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THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.
ONE-DIMENSIONAL TWO-PHASE FLOW IN COMBUSTION CHAMBER
OF SOLID PROPELLANT ROCKETS

ABSTRACT

In this paper, a numerical solution of basic equation for one dimensional two-phase nonequilibrium flow in a combustion chamber of solid propellant rocket motors is discussed in detail, the effect of particle size on flow field in a chamber and pressure-time curves is analyzed, and some useful conclusions are obtained in comparison with results of one dimensional two-phase constant combustion lag flow in a chamber. It is useful for predicting pressure-time curves accurately and providing accurate boundary conditions for the calculation of two-phase flow through the nozzle.
Nomenclature

A - duct cross section area
\( A_b \) - Burning area of charge
\( A_Q \) - Thermal equivalent of work
\( A_t \) - Area of the nozzle throat
\( \bar{M} \) - Mean molecular weight of gas

n - Pressure exponent
P - Pressure
\( P_s \) - Total pressure

b - Burning speed coefficient

C* - Characteristic speed of propellant

\( C_t \) - Particulate specific heat

\( C_{pg} \) - Specific heat of gaseous phase at constant pressure

\( g \) - Gravitational acceleration
h - Enthalpy per unit mass

\( H_s \) - Total enthalpy in 1 kg of two-phase mixture

\( H_{sg} \) - Total enthalpy in 1 kg of gas

\( H_{sp} \) - Total enthalpy in 1 kg of liquid

\( \kappa \) - Ratio of specific heat of gas

K - Particle velocity lag coefficient

\( K_1 \) - Surface - throat ratio

\[ K_1 = \frac{A_b}{A_t} \]

l - Charge length

L - Particle temperature lag coefficient

\( \dot{m} \) - Mass flow rate of two-phase mixture

\( \dot{m}_g \) - Mass flow rate of gas

q - Heat flux of particles per unit mass

r - Burning rate
\( r_p \) - Radius of particles

\( R_g \) - Gas constant of gaseous phase

\( S \) - Duct circumference

\( t \) - Time

\( T \) - Temperature

\( T_0 \) - Total temperature of charge head gaseous phase

\( T_s \) - Total temperature of gaseous phase

\( v \) - Velocity

\( x \) - Axial coordinate

\( X \) - Particle resistance per unit mass

\( Y_C \) - Outer radius of charge

\( \rho \) - Density

\( \rho_t \) - Density of propellant

\( \dot{m}_p \) - Mass flow rate of condensed phase

\( M_g \) - Mach number of gaseous phase

\( \rho_{mp} \) - Density of \( Al_2O_3 \) material

\( \varepsilon \) - Fraction of particle mass flow rate, \( \varepsilon = \frac{\dot{m}_p}{\dot{m}} \)

\( \lambda_g \) - Coefficient of thermal conductivity of gaseous phase

\( \mu_g \) - Coefficient of dynamic viscosity of gaseous phase

**SUBSCRIPTS**

\( g \) - Gas phase

\( 0 \) - Cross section of charge head

\( t \) - Cross section of nozzle throat

\( P \) - Condensed phase

\( L \) - Cross section at the charge tip

\( i \) - Initial value
1. PREFACE

Modern composite solid propellants are mixed with a certain amount of aluminum powder for increasing energy and decreasing unsteady burning. When the aluminum mixed propellant burns, Al$_2$O$_3$ particles are formed in liquid phase. The weight fraction can be up to 30-40%. Therefore, in combustion chambers and jet nozzles, the products of combustion actually are a mixture of gas and liquid.

There have been a lot of publications regarding two-phase flow in jet nozzles of a rocket motor. Most of them are devoted to one dimensional two-phase flow and two dimensional axial symmetrical two-phase flow in nozzles. There has been some progress. Two-phase flow in a combustion chamber is involved with mass, and has some new characteristics. The author has done research on one dimensional two-phase constant lag flow in a combustion chamber [4], analyzing the particle velocity lag effects on the combustion chamber processes. This article studies the one dimensional two-phase nonequilibrium flow in a combustion chamber. Based upon the principle equations of one dimensional mass involving two-phase flow in a combustion chamber, it discusses the numerical solutions of the equations in detail, analyzes particle size effect on the chamber flow field and the pressure-time curve, and compares the results with that of constant lag flow. Some practical conclusions are reached. This gives precision in predicting pressure-time curve, and more accurate boundary conditions for calculation of two-phase flow in jet nozzles.
II. FUNDAMENTAL EQUATIONS

Assume:

1. Flow is one-dimensional and steady-state;
2. The friction and heat loss on the duct wall are negligible;
3. \( \text{Al}_2\text{O}_3 \) particles are spherical, uniform, and in the liquid phase; the particle volume and the Brownian motion effects on pressure are negligible;
4. The gas phase is an ideal gas at freezing point except in contact with particles and otherwise is inviscid;
5. No mass exchange between two phases;
6. The specific heat of gas and particles is constant

Based upon the above assumptions, the fundamental equations of two-phase flow in the combustion chamber are obtained as follows [11] and [33]:

Gas phase -

Mass Equation:

\[
\frac{d}{dx}(\rho_r v_r A) = (1 - \epsilon) \rho_r r S
\]

Momentum equation:

\[
\frac{d}{dx}(\rho_r v_r) = -A \frac{dP}{dx} - X \rho_r A
\]

Energy equation:

\[
\frac{d}{dx}[\rho_r g v_r A (h_r + \frac{A_0 v_r^2}{2g})] = (1 - \epsilon) \rho_r r S g H_{ss} - A_0 X \rho_r v_r A + q \rho_r A
\]

in which

\[
r = \beta P^{
}

\[
h_r = C_{ss} T_s
\]

Condensed phase -

Mass Equation:

\[
\frac{d}{dx}(\rho_v v_r A) = \epsilon \rho_r r S
\]
Momentum equation:
\[
\frac{d}{dx}(\rho v, A) = X \rho A
\]  
(5)

Energy equation:
\[
\frac{d}{dx} \left[ \rho g v, A (h, + A_0 \frac{v^2}{2g}) \right] = \rho r S g H_2 + A_0 X \rho v, A - q \rho A
\]  
(6)

Two-phase mixture—

Mass equation:
\[
\frac{d M}{dx} = \rho, r S
\]  
(7)

Momentum equation:
\[
\frac{d}{dx}(\rho v, A + \rho v, A) = \frac{d P}{dx}
\]  
(8)

Energy equation:
\[
\frac{d}{dx} m_s \left( h_s + A_0 \frac{v^2}{2g} \right) + m_r \left( h_r + A_0 \frac{v^2}{2g} \right) = \rho r S H_2
\]  
(9)

in which
\[
m_s = \rho s v_s A
\]
\[
m_r = \rho r v_r A
\]
\[
m = m_s + m_r
\]

III. SOME SUPPLEMENTARY RELATIONSHIPS

Equations (1)-(6) are not closed, therefore, we introduce the following supplementary relationships:

1. Gas-phase condition equation
   for an ideal gas --
   \[
P = \rho s g R_s T_s
\]  
(10)
2. Condensed-phase condition equation.

