent can either acquiesce (play 'soft') or respond aggressively (play 'tough'). We describe two examples, the second of which is developed in more detail later.

◇ Example 1.1: The War Chest. In this first example, suppose that the entrant can survive an aggressive response for only a limited number of periods, say  $n$ , which might be very large. Assume that both the incumbent and the entrant know what  $n$  is, that the horizon is unlimited, and that the incumbent is surely willing to respond aggressively if the entrant can be driven out in one period. We claim that in this situation the entrant's optimal strategy is not to enter, and if it does it capitulates immediately and exits; for, the incumbent responds aggressively.

To demonstrate this claim we argue by induction. The claim is surely true if the number of periods remaining before the entrant is exhausted is small enough, say not more than  $m > 1$ , that the incumbent's losses from fighting are outweighed by the subsequent monopoly profits after the entrant is driven out. So, we conclude that when funds for  $m$  periods remain in its war chest the entrant can expect an aggressive response from the incumbent thereafter and therefore its

optimal strategy is to exit. But then with  $m + 1$  periods remaining the incumbent needs to be aggressive for only one period to drive out the entrant, so it will surely fight, and anticipating this the entrant will exit. Continuing this argument for  $m + 2, \dots, n$  periods remaining, we conclude that also when all  $n$  periods are available to the entrant, it wants to exit -- or better, not to enter initially.

The striking paradox inherent in this result is that it holds whatever might be the size  $n$  of the entrant's war chest. Later we unravel such paradoxes by showing that the induction argument is invalid if there is even the slightest violation of the assumption that all data of the problem are common knowledge between the incumbent and entrant.

◇ Example 1.2: The Chain Store Game. In this example we model the competitive situation somewhat differently. Rather than supposing that the entrant's resources are limited, we assume that the present values of the entrant's and the incumbent's payoffs in the three possible scenarios are those displayed in Table 1.1 which follows. Notice in the Table that without the entrant present the incumbent enjoys monopoly profits of \$3, whereas with the entrant present he can either acquiesce in splitting the market (\$1 each) or respond aggressively to force losses on the incumbent (-\$1) and eliminate his own profits (\$0).

**Table 1.1**  
**Payoffs for the Entrant and the Incumbent**

Actions		Payoffs	
Entrant	Incumbent	Entrant	Incumbent
Stays Out		\$0	\$3
Enters	Soft	1	1
	Tough	-1	0

It appears clear in this situation that the entrant wants to enter, expecting the incumbent to act in his own best interest to share the market. Moreover, if this situation is repeated between the same incumbent and entrant then this outcome persists: the two firms adapt to co-existence.

Or is it so clear? Suppose that in fact this market is one of many, say ten, similar ones in which the incumbent operates, and in every one there is a potential entrant. If it is true in this market that the incumbent will acquiesce then surely it is so in every market. Playing soft in all markets yields the incumbent \$10. But suppose he were tough in a few, say 4, and this salutary action sufficed to deter entry in the other 6 markets: then his profit would be  $6 \times \$3 = 18$  -- well worth the effort to preserve his dominant position. If the number  $n$  of markets is larger, say 100 or 1000, then intuitively one expects the incumbent to establish a reputation for being tough on entrants. The cost of

clobbering a few hapless entrants initially is well worth its demonstration effect on the subsequent ones.

How are we to make sense of this seeming contradiction between elementary logic and the intuitive appeal of the 'tough guy' gambit? There are some ready answers that rely heavily on intuition. Perhaps the incumbent likes to brutalize, maybe he learned rules of thumb ("be tough") at his father's knee, maybe his friends would ridicule any signs of passivity, or maybe his belief in the powers of intimidation transcends accounting statements. The feature shared by all these explanations is the appeal to some maybe. Even though this is supposedly a game with complete information, one can conjure uncertainties that allow reputational effects. As long as there is some chance of unobservable psychic or economic factors that could influence the incumbent's choices, then one can interpret tough play as the natural mode of reinforcing in the entrants' perceptions the likelihood that the incumbent has a penchant for snuffing out trespassers. Building a reputation for being tough becomes a sane strategy, even if one is ordinary. It is this approach to the 'paradox' above that is pursued below.

## 1.2 Uncertainty About Preferences

The simplest way to enable reputational effects is to dispense with the assumption that the features of the situation are common knowledge. To illustrate this we analyze an example in which the incumbent is uncertain whether one of the entrants knows the facts of the situation. For simplicity we suppose that there are only two potential entrants,

each of whom has an opportunity to enter one of the incumbent's markets, and these opportunities occur in sequence.

Suppose that it is common knowledge among the entrants that the possible actions and the resulting payoffs are as described in Table 1.1. The incumbent knows these facts too but he is uncertain whether the second entrant knows them. To be specific, suppose that the incumbent assigns a probability 0.6 to the prospect that the second entrant assigns a probability exceeding 0.5 to the notion that the incumbent's predictable or only possible response to entry is to play tough.

The claim we make is that in this circumstance an equilibrium includes the following. The incumbent is prepared to play tough if the first entrant enters, but not if the second enters. The first entrant, anticipating the incumbent's tough response, stays out. The second entrant, on the other hand, anticipates a soft response and decides to enter. Thus the key result is that the incumbent plays tough against the first entrant although he foregoes an immediate profit of \$1 by doing so. We first explain this equilibrium in an informal way, and then undertake a more complete analysis.

Informal Explanation: The entrants' actions are clearly their best choices in response to the incumbent's strategy, so we focus on an explanation of the incumbent's reasoning. If the incumbent meets the first entry by playing tough then his payoff is \$0 plus what he can expect in his match with the second entrant, and similarly if he plays soft it is \$1 plus the follow-on profit. He realizes, however, that if he plays soft then the second entrant will recognize that playing tough

is not his only feasible action and so the second entrant will surely enter (yielding \$1 for the incumbent, who will respond softly); whereas if he plays tough then (in the incumbent's view) there will remain a 0.6 probability that the second entrant ascribes a high likelihood to the possibility that the incumbent will surely respond to entry by playing tough, and in this event the second entrant will not enter (yielding \$3 for the incumbent). Thus, the incumbent calculates that a tough response to the first entry yields  $\$0 + .6 \times \$3 + .4 \times \$1 = \$2.2$ , whereas a soft response to the first entry yields  $\$1 + \$1 = \$2.0$ . Choosing between these, the incumbent opts to meet entry the first time with tough play -- simply on the chance that this will deter the second entrant. Notice that the incumbent's calculations are wrong, in a sense, since in fact neither entrant assigns any chance to the incumbent's having only the tough response available, and they know that the incumbent is wrong. Nevertheless, the first entrant knows the inexorable consequence of the incumbent's uncertainty about the second entrant's beliefs, which is that he will clobber the first entrant to avoid possibly revealing information to the second entrant; so, the first entrant is deterred from entering.

Formal Analysis: To build up a game-theoretic model of this problem, assume that there are three possible states of the world, labeled A, B, and C. The true state happens to be C, which the entrants know, but the incumbent knows only that it is one of B or C, and it is B that represents the possibility entertained by the incumbent that the second entrant is unsure whether the incumbent's only

feasible action is to play tough, which is represented by A. Thus, in A the incumbent can only play tough, whereas he has both actions available in B and C. The incumbent knows only that the true state is A or not-A; the first entrant knows whether it is A, B, or C; and the second entrant knows whether it is C or not-C. Assume that the prior probabilities of A, B, and C are 0.5, 0.3 and 0.2 respectively. For example, if the true state were B then the second entrant, who would know that it is not C, would assign probability  $.5/ [.5 + .3] = .625$  to state A; and the incumbent would assign probability  $.3/ [.3 + .2] = .6$  to state B, as we initially assumed. The strategies in this setup are:

- The first entrant stays out in any event.
- The second entrant enters only if either he knows that C obtains or he observed the first entrant to enter and the incumbent to play soft, in which case he infers that state B obtains.
- The incumbent counters the first entry with tough play in any event, and he counters the second entry with tough play only if he knows that A obtains.

The second entrant does not enter if he knows only that C does not obtain, because it yields a negative expected profit:

$.625 \times (-\$1) + .375 \times \$1 < \$0$ . On the other hand, he enters if the incumbent's play of soft reveals that the state is B rather than A. Anticipating this, the incumbent, if not-A obtains, prefers to play

tough against the first entrant, based on the calculation in the previous paragraph.

There is, of course, another equilibrium in which the incumbent threatens to meet all entry with tough play. This threat is not credible, however, since it violates sequential rationality. That is, once the second entrant enters the incumbent's best strategy in the subgame remaining is to play soft. Realizing this the second entrant who knows that C obtains has every incentive to call the incumbent's bluff.

There is, however, another sequentially rational equilibrium in which in states B and C both entrants enter and the incumbent acquiesces. In this equilibrium, if the second entrant is unsure whether A or B obtains but he sees the first entrant enter then he infers that the true state is actually B and so he enters also. Anticipating this, the first entrant does indeed enter if the state is B or C, since he realizes that in state B the signal to the second entrant that his own entry provides precludes any motive for the incumbent to play tough so as to influence the second entrant's decision. Similarly, if the first entrant's information is the same as the second entrant's, then he enters in state C (but not in not-C) since he anticipates that his entry signals to the incumbent that B is not possible, so tough play to influence the second entrant is useless. However, these equilibria with entry in the first period are eliminated if the first entrant can not distinguish among the states.

### 1.3 Incomplete Information

We now elaborate the chain-store game to explore further aspects of the problem. Keep the same formulation, except suppose that there are  $n$  potential entrants, each with a single opportunity to enter in sequence. And, suppose that initially each entrant entertains a small probability  $p$  that the incumbent is sure to respond to entry with tough behavior. The origin of this possible adherence to tough is immaterial, but here are several possibilities: (1) the incumbent's payoffs might be reversed, so that tough is more rewarding than soft (possibly for non-financial reasons); (2) the incumbent might be acting under instructions that commit him to respond aggressively; (3) he might be irrational, or addicted to a rule-of-thumb, or simply blind to the evident financial consequences. If the probability  $p$  is small enough, say  $p = .0001$ , perhaps one would be willing to assign that the probability to one's twin acting so weirdly. One other ingredient is necessary: assume that if the incumbent ever responds to entry with soft then thereafter each entrant resets  $p$  to zero and keeps it there regardless of the incumbent's subsequent actions. This is merely an application of Bayes' Rule if one believes that an incumbent committed to tough can never play soft.

The entrants will employ their observations of the incumbent's behavior to update their probability assessments that the incumbent is committed to tough. We use  $p_k$  to indicate this probability assessment when there are  $k$  entrants remaining with opportunities to enter. Initially  $p_n = p$  and thereafter  $p_k$  is a state variable that effec-

tively summarizes the consequences of past history for divining the future. We call  $p_k$  the probability that the incumbent is 'strong' (i.e., committed to tough) and then  $1 - p_k$  is the probability that the incumbent is 'weak' (i.e., an ordinary mortal) when  $k$  encounters remain.

Before proceeding we caution the reader that we allow the weak incumbent to randomize his actions between tough and soft. This is done for simplicity in this exposition. In practice no sensible executive flips a coin to choose his action; rather, his choice is decided by recourse to further information, of which there is always plenty (at the very least the choice can depend on his bank account, the contents of the morning paper, etc.). There would be no role here for randomization if we used a model with a finer representation of the incumbent's private information, say allowing a continuum of types for the incumbent instead of just two. In any case the actor need not actually randomize. It is only required that he is indifferent as to which of the 'randomized' actions he takes, and the opponent assesses probabilities for the actions that conform to the randomization.

The use of Bayes' Rule to update the probability assessment is straightforward so long as no events are observed that were assigned zero probability. The rule is that (1)  $p_{k-1} = p_k$  if nothing was observed (no entry occurred), (2)  $p_{k-1} = 0$  if the incumbent played soft after an earlier entry, and (3) if the incumbent played tough then  $p_{k-1} = p_k/q_k$ , where  $q_k$  is the marginal probability that an incumbent of either type would have played tough. If  $x_k$  is the probability that

a weak incumbent would have played tough against the  $k$ -th from last entrant, then  $q_k = 1 \cdot p_k + x_k \cdot [1 - p_k]$ .

