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# U.S. Army Reenlistment and Extension by Occupation: A Reduced-Form Trinomial Probit Approach

Joseph V. Terza and Ronald S. Warren  
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U.S. Army

Research Institute for the Behavioral and Social Sciences

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20. (continued)

extension and reenlistment alternatives are perceived to be closer substitutes than either extension and separation or reenlistment and separation. Consequently, the multinomial logit model would almost surely predict too high a joint probability of extending or reenlisting.

The probit estimates were compared with those obtained with the multinomial logit estimator and a variant of the latter, which imposes the constraints on the reduced-form coefficients that are implied by the structure of the underlying utility equations. Unfortunately, the model performed poorly across all three estimation techniques, so it was difficult to evaluate them unambiguously. Only the coefficient on the selective reenlistment bonus variable consistently displayed the expected sign and, simultaneously, was significantly different from zero. Especially disappointing was the performance of the military-civilian relative pay variable, in light of its success in the Lakhani-Gilroy study. On the basis of predictive accuracy, the constrained logit model dominated both the unconstrained logit and probit formulations. On the other hand, tests of the null hypotheses that (1) the constraints are empirically valid and (2) the IIA assumption characterizes the data were both decisively rejected. As a whole, these results suggest that the model is misspecified and that both the constraints and the functional form affect its performance. Consequently, further development of the theoretical model and an investigation into the computational feasibility of a constrained probit estimator would seem to be warranted. *Keywords:*

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# **U.S. Army Reenlistment and Extension by Occupation: A Reduced-Form Trinomial Probit Approach**

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## FOREWORD

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The Manpower and Personnel Policy Research Group of the U.S. Army Research Institute (ARI) performs research in the economics of manpower and personnel issues of particular significance to the U.S. Army. Questions have recently arisen regarding the ability of the Army to increase reenlistment and extension rates by providing economic incentives in the form of selective reenlistment bonuses. This report was prepared as part of ARI's continuing support to the office of the Deputy Chief of Staff for Personnel.

The research presented in this report quantifies several economic and demographic variables thought to affect reenlistment and extension decisions in the Army and contributes to the ongoing theoretical and empirical discussion of military manpower modelling.



EDGAR M. JOHNSON  
Technical Director

## ACKNOWLEDGMENTS

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U.S. ARMY REENLISTMENT AND EXTENSION BY OCCUPATION:  
A REDUCED-FORM TRINOMIAL PROBIT APPROACH

EXECUTIVE SUMMARY

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Requirement:

Questions have been raised about the Army's ability to increase reenlistment and extension rates in specific military occupations and/or Career Management Fields (CMFs). The authors have examined some economic and demographic variables that affect reenlistment and extension decisions that can have a significant impact on the long-term readiness of an experienced Army. Special attention is paid to the impact of the selective reenlistment bonus on the individual reenlistment decision.

Procedure:

The authors use a reduced-form trinomial probit approach to explain reenlistment and extension decisions of first-term soldiers in FY 1981 in specific CMFs. This procedure represents an improvement over earlier research that employed a multivariate logit model with its restrictive assumption of "independence from irrelevant alternatives."

Findings:

The results reveal that reenlistment probabilities in specific "shortage" and "critical" CMFs can be increased by increasing selective reenlistment bonuses (SRBs) paid by the Army.

Utilization of Findings:

Results of the research seem to show that voluntary reenlistment in specific "shortage" and "critical" CMFs can be increased significantly by increasing SRBs. Moreover, since the impact of SRBs on different CMFs is different, there is a case, then, for reallocating SRBs from where elasticities are high to where they are low. In this way, imbalances across CMFs might be reduced.

**U.S. ARMY REENLISTMENT AND EXTENSION BY OCCUPATION:  
A REDUCED-FORM TRINOMIAL PROBIT APPROACH**

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## 1. Introduction

Since the advent of the All Volunteer Force (AVF) in 1973, considerable attention has been accorded the demographic and, especially, economic factors which affect the decisions of first-term enlistees to remain in or separate from the Armed Services. Special emphasis has been placed on estimating the responsiveness of retention rates to changes in both the ratio of regular military compensation to the civilian wage opportunity and the amount of the reenlistment bonus paid to servicemen in Military Occupation Specialties (MOS) deemed either in short supply or essential for combat readiness. The retention of skilled personnel beyond the first term is of central importance to the fiscal viability of a high-quality AVF and allows the services to amortize their share of the cost of investment in the general and specific human capital provided to new recruits.

Of more recent concern has been the decomposition of retention into reenlistments for periods of three or more years and extensions of less than three years. This disaggregation is conceptually important for at least two reasons. First, these two components of retention may respond differently to various pecuniary incentives. For example, since the Selective Reenlistment Bonus (SRB) is only received by eligibles who reenlist, and not by those who extend, an increase in the SRB will, other things equal, most likely increase the reenlistment rate and decrease the extension rate. Second, as Warner (1984) notes, extensions have constituted a rising proportion of the total retention rate during the last decade. This shift in the mix of retention toward extensions and away from reenlistments implies a lower stock of contracted man-years and, hence, a smaller and less stable enlisted force over time. To counteract this trend, a change in the composition of the military wage bill away from regular compensation and toward SRB payments may be warranted. The magnitude of the required change, and its implications for the total labor cost of the AVF, depend crucially on the elasticities of reenlistment and extension with respect to pay and bonus payments.

Despite a general recognition of the importance of disaggregating retentions into reenlistments and extensions,<sup>1</sup> only a handful of studies have estimated the implied trichotomous choice model using data on individual enlistees' decisions. This dearth of empirical analyses can be explained in part by the theoretical complexity and computational expense of estimating general models of this sort with the large sample sizes characteristic of micro data sets. To circumvent these problems, researchers have typically formulated and estimated either a multinomial logit model, which imposes the restrictive assumption of the "independence from irrelevant alternatives" (IIA), or the sequential logit model, which assumes that the random components of the sequential choices are independent. For example, Goldberg and Warner (1982a) estimated a multinomial logit model of extensions and reenlistments among Navy enlisted personnel. Lakhani and Gilroy (1984) reported the results from estimating a multinomial logit model of Army extension and reenlistment

decisions. Unfortunately, the IIA assumption embedded in the multinomial logit model implies, in the context of the present study, that the relative probabilities of choosing to reenlist in or separate from the Army are unaffected by the presence of a third option, to extend one's enlistment contract for less than three years. Since this implication would seem to be unduly restrictive, the applicability of the multinomial logit framework for modelling this problem may be severely limited. Goldberg and Warner (1982b) specified a sequential logit model of Marine Corps retention behavior in which a decision is made first whether or not to remain in the Corps and, if the answer is affirmative, whether to reenlist or extend. However, this model imposes a recursive structure on the decision process which implies not only that the choice probability in the second stage is independent of the choice made at the first stage but also that the random factors influencing the decisions in the two stages are independent. Thus, the sequential logit model would also seem to require the untenable IIA assumption.

The problem, then, is to devise an approach to estimating a model of the reenlistment, extension, or separation choice which avoids the IIA hypothesis and yet is computationally feasible. There are two existing approaches which, in principle, are capable of solving this problem. The first is McFadden's (1981) Generalized Extreme Value (GEV) model, which would circumvent the IIA assumption by allowing for correlation between the stochastic error terms affecting the extension and reenlistment probabilities, while maintaining independence between the random components associated with separation and retention.<sup>2</sup> Although the GEV model incorporates the correlation between the two similar alternatives, it does so in a very simple way by introducing a single covariance parameter. This is both an advantage, because fewer parameters must be estimated, and a disadvantage, since the true correlation structure among the three options may be more complicated. Nevertheless, the GEV model is quite cumbersome computationally and its parameters are difficult to interpret. The second approach is multinomial probit, which relaxes the IIA assumption by assuming that the structural errors are jointly normally distributed.<sup>3</sup> Until recently, the multinomial probit model has been used infrequently because of its computational difficulty. Moreover, the structural parameters of this model are not identified without the imposition of a priori restrictions on the structural coefficient vectors or the covariance matrix. Often, however, such restrictions are unavailable, untenable, or insufficient to ensure structural identification. Terza (1985) has developed an identified reduced form for the multinomial probit model which uses admissible normalizations of, rather than a priori restrictions on, the structural parameters. Estimates of this reduced form are sufficient to (1) predict the probabilities of separating, extending, or reenlisting for a serviceman with given characteristics and (2) evaluate the effects of various explanatory variables on the probability of choosing a given alternative.

This report presents estimates of the Lakhani-Gilroy (1984) model of the extension and reenlistment choices of first-term Army enlistees, using the reduced-form trinomial probit approach devised by Terza (1985). These estimates are compared with those obtained with the multinomial logit estimator and a variant of the latter, which imposes the constraints on the reduced-form coefficients that are implied by the structure of the

underlying utility equations. To preview our results, we find that - on the basis of predictive accuracy and the signs and significance levels of the estimated coefficients - the model performed poorly with all three estimation techniques. Consequently, a clear-cut comparative evaluation of the estimators was not possible. Taken together, however, our results suggest that further development of the theoretical model and an investigation into the computational feasibility of a constrained probit approach may be warranted.

The report is organized as follows: Section 2 provides a brief review of previous research on reenlistment behavior in the AVF era; Section 3 presents a detailed discussion of the theoretical models and their corresponding estimators; Section 4 discusses the data sources and estimation algorithms used and Section 5 presents the empirical results; finally, Section 6 summarizes the purpose and method of the study, highlights the principal empirical findings, and suggests several potentially fruitful directions for future research on this topic.

## 2. Summary of Previous Research

This section presents a brief summary of previous research on the reenlistment behavior of first-term servicemen. A review of the voluminous literature on first-term enlistment supply to the AVF is beyond the scope of (and not especially germane to) the present study and, in any event, has been ably provided by Morey and McCann (1981) and Perelman (1983). Moreover, we deliberately avoid discussing studies of second-term and third-term reenlistment decisions, which involve considerations such as promotion and retirement that are largely irrelevant to an investigation of first-term retention. Since Lakhani and Gilroy (1984) and Baldwin and Daula (1985) have recently surveyed the range of point estimates of the reenlistment elasticities reported in studies of first-term reenlistment, we confine our discussion to various methodological issues.

For the purpose of organizing this review, we classified the existing literature into two groups: (1) a "first generation," which estimated the impact of specific policies, like relative pay and reenlistment bonuses, on the retention of enlisted personnel using the grouped logit technique on observations formed by aggregating data on individuals to create cells that were defined by various demographic and job-specific characteristics; and (2) a "second generation," which applied different logit or probit estimators directly to observations on the choices of individuals and their demographic and economic determinants. We conclude our summary by extrapolating recent developments in reenlistment research, along with those in the relevant econometric literature, in order to speculate on the theoretical models and empirical methods that will most likely characterize an emerging "third generation" of studies.

First generation research includes the early studies by Kleinman (1975) and Enns (1975), as well as the more recent investigations by Atwater and Rowe (1982), Bowman and Thomas (1982), Zulli (1982), Hosek and Peterson (1984), and Goldberg (1985). This group of studies has in common the feature that the unit of observation on the dependent variable is the

sample proportion of the first-term servicemen who (1) choose either to separate from or remain in the service, if the variable is dichotomous or (2) decide among separation, extension, or reenlistment, if the variable is trichotomous. Similarly, in order to ensure a sufficient number of observations in each cell, the explanatory variables are typically partitioned into discrete intervals whose sample means then replace the actual values in the estimating equations.

Kleinman (1975) examined the effect of reenlistment bonuses on first-term reenlistment rates by applying a logit estimator to data on Navy personnel who were grouped by skill. His estimates are of limited usefulness, however, since they measure only the total, rather than the marginal, effects of bonus payments on reenlistments. Enns (1975) specified the dependent variable as the logarithm of the reenlistment rate within a group and applied ordinary least squares to data from all services for the year 1971. This study is especially noteworthy for two reasons. First, the potential civilian wage variable was estimated from earnings data reported by veterans ten months after separation from the service. This procedure avoids the sample selection bias inherent in using civilian wage estimates obtained either from non-veterans or from the civilian population as a whole. Second, on the other hand, having been generated by data aggregated over all branches of the armed services, the results obtained by Enns (1975) are subject to aggregation bias since Warner and Simon (1978) have shown that reenlistment behavior varies substantially across military occupations. Enns (1977) followed up the previously mentioned study by estimating a reenlistment model with a logit technique, again using grouped data for all services. Bowman and Thomas (1982) avoided the aggregation bias associated with Enns' use of data from all services by applying the logit estimator to grouped data from the Air Force. Similarly, Atwater and Rowe (1982) investigated retention probabilities in three Navy occupations with a probit function. All of these studies focused on retention behavior and, thus, failed to distinguish between extensions and reenlistments.

Studies by Zulli (1982), Hosek and Peterson (1984), and Goldberg (1985) used grouped data but improved upon previous research by recognizing the importance of discriminating between extensions and reenlistments among retention choices. Zulli (1982) used a sequential logit model with grouped data from the Navy to analyze the retention-separation decision and, among stayers, the choice between extending and reenlisting. However, the sequential logit model imposes a recursive two-stage structure on the decision process and, thus, requires an assumption that the choice probabilities at each stage are independent. In addition, the error terms in the choice equations at each stage are assumed to be independent.<sup>4</sup> Hosek and Peterson (1984) attempted to circumvent the latter problem by applying Zellner's Seemingly Unrelated Regressions (SUR) estimator to the two log-odds equations representing, respectively, the choices between leaving and staying in the Army and between extending and reenlisting. Although this estimation technique renders the model triangular rather than recursive and, therefore, is somewhat more general than that of Zulli (1982), it is still inappropriate if the choice among the three alternatives is made simultaneously.<sup>5</sup> Goldberg (1985) incorporated the extension-reenlistment choice by estimating a model using multinomial

logit, nested logit, and universal logit techniques on grouped data from the Navy. The three sets of parameter estimates were subjected to various specification tests. The nested logit model fit the data better than multinomial logit since the cross-equation restriction on the coefficients of the relative pay variable implied by the latter can be rejected with a likelihood ratio test. However, the universal logit model (which specifies the relative probabilities as functions of all the variables) dominated the nested logit estimates, again using the likelihood ratio criterion. These results are both interesting and troublesome. They are interesting because they imply that the multinomial logit model, with its embedded IIA assumption, may be an inappropriate framework for analyzing the choice among separating, extending, and reenlisting. They are disturbing because the nested logit and universal logit models are so unrestrictive that they contain features that may be inconsistent with utility maximization. For example, the free coefficients on the pay variable in the two equations of the nested logit model allow the marginal utility of income to be nonunique, which would seem to violate the rationality postulate.<sup>6</sup> Nevertheless, the Goldberg (1985) study represents an important advancement since it explores formulations which do not impose the IIA assumption. Unfortunately, these generalizations were applied to grouped data, so the Goldberg estimates suffer from potential aggregation bias, as well as informational inefficiency.

