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THE ACCURACY OF TANK MAIN  
ARMAMENTS

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## INTRODUCTION

The authors and numerous other armored systems analysts use accuracy data in their studies, but we have noted that there is no single reference that discusses armored cannon and missile accuracy at a level useful to us. This report is an attempt to remedy that situation.

The report summarizes the available knowledge about gun and missile accuracy that would be of interest to an analyst. Accuracy data can be used four ways:

- A. To compare the accuracy data between systems,
- B. To draw random errors for monte-carlo simulations,
- C. To calculate hit probabilities, and
- D. To find mean dispersion for interpolating in lethality tables.

We discuss and give examples for these last three uses.

This chapter gives background information that applies to both guns and missiles. Chapter 2 discusses the accuracy data produced by The Army Materiel Systems Analysis Activity (AMSAA), at Aberdeen Proving Ground, MD. Chapter 3 tells how that data should be used in a stochastic model, and Chapter 4 shows how to use the data to calculate hit probabilities. Chapter 5 gives a summary of the mathematics used. Appendix A is a more detailed discussion of the components of tank cannon error. Appendix B presents a Fortran program for calculating hit probabilities.

To keep the report unclassified, the data shown in the report is for unidentified obsolete systems or is hypothesized data.

AMSAA produces large volumes of accuracy data for weapons systems. They have published a reference handbook<sup>1</sup> containing those portions of the data most useful to the weapons system analysis community. If you need accuracy data, you should obtain the latest version of these reference documents and then consult AMSAA if you need further data.

AMSAA provides accuracy data based on range for armor systems of the US and USSR. This data comes in the form of tables found in AMSAA's handbooks. A set of tables is made for each of the many different combinations of tank, fire control system, and round type chosen by AMSAA to constitute a firer. The mil, 1/6400th of a circle, is the unit used for these errors. The data is given for stationary-firer versus stationary-target (SS), stationary-firer versus moving-target (SM), and moving-firer versus stationary-target (MS).

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<sup>1</sup> Brewer, Jesse W., et al. "Delivery Accuracy, Range of Fire, and Terminal Effects Handbook Update For Some Large Caliber Armor/Anti-armor Weapon Systems," Number 1, Vol 1: Delivery Accuracy and Rate of Fire, USAMSAA, APG, MD, February 1980.

## Preliminaries

**Aim point.** A target may be totally exposed or partially exposed to any degree, but the most common cases analyzed are for a target that is fully exposed or in hull defilade (only the turret exposed.) In the past, analysts chose the center of the turret ring as the aim point on the fully exposed target. Since tank gunners are now trained to aim at the center of mass, it is generally considered more appropriate to treat the center of mass as the aim point. This is a tedious calculation for an irregular three dimensional shape such as a tank, so a point 1 foot (or 0.3 meters) below the center of the turret ring is often chosen as a reasonable approximation for the center of mass. The approximation is fairly good for a frontal shot, less so for a shot off the front of the target.

For a hull defilade target, the aim point has been taken as a point directly above the center of the turret ring at one half the turret height; again, this is not exactly the center of mass, but analysts commonly treat it as the aim point on the turret.

To model the gunner's aiming process, we must know exactly what the gunner does. For the first round this is quite simple: he lays the cross hairs on the center of mass of the target and pulls the trigger. Now things get complicated, depending on whether the shot is a hit, a sensed miss, or a lost miss.

If the gunner hits the current target and has reason to believe the target is dead or a more dangerous target threatens, he will stop firing at the current target. Otherwise, with a full up system, he will re-lay on the current target and fire again. (He must re-lay because recoil forces knock the cross hairs somewhat away from the desired aim point.)

The reader is cautioned that there are cases where the aim point will be different than we discussed above. This occurs if the previous round has missed the target and the fire control system is degraded. We will discuss this, but a presentation of the mathematics is beyond the scope of this report.

The system is degraded if, for example, the laser ranger is out or the fire control computer is down as indicated by an 'F' in the gunners' primary sight.

If the prior round missed the target, either the gunner or the tank commander may sense the impact location and a sensed miss has occurred; if neither senses then a lost miss has occurred.

How often is a miss sensed? It depends on the type of round. Tankers almost never sense KE misses but can often sense HEAT round misses. The impact point of a KE miss is almost impossible to detect<sup>2</sup> because no explosion occurs and the time of flight is so short that muzzle smoke and dust obscuration usually haven't dissipated

---

<sup>2</sup> Tankers disagree with this; they say a short (low) KE round can sometimes be seen if it kicks up dirt. So they can use the burst-on-target technique on a sensed miss and drop a half mil on a lost miss just as they would with a HEAT round.

before impact occurs. The impact point of a HEAT miss is easier to detect because of the explosion at impact, and because the time of flight is longer so more of the muzzle smoke and dust has dissipated before impact. The tank commander senses misses more often than the gunner because his line of sight is farther off line from the gun tube than is the gunner's so smoke and dust interrupts his line of sight less.

If the fire control is degraded and the prior round was a sensed miss, the gunner may attempt to move the reticle so that the point on the reticle at which he observed the burst of the prior round now lies on the target at the desired aim point. This is called the burst-on-target technique.

If the fire control is degraded and the prior round was a lost miss, the gunner may attempt to move the reticle down a half mil. The assumption is that the prior round was not sensed because it flew over the target and landed beyond with the target blocking the line of sight to the impact point.

Converting to linear error. The angular errors from the AMSAA tables must be converted to linear errors to find miss distances or hit probabilities. The angular errors are first converted from mils to radians using the following equation:

$$x_{\text{radians}} = x_{\text{mils}} 2\pi / 6400$$

Then the angular error in radians is converted to a linear error in meters using the following:

$$x_{\text{meters}} = r \operatorname{atan}(x_{\text{radians}}) \approx r x_{\text{radians}}$$

Where  $r$  is the target range in meters, and the approximation is good at small angles.

*The rule of thumb is that a mil of error produces a meter of error per kilometer of range. More accurately:*

$$x_{\text{meters}} = 0.9817 r_{\text{km}} x_{\text{mils}}$$

Kinds of error. All errors may be divided into fixed biases  $\mu$ , variable biases  $\nu$ , and random errors  $\sigma$  as shown in Figure 1. The  $v_i$  vectors are random draws from  $\nu$ , and the  $s_i$  vectors are random draws from  $\sigma$ . Variable biases only occur when a stationary gun fires at a stationary target, while fixed biases and random errors are found in all situations. Each of these will be discussed later for specific situations.

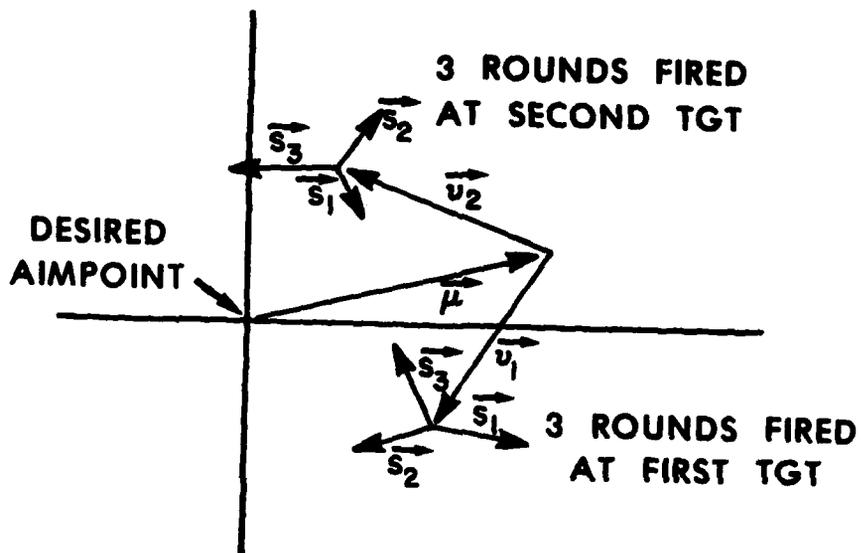


Figure 1. Kinds of Errors

## TANK CANNON ACCURACY DATA

The tank cannon accuracy data that we will describe comes from AMSAA. AMSAA's model for tank gun accuracy has two main assumptions: independence and normality. The first assumption means that the individual errors which make the error budget are assumed not to affect each other. That is, the error due to wind does not affect the parallax error. Also the horizontal and vertical components of the errors are assumed to be independent, meaning that the cannon shooting too high or low has nothing to do with shooting too far to the left or right. The second assumption simply says that all errors of the error budget follow a normal distribution. This allows the standard deviation of the distributions of the individual errors to be root summed squared together to find the standard deviation of the distribution of the total error. (Assuming no error dominates.)

AMSAA divides errors into three types: fixed biases, variable biases, and random errors. Fixed biases are errors which are constant for a given range and target speed. An example of a fixed bias is parallax. Variable biases are errors which remain constant for a particular occasion, that is, a particular place/time set. Cant is a variable bias if a tank remains in the same position. Random errors are errors which change with every shot. An example is lay error.