We claim that the following are sequentially rational equilibrium strategies:

- The  $k$ -th from the last entrant stays out if  $p_k > .5^k$ , enters if  $p_k < .5^k$ , and flips a coin to decide if  $p_k = .5^k$ .
- The strong incumbent is always tough on entrants, by assumption. The weak incumbent is tough if  $p_k > .5^{k-1}$  and otherwise he randomizes so that if he were to be tough then the next entrant's probability assessment would be  $p_{k-1} = .5^{k-1}$ .

We first trace the ramifications of these strategies. Table 1.2 gives a few values of the critical probability above which the current entrant is deterred from entering. Suppose that the number of entrants is  $n = 10$  and that initially the probability of a strong incumbent is  $p_{10} = 0.001$ . A weak incumbent will surely be tough on each of the first three entrants if they dare to enter, since  $p_k = .001 > .5^{k-1}$  for  $k = 10, 9, 8$ . Anticipating this they stay out so the probability that the incumbent is strong remains at 0.001. Indeed, this is true for the fourth entrant as well since he anticipates a probability greater than 0.5 (as explained later) that the incumbent will respond with tough. The fifth entrant enters but he may be met with a tough response; if so then the sixth is indifferent whether he enters (his expected profit is zero, since he sees a 50-50 chance that he will get a tough response). This pattern continues until some entrant enters and is met with a soft response;

thereafter, the remaining entrants enter in turn. An interesting feature is that if there were, say, 100 entrants then again it would be the sixth from the last, the 95th, who would dare enter first.

Now we verify that these strategies are optimal for each firm. An entrant must foresee a probability  $q_k$  of a tough response that is less than one-half to want to enter. Now  $q_k$  is less than or greater than

**Table 1.2**  
**Critical Probabilities in the Repeated Chain Store Game**

$k$ :	1	2	3	4	5	6	7
$.5^k$ :	.5	.25	.125	.0625	.03125	.015625	.0078125

one-half according to whether  $p_k$  is less or greater than  $.5^k$ ; hence, the  $k$ -th from last entrant prefers to enter precisely when  $p_k < .5^k$ , as indicated in the equilibrium strategy. For the weak incumbent the choice is whether to forego \$1 by being tough on the current entrant in exchange for the chance of deterring subsequent entrants. If  $p_k > .5^{k-1}$  then playing tough will surely deter the next entrant, since then  $p_{k-1} > .5^{k-1}$ , and this is worth a gain of  $\$3 - \$1 = \$2$ . If  $p_k < .5^{k-1}$  then there is a 50-50 chance of deterring the next entrant, since after the incumbent plays tough the next entrant will set  $p_{k-1} = .5^{k-1}$  and thereby be led to flip a coin to decide entry. This is worth an expected gain of \$1, and therefore the weak incumbent is indifferent. (There are no subsequent gains to deterrence because the weak incumbent anticipates that thereafter he will be indifferent about responding aggressively.) Thus the incumbent's strategy is also optimal.

This equilibrium is robust to various amendments in the formulation. If the incumbent discounts his profits from successive engagements by a factor exceeding one-half then the equilibrium is unchanged except that the entrants' randomizations when  $p_k = .5^k$  are altered; if the discount factor is less than one-third, however, the weak incumbent immediately resorts to a soft response. The equilibrium is unchanged also if it is always the same firm that is the potential entrant; in this case the results indicate that the entrant defers entry until near the end of the life of the market. A slight alteration of the equilibrium is required if the incumbent must make his choice before he knows the entrant's decision, assuming as before that playing tough (e.g., cutting price) costs \$1. We discuss in §2.3 what happens when the incumbent is uncertain about the entrant's payoffs.

The overriding conclusion from this example is that predatory behavior can be a sane and rational strategy if there is some opportunity for the incumbent to sustain a reputation for aggressive behavior. Typically it is incomplete information, some fuzziness in the entrants' perceptions of the incumbent, that provides this opportunity. The probability assigned to the prospect that the incumbent enjoys or is committed to aggressive behavior can be arbitrarily small provided there are many encounters anticipated, so that the incumbent can fully realize the benefits of a reputation for toughness. That tiny probability suffices because it is the lever that enables the weak incumbent to sustain the entrants' beliefs that he might be strong by imitating the behavior of a strong incumbent.

2. Exit and Attrition

We now examine more closely a battle for survival between two firms locked in a competition for market share within a single industry, with the possibility that one could be driven out of the market. The aim is to elaborate some of the dynamic features of the competitive process.

The main idea explored is that competition is much like bargaining. Words are cheap in bargaining; what counts is one's willingness to forego an agreement rather than accept unfavorable terms. Price wars and battles for market share have this feature. What matters is one's ability or willingness to sustain extra costs or lower prices, and the prize goes to the firm that can sustain the struggle longer. Low prices and other costly maneuvers are the language of the negotiation over who wins. They are a meaningful language because the strong -- the firm with the better product, the lower costs, or the greater resources -- can sustain them longer than can the weak. Often they are the only possible language; few other signals are credible. Tenacity, or the resource advantage that makes it possible, is the ultimate determinant of the outcome. This is not to say that business competition is necessarily or entirely Darwinian, since other modes of encounter are possible, such as product differentiation. Also, more or less collusive behavior is possible, as we study later. But when competition is overtly aggressive it is rare that the weaker firm has a chance to win.

## 2.1 Attrition and Auctions

To introduce the basic ideas, we start with the simplest war of attrition in which the combatants incur costs (or opportunity costs) until one gives up and the other wins a prize. Here, the two parties are duopolists in a market, giving up means exiting the market, and winning means attaining a monopoly position in the market. The value of the prize is standardized so that it is \$1 for each firm. The feasible choice for the firms at each instant are simply either to fight or to exit; once a firm exits the other receives the prize and the exiting firm gets nothing. As long as they are both fighting each firm incurs a cost that we designate as  $c_1$  for firm 1 and  $c_2$  for firm 2. These costs are measured in dollars per unit time, so that total cost incurred by firm  $i$  if the fighting continues for a time  $t$  is  $c_i \cdot t$ . Neither these costs nor the value of the prize is discounted in this simple formulation.

The key assumption is that the firms' costs are privately known. One can easily see that there is no reason to fight if the costs are common knowledge: the one with the higher cost exits immediately. Assume that each firm assesses a probability distribution for the other firm's costs that has a cumulative distribution function  $F$  and a positive density  $f$  over an interval of possible costs that are positive. Assume that the firms' costs are independent and that this distribution function is common knowledge.

As the fighting continues each firm learns only that the other firm has not exited, so the time that has elapsed fully summarizes the

history of the game so far. A firm's strategy prescribes at each instant whether to continue fighting or not, which can be conveniently summarized by the time at which to exit if the other has not capitulated previously. Since the firms are symmetric, it suffices to consider a symmetric equilibrium. Thus an equilibrium is specified by a function  $T$  that indicates that if one's cost is  $c$  then exit at time  $T(c)$ . One can show that  $T$  must be non-increasing, and here we assume that it decreases smoothly. Conversely, use  $C(t)$  to denote the cost of a firm that chooses to exit at time  $t$ . For example, one firm initially assesses the probability  $F(C(t))$  that the other firm will exit after time  $t$ , since this is the probability that the other firm has costs low enough to choose an exit time exceeding  $t$ .

If firm  $i$  expects the other to use the strategy  $T$ , and  $i$  were to pick the stopping time  $t$ , then it would assign probability  $1 - F(C(t))$  to winning the prize at some time  $t'$  before  $t$ , and incurring the cost  $c_i \cdot t'$ , and otherwise that it will exit at  $t$  having incurred the cost  $c_i \cdot t$ . To pick its optimal stopping time it can reason that for each small interval  $\Delta$  of time that it continues fighting it incurs the cost  $c_i \cdot \Delta$  in exchange for the probability that the other firm will exit in that interval. Conditional on the other firm lasting until  $t$ , this probability is approximately  $-\Delta \cdot f(C(t))C'(t)/F(C(t))$ . Thus, it continues until that time  $t$  at which  $c_i = -f(C(t))C'(t)/F(C(t))$  and then exits. An equilibrium requires, of course, that this value of  $t$  is precisely  $T(c_i)$  so that the other firm's expectation about its

strategy is correct. Using this requirement the condition for an optimal stopping time can be written as

$$T'(c) = - \frac{f(c)}{cF(c)} .$$

This condition defines a differential equation that characterizes the equilibrium strategy  $T$ . The appropriate boundary condition is that  $T(c^*) = 0$  if  $F(c^*) = 1$ ; i.e.,  $c^*$  is the maximum possible cost (possibly infinite) that a firm might have: this is because a firm with the highest possible cost anticipates that it will surely lose if it fights and so it exits immediately. Combining these results, the equilibrium strategy is:

$$T(c) = \int_c^{c^*} \frac{f(x)}{xF(x)} dx.$$

That is, each firm stops at the time that is computed as the area under portion of the curve  $f(x)/xF(x)$  that lies to the right of its cost parameter.

◇ Example 2.1: Suppose that each firm's cost is uniformly distributed between zero and one. Then  $T(c) = (1/c) - 1$ . The stopping times for a few values of the cost parameter are tabulated in Table 2.1. The times are measured in whatever units of time are used to measure the cost, such as dollars per year. Referring to the Table, a firm with a cost \$.20 per unit time plans to exit when 4 units of time have elapsed, if the other firm has not exited previously. Initially it assesses a probability of 0.8 that it will win the prize, since that is

the probability that the other firm's cost is higher, and overall its expected profit is \$.48.

**Table 2.1**  
**Stopping Times in the War of Attrition**  
**Uniformly Distributed Costs**

$c$ :	0	.2	.4	.6	.8	1.0
$T(c)$ :	$\infty$	4.00	1.50	0.67	0.25	0
Exp. Cost(c) :	\$0.00	\$0.32	\$0.37	\$0.31	\$0.18	\$0
Exp. Profit(c) :	\$1.00	\$.48	\$.23	\$.09	\$.02	\$0

These stopping times may seem surprisingly large: could it be optimal to sustain losses for so long a duration before calling it quits? The firm with a cost of \$.20 loses \$.80 in its quest for the \$1 prize before calling it quits. But such is life: hope springs eternal. Note too that on average a large proportion of the value of the prize is expended by the two firms in their competition to capture it. This proportion is roughly 45% in this example, averaging over the possible types of both firms. This feature, that competition dissipates profits, is the bane of entrepreneurs, but customers benefit, which is a reason it is encouraged socially.

An interesting way of thinking about games of this sort is that a kind of auction is played. Each firm's bid is the maximal time it is willing to endure fighting. The prize is awarded to the highest bidder at the second highest bid; that is, the winning bidder needs to endure

fighting only as long as its opponent does. The rub is that the losing bidder also pays the amount of its bid.

### 2.1.1 Brinkmanship

A war of attrition can also be used to describe competition among firms in various other contexts. An example is the provision of a common good among the firms. Suppose that each firm in an industry obtains a benefit worth  $V = \$1$  if some one firm develops a (non-patentable) innovation that becomes available to all firms. In this case one anticipates that each firm plays a waiting game, hoping that another firm will incur the cost of development. However, if each firm is impatient then a firm's waiting time is limited and at some point it is willing to proceed with the development if no other firm has taken the initiative. We develop briefly one model of this competitive process.

In the interest of simplicity, assume that all firms have the same impatience, represented by an interest rate  $r$ . By choosing the units of time appropriately this interest rate can be chosen to be one:  $r = 1$ . The firms are symmetric except that each firm is characterized by its privately known cost  $c$  of developing the innovation. Assume that each firm assesses a probability distribution function  $F(c)$  that any other firm's cost is less than  $c$ , and that the firms' costs are independently distributed. In this case the minimum of the  $n - 1$  others firms' costs has a probability distribution with the distribution function  $1 - [1 - F(c)]^{n-1}$ . All of these features are common knowledge among the firms.