The second generation of research on retention behavior takes the individual serviceman as the unit of observation, thereby avoiding the loss of valuable information that accompanies the aggregation of micro data into groups. However, the benefits of having a richer set of observations are obtained at the (computational) expense of correspondingly larger sample sizes. This is an especially critical issue in those studies which deem important the disaggregation of retentions into extensions and reenlistments. When coupled with the desirability of relaxing the IIA assumption and imposing the structural restrictions implied by utility theory, the use of individual observations strains the computational limits of existing optimization routines.

Warner and Simon (1978) avoided the problems involving the use of grouped data by analyzing the choices of individual Navy enlisted men. They estimated a model of the retention decision, using the binary probit technique. Unfortunately, they ignored the distinction between extending and reenlisting and so provided estimates of the relative pay elasticity but not the SRB elasticity. Warner and Goldberg (1982) estimated a sequential logit model with data on individuals in the Marine Corps. This model incorporated the extension-reenlistment margin of choice and avoided the aggregation issues arising in the first generation studies but imposed the recursive structure on the decision process that characterized the studies by Hosek and Peterson (1984) and Zulli (1982), discussed above. Goldberg and Warner (1982) and Lakhani and Gilroy (1984) included extensions explicitly in their trichotomous models and recognized the simultaneity of the choices by estimating multinomial logit models using data on individuals in the Navy and Army, respectively. Of course, multinomial logit imposes the undesirable IIA assumption so the informational detail and expanded choice set embodied in these two studies is attained at the cost of a restrictive theoretical framework. Finally,

Baldwin and Daula (1985) estimated a model of individual retention behavior among Army enlistees, using binary probit techniques. The novelty of their study is the explicit correction for sample selectivity bias in the construction of the estimated civilian wage variable.

The major developments in research on the retention of first-term enlistees have been: (1) the use of data on the choices of individual servicemen, which avoids the aggregation biases and information losses associated with grouped data; (2) recognition of the importance of disaggregating retention into reenlistment and extension decisions; (3) the imposition of constraints on the estimated reduced-form coefficients that reflect theoretically appropriate zero or cross-equation restrictions on the structural parameters in the utility equations; (4) the construction of the potential civilian wage variable in a manner that corrects for the sample selection bias arising from the optimal sorting of individuals between the military and civilian sectors; (5) the application of estimation techniques (like probit, nested logit, or universal logit) which avoid the assumption of the independence from irrelevant alternatives. Although each of the second generation studies cited above reflects at least one of these improvements and several papers embody two, none - including the present study - incorporates all of them. Recent advances in econometric theory and computational efficiency have made feasible a "third generation" of reenlistment research which can address simultaneously all of these conceptual and statistical issues. This research can be expected to provide more reliable estimates to Army manpower planners of the responsiveness of reenlistment and extension rates to changes in economic incentives and demographic trends.

### 3. Theoretical Models

In this section, three alternative models are described which represent the (stochastic) utility maximizing choice among separation from the service, extension for a period of up to three years, and reenlistment for a period of from three to six years.

The first model discussed is the familiar (unconstrained) multinomial logit model, which was given a stochastic utility maximization justification by McFadden (1974). The unconstrained multinomial logit approach has the advantage of being computationally inexpensive and its parameters are easily interpreted as marginal effects of explanatory variables on (the natural logarithm of) the ratios of the odds of choosing the pairwise alternatives. This model suffers from two potentially serious drawbacks, however. First, each explanatory variable appears in every equation determining the level of utility associated with a particular chosen alternative, even when the underlying theory implies that one or more of the variables is alternative-specific in its effect. In the present study, for example, the selective reenlistment bonus (SRB) affects the utility levels of those eligible servicemen who choose to reenlist but is not available to, and therefore does not affect the utility of, extendees or, of course, those who separate from the Army. These zero restrictions on the structural coefficients imply a corresponding zero restriction on one of the reduced-form coefficients of the SRB variables

in the multinomial logit equations that are estimated. Second, as is well known [see Amemiya (1981, p. 1518) and Maddala (1983, pp. 61-2)], the multinomial logit model embodies the assumption of the independence from irrelevant alternatives (IIA). Unfortunately, the IIA assumption imposes severe restrictions on the structure of preferences. Indeed, Samuelson (1985) has shown, in the context of general probabilistic choice models in which preferences are assumed to satisfy only the usual conditions for being well-behaved (completeness and transitivity of the ordering from among feasible alternatives), that the IIA assumption will almost always fail to hold. McFadden (1976) and Hausman and McFadden (1984) have developed and implemented diagnostic tests of the IIA hypothesis for the multinomial logit model. However, the power of these tests (to reject correctly the IIA assumption) may be small or unknown so that, in light of Samuelson's above-mentioned theoretical result and the availability of computationally feasible alternatives which do not impose the IIA hypothesis, the use of the multinomial logit model cannot be strongly encouraged.

The second model discussed in detail in this section is the constrained multinomial logit model mentioned in the preceding paragraph. This model shares with the unconstrained multinomial logit model the defect of imposing the IIA assumption but remedies the problem the latter has of being inconsistent with the utility structure which underlies the stochastic optimization model. Thus, the constrained multinomial logit model represents a computationally feasible improvement over the unconstrained model. The empirical validity of the theoretical restrictions on the structural coefficients can be examined by computing a likelihood ratio statistic, the value of which will be large when the restrictions are invalid.

The third model described in this section is a reduced-form variant of the multinomial probit model popularized by Daganzo (1979). The multinomial probit model allows for nonzero covariances among the stochastic utility indexes and, thus, avoids the IIA assumption that plagues the multinomial logit formulations. This feature makes the probit approach especially attractive from a theoretical perspective but there are two noteworthy drawbacks. First, the structural parameters of the multinomial probit model are not identified. Typically, this problem is solved by imposing prior restrictions on the structural coefficient vectors and the covariance matrix. Often, however, such identifying structural restrictions are unavailable so identification of the associated reduced-form model is attempted. This is the approach taken by Terza (1985) and McElroy (1985) and followed here. The second disadvantage of the multinomial probit model is that it is computationally burdensome since, in the trinomial choice problem, the bivariate normal distribution function must be iteratively approximated. Although, at present, estimation of the unconstrained reduced form of the trinomial probit model is barely feasible with data sets of the size mandated by the investigation at hand, imposition of the restrictions implied by the underlying utility structure would require considerable programming and computational expense. Formulation and estimation of a constrained reduced-form trinomial probit model of reenlistment choice would be a potentially important advance, since the IIA assumption could be relaxed and, simultaneously, the restrictions implied by stochastic utility theory could be imposed. However, these tasks are well beyond the scope of the present study.

### 3a. Unconstrained Trinomial Logit Model

Each first-term enlisted serviceman who is eligible for reenlistment is faced, toward the end of his or her term, with a choice among three discrete, mutually exclusive and exhaustive alternatives: (1) separation from the service at the end of the first term; (2) a short-term extension of the present enlistment contract for a period of up to three years; and (3) a long-term reenlistment for a period of three to six years. The serviceman is assumed to choose the alternative from which he will derive the maximum utility or satisfaction. Each alternative, then, has a utility level associated with it that has two components: (1) a systematic or deterministic part which is a function of both the attributes (consequences) of the choice and the characteristics of the serviceman; and (2) a stochastic component, representing person-specific characteristics [like the taste (or distaste) for Army life, patriotic zeal, and innate ability in or aptitude for service-connected tasks] that cannot be readily measured or observed.

Formally, let  $j = 1, 2, 3$  index the three alternatives, so that  $j = 1$  if the serviceman chooses to separate from the Army,  $j = 2$  if he or she decides to extend, and  $j = 3$  if reenlistment is chosen. Furthermore, denote the utility level associated with the  $j$ -th choice as  $U_j$  and assume that it is linear in the attributes of the choice and  $j$  characteristics of the serviceman and the stochastic component,  $\epsilon_j$ . Then, for the  $j$ -th alternative faced by the  $t$ -th serviceman, we have

$$(1) \quad U_{jt} = X_{jt}\beta_j + \epsilon_{jt} \quad t = 1, \dots, T$$

where  $X_{jt}$  is a  $(1 \times (M+1))$  vector of explanatory variables with corresponding  $((M+1) \times 1)$  fixed coefficient vector  $\beta_j$ , and  $\epsilon_{jt}$  is an alternative-specific error term with  $E[\epsilon_{jt}] = 0$ ,  $\text{cov}(\epsilon_{jt}, \epsilon_{kt}) = 0$  for  $j \neq k$ , and  $\text{cov}(\epsilon_{jt}, X) = 0$ . The  $t$ -th serviceman will choose alternative  $j$  among the three available options if and only if

$$(2) \quad U_{jt} > U_{kt} \text{ for all } k \neq j.$$

Of course, we cannot observe the utility levels associated with the three alternatives but only the choices themselves, represented by the random variable  $y_t$ , where

$$y_t = \begin{array}{l} 1 \text{ if separation is chosen} \\ 2 \text{ if extension is chosen} \\ 3 \text{ if reenlistment is chosen.} \end{array}$$

Substituting (1) into (2), alternative  $j$  will be chosen ( $y = j$ ) if and only if

$$(3) \quad \epsilon_{kt} - \epsilon_{jt} < X_{jt}\beta_j - X_{kt}\beta_k \quad j \neq k$$

Alternatively, the probability that the  $j$ -th alternative is chosen  $P(y = j)$  is

$$(4) \quad P(y = j) = P(\epsilon_{kt} - \epsilon_{jt} < X_{jt}\beta_j - X_{kt}\beta_k) \quad \forall k \neq j.$$

Since we cannot observe  $\epsilon_t = [\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}]$  we are unable to calculate  $\epsilon_{kt} - \epsilon_{jt}$  and, therefore, cannot determine whether or not the inequality in (3) holds. If, however, we are willing to make an explicit assumption about the probability distribution of  $\epsilon_t$  in the population, then we can estimate the probability function (4). Let us follow McFadden (1974) in this regard and assume that the  $\epsilon_{jt}$  are independently and identically distributed according to the Gumbel (extreme value) distribution, the density function for which is

$$(5) \quad f(a) = \exp\{-e^{-a}\}.$$

Then the difference  $Z_{jt} = \epsilon_{jt} - \epsilon_{kt}$  has a Sech<sup>2</sup> distribution with density function

$$(6) \quad f(Z_{jt}) = \exp\{Z_{jt}\} / (1 + \exp\{Z_{jt}\})^2$$

and a logistic cumulative distribution function. Therefore, the (logistic) probability that alternative  $j$  will be chosen by the  $t$ -th individual is given by

$$(7) \quad P_{jt} = \exp\{X_{jt}\beta_j\} / \left( \sum_{k=1}^3 \exp\{X_{kt}\beta_k\} \right) \quad \forall j \neq k.$$

In order to identify the parameters in the choice probability equations, the normalization  $\beta_{1m} = 0$  ( $m = 0, 1, 2, \dots, M$ ) was chosen. This normalization sets the coefficients of the explanatory variables for the base group (those who separate) equal to zero so that the coefficients of the determinants of the other two choice alternatives are interpreted relative to those of the first.

The specification of the vector of individual traits and economic incentives ( $X_{jt}$ ) affecting the serviceman's decision is taken from Lakhani and Gilroy (1984). Three economic variables appear in their model. The first is the selective reenlistment bonus (SRB) for which certain first-term servicemen are eligible. The amount of this bonus varies across eligible individuals for three reasons. First, the bonus varies across military occupation specialties (MOS) according to the degree to which the MOS skill is presently deemed in critical demand or essential for combat readiness. Second, the amount of the bonus varies directly with the size of the serviceman's monthly basic pay. Third, the bonus increases with the years of additional obligated service upon reenlistment. Lakhani and Gilroy (1984) calculated an SRB amount not only for the reenlistees, who actually received it, but also for those eligible first-term enlistees who had an opportunity to reenlist but decided instead to extend or separate. The estimation of the SRB for separatees was based on the assumption that they would have reenlisted for the average term in their specific Career Management Field (CMF). Lakhani and Gilroy (1984) argued that an increase in the SRB is expected to decrease the extension probability but increase the probability of reenlistment since extendees are ineligible for the SRB.

The second economic determinant of reenlistment choice in the Lakhani-Gilroy model is the ratio of military pay to the civilian wage alternative (RMCCW). For each serviceman in the sample, military pay was

defined as his or her Regular Military Compensation (RMC) and is comprised of basic pay, basic subsistence and quarters allowances, the variable housing allowance, the estimated federal tax advantage of military service, and adjustments for pay grade, years of service, and marital status. The civilian wage alternative for each enlistee was proxied by the predicted values of a standard wage model estimated with data from the National Longitudinal Survey (NLS) on males aged nineteen through twenty-two who have completed no more than two years of college. An increase in the relative pay variable is expected to have a positive effect on both reenlistment and extension probabilities.

The third and final economic variable assumed to affect the reenlistment decision is the unemployment rate of the state of residence of the individual at the time of enlistment (HU3). The home-state unemployment rate was chosen, rather than the unemployment rate of the state in which the last tour of duty occurred, in order to reflect the assumption that servicemen who separate after the first term are more likely to return to their original state of residence than to any other single destination. Since the reenlistment or extension decision must be made several months prior to the end of the term of service, Lakhani and Gilroy (1984) lagged this variable three months and predicted that it would positively affect the probabilities of both extending and reenlisting.