In addition, AMSAA defines two other errors: dispersion and moving firer add-on dispersion. Dispersion error is simply the combination of the variable bias and the random error. Moving firer add-on dispersion errors are errors associated with the base motion of a moving firer.

The numbers in the AMSAA handbook tables titled "Random Error", "Variable Biases", "Total Dispersion", and "Moving Firer Add-on Dispersions" are the standard deviations of their respective error distributions. The actual errors must be drawn from these distributions. On the other hand, the numbers labeled "Fixed Biases" are the actual fixed bias errors. One other note, the probability of hit numbers are for the NATO standard 2.3m X 2.3m square target.

After working in the field, one sees that terms such as "random error" and "variable bias" can have several meanings. For example, a "variable bias" could be a description of an error, as in, "Wind is a variable bias." The "variable bias" could be the standard deviation of the variable bias distribution such as is the case for the AMSAA tables. In addition, "variable bias" could be a draw from the variable bias distribution: this draw being used to simulate a shot fired from a tank. We point out this problem of language but make no attempt to correct it.

The following discusses how to use the AMSAA data available for different scenarios.

### Stationary Firer Versus Stationary Target

Table 1 shows typical accuracy data for the first shot at a target. It's in the usual AMSAA format and gives fixed biases, variable biases, and random errors both horizontally and vertically. It also gives total dispersion (the root sum square of the variable bias and the random error.) The table contains enough information to generate stochastic errors for the first and subsequent rounds. The information is also sufficient to calculate hit probabilities for the first round but not for subsequent rounds.

AMSAA provides subsequent round accuracy data which may be of interest if you wish to calculate hit probabilities for those SS cases.

**TABLE 1. First Round Accuracy  
Stationary Firer vs Stationary Target**

First Round Biases, Dispersions, and First Round Probability of Hit									
Range (Meters)	Fixed Biases (mils)		Horizontal (mils)			Vertical (mils)			$P_{H1}$
	Horizontal	Vertical	Random Error	Variable Biases	Total Dispersion	Random Error	Variable Biases	Total Dispersion	
250	1.072	0	1.3702	.5728	1.4272	1.3702	.6284	1.4504	.9927
500	.357	0	.7260	.6940	1.0043	.7260	.8572	1.1233	.9343
1000	.000	0	.4652	1.1345	1.2262	.4652	1.8468	1.9045	.3019
1500	-.119	0	.2929	1.7860	1.8287	.3929	3.4496	3.4719	.0580
2000	-.178	0	.3621	2.6669	2.6914	.3621	6.2610	6.2715	.0126
2500	-.214	0	.3459	3.8062	3.8757	.3459	11.1232	11.1286	.0032
3000	-.238	0	.3362	5.4729	5.4832	.3362	19.1972	19.2001	.0009

### Stationary Firer Versus Moving Target

Table 2 shows typical data for a stationary tank firing at a moving target. In this case, accuracy is a function of range, target speed, crossing angle and evasiveness. Such tables contain the bias and dispersion errors as well as the probability of hit for targets with speeds of 2, 10, 20, 30, and 40 km/hr.

TABLE 2. Stationary-Firer vs. Moving-Target Accuracy

Stationary Firer vs. Moving Target								
Evasive Factor = .25								
Target Crossing Direction = counterclockwise								
Target Crossing Angle = 0 degrees								
Bias and Dispersion in mils								
Target Speed (KPH)		ACCURACY DATA AS A FUNCTION OF RANGE (METERS)						
		250	500	1000	1500	2000	2500	3000
2	H BIAS	1.0547	.3194	-.0903	-.2843	-.4510	-.6415	-.8751
	V BIAS	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	H DISP	.6466	.7561	1.1735	1.8110	2.6837	3.8718	5.4811
	V DISP	.6994	.9107	1.8725	3.4636	6.2689	11.1277	19.1999
	P(H)	1.000	.9853	.3156	.0581	.0125	.0032	.0009
10	H BIAS	.7800	-.2740	-1.4549	-2.5467	-3.5063	-3.8250	-3.2910
	V BIAS	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	H DISP	.6472	.7566	1.1739	1.8114	2.6840	3.8721	5.4983
	V DISP	.7693	.9702	1.9082	3.4875	6.2855	11.1399	19.2091
	P(H)	1.000	.9801	.1784	.0231	.0055	.0020	.0008
20	H BIAS	.7794	-.2785	-1.5177	-2.8526	-4.5495	-6.6631	-8.8527
	V BIAS	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	H DISP	.6490	.7583	1.1752	1.8124	2.6849	3.8729	5.4821
	V DISP	.9552	1.1361	2.0154	3.5609	6.3371	11.1775	19.2378
	P(H)	1.000	.9560	.1617	.0178	.0031	.0007	.0002
30	H BIAS	.7793	-.2806	-1.5296	-2.9125	-4.7684	-7.3413	-10.6929
	V BIAS	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	H DISP	.6521	.7611	1.1774	1.8142	2.6864	3.8742	5.4833
	V DISP	1.2028	1.3685	2.1824	3.6799	6.4129	11.2398	19.2852
	P(H)	.9999	.9076	.1491	.0164	.0026	.0005	.0001
40	H BIAS	.7792	-.2809	-1.5338	-2.9337	-4.8470	-7.5917	-11.4060
	V BIAS	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	H DISP	.6563	.7650	1.1805	1.8166	2.6885	3.8761	5.4851
	V DISP	1.4814	1.6393	2.3965	3.8400	6.5387	11.3262	19.3512
	P(H)	.9983	.1411	.1365	.0155	.0025	.0005	.0001
H - HORIZONTAL								
V - VERTICAL								
P(H) - PROBABILITY OF HIT AGAINST A 2.3M X 2.3M VERTICAL MOVING TARGET								

This particular table is for a target making a counterclockwise circular turn with an evasive factor of .25. (An evasiveness factor of one is 0.7 G's or 6.88 m/s<sup>2</sup>.) Refer to Figure 2.

The horizontal biases in Table 2 are calculated assuming that the target begins a circular evasive maneuver (either clockwise or counter-clockwise) at the moment of firing. That is, part or all of the horizontal fixed biases are target induced. It is not clear to the authors that the target will take such maneuvers, and it is certain that the target will not begin to maneuver at the instant it is fired upon due to the reaction time of the crew and the ability of the crew to detect the shot.

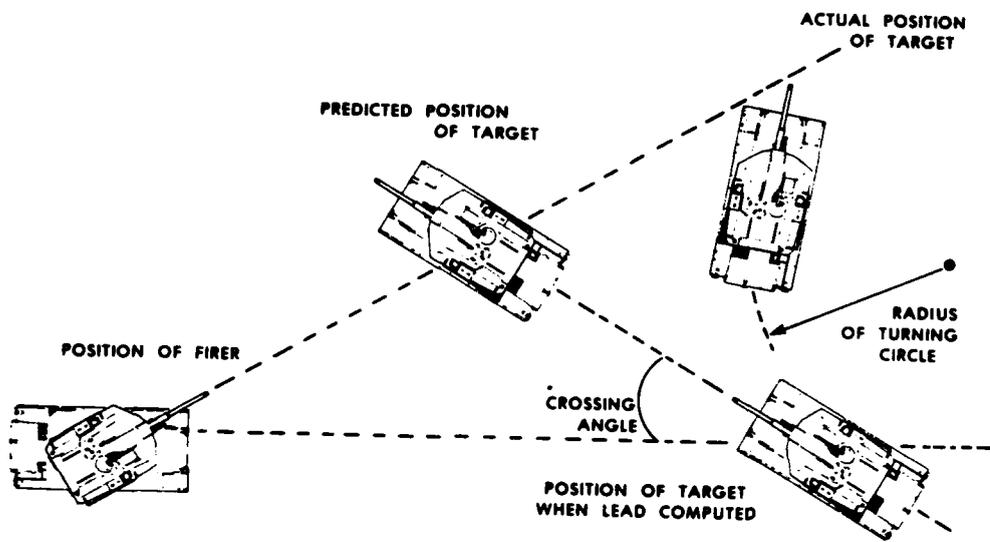


Figure 2. Error in Using Linear Predictor for Accelerating Target.

AMSAA accuracy experts have stated that normal motion of a tank to avoid rocks, pits, and other minor obstacles generates biases roughly equal to the biases for 0.25 evasiveness factor. Their handbook contains only SM accuracy data for this level of evasiveness. For data at other levels of evasiveness, they must be contacted directly.

To select the appropriate subset of the SM data, one must know how fast a tank will move during combat. If firepower killed, it will probably move at maximum cross country speed seeking cover. But when firing, it may move much slower to maintain accuracy. Speeds of 10 and 20km/hr have been used in simulating tanks firing on the move.