When there are  $n$  firms each firm's strategy can be represented by a function  $T_n$  indicating that it will stop waiting and supply the innovation at time  $T_n(c)$  if its cost is  $c$  and no other firm has previously volunteered. One expects that  $T_n(0) = 0$  and that the function  $T_n$  is increasing. If all other firms are using such a strategy, then a firm with the cost  $c$  that plans to stop waiting at time  $t$  has an expected present value that is the sum of its gains from providing the innovation and from obtaining it free:

$$[1 - c]e^{-t}[1 - F(C_n(t))]^{n-1} + \int_0^{C_n(t)} e^{-T_n(x)} d\{1 - [1 - F(x)]^{n-1}\},$$

where  $C_n(t)$  is the cost that would lead another firm to stop waiting at  $t$ . An equilibrium requires that this expected present value is maximized by choosing  $t = T_n(c)$ . If the distribution function  $F$  has a density function  $f$ , then this criterion leads to the following differential equation that characterizes the optimal waiting time strategy:

$$T'_n(c) = [n - 1] \frac{c}{1 - c} \frac{f(c)}{1 - F(c)}.$$

This characterization implies immediately that the  $n$ -firm case is related to the 2-firm case by the simple relationship that  $T_n(c) = [n - 1] \cdot T_2(c)$ . For example, if a firm has eight competitors then it waits eight times as long as it does with a single competitor. However, as one might expect it can be shown that the expected time until the innovation is completed decreases as the number of firms increases. The least-cost firm is always the one that provides the

innovation; consequently, a firm's expected present value is an increasing function of its cost of supply.

◇ Example 2.2: Suppose that each firm's cost of development is distributed uniformly between \$0 and \$1. Then the equilibrium waiting time for a firm with the cost  $c$  is

$$T_n(c) = [n - 1] \left\{ c \frac{1}{1 - c} - \log\left(\frac{1}{1 - c}\right) \right\}.$$

One can then calculate that the expected time until the firm with the lowest cost undertakes the development is  $1/n$ . The waiting times for the case of two firms are tabulated in Figure 2.2. The rather long waiting times when the cost is high reflect the free-rider phenomenon: a high cost firm waits in hopes that another will take the initiative first.

## 2.2 Attrition and Selection

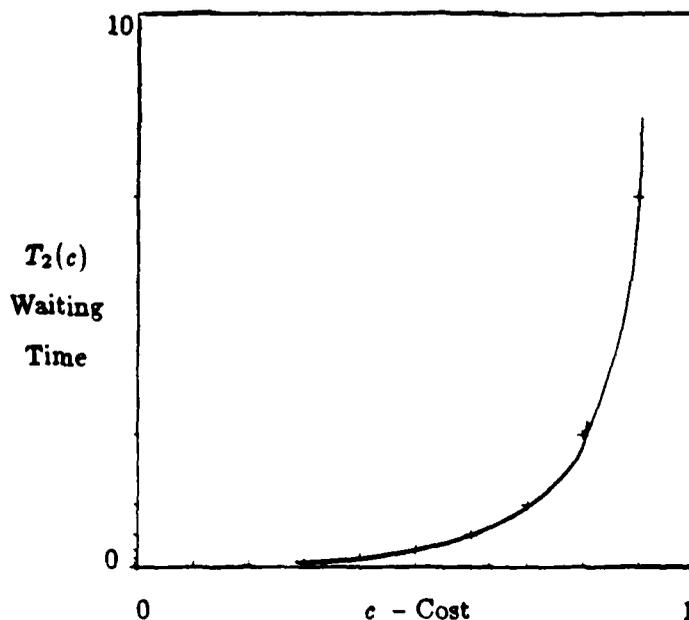
The previous examples can be enriched by adding a few ingredients from a realistic situation. Suppose that two firms are in a market that might possibly sustain only one of them with positive profits. Indicate the firms by  $i = 1$  and  $2$  and to be specific suppose that at each time  $t$  firm  $i$  can earn a gross profit rate of  $M_i(t)$  as a monopolist or

**Equilibrium Waiting Times**

$(F(c) = c/V, r = 1, n = 2)$

**Table 2.2**

$c/V$	$T_n(c) \cdot r/[n - 1]$
0	0
.1	.006
.2	.027
.3	.072
.4	.156
.5	.307
.6	.584
.7	1.129
.8	2.391
.9	6.697
1.0	$\infty$



$D_i(t)$  as a duopolist. Of course a firm's duopoly profit is less than its monopoly profit. It may be that these profits vary with time, increasing if the market is expanding or if the firm's costs reflect learning curve effects, or decreasing in the case of a declining industry (for this analysis both firms' profits must move together, up or down). In addition each firm incurs a fixed cost of staying in the market, so many dollars per year: let  $F_i$  be firm  $i$ 's fixed costs. Adopting the convention that a firm earns zero if it drops out of the market, interpret the fixed cost as cash outlays for overhead, etc., plus an opportunity cost reflecting foregone opportunities to engage in other markets.

Thus, if firm 1 drops out after a duration  $T$  then its net profit is the present value of the stream  $D_1(t) - F_1$  for  $0 < t < T$ , whereas 2 gets the stream  $D_2(t) - F_2$  over this period plus its subsequent monopoly profit  $M_2(t) - F_2$  for each time  $t > T$ . If  $r$  is the continuous-time interest rate then 2's present value is given by the formula:

$$\int_0^T [D_2(t) - F_2] e^{-rt} dt + \int_T^\infty [M_2(t) - F_2] e^{-rt} dt,$$

of which it obtains only the first term if it drops out at time  $T$ . The key feature assumed is that each firm's fixed cost is privately known, whereas all the other data of the situation are common knowledge. This is fairly realistic, since a firm's opportunity cost is often not observable, in contrast to nominal accounting profits. The other firm's probability assessment of firm  $i$ 's fixed cost can be represented by a distribution function  $G_i$  having a density function  $g_i$  on an interval of fixed costs. For technical reasons, assume this interval is wide enough to allow the possibility that  $i$  might be viable as a duopolist were the other never to drop out, and possibly not viable as a monopolist. The highest fixed cost that allows a monopoly profit is

$$F_i^* = r \int_0^\infty M_i(t) e^{-rt} dt.$$

A repetition of the reasoning used in the previous example suffices to characterize the firms' equilibrium strategies. At time  $t$  firm  $i$ 's rate of net profit from continuing as a duopoly is  $D_i(t) - F_i$ ,

plus the chance that it will win the subsequent monopoly stream with the value

$$V_1(t) = \int_t^{\infty} [M_1(s) - F_1] e^{-rs} ds.$$

This chance has the probability that one computes using Bayes' Rule, to incorporate the information that the other firm has already lasted a duration  $t$ , and one's hypothesis about the other's stopping rule. Suppose the other's stopping rule is to stop at time  $T$  if its fixed cost is  $C_j(T)$ , where  $j$  indicates the other firm. Then the probability density of its stopping shortly after  $t$  (conditional on having lasted to  $t$ ) is

$$P_j(t) = [1 - G_j(C_j(t))]' / G_j(C_j(t)).$$

Combining these components, firm  $i$  wants to stop as soon as its net profit rate declines to zero: namely, when  $t$  satisfies

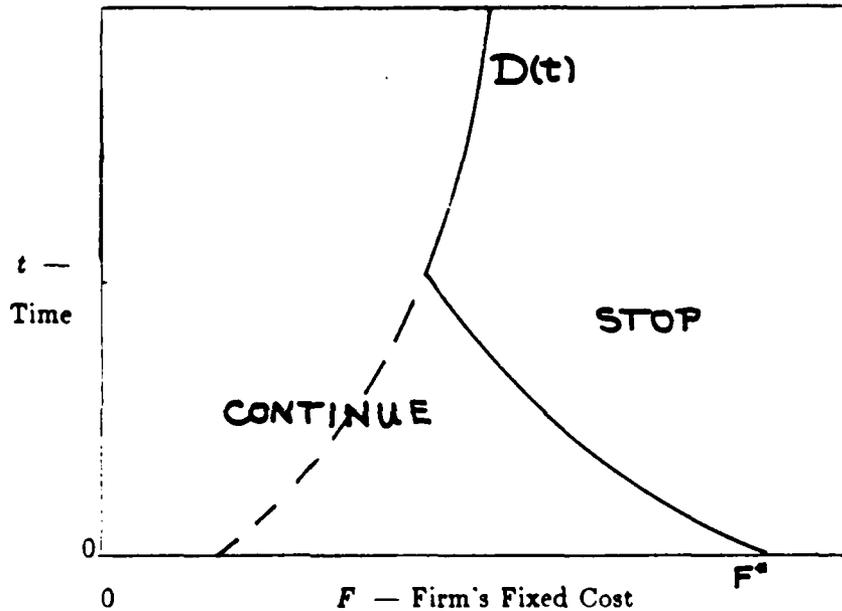
$$D_1(t) - F_1 + P_j(t)V_1(t) = 0.$$

An equilibrium requires that this decision to stop occurs at the time  $t$  for which  $F_1 = C_1(t)$  so that the other firm's hypothesis about  $i$ 's strategy is correct. This condition and a similar one for the other firm specify two differential equations that characterize the two firms' strategies. There is also a boundary condition: a firm wants to drop out immediately if its cost exceeds  $F_1^*$ , which is the maximal viable fixed cost even with a monopoly.

The form of the equilibrium strategies is represented schematically in Figure 2.3 for the case that the profitability of a presence in the market is increasing over time and the same for both firms.

**Equilibrium Strategies**  
**Case I: Increasing Profitability**

Figure 2.3



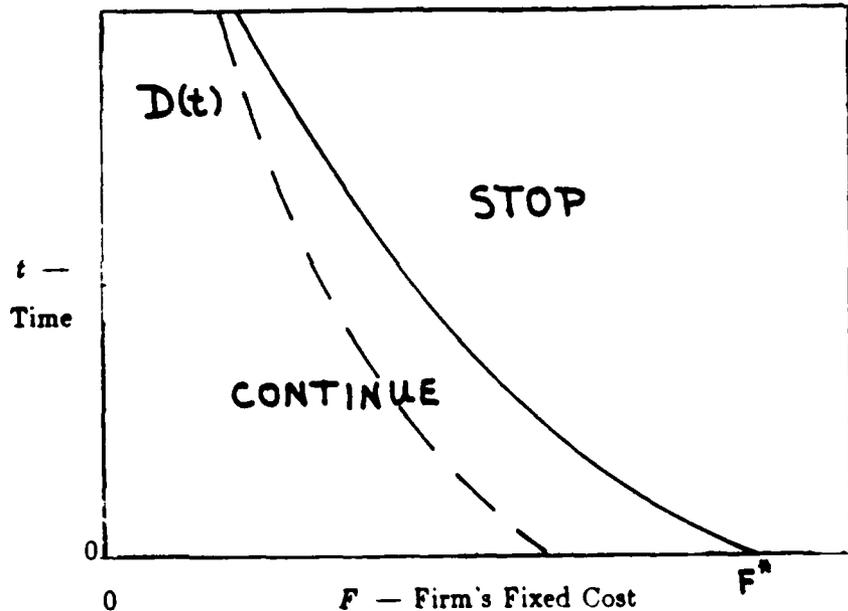
Similarly, Figure 2.4 depicts the case that profitability is declining over time.

◇ Example 2.3: To illustrate, consider an example in which the firms are symmetric and profits do not depend on time. If the demand curve is linear, say  $p = A - Bq$  is the price in excess of the firms' marginal cost when a quantity  $q$  is offered, then in a Cournot model the monopoly gross profit is  $M = A^2/4B$  (so  $F^* = A^2/4B$ ) and the duopoly gross profit is  $D = A^2/9B$ . Assume that the fixed costs have a uniform distribution between zero and some upper limit. Computing the

symmetric equilibrium stopping-time strategy for the case that the interest rate is  $r = 1$  and the choke price is  $A = 1$  with  $B = 1$  yields the results shown in Table 2.5. Note that the present value is calculated at a time just after zero, so that an opponent who has not immediately dropped out is known to have a standardized fixed cost that is less than 0.25; i.e.,  $G(F) = F/.25$ .

**Equilibrium Strategies**  
**Case II: Decreasing Profitability**

Figure 2.4



For example, if  $A = \$1000$  per unit, the slope is  $B = 1$ , the interest rate is  $r = 0.1$  per year, and the fixed cost is  $F = \$150,000$  per year, then  $F \cdot B/A^2 = .15$  and according to the Table,  $T \cdot r = 0.442$ , so the indicated stopping time is  $T = 4.42$  years, yielding an expected present value of \$264,000. Comparing this to the monopoly present value of \$1,000,000 in this case, one sees that again a considerable portion

of the prize is dissipated in the competitive process. If the fixed cost were \$200,000 per year the stopping time would be a fraction 0.558 of a year, and the expected present value would be only \$56,900. Note that firms with fixed costs less than \$111,111 per year stay forever and those with fixed costs exceeding \$250,000 per year exit immediately; thus there is a fairly narrow window of fixed costs that generate battles with positive but finite resolutions.

