SUPervisory Manipulation for Assembling Mechanical Parts While Compensating Uncertainty Masschusetts Institute of Technology Dept of Mechanical Engineering Hirabayashi JUN 81 W0014-77-C-0256 NCLNL
SUPERVISORY MANIPULATION FOR ASSEMBLING MECHANICAL PARTS WHILE COMPENSATING FOR RELATIVE MOTION

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JUNE 1981

Hitachi, Ltd.
Japan

Grant 04-7-150-1579
MIT Sea Grant Program
Office of Sea Grant
National Oceanic and Atmospheric Administration
Department of Commerce

Contract N00014-79-C-0006
Work Unit Number 00-152-152
Engineering Psychology Program
OFFICE OF NAVAL RESEARCH

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Supervisory Manipulation for Assembling Mechanical Parts While Compensating for Relative Motion

by

Hisaaki Hirabayashi

Kogakushoshi, Waseda University (1972)
Kogakushushi, Waseda University (1974)

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS OF THE DEGREE OF MASTER OF SCIENCE IN MECHANICAL ENGINEERING at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY June 1981

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ABSTRACT

This research is to develop and demonstrate the capability for a manipulator system to automatically compensate for random motion of the object being manipulated. This is done by means of a computer and a "measurement arm", a multi-degree-of-freedom position sensor independent of the manipulator itself.

Following preliminary experiments of Dr. K. Tanaka which presupposed perfect measurement, we developed the position sensor and the Jacobian matrices of approximation necessary to interject and transform the measurement to enable control. This report describes the interaction of the 6 degree-of-freedom sensor, and the Jacobian matrices of first order approximation. Evaluation tests were done for simple motions. As the result of the tests, we found the errors acceptable, and believe that this technique is useful for this type of compensation.

Thesis Supervisor: Dr. Thomas B. Sheridan
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ACKNOWLEDGMENTS

First I most gratefully acknowledge the instruction and guidance of Professor Thomas B. Sheridan. I am also thankful to Mr. Don Fyler and Mr. Ahmet Buharali for the kind assistance in using the computer, Mr. Dana Yoerger for his constructive comments, Dr. Anatoly Farberov for his aid to mechanism and the staff and the students of the Man-Machine Laboratory, and Dr. Kazuo Tani who performed the preliminary experiments which led to my thesis.

I would like to acknowledge Hitachi, Ltd. Japan, for supporting me financially and finally express my appreciation to my wife Tomoko for her encouragement and continual support during my study at M.I.T.
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1.1 Supervisory Control

Recently the need has increased for operating vehicles and devices far from the surface of the earth, such as underwater submersibles and space shuttles. These operations are so difficult that fully unmanned systems cannot carry out them. Consequently, for these operations the man is required to be within the systems to make some decisions. These manned systems are divided into two categories: one is direct control and another is supervisory control. We can explain the difference as follows, by making use of Fig. 1.1.[1] In direct control, the human directly controls, over the communication link, the separate propulsive actuators of the vehicle, the actuators for the separate degrees of freedom of the manipulator and the actuators of the video camera. The video picture is sent back directly to the operator. The "hand control" can be a master-slave positioning replica or a rate joystick. In supervisory control a computer is added to the teleoperator, and for short periods and limited circumstances the teleoperator can function autonomously. Then the terminology, teleoperator, is defined as follows: A vehicle having sensors and actuators for mobility and/or manipulation, remotely controlled by a human operator, and thus enabling him to extend his sensory-motor function to remote or hazardous environments.

One example of direct control is the conventional master-slave arm system, where the operator has to make all decisions for control on the basis of all information from teleoperator in a short time. It will
a) Direct Control

b) Supervisory Control

Fig. 1.1 Direct and Supervisory Control of a Teleoperator
make him do too much work and might lead to misoperations in complicated systems. Alternatively, supervisory control may be advantageous to achieve faster or more accurate control, or to control simultaneously in more degrees of freedom than the operator can achieve by direct control, or relieve him of tedium. This is why a computer is included to carry out a part of the work in addition to the operator. Strictly speaking these computer roles are divided into four categories, as in Fig. 1.2 [1]:

1) it can extend his capabilities to help the teleoperator accomplish more than he alone were in control.

2) it can relieve him of some control tasks.

3) it can provide back-up by taking over control for a short time if feedback is lost.

4) it can replace him when a task is too dull.

We have explained supervisory control so far in comparison with the counterpart which is the manned direct control without computer. Next it is also important to explain supervisory control in comparison with the work of divers or manned submersibles in the sea.

About ten years ago divers seemed to have an advantage over manned work - vehicles with manipulators in terms of maneuverability, manipulation, tactile sensing, and covertness. Because of smaller unmanned vehicles and computers, only manipulation, sensing and cognition still remain the primary advantage for the divers.

As to the comparison between teleoperators and manned submersibles, because of the remarkable progress in television cameras and communication channels, the major difference remaining between manned submersibles and
Roles of Computer
(L-load or task, H-human, C-computer)

"Sharing"  "Trading"

Fig. 1.2 Roles of Computer
teleoperators are cost and safety. The pressure vessel and life-support equipment make the manned submersible much more costly than the same vehicle without the pressure vessel and life-support equipment but with remote control instead.

The above-mentioned explanations show that supervisory control is getting to have an advantage in terms of technology and cost compared with divers and manned submersibles.

1.2 Compensation

There are many problems to be solved in the field of the supervisory control. As an example, in the project we concentrate on remote manipulation by the supervisory control, particularly on manipulation with automatic compensation, which belongs to the 1st category in computer roles said in 1.1, for moving targets.

As far as compensation is concerned, a first step has been achieved so far in our laboratory by Tani.[2] His work demonstrated that automatic manipulator compensation for relative motion between manipulated object and manipulator base made master-slave manipulation easier. For his work, he used a hardware system which consisted of master/slave manipulator, a moving table for the moving objects, and a computer controlling both the manipulator and the table. By means of the method of resolved motion rate control, his software system allowed the master/slave operation with object motion compensation under computer control. Tani's experiments were of three kinds: no object motion, compensation for the object motion, and no compensation. The comparison of the situations with the compensation without it showed that the compensation reduced the operation time or increased the accuracy in some tasks.
2.1 Compensation with Position Measuring

The manipulator compensation done so far presupposed a perfect measurement of the relative motion between manipulated object and manipulator base. The purpose of this project is to extend the compensation by using an experimental 6 degree-of-freedom passive "measurement arm" having a simple gripper, but otherwise flaccid. The operator positions it with the actual manipulator. We explain it in Fig. 2.1.

\[ \hat{A} = \hat{T} + \hat{R} \]  

(2.1)

where

- \( \hat{A} \): Absolute Position of the Object with respect to the Manipulator Base
- \( \hat{T} \): Table Position with respect to the Manipulator Base
- \( \hat{R} \): Relative Position of the Object with respect to the Table

As shown in Fig. 2.1.a, in the case that the table where the object is mounted is fixed, all the operator has to do is to pick up and place the object with direct visual feedback. On the other hand, as shown in Fig. 2.1.b, in the case that the table moves, he is required to do more to achieve the same operation. That is, as we showed in Fig. 2.1.c he has to pick it up and to place it by taking account of the movement of both table and object. If there were some function which enabled him to operate as if the table were fixed, it would be very convenient for him.
Fig. 2.1 Compensation with Position Measuring

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In this project, I define compensation as this function. According to the above mentioned notation, the compensation means to change \( A \) from \( \hat{T} + \hat{R} \) to \( \hat{A} \). In other words it means subtraction of \( \hat{T} \) from \( \hat{A} \).

2.2 Function of Measurement Arm

When it comes to subtraction of \( \hat{T} \) from \( \hat{A} \), \( \hat{T} \) has to be measured while \( \hat{A} \) can be controlled by the operator. In order to measure \( \hat{T} \), that is, table position with respect to the manipulator base, we introduced the measurement arm to the system. Fig. 2.2 by means of 6 potentiometers, are set in each joint of the arm, we can measure each joint angle; therefore, we can estimate the position and orientation of the table with the help of some geometrical calculations. With the aid of computation and by making use of the measurement arm, the operator can pick up and place the object on the moving table as if the table were fixed.
Fig. 2.2 Measurement Arm
CHAPTER 3

METHOD

3.1 Control Theory of Manipulation

At first, we explain the control of manipulation. It is the control of $A$: Absolute Position of the Object with respect to the Manipulation Base, illustrated in Chapter 2. In Sec. 3.1 the general method is stated and in Sec. 3.2 the approximated method we used is stated.

Generally speaking, an unconstrained rigid body has six independent degree of freedom: three independent translation components and three independent rotation components.[3]

Since the hand of a manipulation is rigid body, it needs six independent components to be fixed in the space. We define the following six components: $x$, $y$ and $z$ which show the translation $P$ of the hand and $\alpha$, $\beta$ and $\gamma$ which show the orientation or rotation of the hand. In order to describe these six parameters, we use the coordinate system whose notation is given by T. Brooks.[4] Matrix $^m_A$ means the transformation from the $m$th frame to the $n$th frame. For example the transformation from the hand frame (6th) to the vehicle frame (0th) is given as

$$0_A^6 = 0_A^1 A_1^2 A_2^3 A_3^4 A_4^5 A_5^6$$

(3.1)

Each frame is defined in Fig. 3.1 and components of each matrix are shown in Table A.1.

According to this notation, $x$, $y$ and $z$ are the vehicle coordinates at point $P$. With the definitions that the $x_m$ axis means the $x$ axis in the
Angles $\theta_k$ are the rotations of coordinate frame $k$. Angles are assumed zero as shown.

Fig. 3.1 Definitions of Coordinate Systems and Rotation Angles of the Manipulator (from Brooks [4])
m\textsuperscript{th} frame and the $x_m y_m$ plane means the plane containing the $x_m$ axis and the $y_m$ axis, we define the rotation $\alpha, \beta$ and $\gamma$ in Fig. 3.2. The rotation $\alpha$ is the angle between the $y_0 z_0$ plane and the plane which contains the $y_6$ axis and which is perpendicular to the $x_0 y_0$ plane. The rotation $\alpha, \beta$ and $\gamma$ are defined in Fig. 3.2.