When the $\text{Al}_2\text{O}_3$ particle temperature is greater than the melting point ($T_m=2318^\circ\text{K}$) assuming its specific heat is constant, then

$$h_r = h_{pm} + C_1 (T_r - T_m)$$

(11)

in which

$C_1$ - Specific heat of liquid $\text{Al}_2\text{O}_3$ particle, 0.34327 KCal/kg.

degree K;  

$h_{pm}$ - Enthalpy of liquid $\text{Al}_2\text{O}_3$ at $T_{pm}$, 876.9498 KCal/kg

From (11) we have

$$dh_r = C_1dT_r$$

(12)

3. The particle resistance $X[1,2]$ per unit mass

$$X = A_r (v_s - v_p)$$

(13)

Under the condition of combustion chambers, the particles carryout Stokes flow, at this moment

$$A_r = \frac{9}{2} \frac{\mu_s}{r^2 \rho_p}$$

in which $\mu_s = 1.208 \times 10^{-5} T_s^4 \text{N\cdotm}^2$ (kg/sec/m$^2$)

4. The heat flux $q[1,2]$ of particles per unit mass

$$q = B_r (T_r - T_s)$$

(14)

in which

$$B_r = \frac{3\lambda_s}{r^2 \rho_p}$$

5. The total enthalpy $H_s$ in 1 kg of two-phase mixture $H_s = (1-\epsilon)H_{s\rho} + \epsilon H_{s\rho}$.

From the assumption 2, $H_s$ is constant along the channel, therefore, it is convenient to use the parameters ($v_s = v_p = 0, T_s = T_r = T_s$) at the charge origin ($x=0$), to express $H_s$. Therefore

$$H_s = (1-\epsilon)C_s T_s + \epsilon [h_{pm} + C_1 (T_r - T_m)]$$

(15)
IV. COMPUTATIONAL EQUATIONS

Let

\[ K = \frac{v_p}{v_o} \quad (0 \leq K \leq 1) \]  \hspace{1cm} (16)

\[ L = \frac{T_p - T_o}{T_o - T_s} \quad (0 \leq L \leq 1) \]  \hspace{1cm} (17)

Therefore particle velocity lag \( \frac{v_p - v_o}{v_o} = 1 - K \)

particle temperature lag \( \frac{T_p - T_o}{T_o - T_s} = 1 - L \)

Here K, L are defined as particle velocity lag coefficient and temperature lag coefficient respectively.

Ignoring the effect of burning erosion, assuming the cross section area A of the charge duct is constant along the longitudinal axis, after an elaborate manipulation, the following numerical solution is obtained based upon the fundamental equations:

\[
\frac{dv_p}{dx} = A_p \left( \frac{v_p - v_o}{v_o} - \frac{\rho_r S}{\rho_p A} \frac{\epsilon}{v_o} \right)
\]

\[
\frac{dv_o}{dx} = -\frac{R_s}{\rho_p (v_o C_p - g R_s T_s C_p - A_0 R_s v_o)} \left[ g \rho_r h_v A_p \frac{v_p - v_o}{v_o} \right.
\]

\[
+ \rho_r B_r (T_p - T_o) A_0 \rho_p A_p (v_p - v_o)^2 + (1 - \epsilon) \rho_r r \frac{S}{A} g
\]

\[
\cdot \left( H_v + h_v + A_0 \frac{v_o^2}{2 g} \right) \left( 1 - \epsilon A_p \frac{v_p - v_o}{v_o} \right) - \frac{(1 - \epsilon) \rho_r S}{\rho_p A} \frac{\epsilon}{1 - \epsilon A_p} \frac{v_p - v_o}{v_o}
\]

\[
\frac{dm}{dx} = \rho_r r S
\]

\[
\frac{\rho_r}{v_o A} = \frac{e m}{v_o A}
\]

\[
\rho_s = \frac{(1 - \epsilon) m}{v_o A}
\]

\[
P = P_r - (\rho_r v_r + \rho_p v_p)
\]

\[
T_r = \frac{P}{\rho_r g R_s}
\]

\[
T_s = \frac{1}{\epsilon C_i} \left[ \epsilon C_i T_o + (1 - \epsilon) C_i T_s - (1 - \epsilon) C_p T_s - (1 - \epsilon) C_p T_s \right.
\]

\[
- (1 - \epsilon) A_0 \frac{v_o^2}{2 g} - \epsilon A_o \frac{v_o^2}{2 g} \]

\[
M_s = \frac{\sqrt{g R_s T_s}}{\sqrt{g R_s T_s}}
\]

\[
T_s = T_s \left( 1 + k \frac{1}{2} M_s^2 \right)
\]

\[
P_s = \left( 1 + k \frac{1}{2} M_s^2 \right)^{1 - \epsilon}
\]
in which \( H_{ss} = C_{rs} T_s \)

The system of equation (18) contains 3 ordinary differential equations and 8 algebraic equations, with the unknowns \( u_r, T^e, \rho_s, v_r, T_s, \rho_s, P, P_s, T, M, \) under the given boundary conditions, which can be solved by the Runge-Kutta numerical method.

V. INITIAL CONDITION AND BOUNDARY CONDITIONS

The initial condition of the equation:

When \( t=0, S=S_i, A=A_{i0} \). The value of \( A_i, S_i \) can be obtained by the actual shape of a charge duct, for a circular cross section of the duct.

\[
S_i = 2\pi y_i, \\
A_i = \pi y_i^2.
\]

\( y_i \) is the duct initial radius.