#### Moving Firer Versus Stationary Target

Table 3 contains "add-on dispersion" for moving firers. These dispersions are to be added to the horizontal and vertical dispersions of the stationary firer versus stationary target errors. The AMSAA handbook contains MS tables for various tank systems. The tables have add-on dispersions for the firer traveling at speeds of 4, 8, 12, 16, 20, 24, 32, and 40 KPH, and for six terrain types. The terrain types are level farmland meadows, fields with overpass roads, frozen plowed fields with crossings, rolling meadows, stony farmland with crossings, and heavily used tank roads. The total error for this scenario is

$$\text{Total Error} = \mu + \sqrt{\sigma^2 + \nu^2 + A^2}$$

where  $\mu$ ,  $\sigma$ , and  $\nu$  are the fixed bias, variable bias, and random error, respectively, from the SS table, and  $A$  is the add-on dispersion from the MS table.

TABLE 3. Add-on Dispersions For Moving Firers

MOVING FIRER ADD-ON DISPERSION(mils) ESTIMATE												
Velocity (KPH)	TT I		TT II		TT III		TT IV		TT V		TT VI	
	H	V	H	V	H	V	H	V	H	V	H	V
4	.40	.40	.49	.61	4.8	6.0	.40	.40	8.5	10.6	.40	.49
8	.40	.40	.49	.61	-	-	.40	.40	-	-	.84	1.04
12	.40	.49	.78	.97	-	-	.40	.40	-	-	.83	1.04
16	.45	.56	1.15	1.44	-	-	.43	.66	-	-	1.75	2.20
20	.54	.67	4.30	5.50	-	-	.91	1.14	-	-	-	-
24	.76	.95	-	-	-	-	1.45	1.80	-	-	-	-
32	1.70	2.10	-	-	-	-	10.9	13.7	-	-	-	-
40	-	-	-	-	-	-	-	-	-	-	-	-

H = HORIZONTAL  
V = VERTICAL  
TT = TERRAIN TYPE  
TERRAIN TYPES:  
I - Level farmland meadows  
II - Field with overpass roads  
III - Frozen plowed fields with crossings  
IV - Rolling meadows  
V - Stony farmland with crossings  
VI - Heavily used tank road

Missile Accuracy Data

Table 4 shows typical missile accuracy data available from AMSAA. Notice that the random errors and variable biases are undefined. The data available is for stationary firers since missile systems must generally halt to fire. A single table can be used for both stationary and moving targets as well as first and subsequent shots. This is because missiles, unlike cannon rounds, can be corrected in flight.

TABLE 4. MISSILE ACCURACY DATA

First and Subsequent Round Biases, Dispersions, and First Round Probability of Hit										
Range (Meters)	Fixed Biases (mils)		Horizontal (mils)			Vertical (mils)			$P_{H1}$	
	Horizontal	Vertical	Random Error	Variable Biases	Total Dispersion	Random Error	Variable Biases	Total Dispersion		
250	.727	.431			1.893			1.098	.70	
500	.378	.215			.946			.542	.68	
1000	.189	.108			.473			.274	.66	
1500	.126	.072			.316			.183	.66	
2000	.095	.054	Not Available			Not Available			.137	.66
2500	.076	.043			.189			.110	.67	
3000	.063	.036			.158			.092	.68	
3750	.051	.029			.128			.074	.68	

## SIMULATING ERRORS STOCHASTICALLY

Knowing the error distributions, how does one find the position of the round as it passes through the target plane? The proper method depends on whether the firer or target is moving and whether the round is a missile or a ballistic round. Note that the methods presented here must be modified to treat burst fire, a subject in it's own right.

We assume that the horizontal and vertical errors are independently normally distributed, so the methods of finding the horizontal position and the vertical position are identical, except that different distributions are used.

Subsequent discussion uses the following notation:

$\sigma$	random error
$\nu$	variable bias
$\mu$	fixed bias
$A$	add on dispersion for moving firer
$s$	draw from random error distribution
$v$	draw from variable bias distribution
$a$	total angular error
$l$	total linear error
$r$	range
$[N]$	a draw from the Standard Normal Distribution.

Square brackets around values is a reminder that a new random value is chosen for each occurrence. An 'x', or 'y' subscript will denote a horizontal or vertical error respectively.

The method of finding errors stochastically is as follows:

1. randomly draw a variable bias,
2. randomly draw a random error for each shot,
3. sum these and the fixed bias,
4. convert to linear error.

The appropriate equations are:

$$\begin{aligned}v &= [N]\nu \\s &= [N]\sigma \\a &= \mu + v + s \\l &= 0.9817ra\end{aligned}$$

### Stationary Firer Versus Stationary Target

To simulate a shot, find the fixed biases, variable biases, and random errors (interpolating if necessary) from SS tables of first round errors similar to Table 1. For a target at 500 meters, the fixed biases, variable biases, and dispersions are:

$$\begin{aligned}\mu_x &= 0.357 \text{ mils} \\ \mu_y &= 0.0 \text{ mils} \\ \nu_x &= 0.6940 \text{ mils} \\ \nu_y &= 0.8572 \text{ mils} \\ \sigma_x &= 0.7260 \text{ mils} \\ \sigma_y &= 0.7260 \text{ mils}\end{aligned}$$

Next, draw random values from the standard normal distribution and multiply by the standard deviation of the variable biases to produce randomly drawn variable biases. These two values will not change while shots are being fired at a single target, but will change when a new target is chosen.

$$\begin{aligned}v_x &= [N]\nu_x = [0.502](0.6940) = 0.3484 \text{ mils} \\ v_y &= [N]\nu_y = [-0.347](0.8572) = -0.2974 \text{ mils}\end{aligned}$$

Once the specific variable biases are determined, the random errors may be found. They must be recomputed for each shot at a target. They are randomly drawn as follows:

$$\begin{aligned}s_x &= [N]\sigma_x = [-1.114](0.7260) = -0.8088 \text{ mils} \\ s_y &= [N]\sigma_y = [0.086](0.7260) = 0.0624 \text{ mils}\end{aligned}$$

Next, combining the fixed biases, specific variable biases, and specific random errors, find the actual angular errors for the first shot.

$$\begin{aligned}a_x &= \mu_x + v_x + s_x = 0.357 + 0.3484 - 0.8088 = -0.1034 \text{ mils} \\ a_y &= \mu_y + v_y + s_y = 0.0 - 0.2974 + 0.0624 = -0.2350 \text{ mils}\end{aligned}$$

Finally, convert to linear error using the approximation given earlier:

$$\begin{aligned}l_x &= 0.9817ra_x = 0.9817 \cdot 0.5 \cdot -0.1034 = -0.0508 \text{ meters} \\ l_y &= 0.9817ra_y = 0.9817 \cdot 0.5 \cdot -0.2350 = -0.1153 \text{ meters}\end{aligned}$$

The first round is 0.05 meters to the left and 0.12 meters below the aim point.

For the second and subsequent shots at this target, only the random errors are redrawn.

### Stationary Firer Versus Moving Target

To simulate single rounds, interpolate for range and speed in a table of data such as is shown in Table 2. For example, shots fired at a target moving at 10 km/hr at 500 meters range have the following fixed biases and dispersions:

$$\begin{aligned}\mu_x &= -0.2740 \text{ miles} \\ \mu_y &= -0.0 \text{ miles} \\ \sigma_x &= 0.7566 \text{ miles} \\ \sigma_y &= 0.9702 \text{ miles}\end{aligned}$$

To draw errors for a single shot, do the following:

$$\begin{aligned}a_x &= \mu_x + [N]\sigma_x = -0.2740 + [0.43] \times 0.7566 = 0.0514 \\ a_y &= \mu_y + [N]\sigma_y = 0.0 + [-1.62] \times 0.9702 = -1.5717 \\ l_x &= 0.9817 \times 0.5 \times 0.0514 = 0.0252 \\ l_y &= 0.9817 \times 0.5 \times -1.5717 = -0.7715\end{aligned}$$

The round is 0.03 meters to the right of the aim point and 0.77 meters below it.

### Moving Firer Versus Stationary Target

Random deviates from the appropriate add-on dispersions are added to the deviates for the stationary-firer versus stationary target case to find the total dispersions for the moving firer.

Take the case of a firer moving 16 km/hr across a rolling meadow while firing at a target 500 meters away. From Table 1, we have:

$$\begin{aligned}\mu_x &= 0.357 \text{ miles} \\ \mu_y &= 0 \text{ miles} \\ \nu_x &= 0.6940 \text{ miles} \\ \nu_y &= 0.8572 \text{ miles} \\ \sigma_x &= 0.7260 \text{ miles} \\ \sigma_y &= 0.7260 \text{ miles}\end{aligned}$$

From Table 3 (terrain type IV) we have:

$$\begin{aligned}A_x &= 0.43 \\ A_y &= 0.66\end{aligned}$$

Root sum squaring the appropriate values we have:

$$\sigma'_x = \sqrt{\nu_x^2 + \sigma_x^2 + A_x^2} = 1.091 \text{ miles}$$

$$\sigma'_y = \sqrt{\nu_y^2 + \sigma_y^2 + A_y^2} = 1.303 \text{ miles}$$

$$\begin{aligned}
a_x &= \mu_x + [M]\sigma_x' = 0.357 + [0.912] \times 1.091 = 1.3520 \text{ miles} \\
a_y &= \mu_y + [M]\sigma_y' = 0.0 + [-0.284] \times 1.303 = -0.3701 \text{ miles} \\
l_x &= 0.9817 \times 0.5 \times 1.3520 = 0.6636 \text{ meters} \\
l_y &= 0.9817 \times 0.5 \times -0.3701 = -0.1817 \text{ meters}
\end{aligned}$$

The round strikes 0.66 meters right of the aim point and 0.18 meters below.