If all joints $\theta_k$ through $\theta_6$ ($\theta_k$ specifies the rotation of the $k$\textsuperscript{th} frame with respect to the $k-1$ frame) are given, these six parameters are found as follows.

$$\begin{bmatrix}
x \\
y \\
z \\
1 
\end{bmatrix} = 0A_6 \begin{bmatrix}0 \\
0 \\
0 \\
1 
\end{bmatrix} = 0A_4 \begin{bmatrix}0 \\
0 \\
0 \\
1 
\end{bmatrix} \tag{3.2}$$

$$\alpha = \tan^{-1} \left( \frac{x_{hy} - x}{y_{hy} - y} \right) \tag{3.3}$$

$$\beta = \tan^{-1} \left( \frac{z_{hy} - z}{\sqrt{(x_{hy} - x)^2 + (y_{hx} - y)^2}} \right) \tag{3.4}$$

$$\gamma = \tan^{-1} \left( \frac{z_{hx} - z}{\sqrt{(x_{hx} - x)^2 + (y_{hx} - y)^2}} \right) \tag{3.5}$$
Fig. 3.2 Orientation of Hand
where

\[
\begin{bmatrix}
x_{hy} \\
y_{hy} \\
z_{hy} \\
1
\end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} A_6 = y_6 
\]  
(3.6)

\[
\begin{bmatrix}
x_{hx} \\
y_{hx} \\
z_{hx} \\
1
\end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} A_6 = x_6 
\]  
(3.7)

In contrast with above procedure, it is difficult to find \( \hat{e}_1 \) through \( \theta_6 \) when \( x, y, z, \alpha, \beta \) and \( \gamma \) are given. Since we want to keep the orientation of the hand parallel to the \( x_0y_0 \) plane, we have to give

\[
\beta = 0 \quad \text{(3.8)}
\]

\[
\gamma = 0 \quad \text{(3.9)}
\]

From those equations, we find \( \theta_5 \) and \( \theta_6 \) as follows. From equations (3.4) and (3.5), \( \beta = 0 \) and \( \gamma = 0 \) mean \( z_{hy} - z = 0 \) and \( z_{hx} - z = 0 \), respectively. That is,
\[
\begin{align*}
\mathbf{z}_{hy} - \mathbf{z} &= (0, 0, 1, 0) \mathbf{A}_0^1 \begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix} - (0, 0, 1, 0) \mathbf{A}_0^0 \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} \\
&= (0, 0, 1, 0) \mathbf{A}_0^1 \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} = a_{32} = 0
\end{align*}
\]

:. \theta_5 = \tan^{-1} \left\{ (\sin^3 \theta_1 \cos \theta_3 \cos \theta_4 + \cos^3 \theta_1 \cos \theta_2 \sin \theta_3 \cos \theta_4 \\
\quad + \cos \theta_1 \sin \theta_2 \sin \theta_4) / (\cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_1 \sin \theta_3) \right\} \tag{3.10}

\[
\begin{align*}
\mathbf{z}_{hx} - \mathbf{z} &= (0, 0, 1, 0) \mathbf{A}_0^1 \begin{bmatrix}
1 \\
0 \\
0 \\
1
\end{bmatrix} - (0, 0, 1, 0) \mathbf{A}_0^0 \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} \\
&= (0, 0, 1, 0) \mathbf{A}_0^1 \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} = a_{31} = 0
\end{align*}
\]

:. \theta_6 = \tan^{-1} \left\{ (-\cos \theta_1 \sin \theta_2 \cos \theta_4 + \sin \theta_1 \cos \theta_3 \sin \theta_4 \\
\quad + \cos \theta_1 \cos \theta_2 \sin \theta_3 \sin \theta_4) / (\cos \theta_1 \cos \theta_2 \cos \theta_3) \right\}

21
\[
- \sin \theta_1 \sin \theta_3 \cos \theta_5 - (\sin \theta_1 \cos \theta_3 \cos \theta_4 + \\
\cos \theta_1 \cos \theta_2 \sin \theta_3 \cos \theta_4 + \cos \theta_1 \sin \theta_2 \cos \theta_4)
\]

\[
\sin \theta_5
\]

(3.11)

Where \( a_{mn} \) represents an element in matrix \( A^0_6 \). On the other hand, from (3.3) \( \alpha \) is expressed as

\[
\tan \alpha = \frac{(1,0,0,0) A^0_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - (1,0,0,0) A^0_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}{(0,1,0,0) A^0_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} - (0,1,0,0) A^0_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix})
\]

\[
= \frac{a_{12}}{a_{22}} = (\sin \theta_2 \sin \theta_3 \cos \theta_4 - \cos \theta_2 \sin \theta_4 + \sin \theta_2 \cos \theta_3 \tan \theta_5) / [\cos \theta_1 \cos \theta_3 \cos \theta_4
\]

\[
- \sin \theta_1 \sin \theta_2 \sin \theta_4 - \sin \theta_1 \cos \theta_2 \sin \theta_3 \cos \theta_4
\]

\[-(\cos \theta_1 \sin \theta_3 + \sin \theta_1 \cos \theta_2 \cos \theta_3) \tan \theta_5 \]

(3.12)
To eliminate $\theta_5$, substitution of (3.10) to (3.12) gives

$$\tan \alpha = (-\cos \theta_1 \cos \theta_3 \sin \theta_4 - \sin \theta_1 \sin \theta_2 \cos \theta_4$$

$$+ \sin \theta_1 \cos \theta_2 \sin \theta_3 \sin \theta_4) / (\cos \theta_2 \cos \theta_4$$

$$+ \sin \theta_2 \sin \theta_3 \sin \theta_4)$$

$$= S \quad (3.13)$$

By means of $S$ instead of $x$, we define $P$.

$$P = S(\cos \theta_2 \cos \theta_4 + \sin \theta_2 \sin \theta_3 \sin \theta_4)$$

$$+ \sin \theta_1 \sin \theta_2 \cos \theta_4 + \cos \theta_1 \cos \theta_3 \sin \theta_4$$

$$- \sin \theta_1 \cos \theta_2 \sin \theta_3 \sin \theta_4 \quad (3.14)$$

Now that we give four parameters: $x$, $y$, $z$ and $P$, to find $\theta_1$ through $\theta_4$, the equations are rearranged as

$$x = 1.39 (\sin \theta_2 \sin \theta_3 \cos \theta_4 - \cos \theta_2 \sin \theta_4) - 40 \sin \theta_2 \cos \theta_3$$

$\quad (3.15)$
\[ y = 1.39 \left( \cos^2_1 \cos^2_3 \cos^2_4 - \sin^2_1 \cos^2_2 \sin^2_3 \cos^2_4 \right. \]
\[ \left. - \sin^2_1 \sin^2_2 \sin^2_4 \right) + 40 \left( \sin^2_1 \cos^2_2 \cos^2_3 \right. \]
\[ \left. + \cos^2_1 \sin^2_3 \right) + 18 \cos^2_1 \]  
(3.16)

\[ Z = 1.39 \left( \sin^2_1 \cos^2_3 \cos^2_4 + \cos^2_1 \cos^2_2 \sin^2_3 \cos^2_4 \right. \]
\[ \left. + \cos^2_1 \sin^2_2 \sin^2_4 \right) + 40 \left( -\cos^2_1 \cos^2_2 \cos^2_3 \right. \]
\[ \left. + \sin^2_1 \sin^2_3 \right) + 18 \sin^2_1 \]  
(3.17)

\[ P = \sin^2_1 \cos^2_2 \cos^2_4 + \sin^2_2 \sin^2_3 \sin^2_4 \]
\[ + \sin^2_1 \sin^2_2 \cos^2_4 + \cos^2_1 \cos^2_3 \sin^2_4 \]
\[ - \sin^2_1 \cos^2_2 \sin^2_3 \sin^2_4 \]  
(3.18)

It is quite difficult to solve equations (3.15)-(3.18), then we introduce the relation between the total differential and the partial differential of a function. [5]

The total differential of \( f \) is defined by the equation, [6]

\[ df = \frac{\partial f}{\partial x} \, dx + \frac{\partial f}{\partial y} \, dy + \frac{\partial f}{\partial z} \, dz \]  
(3.19)
where \( f \) is a function of three variables \( x, y \) and \( z \), and \( \frac{\partial f}{\partial x} \) is the partial derivative with respect to \( x \).

If \( x, y \) and \( z \) are all functions of a single variable, say \( t \), then the dependent variable \( f \) may also be considered as truly a function of the one independent variable \( t \).

Since only one independent variable is present, \( df/dt \) has a meaning and it can be shown, by appropriate limiting processes, that

\[
\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}.
\]  

(3.20)

For a short period of time \( \Delta t \), it can be replaced by

\[
\frac{\Delta f}{\Delta t} = \frac{\partial f}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y} \frac{\Delta y}{\Delta t} + \frac{\partial f}{\partial z} \frac{\Delta z}{\Delta t}.
\]  

(3.21)

Consequently, we get

\[
\Delta f = f_x \Delta x + f_y \Delta y + f_z \Delta z.
\]  

(3.22)

where \( f_x = \frac{\partial f}{\partial x} \).

Applying the relation to our problem gives

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta p
\end{bmatrix} = \begin{bmatrix}
x_1 & x_2 & x_3 & x_4 \\
y_1 & y_2 & \cdots & \cdots \\
z & \cdots & \cdots \\
p_1 & \cdots & \cdots & p_4
\end{bmatrix} \begin{bmatrix}
\Delta \theta_1 \\
\Delta \theta_2 \\
\Delta \theta_3 \\
\Delta \theta_4
\end{bmatrix}.
\]  

(3.23)
Provided that \(|J(\hat{\theta})| \neq 0\), we have

\[
\begin{bmatrix}
\Delta \theta_1 \\
\Delta \theta_2 \\
\Delta \theta_3 \\
\Delta \theta_4
\end{bmatrix} = J(\hat{\theta})^{-1}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta p
\end{bmatrix}
\]

(3.24)

Each component of Jacobian matrix is shown in Table A.3.

