UNIVERSITY OF MINNESOTA
ST. ANTHONY FALLS HYDRAULIC LABORATORY

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AN EXPERIMENTAL INVESTIGATION OF THE
INFLUENCE OF AIR BUBBLES ON THE ACOUSTIC RADIATION
EFFICIENCY OF TURBULENT SHEAR FLOW

by

Roger E. A. Arndt
Principal Investigator

Prepared for

DEPARTMENT OF THE NAVY
David W. Taylor Naval Ship Research & Development Center
Washington, D. C.

under
Office of Naval Research
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The University of Minnesota is committed to the policy that all persons shall have equal access to its programs, facilities, and employment without regard to race, creed, color, sex, national origin, or handicap.
SYNOPSIS

The objective of this program is to experimentally examine the interaction of a turbulent mean flow with entrained air bubbles. Particular attention has been paid to the determination of the relative acoustic radiation efficiencies of bubbles undergoing simple harmonic oscillation and those undergoing splitting. There were three phases to this program:

1) Design and fabrication of a bubble injection system. Redesign and quieting of a turbulent jet facility.

2) Design and fabrication of a facility to study bubble formation noise. Acoustic and high speed observations were made and correlated with new theory developed in this program.

3) Experimental determination of the bubble splitting noise over a range of velocity. Stability theory is being used to explain the results.

It can be concluded that splitting noise is the most potent noise source when it occurs. An average increase of 20 dB over single phase jet noise is common. The time scale of each noise pulse is independent of velocity, leading to the conjecture that the splitting process is triggered by turbulence, but the resulting unstable motion is a function of only bubble geometry. The peak sound pressure resulting from the formation of bubbles is dependent on the details of how the bubble was formed. A simple model of this process correlated well with the measured data.

This work is continuing. Efforts are underway to form a theoretical basis for the observed instabilities which lead to splitting. Additional experiments are now being run to more clearly elucidate the complex physics involved.
INTRODUCTION

The noise generated by multiphase flows has been studied by many researchers. Cavitation produces especially high noise levels, showing some correlation with damage resulting from prolonged exposure to such conditions. Substantial noise radiation is also possible in non-cavitating air-water mixtures. The highest noise levels result from the splitting of air bubbles when the two-phase mixture is exposed to turbulent shear flow conditions.

Hinze [1955] was one of the first to describe the splitting process of drops and bubbles in turbulent flow fields. Many others have followed, including Collins and Knudson [1970], Azbel [1981], Sevik and Park [1973] and most recently Bentley and Leal [1986] in a series of finely controlled experiments. While these researchers were interested in splitting mechanics, others have been concerned with noise generation due to the splitting. Strasberg [1956] was one of the first to postulate on noise production due to many different fundamental processes in bubble dynamics, including splitting. Blake [1976] and Gavigan et al. [1974] have presented experimental data on noise generated by gas jets in a turbulent wake. In spite of the numerous literature on the general topic, the noise associated with the splitting of single bubbles has not yet been reported. In addition, the noise due to formation of bubbles has only been given rudimentary treatment. Both issues were treated in this program.
TEST FACILITIES

A submerged jet facility at the St. Anthony Falls Hydraulic Laboratory, University of Minnesota, was used for the splitting experiments. A 25.4 mm diameter jet issued vertically from a nozzle into a tank of quiescent water. Degassed water (5-7 ppm total dissolved gas) was pumped through the closed loop system, with jet velocities of up to 20 m/s possible. The ambient pressure in the tank could be controlled between 0.1 atm and 1.7 atm. The turbulence level at the nozzle exit plane was determined to be 0.6 percent using a two component laser Doppler velocimeter. Figure 1 shows the test facility.

Air bubbles were introduced into the flow just downstream of the turbulence management section. Dried, compressed (4-6 psig) air was passed through a 27-gauge (0.203 mm I.D.) hypodermic needle, resulting in bubbles of $1.1 \pm 0.1$ mm in diameter. The bubbles were injected along the centerline and carried with the flow into the potential core of the jet. Splitting of the bubbles occurred in the shear layer of the jet, usually near the end of the region of flow development.

The acoustic signals were measured with a miniature hydrophone positioned in the tank at four jet diameters downstream from the jet exit plane. The sound pressure signal was then passed through a low noise amplifier into an analog anti-aliasing filter and finally into a digital oscilloscope. The oscilloscope was set to capture the transient bubble splitting events at a data rate of 500 kHz. The waveforms were then transferred to a microcomputer for storage and further analysis.