The boundary conditions of equations:

If the charge fills the annular space between two closed end coaxial cylinders, the boundary conditions are (refer to Figure 1): at the charge head (x=0) cross section

\[
\begin{align*}
 v_{rs} &= v_s = 0 \\
 m_s &= 0 \\
 T_{ss} &= T_{rs} = T \\
 \rho &= \rho_s \\
 \rho_{ss} &= \frac{\rho_s}{gR_s T_s} \\
 \rho_{rs} &= \frac{\varepsilon}{1-\varepsilon} v_{rs} \rho_{ss} &= \frac{\varepsilon}{1-\varepsilon} \rho_{ss} K_s.
\end{align*}
\]
in which, the total temperature of the charge head gaseous phase $T_0$ is equal to the burning temperature of the charge. $P_0$ is the corresponding pressure. Before solving the equation, it is unknown. Therefore, its value has to be determined by the iteration process in the numerical solution. The particle density $\rho_{ps}$ at the charge head is an uncertain value. To determine this value, the value of $K_0$ has to be obtained beforehand.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Figure 1.}
\end{figure}

At the charge end ($x=1$) cross section the mass flow rate $\dot{m}$ of the two-phase mixture passing through the cross section area at the charge end ($x=1$), should be equal to the mass flow rate $\dot{m}_t$ of the two-phase mixture passing through the throat of the jet nozzle, that is

$$m_L = \dot{m}_t \tag{20}$$

$\dot{m}_L$ can be determined by the parameters of the $x=1$ cross section, that is

$$m_L = A(\rho_s \nu_s + \rho_r \nu_r) \tag{21}$$
The present article is mainly about two-phase nonequilibrium flow in a combustion chamber. To avoid the numerical solution for one-dimensional nonequilibrium two-phase flow in the jet nozzle, in the determination of the boundary condition of the charge end, it is assumed that the flow in the jet nozzle is a one-dimensional two-phase constant lag flow, therefore:

\[ m_i = \frac{1}{1 - \varepsilon} \sqrt{\frac{\bar{p}}{gR_s T_i C}} P_s A_i \sqrt{\frac{h}{r}} \]  

(22)

in which

\[ C = 1 + \frac{\varepsilon}{1 - \varepsilon} (\delta L) \]

\[ D = \frac{1 + \frac{\varepsilon}{1 - \varepsilon} \delta L}{1 + \frac{\varepsilon}{1 - \varepsilon}} \]

\[ \delta = \frac{C_i}{C_{ps}} \]

\[ s = 1 + (k-1) \frac{D}{C} \]

\[ \bar{p} = \sqrt{s} \left( \frac{2}{s+1} \right) \frac{r_s}{n(r-1)} \]

where the value of K, L should be taken as the corresponding value at the end of the field length.

VI. THE DETERMINATION OF K₀, L₀

For obtaining the distribution of particle speed lag and temperature lag along the duct, and the numerical solution for a system of equations (18), the values of K, L at the
charge head (x=0) have to be determined.

However, at x=0, both K and L are indeterminate forms, therefore, it is necessary to consider that at the charge head

\[ K_s = K_{s+} \]
\[ L_s = L_{s+} \]

1. The determination of \( K_0 \)

from l'Hopital's rule

\[ K_s = \frac{v_s'|_{x \to +}}{v_s'|_{x \to -}} \] (a)

After transforming the first equation in the system of equations (18), we find that, at \( x \rightarrow 0^+ \), \( v_s'|_{x \to +} \) is also an indeterminate form of \( \frac{0}{0} \). So

\[ v_s'|_{x \to +} = \frac{v_s'|_{x \rightarrow +} - v_s'|_{x \rightarrow -}}{v_s'|_{x \rightarrow +} - v_s'|_{x \rightarrow -}} \]

For convenience, the subscript \( x \rightarrow 0^+ \) has been omitted in the following expression. After rearrangement it reads

\[ \frac{2}{A_p} v_s'' + v_s' - v_s = 0 \] (b)

With the help of the boundary condition (19), from the second equation in the system of equations (18) the following is derived that at \( x \rightarrow 0^+ \)

\[ v_s' = \frac{(1-\varepsilon) \rho \tau_s S}{\rho \varepsilon A} \] (c)

in which \( r_s = b \rho \)

Substitute (c) into (b), it is obtained that at \( x \rightarrow 0^+ \)

\[ v_s' = \frac{-1 \pm \sqrt{1 + \frac{8}{A_p} \frac{(1-\varepsilon) \rho \tau_s S}{\rho \varepsilon A}}}{A_p} \] (d)
Because in a combustion chamber the flow involves
an additional mass of particles, if the speed of a particle $v_x$
increases, then $v'_x > 0$, therefore, (d) should be positive
which results in the sign in front of the square root being
"+".

Substitute (d), (c) into (a), and set

$$a = \frac{A}{\rho_0} \left( \frac{1-\varepsilon_0}{\rho_0 + A} \right)$$

then obtain

$$K_* = -\frac{1 + \sqrt{1 + 2a}}{a}$$

(24)

2. The determination of $L_0$:

Same as above, from l'Hopital rule

$$L_* = \frac{T'_i|_{x=x^*}}{T'_i|_{x=x^*}}$$

(e)

It can be derived from the fundamental equations (1)-(6)

$$\frac{dT'_p}{dx} = T'_i = \frac{1}{x C_i} \left( H_{s'} - h_x + A_0 \frac{v_x^2}{2g} \right) - \frac{B_p(T_x - h_x)}{v_x g C_i}$$

herein, $T'_i|_{x=x^*}$ is also an indeterminate form of $0$.

From l'Hopital rule it is obtained as

$$T'_i|_{x=x^*} = -T'_i|_{x=x^*} - \frac{B_p}{g C_i} \frac{T'_i|_{x=x^*} - T'_i|_{x=x^*}}{v_x|_{x=x^*}}$$

Omitting the subscript $x \to 0^+$, then

$$2T'_i = -\frac{B_p}{g C_i} \frac{T'_i - T'_i}{v_x}$$

let

$$\beta = \frac{B_p}{g C_i} \frac{1}{v_x}$$

(f)
from equation (f) it is obtained

$$L_0 = \frac{\beta}{2+\beta}$$  \hspace{1cm} (25)

With the expressions of equations (d) and (23), the following is obtained:

$$\beta = \frac{4}{gC_i} \frac{B_p}{A_p} \left( \frac{1}{-1+\sqrt{1+2g}} \right)$$  \hspace{1cm} (26)

VII. NUMERICAL SOLUTION OF THE SYSTEM OF EQUATIONS

Under the boundary conditions (19), (20) and the initial condition, but use of the fourth order Runge-Kutta method the solution for the system of equations (18) can be obtained as follows:

1. At a fixed time $t$, the gas flow parameters $(v, T, \rho, P, P_s, T_s, M_s)$, the particle flow parameters $(v_r, T_r, \rho_r)$ and the particle lag factor $K$, $L$ distribution along the duct.

2. The pressure variation with respect to time in the combustion chamber.

In the boundary condition (19), the head pressure $P_0$ is unknown apriori, and has to be determined with the numerical solution process. The procedure is the following:
(1) Form \( P_0^{(1)} = (C^* \rho_t b K_t)^{\frac{1}{\gamma_t}} \) to compute the first approximate value;

(2) With the value of \( P_0^{(1)} \) solve for the system of equations (18), and obtain the variation of parameters of the gas and the particle along the x-axis, then form equation (21), (22) and solve for \( \dot{m}_L \) and \( \dot{m}_T \);

(3) Let \( \Delta m = m_n - m_{n-1} \), and make a judgment on

\[ \left| \frac{\Delta m}{m_n} \right| < \varepsilon, \]

(27)

(\( \varepsilon \) is a given allowable error, for instance \( \varepsilon = 0.01 \)) to see whether it is satisfied.