3.2 Jacobian of 1st Order Approximation

By looking over components of the Jacobian matrix, we find it contains many trigonometric functions. Decreasing the trigonometric functions in number will cause more efficiency in terms of mechanism and time for calculation. Then we introduce the appropriate Jacobian matrix so that we may eliminate the trigonometric functions, provided that \(\theta\) is limited in the neighborhood of \(\theta_0\); \(\theta_s (= \theta - \theta_0)\) is small.

At first we consider the 0th order approximation: \(\sin \theta_s\) and \(\cos \theta_s\) are expressed as \(\sin \theta_s = 0\) and \(\cos \theta_s = 1\), that is,

\[
\begin{align*}
\sin \theta &= \sin (\theta_0 + \theta_s) \approx \sin \theta_0 \\
\cos \theta &= \cos (\theta_0 + \theta_s) \approx \cos \theta_0
\end{align*}
\]

(3.25)

The 0th order approximation makes Jacobian matrix constant so that we do not need estimate it after the manipulator starts moving as well as we get \(\hat{\theta}\) with ease. But it causes much error.
The 1st order approximation: \( \sin \theta \) and \( \cos \theta \) are expressed as

\[
\sin \theta = \sin (\theta_o + \theta_s) = \sin \theta_o + \theta_s \cos \theta_o
\]

\[
\cos \theta = \cos (\theta_o + \theta_s) = \cos \theta_o - \theta_s \sin \theta_o
\]

(3.26)

This approximation has less complicated calculation than the conventional one and less error than the 0th order approximation. The final form we get is expressed as

\[
\Delta \theta_n = \frac{(\Delta x, \Delta y, \Delta z, \Delta p)[C_n]}{\theta_{1s} \theta_{2s} \theta_{3s} \theta_{4s}} = \frac{-1}{\Delta x [C_n] \theta_s}
\]

(3.27)

where

\[
\hat{\theta}_s = \theta - \hat{\theta}_o
\]

\[
J(\hat{\theta}) \hat{\theta} = [C_n]^{-1} \theta_s
\]

[\( C_n \)] is constant
Next, as an example of higher order approximation, we consider the 2nd order approximation $\sin_{s}^{2}$ and $\cos_{s}^{2}$ are expressed as

$$
\sin_{s}^{2} = \sin^{2}(\theta_{0} + \theta_{s}) = (1 - \frac{\theta_{s}^{2}}{2}) \sin \theta_{0} + \theta_{s} \cos \theta_{0}
$$

$$
\cos_{s}^{2} = \cos^{2}(\theta_{0} + \theta_{s}) = (1 - \frac{\theta_{s}^{2}}{2}) \cos \theta_{0} - \theta_{s} \sin \theta_{0}
$$

These equations prove that the higher order approximation needs more complicated calculation compared with the 0th and 1st order ones, though it is more accurate. Accordingly, we conclude that the 1st order is the most appropriate in terms of simplicity and accuracy. The components of the 1st order approximation are expressed as

$$
\begin{bmatrix}
x_{1} \\
x_{2} \\
x_{3} \\
x_{4}
\end{bmatrix} =
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
x_{41} & \ldots & \ldots & \ldots & x_{45}
\end{bmatrix}
\begin{bmatrix}
\theta_{1s} \\
\theta_{2s} \\
\theta_{3s} \\
\theta_{4s} \\
1
\end{bmatrix}
$$

$$
\begin{bmatrix}
y_{1} \\
y_{2} \\
y_{3} \\
y_{4}
\end{bmatrix} =
\begin{bmatrix}
y_{11} & \ldots & \ldots & \ldots & y_{15} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
y_{41} & \ldots & \ldots & \ldots & y_{45}
\end{bmatrix}
\begin{bmatrix}
\theta_{1s} \\
\theta_{2s} \\
\theta_{3s} \\
\theta_{4s} \\
1
\end{bmatrix}
$$
where $x_1 = 3x/3\theta_1$.

Details are shown in Table A.4. Consequently, we have $\theta$, through $\Delta\theta_4$

$$\Delta\theta_n = \Delta / \Delta \quad (n = 1, \ldots, 4)$$

where

$$\Delta = \begin{bmatrix}
    x_1 & x_2 & \cdots & x_4 \\
    y_1 & & & \\
    z_1 & & & \\
    p_1 & \cdots & p_4 \\
\end{bmatrix}$$

$$\Delta n = \begin{bmatrix}
    x_1 & \Delta x & \cdots & x_4 \\
    y_1 & \Delta y & & \\
    z_1 & \Delta z & & \\
    p_1 & \Delta p & \cdots & p_4 \\
\end{bmatrix}$$
In reference to $\theta_5$ and $\theta_6$, they are much influenced by the posture of the hand so that they change more than others. Therefore, introducing the approximate calculus for $\theta_5$ and $\theta_6$ is not suitable. So we get $\theta_5$ and $\theta_6$ by the basic expression.

$$\theta_5 = f(\theta_1, \theta_2, \theta_3, \theta_4), \quad \theta_6 = f(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) \quad (3.35)$$

According to those methods, we can get ($\theta_1$, $\theta_2$, $\ldots$, $\theta_6$), consequently $x$, $y$, $z$ and $S$ by means of equations (3.15) - (3.18).

As the result, we can control $A$.

### 3.3 Compensation

According to Chapter 2, $T$ is measured by the measurement arm.

We show the basic idea of measuring the position and orientation of the table. The measurement arm consisted of 6 joints all of which are rotational pairs. In practical use, the extreme distal link is kept perpendicular to the horizontal plane. On this condition, we can get $x$, $y$ and $z$ coordinates of point $Q$ and angle $\alpha$. For convenience, we define angles $\theta_1'$, $\theta_2'$ and length $\lambda$ as in Fig. 3.4 and Fig. 3.5.

Accordingly, point $Q$ is determined by

$$\begin{align*}
Q_x &= \lambda \cos \theta_2' \\
Q_y &= \lambda \sin \theta_2' \\
Q_z &= \lambda \sin \theta_1' - (b \cos(\theta_1' + \theta_4') + c)
\end{align*} \quad (3.36)$$
where

\[ l = a \cos \theta_1' + b \sin(\theta_1' + \theta_4) \]

\[ \theta_1' = \tan^{-1}\left(\frac{1}{\tan \theta_1 \tan \theta_2}\right) \]

\[ \theta_2' = -\tan^{-1}\left(\frac{\tan \theta_2}{\sin \theta_1}\right) \]

These coordinates are determined in terms of the coordinate system which is fixed in the measurement arm. However, they need to be expressed by the common coordinate system which the slave arm can use. In terms of the common system, point \( Q \) is expressed by

\[
\begin{align*}
Q_x &= Q_x + \Delta x \\
Q_y &= Q_y + \Delta y \\
Q_z &= Q_z + \Delta z
\end{align*}
\]  \hspace{1cm} (3.37)

Where \((\Delta x, \Delta y, \Delta z)\) means the vector which shows the distance between two origins of coordinate system.

\[ \alpha = \frac{\pi}{2} - (\theta_2' - \theta_6) \]  \hspace{1cm} (3.38)
Angles are assumed zero as shown.

Fig. 3.3 Rotational Angles of the Measurement Arm
Fig. 3.4 Rotational Angles in Practical Use
a) Two Coordinate Systems

b) Plane including all Links of Measurement Arm

Fig. 3.5 Rotational Angles and Some Other Parameters
CHAPTER 4

EXPERIMENTS

We mainly had two experiments: one was to measure the accuracy of the Measurement Arm; another was to measure the accuracy of the first order of approximation of Jacobian Matrices used for the straight-line motion of Slave Arm.

As to the first one, we combined the Measurement Arm with the Table illustrated in Fig. 2.1.b. and made the table move with the distance of $\Delta x = (x, y, z)$ by means of the program MSURE. As illustrated in Figs. 4.1 and 4.2, we got good linearity, with errors within $\pm 0.15$ inch. This is a good result, considering the experimental mechanism of the Measurement Arm.

Next, we tried to make a straight-line motion of the Slave Arm by means of program MAIN. As illustrated in Fig. 4.3, we make the arm move from origin to point $p_1$, the distance $\hat{x}_1$. The results are shown in Tables 4.3, 4.4 and 4.5. For example, as to the reference $\hat{x}_1(Ax_1, 0, 0)$ in Table 4.3, we have the errors within 0.09 inch, 0.19 inch and 0.21 inch, along the x-axis, y-axis and z-axis, respectively. This is also a good result, considering that the Slave Arm mechanism has significant backlash. The errors along the y-axis are big compared with others. It is mainly caused by the backlash of the angle $\theta_1$.

In Figs. 4.1.c. and 4.2.c. we can see the increase in error, it is because the z-axis of the Measurement Arm is not in parallel with the z-axis of the Table.
TABLE 4.1
ACCURACY OF MEASUREMENT ARM

a) Measured Data $\lambda_y = 0$, $\lambda_z = 0$

<table>
<thead>
<tr>
<th>$\lambda_x$ inch</th>
<th>x inch</th>
<th>y inch</th>
<th>z inch</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.71</td>
<td>-6.78</td>
<td>3.89</td>
<td>0.870</td>
</tr>
<tr>
<td>1</td>
<td>5.75</td>
<td>-6.84</td>
<td>3.96</td>
<td>0.879</td>
</tr>
<tr>
<td>2</td>
<td>6.73</td>
<td>-6.85</td>
<td>4.03</td>
<td>0.890</td>
</tr>
<tr>
<td>3</td>
<td>7.61</td>
<td>-6.78</td>
<td>4.10</td>
<td>0.908</td>
</tr>
<tr>
<td>4</td>
<td>8.53</td>
<td>-6.75</td>
<td>4.14</td>
<td>0.932</td>
</tr>
<tr>
<td>5</td>
<td>9.49</td>
<td>-6.67</td>
<td>4.19</td>
<td>0.964</td>
</tr>
</tbody>
</table>