The bubble creation experiments were carried out in a small rectangular tank (508 mm by 254 mm by 304 mm), filled with distilled water. Air bubbles were created by passing dry, compressed air through a submerged hypodermic needle. The inlet pressure was regulated to 27.5 kPa and the air flow was controlled with a small needle valve. Air flowrates were measured with a rotameter, water temperature and atmospheric pressure were also recorded. Several needle sizes were tested: 18, 21, 25, 27, and 30 gauge, corresponding to 0.838, 0.508, 0.254, 0.203, and 0.152 mm I.D., respectively.

The sound pressure was measured with a miniature hydrophone. The amplified output was passed through analog anti-aliasing filters and into a digital oscilloscope. The waveform was acquired on the oscilloscope and then transferred to a microcomputer for storage and further analysis. A schematic of the experimental setup is shown in Figure 2.
Figure 1. Schematic of test apparatus.
Figure 2. Schematic of experimental apparatus.
The noise generated by air bubble formation at a nozzle is characterized by a damped sinusoidal oscillation. The sound pressure is a direct result of motion of the bubble wall. Strasberg [1956] has shown that simple volume pulsation, or zero mode oscillation, is dominant in generating sound. The natural frequency of this mode was given by Minnaert [1933].

$$f_o = (3\gamma P_o/\rho)^{1/2}/2\pi R_b$$  \hspace{1cm} (1)

where \( \gamma \) = ratio of specific heats of the gas in the bubble, 
\( P_o \) = ambient pressure, 
\( \rho \) = density of the fluid, and 
\( R_b \) = bubble radius.

These volume oscillations of the bubble are excited by a change of pressure on the bubble, either internally or externally. By solving the linear equation describing bubble motion,

$$M_e \ddot{v} + R \dot{v} + K(v(t) - v_o) = 0$$  \hspace{1cm} (2)

with the initial conditions of \((v(t) - v_o) = \Delta v_o; \) and \( v(0) \), the pressure at a distance \( r \) from the source is

$$P = \frac{\rho \omega^2}{4\pi} A e^{-\pi \delta t} \cos(\omega_o t - \phi) = \frac{\rho \omega^2}{4\pi} A e^{-\pi \delta t} \cos(\omega_o t - \phi)$$  \hspace{1cm} (3)

where: \( A = \Delta v_o (1 + \frac{\delta \dot{v}(o)}{\omega_o \Delta v_o} + (\frac{1}{\omega_o \Delta v_o})^2)^{1/2} \), and 
\( \delta \) = dissipation constant.

In Strasberg's model, the assumptions are that the bubble grows very slowly and the pressure inside the nozzle is constant and equal to \( P_i = P_o + P_+ = P_o + 2S/R_n \). These conditions give a resulting bubble wall
velocity of \( \dot{R} = (2/3 \frac{p_+}{p})^{1/2} \). This value was then used as the initial condition driving the oscillation neglecting \( \Delta v_0 \). The initial volume is then given by

\[
\dot{V}(0) = 4\pi R^2 R^2_b = 4\pi R^2_b \left(\frac{2}{3} \frac{P_+}{P}\right)^{1/2}
\]

resulting in a peak sound pressure of

\[
P_p = \frac{R_b}{r^2} \left(2\gamma \frac{P_0}{P_+}\right)^{1/2}
\]

The purpose of our experiments was to verify this theory, in particular the peak sound pressure level. The oscillation and damping characteristics were also studied.

**Results**

As expected, the noise waveforms were damped sinusoidal oscillations. A typical waveform is shown in Figure 3 and correlated with bubble motions observed with high speed photography. The time at which each photograph was made is marked in letters A through F on the time trace. The small cylindrical shape below each bubble is the hypodermic needle used to produce the bubble. Note that by position F the bubble has moved a considerable distance from the needle. Also note that the peak sound pressure is generated while the bubble is still spherical. Each transient waveform was analyzed for the peak sound pressure and its frequency spectrum. The peak sound pressures generated for each needle size at a distance of \( r = 20 \text{ mm} \) are shown in Figure 4. The two lines indicate an interesting phenomenon...
Peak sound pressures for a range of needle sizes.