### Stationary Missile Firing Versus Stationary or Moving Target.

Successive shots at a stationary or moving target are independent and may be treated identically. The procedure is the same as for S-M cannon fire; interpolate in the table to find the angular errors at the appropriate range, then draw the errors as follows:

$$\begin{aligned}
a_x &= \mu_x + [M]\sigma_x = 0.378 + [0.461](0.946) = 0.8141 \text{ miles} \\
a_y &= \mu_y + [M]\sigma_y = 0.215 + [-0.306](0.542) = -0.0491 \text{ miles}
\end{aligned}$$

Next convert to linear error:

$$\begin{aligned}
l_x &= 0.9817 \times 0.5 \times 0.8141 = 0.3996 \text{ meters} \\
l_y &= 0.9817 \times 0.5 \times -0.0491 = -0.0241 \text{ meters}
\end{aligned}$$

The round strikes 0.40 meters to the right of the aim point, and 0.02 meters below it.

Armor systems firing missiles must generally halt to fire. The treatment of missiles fired from a moving platform may require a different methodology from that of this section.

## CALCULATING HIT PROBABILITY

If the target is a rectangle with edges parallel to the horizontal and vertical, the hit probability is the product of two integrals, as given below. The first integral is the probability that the round is within the left and right boundaries of the target, and the second is the probability that the round is within the lower and upper boundaries of the target. The limits of integration are the coordinates of the edges of the target. These limits are relative to the aim point and are expressed in 'sigmas'.

$$P_{hit} = \left[ \frac{1}{\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-x^2/2} dx \right] \left[ \frac{1}{\sqrt{2\pi}} \int_{y_1}^{y_2} e^{-y^2/2} dy \right]$$

Where,

$$\begin{aligned} x_1 &= (x_{left} - \mu_x) / \sigma_x \\ x_2 &= (x_{right} - \mu_x) / \sigma_x \\ y_1 &= (y_{lower} - \mu_y) / \sigma_y \\ y_2 &= (y_{upper} - \mu_y) / \sigma_y \end{aligned}$$

The integrals may be evaluated manually or with a computer program such as the one shown in Appendix B. (Programs are also available for calculating hit probabilities on other than rectangular targets.) To illustrate the proper method, we will use the computer program. It assumes the target is represented by a turret rectangle centered on top of a hull rectangle, and calculates the probability of hitting a) the upper box, b) either box, and c) a NATO standard 2.3 by 2.3 meter box. The upper-box-only case represents a hull deflade target. Only the turret is exposed, and the aim point is assumed to be the center of the upper box. The two box case represents a fully exposed target, and the aim point is assumed to be a point 0.3 meters below the common edge of the boxes. The third case assumes the aim point is at the center of the NATO standard rectangle.

Note that the methods presented here must be modified to treat burst fire, a subject in it's own right.

To illustrate the methods used, we will assume the target is represented by a turret box and a hull box with the following dimensions:

Turret height 0.8 meters,  
Turret width 2.0 meters,  
Hull height 1.4 meters,  
Hull width 3.2 meters

### Stationary Firer Versus Stationary Target First Round

To find hit probability, find the fixed biases, and total dispersion from SS tables of first round errors similar to Table 1. For a target at 500 meters, the fixed biases and total dispersions are:

$$\begin{aligned}\mu_x &= 0.357 \text{ mils} \\ \mu_y &= 0.0 \text{ mils} \\ \sigma_x' &= 1.0043 \text{ mils} \\ \sigma_y' &= 1.1233 \text{ mils}\end{aligned}$$

The input to the program is:

0.8,2.0	turret height, width (m)
1.4,3.2	hull height, width (m)
0.5,1.0043,1.1233,0.357,0.0,	$r, \sigma_x, \sigma_y, \mu_x, \mu_y$

The program produces the following hit probabilities:

- a) Phit=0.52 for hull deflade
- b) Phit=0.74 for fully exposed
- c) Phit=0.94 for the standard NATO target.

The calculation of subsequent round hit probabilities is a more difficult matter. AMSAA generates tables of subsequent round accuracies and hit probabilities, however, the most accurate way to calculate subsequent round hit probabilities is by Peterson's method.<sup>3</sup>

### Stationary Firer Versus Moving Target

Finding hit probabilities on a moving target is similar to finding hit probabilities on a stationary target. For the target sizes used above, where the target is moving at 10 km/hr and is at 500 meters range, the appropriate values from table 2 are:

$$\begin{aligned}\mu_x &= -0.2740 \text{ mils} \\ \mu_y &= 0.0 \text{ mils} \\ \sigma_x &= 0.7566 \text{ mils} \\ \sigma_y &= 0.9702 \text{ mils}\end{aligned}$$

The errors will be larger than if the target was stationary, but there will be no variable bias so all shots are treated as first round shots.

<sup>3</sup> Peterson, Richard H. "Hit Probabilities Associated with Two Successive Rounds". BRL MR1714. June 1966.

If these values are used as input to the program as before, the program will produce the following hit probabilities:

- a)  $Phit=0.66$  for hull defilade
- b)  $Phit=0.86$  for fully exposed
- c)  $Phit=0.98$  for the standard NATO target.

#### Moving Firer Versus Stationary Target

The sigma values to be used in this case are a composite of the SS total dispersions from Table 1 and the MS add-on dispersions from Table 3. In the previous chapter, for the case of a firer moving across a rolling meadow at 16 km/hr and firing at a target 500 meters away, the values were found to be:

$$\begin{aligned}\mu_x &= 0.357 \text{ mils} \\ \mu_y &= 0.0 \text{ mils} \\ \sigma_x &= 1.091 \text{ mils} \\ \sigma_y &= 1.303 \text{ mils}\end{aligned}$$

If these values are used as input to the program as before, the program will produce the following hit probabilities:

- a)  $Phit=0.46$  for hull defilade
- b)  $Phit=0.70$  for fully exposed
- c)  $Phit=0.89$  for the standard NATO target.

#### Stationary Missile Firer Versus Stationary or Moving Target

The calculation for this case is identical to the calculation for the SM case above. For a target at 500 meters, the appropriate values from Table 4 are:

$$\begin{aligned}\mu_x &= 0.378 \text{ mils} \\ \mu_y &= 0.215 \text{ mils} \\ \sigma_x &= 0.946 \text{ mils} \\ \sigma_y &= 0.542 \text{ mils}\end{aligned}$$

If these values are used as input to the program as before, the program will produce the following hit probabilities:

- a)  $Phit=0.57$  for hull defilade
- b)  $Phit=0.78$  for fully exposed
- c)  $Phit=0.98$  for the standard NATO target.

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## SUMMARY

Obtain the AMSAA accuracy handbook; if you need more data consult AMSAA.

To convert angular error to linear error use the following:

$$l = 0.9817ar$$

Where:

**a** is the angular error in mils,  
**r** is the range in kilometers, and  
**l** is the linear error in meters.

### Simulating Errors Stochastically.

After finding the appropriate linear errors, draw individual errors as follows:

S-S Cannon first round

$$v_s = [N]v_s$$
$$l_s = \mu_s + v_s + [N]\sigma_s$$

S-S Cannon subsequent rounds

$$l_s = \mu_s + v_s + [N]\sigma_s$$

S-M Cannon all rounds

$$l_s = \mu_s + [N]\sigma_s$$

M-S Cannon all rounds

$$\sigma'_s = \sqrt{\sigma_s^2 + v_s^2 + A_s^2}$$
$$l_s = \mu_s + [N]\sigma'_s$$

S-S, S-M Missiles

$$l_s = \mu_s + [N]\sigma_s$$

### Calculating Hit Probabilities.

To find hit probabilities, you can either calculate by hand using the method below, or use a program such as the one in appendix B.

After finding the appropriate linear errors, find hit probabilities on a rectangular target bounded horizontally by  $x_1, x_2$ , and vertically by  $y_1, y_2$  as follows:

$$\begin{aligned}x_1' &= (x_1 - \mu_x) / \rho_x \\x_2' &= (x_2 - \mu_x) / \rho_x \\y_1' &= (y_1 - \mu_y) / \rho_y \\y_2' &= (y_2 - \mu_y) / \rho_y\end{aligned}$$

$$P_x = \frac{1}{\sqrt{2\pi}} \int_{x_1'}^{x_2'} e^{-x'^2/2} dx'$$

$$P_y = \frac{1}{\sqrt{2\pi}} \int_{y_1'}^{y_2'} e^{-y'^2/2} dy'$$

$$P_{hit} = P_x P_y$$

Where, for

S-S Cannon first round  $\rho_x = \sqrt{\sigma_x^2 + \nu_x^2}$ .

S-S Cannon subsequent rounds use Peterson's method.

S-M Cannon  $\rho_x = \sigma_x$ .