b) Modified Data $\lambda_y = 0$, $\lambda_z = 0$

<table>
<thead>
<tr>
<th>$\lambda_x$ inch</th>
<th>x inch</th>
<th>y inch</th>
<th>z inch</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>1.04</td>
<td>-0.06</td>
<td>0.07</td>
<td>0.009</td>
</tr>
<tr>
<td>2</td>
<td>2.02</td>
<td>-0.07</td>
<td>0.14</td>
<td>0.020</td>
</tr>
<tr>
<td>3</td>
<td>2.90</td>
<td>0.00</td>
<td>0.21</td>
<td>0.038</td>
</tr>
<tr>
<td>4</td>
<td>3.82</td>
<td>0.03</td>
<td>0.25</td>
<td>0.062</td>
</tr>
<tr>
<td>5</td>
<td>4.78</td>
<td>0.11</td>
<td>0.30</td>
<td>0.094</td>
</tr>
</tbody>
</table>
Fig. 4.1 Accuracy of Measurement Arm, $l_y = 0$, $l_z = 0$
TABLE 4.2
ACCURACY OF MEASUREMENT ARM

a) Measured Data $\lambda_x = 0$, $\lambda_z = 0$

<table>
<thead>
<tr>
<th>$\lambda_y$ inch</th>
<th>x inch</th>
<th>y inch</th>
<th>z inch</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.74</td>
<td>-12.71</td>
<td>3.70</td>
<td>0.812</td>
</tr>
<tr>
<td>1</td>
<td>3.71</td>
<td>-11.79</td>
<td>3.74</td>
<td>0.815</td>
</tr>
<tr>
<td>2</td>
<td>3.74</td>
<td>-10.80</td>
<td>3.77</td>
<td>0.807</td>
</tr>
<tr>
<td>3</td>
<td>3.79</td>
<td>-9.76</td>
<td>3.79</td>
<td>0.811</td>
</tr>
<tr>
<td>4</td>
<td>3.83</td>
<td>-8.71</td>
<td>3.80</td>
<td>0.823</td>
</tr>
<tr>
<td>5</td>
<td>3.84</td>
<td>-7.77</td>
<td>3.80</td>
<td>0.842</td>
</tr>
</tbody>
</table>

b) Modified Data $\lambda_x = 0$, $\lambda_z = 0$

<table>
<thead>
<tr>
<th>$\lambda_y$ inch</th>
<th>x inch</th>
<th>y inch</th>
<th>z inch</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>-0.03</td>
<td>0.92</td>
<td>0.04</td>
<td>0.003</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>1.91</td>
<td>0.07</td>
<td>-0.005</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>2.95</td>
<td>0.09</td>
<td>-0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>4.00</td>
<td>0.10</td>
<td>0.011</td>
</tr>
<tr>
<td>5</td>
<td>0.10</td>
<td>4.94</td>
<td>0.10</td>
<td>0.030</td>
</tr>
</tbody>
</table>
Fig. 4.2 Accuracy of Measurement Arm, $\lambda_x = 0, \lambda_z = 0$
Fig. 4.3 Accuracy of Approximation Vehicle Frame
TABLE 4.3

Accuracy of Approximation \( \bar{x}_1, \Delta y_1 = \Delta z_1 = 0 \)

a) Measured Data

<table>
<thead>
<tr>
<th>( \Delta x_1 )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.01</td>
<td>-0.13</td>
<td>-0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.49</td>
<td>-0.35</td>
<td>-0.04</td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>-0.17</td>
<td>-0.03</td>
</tr>
<tr>
<td>2</td>
<td>1.98</td>
<td>-0.32</td>
<td>-0.05</td>
</tr>
<tr>
<td>3</td>
<td>2.99</td>
<td>-0.32</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>3.90</td>
<td>-0.32</td>
<td>0.21</td>
</tr>
</tbody>
</table>

b) Modified Data

<table>
<thead>
<tr>
<th>( \Delta x_1 )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>-0.22</td>
<td>-0.04</td>
</tr>
<tr>
<td>1</td>
<td>0.99</td>
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<td>-0.03</td>
</tr>
<tr>
<td>2</td>
<td>1.99</td>
<td>-0.19</td>
<td>-0.05</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>-0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>3.91</td>
<td>-0.19</td>
<td>0.21</td>
</tr>
</tbody>
</table>
### TABLE 4.4
Accuracy of Approximation $x_1$, $\Delta x_1 = \Delta z_1 = 0$

#### a) Measured Data

<table>
<thead>
<tr>
<th>$\Delta y_1$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.00</td>
<td>-0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.02</td>
<td>0.32</td>
<td>-0.03</td>
</tr>
<tr>
<td>1</td>
<td>-0.07</td>
<td>0.88</td>
<td>-0.09</td>
</tr>
<tr>
<td>2</td>
<td>-0.02</td>
<td>1.74</td>
<td>-0.15</td>
</tr>
<tr>
<td>3</td>
<td>-0.07</td>
<td>2.84</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>-0.06</td>
<td>3.87</td>
<td>0.26</td>
</tr>
</tbody>
</table>

#### b) Modified Data

<table>
<thead>
<tr>
<th>$\Delta y_1$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>0.00</td>
</tr>
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<td>-0.02</td>
<td>0.38</td>
<td>-0.02</td>
</tr>
<tr>
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<td>-0.07</td>
<td>0.94</td>
<td>-0.08</td>
</tr>
<tr>
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<td>-0.02</td>
<td>1.30</td>
<td>-0.14</td>
</tr>
<tr>
<td>3</td>
<td>-0.07</td>
<td>2.90</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>-0.06</td>
<td>3.93</td>
<td>0.27</td>
</tr>
</tbody>
</table>
TABLE 4.5
Accuracy of Approximation $\tilde{x}_1$, $\Delta x_1 = \Delta y_1 = 0$

a) Measured Data

<table>
<thead>
<tr>
<th>$\Delta z_1$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.00</td>
<td>-0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>0.2</td>
<td>0.03</td>
<td>-0.22</td>
<td>0.13</td>
</tr>
<tr>
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<td>-0.04</td>
<td>-0.16</td>
<td>0.45</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
<td>-0.31</td>
<td>0.97</td>
</tr>
<tr>
<td>2</td>
<td>-0.05</td>
<td>-0.30</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>-0.55</td>
<td>2.90</td>
</tr>
</tbody>
</table>

b) Modified Data

<table>
<thead>
<tr>
<th>$\Delta z_1$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.03</td>
<td>-0.16</td>
<td>0.14</td>
</tr>
<tr>
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<td>-0.04</td>
<td>-0.10</td>
<td>0.46</td>
</tr>
<tr>
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<td>0.01</td>
<td>-0.25</td>
<td>0.98</td>
</tr>
<tr>
<td>2</td>
<td>-0.05</td>
<td>-0.24</td>
<td>1.98</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>-0.49</td>
<td>2.91</td>
</tr>
</tbody>
</table>
TABLE 4.6
Accuracy of Approximation $\bar{x}_2$, $y_2 = z_2 = 0$

a) Measured Data

<table>
<thead>
<tr>
<th>$\Delta x_2$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.92</td>
<td>-2.22</td>
<td>1.15</td>
</tr>
<tr>
<td>0.5</td>
<td>3.46</td>
<td>-2.50</td>
<td>1.20</td>
</tr>
<tr>
<td>1</td>
<td>3.93</td>
<td>-2.19</td>
<td>1.14</td>
</tr>
<tr>
<td>2</td>
<td>4.90</td>
<td>-2.32</td>
<td>1.28</td>
</tr>
<tr>
<td>3</td>
<td>5.82</td>
<td>-2.26</td>
<td>1.36</td>
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</tbody>
</table>

b) Modified Data

<table>
<thead>
<tr>
<th>$\Delta x_2$</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.54</td>
<td>-0.28</td>
<td>0.05</td>
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<td>1</td>
<td>1.01</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>4</td>
<td>3.96</td>
<td>-0.21</td>
<td>0.48</td>
</tr>
</tbody>
</table>
### TABLE 4.7

Accuracy of Approximation $\hat{x}_2$, $\Delta x_2 = \Delta z_2 = 0$

#### a) Measured Data

<table>
<thead>
<tr>
<th>$\Delta y_2$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.92</td>
<td>-2.12</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>2.94</td>
<td>-1.84</td>
<td>1.09</td>
</tr>
<tr>
<td>1</td>
<td>2.93</td>
<td>-1.43</td>
<td>1.10</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>-0.48</td>
<td>1.11</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>0.73</td>
<td>1.05</td>
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<tr>
<td>4</td>
<td>2.88</td>
<td>1.77</td>
<td>1.16</td>
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</table>

#### b) Modified Data

<table>
<thead>
<tr>
<th>$\Delta y_2$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.02</td>
<td>0.28</td>
<td>-0.05</td>
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<tr>
<td>1</td>
<td>0.01</td>
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<td>-0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>1.64</td>
<td>-0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.08</td>
<td>2.85</td>
<td>-0.09</td>
</tr>
<tr>
<td>4</td>
<td>-0.04</td>
<td>3.89</td>
<td>0.02</td>
</tr>
</tbody>
</table>
### TABLE 4.8
Accuracy of Approximation $\Delta x_2$, $\Delta y_2 = \Delta y_2 = 0$

**a) Measured Data**

<table>
<thead>
<tr>
<th>$\Delta z_2$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-2.32</td>
<td>1.19</td>
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<td>2.87</td>
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<td>1.58</td>
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<tr>
<td>1</td>
<td>2.96</td>
<td>-2.30</td>
<td>2.05</td>
</tr>
<tr>
<td>2</td>
<td>2.92</td>
<td>-2.64</td>
<td>3.10</td>
</tr>
<tr>
<td>3</td>
<td>2.86</td>
<td>-2.81</td>
<td>4.08</td>
</tr>
<tr>
<td>4</td>
<td>2.82</td>
<td>-3.06</td>
<td>5.04</td>
</tr>
</tbody>
</table>

**b) Modified Data**

<table>
<thead>
<tr>
<th>$\Delta z_2$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.14</td>
<td>0.07</td>
<td>0.39</td>
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<tr>
<td>1</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>-0.08</td>
<td>-0.32</td>
<td>1.91</td>
</tr>
<tr>
<td>3</td>
<td>-0.14</td>
<td>-0.49</td>
<td>2.89</td>
</tr>
<tr>
<td>4</td>
<td>-0.18</td>
<td>-0.74</td>
<td>3.85</td>
</tr>
</tbody>
</table>
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS
FOR FUTURE WORK

The purpose of this research was to extend the compensation done previously by Tani which presupposed a perfect measurement by using a sensor.