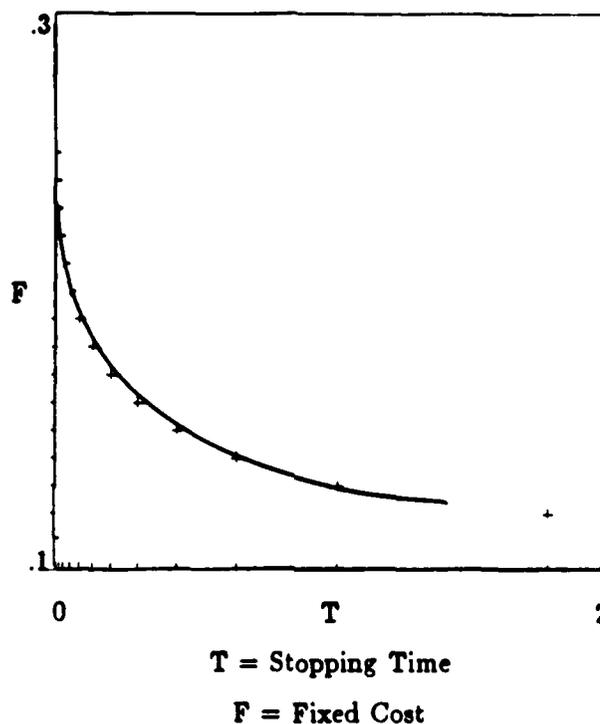
The equilibrium shown in the Table allows that the firms might sustain battle forever as a duopoly. This feature is reinforced if gross profits are increasing over time. If the duopoly profit  $D(t)$  increases fast enough so that the curve  $C(t)$  enters the area in which the present value of continuing net profits as a duopoly are at least zero (i.e., fixed costs are small enough in relation to future duopoly gross profits), then after the time at which it is certain that both can survive profitably as duopolists the firms settle down to continuing duopoly competition with no further chance that either will exit. (In the graph this area lies under the horizontal line at height 0.1111 and the curve intersects this area only asymptotically, but in a general case this intersection might occur at some finite time.)

**Stopping Times in the War of Attrition**

( $p = 1 - q$ ,  $G(F) = F/.25$ , and  $r = 1$ )

**Table 2.5**

Fixed Cost $F \cdot B/A^2$	Stop Time $T \cdot r$	Present Value $PV \cdot rB/A^2$
.1111	$\infty$	.0579
.12	1.785	.0493
.13	1.023	.0407
.14	.658	.0331
.15	.442	.0264
.16	.301	.0208
.17	.205	.0159
.18	.137	.0118
.19	.0896	.00843
.20	.0558	.00569
.21	.0323	.00354
.22	.0166	.00194
.23	.00675	.00084
.24	.00155	.00020
.25	0	0



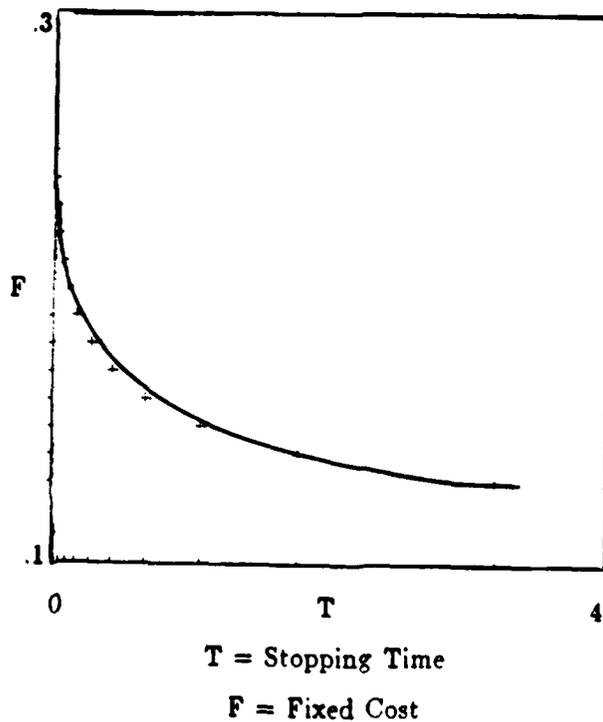
The numerical results are fairly sensitive to the assumption that there is a substantial chance that a competitor has fixed costs low enough to sustain a duopoly forever. Table 2.6 shows the strategies that result when  $A = B = r = 1$  and it is known that a firm incurs a cash outflow of 0.10 in addition to a privately known opportunity cost

that is uniformly distributed between zero and 0.15 (the technical assumption that represents this feature is that  $G(F) = [F - .1]/.15$ ). Note that in the figure the scale of the horizontal axis has been compressed compared to the previous figure.

**Stopping Times in the War of Attrition**  
 ( $p = 1 - q$ ,  $G(F) = [F - .1]/.15$ , and  $r = 1$ )

**Table 2.6**

Fixed Cost $F \cdot B/A^2$	Stop Time $T \cdot r$	Present Value $PV \cdot rB/A^2$
.1111	$\infty$	.0796
.12	7.160	.0708
.13	3.211	.0607
.14	1.784	.0510
.15	1.081	.0417
.16	.682	.0333
.17	.436	.0258
.18	.279	.0194
.19	.174	.0139
.20	.105	.00942
.21	.0589	.00588
.22	.0294	.00323
.23	.0117	.00140
.24	.0026	.00034
.25	0	0



For example, suppose as before that  $A = \$1000$  per unit,  $B = 1$ , and the interest rate is  $r = .1$ . A firm with the fixed cost  $F = \$150,000$  per year (i.e., a cash outlay of  $\$1,000,000$  plus an opportunity cost of  $\$50,000$ ) now waits  $1.081/.1 = 10.81$  years before exiting. The expected present value is increased to  $\$417,000$ . If the opportunity costs were doubled to  $\$100,000$  then it would exit after 1.05 years; the expected present value is  $\$94,200$ . The firm's greater patience stems from the greater perceived chance that the opponent will drop out first, given that it has to meet the continuing cash outlay for overhead of  $\$100,000$  per year.

### 2.3 Entry and Exit

We turn now to a slightly more realistic depiction of the firms' costs and include as well the decision of the second firm to enter the market and engage in combat. For this purpose we study in more detail the Chain Store game introduced in §1. Before we allowed that the incumbent might be either weak or strong, and now we allow also that the entrant might be either weak or strong. Assume that there is only a single entrant with repeated opportunities to enter.

Initially the entrant assigns probability  $p$  that the incumbent is strong, and the incumbent assigns probability  $q$  that the entrant is strong. To represent the idea that the strong incumbent is one with low costs, so that it prefers to price low (i.e., play tough) in response to entry, we use the payoffs in Table 2.7 for each of the possible outcomes in each period. Note that compared to the payoffs used in §1 we have

subtracted 1 from the incumbent's payoffs and divided the entrant's by 2; since these modifications affect only the origin and scale of the payoffs they have no effect on the equilibria of the game.

**Table 2.7**  
**Payoffs in the Repeated Chain Store Game**  
**with Two-Sided Uncertainty**

	Incumbent's Payoffs		Entrant's Payoffs	
	Weak	Strong	Weak	Strong
No Entry :	\$2	\$2	\$0	\$0
Enter, Soft :	0	-1	.5	1
Enter, Tough :	-1	0	-.5	.5

Assume that there are  $n$  periods in which the situation is repeated and that each firm seeks to maximize the sum of its expected profits (the results are hardly changed if the firms discount future profits moderately).

Note that the strong entrant prefers to enter even if the incumbent plays tough. Following the convention adopted in §1, assume that an incumbent who plays soft or an entrant who fails to enter is thereafter branded as weak, namely his opponent resets  $p = 0$  or  $q = 0$  as the case may be, and it remains there regardless of the subsequent actions chosen.

With this convention we study an equilibrium in which the strong entrant always enters and the strong incumbent is always tough. The strategy for the weak entrant therefore specifies whether and when he

will enter and for how long he is prepared to fight before exiting if the incumbent has not reverted to soft play by then. Actually, it is easy to see that a weak entrant will either enter immediately or wait until near the end of the game. Waiting, or entering and then exiting, identifies him as weak, so thereafter the game is exactly as described in §1 where it was assumed that the entrant was known to be weak. Recall that he may re-enter near the end of the market when the chance increases that a weak incumbent will respond softly.

The strategy for the weak incumbent specifies for how long he is prepared to play tough against an entrant who has entered. Once he capitulates by playing soft he is identified as weak and thereafter the entrant, whether strong or weak, stays in the market and the incumbent's best response is to respond softly, as we saw in the initial analysis of the Chain Store game when the incumbent was known by the entrant to be weak.

In sum, then, the strong entrant and the strong incumbent never stop; the weak ones pick stopping times for how long they are prepared to fight; and lastly, the weak entrant may choose to enter initially or to enter near the end.

Suppose now that the entrant has entered. As the battle continues to firms' probability assessments about each other evolve according to Bayes' Rule. To whatever extent a weak entrant would be likely to have exited, the incumbent assesses an increasingly higher probability that he is facing a strong one; similarly, to whatever extent a weak incumbent would have reverted to a soft response the entrant assesses an increasingly higher probability that he faces a strong one. Thus if

$(p_k, q_k)$  indicates their probability assessments when  $k$  encounters remain, then as the fight continues and  $k$  decreases the pair  $(p_k, q_k)$  traces a locus of increasing values. If both are strong then eventually  $(p_0, q_0) = (1, 1)$  when no periods remain (actually before that as it turns out). If one or both are weak, then eventually one gives in: if it is, say, the incumbent who is weak and acquiesces then there is a jump to  $(p_k, q_k) = (0, q_k)$  and thereafter the incumbent plays soft in response to the continuing entry; whereas if it is the entrant who is weak and exits then there is a jump to  $(p_k, q_k) = (p_k, 0)$  and thereafter the entrant stays out (expecting tough play) until near the end when the entrant may again test the incumbent's response.

As we saw in §2, if the firms' possible types are a continuum then each selects a stopping time depending on his type. Here, however, each firm has only two possible types so an equilibrium involves each of the weak types selecting a randomly determined stopping time. Computing these equilibrium strategies is complicated so we will not do it here. The key idea is simple enough, however, to deserve mention. Since the stopping time is random, it must be that the firm is indifferent as to the outcome of the randomization; e.g., whether to stop now or a little later. Thus, each firm's randomization is determined so as to make the other firm indifferent about whether to stop now or a little later, and this is true at all times.