Figure 4.
associated with the addition of a light coating of oil onto the needle surface. A localized change in surface tension is noted by a slight increase in bubble size for a given needle diameter, but perhaps more interesting is the substantial drop in peak sound pressure.

The frequency analysis was carried out with an FFT (Fast Fourier Transform) routine on 8192 data points. The results for all cases was a nearly singular frequency component at the natural frequency of the volume pulsation of the bubble. The exponential decay of the sound pressure was best characterized by the value of \( \delta \), the dissipation constant, given by Meyer and Skudrzyk [1953]. This constant, \( \delta = 0.014 + 1.1 \times 10^{-5} f_0 \), when input into the term \( e^{-\delta t} \), describes the decaying envelope very well for all bubble sizes. Additional tests validated the \( 1/r \) dependence on the peak sound pressure.

**Discussion**

The classic theory for bubble formation at a nozzle, as given by Strasberg, does not give a fully satisfactory result for our experimental data. While the frequency and decay match quite well, the peak sound pressure is on average 40 dB below the value predicted by Strasberg's theory. Since frequency and damping are predicted correctly, the difference in the peak amplitude can be tied directly to the initial conditions \( \Delta v_0 \) and \( \dot{v}(0) \), used in solving the linear differential equation for bubble motion. There is some prior evidence that the value of \( \dot{v}(0) \) predicted by hydrodynamic theory is not valid. When compared to the measured value Strasberg presents in his paper, the predicted value is one order of magnitude higher; however, no comment is made on this discrepancy.

The need for a slightly different model is evident. The main area of question is in the assumption of constant internal pressure. The excess pressure \( P_+ \) initially has to be at least \( 2s/R_b \); however, as the bubble grows, the magnitude of \( P_+ \) does not necessarily remain constant. Much less pressure is required for bubble growth than is required to initiate the formation. If \( P_+ \) decreases, so does the bubble wall velocity \( \dot{v}(0) \), and in turn the peak sound pressure is also decreased. As the bubble detaches from the needle, the bubble surface must close in order to seek its minimum energy state. This closure imparts some kinetic energy into the bubble oscillation. By equating the work done to close the bubble with the kinetic energy of the resulting oscillation, one can solve for a new value of \( \dot{v}(0) \) of \( v(0) \). Figure 5 identifies the variable used in this formulation.

The work done in closing the bubble can be given as

\[
W = 2\pi R_n \int_0^R \sigma \left( P_i - P_o \right) R \, dR = 2\pi R_n \sigma \frac{R}{8} \left( P_i - P_o \right)
\]

where \( P_i - P_o = P^* = 2s/R_b \). The kinetic energy of the oscillation is given by:

\[
E_k = \frac{1}{2}(m_b + m_{add})R^2
\]
Figure 5. Important variables in the new closure model.
By equating equations 6 and 7, and solving for the volume velocity, we get
\[ \dot{V}(0) = 4\pi R_n^3 (S/\rho)^{1/2} \]. Using this initial condition, new values of peak sound pressure can be calculated, which are of the same order as our measurements. The results are summarized below, again for \( r = 20 \text{ mm} \) and without oil.

<table>
<thead>
<tr>
<th>Needle size (mm I.D.)</th>
<th>Bubble size (mm dia.)</th>
<th>( P_{\text{meas.}} ) (dB re 1( \mu )Pa)</th>
<th>( P_{\text{model}} ) (dB re 1( \mu )Pa)</th>
<th>( P_{\text{theory}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.838</td>
<td>3.812</td>
<td>147.3</td>
<td>153.5</td>
<td>179.4</td>
</tr>
<tr>
<td>0.508</td>
<td>3.048</td>
<td>143.9</td>
<td>148.9</td>
<td>179.6</td>
</tr>
<tr>
<td>0.254</td>
<td>2.370</td>
<td>139.6</td>
<td>142.1</td>
<td>180.5</td>
</tr>
<tr>
<td>0.203</td>
<td>2.052</td>
<td>137.9</td>
<td>140.4</td>
<td>180.2</td>
</tr>
<tr>
<td>0.152</td>
<td>1.778</td>
<td>139.1</td>
<td>137.9</td>
<td>180.2</td>
</tr>
</tbody>
</table>
SPLITTIN4G NOISE

Experiments

The splitting noise experiments were initially designed utilizing critical Weber number criteria developed by Hinze [1955] and further tested for a submerged jet by Sevik and Park [1973]. For a given bubble injector diameter, it is possible to go from a non-splitting to a splitting regime by simply varying the jet velocity. Figure 6 shows this concept for a 0.1 mm diameter bubble injector and a 25.4 mm diameter jet. The details of the calculation can be found in Frizell [1987]. The data presented in this paper cover jet velocities from 2.7 m/s to 17.5 m/s. Bubble splitting was observed at all velocities tested. Single phase jet noise levels along with fifteen splitting events were recorded at each velocity. Photographs were taken to document each test run. The results are discussed below.