For this purpose, as a sensor we developed a 6 degree-of-freedom passive Measurement Arm having a simple gripper but otherwise flaccid. In addition to developing the sensor, we extended the control method by considering some approximated Jacobian matrices instead of complicated strict Jacobian matrices. Consequently, we developed the Jacobian matrices of first order approximation with no trigonometric functions.

We performed two experiments: one was to measure the accuracy of the Measurement Arm by moving the table; another was to measure the accuracy of the Jacobian matrices of first order approximation by moving the Slave Arm. The result showed the capability of compensation with the Measurement Arm as a sensor.

Having developed the basic hardware and software for compensation by means of the sensor, we intend to improve this compensation by using these techniques in the near future.

As an example of future work, I show the following two programs. One is the program named TTT, which will keep the same orientation of the hand of the Slave Arm and the same distance between the table and Slave Arm, regardless of the position of the table. This is to test the total error caused by the hardware and software of the system. Another is the program named HICOM which is to extend Tani’s compensation by using the
sensor. These are almost completed but not yet perfect.

In order to extend the compensation with sensor, we suggest developing a program which considers the errors caused by mechanisms such as backlash in addition to present considerations.
REFERENCES


APPENDIX I

TRANSFORMATION MATRICES
### TABLE A.1

Frame Transformation

\[
0_{A_1} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_1 & -\sin \theta_1 \\
0 & \sin \theta_1 & \cos \theta_1 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
1_{A_2} = \begin{bmatrix}
\cos \theta_2 & 0 & \sin \theta_2 \\
0 & 1 & 0 \\
-\sin \theta_2 & 0 & \cos \theta_2 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
2_{A_3} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_3 & -\sin \theta_3 \\
0 & \sin \theta_3 & \cos \theta_3 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
3_{A_4} = \begin{bmatrix}
\cos \theta_4 & -\sin \theta_4 & 0 & -1.39 \sin \theta_4 \\
\sin \theta_4 & \cos \theta_4 & 0 & 1.39 \cos \theta_4 \\
0 & 0 & 1 & -40 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
4_{A_5} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_5 & -\sin \theta_5 \\
0 & \sin \theta_5 & \cos \theta_5 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
5_{A_6} = \begin{bmatrix}
\cos \theta_6 & 0 & \sin \theta_6 \\
0 & 1 & 0 \\
-\sin \theta_6 & 0 & \cos \theta_6 \\
0 & 0 & 1
\end{bmatrix}
\]
### TABLE A.2

Transformation from Hand to Vehicle

\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} \\
  a_{21} & a_{22} & a_{23} & a_{24} \\
  a_{31} & a_{32} & a_{33} & a_{34} \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{align*}
a_{11} &= (C2C4 + S2S3S4) C6 + (S2S3C4S5 - C2S4S5 - S2C3C5) S6 \\
a_{12} &= (S2S3C4 - C2S4) C5 + S2C3S5 \\
a_{13} &= (C2S4S5 - S2S3C4S5 + S2C) C6 + (C2C4 + S2S3S4) S6 \\
a_{14} &= 1.39 (S2S3C4 - C2S4) - 40 (S2C3) \\
a_{21} &= (C1C3C4 - S1C2S3C4 - S1S2S4) S5S6 + (C1S3 + S1C2C3) C5S6 + (C1C3 - S1C2S3) S4C6 + S1S2C4C6 \\
a_{22} &= (C1C3C4 - S1S2S4 - S1C2S3C4) C5 - (C1S3 + S1C2C3) S5 \\
a_{23} &= (S1S2S4 - C1C3C4 + S1C2S3C4) S5C6 - (C1S3 + S1C2C3) C5C6 + (C1C3 - S1C2S3) S4S6 + S1S2C4S6 \\
a_{24} &= 1.39 (C1C3C4 - S1C2S3C4 - S1S2S4) + 40 (C1S3) \\
a_{31} &= (C1S2S4 + S1C3C4 + C1C2S3C4) S5S6 + (S1S3 - C1S2C3) C5 + (C1C2S3 + S1C3) S4C6 - C1S2C4C6 \\
a_{32} &= (C1S2S4 + S1C3C4 + C1C2S3C4) C5 + (C1C2C3 - S1S3) \\
a_{33} &= -(C1S2S4 + S1C3C4 + C1C2S3C4) S5C6 + (S1C3 + C1C2S3) + (C1C2C3 - S1S3) C5C6 - C1S2C4S6 \\
a_{34} &= 1.39 (S1C3C4 + C1C2S3C4 + C1S2S4) + 40 (S1S3 - C1C2C3) + 18S1
\end{align*}
\]

where \( S1 = \sin \theta_1 \), \( Cl = \cos \theta_1 \)
APPENDIX II

JACOBIAN MATRICES
TABLE A.3

Jacobian Matrix

\[
J(\theta) = \begin{bmatrix}
  x_1 & x_2 & x_3 & x_4 \\
  y_1 & y_2 & y_3 & y_4 \\
  z_1 & z_2 & z_3 & z_4 \\
  p_1 & p_2 & p_3 & p_4
\end{bmatrix}
\]

where \( x_1 = \partial x / \partial \theta_1 \).

\[
\begin{align*}
  x_1 &= 0 \\
  x_2 &= 1.39 (C_2S_3C_4 + S_2S_4) - 40C_2C_3 \\
  x_3 &= 1.39S_2C_3C_4 + 40S_2S_3 \\
  x_4 &= -1.39 (C_2C_4 + S_2S_3S_4) \\
  y_1 &= -1.39 (S_1C_3C_4 + C_1C_2S_3C_4 + C_1S_2S_4) \\
  &\quad + 40 (C_1C_2C_3 - S_1S_3) - 18S_1 \\
  y_2 &= 1.39 (S_1S_2S_3C_4 - S_1C_2S_4) - 40S_1S_2C_3 \\
  y_3 &= -1.39 (S_1C_2C_3C_4 + C_1S_3C_4) + 40 (C_1C_3 - S_1C_2S_3) \\
  y_4 &= 1.39 (-S_1S_2C_4 - C_1C_3S_4 + S_1C_2S_3S_4) \\
  z_1 &= 1.39 (C_1C_3C_4 - S_1C_2S_3C_4 - S_1S_2S_4) \\
  &\quad + 40 (S_1C_2C_3 + C_1S_3) + 18C_1 \\
  z_2 &= 1.39 (-C_1S_2S_3C_4 + C_1C_2S_4) + 40C_1S_2C_3 \\
  z_3 &= 1.39 (C_1C_2C_3C_4 - S_1S_3C_4) + 40 (S_1C_3 + C_1C_2S_3) \\
  z_4 &= 1.39 (C_1S_2C_4 - S_1C_3S_4 - C_1C_2S_3S_4) \\
  p_1 &= 0 + C_1S_2C_4 - S_1C_3S_4 - C_1C_2S_3S_4 \\
  p_2 &= S (-S_2C_4 + C_2S_3S_4) + (S_1C_2C_4 + S_1S_2S_3S_4) \\
  p_3 &= S (S_2C_3S_4) + (-S_1C_2C_3S_4 - C_1S_3S_4)
\end{align*}
\]
Table A.3 (Continued)

\[ p_4 = S (S_2S_3C_4 - C_2S_4) + (C_1C_3C_4 - S_1C_2S_3C_4 - S_1S_2S_4) \]

where \( S_1 = \sin \theta_1 \) and \( C_1 = \cos \theta_1 \).
TABLE A.4

Components of Matrix

\[ \begin{align*}
&x_{11} = 0 \\
&x_{12} = 0 \\
&x_{13} = 0 \\
&x_{14} = 0 \\
&x_{15} = 0 \\
&x_{21} = 0 \\
&x_{22} = 1.39 \left( -S_{30} - C_{40} \right) S_{20} + S_{40} C_{20} + 40C_{20}S_{20} \\
&x_{23} = 1.39 \left( C_{20} + C_{40} \right) C_{30} + 40C_{20}C_{30} \\
&x_{24} = 1.39 \left( -C_{20} - S_{30} \right) S_{40} + S_{20}C_{40} \\
&x_{25} = 1.39 \left( C_{20}S_{30}C_{40} + S_{20}S_{40} \right) - 40C_{20}C_{30} \\
&x_{31} = 0 \\
&x_{32} = 1.39 \left( C_{20}C_{30}C_{40} \right) + 40C_{20}S_{30} \\
&x_{33} = -1.39 S_{20}S_{30}S_{40} + 40S_{20}C_{30} \\
&x_{34} = -1.39S_{20}C_{30}S_{40} \\
&x_{35} = 1.39S_{20}C_{30}C_{40} + 40S_{20}S_{30} \\
&x_{41} = 0 \\
&x_{42} = 1.39 \left( S_{20}C_{40} - C_{20}S_{30}S_{40} \right) \\
&x_{43} = 1.39S_{20}C_{30}S_{40} \\
&x_{44} = 1.39 \left( C_{20}S_{40} - S_{20}S_{30}C_{40} \right) \\
&x_{45} = -1.39 \left( C_{20}C_{40} + S_{20}S_{30}S_{40} \right) \\
\end{align*} \]

where \( C_{10} = \cos \theta_{10} \), \( S_{10} = \sin \theta_{10} \)
Table A.4 (Continued)