When time is continuous it is considerably easier to compute the firms' strategies; partly this is because with continuous time the weak entrant never re-enters until the last instant. We outline the con-

struction briefly. Assume that the game lasts for a duration  $T$  and let the pair  $(p_t, q_t)$  be the entrant's and the incumbent's assessed probabilities that the other is strong when a duration  $t$  remains. Our aim is to find the locus this pair traces after the entrant enters as  $t$  declines from  $T$  to  $0$ , and to derive the equilibrium strategies in the form of random stopping times. Suppose that when a duration  $t$  remains the entrant assesses a conditional probability  $r_t h$  that the weak incumbent will capitulate in the next small interval of time of length  $h$ ; similarly, the incumbent assesses a conditional probability  $s_t h$  that the weak entrant will exit, given that both have been hanging tough so far. Recall that the incumbent gets zero thereafter once he plays soft, whereas he gets  $2t$  if the entrant exits with  $t$  remaining. Since the weak incumbent is indifferent between continuing tough play or not, it must be that his cost of continuing for the interval  $h$  equals his expected profit from the chance that the weak entrant will exit:  $h = 2ts_t h(1 - q_t)$ . Similarly, for the weak entrant we get:  $.5h = .5tr_t h(1 - p_t)$ . Thus, we know that  $r_t = 1/t(1 - p_t)$  and  $s_t = .5/t(1 - q_t)$ , which tells us what the probability distribution of stopping times must be. It remains to describe the locus of probability assessments. If the incumbent continues tough during the interval  $h$  then the entrant's posterior probability that he is strong is given by Bayes' Rule:  $p_{t-h} = p_t/[1 - r_t h(1 - q_t)]$ ; and similarly one gets  $q_{t-h} = q_t/[1 - s_t h(1 - p_t)]$ . Interpreting  $h$  as infinitesimally small, these two equations imply a differential equation that has the unique solution  $q_t = \sqrt{p_t}$  satisfying the restriction that  $(p_0, q_0) = (1, 1)$ ;

this restriction is necessary since if the firms hang tough until the end then they are surely both strong. Thus, we know the locus along which the probability assessments evolve. Further, in order to satisfy the results of the Bayes' Rule calculations above it must be that these assessments evolve according to the formulas:  $p_t = a/t$  and  $q_t = \sqrt{a/t}$ , where  $a = p_T T = q_T^2 T$  is constant determined from the initial probability assessments from which the process starts. For example, if initially  $q < \sqrt{p}$  then the weak entrant randomizes as to whether he will enter, and if he does enter then the battle begins with the initial assessment  $(p_T, q_T) = (p, \sqrt{p})$ ; similarly, if  $p < q^2$  then the entrant enters and the weak incumbent randomizes whether to do battle, and if it does respond initially with tough then the process starts from  $(q^2, q)$ . Note that it becomes certain that the firms are both strong after a time has passed that is less than the full duration  $T$ ; that is,  $(p_t, q_t) = (1, 1)$  at  $t = a$ . At any time before this ( $t > a$ ) the weak incumbent and the weak entrant have conditional probabilities of stopping (per unit time) of  $r_t = 1/(t - a)$  and  $s_t = .5/(t - \sqrt{at})$ , respectively.

These strategies are depicted schematically in Figure 2.8. Recall that if initially  $(p, q)$  is below the curve and the entrant enters then there is a jump upwards to the curve, and if it is above the curve and the incumbent plays tough then there is a jump right to the curve; thereafter, the probability assessments move along the curve until one capitulates, producing a jump to the lower boundary if the entrant exits and to the left boundary if the incumbent acquiesces.

This yields the solution  $Q = 54$  and an expected price of \$46, exactly as the equilibrium specifies. A similar analysis justifies the high-cost incumbent's choice of the supply quantity  $Q = 44$  with an expected price of \$56. It is important to note that in the above calculation it is the incumbent's anticipation that the entrant's price observation will influence his entry decision that motivates the choice of an expected price below the monopoly price. Otherwise the low-cost incumbent would just choose  $Q = 45$  units to solve the simple first period marginal profit condition:  $90 - 2Q = 0$ .

The thrust of this example of limit pricing is to substantiate the intuitive appeal of the argument that limit pricing is intrinsic to situations with threatened entry. Even with noisy observations, the entrant must anticipate limit pricing, lest the incumbent deter profitable entry by misleading action; and in turn, the incumbent must price below the monopoly price, lest the entrant infer that entry is more profitable than in fact it is.

#### 4. Price Wars

The previous sections studied competitive battles precipitated by entry or, as in wars of attrition, the pressure to cover fixed costs in order to survive profitably. In some cases, however, price wars are precipitated by one firm's misguided efforts to enlarge its market share -- or by simple mistakes. In other cases price wars are undertaken intentionally to police the terms of collusive agreements: deviant members are punished for their attempts to enlarge their share of the pie. In

this section we examine some examples that illustrate how such episodes arise. The first example shows the role of mistakes in precipitating price wars.

◇ Example 4.1: Consider a market shared by two firms that are alike in all respects. Demand is such that at a price  $p$  the quantity sold is  $q = 1 - p$ . We interpret this price as net of the firms' marginal cost, so the industry net revenue is  $p[1 - p]$ . If the firms offer the same price they share the market equally, otherwise the firm with the lower price captures all of the demand. Each firm discounts its profits for all future periods using the discount factor  $d < 1$ . Two special technological features affect this market. First, the firms select their prices alternately, and each firm is committed to its chosen price for two periods. Second, the only feasible prices are  $\$0, .1, .2, .3$ , etc. As we shall see these two features combine to enable the two firms to share the monopoly profit.

The optimal monopoly price is  $p = \$.5$  and at this price the profit shared by the two firms is  $\$.25$  in each period. This allows each firm a profit of  $\$.125$  and over an infinite horizon this profit stream has a present value of  $\$.125/[1 - d]$ . To be specific we will specify that  $d = .9$  so that this present value is  $\$1.25$ .

In order to show that the firms will maintain the monopoly price we must verify that if one adopts that price then the other's best response is to follow suit. Evidently, one firm can cut its price and gain in the short run, so the incentive to maintain price must lie in the expectation that one price cut will be followed by another. An equilibrium

private information about its unit cost of production. In general there are recurrent episodes in which the incumbent first makes a choice about its price or quantity, and then the entrant, having observed this choice more or less precisely, decides whether to enter the market or to defer until the next episode (if any). If he enters then the two firms constitute a duopoly thereafter, or they play out a battle as in §2. The incumbent's payoff is the present value of his immediate profit depending on his choice and the subsequent profit depending on the entrant's choice as to whether they continue as a monopoly or a duopoly. In the latter case the entrant's payoff is the present value of his duopoly profit less a sunk cost of entry (discounted by one period to account for the delay in entry), whereas it is zero until he opts for entry. We assume, of course, that the incumbent's profits are always higher in a monopoly, and that the entrant's profits are lower the lower is the incumbent's unit cost.

### 3.1 Information, Signalling, and Noise

The basic idea of limit pricing can be stated simply as the observation that if the incumbent perceives that the chance of entry increases with his price then he wants to shave his price somewhat below the optimal monopoly price in order to deter entry. This idea is subtle, however, since it is clear that if the entrant knows the incumbent's cost then no observation of the incumbent's action can influence his decision about entry: he enters or not based on his assessment of his profits in a duopoly. Thus the chance of entry is fixed independently

of the incumbent's choice and therefore no limit pricing is worthwhile. Thus some uncertainty about the profit from entry in the mind of the entrant is essential to the existence of limit pricing.

A second observation is that there may be more entry with limit pricing, in the context we described, than there would be if both firms had perfect information. To see this, suppose that an entrant gets increasingly finer information about the incumbent's cost as he accumulates observations from successive episodes. An entrant that defers entry for a long time gains nearly perfect information and therefore enters or not in precisely the same circumstances (though much later) as it would if it had perfect information initially. On the other hand, an entrant that enters early is likely to make a mistake, so overall there is a greater likelihood of entry. In an extreme case there is only a single episode possible, but suppose that the entrant's observation enables him to infer what cost led the incumbent to make that choice: in this case the entrant enters under precisely the same circumstances as he would with perfect information initially. In sum, then, limit pricing does not necessarily restrict entry below that expected with perfect information.

Why, then, would the incumbent engage in limit pricing? The answer unveils a further subtlety in the basic idea with which we began. If the entrant anticipates that the incumbent will engage in limit pricing, then the incumbent is forced to do so, since otherwise a high price would induce a greater chance of entry. That is, the entrant will take the incumbent's price as a signal about his cost, and a high price

signals a high cost and a high duopoly profit for the entrant. On the other hand, the entrant can not afford to reject the hypothesis of limit pricing, since otherwise it can be used against him to induce him to stay out when in fact entry is profitable. Thus, limit pricing is intrinsic when there is a potential entrant lurking about. The mere potentiality of entry depresses the monopolist's price, and it may even increase the chance of entry above the perfect information case.

If the entrant's observations are noisy (he observes only some portion or some garbled version of the incumbent's decision), then he has an incentive to defer entry until he accumulates enough observations to reduce his risk of entering an unprofitable duopoly. This benefits the incumbent, of course, since he maintains his monopoly longer, but as mentioned above noisy observations increase the chance that the entrant will enter mistakenly and decrease both their profits.

After these initial remarks we turn to a worked out example that illustrates limit pricing.

◇ Example 3.1: Suppose that there is a single episode with two periods. The demand function for the market is assumed to be linear of the form  $P = \$100 - \$1 \cdot Q + \epsilon$ , indicating that the expected price declines by one dollar for each unit of output offered by either firm. The noise term  $\epsilon$  is uniformly distributed between -20 and +20. [One can equally well suppose that the demand function is  $P = \$100 - \$1 \cdot Q$ , and that  $\epsilon$  is a noise term in the entrant's observation of the

incumbent's average price. In a model of this sort the incumbent can choose either the price or the quantity.]

The unit cost of the incumbent can be either low (\$10) or high (\$30) and these are perceived by the entrant to be equally likely. The low-cost incumbent's optimal monopoly output is 45 units, yielding an expected profit of \$2025, and for the high-cost incumbent it is 35 units, yielding an expected profit of \$1225. The entrant's cost can also be either low (\$25) or high (\$30) and the incumbent sees these as equally likely. To describe the post-entry profits we will assume for simplicity that the incumbent and the entrant immediately learn each other's costs and then select their supply quantities in the duopoly so that each picks a best response to the other's choice. After some calculation this yields the post-entry profits shown in Table 3.1A,B.

**Table 3.1**

**A. Post-Entry Profit for the Incumbent**

Incumbent	Entrant	
	Low Cost	High Cost
Low Cost :	\$1225	\$1344
High Cost :	469	544

**Table 3.1**

**B. Post-Entry Profit for the Entrant**

Incumbent	Entrant	
	Low Cost	High Cost
Low Cost :	\$400	\$278
High Cost :	711	544

Assume also that the entrant incurs a sunk cost of \$500 to enter. From these data it is straightforward to see that with perfect information either type of entrant would enter against the high-cost incumbent but not the low-cost one. Also, with incomplete information and unable to make any observation, only the low-cost entrant would enter.

Now suppose that, as in the case of limit pricing, the incumbent chooses the quantity he will supply as a monopolist in the initial period, after which the entrant gets to observe the resulting price (which, due to the noise term, garbles the incumbent's choice) and then to decide whether to enter for the second period. The equilibrium in this case is the following:

- The low and high cost incumbents choose to supply 54 and 44 units respectively. These choices correspond to expected prices of \$46 and \$56 respectively.
- The low and high cost entrants choose to enter only if the price exceeds \$36 and \$66 respectively.

Notice that either type of incumbent increases his output by 9 units and lowers his expected price by \$9 compared to the optimal monopoly choice.

In order to understand the factors that motivate this behavior we begin by examining the entrant's decision process. The entrant anticipates prices in the range from \$26 to \$66 from the low-cost incumbent, and in the range from \$36 to \$76 from the high-cost incumbent. Prices at the low end from \$26 to \$36 therefore identify the incumbent as one with low costs, and prices at the high end from \$66 to \$76 identify a

high-cost incumbent. In the middle range from \$36 to \$66 it is equally likely that the price originated from a low-cost or a high-cost incumbent. Since the low-cost entrant is not willing to enter against the low-cost incumbent, but is willing if the odds are 50-50, he will enter if the price is in the mid range or above. Similarly, the high-cost entrant is willing to enter only against the high-cost incumbent, and not willing if the odds are 50-50, so he will enter only if the price is in the high range above \$66.

The low-cost incumbent seeks to maximize the sum of his expected profits from the first and second periods. In the first his expected profit margin is  $\$90 - \$1 \cdot Q$  per unit, so his marginal profit is  $\$90 - \$2 \cdot Q$ . As for the second period, he thinks there are equal chances that he faces a low-cost entrant who will enter if the first price exceeds \$36, in which case the second period profit will be \$1225 instead of the maximum monopoly profit of \$2025 (i.e., \$800 less), and that he faces a high-cost entrant who will enter if the price exceeds \$66, in which case the second period profit will be \$1344 instead of his hoped-for \$2025 (i.e., \$681 less). Realizing that the probability of a price realization in excess of \$36 declines at the rate of  $1/40$  as  $Q$  increases (i.e., the probability density of the noise term is  $1/40$ ), the supply at which his combined marginal profit is driven to zero is the one satisfying the equation

$$90 - 2Q + \frac{1}{40} \{ .5 \cdot 800 + .5 \cdot 681 \} = 0.$$

This yields the solution  $Q = 54$  and an expected price of \$46, exactly as the equilibrium specifies. A similar analysis justifies the high-cost incumbent's choice of the supply quantity  $Q = 44$  with an expected price of \$56. It is important to note that in the above calculation it is the incumbent's anticipation that the entrant's price observation will influence his entry decision that motivates the choice of an expected price below the monopoly price. Otherwise the low-cost incumbent would just choose  $Q = 45$  units to solve the simple first period marginal profit condition:  $90 - 2Q = 0$ .