M-S Cannon  $\rho_x = \sqrt{\sigma_x^2 + \nu_x^2 + A_x^2}$ .

S-S, S-M Missiles  $\rho_x = \sigma_x$ .

The variable bias for the subsequent rounds is not normally distributed. For calculating the hit probability of subsequent rounds use Peterson's method.

#### Finding Mean Dispersion.

For all situations, the mean dispersion is  $\rho = \sqrt{(\rho_x^2 + \rho_y^2)/2}$ , where  $\rho$  is as defined above.

## ACRONYMS AND SYMBOLS

<b>AMSAA</b>	<b>Army Materiel Systems Analysis Activity (APG, MD)</b>
<b>BOT</b>	<b>Burst On Target (aiming technique)</b>
<b>BRL</b>	<b>Ballistics Research Laboratory (APG, MD)</b>
<b>GAS</b>	<b>Gunner's Auxiliary Sight</b>
<b>GCI</b>	<b>Group Center of Impact</b>
<b>GPS</b>	<b>Gunner's Primary Sight</b>
<b>HEAT</b>	<b>High Explosive Anti-Tank round</b>
<b>IUA</b>	<b>Independent Unit Action (lethality table)</b>
<b>KE</b>	<b>Kinetic Energy round</b>
<b>MS</b>	<b>Moving-firer Stationary-target</b>
<b>PAT</b>	<b>Precision Aim Technique</b>
<b>SABOT</b>	<b>Kinetic energy round with sabot</b>
<b>SM</b>	<b>Stationary-firer Moving-target</b>
<b>SS</b>	<b>Stationary-firer Stationary-target</b>
<b>TC</b>	<b>Tank Commander</b>
<b>A</b>	<b>Add-on error for a moving firer</b>
$a_x$	<b>Horizontal angular error</b>
$l_x$	<b>Horizontal linear error</b>
<b>N</b>	<b>A random draw from the standard normal distribution</b>
<b>s</b>	<b>A specific random error</b>
<b>v</b>	<b>A specific variable bias</b>
$\mu$	<b>Fixed bias</b>
$\nu$	<b>Standard deviation of the variable bias</b>
$\sigma$	<b>Standard deviation of the random error</b>

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## APPENDIX A

### CLASSES AND SOURCES OF GUN ERROR

The components of tank cannon error are generally divided into three categories for a stationary firer versus a stationary target. In addition to these errors, there are errors due to the motion of a target, and errors due to the motion of the firer. We discuss the stationary-stationary errors and those due to the target motion but not those due to firer motion.

The three categories for the stationary-stationary case are: fixed biases, variable biases, and random errors. Fixed biases are errors which are usually caused by the weapon, the ammunition, or the fire control system design. These errors are constant and predictable at any range for all firing occasions where an occasion is a particular place/time set.<sup>4</sup> All other things being equal, they will remain the same even when a gunner switches targets. Variable biases are errors which remain nearly constant during a particular firing occasion, but which vary considerably from occasion to occasion.<sup>5</sup> If the gunner switches to a new target the variable biases change. Random errors are errors whose magnitude and direction vary considerably from round-to-round even during a single occasion. Fixed biases can be treated as either individual errors or as an aggregated set of errors which are divided into horizontal and vertical components. Variable biases are aggregated into horizontal and vertical components. Random errors are usually treated individually.<sup>6</sup> Random errors are errors that change from round to round. We will now look at the individual errors which make up these three types of errors.

#### Fixed Biases

Fixed bias errors can be divided into parallax, drift, and mean jump.

Parallax. The optics of a tank are offset from the gun barrel horizontally and to a lesser extent vertically. If the gunner has accurately zeroed the gun, the optical axis will cross the gun axis at 1200 meters range, that being the standard zeroing range. The first error, parallax, is the angular difference between the axis of the gun barrel and the axis of the line of sight of what the gunner sees when looking at the target.

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<sup>4</sup> Goulet, Bernard N. *User and Analyst Manual For a FORTRAN Computer Program Simulating the Engagement of a Stationary Point Target By a Stationary Direct Fire Weapon*, Tech Report 923, December 1980, p. 11.

<sup>5</sup> Goulet, p.11.

<sup>6</sup> Goulet, p. 11.

A round fired will have parallax error unless the target is at the point of parallax (or unless the fire control is capable of correcting this). The point of parallax is where the gun axis and the gunner's lines of sight axis cross as shown in Figure 3. The linear error in the y direction is found by the following equation:

$$le_y = \frac{x_z - x_t}{x_z} d,$$

where

$x_t$  = distance between firer and target

$x_z$  = zeroing range (where gun axis and line of sight cross)

$d$  = distance between gun and optics

$le_y$  = linear error distance between target and round at target distance.

Assuming<sup>7</sup>  $x_z = 1200\text{m}$ , then

If  $d=1\text{m}$  and  $x_t = 600\text{m}$  then  $le_y = .5\text{m}$  to the left

If  $d=1\text{m}$  and  $x_t = 1200\text{m}$  then  $le_y = 0$

If  $d=1\text{m}$  and  $x_t = 2400\text{m}$  then  $le_y = 1\text{m}$  to the right

**Drift.** The next error, drift, is caused by rifled barrels usually found on older tanks. The rifling causes the projectile to spin for stabilization, but it also causes the rounds to "claw" to one side or the other. Spin causes drift because of an aerodynamic effect known as the over turning moment. If there is no spin, then the over turning moment is seen as the tail trying to move past the nose. But when spin is induced, instead of the tail moving past the nose, the whole projectile moves. There are components of drift in both the vertical and the horizontal directions. If the spin is clockwise as viewed from the rear, the projectile will move toward the right, and conversely if the spin is counter-clockwise as viewed from the rear, the projectile will move to the left. (American guns are rifled clockwise, while British guns are rifled counter-clockwise.)

**Mean Jump.** The last error, mean jump, is the angular deviation from where the group center of impact, GCI, is expected. (The GCI is the point found by taking the average of the vertical and the average of the horizontal coordinates of a set of impact points of rounds fired at a target by a tank on an occasion.) This jump is caused by unknown factors, but it may be predictable for a class of rounds, and if so, this predictable jump is called mean jump. Mean jump can be "zeroed out" for an individual weapon by firing several rounds at a target and then adjusting the cross hairs of the sight so that they lay on the GCI of the rounds just fired. Jump differs among different types of ammunition.<sup>8</sup>

<sup>7</sup> *Boresighting is a passive method of 'zeroing', while zeroing involves the firing of rounds. For KE rounds, zeroing is often done at 1200 meters, but this is scenario dependent. For Heat rounds 900 meters is sometimes used.*

<sup>8</sup> *Sissom, B.D., "Comments on Tank Weapon System Accuracy", Special Publication ARBRL-SP-00018, September 1980, p.9.*

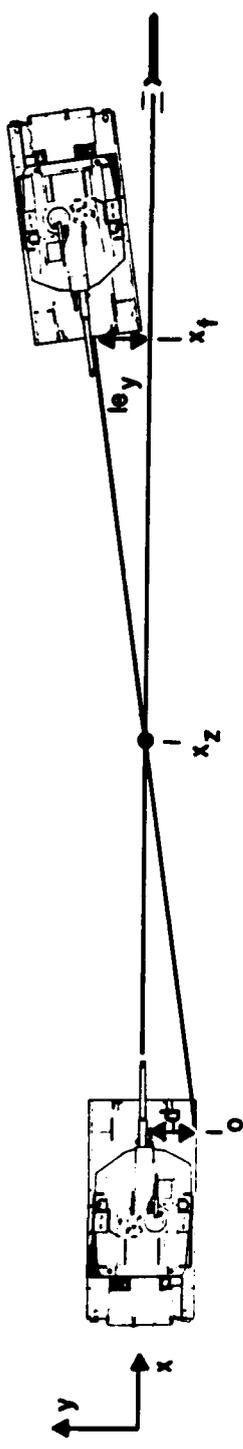


Figure 3. Error Due to Parallax

## Variable Biases

Variable biases contain the largest number of sources of error. The particular variable biases used in an error budget vary. The following variable biases are from a conglomerate of different error budgets.

Cant. Cant is the tilting angle of the gun tube. Cant exists if the axis perpendicular to the turret ring is not perpendicular to what would be considered level ground. If the tank is rolled, then there will be both a vertical and a horizontal error as is shown in Figure 4.

$$\begin{aligned} E_h &= r \sin \Phi = \text{Horizontal Cant Error}^9 \\ E_v &= r(-1 + \cos \Phi) = \text{Vertical Cant Error, where} \\ &\quad \Phi = \text{angle of roll} \\ &\quad r = R \tan \theta = \text{drop of projectile, where} \\ &\quad \quad R = \text{Range} \\ &\quad \quad \theta = \text{correct superelevation} \\ \text{or } r &= gt_f^2, \\ &\quad \text{where} \\ &\quad t_f = \text{time of flight} \\ &\quad g = \text{gravitational acceleration} \end{aligned}$$

The # and + would be shifted together up or down if the tank was pitched as well as rolled. As can be seen in the error equations for  $\Phi < 10$  degrees, the horizontal error will be much larger than the vertical error.