(4) If equation (27) is not true, and \( \Delta m > 0 \), take \( P_0^{(n)} = P_0^{(n-1)} - \Delta P \) (\( \Delta P \) is the given increase in pressure, for example \( \Delta P = 1 \text{ kg/cm}^2 \)), otherwise take \( P_0^{(n)} = P_0^{(n-1)} + \Delta P \), repeat the procedure for solving the system of equations (18), if the mass flow rate satisfies (27), then \( P_0^{(2)} \) is the true value of the pressure at the charge head \( P_0 \);

(5) If the condition (27) still cannot be satisfied, the following interpolation equation can be used for computing \( P_0^{(n)} \):

\[ P_0^{(n)} = P_0^{(n-1)} + \Delta m^{(n-1)} \frac{P_0^{(n-1)} - P_0^{(n-1)}}{\Delta m^{(n-1)} - \Delta m^{(n-2)}} \quad (n = 3, 4, 5, \ldots) \]

then solve for the system of equations (18), till the mass flow rate satisfies equation (27). Figure 2 shows the flow chart of the numerical solution process, in which \( \tau_{cp} \) is the average burning speed along the length of the duct.
Figure 2.

Key: (1) start; (2) input; (3) compute $P_o$; (4) furnish boundary conditions; (5) Runge-Kutta method for solving the system of equations (18), (6) output, (7) adjust the head pressure $P_o$, (8) No, (9) Yes, (10) Yes, (11) No, (12) Stop, (13) Flow Chart of Computation.
VIII. THE EFFECT OF THE PARTICLE SIZE ON
INTERNAL BALLISTIC PROPERTIES

As to a certain solid propellant rocket motor ($\varepsilon = 0.26$),
the pressure-time curve and the flow field in the combustion
chamber have been computed for different sizes of particles.

Figure 3 shows the effect of the particle size on the Pressure-
Time curve. It can be observed in the figure that the pressure
in the combustion chamber decreases as the particle size increases.
At $t=0$ sec., the variation of the combustion chamber pressure
$P_0$ with respect to the particle radius is shown in Figure 4.
The decrease of the combustion chamber pressure will lead to
a slowdown of the burning speed, increase the charge burning
period and decrease the mass flow rate.

At $t=0$ sec., the variation of the gas parameters along the
$x$-axis for the different particle sizes is shown in Figure 5.
$M_g$ solely increases with the distance, the rest of the parameters
decrease with the distance. Because $P_0$ decreases when the particle
size increases, therefore $P, P_s, \rho_g$ are affected by the particle
size distinctively. The larger the particle size, the smaller
are the values of them. But the particle size effect on $T_g$ is
negligible.

Figure 6 shows at $t=0$ sec. the distribution of the gas speed
$\nu_g$ and the particle speed $\nu_p$ along the duct under
different particle size conditions. Because the flow
in the combustion chamber is involved with an increase in mass,
both $\nu_g$ and $\nu_p$ increase along the length of the duct. But the particle
size effect on $\nu_g$ and $\nu_p$ is different when particle size increases: $\nu_g$ increases, but $\nu_p$ decreases. This is because the particle of unit mass has a resistance to the gas of $\frac{1}{r_n}$. When the particle size increases, $X$ decreases, so the gas speed increases. At this moment the torque exerted on the particle by gas decreases. This results in the particle acceleration decreasing, therefore $\nu_p$ becomes small. The density and temperature distribution of the particle are shown in Figure 7. It can be observed that $\rho_p$ hardly decreases along the duct and $T_p$ basically stays constant.

The effect of the particle size on the value of $\rho_p$ is significant: when particle size increases, $\rho_p$ increases too. But the effect on the value of $T_p$ is negligible.

At $t=0$ sec., the particle speed lag factor $K$ and temperature lag factor $L$ distributions along the duct are shown in Figure 8. It can be observed that the value of $K$ slightly decreases along the distance, but the value of $L$ fundamentally stays constant. This indicates that in the combustion chamber, the particle speed lag $(1-K)$ increases slightly along the length of the duct, but the temperature lag $(1-L)$ basically stays constant. The smaller is the particle size, the smaller is the variation of $K$ value along the distance. (Refer to Table 1.) Therefore when the particle size is small, the two-phase flow in the combustion chamber can be treated as the constant lag flow motion.
Table 1.

<table>
<thead>
<tr>
<th>细子半径 $r_p$（微米）</th>
<th>$K_s$</th>
<th>装药末端 $K$ 值降低 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9958</td>
<td>0.220</td>
</tr>
<tr>
<td>5</td>
<td>0.9752</td>
<td>0.082</td>
</tr>
<tr>
<td>10</td>
<td>0.9123</td>
<td>0.068</td>
</tr>
<tr>
<td>20</td>
<td>0.7538</td>
<td>0.047</td>
</tr>
</tbody>
</table>

Key:
1. The radius of the particle $r_p$ (micrometer),
2. The decrease of $K$ at the charge end.

Table 2 lists, for different particle sizes, the comparison between the numerical solutions and constant lag flow computations of some important parameters in two-phase flow in a combustion chamber. Compared with the numerical solution, the combustion chamber pressure $P_0$ and the total pressure at the end charge end obtained from the constant lag flow computation are comparatively low, and the rest of the parameters are comparatively high, but the deviation is less than 1%.

Table 2.

<table>
<thead>
<tr>
<th>$r_p$ (微米)</th>
<th>$P_{0}$ (公斤/厘米²)</th>
<th>$P_{st}$ (公斤/厘米²)</th>
<th>$V_{st}$ (米/秒)</th>
<th>$V_{st}^*$ (米/秒)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>18.66</td>
<td>18.57</td>
<td>18.42</td>
<td>18.38</td>
</tr>
<tr>
<td>10</td>
<td>18.42</td>
<td>18.87</td>
<td>18.20</td>
<td>18.18</td>
</tr>
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<td>20</td>
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Key:
2. Micrometer, 3. Kg/cm², 4. kg/cm², 5. m/sec, 6. m/sec, 7), 9), 11), 13) numerical solution, 8), 10), 12), 14) constant lag flow.
From equation (24),(25) it becomes known the values of $K_0$, $L_0$ are functions of the particle radius $r_p$. Their variation with respect to $r_p$ is shown in Figure 9. It can be seen in the Figure $K_0$, $L_0$ values decrease when the particle size increases, which means, the particle speed lag and temperature lag increases as the particle size increases.

Besides, $K_0 < L_0$, in the nonequilibrium flow in the combustion chamber, the lag of the particle speed is greater than the lag of the particle temperature.

IX. CONCLUSION

Through the above discussion the following conclusions are therefore obtained:

1. Because of the effect of the two-phase flow, the pressure of the combustion chamber decreases, and the larger the particle size, the smaller is the combustion chamber pressure.