The person-specific demographic characteristics which Lakhani and Gilroy (1984) included in their model were the mental category of the serviceman (CAT), whose value equals one for individuals who scored in the upper half of the Armed Forces Qualification Test and zero otherwise; a dichotomous variable (ETHNIC) set equal to one if the enlistee is black and zero otherwise; and number of dependents (DEPENDS). Lakhani and Gilroy (1984) argued that the number of dependents should be positively related to both reenlistment and extension because servicemen with dependents are more likely than singles to want to avoid the geographical relocation that typically occurs with separation from the military. They contended, furthermore, that the coefficient on the ETHNIC variable should be positive since blacks may face greater discrimination in the civilian labor market than in military service and thus would be more likely than whites to remain in the Army, other things being equal. Finally, the effect of mental ability category (CAT) on the probability of reenlistment or extension is theoretically indeterminate.

As a consequence of these specification decisions by Lakhani and Gilroy (1984), the underlying utility structure for the unconstrained trinomial logit model can be written as follows. The index ( $t = 1, 2, \dots, T$ ) of observations has been suppressed for notational convenience and the first and second subscripts on the structural coefficients represent, respectively, the  $j$ -th alternative and the  $m$ -th explanatory variable ( $m = 0, 1, 2, \dots, M$ )

(8)

$$U_1 = \beta_{10} + \beta_{11}SRB + \beta_{12}CAT + \beta_{13}ETHNIC + \beta_{14}DEPENDS + \beta_{15}HU3 + \beta_{16}RMCCW + \epsilon_1$$

(9)

$$U_2 = \beta_{20} + \beta_{21}SRB + \beta_{22}CAT + \beta_{23}ETHNIC + \beta_{24}DEPENDS + \beta_{25}HU3 + \beta_{26}RMCCW + \epsilon_2$$

(10)

$$U_3 = \beta_{30} + \beta_{31}SRB + \beta_{32}CAT + \beta_{33}ETHNIC + \beta_{34}DEPENDS + \beta_{35}HU3 + \beta_{36}RMCCW + \epsilon_3.$$

This utility structure is the operational version, for the Lakhani-Gilroy specification, of the theoretical model given in (1) above. Because of the normalization  $\beta_{1m} = 0$  ( $m = 0, 1, \dots, M$ ), the coefficients  $\beta_{jm}$  ( $j = 2, 3; m = 0, 1, \dots, M$ ) are interpreted as marginal effects on the utility of the  $j$ -th alternative relative to separation. This is the (unconstrained) trinomial model of extension and reenlistment specified and estimated by Lakhani and Gilroy (1984). The results from reestimating this model with the revised data provided by the Army Research Institute are presented in Tables 3, 4 and 5 and are discussed in Section 5 below.

### 3b. Constrained Trinomial Logit Model

When the underlying utility structure of a multinomial logit model contains explanatory variables which are specific to particular choice alternatives, it is intuitively appealing to consider imposing zero restrictions on the structural coefficients of the alternative-specific variables. Although this seems to be a natural course to follow, it has apparently not been standard practice in the literature on the stochastic utility model. Because these restrictions are seldom imposed, it is instructive to consider explicitly their theoretical justification and implications in the context of the present study.

Consider first the explanatory variable which measures the amount of the selective reenlistment bonus (SRB). Since, as noted above, the SRB is received only by those eligible enlistees who reenlist ( $y_t = 3$ ) and therefore is not available to those who separate ( $y_t = 1$ ) or extend ( $y_t = 2$ ), an increase in the SRB affects  $U_3$  but does not affect either  $U_1$  or  $U_2$ . Consequently, the structural coefficients  $\beta_{11}$  and  $\beta_{21}$  - which capture, respectively, the partial equilibrium effects of an increase in the SRB on the individual's utility when he or she separates or extends - must be zero. That is, the restriction  $\beta_{11} = \beta_{21} = 0$  should logically be imposed on the utility structure specified in equations (8) - (10). The reduced-form coefficients obtained upon implementing the identifying normalization are  $\pi_{2m} = \beta_{2m} - \beta_{1m}$  and  $\pi_{3m} = \beta_{3m} - \beta_{1m}$  ( $m = 0, 1, 2, \dots, M$ ). Therefore, the structural constraints  $\beta_{11} = \beta_{21} = 0$  translate into the reduced-form constraint  $\pi_{21} = 0$ .

Another alternative-specific explanatory variable in the Lakhani-Gilroy model is the lagged home-state unemployment rate (HU3). The theoretical justification for the inclusion of this variable in the model is that individuals who are comparing the attractiveness of remaining in the Army (by either extending or reenlisting) relative to

separating from the military will consider the unemployment risk associated with the civilian labor market. However, this risk is relevant only to those who actually choose to separate ( $y_t = 1$ ); increases in civilian unemployment risk do not affect the utility levels attained by individuals who choose to extend or reenlist. Therefore, it would seem necessary to impose the (structural) zero restrictions  $\beta_{25} = \beta_{35} = 0$ . These structural restrictions imply the reduced-form constraint  $\pi_{25} = \pi_{35}$ , since  $\pi_{25} = \beta_{25} - \beta_{15}$  and  $\pi_{35} = \beta_{35} - \beta_{15}$ .

Finally, the ratio of military compensation to estimated civilian wage (RMCCW) captures an important aspect of the relative desirability of military and civilian employment. However, a change in RMCCW does not affect the enlistee's relative evaluation of the reenlistment and extension options. As a consequence, the coefficients on this variable in the equations that determine the utility levels associated with these two alternatives should be equal. Since the reduced-form coefficients on RMCCW are related to the structural coefficients by the equations  $\pi_{26} = \beta_{26} - \beta_{16}$  and  $\pi_{36} = \beta_{36} - \beta_{16}$ , the equality restriction  $\beta_{26} = \beta_{36}$  on the latter implies the equality constraint  $\pi_{26} = \pi_{36}$  on the former.

With the theoretically appropriate zero and equality restrictions imposed, the utility structure for the enlistee's choice problem can be written

(11)

$$U_1 = \beta_{10} + \beta_{11}CAT + \beta_{12}CAT + \beta_{13}ETHNIC + \beta_{14}DEPENDS + \beta_{15}HU3 + \beta_{16}RMCCW + \epsilon_1$$

(12)

$$U_2 = \beta_{20} + \beta_{21}SRB + \beta_{22}CAT + \beta_{23}ETHNIC + \beta_{24}DEPENDS + \beta_{25}HU3 + \beta_{26}RMCCW + \epsilon_2$$

(13)

$$U_3 = \beta_{30} + \beta_{31}SRB + \beta_{32}CAT + \beta_{33}ETHNIC + \beta_{34}DEPENDS + \beta_{35}HU3 + \beta_{36}RMCCW + \epsilon_3.$$

Now denote the utility difference  $U_{jt} - U_{1t}$  by  $V_{jt}$  ( $j = 2, 3$ ). Then the corresponding constrained reduced form of this model is

(14)

$$V_2 = U_2 - U_1 = \pi_{20} + \pi_{21}SRB + \pi_{22}CAT + \pi_{23}ETHNIC + \pi_{24}DEPENDS + \pi_{25}HU3 + \pi_{26}RMCCW + Z_2$$

(15)

$$V_3 = U_3 - U_1 = \pi_{30} + \pi_{31}SRB + \pi_{32}CAT + \pi_{33}ETHNIC + \pi_{34}DEPENDS + \pi_{35}HU3 + \pi_{36}RMCCW + Z_3.$$

The estimates of this constrained logit model appear in Tables 6, 7, and 8 and are compared in Section 5 below with those obtained from the unconstrained model.

### 3c. Reduced-Form Trinomial Probit Model

Although the constrained multinomial logit model represents an improvement over the unconstrained model, in the sense that the former approach is more consistent than the latter with the utility-theoretic structure of the choices, it shares the drawback of imposing the restrictive preference property of the independence from irrelevant alternatives (IIA). In the context of a trinomial choice problem, this property requires that the relative probabilities of any pair of alternatives be determined in the model independently of the nature of the third alternative. Although this feature may be innocuous when the choices are qualitatively dissimilar, it is clearly untenable when one or more pairs of alternatives share the same basic characteristics. In the present study, for example, it may be reasonable to view extension and reenlistment as similar alternatives when focussing on the effects of increases in the home-state unemployment rate or decreases in the ratio of military compensation to the estimated civilian wage on the enlistee's decision. Alternatively, when examining the effect of an increase in the SRB (which is paid out only to those who choose to reenlist) it may be sensible to categorize extension and separation as dissimilar in nature. With the multinomial logit model in either the unconstrained or constrained version, it is impossible to incorporate these types of interdependencies since the error terms in the utility equations are assumed to be uncorrelated.

One important implication of IIA and, thus, the multinomial logit model is that the true probability of choosing the relatively dissimilar alternative will be underestimated by the probability calculated under the IIA assumption. Suppose, for example, that extension and reenlistment are, taken together, regarded as being more similar in essential features than the pairs extension and separation or reenlistment and separation. Indeed, much of the previous empirical literature on military reenlistment assumed implicitly that extension and reenlistment were identical, since the enlistee's problem was often formulated as the binary choice between retention and separation. Then the probability of separating from the Army [ $P_1 = P(U_1 > U_2, U_1 > U_3)$ ] calculated from the multinomial logit estimates would almost surely underestimate the true probability of separation, since the IIA assumption ignores the fact that the evaluation  $U_1 > U_2$  makes the evaluation  $U_1 > U_3$  more likely to occur. In view of this bias implied by the IIA, the multinomial logit approach would seem to be inappropriate for modelling the enlistee's choice among separation, extension, and reenlistment. As McFadden (1974, p. 113) has argued, "applications of the [multinomial logit] model should be limited to situations where the alternatives can plausibly be assumed to be distinct and weighed independently in the eyes of each decision-maker." This stricture, when coupled with Samuelson's (1985) more general theoretical objections to the IIA assumption, severely limits the attractiveness of the multinomial logit model as a framework for representing the enlistee's choice problem.

Multinomial probit is a conceptually attractive alternative to multinomial logit in which the stochastic utility indices are assumed to be multivariate normally distributed. This assumption makes multinomial probit an especially desirable approach to the present problem since, unlike multinomial logit, it allows for nonzero covariances among the stochastic utility indices. Because of this feature, multinomial probit avoids the theoretically objectionable IIA assumption and its attendant bias. Multinomial probit was first proposed by Aitchison and Bennett (1970) but has until recently rarely been used in empirical applications because of its computational complexity. For example, in the trinomial case at hand in which there are  $J = 3$  alternatives, maximum likelihood estimation of the choice probabilities requires, at each step of the iterative process, multiple evaluations of the bivariate normal cumulative distribution function (cdf). This is particularly troublesome because a closed-form expression for the bivariate normal cdf does not exist. This function must instead be numerically approximated. Another difficulty with multinomial probit is that, without the imposition of some a priori restrictions or normalizations, the parameters of the utility structure are not identified. For example, in all of the specifications discussed by Daganzo (1979), restrictions are placed on the structural covariance matrix. Often, however, either such restrictions are unavailable or the available restrictions are insufficient to ensure identification. To deal with the latter situation, Daganzo (1979, pp. 93 - 94) suggested finding an identified reduced form of the structural model with appropriate normalizations or other combinations of the structural parameters. Following and generalizing upon this suggestion, Terza (1985) derived an admissible reduction of the unrestricted structural model. Terza's approach is best viewed, then, as an extension to the trinomial case of the unrestricted reduced-form estimator developed by Finney (1971) for the binomial probit model. In this regard, it also differs in important respects from the trinomial probit specification developed by Hausman and Wise (1978), in which the elements of the utility covariance matrix are functions of the observed attributes of the alternatives. In the Hausman-Wise model, therefore, the multinomial probit covariance matrix differs across observations. Terza's (1985) model, in contrast, assumes a fixed utility covariance matrix and considers only the reduced form of the unrestricted structure.

To denote the enlistees' observed choices within the trinomial probit context, we use the random variable  $y_t$  defined in Section 3a as

$$y_t = \begin{cases} 1 & \text{if separation is chosen} \\ 2 & \text{if extension is chosen} \\ 3 & \text{if reenlistment is chosen.} \end{cases}$$

The utility derived by the  $t$ -th enlistee from the  $j$ -th alternative is assumed to take the same form as that of equation (1) with all corresponding assumptions intact except for the zero covariances among the elements of the random error vector  $\epsilon_t$  and its hypothesized distribution. In the trinomial probit model,  $\epsilon_t$  is assumed to be trivariate normally distributed with mean vector zero and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix} .$$

In vector form, the structural parameters of the trinomial probit model can then be written

$$\theta = [\beta_1' \mid \beta_2' \mid \beta_3' \mid \sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}]$$

The probability density function (pdf) of  $Y_t$  is

$$f(y_t, \theta) = \begin{cases} P_{1t} & \text{if } y_t = 1 \\ P_{2t} & \text{if } y_t = 2 \\ P_{3t} & \text{if } y_t = 3 \end{cases}$$

where the choice probabilities are given by

$$P_{1t} = \int_0^{\infty} \int_0^{\infty} \Phi(X_t(\beta_1 - \beta_2), X_t(\beta_1 - \beta_3); \Omega_1) dQ_{12}dQ_{13}$$

$$P_{2t} = \int_0^{\infty} \int_{-\infty}^0 \Phi(X_t(\beta_1 - \beta_3), X_t(\beta_2 - \beta_3); \Omega_2) dQ_{12}dQ_{23}$$

$$P_{3t} = \int_{-\infty}^0 \int_{-\infty}^0 \Phi(X_t(\beta_1 - \beta_3), X_t(\beta_2 - \beta_3); \Omega_3) dQ_{13}dQ_{23}$$

and

$$\Omega_1 = \begin{bmatrix} \sigma_{11} + \sigma_{22} - 2\sigma_{12} & \\ \sigma_{11} - \sigma_{12} - \sigma_{13} + \sigma_{23} & \sigma_{11} + \sigma_{33} - 2\sigma_{13} \end{bmatrix}$$

$$\Omega_2 = \begin{bmatrix} \sigma_{11} + \sigma_{22} - 2\sigma_{12} & \\ -\sigma_{22} + \sigma_{12} - \sigma_{13} + \sigma_{23} & \sigma_{22} + \sigma_{33} - 2\sigma_{23} \end{bmatrix}$$

$$\Omega_3 = \begin{bmatrix} \sigma_{11} + \sigma_{33} - 2\sigma_{13} & \\ \sigma_{33} + \sigma_{12} - \sigma_{13} - \sigma_{23} & \sigma_{22} + \sigma_{33} - 2\sigma_{23} \end{bmatrix}$$

$Q_{jk} = U_{jt} - U_{kt}$  ( $j, k = 1, 2, 3$ ) and  $\phi(a, b; C)$  denotes the bivariate normal pdf with mean  $[a, b]'$  and covariance matrix  $C$ .