Cross Wind and Range Wind. Cross wind is the component of wind perpendicular to the path of flight, and range wind is the component of wind parallel to the path of flight. Crosswind will cause a projectile to move to the left or the right as it travels while range wind will cause a drag which will slow a projectile. The classical cross wind equation is

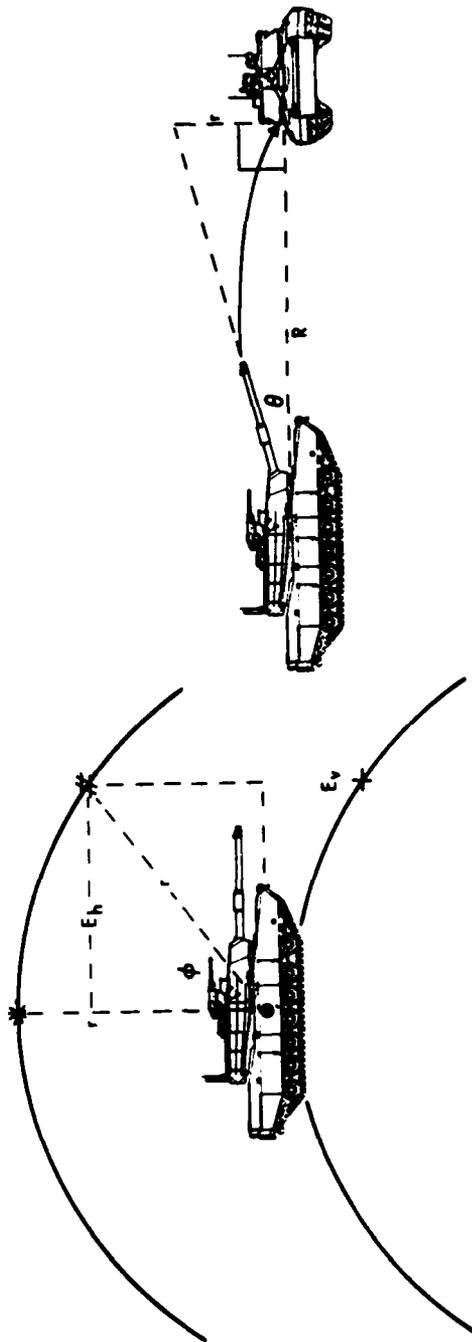
$$D = W \left( t - \frac{R}{V_m} \right)$$

where

$D$  = cross wind drift of the projectile

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<sup>9</sup> Dick Norman, Letter to President, US Army Armor and Engineer Board, with regard to Formulas Being Used to Compute Errors and Biases HPC as Compared With Formulas in R-1380 and R-1937, 22 Oct 79.



- \* Where gun should point if on level ground
- . Where round will hit if on level ground
- e Where gun would be pointed if firer is rolled (without correction)
- + Where round would hit if firer is rolled (without correction)

Figure 4. Error Due to Cant

$W$  = the velocity of the cross wind  
 $t$  = time of flight  
 $R$  = range  
 $V_m$  = the projectile muzzle velocity<sup>10</sup>

Notice that  $\frac{R}{V_m}$  is the time of flight for the projectile in a vacuum. In other words, the displacement caused by the cross wind is equal to the velocity of the cross wind multiplied by extra time the projectile is in the air due to the friction of the atmosphere.

The force of drag equation is

$$\text{Drag} = K_D \rho d^2 (V_p + V_R)^2$$

where

$\text{Drag}$  = drag  
 $K_D$  = drag coefficient (dimensionless)  
 $\rho$  = air density  
 $d$  = diameter of projectile  
 $V_p$  = velocity of projectile without wind  
 $V_R$  = velocity of range wind<sup>11</sup>

The cross wind equation shows that the faster a round is, the less it will drift. (This does not mean that the faster of two dissimilar rounds has less drift.) In any case range wind has no significant effect on the accuracy of tank rounds, while cross winds can cause large dispersions away from the point of aim.<sup>12</sup> Crosswind error correction may be incorrect because cross wind sensors often fail and because the cross wind sensor measures the cross wind at the firer's position and not along the path of flight of the projectile.

**Windage Jump.** Windage jump is the initial yaw given by a cross wind to a projectile as it exits the muzzle. If the cross wind is from left to right, then the windage jump will be up. This jump, however, is negligible for tank guns.

**Jump.** Jump is a variable bias if it is not predictable from occasion to occasion, but is predictable from round to round within an occasion. Jump can be substantially reduced by referencing the muzzle of the weapon.<sup>13</sup>

<sup>10</sup>Sauerborn, Geoffrey C. and Bunn, Fred L. *The Value of Fire Control Sensors in Armored Systems*, Technical Report BRL-TR-2639, February 1985, p. 53.

<sup>11</sup>Sauerborn and Bunn, p. 53.

<sup>12</sup>Sauerborn and Bunn, p. 53.

<sup>13</sup>Sissom, p. 8.

**Range Estimation.** Range estimation error differs with the type of fire control. Error in range estimation has both vertical and horizontal effects. A bad range estimate will cause a wrong superelevation to be calculated as well as a wrong lead error. With the older tanks using iron sights, the probability distribution for range estimation is broad but single modal, whereas with newer tanks using lasers, the probability distribution is very tight, but is multi-modal. This multi-modalness is caused by the laser hitting objects other than the target. But with the gunner dumping what he feels are bad readings, the correct range is likely to be found.<sup>14</sup>

**Fire Control.** Another variable bias error is fire control error which is a measure of a system's inability to put the gun in the proper direction relative to the sight.<sup>15</sup> This error can be broken into computational error and implementation error. Computational error is caused by not including all the terms in the formula for the elevation, round-off error, or other errors associated with the computation of the fire control equations. Implementation error is caused by the error in implementing what the fire control computer says to implement. This can be caused by, say, worn out parts or play in the system. An example of fire control error is system drift, which is reticle in the sight moving even though the gunner is not moving his controls. Fire control error can also be broken down as computer error, ballistic solution error, boresight retention, eye/sight parallax and elevation axes alignment.<sup>16</sup>

**Air Temperature.** Air temperature affects air density which affects the drag of the projectile in two ways: one, it inversely changes air density and two, it influences the speed of sound or Mach number as shown in the following equation:

$$Mach = \frac{v}{331.3 \sqrt{\frac{T}{273.13}}}$$

where

$v$  = projectile velocity (m/s)

$T$  = air temperature (K)

331.3 = speed of sound at standard temperature and pressure (m/s)

273.13 = temperature in kelvin = 0 degrees C

The Mach number affects the coefficient  $K_D$  in the already mentioned drag equation.

<sup>14</sup> Sauerborn and Bunn, p. 13.

<sup>15</sup> Nolan, Thomas "M60A1 Delivery Accuracy Estimates", US AMSAA, JMEM/SS-DAWG December 1, 1972, p. 22.

<sup>16</sup> Nolan, p. 22.

Air temperature does not affect KE round accuracy. HEAT round accuracy is not affected if the air temperature is within 20° F of the standard operating temperature. And if the range is less than 2 km, then HEAT round is not affected if the air temperature is within 30° F of the standard operating temperature.<sup>17</sup>

**Barometric Pressure.** Barometric pressure changes cause air density changes. Barometric pressure changes with weather, altitude, and latitude. In the US at 40° latitude, the pressure change will be about .6% in a given day, whereas a change in altitude of 100m will lower pressure by 1% and a change in altitude of 1 mile will lower the pressure by 18%. KE rounds are highly insensitive to changes in pressure (air density) since they are designed as low drag, high velocity projectiles. HEAT rounds under one kilometer are not affected much, but over two kilometers, the effect can be noticed.<sup>18</sup>

**Humidity.** Humidity affects the air density. The more moisture in the air, the greater the density. Humidity does not have much of an effect on tank rounds. In the most extreme case: HEAT at 3 km in 100% humidity the round lands only .15m above the point of aim.<sup>19</sup>

**Optical Path Bending.** Optical path bending is caused by the sunlight heating the air or the lack of sunlight cooling the air through which the light passes and refracts. This causes the image of the target to be either at a different place than the target, to be shimmering, that is, oscillating, or to be broken. The light from the laser range finder will hit the target since it will refract in the same manner as the optical light, but the projectile will not be refracted, of course, and thus there is error. The significance of this error source has been debated for years. There are those who feel that the error is not significant at two kilometers, significant at three kilometers, and very significant beyond three and a half kilometers. Others feel the error is significant at shorter ranges. During the day the optical light bends upward causing a gunner to hit low, and at night the optical light bends downward causing a gunner to hit high.

**Distortion of Gun Tubes.** Tubes are crooked, but it appears that shot departure is consistent and reproducible if the weapon is only slightly bent. Tube bends are caused by temperature differentials, which are caused by sun, rain, and cross winds against heated barrels. A rain shower will cause a hot tube to curl upwards rather quickly.<sup>20</sup>

**Muzzle Velocity Variation.** Muzzle velocity errors affect the vertical components of error. Muzzle velocity errors can be broken into the following parts: tube wear, occasion-to-occasion/tube-to-tube errors, charge temperature, and muzzle velocity temperature sensitivity.<sup>21</sup> The muzzle velocity variation (excluding tube wear) can be

<sup>17</sup> Sauerborn and Bunn, p. 47.