Discussion

The sound pressure waveform of a splitting event is characterized by a dumped, multi-frequency oscillation (see Figure 7). In most cases, the results indicated that there was some type of volume oscillation present, although the frequencies of the oscillations generally predicted a bubble size closer to the initial bubble diameter rather than the size resulting after splitting. A comparison of the acoustic intensity in 1/3 octave bands shows a general increase in level and a broadening of the spectra at the higher frequencies (see Figure 8).

Some of the broadening effects can be explained by the measurement position. At 5 kHz, for example, the hydrophone is positioned in the acoustic near field. On the other hand, the measurement position is marginally in the acoustic far field at 50 kHz. The acoustic intensity is expected to vary with $U^n$ in the near field and to a higher power of $U$ in the acoustic far field, hence a possible reason for the broadening of the spectrum with increasing velocity. Since the bulk of the acoustic energy is centered about frequencies corresponding to the acoustic near field, the peak pressure follows a $U^n$ relationship ($U^n$ for intensity) as compared to $U^3$ for Blakes [1976] measurements that were made in the acoustic far field. This dependence, along with a comparison to single phase jet noise and bubble creation noise, is shown in Figure 9.

Further analysis of the splitting waveforms yields interesting information about the characteristic time scale of the events. If the splitting is to be an event dominated by the turbulent field, one would expect that the time scale of the splitting would be of the same order as the characteristic time scale of the turbulence. However, it was found that the time scale of the splitting was three orders of magnitude smaller than that of the turbulence. This comparison is made using time scales.
Figure 6. The solid line in the figure is critical Weber number versus velocity. The dotted line is a prediction of bubble size for an 0.1 mm injector.
Figure 7. typical noise pulse.
Figure 8. Bubble noise spectra at different velocities.
Figure 9. Comparison of peak sound pressure in a jet with and without bubbles.
typical of turbulent fluctuations that are spatially coherent over one bubble diameter. Following Sevik and Park [1973], the following definition is used:

$$\tau_d \sim \left( \frac{d^2}{\varepsilon} \right)^{1/3}$$  \hspace{1cm} (9)

where $$\varepsilon$$ is the dissipation rate and $$d$$ is the bubble diameter. In addition, the splitting time scale did not vary with jet velocity (see Figure 10). This fact may lead to a new postulation that while the turbulence is responsible for exciting the bubbles into oscillations, resulting in severe deformation, the actual splitting is the direct result of an instability.
Figure 10. Comparison of noise pulses generated at different velocities.
CONCLUSIONS

The results of the splitting experiments have shown that the noise generated by single bubbles splitting in a turbulent shear flow can be substantial. An average rise of 20 dB above single phase jet noise is common.

Comparison of the time scale of a splitting event with typical turbulence time scales leads to the conjecture that splitting is the result of an instability that is triggered by turbulence, but the final stages of the process are a function of bubble geometry.

The formation of a bubble at a nozzle generates a sound pulse in the form of a damped sinusoidal oscillation. The frequency of oscillation is associated with the volume pulsation of the bubble. The damping characteristics in the frequency range studied are dominated by acoustic and thermal damping, and are well defined by theory. The peak sound pressure is a function of the method used to form the bubble, the initial wall velocity being the most important parameter. The values predicted by Straaberg's model did not match our experimental data. The assumptions were reviewed and a simple model formulated to predict the initial volume velocity. The results of this model predicted our experimental data usually within 3 to 6 dB. The influence of a coating of oil on the needle surface is noticed in two ways: 1) a change in the localized surface tension, resulting in a slightly larger bubble than for the nonoilled needle, and 2) approximately a 10 dB drop in the peak sound pressure level. This drop in noise has not been explained as of yet, but opens an area for further research.
REFERENCES