Before proceeding, it will be helpful to establish definitions concerning parametric models in general. Define  $y$  to be a random vector. The pdf of  $y$  is assumed to be uniquely determined by the value of a parameter vector  $\theta$  and is denoted  $f(y, \theta)$ . Each specific value of  $\theta$  is called a structure and the set  $A \subseteq R^n$  containing all possible structures is called a model. Within this context we have the following definitions:

DEFINITION 1: The vector-valued function,  $\tau$ , from  $A$  to  $R^p$  ( $p < n$ ) is said to be a reduction of the model  $A$  if for every structure  $\theta \in A$ : 1)  $\tau = \tau(\theta)$  defines a unique pdf,  $f^*(y, \tau)$ ; and 2)  $\tau$  does not have a unique image vector in  $A$ .

DEFINITION 2: The reduction  $\tau$  is said to be admissible for the model  $A$  if for all  $\theta \in A$   $f(y, \theta) = f^*(y, \theta)$  where  $\tau = \tau(\theta)$  and  $f^*$  is the pdf described in Definition 1.

Now consider the reduction  $\tau$  defined in the following way:

$$\tau_1(\theta) = (1/\sqrt{\sigma_3}) \sigma_1 = \tau_1$$

$$\tau_2(\theta) = (1/\sqrt{\sigma_3}) \sigma_2 = \tau_2$$

$$\tau_3(\theta) = \sigma_4 / \sigma_3 = \tau_3$$

$$\tau_4(\theta) = \sigma_5 / \sigma_3 = \tau_4$$

where

$$\sigma_1 = \beta_2 - \beta_1$$

$$\sigma_2 = \beta_3 - \beta_1$$

$$\sigma_3 = \sigma_{11} - \sigma_{12} - \sigma_{13} + \sigma_{23}$$

$$\sigma_4 = -\sigma_{22} + \sigma_{12} - \sigma_{13} + \sigma_{23}$$

$$\sigma_5 = \sigma_{33} + \sigma_{12} - \sigma_{13} - \sigma_{23}$$

To show that the reduction  $\tau$  is an admissible reduction, note that  $f(y_t, \theta) = f^*(y_t, \tau)$  for all  $y_t$  where  $\tau' = [\tau_1' \mid \tau_2' \mid \tau_3' \mid \tau_4']$

$$f^*(y_t, \tau) = \begin{cases} P_{1t}^* & \text{if } y_t = 1 \\ P_{2t}^* & \text{if } y_t = 2 \\ P_{3t}^* & \text{if } y_t = 3, \end{cases}$$

$$(16) \quad P_{1t}^* = N \left[ x_t \frac{-\tau_1}{\sqrt{1-\tau_3}}, x_t \frac{-\tau_2}{\sqrt{1+\tau_4}}; \frac{1}{\sqrt{(1-\tau_3)(1+\tau_4)}} \right]$$

$$(17) \quad P_{2t}^* = N \left[ x_t \frac{\tau_1}{\sqrt{1-\tau_3}}, x_t \frac{-(\tau_2-\tau_1)}{\sqrt{\tau_4-\tau_3}}; \frac{-\tau_3}{\sqrt{(1-\tau_3)(\tau_4-\tau_3)}} \right]$$

$$(18) \quad P_{3t}^* = N \left[ x_t \frac{\tau_2}{\sqrt{1+\tau_4}}, x_t \frac{(\tau_2-\tau_1)}{\sqrt{\tau_4-\tau_3}}; \frac{\tau_4}{\sqrt{(1+\tau_4)(\tau_4-\tau_3)}} \right],$$

and  $N[a,b;r]$  denotes the bivariate standard normal cdf evaluated at  $[a,b]$  with correlation  $r$ . An estimate of the vector  $\tau$  is obtained from a sample of size  $T$  by maximizing the following likelihood function,

$$(19) \quad L(\tau) = \prod_{t=1}^T P_{y_t}^*$$

As in Finney's (1971) binomial probit model, the estimated reduced-form parameters enable us to evaluate (for a given  $x_t$ ): 1) the choice probabilities, and 2) the effects of the explanatory variables on the choice probabilities.

Since the resulting estimates of the parameter vector  $\tau$  are maximum likelihood, they have the theoretically desirable properties of consistency, asymptotic normality, and asymptotic efficiency. Unfortunately, maximization of the likelihood function (19) is computationally burdensome since the bivariate normal probabilities in (16)-(18) must be evaluated at each iteration of the maximization procedure. Arza (1985) has demonstrated the computational feasibility of the maximum likelihood estimator of the reduced-form trinomial probit model but at the same time noted its expense in terms of the number of iterations and central processor time required. It would, of course, be highly desirable to impose on the reduced-form trinomial probit parameters the constraints implied by the theoretical restrictions on the underlying utility structure which were discussed in Section 3b in connection with the multinomial logit model. In that way, not only could the untenable IIA assumption be avoided but also the restrictions implied by stochastic utility theory could be imposed. Unfortunately, estimation of such a constrained reduced-form trinomial probit model would require considerable developmental efforts in programming the computer software in a computationally efficient and theoretically consistent manner. While such efforts would yield an algorithm which would be extremely useful in

estimating a theoretically sound model of the relevant choice problem faced by first-term Army enlistees, they are well beyond the scope of the present study.

Before turning to the empirical implementation of these models, it is interesting and important to note that under the IIA assumption the values of the two covariance parameters  $\tau_3$  and  $\tau_4$  in the trinomial probit model would be -1 and +1, respectively. To see this, recall that  $\tau_3 = \sigma_4/\sigma_3$  and  $\tau_4 = \sigma_5/\sigma_3$ . However, under the assumption that the structural equation error terms are identically and independently distributed (which is equivalent to the IIA hypothesis),  $\sigma_3 = \sigma_5 = -\sigma_4$ , so that  $\tau_3 = -1$  and  $\tau_4 = 1$ . The IIA assumption, then, is a testable hypothesis within the context of the reduced-form probit model.

#### 4. Data Base and Estimation Technique

To estimate the effects of demographic and economic variables on the probabilities of reenlisting, extending, or separating from the U.S. Army, data were obtained primarily from the Enlisted Master File (EMF) for fiscal years 1980 and 1981. From a special match of these files carried out by the U.S. Army Research Institute, the first-term enlistees who were eligible for reenlistment in FY 1981 were determined. These enlistees could choose, during an "open window" period of between twenty-seven and thirty-six months of service, to extend their current enlistment for a period of up to three years, reenlist for a period of between three and six years, or separate from service in the Army.

In FY 1981, there were more than three hundred Military Occupation Specialties (MOS), of which one hundred and thirty-one were eligible for the first-term SRB. In order to reduce the number of separate equations to be estimated to a manageable number, conserve on degrees of freedom within each occupation, and provide more variability across the occupations, the MOS were grouped into fifteen Career Management Fields (CMF), which were constructed to be as occupationally homogeneous as possible. Only those MOS for which a SRB was actually paid were included in the sample. The resulting MOS, along with their corresponding CMF, are listed in Table 1.

The variable  $y$  (defined in Section 3) was used to represent the serviceman's choice among separation from the Army, extension for up to three years, and reenlistment for a period of from three to six years. Recall that this dependent variable is defined as follows:  $y = 1$  (separation),  $y = 2$  (extension), and  $y = 3$  (reenlistment). The set of explanatory variables ( $X$ ) used to predict the choice among the three alternatives was discussed briefly in Section 3a above and in more detail by Lakhani and Gilroy (1984).

Sample means for these variables are presented in Table 2. The sample sizes for each CMF are uniformly lower than those given in Lakhani and Gilroy (1984) for two reasons. First, errors in the earlier categorization of some servicemen into high and low mental ability (CAT) were discovered by the Army Research Institute and these observations were deleted from the sample. Second, observations with missing or obviously

TABLE 1

## MILITARY OCCUPATIONAL SPECIALITIES (MOS) AND CAREER MANAGEMENT FIELDS (CMF)

Number	MOS Description	CMF Number
05D	EW/SIGINT Ident/Loc.	98
05G	SIGSEC Specialist	98
05H	EW/SIGINT Morse Interceptor-IMC	98
05K	EW/SIGINT NMors Interceptor	98
11B	Infantryman	11
11C	Indirect fire infantryman	11
11H	Heavy antiarmor wpns infantryman	11
12B	Combat engineer	12
12C	Bridge crewman	12
12E	ADM (Atomic demol mun) spec	12
12F	Engr TRVEH (tracked veh) crewman	12
13B	Cannon crewman	13
13C	TACFIRE Opns Spec	13
13E	Cannon FD (fire dir) spec	13
13F	Fire support spec	13
13R	Firefinder radar opr	13
15D	Lance missile crew mbr/MLRS Sgt	13
15E	PERSHING missile crew mbr	13
15J	MLRS/LANCE op/fire dir spec	13
16B	HERCULES missile crew mbr	16
16C	HERCULES fire control crew mbr	16
16D	HAWK missile crew mbr	16
16E	HAWK fire control crew mbr	16
16R	ADA Short Range Gunnery crewman	16
19D	Cavalry scout	19
19E	M48-M60 armor crewman	19
19F	MR8/60 tank driver	19
19G	Armor recon veh crewman	19
19H	Armor recon veh crewman	19
19J	M60A2 armor crewman	19
19K	XMI armor crewman	19
24E	IH fire control mech	23
24G	IH (info) coordinator Cen Mech	23
24H	IH fire control repairer	23
24K	IH CW radar repairer	23
24N	CHAPPARAL sys mech	27
24U	HERCULES elect mech	23
26B	Wpns Spt Rdr Rpr	29
26E	Aerl survival sensor rpr	28
26Q	Tac microwave sat sys opr	31
26R	Strtgc microwave sat sys opr	31
26V	Strtgc microwave sys rpr	29
26Y	SATCOM equip rep	29
27E	TOW/DRAGON rpr	27
27F	VULCAN repairer	27
27G	CHAPARRAL/REDEYE rpr	27
27N	FAAR rep	27
31J	Telotypewriter rep	29

TABLE 1 (continued)

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31S	Field gen COMSEC rep	29
31T	Field sys COMSEC rep	29
31V	Tac comm sys op/mech	31
32D	Statn tech controller	31
32F	Fixed ciphony rep	29
35H	Calibration specialist	29
35L	Avionic comm equip rpr	28
35M	Avionic Nav/fit con eq rpr	28
35R	Avionic special equip rpr	28
35U	Biomed Equip Sp Adv	91
36K	Tac wire op sp	31
36L	Elec switching sys rep	29
44E	Machinist	63
45D	SPFA (Field Artillery) turret mech	63
45E	M1 ABRAMS turret mech	63
45G	FC systems rep	63
45K	Tank repr	63
45N	M60A1/A3 turret mech	63
45T	Itv/ifv/cfv turret mech	63
63B	Lt wt veh & pwr gen mech	63
63D	Sp FA system mech	63
63E	M1 ABRAMS tank sys mech	63
63N	M60A1/A3 tank sys mech	63
63S	Hvy wheel veh mech	63
63T	ITV/IFV/CFV sys mech	63
63Y	Track veh mech	63
91B	Medical specialist	91
91C	Patient care specialist	91
91D	Operating room specialist	91
91F	Psychiatric specialist	91
91G	Behavioral science specialist	91
91H	Orthopedic specialist	91
91J	Psychiatric therapy specialist	91
91Q	Pharmacy specialist	91
91R	Veterinary specialist	91
91S	Environmental health specialist	91
91U	ENT specialist	91
91V	Respiratory specialist	91
91W	Nuclear med specialist	91
93F	FA met crew mbr	13
93H	ATC tower operator	64
93J	ATC radar controller	64
96B	Intelligence analyst	96
96C	Interrogator	96
96D	Image interpreter	96
96H	Aer SNS Sp OV-ID	96
97B	CI (Central Intelligence) agent	96
98C	EW/SIGINT analyst	98
98G	EW/SIGINT voice intep	98
98J	EW/SIGINT NC (Non Com) intecp	98

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TABLE 2

DESCRIPTIVE STATISTICS  
(Mean Values of Variables)

CMF	ELIGIBLES	REENLIST (%)	EXTEND (%)	SRB (\$)	RMCCW	HU3	CAT (%)	ETHNIC (%)	DEPENDS
11	4144	23.2	17.8	3785	1.10	6.81	37.0	43.7	0.57
12	1061	19.1	16.6	3722	1.11	7.34	35.2	24.2	0.47
13	2025	25.1	18.5	3725	1.10	6.91	30.0	53.4	0.60
16	548	20.3	14.2	3764	1.09	7.08	19.1	59.6	0.51
27	159	18.9	19.5	3732	1.11	7.34	57.9	30.0	0.51
29	240	22.9	12.9	3743	1.11	7.01	70.0	21.3	0.43
31	1069	21.7	16.8	3776	1.08	6.81	27.3	54.6	0.41
63	2354	20.9	17.2	3682	1.11	7.14	28.2	30.8	0.53
64	111	18.9	23.4	3898	1.10	7.44	80.2	19.8	0.69
91	1557	21.5	14.7	3829	1.09	7.27	61.6	32.1	0.54
96	175	23.4	13.7	3892	1.10	7.34	78.3	13.1	0.57
98	589	18.5	17.2	3630	1.11	7.14	77.8	16.0	0.46

incorrect values for one or more variables (for example, having more than seven dependents) were removed from the data set. Because of these deletions, of course, sample means for each occupation reported here differ slightly from those implied by Tables 3 and 4 of Lakhani and Gilroy (1984).