<sup>18</sup> Sauerborn and Bunn, p. 50-51.

<sup>19</sup> Sauerborn and Bunn, p. 47.

<sup>20</sup> Sissom, p. 9

<sup>21</sup> Report on the Trilateral Tank Main Armament Evaluation, p. 3.280.

calculated using the following formula:

$$\sigma_v = \left[ (K(\sigma_{\mu}))^2 + (\sigma_{u^2}) \right]^{\frac{1}{2}}$$

where

$\sigma_v$  = muzzle velocity variation

$K$  = constant = 2 for HEAT rounds

$\sigma_{\mu}$  = muzzle velocity variation due to propellant temperature

$\sigma_u$  = muzzle velocity lot-to-lot variation

**Tube Wear.** Tube wear will cause a slower muzzle velocity because some of the gasses will escape around the round, and tube wear will also cause the round to wiggle as it moves through the tube. These two effects cause a higher dispersion for the rounds fired.

**Occasion-to-Occasion/Tube-to-Tube Error.** Unknown factors cause dispersion to vary from occasion to occasion. Variations in manufacture and use of cannon cause the dispersion to vary from tube to tube.

**Charge Temperature.** Charge (propellant) temperature affects muzzle velocity, and this effect is called the muzzle velocity temperature sensitivity. The change in the temperature of the charge will change the kinetic energy which will change the velocity of the shell, that is, the muzzle velocity. If the charge temperature is higher than the standard temperature used in the firing tables, then the shell will hit higher than predicted by the firing tables; conversely, if the charge temperature is lower than the standard temperature used in the firing tables, then the shell will hit lower than predicted by the firing tables. (Standard charge temperature of the firing tables are around 70 degrees Fahrenheit.) The muzzle velocity of a projectile is roughly linear in correlation with respect to charge temperature.

**Live Fire Zeroing.** Live fire zeroing is zeroing out the fixed bias jump by firing rounds at a target and lining the cross hairs of the sight on the GCI of the impact points of the rounds fired. Live fire zeroing addresses the first round hit probability of a new target in a new position. Live fire zeroing errors include cant, cross wind, and fire control which have already been mentioned, and parallax and drift compensation, GCI, and observation of the GCI.<sup>22</sup>

**Parallax and Drift Compensation.** Parallax and drift compensation involves moving the gun to the left or right so that error due to parallax and drift is taken out. In order to make the correct compensation, the true range must be known. An error in the

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<sup>22</sup>Report on the Trilateral Tank Main Armament Evaluation, p. 3.280.

range estimate causes parallax and drift compensation error as shown below:

**Range Estimate Short**

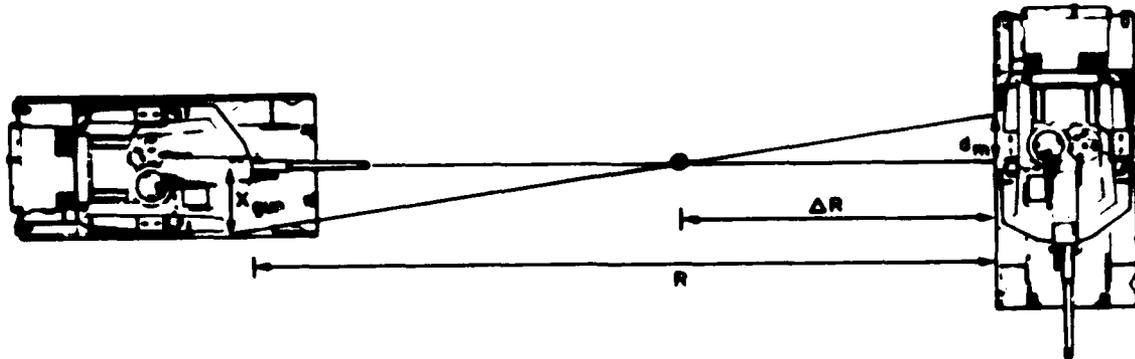


Figure 5. Parallax and Drift Compensation Error Due to Short Range Estimate

$$\frac{x_{gun}}{R-\Delta R} = \frac{d_m}{\Delta R}$$

**Range Estimate Long**

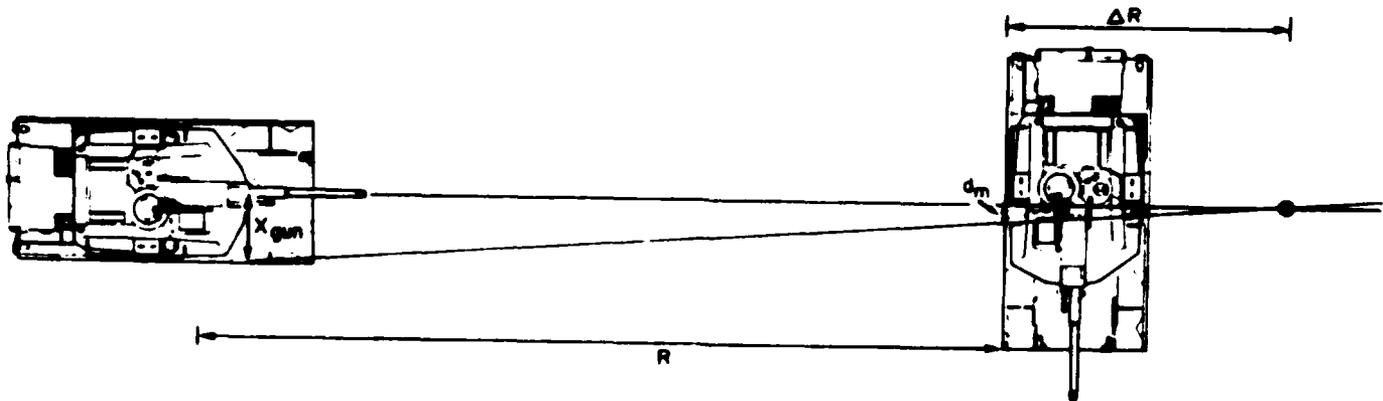


Figure 6. Parallax and Drift Compensation Error Due to Long Range Estimate

$$\frac{x_{gun}}{R+\Delta R} = \frac{d_m}{\Delta R}$$

$x_{gun}$ =distance between gun and sight

E=Error

R=range

$\Delta R = \text{Error in Range}$

$$E_{avg} = \frac{E_{short} + E_{long}}{2} = z_{90\%} \frac{R \Delta R}{R^2 - (\Delta R)^2}$$

**Group Center of Impact.** GCI errors comes from not knowing the true GCI since the GCI is estimated, at least when zeroing, by firing three rounds. Error in observing the GCI is simply the error in guessing where the middle of the three shots are.

**Silent Zeroing.** Silent zeroing is zeroing a tank gun without firing. It involves lining the gun using a number obtained from exhaustive firings. Silent zeroing errors can be divided into the following errors: bore sight eccentricity, errors in sight, gun alignment, and sight alignment.<sup>23</sup> (The boresight is the axis of the gun tube.)

**Other Errors.** Three other bias sources are bore sight eccentricity, errors in sight, and gun and sight alignment errors.

### Random Errors

Random errors are divided into round-to-round or ballistic errors and lay errors.

**Ballistic Error.** Ballistic errors result from the difference between individual rounds. The major components of ballistic error are angle of departure, differences in muzzle velocity, drag differences, and cross wind. The angle of departure is the angle between the centerline of bore and the path of the bullet. This is caused by the reaction of the tube to the bullet moving through it. Drag differences are caused by two things. One is the manufacturing process which involves differences in the shape of the projectile and the composition of the propellant, and the other is a non-zero yaw which will cause a different effective drag. The manufacturing process has little effect on ballistic error, whereas crosswind and differences in muzzle velocity are the major contributors to ballistic error.

The round to round dispersion of earlier tank weapons was smaller than present tank weapons. The reason for the increased dispersion is not fully known but is related in part to higher velocity, type of rotating band (particularly with discarding sabot and fin stabilizing rounds), and longer and more slender tubes.<sup>24</sup>

**Lay Errors.** Lay errors are the random errors associated with the fine lay made by the gunner before firing. On a test range where there is no hurry, the lay errors will be small and without variability. Under combat, the errors will be large since the gunner is in a hurry to hit the target. The gunner will consider the lay to be "good enough for

<sup>23</sup> *Report on the Trilateral Tank Main Armament Evaluation*, p. 3.280.

<sup>24</sup> *Sissom*, p. 7.

now." The standard value used for lay error is a combination of a 0.3 meter linear error and a 0.05 mil angular error.

### Errors Due to Target Motion.

The errors of a round fired at a moving target are a combination of the difference between the actual target position at impact and the predicted position. A second large factor is called drift; this is probably related to the inability of the gunner to keep the cross hairs from wandering around the aim point. The fire control needs angular rates for a fraction of a second or more to calculate lead angle. If these angular rates are corrupted by drift, the lead angle will be in error. The errors of a round fired at a stationary target and several new errors. AMSAA has developed a mathematical methodology for calculating drift which has been placed in a computer program. Unfortunately, the computer program is not understandable.