The thrust of this example of limit pricing is to substantiate the intuitive appeal of the argument that limit pricing is intrinsic to situations with threatened entry. Even with noisy observations, the entrant must anticipate limit pricing, lest the incumbent deter profitable entry by misleading action; and in turn, the incumbent must price below the monopoly price, lest the entrant infer that entry is more profitable than in fact it is.

#### 4. Price Wars

The previous sections studied competitive battles precipitated by entry or, as in wars of attrition, the pressure to cover fixed costs in order to survive profitably. In some cases, however, price wars are precipitated by one firm's misguided efforts to enlarge its market share -- or by simple mistakes. In other cases price wars are undertaken intentionally to police the terms of collusive agreements: deviant members are punished for their attempts to enlarge their share of the pie. In

this section we examine some examples that illustrate how such episodes arise. The first example shows the role of mistakes in precipitating price wars.

◇ Example 4.1: Consider a market shared by two firms that are alike in all respects. Demand is such that at a price  $p$  the quantity sold is  $q = 1 - p$ . We interpret this price as net of the firms' marginal cost, so the industry net revenue is  $p[1 - p]$ . If the firms offer the same price they share the market equally, otherwise the firm with the lower price captures all of the demand. Each firm discounts its profits for all future periods using the discount factor  $d < 1$ . Two special technological features affect this market. First, the firms select their prices alternately, and each firm is committed to its chosen price for two periods. Second, the only feasible prices are  $\$0, .1, .2, .3$ , etc. As we shall see these two features combine to enable the two firms to share the monopoly profit.

The optimal monopoly price is  $p = \$.5$  and at this price the profit shared by the two firms is  $\$.25$  in each period. This allows each firm a profit of  $\$.125$  and over an infinite horizon this profit stream has a present value of  $\$.125/[1 - d]$ . To be specific we will specify that  $d = .9$  so that this present value is  $\$1.25$ .

In order to show that the firms will maintain the monopoly price we must verify that if one adopts that price then the other's best response is to follow suit. Evidently, one firm can cut its price and gain in the short run, so the incentive to maintain price must lie in the expectation that one price cut will be followed by another. An equilibrium

must specify, therefore, how each firm will respond to each possible price of the other firm. Only in this way can one trace the price war that ensues when one firm deviates, thereby establishing the incentive to maintain the monopoly price, as well as trace the process by which the monopoly price is initially attained or subsequently regained. In this simple example it is easy to calculate the equilibrium presented in Table 4.1.

**Table 4.1**  
**Equilibrium Pricing Strategies**

$$q = 1 - p \text{ and } d = .9$$

Competitor's Current Price :	\$0	\$.1	\$.2	\$.3	\$.4	\$.5
Firm's Best Response :	.5	.5 or .1	.1	.1	.1	.5
Firm's Present Value :	1.125	1.125	1.17	1.17	1.17	1.25

The calculation of each firm's present value shown in the table will be explained below. Note that in the table there are two best responses if the competitor's price is \$.1, namely the firm can respond with either the same price or revert to the monopoly price. The equilibrium requires that this choice be 'randomized' (or at least that the competitor entertains a probability assessment as though the choice were randomized). The requisite probability is that the reversion to the monopoly price has probability .905. This probability is selected so that either firm is indifferent about reverting to the monopoly price when its competitor's current price is \$.1.

The verification that this strategy is optimal for each firm if the other is using it is displayed in Table 4.2, which shows the firm's

present value for each possible response it might choose. The computation assumes that the indicated equilibrium strategy is followed in subsequent periods; clearly, if the strategy were not optimal then some single-period deviation from it would be profitable.

**Table 4.2**  
**Verification of the Equilibrium**  
**Present Value From Each Response**

Competitor's Price	Firm's Response				
	<u>\$.1</u>	\$.2	\$.3	\$.4	\$.5
\$.1	<u>1.125</u>	.91125	.91125	.91125	<u>1.125</u>
\$.2	<u>1.17</u>	.99125	.91125	.91125	1.125
\$.3	<u>1.17</u>	1.07125	1.01625	.91125	1.125
\$.4	<u>1.17</u>	1.07125	1.12125	1.03125	1.125
\$.5	1.17	1.07125	1.12125	1.15125	<u>1.25</u>

In each row of the table an underlined present value figure indicates the firm's best response to the competitor's price. To see how these figures are obtained, take the example in which the competitor's current price is \$.2 and the firm is contemplating offering the price \$.1. By doing so it gets the whole market this period for an immediate profit of \$.09. It expects the competitor to respond in the next period with either the same price (yielding a profit of \$.045 discounted by the factor .9) or the monopoly price (yielding a profit of \$.09 discounted by .9). If the price remains at \$.1 then the value of continuation two periods hence is the indicated present value of \$1.125

discounted further by  $.9^2$ ; whereas, if the competitor reverts to the monopoly price then the value of continuation is \$1.25 again discounted by  $.9^2$ . All told, then, the present value of offering the price \$.1 in response to the competitor's price of \$.2 is

$$$.09 + .9 \cdot \{ .095 \cdot [$.045 + .9 \cdot \$1.125] + .905 \cdot [$.09 + .9 \cdot \$1.25] \} = \$1.17$$

The complexity of this computation stems from the fact that the firms alternate moves and each is committed to its price for two periods.

Now we can see how this equilibrium describes a price war in the event that either firm mistakenly or intentionally deviates from the equilibrium. Suppose that firm 1 cuts its price to \$.4. Then next period firm 2 will cut its price to \$.1, following the indicated strategy. Assuming that firm 1 now recognizes its mistake and wants to resume the equilibrium, it is now indifferent between maintaining the current price of \$.1 or reverting to the monopoly price of \$.5. If it keeps the current price then firm 2 finds itself in the same situation. Thus the situation continues until one firm opts to resume monopoly pricing: this costs it in the short run (initially it has no customers) but the deficiency is recouped when the other firm joins in and thereafter they expect to divide the monopoly profit.

The way to imagine this equilibrium is that it is like a yo-yo. The price tumbles down; how long it stays before it pops back up is uncertain. When the price is at the bottom (\$.1) each firm is indifferent between continuing there (hoping the other firm will take the initiative to raise its price and forego profits for one period) or venturing to be

first to raise its price. The mean time that the price persist at the bottom can be calculated: it is 1.111 periods. Indeed this is all that really matters: the equilibrium can be described equally well by saying that each firm picks a (random) stopping time indicating how many periods it is willing to persist with the bottom price if the other does not earlier revert to the monopoly price. An intriguing feature is that the firms' ability to commit in alteration for two periods to their prices provides enough stickiness to sustain the monopoly price in equilibrium.

There are, of course, other ways that firms can sustain monopoly prices if commitment is possible. The usual cartel agreement is an example: the firms plan to share the market at a uniform high price until any one member deviates, after which they adopt competitive pricing for some period. The punishment period is chosen long enough to deter deviations; it may be forever if deviation dissolves the cartel. The usual problem with such agreements is familiar from the experience of the OPEC oil cartel: typically it is hard to detect deviations since the price cutting is done secretly to selected customers or in invisible ways such as improvements in product quality. Responding to this problem, cartels adopt policing measures that rely on accessible information; for example, if anticipated demand falls short of expectations then a competitive episode ensues even if no firm was secretly cutting price. Consequently, depending on the noise level in the observations that can be made, there are recurrent episodes of competitive pricing, many of which may be triggered by errors of observation. This scenario

is consistent with the view that collusive agreements pose a problem of organization design, namely, to design detection and enforcement procedures that deter cheating. In most cases a period of competitive pricing is the effective punishment.

5. Commitment and Sunk Costs

The pivotal idea in the discussion of predatory behavior in §1 was that a firm could plausibly imitate commitment to aggressive behavior when in fact it was not committed. There are other cases in which commitment is real and obvious, and it is to these situations that we now turn. Our focus is on the role of sunk costs in tangible equipment, advertising, etc.

It is clear that the structure of costs has a lot to do with entry. Economies of scale and indivisibilities in the size of plants can make it unprofitable for newcomers to enter on a small scale. If there are no fixed or sunk costs then an entrant may have little to lose by invading a market: if entry is free then presumably the market will fill up with competitors. On the other hand, asymmetries among firms' fixed and sunk costs can be a major determinant of the outcome of competitive battles. The classic example is the hypothetical battle between a railroad and a shipping line for coastal freight traffic in the absence of regulatory and legal restraints. The railroad's costs are mostly sunk into irreversible investments in track, whereas the shipping line can freely transfer its ships to other routes. With few other options, the railroad is willing to cut prices to the bone to survive;

the shipping line presumably must include in its costs the opportunity cost of the profits foregone on other routes for every day it sustains the battle with the railroad. The plausible outcome is that the shipping line will exit this market, or better, never enter in the first place given that it sustains fixed costs in maintaining a presence in the market.

Our interest here is in firms' strategic decisions about commitments to alternative cost structures via choices about capacity. An irreversible commitment to the type and amount of capacity effectively sets the rules of the game that ensues. As before the concern is to elucidate the effects on entry and exit.

### 5.1 The Incumbent's Defense

We first examine markets that are natural monopolies and study some of the ways that an incumbent firm can deter entry. By a natural monopoly we mean that the present value of monopoly profits more than cover the cost of capital and/or other fixed costs, but a duopolist's profits are insufficient to recover these costs.

A first possibility is that the firm's capital stock is, like the railroad's track, long-lived and irreversible. In this case it is clear that an entrant is deterred from entering by the fact that it can not recover the requisite capital and fixed costs. But what if capital equipment is short lived (and for simplicity there are no other fixed costs)? As the time nears for reconstruction of the incumbent's equipment an entrant could perceive a chance to enter profitably by moving

first to install capacity and thereby deter the incumbent from refurbishing its plant. The incumbent in this case is motivated to replace its capital stock early in order to deter the entrant: the optimal time to renew its capital equipment is calculated so that if the entrant were to enter any time sooner its losses in the period of the duopoly would outweigh its subsequent profits were it to gain the monopoly upon the demise of the incumbent's plant. This optimal strategy reveals that the incumbent's profit is precisely the present value of the difference between the monopoly and duopoly profits over the period until replacement of the capital: were it more the entrant would enter profitably and recover its investment by waiting out the incumbent.

An interesting aspect of this analysis is revealed by supposing that the lifetime of equipment is very short (and then so too is the replacement time): in this case the incumbent's profit is nearly zero. This result reflects the general proposition that the profitability derived from the power to make long commitments via irreversible investments diminishes to zero as the period of the commitment shortens. Monopolies in which the brevity of the commitments wipes out profits in the attempt to deter entry are said to be perfectly contestable: monopolies they may be, but monopoly profits are precluded by the threat of entry that would eject the incumbent.

The analysis is similar if there are no irreversible capacity costs but there are continuing fixed costs  $F$  that must be covered. In this case the market is a natural monopoly if the gross profit of a duopolist is insufficient to cover this fixed cost; here we assume the stronger

property that even the monopoly gross profit is insufficient to cover the fixed costs of two firms (to exclude the case that the firms might collude explicitly or implicitly to share the monopoly profit). To represent the power of commitment suppose that each firm commits to its output level for two periods, and in a duopoly they alternate in choosing their output levels. In this case an incumbent monopolist can deter entry by choosing its output level sufficiently large so that an entrant entering in the second period would incur losses that outweigh subsequent profits even were it to succeed in driving out the incumbent. For example, in a symmetric model let  $P(q_1, q_2)$  be the period profit of one firm if its output is  $q_1$  and the other's is  $q_2$ ; and let  $d$  be the discount factor. The output level that deters entry is the largest value of  $q$  satisfying:

$$P(q, q) + \frac{d}{1-d} P(q, 0) = \frac{F}{1-d} .$$

That is, if  $q$  is the optimal then the entrant will also select  $q$  when it enters to deter the incumbent from continuing; consequently, the entrant's profit is  $P(q, q)$  the first period it enters (since the incumbent committed for two periods), and  $P(q, 0)$  thereafter (since the incumbent withdraws), and optimally the present value of this profit stream must be no greater than the present value of the stream of fixed costs (to deter the entrant's entry), and preferably no less (or the incumbent would be sacrificing profits with excessive deterrence). Again we see the feature that if the power of commitment is strong (the discount factor is small, indicating that each period is long) then the

incumbent's profit  $P(q,0) - F$  per period is large, but if the power of commitment is weak (the discount factor is large, approaching 1) then the incumbent's profit  $P(q,0) - F = 0$  per period is wiped out and the market is perfectly contestable.