Before the estimation techniques used in this study are discussed, mention should be made of a potential downward bias in the construction of the variable representing the ratio of actual military compensation to potential civilian pay, RMCCW. In order to estimate the civilian wage alternative for enlisted personnel on the verge of their reenlistment decision, Lakhani and Gilroy (1984) obtained data from the National Longitudinal Surveys on full-time employed males aged 19 to 22 who had completed no more than two years of college and who earned at least \$1000 in the survey year. A conventional wage equation was estimated with these data and the significant coefficients from the estimated equation were multiplied by the values of the explanatory variables for each of the enlistees in the EMF sample. These products were then summed across explanatory variables (including the estimated intercept term) to arrive at an imputed civilian wage for each enlistee. One potentially important problem with this imputation procedure is that it assumes that individuals with a given set of values of the observable explanatory variables are randomly distributed between the civilian and military sectors. In the context of an all-volunteer Army, however, this is clearly incorrect. Presumably, individuals sort themselves optimally on the basis of comparative advantage in the two sectors. As a result, the imputed wages obtained with data on individuals in the civilian sector would systematically overstate the true civilian wage potential of the representative enlisted man and the RMCCW variable would be biased downward. Consequently, the estimated coefficient on the RMCCW variable constructed in this manner would be biased upward.

The parameters of the unconstrained and constrained multinomial logit models, as well as the reduced-form trinomial probit model, were estimated using maximum likelihood techniques. Estimates of the logit models were obtained with the LOGIT procedure in the LIMDEP program developed by Greene (1983). This procedure secures identification of these models by imposing the normalization that the structural coefficients of the explanatory variables in the equation determining the utility associated with separating from the military are all zero. The zero and equality restrictions on the structural parameters in the constrained logit model were imposed with the FIX and CNSTRN options, respectively. Newton's method was used to solve iteratively for the parameter values which maximize the likelihood function. This algorithm, which requires the calculation of the Hessian (matrix of second-order partial derivatives) of the function, is the default option for the LOGIT procedure in LIMDEP and was judged most likely among available alternatives to converge quickly at the least expense. The probit model was estimated using an algorithm developed and discussed by Terza (1985). The Davidon-Fletcher-Powell (DFP) technique was employed to maximize the likelihood function given in (19) above. Because of the complexity of this likelihood function and the associated computational expense, the two largest occupational samples, CMF11 (infantry) and CMF63 (armored), were split into four and two equal-sized subsamples, respectively, which were then estimated

separately. Finally, three occupations (CMF19, CMF23, and CMF28) analyzed by Lakhani and Gilroy (1984) were deleted from this study because either small sample sizes (CMF23 and CMF28), missing observations in one or more of the categories of the dependent variable (CMF19), or an ill-conditioned data matrix prevented the nonlinear optimization routines from converging to a stable set of parameter values prior to twenty-five iterations. Lakhani and Gilroy (1984, p. 17) also reported that their unconstrained multinomial logit model would not converge for CMF23 and CMF28 and attributed this to multicollinearity.

## 5. Empirical Results

### 5a. Unconstrained Trinomial Logit

The first set of results reported (in Tables 3, 4, and 5) are those obtained with the unconstrained multinomial logit model.<sup>8</sup> These estimates provide a useful comparison with the results reported by Lakhani and Gilroy (1984), as well as a benchmark against which the constrained multinomial logit and reduced-form trinomial probit results can be compared. Tables 3 and 4 contain estimates of the parameters of the model for first-term enlistees who chose to extend and reenlist, respectively. The most striking feature of these results, in comparison with those obtained by Lakhani and Gilroy (1984), is the number of estimated coefficients that are not significantly different from zero. Although it is impossible to determine precisely the reason for this diminished performance of the model, the reduced sample sizes and consequent decrease in the variability across observations in the values of the explanatory variables may have contributed to this difference.

The economic variables (SRB, HU3, and RMCCW) rendered a mixed performance in explaining both the extension and reenlistment decisions. Clearly, the most successful of these is the SRB variable which, with one exception (CMF96), exhibited the theoretically anticipated signs for both extension and reenlistment and was almost uniformly significantly different from zero. It should be mentioned, however, that the separation elasticities (the percentage change in the probability of separating from the Army of a one percent increase in the SRB, evaluated at the sample means of the other explanatory variables) are uniformly positive, as indicated in Table 5, in contrast to the negative effect expected theoretically. Since Lakhani and Gilroy (1984) did not report estimated elasticities for the separation category, this anomalous result has no explicit counterpart in their study. The elasticities of extension with respect to SRB are comparable in magnitude to those given by Lakhani and Gilroy (1984, Table 9). In contrast, however, the reenlistment elasticities are generally lower than theirs but higher than those presented by Goldberg and Warner (1982). The most disappointing set of results among the economic variables is the estimated coefficients on the relative pay variable, RMCCW. These coefficients (and the associated elasticities reported in Table 5), which Lakhani and Gilroy (1984) expected to be positive for both extendees and reenlistees, change sign frequently across occupations and are generally not significantly

TABLE 3

## UNCONSTRAINED LOGIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE EXTENSION EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW
11	-1.584 (-1.4341) [-0.112]	-0.564 (-20.36) [-0.061]	-0.001 (-0.009) [0.002]	0.274 (2.439) [0.022]	0.185 (2.461) [0.017]	-0.063 (-2.6033) [-0.005]	2.041 (2.050) [0.192]
12	-2.593 (-1.637) [-0.231]	-0.508 (-9.551) [-0.054]	0.238 (1.196) [0.021]	0.466 (1.951) [0.041]	0.188 (1.392) [0.018]	-0.035 (-0.727) [-0.003]	2.409 (1.738) [0.248]
13	1.039 (0.772) [0.202]	-0.586 (-14.433) [-0.066]	-0.334 (-2.055) [-0.032]	0.259 (1.644) [0.016]	0.032 (0.334) [-0.002]	-0.062 (-1.941) [-0.007]	-0.079 (-0.066) [-0.050]
16	-3.916 (-1.349) [-0.208]	-0.514 (-6.510) [-0.047]	0.525 (1.394) [0.036]	0.681 (1.961) [0.055]	0.118 (0.550) [0.007]	-0.013 (-0.199) [0.001]	3.096 (1.186) [0.170]
27	0.912 (0.174) [0.014]	-0.445 (-3.513) [-0.057]	1.090 (2.090) [0.147]	-0.434 (-0.767) [-0.014]	-0.333 (-0.799) [-0.043]	0.021 (0.146) [0.002]	-1.185 (-0.249) [-0.027]
29	2.269 (0.478) [0.196]	-0.546 (-4.449) [-0.041]	0.096 (0.191) [0.006]	0.317 (0.512) [0.010]	0.315 (0.946) [0.022]	-0.034 (-0.291) [-0.001]	-2.158 (-0.509) [-0.167]
31	4.059 (2.024) [0.386]	-0.544 (-9.931) [-0.057]	-0.150 (-0.659) [-0.004]	0.182 (0.803) [0.016]	-0.262 (-1.699) [-0.023]	0.061 (1.288) [0.007]	-3.803 (-2.060) [-0.340]
63	-1.442 (-1.046) [-0.108]	-0.524 (-14.685) [-0.058]	0.085 (0.608) [0.019]	0.372 (2.398) [0.033]	0.159 (1.625) [0.017]	-0.073 (-2.305) [-0.005]	1.754 (1.434) [0.162]
64	-3.248 (-0.576) [-0.375]	-0.676 (-3.768) [-0.087]	-0.220 (-0.274) [-0.011]	0.085 (0.121) [0.045]	-0.293 (-0.711) [-0.034]	0.142 (0.895) [0.022]	3.356 (0.642) [0.385]
91	-0.292 (-0.188) [-0.011]	-0.486 (-10.978) [-0.046]	-0.035 (-0.207) [0.001]	0.347 (1.869) [0.027]	0.055 (0.494) [0.005]	0.010 (0.230) [0.001]	0.079 (0.056) [0.022]
96	-3.557 (-0.778) [-0.232]	-0.574 (-4.050) [-0.044]	0.579 (0.856) [0.058]	1.118 (1.402) [0.086]	0.189 (0.572) [0.017]	-0.058 (-0.463) [-0.007]	3.211 (0.805) [0.227]
98	1.426 (0.914) [0.174]	-0.540 (-7.470) [-0.059]	0.026 (0.082) [0.007]	-0.097 (-0.264) [-0.016]	-0.093 (-0.530) [-0.009]	-0.063 (-0.911) [-0.010]	-0.631 (-0.490) [-0.070]

TABLE 4

## UNCONSTRAINED LOGIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE REENLISTMENT EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	Pred. Acc.
11	-1.875 (-2.034) [-0.299]	0.068 (4.099) [0.029]	-0.072 (-0.809) [-0.013]	0.244 (2.568) [0.037]	0.082 (1.293) [0.010]	-0.050 (-2.460) [-0.007]	0.787 (0.944) [0.085]	56.20
12	-1.538 (-0.936) [-0.183]	0.083 (2.450) [0.025]	0.136 (0.770) [0.016]	0.289 (1.366) [0.035]	0.037 (0.294) [0.002]	-0.016 (-0.350) [-0.002]	-0.048 (-0.032) [-0.062]	62.57
13	-2.753 (-2.312) [-0.566]	0.081 (3.347) [0.035]	-0.131 (-0.999) [-0.015]	0.364 (2.784) [0.062]	0.166 (2.024) [0.031]	-0.007 (-0.271) [0.001]	1.273 (1.201) [0.249]	54.78
16	-7.052 (-2.910) [-1.022]	0.178 (3.912) [0.037]	0.516 (1.694) [0.070]	0.200 (0.713) [0.019]	0.184 (1.019) [0.026]	-0.142 (-2.345) [-0.022]	5.307 (2.443) [0.766]	65.50
27	4.027 (0.840) [0.546]	0.131 (1.374) [0.029]	-0.604 (-1.271) [-0.112]	-1.596 (-2.492) [-0.215]	0.101 (0.320) [0.022]	0.031 (0.243) [0.004]	-4.842 (-1.113) [-0.655]	57.86
29	-1.420 (-0.383) [-0.299]	0.039 (0.572) [0.017]	0.065 (0.167) [0.010]	0.708 (1.556) [0.121]	0.053 (0.199) [0.004]	-0.059 (-0.670) [-0.010]	0.366 (0.113) [0.107]	65.83
31	1.028 (0.600) [0.074]	0.071 (2.113) [0.026]	-0.458 (-2.279) [-0.076]	0.091 (0.468) [0.011]	-0.147 (-1.133) [-0.019]	-0.040 (-0.926) [-0.008]	-1.824 (-1.157) [-0.219]	59.25
63	-1.756 (-1.463) [-0.255]	0.103 (4.556) [0.030]	-0.395 (-2.999) [-0.068]	0.248 (1.858) [0.032]	0.010 (0.111) [-0.002]	-0.110 (-3.844) [-0.016]	0.930 (0.870) [0.110]	58.60
64	-1.200 (-0.186) [-0.092]	0.090 (0.875) [0.032]	-0.602 (-0.881) [-0.085]	-1.252 (-1.460) [-0.191]	-0.094 (-0.223) [-0.006]	-0.150 (-0.906) [-0.026]	1.347 (0.228) [0.111]	65.76
91	-0.689 (-0.535) [-0.112]	0.062 (2.382) [0.022]	-0.205 (-1.481) [-0.035]	0.208 (1.342) [0.028]	-0.011 (-0.123) [-0.003]	0.014 (0.415) [0.002]	-0.646 (-0.551) [-0.113]	62.98
96	-2.974 (-0.866) [-0.465]	-0.163 (-1.797) [-0.017]	-0.430 (-0.979) [-0.091]	0.325 (0.523) [0.035]	-0.057 (-0.205) [-0.015]	0.112 (1.214) [0.022]	1.906 (0.637) [0.277]	62.85
98	-0.880 (-0.572) [-0.170]	0.042 (0.823) [0.019]	-0.160 (-0.578) [-0.025]	0.254 (0.818) [0.042]	-0.033 (-0.211) [-0.003]	-0.074 (-1.214) [-0.010]	0.077 (0.059) [0.027]	61.46

TABLE 5

## UNCONSTRAINED LOGIT ELASTICITY ESTIMATES

CMF	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW
SEPARATION						
11	0.190	0.638	-4.043	-0.024	0.128	-0.478
12	0.157	-1.891	-2.670	-0.014	0.053	-0.300
13	0.188	2.296	-6.782	-0.028	0.068	-0.356
16	0.053	-2.839	-6.186	-0.024	0.209	-1.431
27	0.152	-2.946	9.986	0.016	-0.064	1.100
29	0.131	-1.638	-4.081	-0.016	0.113	0.097
31	0.177	3.302	-2.229	0.026	0.010	0.913
63	0.154	2.065	-2.991	-0.012	0.224	-0.451
64	0.321	11.523	4.327	0.041	0.045	-0.817
91	0.136	3.100	-2.613	-0.002	-0.032	0.147
96	0.356	3.876	-2.378	-0.002	-0.165	-0.832
98	0.212	2.043	-0.607	0.008	0.208	0.070
EXTENSION						
11	-1.932	0.619	8.044	0.081	-0.285	1.767
12	-1.736	6.384	8.569	0.073	-0.190	2.378
13	-1.990	-7.772	6.917	-0.010	-0.392	-0.445
16	-1.860	7.231	34.473	0.038	0.074	1.949
27	-1.498	59.918	-2.957	-0.154	0.103	-0.211
29	-1.898	5.195	2.634	0.117	-0.087	-2.293
31	-1.874	-0.951	7.606	-0.082	0.415	-3.197
63	-1.776	4.455	8.451	0.075	-0.297	1.495
64	-2.302	-5.989	6.049	-0.159	1.111	2.875
91	-1.707	0.597	8.399	0.026	0.070	0.232
96	-1.854	49.158	12.195	0.105	-0.556	2.703
98	-1.750	4.451	-2.092	-0.034	-0.583	-0.635
REENLISTMENT						
11	0.452	-1.982	6.663	0.023	-0.196	0.385
12	0.476	2.882	4.334	0.005	-0.075	-0.352
13	0.497	-1.716	12.622	0.071	0.026	1.044
16	0.726	6.967	5.901	0.069	-0.812	4.351
27	0.637	-38.154	-37.949	0.066	0.173	-4.278
29	0.270	2.973	10.946	0.007	-0.298	0.504
31	0.439	-9.278	2.686	-0.035	-0.244	-1.058
63	0.525	-9.112	4.683	-0.005	-0.543	0.580
64	0.676	-36.937	-20.491	-0.022	-1.048	0.662
91	0.381	-9.748	4.064	-0.007	0.066	-0.557
96	-0.275	-29.564	1.902	-0.035	0.670	1.264
98	0.359	-10.119	3.496	-0.007	-0.371	0.156

different from zero. The favorable exceptions are the estimated coefficients on RMCCW in the extension equations for CMF11 and CMF12, which have the expected positive sign and are significantly different from zero, and the RMCCW coefficient in the reenlistment equation for CMF16, which is also significantly positive. For extenders in CMF31, however, the estimate of the coefficient on RMCCW is unexpectedly negative and significantly different from zero. The results for the RMCCW variable stand in marked contrast to those obtained by Lakhani and Gilroy (1984), who reported coefficient estimates which were, with one exception, uniformly positive as expected and typically significantly different from zero. The third economic variable, home-state unemployment (HU3), usually has no significant explanatory power in either the extension equation or the reenlistment equation. When the coefficient on HU3 is statistically different from zero, it is always negative which is at variance with the theoretical expectation. This latter finding accords with that of Lakhani and Gilroy (1984), who attribute the unexpected signs to the presence of measurement error.