2. The two-phase flow affects the flow field in the combustion chamber greatly. When the particle size increases, the gas speed increases, the pressure, total pressure and the gas density all decrease. When the particle speed decreases, the density increases. The effect of the particle size on the temperature of both gas and particle is very small.

3. In the combustion chamber the lag of the particle speed increases along the length of the duct, the lag of the temperature basically keeps constant. Besides, the particle speed lag is greater
than the temperature lag. When the particle size is small, the two-phase flow in the combustion chamber can be treated as the constant lag flow.

Figure 3. Pressure-Time Curve

Figure 4. Effect of the particle size on \( P_0 \)

Figure 5. Distribution of gas parameters in the two-phase flow.

Figure 6. Speed distribution of the gas and the particle in two-phase flow.
Figure 7. Distribution of the particle parameters in two-phase flow.

Figure 8. Particle speed lag factor and temperature lag factor distribution.

Figure 9. The effect of the particle size on the values of $K_o$, $L_o$. 
REFERENCES

PRESSURE COUPLED RESPONSE FUNCTION
OF SOLID PROPELLANTS INCLUDING THOSE WITH
NEGATIVE PRESSURE EXPONENTS

XU Weng-an
SUMMARY

On the basis of the evidence presented, a steady state combustion model (1) of solid state propellents with negative pressure coefficient burn speed characteristics allows us to derive a new pressure response function formula. This can be used to explain pressure pairing phenomena for propellents whose combustion speed pressure exponents are zero, positive, and negative. The burning propellent is divided into two sections: the first section is a structure composed of oxidizing agent covered by molten binding agent and the corresponding binding agent surface. The other section is formed by uncovered oxidizer surface and remaining binding agent surface. This model is different from the various types of models in the past. In the combustion on the surface of the first type of section described above, consideration has been given to the oxidizer, under conditions in which it is covered by molten binding agent, so that it is considered to be in a state of opposed gasification and congealed phase reaction. Therefore, the real section of the pressure response function which is obtained, when the pressure exponent of the propellent steady state fuel speed is zero or has a negative value, is also capable of being a positive value. When we made use of the expression obtained for the pressure response function in experimentation with the propellent (S04-5A) and made qualitative calculations, the results of these calculations satisfactorily explain the phenomenon of propellents with negative pressure exponents still being unstable in combustion when most of the surface area of the oxidizer is covered by molten binder agent. This not only overcomes the weaknesses which all expressions for pressure response functions had in the past when used with negative pressure exponent propellents, but also, in a way, reflects the accuracy of the solid fuel propellent steady state combustion model (1) in its combustion speed characteristics for positive and negative pressure exponents.

Explanation of Symbols

\[ A = \frac{E}{R^2 T} \left(1 - \frac{T}{T_f}\right) \]
A. Oxidizer Gasification Rate Indicator Prefactor

\[ B = \frac{2RT_0}{(T_0 - T_1)E_f} \]

B. Indicator Prefactor in Oxidation Agent Equilibrium Evaporation Pressure Formula

\[ C = B_1 \left( 1 + \frac{G}{1 - G} \frac{W_{\text{ave}}}{W_e} \right) \frac{P_{\exp}}{\frac{q}{RT_0}} - 1 \]

\( B_1 \)

\( c \) Specific Heat

\[ D = A + \lambda(AB - A) \]

\( D \)

\[ F = \frac{C}{C + q/E_*} \Omega + \lambda(AB - A) \]

\( E \) Energy of Activation for Oxidizer Agent Gasification Reaction

\( E_1 \) Energy of Activation for the Propellant Gas Phase Combustion Process

\[ F = \Omega + \lambda(AB - A) \]

\( G \) Mass Fraction of Oxidizer Agent for Completion of Condensation Reaction

\( K \) Speed Constant for Gas Phase Reaction
m Mass Flow Rate

n Combustion Speed Pressure Exponent

p Pressure

Q_\text{r} Unit Mass Binder Decomposition Reaction Heat

Q_\text{r} Boundary Layer Reaction Heat of Unit Mass Oxidizer

q Heat of Evaporation for Unit Mass Oxidizer

q Heat Flow for Gas Phase Boundary Surface Flow

Q Total Amount of Heat Released by Unit Mass Gas Phase Reaction

q Rate of Release for the Heat of Purification from Boundary Surface Reaction

Q Real Section

R Universal Gas Constant

\text{R}_r Pressure Response Function \quad \frac{m}{p}

\text{S} = i \omega \frac{p}{m_c} = i \Omega \quad (i is the imaginary number unit \sqrt{-1})

T Temperature

T_\text{i} Absolute Temperature of the Propellent Flame

T_\text{i} Initial Propellent Temperature

T_\text{s} Propellent Surface Temperature

t Time
\( W_{an} \) Gram molecular weight of oxidizer evaporation gas

\( W_c \) Gram Molecular Weight of Gaseous Products from Congealed Phase Oxidizer Reaction

\( x \) Distance

\( \sigma \) Mass Fraction of Oxidizer in the Propellant

\( \gamma \) Surface Area Fraction Composed of Oxidizer Agent Surface Covered by Molten Binder Agent

\( \lambda \) Congealed and Solid Phase Heat Conduction Coefficient

\( \ell \) Nondimensional Distance \[ \ell = \frac{\lambda}{m} \]

\( \rho \) Congealed and Solid Density

\( \Omega \) Nondimensional Frequency \[ \Omega = \frac{\rho}{m c} \omega \]

\( \omega \) Angular Frequency

**SUPERSCRIPETS**

- Steady State Value or Average Value
- Perturbation Value
- Complex Amplitude of Perturbation
1. Introduction

Sonic instability is the primary form of combustion instability of solid propellant rocket motors. Moreover, this type of sonic instability has as its primary source the combustion response of solid fuel propellants. Because of this, in the test construction of solid rocket motors, the sonic instability which they show is extremely important, and the theory and measurements associated with combustion response are indispensable. In linear sonic analysis, one can see whether or not a solid rocket motor will give a timely, consistent pressure perturbation enlargement, forcing a sonically unstable fuel. In this type of analysis, the pressure response function must already be known. The problem of solid propellant pressure response has received large amounts of research work in both the U.S. and the Soviet Union.
F.E.C. Culick (2) and N.S. Cohen (3) have already done excellent critiques of this problem. Concerning "flatbed" propellent pressure response functions, F.E.C. Culick and others (2) (4) have also done large amounts of research. However, the author recognizes that the steady state combustion model and several assumptions which form the basis of the research are still short on experimental data to support them. Moreover, several points of theory are still doubtful. For example, in order to explain the phenomenon of sonic vibrations in the combustion of flatbed propellents, we take the mass gasification rate for solid phase reaction areas and change it for use in the expression below:

\[ m = A, P^x e^{\frac{E_s}{R T}}. \]