\[ y_{11} = -1.39 \ (ClOC3OC40 - S10C20S30C40) \]
\[ + 40 \ (-S10C20C30 - Cl0S30) - 18C10 \]
\[ y_{12} = -1.39 \ [C10 (-S20) S30 C40] + 40 \ [C10 (-S20) C30] \]
\[ y_{13} = -1.39 \ [S10 (-S30) C40 + C10C20C30C40] \]
\[ + 40 \ [Cl0C20 (-S30) - S10C30] \]
\[ y_{14} = -1.39 \ [S10C30 (-S40) + C10C20S30 (-S40)] \]
\[ y_{15} = -1.39 \ (S10C30C40 + C10C20S30C40) \]
\[ + 40 \ (C10C20C30 - S10S30) - 18S10 \]
\[ y_{21} = -1.39 \ (S10S20S30C40 - C10C20S40) \]
\[ - 40C10S20C30 \]
\[ y_{22} = -1.39 \ [S10C20S30C40 - S10 (-S20) S40] \]
\[ - 40S10C20C30 \]
\[ y_{23} = 1.39S10S20C30C40 - 40S10S20 (-S30) \]
\[ y_{24} = -1.39 \ [S10S20S30 (-S40) - S10C20C40] \]
\[ y_{25} = -1.39 \ (S10S20S30C40 - S10C20S40) \]
\[ - 40S10S20C30 \]
\[ y_{31} = -1.39 \ (C10C20C30C40 - S10S30C40) \]
\[ + 40 \ (-S10C30 - C10C20S30) \]
\[ y_{32} = -1.39 \ [S10 (-S20) C30C40] - 40S10 (-S20) S30 \]
\[ y_{33} = -1.39 \ [S10C20 (-S30) C40 + C10C30C40] \]
\[ + 40 \ [C10 (-S30) - S10C20C30] \]
\[ y_{34} = -1.39 \ [S10C20C30 (-S40) + C10S30 (-S40)] \]
\[ y_{35} = -1.39 \ (S10C20C30C40 + C10S30C40) \]
\[ + 40 \ (C10C30 - S10C20S30) \]
\[ y_{41} = 1.39 \ (-C10S20C40 + S10C30S40 \]
\[ + C10C20S30S40) \]
\[ y_{42} = 1.39 \ [-S10C20C40 + S10 (-S20) S30S40] \]
\[ y_{43} = 1.39 \ [-C10 (S30) S40 + S10C20C30S40] \]
Table A.4 (Continued)

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{44}$</td>
<td>$1.39 \left[ \begin{array}{l} -S1O20 \ (-S40) - C10C30C40 \ + S1O20S30C40 \end{array} \right]$</td>
</tr>
<tr>
<td>$y_{45}$</td>
<td>$1.39 \left(\begin{array}{l} -S1O20C40 - C10C30S40 \ + S1O20S30S400 \end{array} \right)$</td>
</tr>
<tr>
<td>$z_{11}$</td>
<td>$1.39 \left[ \begin{array}{l} -S1O30C40 - C10C20S30C40 - C10S20S40 \end{array} \right] + 40 \left(\begin{array}{l} C1O20C30 - S1O20S30 \end{array} \right) + 18 \left(\begin{array}{l} -S1O \end{array} \right)$</td>
</tr>
<tr>
<td>$z_{12}$</td>
<td>$1.39 \left[ \begin{array}{l} -S1O \ (-S20) S30C40 - S1O20S40 \end{array} \right] + 40 \left[\begin{array}{l} S1O \ (-S20) C30 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{13}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O \ (-S30) C40 - S1O20C30C40 \end{array} \right] + 40 \left[\begin{array}{l} S1O20 \ (-S30) + C1O2C30 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{14}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O2C30 \ (-S40) - S1O20S30 \ (-S40) \ - S1O20C40 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{15}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O2C30C40 - S1O20S30C40 - S1O20S40 \end{array} \right] + 40 \left[\begin{array}{l} S1O20C30 + C1O20S30 \end{array} \right] + 18C1O$</td>
</tr>
<tr>
<td>$z_{21}$</td>
<td>$1.39 \left[ \begin{array}{l} S1O20S30C40 - S1O20S40 \end{array} \right] + 40 \left(\begin{array}{l} - S1O20C30 \end{array} \right)$</td>
</tr>
<tr>
<td>$z_{22}$</td>
<td>$1.39 \left[ \begin{array}{l} -C1O2C30S30C40 + C1O \ (-S20) S40 \end{array} \right] + 40C1O2C20C30$</td>
</tr>
<tr>
<td>$z_{23}$</td>
<td>$1.39 \left(\begin{array}{l} -C1O \ S20C30C40 + 40C1O20S20 \ (-S30) \end{array} \right)$</td>
</tr>
<tr>
<td>$z_{24}$</td>
<td>$1.39 \left[ \begin{array}{l} -C1O2S20S30 \ (-S40) + C1O2C20C40 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{25}$</td>
<td>$1.39 \left[ \begin{array}{l} -C1O2S20S30C40 + C1O2C20S40 \end{array} \right] + 40C1O2S20C30$</td>
</tr>
<tr>
<td>$z_{31}$</td>
<td>$1.39 \left[ \begin{array}{l} -S1O2C20C30C40 - C1O2S30C40 \end{array} \right] + 40 \left[\begin{array}{l} C1O2C30 - S1O20S30 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{32}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O \ (-S20) C30C40 + 40C1O \ (-S20) S30 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{33}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O2C20 \ (-S30) C40 - S1O2C30C40 \end{array} \right] + 40 \left[\begin{array}{l} S1O \ (-S30) + C1O2C20C30 \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{34}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O2C20C30 \ (-S40) - S1O2S30 \ (-S40) \end{array} \right]$</td>
</tr>
<tr>
<td>$z_{35}$</td>
<td>$1.39 \left[ \begin{array}{l} C1O2C20C30C40 - S1O2S30C400 \end{array} \right] + 40 \left[\begin{array}{l} S1O2C30 + C1O2C20S30 \end{array} \right]$</td>
</tr>
</tbody>
</table>
Table A.4 (Continued)

<table>
<thead>
<tr>
<th>( z_{41} )</th>
<th>( 1.39 ) ((-S10S20C40 - C10C30S40 + S10C20S30S40))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{42} )</td>
<td>( 1.39 ) ([C10C20C40 - C10 (-S20) S30S40])</td>
</tr>
<tr>
<td>( z_{43} )</td>
<td>( 1.39 ) ([-S10 (-S30) S40 - C10C20C30S40])</td>
</tr>
<tr>
<td>( z_{44} )</td>
<td>( 1.39 ) ([C10S20 (-S40) - S10C30C40 - C10C20S30C40])</td>
</tr>
<tr>
<td>( z_{45} )</td>
<td>( 1.39 ) ((C10S20C40 - S10C30S40 - C10C20S30S40))</td>
</tr>
<tr>
<td>( p_{11} )</td>
<td>(-S10S20C40 - C10C30S40 + S10C20S30S40)</td>
</tr>
<tr>
<td>( p_{12} )</td>
<td>(C10C20C40 - C10 (-S20) S30S40)</td>
</tr>
<tr>
<td>( p_{13} )</td>
<td>(-S10 (-S30)S40 - C10C20C30S40)</td>
</tr>
<tr>
<td>( p_{14} )</td>
<td>(C10S20 (-S40) - S10C30C40 - C10C20S30C40)</td>
</tr>
<tr>
<td>( p_{15} )</td>
<td>(C10S20C40 - S10C30S40 - C10C20S30S40)</td>
</tr>
<tr>
<td>( p_{21} )</td>
<td>(C10C20C40 + C10S20S30S40)</td>
</tr>
<tr>
<td>( p_{22} )</td>
<td>(S ([-C20C40 - S20S30S40]) + S10 (-S20) C40 + S10C20S30S40)</td>
</tr>
<tr>
<td>( p_{23} )</td>
<td>(S [C20C30S40] + S10S20C30S40)</td>
</tr>
<tr>
<td>( p_{24} )</td>
<td>(S [-S20 (-S40) + C20S30C40]) + S10C20 (-S40) + S10S20S30C40)</td>
</tr>
<tr>
<td>( p_{25} )</td>
<td>(S [-S20C40 + C20S30S40]) + S10C20C40 + S10S20S30S40)</td>
</tr>
<tr>
<td>( p_{31} )</td>
<td>(-C10C20C30S40 + S10S30S40)</td>
</tr>
<tr>
<td>( p_{32} )</td>
<td>(S [C20C30S40] - S10 (-S20) C30S40)</td>
</tr>
<tr>
<td>( p_{33} )</td>
<td>(S [S20 (-S30) S40] - S10C20 (-S30) S40 - C10C30S40)</td>
</tr>
<tr>
<td>( p_{34} )</td>
<td>(S [S20C30S40])</td>
</tr>
<tr>
<td>( p_{35} )</td>
<td>(S [S20C30S40] - S10C20C30S40 - C10S30S40)</td>
</tr>
</tbody>
</table>

59
\[
\begin{align*}
P_{41} &= - \text{SOC}\text{OCR}_3\text{OCR}_4\text{OCR}_5 \quad \text{SOC}\text{OCR}_2\text{OCR}_3\text{OCR}_4 \quad \text{SOC}\text{OCR}_1\text{OCR}_2\text{OCR}_4 \\
P_{42} &= S \left[ \text{SOC}\text{OCR}_3\text{OCR}_4\text{OCR}_5 + \text{SOC}\text{OCR}_2\text{OCR}_4 \right] \\
P_{43} &= S \left[ \text{SOC}\text{OCR}_3\text{OCR}_4 \right] \\
&\quad + \text{SOC}\text{OCR}_1 \left[ \text{SOC}\text{OCR}_2\text{OCR}_3 \right] \quad \text{SOC}\text{OCR}_1\text{OCR}_2\text{OCR}_4 \quad \text{SOC}\text{OCR}_1\text{OCR}_2\text{OCR}_3\text{OCR}_4 \\
P_{44} &= S \left[ \text{SOC}\text{OCR}_3\text{OCR}_4 \right] \\
&\quad + \text{SOC}\text{OCR}_1 \left[ \text{SOC}\text{OCR}_2\text{OCR}_3 \right] \quad \text{SOC}\text{OCR}_1\text{OCR}_2\text{OCR}_3\text{OCR}_4 \\
P_{45} &= S \left[ \text{SOC}\text{OCR}_3\text{OCR}_4 \right] \\
&\quad + \text{SOC}\text{OCR}_1 \left[ \text{SOC}\text{OCR}_2\text{OCR}_3 \right] \quad \text{SOC}\text{OCR}_1\text{OCR}_2\text{OCR}_3\text{OCR}_4 \\
\end{align*}
\]
APPENDIX III

LIST OF COMPUTER PROGRAMS
"MAIN": Straight-line motion of Slave Arm according to the reference \((\Delta x, \Delta y, \Delta z, \Delta \alpha)\) in Figs. 4.3 and 3.2, by means of Jacobian Matrices of 1st Order Approximation in Secs. 3.1 and 3.2.