The demographic variables (CAT, ETHNIC, and DEPENDS) are most often not significantly different from zero. Among these three determinants of extension and reenlistment, the most successful - in terms of statistically significant conformance with the theoretical prediction - is ETHNIC, which equals one if the individual is black and zero otherwise. Other things equal, being black significantly increases the probability of extension, as expected, in five of the twelve occupational groups. The coefficients on ETHNIC in the remaining seven equations predicting extension are more often positive than negative but are, nevertheless, not significantly different from zero. The ETHNIC variable is less helpful in predicting the probability of reenlistment; in only three of the twelve estimated equations is the coefficient on this variable positive and significantly different from zero. In one occupation (CMF27), black enlistees are significantly less likely to reenlist, relative to separate, other factors held constant. The estimated coefficients on this variable in the other eight equations are almost always positive, as expected, but are not significantly different from zero. Taken together, the three demographic factors do not appear to contribute importantly to the explanation of the extension and reenlistment decisions. These results are somewhat surprising in view of the findings of Lakhani and Gilroy (1984) that, with one notable exception (CMF16 reenlistment), the estimated coefficients on each of these three variables have the expected positive signs and are significantly different from zero.

The accuracy of the unconstrained multinomial logit model in predicting the extension and reenlistment decisions of the individuals in the sample ranges between fifty-six percent and sixty-six percent. This relatively low accuracy rate, when combined with the disappointing performances of most of the explanatory variables (SRB excepted), would seem to indicate that the model is seriously misspecified. One possibility is to use a more comprehensive and judiciously specified set of economic and demographic determinants of extension and reenlistment. Given the limited scope of the present investigation, however, alternative specifications of the model were not attempted. Another approach, and the one followed here, is to retain the explanatory variables specified by Lakhani and Gilroy (1984) and (1) impose the constraints on the

reduced-form coefficients of the logit model that are implied by the underlying utility structure and (2) relax the restrictive IIA assumption embodied in the logit formulation by estimating an identified reduced form of the corresponding multinomial probit model. The empirical consequences of pursuing these alternative strategies are reported in Tables 6, 7, and 8, and Tables 9, 10, and 11, respectively.

#### 5b. Constrained Trinomial Logit

Tables 6 and 7 contain estimates of the reduced-form coefficients of the extension and reenlistment equations, respectively. These coefficients reflect the zero and equality restrictions implied by the theoretical constraints on the underlying utility structure discussed in Section 3b above. In particular, the SRB coefficients in Table 6 are all zero as a result of the structural restrictions that the marginal effects of a change in the SRB on the utilities associated with separating and extending be zero. Additionally, the coefficients on the HU3 variables in Tables 6 and 7 are equal in order to reflect the fact that changes in home-state unemployment do not affect the utilities associated with extending or reenlisting. Finally, the coefficients on RMCCW in the two tables are equal since a change in the ratio of military to civilian pay does not affect the enlistee's relative evaluation of the extension and reenlistment options.

A comparison of the overall performance of the constrained logit model with that of the unconstrained model leads to varying relative evaluations, depending on the criterion used. In terms of the goodness-of-fit of each model, as measured by the within-sample prediction accuracy, the constrained logit model clearly performs better than the unconstrained model. Specifically, the prediction accuracy achieved when the theoretical constraints are imposed is higher in eleven of the twelve estimated equations. Perhaps more importantly, the signs of the elasticities of the separation probability with respect to the SRB variable (reported in Table 8), are almost uniformly negative, as expected, in contrast to the overwhelmingly positive SRB elasticities estimated by unconstrained logit. Unfortunately, the null hypothesis that the constraints on the reduced-form coefficients are valid can be rejected in all twelve equations. In particular, under this hypothesis  $-2\ln(\text{RATIO})$  has a chi-square distribution with degrees of freedom equal to the number of constraints imposed (five), where RATIO is the ratio of the value of the likelihood function when the constraints are imposed to its value when the function is unconstrained. The critical value of a chi-square variate with 5 degrees of freedom at the 0.05 level of significance is 15.1. Since the calculated value of the likelihood ratio statistic is larger than this critical value for every one of the twelve estimated equations, we can decisively reject the theoretical restrictions on the reduced-form coefficients.

An assessment of the explanatory power and conformance with theoretical expectations of the economic and demographic variables provides little basis for ranking the performances of the unconstrained and constrained models. Evaluating first the two extension equations

TABLE 6

## CONSTRAINED LOGIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE EXTENSION EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	L. R.
11	-1.758 (-2.281) [-0.174]	0.0 (0.0) [-0.006]	-0.018 (-0.182) [3.E-04]	0.246 (2.451) [0.026]	0.092 (1.546) [0.010]	-0.062 (-3.613) [-0.007]	0.740 (1.065) [0.078]	558.00
12	-2.762 (-2.348) [-0.284]	0.0 (0.0) [-0.005]	0.172 (0.926) [0.020]	0.364 (1.697) [0.039]	0.097 (0.844) [0.010]	-0.033 (-0.952) [-0.004]	1.305 (1.252) [0.141]	121.36
13	-2.045 (-2.071) [-0.198]	0.0 (0.0) [-0.008]	-0.200 (-1.346) [-0.025]	0.346 (2.461) [0.037]	0.087 (1.093) [0.007]	-0.025 (-1.121) [-0.003]	0.838 (0.952) [0.089]	228.00
16	-5.960 (-2.946) [-0.567]	0.0 (0.0) [-0.006]	0.462 (1.283) [0.043]	0.630 (2.003) [0.073]	0.181 (1.054) [0.020]	-0.079 (-1.670) [-0.008]	4.081 (2.245) [0.394]	57.20
27	2.976 (0.765) [0.352]	0.0 (0.0) [-0.006]	1.047 (2.077) [0.174]	-0.490 (-0.915) [-0.027]	-0.469 (-1.326) [-0.074]	0.005 (0.050) [0.001]	-4.063 (-1.155) [-0.485]	16.36
29	-0.776 (-0.245) [-0.069]	0.0 (0.0) [-0.003]	0.009 (0.018) [1.E-04]	0.041 (0.070) [-0.013]	0.252 (0.954) [0.028]	-0.029 (-0.393) [-0.002]	-0.689 (-0.248) [-0.057]	26.08
31	1.863 (1.293) [0.203]	0.0 (0.0) [-0.005]	-0.143 (-0.675) [-0.003]	0.109 (0.538) [0.013]	-0.219 (-1.705) [-0.023]	0.003 (0.096) [4.E-04]	-2.877 (-2.174) [-0.302]	133.42
63	-2.589 (-2.651) [-0.271]	0.0 (0.0) [-0.006]	0.097 (0.747) [0.027]	0.352 (2.568) [0.039]	0.128 (1.582) [0.017]	-0.095 (-4.142) [-0.010]	1.596 (1.833) [0.174]	284.40
64	-4.292 (-0.936) [-0.600]	0.0 (0.0) [-0.007]	0.322 (0.462) [0.081]	-0.091 (-0.148) [0.032]	-0.033 (-0.098) [-0.007]	-0.063 (-0.531) [-0.009]	3.317 (0.784) [0.469]	23.99
91	-1.472 (-1.374) [-0.141]	0.0 (0.0) [-0.004]	-0.008 (-0.048) [0.005]	0.327 (1.919) [0.034]	0.023 (0.250) [0.002]	0.010 (0.333) [0.001]	-0.164 (-0.168) [-0.016]	155.00
96	-6.268 (-2.148) [-0.589]	0.0 (0.0) [0.002]	0.319 (0.521) [0.050]	0.534 (0.799) [0.053]	0.317 (1.204) [0.036]	0.081 (1.010) [0.007]	3.294 (1.300) [0.284]	22.30
98	-0.423 (-0.343) [-0.034]	0.0 (0.0) [-0.004]	-0.130 (-0.449) [-0.013]	-0.115 (-0.338) [-0.024]	-0.120 (-0.759) [-0.015]	-0.067 (-1.371) [-0.007]	-0.227 (-0.219) [-0.025]	72.70

TABLE 7

## CONSTRAINED LOGIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE REENLISTMENT EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	Pred. Acc.
11	-2.055 (-2.664) [-0.289]	0.149 (9.490) [0.026]	-0.073 (-0.820) [-0.012]	0.253 (2.728) [0.034]	0.078 (1.367) [0.010]	-0.062 (-3.613) [-0.008]	0.740 (1.065) [0.100]	58.94
12	-3.209 (-2.711) [-0.398]	0.149 (4.587) [0.022]	0.140 (0.788) [0.016]	0.358 (1.746) [0.043]	0.123 (1.122) [0.016]	-0.033 (-0.952) [-0.004]	1.305 (1.252) [0.156]	64.16
13	-2.468 (-2.492) [-0.363]	0.171 (7.519) [0.032]	-0.124 (-0.936) [-0.014]	0.348 (2.703) [0.049]	0.144 (1.938) [0.023]	-0.025 (-1.121) [-0.003]	0.838 (0.952) [0.117]	57.59
16	-6.257 (-3.083) [-0.791]	0.224 (4.967) [0.034]	0.530 (1.731) [0.068]	0.192 (0.697) [0.012]	0.111 (0.665) [0.012]	-0.079 (-1.670) [-0.010]	4.081 (2.245) [0.511]	66.30
27	3.090 (0.794) [0.334]	0.193 (2.168) [0.026]	-0.582 (-1.238) [-0.111]	-1.548 (-2.496) [-0.198]	0.147 (0.518) [0.034]	0.005 (0.050) [0.001]	-4.063 (-1.155) [-0.435]	61.63
29	-0.618 (-0.195) [-0.085]	0.098 (1.537) [0.017]	0.032 (0.082) [0.005]	0.622 (1.410) [0.107]	-0.002 (-0.007) [-0.008]	-0.029 (-0.393) [-0.004]	-0.689 (-0.248) [-0.100]	70.83
31	1.644 (1.141) [0.207]	0.146 (4.582) [0.024]	-0.480 (-2.362) [-0.075]	0.056 (0.294) [0.005]	-0.206 (-1.714) [-0.026]	0.003 (0.096) [4.E-04]	-2.877 (-2.174) [-0.375]	61.40
63	-2.903 (-2.959) [-0.374]	0.173 (8.002) [0.028]	-0.383 (-2.895) [-0.064]	0.332 (2.575) [0.041]	0.046 (0.570) [0.003]	-0.095 (-4.142) [-0.012]	1.596 (1.833) [0.199]	62.51
64	-4.457 (-0.965) [-0.453]	0.182 (1.823) [0.026]	-0.563 (-0.838) [-0.092]	-1.200 (-1.415) [0.165]	0.288 (0.080) [0.005]	-0.063 (-0.531) [-0.006]	3.317 (0.784) [0.332]	59.45
91	-1.430 (-1.329) [-0.192]	0.120 (4.803) [0.020]	-0.193 (-1.390) [-0.032]	0.242 (1.592) [0.030]	0.012 (0.148) [0.001]	0.010 (0.333) [0.001]	-0.164 (-0.168) [-0.022]	64.08
96	-4.580 (-1.609) [-0.629]	-0.074 (-0.916) [-0.013]	-0.432 (-0.998) [-0.085]	0.296 (0.485) [0.036]	0.033 (0.126) [-0.004]	0.081 (1.010) [0.012]	3.294 (1.300) [0.477]	64.00
98	-0.853 (-0.688) [-0.113]	0.121 (2.533) [0.018]	-0.180 (-0.646) [-0.023]	0.254 (0.823) [0.041]	-0.072 (-0.432) [-0.007]	-0.067 (-1.371) [-0.008]	-0.227 (-0.219) [-0.027]	64.34