Note that there are three levels of error: that with optical ranging and a mechanical fire control computer, that of the M60A3, and that with laser ranging and a solid state computer.

The target induced error is easy to calculate. If a target is moving at an angle to the line of sight, and begins to turn at the moment of firing, its actual position at the time the bullet passes it will differ from the predicted position at the time the bullet was predicted to pass it. The situation is shown in Figure 7, and calculated below.

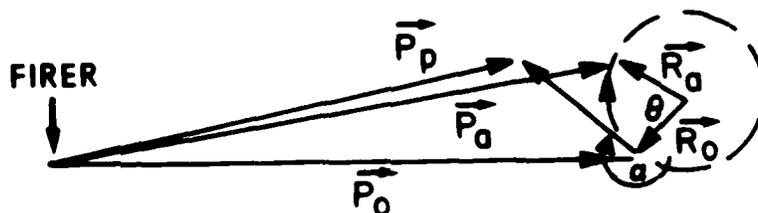


Figure 7. Target Induced Errors

Let

- $\vec{p}_0$  = the position of the target at the time of firing.
- $\vec{p}_p$  = the predicted position of the target.
- $\vec{p}_a$  = the actual position of the target.

$\vec{v}$  = the velocity vector of the target,  
 $\vec{r}_0$  = the position of the target at time zero relative to the center of the turning circle,  
 $\vec{r}_e$  = the actual position of the target at impact time relative to the center of the turning circle,  
 $r$  = the radius of the circle in which the target turns,  
 $a_r$  = the radial acceleration  
 $\alpha$  = the crossing angle of the target as measured clockwise from the line of sight vector.  
 $\theta$  = the angle through which the target turns,  
 $M(\theta)$  is the rotation matrix

$$M(\theta) = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Then

$$r = v^2/a_r$$

$$\vec{r}_0 = (r/v)\vec{v}M(\pi/2) = (v/a_r)\vec{v}M(\pi/2)$$

$$\vec{r}_e = \vec{r}_0M(\theta)$$

$$\vec{p}_p = \vec{p}_0 + \vec{v}t_f$$

$$\vec{p}_e = \vec{p}_0 - \vec{r}_0 + \vec{r}_e$$

$$\phi_e = \text{atan}(p_{e_x}/p_{e_y})$$

$$\phi_p = \text{atan}(p_{p_x}/p_{p_y})$$

And the error is,  $\phi_e - \phi_p$

#### Additional Errors for the Moving Firer.

If the platform is moving then there will be error due to this motion, even if this motion is taken into account. This is because if the platform is moving, there will be an additional wind applied to the projectile due to motion of the platform. There is also alignment error associated with the dynamic flexure of the gun tube while vehicle is on the move.

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## APPENDIX B HIT PROBABILITY PROGRAM

The following program is one of a number of programs used by the authors for calculating hit probabilities. Others include programs for finding hit probabilities against targets of more general shapes: arbitrary polygons and arbitrary polyhedrons. The following one calculates hit probabilities on tanks represented as two rectangles. It produces probabilities for the fully exposed target, the hull defilade target, and the NATO standard 2.3 by 2.3m square, given target dimensions, range, and round errors. The aim point is assumed to be in the center of the appropriate rectangle except that it is 0.3 meters below the turret ring in the fully exposed case.

It is written in Fortran 77 to run under the UNIX operating system, but can easily be modified to run under other operating systems. If the executable code is stored in a file called phtank3, then the command to execute is:

phtank3 -ph

where the h option puts headers on the output and the p option prompts for input. The input is:

Turret width and height (m)  
Hull width and height (m)  
Range to target (km), horizontal dispersion, vertical dispersion  
horizontal fixed bias, vertical fixed bias (mils)  
n more lines of ranges, dispersions, and biases

After reading the first three lines of input, the program integrates the normal distribution in both directions over the area of the target. The routine prints the hit probability and loops back for another range and set of dispersions.

### Sample Run.

Command: phtank3 -h <KE.data

Sample input in KE.data file:

2.35,.75,  
3.55,1.5,  
0.5,0.8032,0.9020,0.,0.,  
1.0,0.5745,0.5775,0.,0.,  
1.5,0.5164,0.5218,0.,0.,  
2.0,0.4957,0.5008,0.,0.,  
2.5,0.4902,0.4916,0.,0.,

Sample output:

PHTANK3 19 Mar 83

rg (km)	dispersn (mils)		bias (mils)		hit prob		
	horiz	vert	horiz	vert	FE	HD	NATO
0.50	0.80	0.90	0.00	0.00	0.99	0.60	0.99
1.00	0.57	0.58	0.00	0.00	0.94	0.47	0.92
1.50	0.52	0.52	0.00	0.00	0.81	0.33	0.75
2.00	0.50	0.50	0.00	0.00	0.66	0.23	0.58
2.50	0.49	0.49	0.00	0.00	0.52	0.16	0.44
3.00	0.49	0.49	0.00	0.00	0.40	0.12	0.33

### The Program.

c Purpose: Find the probability of hitting a FE or HD target.

c The target is represented as two rectangles.

c

logical HEADER, PROMPT, is arg

1 format(6f10.3)

2 format (6f8.2, 2f5.2)

c

PROMPT= is arg('p')

HEADER= is arg('h')

IF (HEADER) THEN

print\*, 'PHTANK3 19 Mar 83'

print\*,

1 ' rg dispersn (mils) bias (mils) hit prob',

2 ' (km) horiz vert horiz vert FE HD NATO'

ENDIF

if (PROMPT) print\*, 'What is turret width & height (m)?'

if (PROMPT) call flush()

read(5,1) tw, th

if (PROMPT) print\*, 'What is hull width & height (m)?'

if (PROMPT) call flush()

read(5,1) hw, hh

c

20 CONTINUE

c Read and convert inputs

if (PROMPT) print\*,

1 ' What are rg (km), disp-h, disp-v, bias-h, bias-v (mils)?'

if (PROMPT) call flush()

read(5,1,END=40) r, sh, sv, bh, bv

c Convert to linear error in meters

xs = sh\*r\*0.9817

```

ys = sv*r*0.9817
xb = bh*r*0.9817
yb = bv*r*0.9817
c Find probability of hitting FE target
ptur = prob(-.5*tw-xb,.5*tw-xb,0.3-yb,th+0.3-yb,xs,ys)
phul = prob(-.5*hw-xb,.5*hw-xb,0.3-h-yb,0.3-yb,xs,ys)
ptank= ptur+phul
c Find probability of hitting HD target
pturHD = prob(-.5*tw-xb,.5*tw-xb,
1      -.5*th-yb,.5*th-yb,xs,ys)
c Find probability of hitting NATO target
pNATO = prob(-1.15-xb,1.15-xb,
1      -1.15-yb,1.15-yb,xs,ys)
write(6,2) r, sh, sv, bh, bv, ptank, pturHD, pNATO
GOTO 20

```

40 CONTINUE

END

FUNCTION PROB(xl,xh,yl,yh,xs,ys)

```

c -----
c find phit
4 format (' p, px, py =',3f8.4)
real ndtr
p1 = ndtr(xh/xs)
p2 = ndtr(xl/xs)
px = p1-p2
p3 = ndtr(yh/ys)
p4 = ndtr(yl/ys)
py = p3-p4
p = px*py
prob=p
END

```

c REAL FUNCTION NDTR (x)

```

c -----
c Integrate the standard normal distribution (sigma=1, mu=0) from
c -infinity to x. This code was adapted from ndtr in the IBM
c Scientific Subroutine Package pg78.

```

```

c
ax = abs(x)
t = 1.0/(1.0+.2316419*ax)
d = 0.0
if(ax.lt.6.0)d = 0.3989423*exp(-x*x/2.0)
p = 1.0 - d*t*(((1.330274*t - 1.821256)*t + 1.781478)*t -
* 0.3565638)*t + 0.3193815)
if (x.lt.0.0) p = 1.-p
ndtr = p
END

```

```

c
LOGICAL FUNCTION IS ARG (ACHAR)
c -----
c Inspect each command line argument that begins with a '-'
c to see if it contains the character in 'achar' and return
c TRUE if a match is found, otherwise return a FALSE.
character*1 achar
character*40 arg
c
is arg = .FALSE.
c Find the number of arguments on the command line.
nargs = iargc()
c Loop through each arguments until ACHAR is found.
DO 30 narg = 1,nargs
call getarg(narg,arg)
c Only examine arguments beginning with '-'
IF ('-' .eq. arg(1:1)) THEN
c Loop through characters 2-39 until ACHAR is found.
DO 20 nchar = 2,39
IF (ACHAR .eq. arg(nchar:nchar)) THEN
is arg = .TRUE.
GOTO 99

ENDIF
20 CONTINUE
ENDIF
30 CONTINUE
99 CONTINUE
END

```

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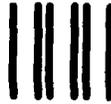
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