From this, we can derive the pressure response function formula:

\[ R_p = \frac{nAB + n_s(\lambda - 1)}{\lambda + \frac{A}{\lambda} - (1 + A) + AB}. \]

Moreover, when we use this relationship to compare with the results of experiments, we select \( n_s = 1.0 \). However, we know that, when the pressure exponent of the mass gasification rate for solid phase reaction areas is a positive value, the pressure exponent for its steady state combustion speed can hardly be zero. This article does not intend to make more criticism of these publications. We will present on the basis of a steady state model (1) of solid fuel propellents in terms of positive and negative pressure exponents for combustion speed characteristics, and derive a new pressure response function formula and force diagram. We will use them to make a complete explanation of the pressure pairing phenomenon which exists in various types of solid propellents when their combustion speed pressure exponents are zero, positive, and negative.
In the same way as was the case with the corresponding steady state model (1), we select for use a one dimensional model and divide the combustion surface of the propellant into two sections. One section is composed of oxidizer agent surface covered by molten binder agent and the corresponding binder agent surface. Let us also stipulate that the fraction of surface area which it occupies out of the whole combustion surface is \( \nu \). The other section is then composed of oxidizer surface which is not covered and the remaining binder agent surface. These two sections are respectively called, for short, Area I and Area II, as shown in Fig. 1. At the same time, we take the combustion process for each solid propellant area and simplify it into three stages, which respectively occur in three different phases (Fig. 2).

1) The solid phase area on the inside of the solid propellant, to which heat is applied, is gasified by the gas phase flame area with the congealed phase reaction area supplying the heat.

2) Within the congealed phase layer between the solid phase and the gas phase one sees the development of an oxidizer boundary layer reaction and binder agent heat of decomposition which are contained in the congealed phase reaction and the gasification reaction. In Area I, unlike Area II, one must consider the molten binder agent liquid layer which covers the surface of the oxidizer as well as the oxidizer underneath it as they exist in the congealed phase reaction related to gasification and its opposite process—opposed gasification.

3) In the gas phase one sees the occurrence of combustion processes which include dispersion, mixing, and chemical reaction.
Fig. 1 One Dimensional Steady State Model

Fig. 2 A Simplified Two Dimensional Model 1. Congealed Phase Layer 2. Solid Phase Area of Increased Heat 3. Gas Phase Reaction Area
3. Mathematical Treatment

For the sake of convenience in the mathematical treatment, we first take the following assumptions:

1) The influence of heat damage is negligible.
2) The solid propellent is noncompressible, of a uniform quality, and of the same nature in all directions.
3) The congealed phase layer between the solid phase and the gas phase is an infinitely thin plane. In this plane one sees occurring the oxidizer congealed phase reaction and the gasification reaction (Area I and Area II are different and each has its own rules of reaction). At the same time, one sees occurring the high temperature decomposition of binder agent. This plane is called the boundary surface.
4) In the three solid, liquid, and gas phases discussed above, the specific heat is always a constant.
5) Gas phase reactions can be seen to be steady state processes.
6) The influence of congealed phase reactions on pressure disturbances is negligible.
7) When the amount of perturbation is sufficiently small, its second and higher order small amounts can all be neglected.

At the same time, in the same fashion as normal models, we choose for use a moving coordinate system. These coordinates take the instantaneous burn speeds of the propellents and move them into the solid phase area. In this way, the origin of the coordinate system can fall entirely on the solid-gas phase boundary surface.

Below we present the respective treatments for Area I and Area II.

Area I:
(1) Basic Equations
1) The unstable thermal conductivity equation in the solid phase heat addition area is:

\[
\frac{\partial T}{\partial t} + c \frac{\partial T}{\partial x} = \rho c \frac{\partial T}{\partial t} \frac{\partial^2 T}{\partial x^2}
\]
2) When we consider the existence of molten boundary agent on the covering of oxidizer surface and the oxidizer under the covering as they affect congealed phase reactions and gasification reactions, from reference article (1), the mass gasification rate of propellant is:

\[ m = \frac{1}{a_1 - G} \exp \left( -\frac{E_{p2}}{RT_f} \right) \left[ 1 - \frac{P_{\exp} \left( \frac{Q_{\exp}}{T_f} \right)}{B_1 \left( 1 + \frac{G}{1 - G} \frac{W_{c}}{W_{f}} \right)} \right] \]  

(2)

3) If we assume a steady state, the mass combustion rate for the gas phase reaction area can be expressed as (5):

\[ m = K P_{\exp} \left( -\frac{E_{p2}}{2RT_f} \right) \]  

(3)

4) Energy equilibrium equation for the inside of the infinitely thin congealed phase layer:

The difference between the heat flow \( q_s \) from the gas phase combustion area and the heat flow \( \lambda \frac{\partial T}{\partial x} \bigg|_{x=0} \) transferred into the solid phase should equal the rate of heat release \( q_s \), which is produced by the reaction in the congealed phase layer of the congealed phase and the gasification reaction, that is,

\[ q_s - \lambda \frac{\partial T}{\partial x} \bigg|_{x=0} = q_s \]

Moreover, using action coordinates for explanation, \( q_s = m[Q_r - c(T_f - T_i)] \).

\[ q_r = m[aQ_r - (1-a)Q_r] \]  

In these equations, \( Q_r \) is the amount of heat released by the unit mass gas phase reaction. \( Q_r \) is the algebraic sum of the effective heats of the oxidizer agent congealed phase reaction and the gasification reaction. \( Q_r \) is the effective heat from the high temperature thermal decomposition of binder agent. If \( q_r \)
substitute the equations above, we get

\[ \lambda \frac{\partial^2 T}{\partial x^2} + m[aQ, - (1-a)Q_x - Q_y + c(T_x - T_y)] = 0 \]  

(4)

(2) Linearization Treatment:

1) First, we carry out a linearization treatment of equation (1). We postulate:

\[
\begin{align*}
m(t) & = m + R_s (\bar{m} e^{i\omega t}) \\
T(x, t) & = T(x) + R_s (T(x) e^{i\omega t})
\end{align*}
\]

(5)

If we substitute in equation (1), and eliminate the high order small quantities, as well as making use of the steady state condition

\[ \lambda \frac{d^2 T}{dx^2} - mc \frac{dT}{dx} = 0 \]  

(6)

then, equation (1) changes to become:

\[
R_s \left\{ \frac{d^2 T}{dx^2} e^{i\omega t} \right\} - R_s \left\{ m e^{i\omega t} \frac{dT}{dx} \right\} - R_s \left\{ m e^{i\omega t} \frac{dT}{dx} \right\} = R_s \{ \rho c T \omega e^{i\omega t} \}.
\]