TABLE 8

## CONSTRAINED LOGIT ELASTICITY ESTIMATES

CMF	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW
SEPARATION						
11	-0.127	0.727	-4.401	-0.019	0.171	-0.329
12	-0.097	-1.950	-3.054	-0.019	0.090	-0.507
13	-0.157	2.053	-8.058	-0.032	0.073	-0.398
16	-0.158	-3.172	-7.579	-0.024	0.191	-1.476
27	-0.114	-5.586	10.336	0.031	-0.022	1.564
29	-0.081	-0.551	-3.088	-0.013	0.065	0.269
31	-0.116	3.430	-1.583	0.032	-0.009	1.178
63	-0.129	1.663	-3.927	-0.017	0.250	-0.660
64	-0.125	1.489	-6.585	0.002	0.188	-1.487
91	-0.095	2.592	-3.202	-0.003	-0.023	0.065
96	0.067	4.295	-1.827	-0.029	-0.219	-1.312
98	-0.078	4.326	-0.420	0.016	0.165	0.089
EXTENSION						
11	-0.128	0.062	6.393	0.032	-0.268	0.483
12	-0.112	4.245	5.691	0.028	-0.177	0.944
13	-0.160	-4.033	10.624	0.023	-0.111	0.526
16	-0.157	5.692	30.154	0.071	-0.393	2.976
27	-0.122	55.088	-4.429	-0.206	0.040	-2.944
29	-0.088	0.055	-2.159	0.094	-0.109	-0.493
31	-0.112	-0.484	4.196	-0.056	0.016	-1.928
63	-0.127	4.390	6.927	0.052	-0.412	1.114
64	-0.114	27.240	2.657	-0.020	-0.281	2.163
91	-0.104	2.082	7.378	0.007	0.049	-0.118
96	0.058	28.988	5.141	0.152	0.380	2.313
98	-0.085	-5.887	-2.235	-0.040	-0.291	-0.162
REENLISTMENT						
11	0.434	-1.960	6.558	0.025	-0.240	0.486
12	0.444	3.053	5.642	0.041	-0.159	0.939
13	0.488	-1.721	10.720	0.057	-0.085	0.527
16	0.683	6.936	3.819	0.033	-0.378	2.974
27	0.591	-39.170	-36.203	0.106	0.045	-2.943
29	0.285	1.567	10.201	-0.015	-0.126	-0.497
31	0.431	-9.748	1.300	-0.051	0.013	-1.928
63	0.518	-9.064	6.342	0.008	-0.430	1.109
64	0.599	-43.616	19.312	0.020	-0.264	2.159
91	0.364	-9.370	4.577	0.003	0.035	-0.114
96	-0.223	-29.343	2.079	-0.010	0.388	2.313
98	0.362	-9.902	3.630	-0.018	-0.316	-0.166

(Tables 3 and 6), we note that the estimated coefficients and t-ratios of the SRB variable cannot be compared between the two models, since the coefficient on the SRB variable in each of the reduced-form equations for extension is constrained to zero. It is noteworthy, however, that the SRB elasticities of extension are uniformly lower in absolute value when the constraints are imposed. When significantly different from zero, the estimated coefficient on HU3 is always negative (contrary to theoretical expectation) in both the unconstrained and constrained logit models. The relative pay variable (RMCCW) is statistically different from zero in only three of the twelve extension equations estimated by either the unconstrained or constrained models and in each case one of the three statistically significant coefficients has the wrong (negative) sign. Among the sets of estimated coefficients on the three demographic variables (CAT, ETHNIC, and DEPENDS), there is once again no basis for discriminating between the two models since, taken together, these variables add very little to the explanation of the decision to extend. The coefficients on the ETHNIC variable are positive and significantly different from zero in about half of the equations estimated by either the unconstrained or constrained models. However, neither the mental ability (CAT) variable nor the number of dependents (DEPENDS) variable has any explanatory power across CMF's, regardless of the model used.

It is similarly difficult to assess the relative merits of the unconstrained and constrained models by examining the estimated coefficients and implied elasticities of the reenlistment equations. With the constraints imposed, the estimated coefficients on the SRB variable are more often significantly greater than zero than in the unconstrained approach but the magnitudes of the SRB elasticities are quite similar. Otherwise the performances of the two reenlistment models are, in this regard, virtually indistinguishable. When the ETHNIC variable has an effect that is significantly different from zero, the effect is positive (as hypothesized), with the exception in both models of the equations for CMF27. The effect of the home-state unemployment rate (HU3) on reenlistment is most often not statistically significant in either the unconstrained or constrained models but, when it is significantly different from zero, the estimated coefficient is always wrong-signed. Neither the number of dependents (DEPENDS) nor the relative pay variable (RMCCW) has much explanatory power across occupations with or without the constraints imposed. Finally, the coefficients on the categorical variable (CAT) measuring mental ability are most often negative in both models but are rarely significantly different from zero in either set of estimated equations.

#### 5c. Unconstrained Trinomial Probit

The final set of empirical results, reported in Tables 9, 10, and 11 were obtained by estimating the model with the reduced-form probit approach developed by Terza (1985). As mentioned previously, the probit function has the virtue of relaxing (and providing a test of) the theoretically unappealing assumption of the independence from irrelevant alternatives. However, because of its computational complexity, this model must at present be estimated without imposing the constraints on the

TABLE 9

## UNCONSTRAINED PROBIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE EXTENSION EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	$\tau_3$	$\tau_4$
11-1	0.110 (0.201) [0.348]	-0.115 (-4.294) [-0.087]	-0.037 (-0.417) [-0.029]	0.093 (1.100) [-0.008]	-0.038 (-0.821) [-0.019]	-0.029 (-1.433) [-0.006]	0.230 (0.474) [0.083]	0.362 (3.787)	0.615 (2.592)
11-2	-0.093 (-0.126) [0.201]	-0.088 (-4.763) [-0.096]	-0.106 (-1.437) [0.011]	0.063 (0.807) [-0.010]	0.074 (1.380) [0.028]	-0.021 (-1.222) [-0.002]	0.292 (0.430) [0.237]	0.466 (6.175)	0.886 (3.506)
11-3	-1.366 (-1.112) [-0.117]	-0.225 (-3.499) [-0.075]	0.011 (0.083) [0.009]	0.251 (1.591) [0.055]	0.097 (0.955) [0.026]	-0.036 (-1.164) [-0.006]	1.624 (1.424) [0.287]	0.585 (2.140)	5.079 (3.476)
11-4	-1.458 (-1.129) [-0.397]	-0.248 (-3.169) [-0.072]	0.056 (0.413) [0.002]	0.157 (1.020) [0.037]	0.172 (1.665) [0.045]	-0.029 (-0.972) [-0.005]	1.596 (1.402) [0.498]	0.482 (1.061)	11.001 (3.132)
12	0.722 (2.129) [0.198]	-0.245 (-5.486) [-0.056]	0.054 (0.456) [0.008]	0.294 (2.047) [0.054]	0.002 (0.028) [-0.003]	-0.046 (-1.458) [-0.009]	-0.427 (-2.750) [-0.061]	-0.145 (-0.396)	2.281 (2.141)
13	0.192 (0.253) [0.216]	-0.234 (-3.667) [-0.074]	-0.129 (-1.214) [-0.032]	0.161 (1.527) [0.027]	0.032 (0.553) [0.001]	-0.024 (-1.098) [-0.007]	0.194 (0.284) [0.003]	0.460 (1.331)	7.250 (3.353)
16	-2.852 (-1.177) [-0.137]	-0.511 (-5.226) [-0.053]	0.525 (1.779) [0.041]	0.471 (1.859) [0.043]	0.058 (0.341) [0.002]	-0.031 (-0.608) [0.0001]	2.349 (1.070) [0.126]	-2.028 (-2.484)	7.799 (1.726)
27	1.056 (1.016) [-0.177]	-0.238 (-2.662) [-0.077]	0.469 (1.387) [0.161]	-0.682 (-1.986) [-0.050]	-0.120 (-0.573) [-0.049]	0.014 (0.156) [0.007]	-1.035 (-1.091) [0.114]	-0.561 (-1.357)	-0.354 (-1.525)
29	1.821 (0.821) [0.185]	-0.548 (2.009) [-0.047]	0.114 (0.311) [0.008]	0.257 (0.600) [0.007]	0.286 (0.933) [0.024]	-0.039 (-0.490) [-0.002]	-1.750 (-0.889) [-0.153]	-2.183 (-0.844)	7.616 (0.819)
31	2.136 (1.655) [0.505]	-0.255 (-4.338) [-0.066]	-0.090 (-0.582) [-0.003]	0.089 (0.595) [0.016]	-0.130 (-1.308) [-0.026]	0.028 (1.021) [0.009]	-1.915 (-1.629) [-0.398]	0.328 (0.886)	6.499 (3.167)

TABLE 9 (Continued)

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	$\tau_3$	$\tau_4$
63-1	-0.160 (-0.477) [0.191]	-0.046 (-3.936) [-0.076]	-0.016 (-0.377) [0.023]	0.080 (1.874) [0.026]	0.023 (0.874) [0.026]	-0.023 (-2.355) [1.E-04]	0.275 (0.928) [0.142]	0.831 (56.697)	4.938 (9.924)
63-2	-0.260 (-0.672) [0.133]	-0.076 (-3.472) [-0.080]	-0.051 (-1.043) [0.037]	0.087 (1.605) [0.060]	0.024 (0.770) [0.009]	-0.024 (-2.142) [-0.006]	0.437 (1.269) [0.197]	0.801 (10.688)	4.490 (2.342)
64	-2.119 (-0.664) [-0.456]	-0.290 (-1.513) [-0.107]	-0.218 (-0.413) [-0.015]	-0.205 (-0.387) [0.048]	-0.167 (-0.629) [-0.061]	0.043 (0.365) [0.034]	2.436 (0.812) [0.522]	0.080 (0.124)	0.358 (0.527)
91	-0.169 (-0.168) [-0.022]	-0.235 (-3.642) [-0.053]	-0.001 (-0.012) [0.007]	0.174 (1.329) [0.031]	0.043 (0.578) [0.011]	0.010 (0.384) [0.001]	0.069 (0.075) [0.062]	0.299 (0.625)	9.100 (3.114)
96	-1.690 (-0.542) [-0.191]	-0.313 (-1.353) [-0.054]	0.320 (0.647) [0.075]	0.564 (0.851) [0.095]	0.096 (0.393) [0.019]	-0.005 (-0.054) [-0.005]	1.480 (0.527) [0.206]	0.270 (0.166)	15.422 (1.267)
98	1.104 (1.067) [0.212]	-0.393 (1.414) [-0.065]	-0.036 (-1.129) [-0.001]	-0.095 (1.314) [-0.022]	-0.090 (-0.362) [-0.014]	-0.041 (-2.056) [-0.004]	-0.500 (0.052) [-0.080]	-0.658 (-1.244)	8.423 (1.493)

TABLE 10

## UNCONSTRAINED PROBIT

MAXIMUM LIKELIHOOD ESTIMATES FOR THE REENLISTMENT EQUATION BY CMF  
 (t-Ratios in Parentheses)  
 [Slopes Evaluated at Regressor Means in Brackets]

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	Pred. Acc.
11-1	-0.574 (-0.848) [-0.354]	-0.011 (-0.364) [0.046]	1.E-04 (-0.004) [0.016]	0.183 (1.707) [0.055]	-0.026 (-0.419) [0.003]	-0.038 (-1.401) [-0.007]	0.225 (0.384) [0.015]	57.23
11-2	-0.616 (-0.514) [-0.252]	0.019 (0.621) [0.051]	-0.245 (-2.051) [-0.067]	0.153 (1.249) [0.044]	0.096 (1.173) [0.011]	-0.039 (-1.466) [-0.009]	0.110 (0.101) [-0.086]	56.37
11-3	-7.133 (-3.241) [-0.872]	0.097 (2.250) [0.022]	-0.125 (-0.827) [-0.017]	0.543 (2.868) [0.060]	0.103 (0.888) [0.009]	-0.114 (-3.279) [-0.013]	5.086 (2.945) [0.593]	59.07
11-4	0.478 (0.348) [0.075]	0.284 (3.664) [0.031]	0.555 (3.050) [0.048]	0.203 (1.216) [0.041]	0.023 (0.241) [-0.002]	-0.093 (-2.450) [-0.008]	-3.359 (-2.269) [-0.334]	57.72
12	-0.945 (-1.547) [-0.198]	0.074 (1.686) [0.024]	0.067 (0.526) [0.009]	0.177 (1.222) [0.015]	0.077 (1.072) [1.E-04]	-0.011 (-0.332) [-0.009]	-0.632 (-2.592) [-0.084]	62.86
13	-4.687 (-3.339) [-0.545]	0.221 (3.826) [0.033]	-0.140 (-1.191) [-0.012]	0.545 (3.944) [0.057]	0.219 (2.737) [0.024]	0.005 (0.206) [0.001]	1.519 (1.471) [0.168]	54.56
16	-11.567 (-2.415) [-1.041]	0.329 (2.628) [0.037]	0.839 (2.326) [0.072]	0.213 (0.827) [0.014]	0.296 (1.417) [0.027]	-0.238 (-2.512) [-0.022]	8.395 (2.248) [0.752]	66.78
27	1.494 (1.073) [0.868]	-0.048 (-0.702) [0.055]	0.071 (0.242) [-0.129]	-0.558 (-1.731) [-0.208]	0.0 (-0.001) [0.048]	-0.004 (-0.063) [-0.009]	-1.317 (-1.090) [-0.723]	61.00
29	-2.646 (-1.486) [-0.297]	0.104 (0.846) [0.017]	0.119 (0.368) [0.011]	1.192 (1.536) [0.121]	0.026 (0.130) [-0.001]	-0.105 (-1.019) [-0.010]	0.527 (0.299) [0.076]	65.83
31	0.100 (0.051) [-0.049]	0.176 (3.431) [0.027]	-0.652 (-3.234) [-0.069]	0.179 (0.895) [0.017]	-0.160 (-1.338) [-0.014]	-0.071 (-1.783) [-0.009]	-2.012 (-1.231) [-0.167]	59.21

TABLE 10 (Continued)