"MSURE": Measurement of Point \(Q (x, y, z, \alpha)\) in Fig. 3.4 which is the extreme end of Measurement Arm, by means of the method in Sec. 3.3.

"TTT": Motion of Slave Arm so as to keep the constant distance between the table measured by "SUBME" and Slave Arm measured by "SUBS1", regardless of the position of table.

"HICOM": Extended Compensation done so far by means of Measurement Arm as a sensor.

Subroutines for "TTT" and "HICOM"

"SUBI", "SUBO": Input and Output of each angle.

"SUBS1" and "SUBM1": Present angle of Slave Arm and Master Arm.

"SUBS2" and "SUBM2": Desired angle according to the reference \((\Delta x, \Delta y, \Delta z, \Delta \alpha)\) for Slave Arm and for Master Arm.

"SUBME": Measurement of the extreme end of Measurement Arm.

62
PROGRAM MAIN

MAIN
COMMON THXSI(7), THXI(7), THXS(7), THX(7)
COMMON IDATA(4)
DIMENSION AX(3), PX(4,5), RX(4,5), S1(4,5), Y(4,5), P(4,5)

THS1=0.0
THS2=0.0
CALL DROR
CALL DOUT(24,0)
VIEW
TYPE *'MANIPULATOR COMPUTER CONTROL'
ACCEPT *
IF(I.NE.1) STOP
CALL AINS(1,29,1,DATA)
CALL AOUTS(1,29,1,DATA)
CALL DOUTS(1,24,0)
CONTINUE

CONTINUE

TYPE *'DO YOU NEED ORIGIN SET? Y'
ACCEPT *
IF(INK.EQ.1) GO TO 500

TYPE *'MCN OF DIVISIBILITY '
ACCEPT *
RN=FLOAT(R)

TYPE *'POSITION INCREMENT DX+DZ IN INCH AND IS IN DEG'
ACCEPT *
DX=FLOAT(DX1)
IS=FLOAT(IS1)

CALL MAINLI

THS1=THXI(5)
THS2=THXI(7)
THS3=THXI(5)-THS3
THS4=THXI(2)
THS5=(THXI(4)+THXI(3))/2.0+0.07*THS3
THS6=(THXI(4)-THXI(3))/1.65

THM1=THM1(5)
THM2=THM1(7)
THM3=THM1(5)-THM1
THM4=THM1(2)
THM5=(THM1(4)+THM1(3))/2.0+0.07*THM3
THM6=(THM1(4)-THM1(3))/1.65

TYPE *'IDATA(1)-3'
TYPE *'IDATA(1):DATA(2)*DATA(3)
TYPE *'THS1=3'
TYPE *'THS1:THS2:THS3

TYPE *'PRESENT THS'
ITHS1=INT(THSI(5)/3.1416)*THS1
ITHS2=INT(THSI(6)/3.1416)*THS2
ITHS3=INT(THSI(7)/3.1416)*THS3
ITHS4=INT(THS4/3.1416)*THS4
ITHS5=INT(THS5/3.1416)*THS5
ITHS6=INT(THS6/3.1416)*THS6
TYPE *'THS1:THS2:THS3:THS4:THS5:ITHS6'

63
TYPE X, XY, Y, Z

X= 1.39*(sin(THS1)*cos(THS3)*cos(THS4) - cos(THS3)*sin(THS4))
   - 40.0*(sin(THS3)*cos(THS4))

Y= 1.39*(sin(THS1)*cos(THS3)*cos(THS4) - cos(THS3)*sin(THS4))
   - 40.0*(sin(THS1)*sin(THS3)*cos(THS4))
   + 18.0*(cos(THS1)*sin(THS3))

Z= 1.39*(sin(THS1)*cos(THS3)*cos(THS4) - cos(THS3)*sin(THS4))
   + 18.0*(cos(THS1)*sin(THS3))

Y=YP - 19.39
Z=ZP + 40.0

TYPE X, XY, Y, Z

TYPE X, Y: (F INITIAL SET NEEDS, INPUT 1)

ACCEPT X: 11

IF (I < 3.1) GO TO 100

THS01=THS1
THS02=THS2
THS03=THS3
THS04=THS4

S1=sin(THS01)
S2=sin(THS03)
S3=sin(THS04)

C1=cos(THS01)
C2=cos(THS03)
C3=cos(THS04)

XX(1,1)=0.0
XX(1,3)=0.0
XX(1,5)=0.0
XX(1,6)=0.0

XX(2,5)=1.39*(C2*S3*C4+S2*S4) - 40.0*(C2*C3)
XX(2,6)=0.0
XX(2,7)=1.39*(-S3*C4*S2+S3*C2) + 40.0*S3*S2
XX(2,8)=1.39*(C2+C4*S3) - 40.0*C2*S3
XX(2,9)=1.39*(-C2-S3)*S4+S2*C4

XX(3,5)=1.39*(S2*C3*C4) + 40.0*(S2*S3)
XX(3,6)=0.6
XX(3,7)=1.39*(C2*C3*C4) + 40.0*(C2*S3)
XX(3,8)=1.39*(S2*S3*C4) - 40.0*(S2*C3)
XX(3,9)=1.39*(S2*S3*C4)

XX(4,5)=-1.39*(C2*C4+S2*S3*S4)
<table>
<thead>
<tr>
<th>( X(2,3) = 1.39 \times 1 \times C2^2 \times S3^3 \times C4^1 \times S2^1 \times S3^3 \times C4^1 \times S1^* \times S2^* \times S3^1 \times C4^1 \times S2^1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y(2,4) = 1.39 \times S1^1 \times S3^1 \times S2^1 \times S3^1 \times S1^* \times S2^* \times S3^1 \times C4^1 \times S1^1 \times S2^1 \times S3^1 \times C4^1 \times S2^1 )</td>
</tr>
<tr>
<td>( Z(2,1) = 1.39 \times S1^1 \times S3^1 \times S2^1 \times S3^1 \times S1^* \times S2^* \times S3^1 \times C4^1 \times S1^1 \times S2^1 \times S3^1 \times C4^1 \times S2^1 )</td>
</tr>
<tr>
<td>( Z(2,2) = 1.39 \times S1^1 \times S3^1 \times S2^1 \times S3^1 \times S1^* \times S2^* \times S3^1 \times C4^1 \times S1^1 \times S2^1 \times S3^1 \times C4^1 \times S2^1 )</td>
</tr>
</tbody>
</table>

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DO 200 I=1,4
DO 201 J=1,5
PP(I,J)=PY(I,J)*SS+PY(I,J)
201 CONTINUE
200 CONTINUE
TSS(1)=THS1-THS01
TSS(2)=THS2-THS02
TSS(3)=THS3-THS03
TSS(4)=THS4-THS04
TSS(5)=1.0

DO 50 I=1,4
DO 51 J=1,5
X(I)=X(I)*TSS(I,J)
Y(I)=Y(I)*TSS(I,J)
Z(I)=Z(I)*TSS(I,J)
   I=I+1
50 CONTINUE
CONTINUE
DO 301 I=2,7
AD(I)=TH(I)*S(I)-TH*SI(I)
CONTINUE

I=1
CONTINUE
DO 302 J=1,7
TH(SU(I)=TH(SI(J)+AD(I)*J)*T*R(I)*F(I)
CONTINUE
CALL MTY623(A1*10631*DATA)

I=1
IF (I.GT.10) 10 TO 311
GO TO 301
311 CONTINUE
GO TO 10

EXECUTE SINGLE COMMON FMTH31
COMMON I(TH)=1,32
COMMON IDATA(1A)
COMMON SIGMAD(11)
COMMON SIGMAH(11)
COMMON SIGMAEX(6)
COMMON SIGMAE(11)
COMMON SIGMAH(11)
COMMON SIGMAEX(6)
COMMON SIGMAE(11)
COMMON SIGMAH(11)
COMMON SIGMAEX(6)
COMMON SIGMAE(11)
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COMMON SIGMAE(11)
COMMON SIGMAH(11)
COMMON SIGMAEX(6)
COMMON SIGMAE(11)
COMMON SIGMAH(11)
COMMON SIGMAEX(6)
COMMON SIGMAE(11)

CONVERT ANGLE RADIANS INTO DEGREES
CALL RTY630(SI*10+DATA)
DO 10 J=1
THI(SI)=RTY630(ADJ(TH)*J*F(I)*T)
CONTINUE
RETURN
ENTRY TANGLO
CONVERT ANGLE RADIANS INTO DEGREE FOR 1
DO 20 J=1
THI(SU(I)=RTY630(ADJ(TH)*J*F(I)*T)
CONTINUE
DO 30 J=1,7
IF (DATA(Y)*J*12.GT.9) DATA(Y)*12
CONTINUE

RETURN
END
DO 130 I=1,6
    VI(I)=I**INT(MI(I))
    CONTINUE
  130 TYPE 1,VI(1),VI(2),VI(3),VI(4),
     I,VI(5),VI(6)
THF=CSEG
CL=3.0
CL=0.0
CL=3.0+39.0
CL=+39.0
GL=+39.0
CL=1.0
CONTINUE

DO 250 I=1,6
    VI(I)=2*V(1)*ATF(IP(I))
    CONTINUE
AA=VI(1)
BB=VI(2)
VI(1)=BB
VI(2)=AA
AA=VI(3)
BB=VI(4)
VI(3)=BB
VI(4)=AA
AA=VI(5)
BB=VI(6)
VI(5)=BB
VI(6)=AA

THD(1)=-20.0*VI(1)-C1
THD(2)=-20.0*VI(2)-C2
THD(3)=-20.0*VI(3)-C3
THD(4)=-20.0*VI(4)-C4
THD(5)=-20.0*VI(5)-C5
THD(6)=-20.0*VI(6)-C6