The thrust of these examples is that the threat of entry is a limiting factor on the profitability of even a natural monopoly. The durability of irreversible investments or other possibilities for commitments that affect adversely the profitability of entry can sustain some portion of the potential monopoly profits, but if these deterrents are weak or costly for the incumbent then it is forced to forego these potential profits to deter entry.

## 5.2 The Entrant's Gambit

Next we consider an entrant's strategy of limiting its capacity in order to avoid provoking a response from the incumbent firm. If an entrant commits to a capacity that is sufficiently small then the incumbent will not have incentive to drive out the entrant, or even to engage it directly with duopolistic pricing. It is a common observation that in many industries "fringe" firms prefer to remain small rather than to risk retaliation by the dominant firms. A useful example is an incumbent hotel in a resort location: an entrant that builds a comparable but small hotel with lower rates (a pension) can expect accommodation from the larger hotel, since it is in the larger one's interest to serve the overflow from the smaller one rather than to cut its price to compete directly. This assumes, of course, that the hotel

can not offer selective price discounts to the other's customers, and that the larger hotel is indeed considerably larger.

To explore the logic of this strategy, consider the following example. Assume that the incumbent has installed a very large capacity and it has constant marginal cost  $c_1$ , whereas the entrant chooses its capacity  $K$  and it has constant marginal cost  $c_2 > c_1$ . Demand is a function  $Q(p)$  of the price charged: assume that the firm with the lower price captures all of the market that it can serve, but that (reflecting the incumbent's prior advantage) if they charge the same price then the demand goes to the incumbent. If the entrant charges a lower price and limits its capacity then its demand must be rationed.

First consider the case that the available capacity is rationed randomly among the customers, in which case the entrant serves  $K$  customers at its price  $p_2$  and the incumbent serves  $[1 - K/Q(p_2)]Q(p_1)$  customers at its price  $p_1$ , assuming that  $p_1 > p_2$ . If the incumbent is accomodating it will, of course, choose to charge its optimal monopoly price to the overflow customers it receives; this is the price  $p_1$  that maximizes its profit contribution  $[p_1 - c_1]Q(p_1)$ , whether or not it obtains the full demand. The entrant's strategy, therefore, must be to curtail its capacity sufficiently to make accomodation better for the incumbent than matching the entrant's price  $p_2$  -- in which case the entrant would obtain no sales at all. Thus, the entrant must choose its capacity and price subject to the constraint that

$$[p_1 - c_1][1 - K/Q(p_2)]Q(p_1) > [p_2 - c_1]Q(p_2).$$

This constraint expresses the requirement that the incumbent's profit from accomodation exceeds its profit from matching the incumbent's price. Subject to this constraint the entrant can choose its capacity and price to maximize its profit contribution. The effect of this constraint is essentially to induce for the entrant a new demand function  $Q^*(p_2|p_1)$ ; this new demand function lies below the original demand function by an amount that leaves for the incumbent a sufficient overflow to encourage accomodation. In general, the entrant chooses a limited capacity and a lower price than the incumbent.

Besides a random rationing of its capacity the entrant might be able to ration its capacity to those customers with the highest valuations of the product. In this case the incumbent's overflow demand will be  $Q(p_1) - K$  at its price  $p_1$ . The incumbent's accomodating response will entail some reduction of its price below the monopoly level, and in turn the entrant's price will be even lower.

A particularly ingenious method of rationing demand to high-valuation customers is for the entrant to issue (transferable) coupons entitling the bearer to service at the price  $p_2$ . If this is done the entrant might garner its profits solely from the coupon sales should the incumbent choose to honor the coupons. The incumbent wants to honor the coupons if  $p_2 > c_1$ , since otherwise it loses this market segment. As for the overflow, the incumbent prefers to accomodate the entrant so long as its optimal price in response exceeds the entrant's price  $p_2$ . For example, if  $p_1 > c_2 > c_1$  then the entrant can issue  $K$  coupons at

a price  $p_2 > c_1$  for which it charges less than  $p_1 - p_2$  per coupon to attract customers.

A useful way to interpret these scenarios is to view the entrant as an extortionist who threatens to produce  $K$  units unless it is bought off. Faced with this threat the incumbent purchases the rights to the entrant's output (by honoring the coupons). The way this can work is for the entrant to issue coupons for the  $K$  units of capacity at the price (slightly more than)  $p_2 = c_1$  by charging (slightly less than)  $p_1 - p_2 = p_1 - c_1$  for each coupon. The incumbent then finds it advantageous to honor these coupons (since  $p_2 > c_1$ ). The cost savings for the industry,  $[c_1 - c_1]K$ , accrues entirely to the entrant via sales of coupons.

In the famous coupon war of 1979 among the major airlines, United issued coupons to recover its market position after a long strike; soon after, the other airlines responded by accepting (and in some cases, issuing) coupons. Eastern did the same in 1980 in its battle with New York Air over the shuttle traffic. Its coupons were valid for a discount on its transcontinental flights as well, and promptly United and American honored the coupons, enabling Eastern to cut back the number of its transcontinental flights in favor of shuttle flights. The airline industry seems particularly suitable for the use of coupons since capacity is mobile: a commitment of capacity to a scheduled route for one period can be reversed later and the equipment redirected to other routes. Thus, the ability to commit capacity for periods of intermediate length may encourage competition via coupons and the like.

Contrast this with a resort hotel where a capacity commitment may be permanent, and having issued coupons honored by the incumbent, the facility may stand empty. The cost of a capacity commitment is relatively less in the airline industry than in the hotel industry because the commitment period is much shorter.

### 5.3 Dynamics of Capacity Expansion

We turn now to an analysis of the incumbent's options in its choice of capacity. In order to examine the dynamic evolution of an industry's capacity levels, we consider a new market in which the incumbent initially has a headstart in the installation of irreversible capacity. The main conclusion will be that the incumbent preserves and exploits its initial advantage by investing more than a subsequent entrant. In the long run, moreover, the entrant is permanently restrained to a lesser capacity by the incumbent's threat to respond to any investment by the entrant with more investment itself.

It is particularly significant that there is a wide range of terminal capacity configurations; indeed, we shall see that the ultimate levels of capacity in the industry depend on the firms' expectations about each other's behavior. This is a familiar phenomenon in industry: each firm's choice of capacity depends on the anticipated actions or reactions of its competitors.

To keep matters simple, assume that the firms' investment process occurs in an interval of time that is short compared to the projected life of the industry. Thus, the firms' discount rates can be ignored

and each is concerned more with the long-run configuration of capacities than with the timing of investments; similarly, depreciation can be ignored. Nevertheless, investment takes time and the rate at which new capacity can be installed is limited, so during the investment phase the firms' decisions interact with one another. Our aim is to analyze this interaction and to identify the final capacities that result.

At any time the firms' two capacities can be plotted as a point  $(K_1, K_2)$  as in Figure 5.1. Shown in the figure is the locus  $R_1$  indicating 1's preferred capacity depending on 2's capacity, and similarly  $R_2$  shows 2's preferred capacity depending on 1's; e.g., at the intersection of these two loci each firm prefers not to change its capacity. Above both loci neither wants to increase capacity, and neither can reduce it. Below both loci one might imagine that both want to increase capacity, but we will see that this is not necessarily the case, due to the effects of their interaction. Also shown on  $R_2$  is 1's preferred point  $S_1$  on that locus, and on  $R_1$  is shown the point  $S_2$  that is 2's preferred point on  $R_1$ . Assume that from any point  $(K_1, K_2)$  there are four possible movements: they can both stop investing, they can both continue (at, say, equal rates), or one can invest while the other stops.

The Firm's Reactions

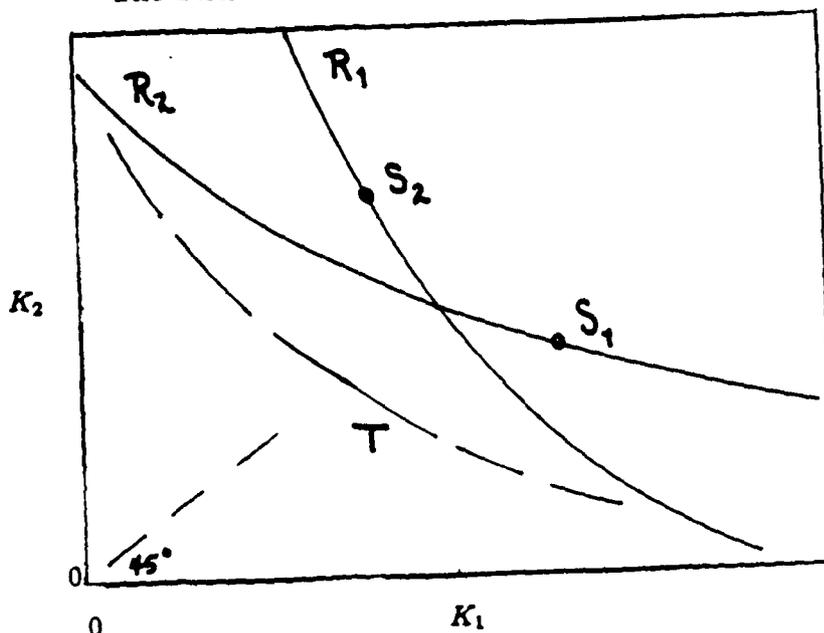


Figure 5.1

$R_1(K_2) = 1$ 's Preferred Response to 2

$R_2(K_1) = 2$ 's Preferred Response to 1

We first argue that if 1 starts initially with a sufficiently large capacity when 2 enters the market then there is an equilibrium that ends up at the point  $S_1$ . To see this consider the following strategy for 1: do not invest if  $(K_1, K_2)$  is above both loci; otherwise invest as rapidly as possible, except stop if  $K_1$  reaches or exceeds the level of 1's capacity at  $S_1$ . The strategy for 2 is similar, except that 2 stops investing when its capacity reaches the level at  $S_2$ . With these strategies, starting from a point on the abscissa with  $K_1 > 0$  and  $K_2 = 0$ , both firms invest rapidly until, if 1 has a sufficient headstart, 1 reaches its optimal capacity at  $S_1$  and stops to wait for 2 to invest until  $S_1$  is reached and 2 stops as well. That these strategies are in equilibrium (each strategy is a best response to the other) is evident

from the fact that below the response curves each expects the other to invest as rapidly as possible to reach its preferred capacity level along its response curve; hence, each must invest rapidly to move the resulting capacity configuration in its preferred direction.

This is not, however, the only equilibrium. To see this, consider the situation that arises when they start at the same time with zero capacities and invest along the  $45^\circ$  line until they reach the intersection of their response curves. In the later stages of this process each is investing because it expects the other to invest: both would prefer to stop with the smaller capacities represented by their preferred points along the line. Indeed, if each expects the other to stop at a smaller capacity (at least as large as both preferred capacities along the line) along the line, then they will invest only up to this point - with the threat that if the other invests beyond this point then they both will and continue on out to the intersection point. Thus, we see that there are many equilibria with outcomes in a range below the response curves, and each equilibrium is described essentially by mutual expectations as to when they will stop.

The general form of such equilibria can be described as follows. Below the upper envelope of the two response curves there will be a locus of terminal capacity configurations that we indicate by the curve T in the figure. This locus is such that either firm is willing to stop at T if the other does. Then, from the initial capacity configuration each firm invests rapidly until either the locus T is reached, or one firm stops if its capacity reaches the level that

corresponds to its preferred point along the locus  $T$ . In the background is the threat that between  $T$  and the response curves each firm invests because it expects the other to invest, until they reach the response curve (as in the previously described equilibrium); and beyond the response curves, of course, neither firm invests.