CMF	Intercept	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW	Pred. Acc.
63-1	-2.581 (-1.705) [-0.336]	0.207 (4.517) [0.035]	-0.285 (-1.322) [-0.037]	0.421 (1.869) [0.047]	-0.011 (-0.086) [-0.005]	-0.173 (-3.450) [-0.021]	1.044 (0.815) [0.107]	61.17
63-2	-2.852 (-2.036) [-0.382]	0.154 (2.339) [0.032]	-0.657 (-2.851) [-0.089]	0.095 (0.507) [0.003]	0.085 (0.817) [0.009]	-0.112 (-2.479) [-0.013]	1.310 (1.345) [0.137]	60.06
64	-1.599 (-0.548) [-0.192]	-0.002 (-0.019) [0.058]	-0.323 (-0.581) [-0.082]	-0.609 (-0.857) [-0.196]	-0.006 (-0.022) [0.032]	-0.087 (-0.757) [-0.043]	1.846 (0.670) [0.224]	60.36
91	-0.657 (-0.558) [-0.058]	0.180 (3.387) [0.022]	-0.357 (-2.674) [-0.033]	0.301 (2.127) [0.025]	-0.086 (-0.954) [-0.009]	0.042 (1.314) [0.004]	-2.360 (-1.746) [-0.223]	63.07
96	-8.973 (-1.586) [-0.657]	-0.204 (-2.000) [-0.011]	-1.256 (-1.672) [-0.099]	0.562 (0.867) [0.035]	-0.097 (-0.470) [-0.009]	0.332 (1.447) [0.025]	4.896 (1.526) [0.351]	62.86
98	-2.187 (-1.116) [-0.211]	0.140 (1.414) [0.019]	-0.287 (-1.129) [-0.025]	0.416 (1.314) [0.038]	-0.048 (-0.362) [-0.003]	-0.117 (-2.056) [-0.010]	0.066 (0.052) [0.014]	61.12

TABLE 11

## UNCONSTRAINED PROBIT ELASTICITY ESTIMATES

CMF	SRB	CAT	ETHNIC	DEPENDS	HU3	RMCCW
SEPARATION						
11	0.303	0.447	-7.591	-0.030	0.195	-1.462
12	0.178	-0.892	-2.489	-0.007	0.097	0.240
13	0.245	2.119	-7.200	-0.024	0.067	-0.302
16	0.085	-3.031	-4.771	-0.021	0.218	-1.344
27	0.127	-2.864	11.963	0.001	0.023	1.045
29	0.163	-1.933	-3.963	-0.014	0.122	0.124
31	0.219	2.921	-2.677	0.024	0.001	0.907
63	0.234	0.613	-3.491	-0.017	0.232	-0.429
64	0.293	11.932	4.494	0.031	0.103	-1.259
91	0.174	2.342	-2.628	-0.002	-0.053	0.257
96	0.371	2.759	-2.501	-0.008	-0.216	-0.900
98	0.242	2.932	-0.371	0.011	0.145	0.106
EXTENSION						
11	-2.704	3.171	22.890	0.141	-0.389	3.007
12	-1.603	2.166	10.052	-0.011	-0.508	-0.521
13	-2.223	-7.742	11.627	0.005	-0.390	0.027
16	-2.100	8.243	26.977	0.011	0.007	1.446
27	-2.686	87.121	-14.019	-0.234	0.480	1.183
29	-2.145	6.829	1.818	0.126	-0.171	-2.071
31	-2.225	-0.731	7.800	-0.095	0.547	-3.838
63	-1.805	4.185	5.166	0.089	0.005	1.017
64	-3.505	-10.109	7.987	-0.354	2.126	4.825
91	-2.029	4.312	9.951	0.059	0.073	0.676
96	-2.388	66.733	14.142	0.123	-0.417	2.575
98	-1.873	-0.617	-2.794	-0.051	-0.227	-0.705
REENLISTMENT						
11	0.357	-2.700	11.253	0.022	-0.380	2.800
12	0.449	1.592	1.824	0.031	0.004	-0.469
13	0.486	-1.423	12.031	0.057	0.027	0.730
16	0.722	7.125	4.323	0.071	-0.807	4.247
27	0.831	-30.239	-25.263	0.099	-0.267	-3.249
29	0.277	3.348	11.206	-0.002	-0.305	0.367
31	0.474	-8.761	4.317	-0.027	-0.288	-0.839
63	0.641	-5.191	7.202	-0.013	-0.746	0.591
64	0.983	-28.593	-16.873	0.096	-1.391	1.071
91	0.390	-9.411	3.715	-0.022	0.135	-1.125
96	-0.185	-33.557	1.985	-0.022	0.794	1.671
98	0.375	-10.571	3.304	-0.007	-0.388	0.084

reduced-form coefficients implied by the structure of the underlying utility model. It is most sensible, therefore, to compare the estimates of the extension and reenlistment equations from the unconstrained probit model with those from the unconstrained logit model.

Comparisons of the overall relative performance of the two unconstrained models can be made with two separate criteria, the within-sample predictive accuracy and hypothesis tests based on point estimates and estimated t-ratios of the two covariance parameters in the probit model. As discussed above in Section 3c, under the null hypothesis of the independence from irrelevant alternatives (IIA),  $\tau_3 = -1$  and  $\tau_4 = 1$ . Since IIA is a logical implication of the multinomial logit model, rejection of the null hypothesis is tantamount to rejection of the logit framework in favor of the less restrictive probit approach. On the basis of these two criteria, the trinomial probit model clearly dominates the trinomial logit specification. To see this, first calculate the mean predictive accuracy for the two occupations (CMF11 and CMF63) which were, for reasons of sample size, subdivided before being estimated by probit. For a majority (seven) of the twelve occupations, the accuracy of the probit model in predicting the extension and reenlistment behavior of the individuals in the sample is greater than that of the logit model, although the margin of relative success for the probit estimator is admittedly small. The more decisive criterion is the outcome of the hypothesis tests involving the two covariance parameters. Of the sixteen occupational samples and subsamples estimated by the probit technique, at least one (and often both) of the estimated covariance parameters was significantly different from its value under the IIA hypothesis, using an appropriate two-tailed test. Thus, in approximately three-quarters of the estimations, we can reject the IIA hypothesis and, by implication, the empirical validity of the logit model.

The explanatory power of the economic and demographic variables continued to be disappointing when the model was estimated with the probit specification of the likelihood function. Moreover, a comparison of coefficient estimates between the probit and unconstrained logit formulations yields mixed results. Overall, more of the logit-estimated coefficients were significantly different from zero and simultaneously had the anticipated sign in the extension equations but the reverse was true for the reenlistment equations. For both the logit and probit models, only the SRB variable had, across occupational equations, a majority of estimated coefficients that were both signed in accordance with the theoretical expectation and significantly different from zero; this result held for both the extension and reenlistment equations. Interestingly, the estimated SRB elasticities of extension obtained with the probit model are uniformly higher in absolute value than those calculated from the logit coefficients. The second-most successful variable according to this criterion was ETHNIC, with a total of ten correctly signed and statistically significant coefficients. The probit estimates of the coefficients on the home-state unemployment variable (HU3) were, like the logit estimates, typically not significantly different from zero. Again, however, when they were different from zero, they always had an unexpected (negative) sign. The results pertaining to the remaining variables (CAT, DEPENDS, and RMCCW) closely parallel those reported for the unconstrained logit model and, thus, provide no guidance in choosing between them.

Direct comparisons of the performance of the unconstrained probit model with that of the constrained logit model are difficult to carry out since - unlike the unconstrained and constrained logit models - the alternatives are not nested. The predictive accuracy of the constrained logit model is typically (but not uniformly) higher than that of the probit model. This result, coupled with the predictive superiority of the constrained logit model over the unconstrained logit model and the greater predictive accuracy of the unconstrained probit model relative to the unconstrained logit formulation, suggests that imposition of the constraints plays a dominant role in determining the relative predictive performance of the three models, while functional form plays a secondary (but nontrivial) role. This suggestion, in turn, raises the possibility that an appropriately constrained reduced-form probit model might exhibit greater accuracy of within-sample predictions than the constrained logit model. Unfortunately, the computational expense and programming complexity of such an experiment is beyond the scope of the present study. Nevertheless, the imposition of the constraints in the probit context, in combination with efforts to develop a more successful specification of the theoretical model, would be an interesting and potentially important direction for future research on the separation, extension, and reenlistment behavior of first-term Army enlistees.

## 6. Concluding Remarks

In this section, we summarize the purpose and methods of the study, highlight the principal empirical findings, and suggest several interesting avenues for possible future empirical research on the reenlistment decision.

### 6a. Summary of Purpose

The primary purpose of this study was to re-estimate the trinomial logit model of Army reenlistment and extension proposed by Lakhani and Gilroy (1984), using the reduced-form trinomial probit estimator developed by Terza (1985). The principal advantage of the probit framework over the logit approach used by Lakhani and Gilroy is that the former does not impose the theoretically restrictive assumption of the independence from irrelevant alternatives (IIA). Indeed, within the context of Terza's formulation of the probit model, the IIA assumption is a testable - rather than maintained - hypothesis. The main disadvantage of the probit function is that, even with only three alternatives (separation, extension, and reenlistment), it is computationally infeasible for the model at hand with sample sizes greater than about two thousand. In fact, this sample-size limitation was binding with the CMF11 (infantry) and CMF63 (armored) occupations; in these two cases, four and two subsamples, respectively, were created in order to obtain the probit estimates. A by-product of this research was the formulation and estimation of a variant of the trinomial logit model which imposes the constraints on the reduced-form coefficients that are implied by theoretically appropriate zero restrictions on the parameters of the underlying utility equations.

These restrictions seem to arise logically in the Lakhani-Gilroy model, since some of their explanatory variables affect the enlistee's well-being only if certain options are chosen.

#### 6b. Highlights of Empirical Results

Overall, the theoretical model did not perform especially well, regardless of the estimation technique that was employed. This was somewhat surprising, given the multinomial logit results reported by Lakhani and Gilroy (1984) with a slightly larger sample from the same Enlisted Master File. The accuracy of the three models in predicting the observed choices among the alternatives ranged between fifty-four and sixty-seven percent, which is relatively low. The signs of the estimated coefficients on many of the explanatory variables were at odds with theoretical expectations and the coefficients often were not significantly different from zero. Especially disappointing was the performance of the military-civilian relative pay variable in view of its success in the extension and reenlistment equations estimated by Lakhani and Gilroy (1984). Only the coefficients on the selective reenlistment bonus variable consistently exhibited the anticipated sign and, at the same time, were significantly different from zero. These results strongly suggest that the model is misspecified.

Because of the rather poor performance of the model across all three estimation techniques, it was difficult to evaluate them on a comparative basis. Indeed, a symptom of this difficulty is that our rankings of the alternative approaches varied with the evaluation criterion employed. For example, on the basis of both the predictive accuracy of the models and the signs and significance levels of the estimated coefficients, as well as the signs of the SRB elasticities of separation, the constrained logit model dominated the unconstrained logit approach. Yet, the null hypothesis that the constraints are empirically valid was decisively rejected, with a likelihood ratio test, for every occupation. The (unconstrained) probit model outperformed the unconstrained logit model in terms of predictive accuracy and, perhaps more importantly, a test of the (null) IIA hypothesis was overwhelmingly rejected, implying that the logit formulation is inappropriate. On the other hand, the constrained logit model was predictively more accurate than the probit function. Taken together, these results suggest that both the constraints and the functional form play important roles in affecting the performance of the model and that a constrained probit approach might be warranted. Unfortunately, the programming and computational complexity of such an estimator is well beyond the scope and resources of this study.

#### 6c. Suggestions for Future Research

The disappointing performance of the theoretical model, and the consequent difficulty of arriving at an unambiguous ranking of the alternative estimation procedures, evokes at least two interesting and potentially useful extensions of the present investigation. First,

considerable effort should be directed towards the development of an improved specification of the model. It is virtually impossible to discriminate empirically among different estimators where there is so little explanatory power of the independent variables. Second, there are forceful a priori arguments for imposing the appropriate restrictions on the reduced-form coefficients of a discrete choice model when one or more of the explanatory variables do not apply or are not observed for some choices. At the same time, Samuelson (1985) has provided a strong theoretical case against choice models (like multinomial logit) which imply the independence from irrelevant alternatives property. These two considerations point to the desirability of modelling the extension and reenlistment decision within a constrained probit framework. A successful effort along these lines, in combination with a more judiciously specified model, would constitute an important contribution to research on the retention of first-term Army enlistees.

## NOTES

<sup>1</sup>For example, see the remarks by Warner (1984) and Hogan (1984) on the papers presented by Daula and Baldwin (1984) and Lakhani and Gilroy (1984), respectively, at the Army Manpower Economics Conference in Williamsburg, VA on December 5, 1984.

<sup>2</sup>The GEV model in its standard form is often called the "nested logit" model. For a more detailed discussion, see Amemiya (1981, pp. 1520 - 22) or Maddala (1983, pp. 70 - 73).

<sup>3</sup>The multinomial probit model was first proposed by Aitchison and Bennett (1970) and has been developed further by Daganzo (1979) and Terza (1985).

<sup>4</sup>For a discussion of the sequential logit model, see Amemiya (1981, pp. 1524 - 25) and Maddala (1983, pp. 49 - 51).

<sup>5</sup>For a discussion of triangular models, see Lahiri and Schmidt (1978).

<sup>6</sup>Hogan (1984) made a similar point about the unconstrained multinomial logit model of Lakhani and Gilroy (1984).

<sup>7</sup>Note that  $\phi_3 = \text{cov}(Q_{12}, Q_{13})$ ,  $\phi_4 = \text{cov}(Q_{12}, Q_{23})$ , and  $\phi_5 = \text{cov}(Q_{13}, Q_{23})$ . Therefore,  $\text{var}(Q_{12}) = \phi_3 - \phi_4$ ,  $\text{var}(Q_{13}) = \phi_3 + \phi_5$ , and  $\text{var}(Q_{23}) = \phi_5 - \phi_4$ . Therefore, (16) - (18) involve standardizations of  $Q_{12}$ ,  $Q_{13}$ , and  $Q_{23}$ .

<sup>8</sup>For CMF11 and CMF63, the elasticities of separation, extension, and reenlistment were calculated only for the subsample with the highest predictive accuracy.

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