Because these terms are set up for all times \( t \), therefore, one can only
\[
\frac{d^2 T}{dx^2} - m_c \frac{dT}{dx} - m_c \frac{dT}{dx} = \omega \rho c T
\]  \hspace{1cm} (7)

Set

\[
\begin{align*}
\xi &= \frac{m_c x}{\lambda} \\
S &= i \omega \frac{\rho c}{m_c} - i \Omega
\end{align*}
\]  \hspace{1cm} (8)

Then equations (6) and (7) can respectively be written

\[
\frac{d^2 T}{d\xi^2} - \frac{dT}{d\xi} = 0
\]  \hspace{1cm} (9)

\[
\frac{d^2 T}{d\xi^2} - \frac{dT}{d\xi} - S T = \frac{m}{m} \frac{dT}{d\xi}
\]  \hspace{1cm} (10)

2) In the same way, in the case of equation (2), one can set

\[
\begin{align*}
m &= m + m' = m + R_s (m e^{i\varphi}) \\
T_s &= T_s + T_s' = T_s + R_s (T_s e^{i\varphi}) \\
P &= P + P' = P + R_s (P e^{i\varphi})
\end{align*}
\]  \hspace{1cm} (11)

Moreover, one can use the steady state condition

\[
m = \frac{1}{\alpha} \cdot \frac{1}{1 - G} A_s \exp \left( - \frac{E_s}{R_s T_s} \right) \left[ 1 - \frac{P \exp \left( \frac{q}{R_s T_s} \right)}{B_s \left( 1 + \frac{G}{1 - G} \frac{W_{s,s}}{W_s} \right)} \right]
\]

Division with equation (2) gives one
\[ 1 + \frac{m'}{m} = \exp \left( \frac{E_{\alpha}}{RT'} \cdot \frac{T'}{T} \right) C + 1 - \left( 1 + \frac{P'}{P} \right) \exp \left[ -\frac{q}{RT'} \cdot \frac{T'}{T} \right] \] (12)

In the equation

\[ C = \frac{B \left( 1 + \frac{G}{1-G} \frac{W_{\alpha}^m \beta}{W_{\alpha}^m} \right)}{P \exp \left( \frac{q}{RT'} \right)} - 1 \]

Also, because of the fact that \( \frac{E_{\alpha}}{RT'} \cdot \frac{T'}{T} \) and \( \frac{q}{RT'} \cdot \frac{T'}{T} \) are both very small \( \left( \frac{E_{\alpha}}{RT'} \text{ and } \frac{q}{RT'} \right) \) are both of the order of magnitude of 10). Moreover, \( \frac{T'}{T} \) is then in the range \( 10^{-1} \sim 10^{-1} \). Therefore, one has

\[ \exp \left( \frac{E_{\alpha}}{RT'} \cdot \frac{T'}{T} \right) \approx 1 + \frac{E_{\alpha}}{RT'} \cdot \frac{T'}{T} \]
\[ \exp \left( -\frac{q}{RT'} \cdot \frac{T'}{T} \right) \approx 1 - \frac{q}{RT'} \cdot \frac{T'}{T} \]

Substituting in equation (12) and eliminating the smaller quantities of higher order, one then obtains

\[ \frac{m'}{m} = \frac{E_{\alpha}}{RT'} \cdot \frac{T'}{T} - \frac{P'}{P} - \frac{q}{RT'} \cdot \frac{T'}{T} \]

As above, because of the fact that all of the equivalent forms for times could be set up, one, consequently has

\[ \frac{m}{m} = \frac{E_{\alpha}}{RT'} \left[ \frac{C + \frac{q}{C} \frac{T}{CE_{\alpha}} \cdot \frac{P}{P}}{C} \right] \] (13)
3) After one carries out the same type of linearization treatment on equation (3), one obtains:

\[ \frac{T_f}{T_i} = \frac{m - np}{E_f \frac{2R^q T_f}{}} \]  

(14)

4) In the same way, equation (4) can be changed to become:

\[ \left( \frac{dT}{d\xi} \right)_i + (T_f - T_0) = \frac{m}{m} \left( \frac{dT}{d\xi} \right) \]  

(15)

(3) Solution of Equations

Because equation (9) and its boundary condition are:

\[
\begin{align*}
\frac{dT}{d\xi} - \frac{dT}{d\xi} &= 0 \\
T &= T_i, \\
T &= T_f \\
T &= T_i \\
T &= T_f
\end{align*}
\]

it follows that its solution is obviously \( T = T_i + (T_f - T_i)e^t \). Moreover, \( \frac{dT}{d\xi} = (T_f - T_i)e^t \). If we take this expression and substitute it into equation (15), we then get

\[ \left( \frac{dT}{d\xi} \right)_i = - (T_f - T_i) + T_i(1 - \frac{T_i}{T_f}) \frac{m}{m} \]  

(16)

Moreover, if we substitute equation (13) and equation (10), equation (10) changes into

\[
\text{dim}^T \frac{dT}{d\xi} - \frac{dT}{d\xi} - S = E_f \left(1 - \frac{T_i}{T_f} \right) \left[ \left( \frac{C + E_f}{C} \right) \frac{T_i}{T_f} - \frac{R^q T_f}{C E_i} \frac{T_i}{T_f} \right]
\]
Its boundary conditions are:

\[
\begin{align*}
\xi = -\infty & \quad \text{time } T = 0 \\
\xi = 0 & \quad \text{time } T = T_0.
\end{align*}
\]

Therefore a general solution for the equation is:

\[
T = c_1 e^{\xi t} - \frac{A^C + \frac{q}{S}}{C} - \frac{T_0}{T} \frac{R_{T_0}}{P} \left[ T_0 e^t \right].
\]

In the equations \( A = \frac{E_{uo}}{R_{T_0}} \left( 1 - \frac{T_0}{T} \right) \). Moreover, \( \lambda \) is then determined from the characteristic equation \( \lambda^2 - \lambda - S = 0 \). Because, when \( \xi = -\infty \), \( T = 0 \); therefore, one only has \( \lambda = \left( 1 + \sqrt{1 + 4S} \right) / 2 \) as a characteristic limit. Again, from the boundary condition \( \xi = 0 \), when \( T = T_0 \), it is possible to obtain a solution to the equation as follows:

\[
T = \left[ T_0 \left( e^{\xi t} + A \frac{C + q}{S} \frac{T_0}{T} \frac{R_{T_0}}{P} \right) \right] \left( e^{\xi t} - e^t \right)
\]

Substituting in equation (16), one then obtains:

\[
\lambda + \frac{A}{S} \left( \frac{C + q}{C} \frac{T_0}{T} \frac{R_{T_0}}{P} \right) \left( \lambda - 1 \right) = (1 - \frac{T}{T_0}) \left( \frac{T}{T_0} \right)^{-1} \left[ \frac{T_0}{T} \cdot \frac{T_0}{T} \cdot \left( \frac{T}{T_0} \right)^{-1} - 1 \right]
\]
From equation (13), one has

$$\left( \frac{T'}{T} \right) = \frac{E_{ss}^*}{R^*T'} \cdot \frac{C + \frac{q}{E_{ss}^*}}{C \frac{m}{m} + \frac{P}{P}} \quad (18)$$