DO 255 I=1,6
    THD(I)=VI(I)**THD(I)
    CONTINUE
TYPE *=THD(1),THD(6),I,THD(5),THD(4)
    TYPE *=THD(1),THD(2),THD(3)

DO 255 I=1,6
    THD(I)=-159/180.0*THD(I)
    CONTINUE

IF THR(1).EQ.0.0 GO TO 270
A=2*ATAN(TAN(THR(1))**2+(TAN(THR(2))**2)
B=3(2)
S(2)=STAN(1.0/S(1))
S(3)=(TAN(THR(2)))/SIN(THR(1))
TH2=E=S(4)
B(4)=ATAN(S(3))
GO TO 260
S(2)=(3.14159/2.0)-THR(2)
IF(THR(2).GT.0.0) S(4)=3.14159/2.0
IF(THR(3).EQ.0.0) S/4=-3.14159/2.0
IF(THR(3).LE.0.0) GO TO 900
CONTINUE
L=5(5)
S(5)=S(5)+COS(S(2))+/D*SIN(S(2)+THR(4))
Y=5-KS
E=5-KS
L=5-KS
A=(5.1-1.5)+C.0-(S(4)-THR(6))
TYPE I=1, V=2, S'
TYPE K=1, S+1
X=K1.0
Y=K1.0
Z=K1.0
F=INIT
F=INIT
F=INIT
F=INIT
F=INIT
F=INIT
TYPE I=1, V=2, I=IA
GO TO 100
STOP
END
PROGRAM TTT

COMMON /A1/ TH3(1),TH3(2),TH3(3),TH3(4),TH3(5),TH3(6)
COMMON /A4/ IDA(1:14)
COMMON /A5/ INCHE,IPHE,INX,IPHE,IPHE
COMMON /A6/ XHFT,TATT,ZHTT,IP

DIMENSION 40(7)

CALL AMINIT

CALL SCUT(24:0)
CALL SCUT(25:0)
TYPE *. COMPUTER CONTROL ?*
ACCEPT #1
IF (1.EQ.1) GO TO 500
CALL ALTSB(1:4,25,10:41)
CALL ALTSB(4:7,130:14)

TYPE *. OPERATION OF DIVISION ?*
ACCEPT #2
THM=FL 04/7

TYPE *. NEED ORIGIN SET ?*
ACCEPT #3
IF (1.EQ.1) GO TO 500

TYPE *. START ?*
ACCEPT #4
IF (1.EQ.1) GO TO 500

CALL IDBI

TH3(1)=TH3(5)
TH3(2)=TH3(3)-TH3(1)
TH3(4)=TH3(2)
TH3(5)=TH3(4)+TH3(3)-2.0*0.2*TH3(3)
TH3(6)=TH3(4)-TH3(3)*3.0 1.5

THM(1)=THM(2)
THM(3)=THM(2)-THM(1)
THM(4)=THM(2)
THM(5)=THM(4)+THM(3)-2.0*THM(3)
THM(6)=THM(4)-THM(3)*3.0 1.5

TYPE *. PRESENT TH3,THM

ITHS1=INT1(17:25)*TH3(1)
ITHS2=INT1(17:25)*TH3(2)
ITHS3=INT1(17:25)*TH3(3)
ITHS4=INT1(17:25)*TH3(4)
ITHS5=INT1(17:25)*TH3(5)
ITHS6=INT1(17:25)*TH3(6)

TYPE *. INPUT THS2,THS3,THS4,THS5,THS6

CALL GBS31

TYPE *. FEED BY XDY,22.03 ?*
ACCEPT #5
IF (1.EQ.1) GO TO 500
IF (I.EQ.1) GO TO 250

100

130

140

150

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260

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290

300
CALL SLOPE2

IT1=INT((THS(1)-TL1)*57.292)
IT2=INT((THS(2)-TL2)*57.292)
IT3=INT((THS(3)-TL3)*57.292)
IT4=INT((THS(4)-TL4)*57.292)
IT5=INT((THS(5)-TL5)*57.292)
IT6=INT((THS(6)-TL6)*57.292)

TYPE # IT1, IT2, IT3, IT4, IT5, IT6

GO TO 600

THS(1)=0.0
THS(2)=0.0
THS(3)=0.0
THS(4)=0.0
THS(5)=0.0
THS(6)=0.0

600 TYPE # 'May I Run?'
ACCEPT # 1
IF(I .NE. 1) GO TO 600

THXSQ(1)=0.0
THXSQ(I)=0.0

DO 300 J=2,7
THXSQ(J)=THS(J)
CONTINUE

300

THXSQ(2)=THS(2)
THXSQ(3)=THS(3)
THXSQ(4)=THS(4)
THXSQ(5)=THS(5)
THXSQ(6)=THS(6)

TYPE # 'Point 1'

DO 251 I=2,7
ADJ = THXSQ(I) - THXI(I)
CONTINUE

251 I=1

TYPE # 'Point 2'

CONTINUE

GO TO 251

CALL SUBO

CALL ACOUTSQ(4.17, 'DATA')

CALL ACOUTSQ(4.17, 'DATA')

CONTINUE

TYPE # 'Point 3'

CONTINUE

TYPE # 'Point 4'

CONTINUE
PROGRAM HICON

COMMON THS(1:9),THX(1:9),THXSO(1:9),THXHD(1:9)
COMMON THS(1:3),SH(1:5),SH(1:3),YSO(1:8),YSD(1:8)
COMMON TH(V(1:9),IM,V(1:9),ID,IV(1:3),ID,V(1:9)
COMMON ICTA(1:3)
COMMON IN(1:3),IN(1:3),IX(1:3),IVY(1:3),IT(1:3)
COMMON VAT(1:5),VAT(1:5)

THXS(1) = 0.0
THX(2) = 0.0

CALL ANINIT

TYPE *, 'KEEP PRESENT POSITION ?'
ACCEPT *,
IF (I.NE.1) GO TO 900

CALL AINSO(I,100,1000)
CALL ACOUTSO(4,17,100)

TYPE *, 'SET A ?'
ACCEPT *,
IF (I.NE.1) GO TO 140

TYPE *, 'A(1,1) = 0', 'A(1,8) = 0', 'A(1,9) = 0.5'
ACCEPT *, 8
A = 1.0 - A

140 TYPE *, 'REA INITIAL SET'
ACCEPT *,
IF (I.NE.1) GO TO 130

IM = 1
CALL SUBH

130 TYPE *, 'MODE WITHOUT TABLE'
ACCEPT *,
IF (I.NE.1) GO TO 900

MODE WITHOUT TABLE
T1 = SEC(2.0)
T3 = SEC(2.0)

CALL SUBI
THS(1) = THSI(5)
THS(2) = THSI(1)
THS(3) = THS(1) + T4S(1)
THS(4) = THS(1)

CALL SUBS1
THM(1) = THMI(2)
THM(2) = THMI(1)
THM(3) = THM(2) - THM(1)
THM(4) = 'H'1(1)

CALL SUBS1
X = SEC(2.0)
Y = SEC(2.0)
Z = SEC(2.0)

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THM11:THmt

170 T4 = SECONDS(T4)
T4 = AINT(T4 * 20.0)
IF (T4 .LT. 1) GO TO 100

C

T2 = SECONDS(T1)
T2 = AINT(T2)
IF (T2 .LT. 60) GO TO 100

C

TYPE *, 'MODE WITHOUT TABLE OR WITH TABLE'
ACCEPT *, I
IF (I .EQ. 1) GO TO 90

C

TYPE *, 'MODE WITH TABLE'
MODE WITH TABLE

C

TYPE *, 'COINCIDENCE OF TWO ORIGINS ?'
ACCEPT *, I
IF (I .NE. 1) GO TO 180

C

TYPE *, 'XMTT < 0.0, XMTT > 0.0, ZMTT < 0.0'
ACCEPT *, XMTT, XMTT, ZMTT

180 CONTINUE

C

TYPE *, 'YS * S * ZS * SS'
IX = AINT(YS)
IY = AINT(IS)
IZ = AINT(ZS)
IS = AINT(SS)
TYPE *, IX, IY, IZ, IS

C

TYPE *, 'XM * XM * ZM * ZM'
IX = AINT(XM)
IY = AINT(YM)
IZ = AINT(ZM)
IS = AINT(ZM)
TYPE *, IX, IY, IZ, IS

C

TYPE *, 'XME, YME, ZME, XME, YME, ZME'
IX = AINT(XME)
IY = AINT(YME)
IZ = AINT(ZME)
IAME = AINT(IAME)
TYPE *, IXME, IYME, ZME, IAME

205 TYPE *, 'TABLE SPEED ?'
ACCEPT *, I
IF (I .NE. 1) GO TO 210

C

TYPE *, 'IVX, IVX, IVX, IVX'
ACCEPT *, IVX, IVX, IVX

210 TYPE *, 'TABLE 3C'
ACCEPT *, I
IF (I .NE. 1) GO TO 205

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78
CALL MOV(IVX,IVY,IVZ)

X1=0.0
Y1=0.0
Z1=0.0

X2=0.0
Y2=0.0
Z2=0.0

III=0
T0=SECONDS(0.0)

CALL MOV(IVX,IVY,IVZ)

TS=SECONDS(0.0)

CALL SUB1

THS(1)=THS(S)
THS(2)=THS(S)
THS(3)=THS(S)-THS(1)
THS(4)=THS(E)

CALL SUBS1

THM(1)=THM(1)/E
THM(2)=THM(1)
THM(3)=THM(1)-THM(1)
THM(4)=THM(1)

CALL SUBm1

XSE=XS-XC
YSE=YS-YC
ZSE=ZS-ZC

IM=2

CALL SUBmE

C-----REAL SPEED

XSD=AMX+AMX+AMX
YSD=AMX-AMX+AMX
ZSD=AMX-AMX+AMX

SSD=AMX+AMX+AMX

XM=XM+XM
YM=YM+YM
ZM=ZM+ZM

IF(III.GT.0) GO TO 710

XSE=XSE
YSE=YSE
ZSE=ZSE
SS=SS

XM2=XM
YM2=YM
ZM2=ZM

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