The important feature of this analysis is that it is indicative of a general feature of capacity expansion over time. Usually there are multiple equilibria representing a range of terminal capacity levels, and each equilibrium corresponds to mutual expectations among the firms as to when they will stop expansion. The formation of these expectations poses a subtle problem of coordination among the firms. In their own interests they would like the terminal locus  $T$  to be as low as possible, keeping capacity low to restrain output and keep prices high, but this likely outcome of implicit collusion is only precariously maintained by the mutual threat to expand capacity further if either firm steps beyond the anticipated locus  $T$  in a short-sighted attempt to expand its market share.

#### 6. Cooperative Behavior

Cooperative behavior in the absence of enforcement mechanisms often seems inexplicable on the hypothesis that the participants are rational and interested solely in their own personal gain. Nevertheless, it occurs so frequently in practice that it calls for an explanation. Rather than address the subject generally, here we concentrate on a particular simplified situation that captures the main features in stark

form. For historical reasons it is called the repeated prisoners' dilemma game; one can as well think of it as depicting the dilemma faced by two firms trying to sustain high prices. It has been the subject of intensive experimental studies.

In each stage the two players move simultaneously, choosing either to cooperate or not, and each then observes the other's choice. If neither cooperates their payoffs are zero. Each gains 3 from the other's cooperation but it costs the other 1 to compete. Each seeks to maximize the sum of his payoffs over  $n$  repetitions of this stage game.

The only sequentially rational equilibrium of this game has both players choosing not to cooperate every time, regardless of the number of repetitions. At the final stage cooperation is disadvantageous for both players; at the penultimate stage, therefore, there is no chance that cooperation will be reciprocated in the next stage and so again cooperation is disadvantageous; and so on ad infinitum. This sort of reasoning backward from the terminus is familiar from Section 1, and as there it depends on the assumption that the data are common knowledge. We therefore explore what happens when there is some private information.

Suppose that player A entertains a positive probability  $p$ , possibly very small, that B is addicted to the strategy tit-for-tat; that is, that B will cooperate unless A does not, in which case his following action will be not to cooperate. Using tit-for-tat, B initially cooperates and thereafter plays whatever A played on the previous round. There are some good reasons for selecting tit-for-tat

as a candidate. Besides the ancient rules of "an eye for an eye" and "do unto others as they do unto you", there is the interesting fact that in two contests conducted by R. Axelrod tit-for-tat won out. Axelrod [1981] solicited entries in the form of computer routines that were pitted against each other in a round-robin series. That they were round-robin series is rather important: tit-for-tat has the property that it always obtains cumulative payoffs that are close to whatever its opponents obtain. That is tit-for-tat always runs a close second, so if its opponents occasionally fare poorly when matched with each other then overall tit-for-tat will tend to win. Indeed, one can show that among a population of rational players an initial minority of tit-for-tat players will tend to survive and grow, in the Darwinian sense. So, tit-for-tat has survival value.

The main result that we verify is that with this small dose of uncertainty, there is a sequentially rational equilibrium in which both players cooperate for all but a relatively few stages. The operative mechanism is that B, whether a tit-for-tat player or not, imitates tit-for-tat, and with this inducement, so does A. The derivation of this result invokes the following argument that we state intuitively and somewhat imprecisely.

Suppose that B is not addicted to tit-for-tat. We argue that for most of the game B will want to imitate tit-for-tat. The first part is trivial: he surely must punish any noncooperative behavior by A since this yields a short-term advantage and does not alter his reputation that he might be a tit-for-tat player. The key, therefore, is to

establish that B will reciprocate A's cooperation if there are many stages remaining. The argument uses the following inequalities, in which  $A(r,s)$  and  $B(r,s)$  represent A's and B's expected payoffs if they use strategies denoted by  $r$  and  $s$  respectively. Also, let  $t$  represent B's tit-for-tat strategy, and let  $(r^0, s^0)$  be some pair of sequential equilibrium strategies.

1.  $B(r^0, s^0) > B(r^0, t)$  since  $s^0$  is B's optimal response to  $r^0$ .
2.  $B(r^0, t) > A(r^0, t) - 4$ , since by using tit-for-tat is assured of a payoff within  $1 + 3$  of A's payoff, regardless of what strategy A uses. Moreover, when using the strategy tit-for-tat B's expected payoff is the same as the contingent payoff depending on B's type, since both types use tit-for-tat.
3.  $A(r^0, t) > A(r^0, s^0)$ , since B's use of tit-for-tat can only benefit A. This uses the fact that  $s^0$  punishes A for noncooperation to the degree that tit-for-tat does, but possibly  $s^0$  does not reciprocate cooperation to the degree that tit-for-tat does.
4.  $A(r^0, s^0) > 2p[n - 1] - 2$ . If A previously cooperated then by continuing cooperation until B does not A's payoff is at least  $2n$  if B is addicted to tit-for-tat and it is at least  $-1$  otherwise. Similarly, if A previously did not cooperate then B is sure to not cooperate, so with this strategy A gets  $-1$  immediately by cooperating plus a continuation value that is at least the

corresponding amounts above:  $2[n - 1]$  or  $-1$ . Either way the expected payoff, using the initial probability assessment, is at least the stated amount.

Combining these results, we see that B's expected payoff is at least  $2p[n - 1] - 6$  when his reputation is  $p$  and  $n$  stages remain. Consequently, when B considers whether to reciprocate cooperation he calculates as follows. He can choose to not cooperate, yielding an immediate payoff of at most 3 but zero thereafter (since his type is revealed the only equilibrium path thereafter has no cooperation). Or he can cooperate, yielding at least  $-1$  immediately and at least  $2p[n - 2] - 6$  in the remaining  $n - 1$  stages (which he will begin with a reputation that is at least  $p$  if he reciprocates cooperation). Thus, if  $n$  and  $p$  satisfy  $-1 + 2p[n - 2] - 6 > 3$ , or  $n > 2 + 5/p$ , then B's best choice is to reciprocate cooperation.

This result can be further strengthened to provide a bound, independent of the length of the game, on the number of noncooperative episodes. The bound we have derived here is quite weak (e.g., if  $p = .01$  then  $n$  must exceed 502) but it serves to illustrate the main point. Cooperative behavior from both players prevails for all but a relatively few stages of a long game. In fact, in experimental settings it is commonly observed that most players cooperate a substantial proportion of the time.

This equilibrium stems from the basic observation that each player prefers to play with another who uses tit-for-tat, will respond in kind if the actions taken are consistent with tit-for-tat, and will initiate

tit-for-tat if there is some chance the other will play tit-for-tat and there are sufficiently many stages remaining.

What is needed to bootstrap the equilibrium is A's assessment that B might be addicted to tit-for-tat, in which case A is willing to cooperate until it is not reciprocated; in turn, B wants to maintain this belief in A's mind by imitating addiction and thus reaping the long term rewards of cooperation. The first time B fails to follow tit-for-tat he ruins the prospects for further cooperation, while as long as there is a chance that B is addicted, so that even if he is not addicted he will imitate addiction, A's optimal response is to cooperate until it is not reciprocated.

A striking aspect of this game is that a little private information is good for both parties. Even before B knows his type, both players prefer that this information not be revealed after it is known to B. For example, A prefers that B has an opportunity to imitate an addiction to tit-for-tat.

## 7. Conclusion

The thrust of these examples is to emphasize some of the strategic aspects of competition among firms. Competitive battles are inherently dynamic, and they are severely affected by informational asymmetries. Entry, contests for market share, and struggles for survival are versions of economic warfare. In addition, they involve the features of bargaining, albeit in a language restricted to credible actions, in which tenacity reveals the stronger contestant. Wars of attrition serve

the social interest both to select the more efficient firm, and to dissipate monopoly rents in the conflict to obtain or retain them. On the other hand, the threat of price wars can sustain monopoly prices, and capacity limitation and collusive behavior generally can be sustained by expectations of reciprocity. The exercise of monopoly power depends substantially on powers of commitment, but this too can be enforced by irreversible investments in durable equipment.

The complex dynamics of competition among firms with differing information present a far different view than envisaged in the traditional static models with complete information. The recent analyses explore the possibility that economic competition has all the complexity that business strategists ascribe to it: "it's a jungle out there" and only the strong and the wily survive. The development of this approach relies on game-theoretic models to elaborate the motivation for firms' strategies, and to provide well-specified formulations that explain reputational effects in both competitive and collusive contexts.

#### 8. Bibliography

Some of the introductory material on predation in §1 is adapted from Frederic Scherer [1980, Chapter 12]. This is an excellent reference on the empirical evidence, legal considerations, and analytical studies on this subject and several of the subsequent topics. The War Chest game (Example 1.1) is studied by Benoit [1983, 1984], and the Chain Store game (Example 1.2) is studied by Selten [1978]. The apparent contradiction between the complete-information game-theoretic analysis of this

finitely repeated game and the intuitive appeal of predatory behavior led Selten to call it a paradox. It is called the Chain Store game after a scenario in which the incumbent is a chain store with shops in several cities, and in each city there is a potential entrant. The idea for the example in §1.2 is due to Milgrom and Roberts [1982b], and the analysis of the incomplete information version of the Chain Store game in §1.3 is based on Kreps and Wilson [1982]; Milgrom and Roberts [1982b] present a similar model with a continuum of types for both the incumbent and the entrants, and a sequentially rational equilibrium is constructed that uses no randomization.

The example of 'brinkmanship' in §2.1.1 is adapted from Bliss and Nalebuff [1984]. The analysis of wars of attrition in §2.2 is based on Fudenberg and Tirole [1983a].

The analysis of entry and exit in §2.3 follows Kreps and Wilson [1982]. Kreps and Fudenberg [1985] provide several major extensions to cases in which the incumbent faces several entrants simultaneously or in succession, and depending on whether entrants who have exited can re-enter if the incumbent is revealed to be weak. In general, reputational effects persist and the equilibria have roughly the same form but the incumbent may be worse off compared to the situation in which contests are not linked informationally by entrants' ability to see how their compatriots are faring or have fared against the incumbent. For example, assuming no re-entry by exited entrants, in the case of sequential play the weak incumbent continues to fight an early strong entrant in order to avoid revealing weakness to later entrants; consequently, depending

on whether the chance that an entrant is strong is low (below  $2/3$  in our example) or high, the weak incumbent's expected profits per entrant increase or decrease as the number of entrant grows, and total profits are bounded in the latter case since the weak incumbent's type is soon revealed. With simultaneous play, and for simplicity a continuum of entrants, the incumbent fights either all or none depending again on whether this chance is high or low.

The material on limit pricing is adapted from Milgrom and Roberts [1982a] and Saloner [1982]. An alternative formulation is adopted by Roberts [1985b] for the case that in an initial phase after entry the incumbent has superior information about demand conditions and its supply quantity is not observable by the entrant. Thus, as in a predation context (§1), the incumbent drives the price down in order to influence the entrant's decision to exit. In equilibrium the entrant makes the correct inferences so the exit decision is not actually biased, but the entrant's anticipation of this behavior by the incumbent tends to deter entry by reducing expected profits in the initial phase. As in the limit pricing context, the incumbent is forced to lower prices in the initial phase lest he encourage the entrant to stay when in fact it is unprofitable. Fudenberg and Tirole [1985] note that possession of superior information by the incumbent is not actually necessary to this result: even if both firms are equally uncertain about demand conditions, the incumbent has an incentive to lower prices in order to affect the price signal observed by the entrant and thereby to influence the exit decision.

The example of a price war in §4.1 is similar to one devised by Maskin and Tirole [1983, 1985] who develop a wide variety of implications of short-term commitments represented by the firms' alternating actions.

The discussion of the incumbent's defense in §5.1 follows Eaton and Lipsey [1980] and Maskin and Tirole [1983]; and the entrant's gambit in §5.2 follows Gelman and Salop [1983]. The discussion of capacity expansion in §5.3 is adapted from Fudenberg and Tirole [1983b].

The analysis in §6 of the repeated prisoner's dilemma game is based on Kreps, Milgrom, Roberts, and Wilson [1982].

For more comprehensive and detailed surveys than attempted here, but without worked-out examples, and different selections of topics and references, the reader can consult the excellent surveys by Roberts [1985a] and Fudenberg and Tirole [1986].

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