Bifurcations in flow between independently rotating circular cylinders are being investigated experimentally, numerically, and theoretically. A nonlinear stability analysis of the primary instability exploits symmetry properties to make predictions about the form of the secondary flows; the predictions for the critical Reynolds numbers, wavespeeds, and wavenumbers of the secondary flows (spirals, ribbons, and Taylor vortices) are in good accord with experiment. In another area of study, measurements of mass transport at high Reynolds numbers show that transport in the axial direction is well-described by an effective diffusion coefficient $D$ that has an exponential dependence on the Reynolds number $R$, $D = R^{1/4}$, while theory suggests that the exponent should be 1 instead of $3/4$. 
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REPORT
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SYMMETRY BREAKING BIFURCATIONS AND THE
GROWTH OF CHAOS IN A ROTATING FLUID

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Primary instabilities and bicriticality in flow between counterrotating cylinders

An extensive experimental and numerical study of the primary instabilities and bicritical curves for flow between counterrotating cylinders is now underway. The computations derive from the Navier-Stokes equations, assuming axial periodicity, and have yielded values of the Reynolds numbers, wavespeeds, and related quantities at the primary transition from Couette flow to spiral flow and ribbons. The ribbon state, which is a standing wave formed by superposition of spirals of opposite helicity, was predicted one year ago by Iooss and collaborators at the Université de Nice; recently we have succeeded in detecting the ribbons in the laboratory.

In both our experiments and simulations particular attention is focused on the bicritical curves that separate (as the magnitude of counterrotation is increased) the transitions from Couette flow to flows with different azimuthal wavenumbers $m$ and $m+1$. The measurements are in good accord with the simulations.

This work is laying the foundation for further analysis of pattern formation and complex dynamics arising from nonlinear mode competition in the neighborhood of the bicritical lines.

Mass transport in turbulent Couette-Taylor flow

We are studying mass transport in a turbulent flow with large coherent structures—turbulent Taylor vortices. A pulse of dye is injected in fluid contained between concentric cylinders (with the inner one rotating and the outer one fixed), and the time dependence of the dye concentration at two axial positions is then determined from optical absorption measurements. Measurements have been made for radius ratios $\eta$ ranging from 0.494 to 0.875, at Reynolds numbers $R$ ranging from 50 to 1000 times that corresponding to the onset of Taylor vortex flow.

Transport in the axial direction is found to be modeled very well by a one-dimensional diffusion process. For a fixed $R$ and $\eta$, $D$ increases linearly with the axial wavelength $\lambda$ of the Taylor vortices. The Reynolds number dependence of the wavelength-independent scaled diffusion coefficient, $D^* = (2d/\lambda)d$ [where $d$ is the gap between the cylinders], is described by a power law, $D^* \sim R^\beta$. Measurements for different parameter regions indicate that $\beta = 0.75\pm0.10$, while theory suggests a larger value, $\beta = 1$. This result indicates that $D^*$ scales as the ratio of the largest length scale, $\lambda$, to the smallest length scale, the Kolmogorov dissipation length.

Wavelength changing instabilities in Taylor-Couette flow

Marcus and his students at Berkeley are examining numerically wavelength changing instabilities of Taylor-Couette flow. In the limit of infinite length cylinders, Taylor-Couette flow is exactly periodic in the axial direction. Therefore, the linear instabilities associated with changes of the axial wavelength can be described by Floquet theory. However, because the primary Taylor-Couette is itself highly nonlinear in the parameter range of interest, the Floquet eigenmodes of an operator require of order $N^3$ numerical operations where $N$ is the number of numerical elements (grid points, spectral modes, etc.) required to resolve accurately the primary flow. Because $N$ is of order 1000 in our flows, the direct methods are unusable, so we have employed the initial-value method used by
Marcus and Tuckerman (to appear, J. Fluid Mech., 1987). We have discovered a new instability that is suggestive of the experimental results of King and Swinney [Phys. Rev. A27, 1240 (1983)]. At low values of the Reynolds number, it can be shown that the most rapidly growing wavelength changing instability is the Ekhaus mode. One signature of the Ekhaus mode is that as its wavenumber goes to zero, its growth rate goes to zero. (In the zero wavenumber limit, the Ekhaus mode corresponds to the infinitesimal translation of the primary Taylor-Couette flow in the axial direction. This translation is always neutrally stable.) The new class of eigenmodes that we have found has velocities that appear to be very similar to the Ekhaus modes, and a plot of the flow field would be hard to distinguish from an Ekhaus eigenmode. However, as the wavenumber of the perturbation goes to zero, the real parts of the growth rates of these modes do not go to zero. The new instability is more rapid than the Ekhaus mode and therefore determines the stability of Couette flow for $2 < R/R_c < 5$; the precise value of $R$ is a function of the radius ratio of the system. We are continuing to examine the physical mechanism of this new instability, and we are now writing a code to examine the wavelength changing instabilities associated with wavy-Taylor-vortex-flow.

**Universal mechanism for instabilities of streamwise vortices in shear flows**

Marcus has hypothesized that many of the secondary instabilities observed in the Taylor-Couette system with a small gap between the inner and outer cylinders are neither centrifugal nor associated with inflection points, but are due to the inflow/outflow jets between the Taylor vortices. In particular, we consider the transitions from Taylor-vortex flow to wavy-vortex flow to modulated-wavy-vortex flow when the outer cylinder is fixed and the inner cylinder is rotating, the transitions to the wavy-inflow state and the wavy-outflow state when the cylinders are co-rotating, and to the spiral flows when the cylinders are counter-rotating. In the small gap limit Taylor-vortex flow looks similar to a unidirectional plane-parallel shear flow with stream-wise vortices alternating in sign. If the centrifugal force in Couette flow is unimportant compared to the inflow/outflow jets in determining the stability of this system, then channel flows with alternating stream-wise vortices should have the same types of instabilities and secondary flows as the Taylor-Couette system. To test this proposal we have written a code for computing the instabilities and secondary flows, where the primary flow consists of a unidirectional shear flow with stream-wise vortices. The primary flow is not a steady solution of the Navier-Stokes equations, so the vortices are maintained by either (1) including the centrifugal forcing derived in the small-gap limit of Taylor-Couette flow, (2) adding an artificial forcing that keeps the vortices fixed for all time, or (3) allowing the primary flow to decay (by the natural action of the Navier-Stokes equation) but since its dissipation rate is slow compared to the growth rate of the instabilities, the secondary flows grow to be large (and indentifiable) before the velocity viscously decays to a Poiseuille-Couette flow.

We are now computing where the secondary flows exist as a function of the amplitudes of the inflow and outflow jets and comparing this phase diagram with the one obtained from experiments of Taylor-Couette flow.
Publications

This report covers only the initial period of the three-year contract (June 1, 1986—December 31, 1986). The research is now well underway, but no publications have yet appeared. The following have been submitted:


Invited Talks (HLS)

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<th>Date</th>
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<tr>
<td>6/23/86</td>
<td>9th Canadian Symposium on Theoretical Chemistry, Toronto, Ontario</td>
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<td>Conference on Physics and Structure of Complexity, Trieste, Italy</td>
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<td>12/14-19/86</td>
<td>Conference on Chaos and Related Phenomena, Jerusalem</td>
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END
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