If one takes equations (14) and (18) and substitutes them with equation (17), one then obtains:

$$\lambda + \frac{1}{S} \left( \frac{C + \frac{q}{E_{ss}^*}}{C \frac{m}{m} + \frac{P}{P}} \right) (\lambda - 1)$$

$$= \frac{A_m}{A_m} \cdot \frac{C + \frac{q}{E_{ss}^*}}{C \frac{m}{m} + \frac{P}{P}} \cdot \frac{m - n}{n} \cdot \frac{\frac{E_f}{E_{ss}^*}}{2R^*T'} \cdot \left[ \frac{m}{m} + \frac{P}{P} \right] \cdot \frac{C + \frac{q}{E_{ss}^*}}{C \frac{m}{m} + \frac{P}{P}} \cdot \left[ \frac{1}{R^*T'} \right]$$

If we make $$R = \frac{2R^*T'}{(T - T')E_f}$$, and we make use of the pressure response function definition $$R_p = \frac{m}{m} / \frac{P}{P}$$, then, the above equation can be arranged so that:

$$R_p = \frac{\frac{nAB}{\lambda} - \frac{\lambda - 1}{C + \frac{q}{E_{ss}^*}}}{A - A + AB + \frac{C}{C + \frac{q}{E_{ss}^*}} (\lambda - 1)}.$$  

Area II:

Based on a method similar to the one above, we only take basic equation (2) and change it to be $$m = A \cdot \exp\left( -\frac{E_{ss}^*}{R^*T'} \right)$$. It is then possible to obtain an expression for the pressure response function for Area II(5):
Again, on the basis of the normal assumption that there is a direct proportion between combustion surface gain and combustion surface area, the pressure response function for the entire combustion surface formed by Area I and Area II should be:

\[
R_p = \frac{n_{AB}}{A - A + A_B + (\lambda - 1)}
\]

Actually, the imaginary section is:

\[
R_p = \frac{\lambda n_A B - \frac{\lambda - 1}{C + q/E_{ss}} + (1 - \gamma) A_{II}}{\lambda A - A + A_B + \frac{C}{C + q/E_{ss}} (\lambda - 1) + (\lambda - 1)}
\]

In these equations,
4. Results of Calculations and Discussion

Making use of the steady state model (1), the various results of initial calculations on S04-5A experiments (Table 1) form the initial data to substitute into the formula for the real section of the pressure response function, yielding results such as those shown in Fig. 3.

![Fig. 3 The Real Portion of Two Low Pressure Response Functions for S04-5A](image)

The formulas obtained and the results of calculations all clearly show that the real sections of pressure response functions for "plateau" and "mesa" propellants whose pressure exponents are zero or negative are all capable of being positive values larger than zero within quite a large range of frequencies. From the standpoint of theory, this explains very well facts associated with severe phenomena of unstable combustion.
and the existence of these two types of propellents (4)(6)(8). Moreover, just as this author calculated in reference (1), due to the fact that the abnormal combustion of oxidizer covered with molten binder agent takes the place of partial shut down, in this way, under the pressure associated with the existence of widespread covering, it is, on the contrary, easy to see the appearance of the results of self-excited oscillation. It is then possible to give a reasonable explanation.

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Table 1 1. Parameter 2. Unit 3. Numerical Value 4. Sample Calculation

What the dotted line in Fig. 3 shows is that, in reference (1), SO4-5A propellant, with a large surface area covered and a pressure of $P=23$ kg/cm$^2$, in experiments with the "T" type engine, shows the appearance of self-excited oscillation. The frequencies involved correspond to the real section values of the response functions for quite high pressures. Because of this, the appearance of self-excited oscillation in these experiments cannot be considered surprising.
Most of the results discussed above were obtained through analysis in Area I. This was done through the use of our steady state model and with consideration given to the covering of the oxidizer surface with molten binder agent and to the existence of a congealed phase reaction as well as opposed gasification. Because of this, the theoretical formula derived for the pressure response function can not only overcome the deficiency associated with the fact that the pressure response function formulas we already have cannot be used with negative propellant pressure exponents, but also, in one respect, it reflects the vitality of the stable state combustion model(1) for solid state propellents, which presents the combustion speed characteristics for positive and negative pressure exponents.

The formula which is derived when the covered surface area fraction is zero, then becomes the "two parameter formula" for propellents with binder agents which are difficult to melt:

\[ R_p = \frac{nAB}{\lambda + A - (1 + A) + AB} \]

Making use of the unified expression we already have for the "gas phase quasi-steady state, uniform propellant unidimensional model"(2), this article consequently derives a formula which is capable of having even broader applicability.

This article, before deriving this formula, made no small number of empirically quite significant assumptions. For example, the assumption that the congealed phase layer which gives rise to the congealed phase reaction and the gasification reaction is an infinitely thin plane is certainly not a good approximation(2). The assumption that the gas phase assumes a steady state has an error less than 10% only when the
frequencies are smaller than 10,000 Hertz (2)(5). The compressibility of propellents also has an influence on response functions which is within 10% (9). Heat radiation losses, under certain conditions, also have an obvious influence on response functions (10)(11). As far as congealed phase reactions are concerned, in our stable state model, they are basically pressure functions (1). However, we also conveniently saw from our mathematical treatment, that, when their influence on pressure diminishes, error is, naturally, even more unavoidable. Although, except for congealed phase reaction problems, the majority of assumptions are obvious, they are also universal in their application. However, this explains the fact that the formulas derived above are still awaiting refinement.

At the same time, formulas derived on the assumption of homogeneity can, of course, only be used in situations where one has homogeneity. However, due to the fact that propellant impurities have already been used by several scholars (12) in methods such as the one that follows; that is, taking a multi-mode composite propellant and viewing it as a finely dispersed oxidizer granule/fuel "surface match up" of random arrangement. When one assumes that each "pairing" is mutually independent, then it also becomes possible to take the propellant surface and rearrange it into a family of hypothetically dispersed units of propellant. This is what is called a hypothetical propellant. And, the pressure responses of these hypothetical fuels are calculated using formulas deduced from homogeneous theory. Moreover, from this one obtains the pressure response for the whole composite propellant. Just as is pointed out in reference (3), although this method has the drawbacks of two different models in its application, it is still currently being followed. Because of this, this model certainly does not fail to explain pressure pairing problem values for composite solid propellents because of its adoption of a homogeneity hypothesis.

5. Conclusions

By using a steady state combustion model for positive and negative pressure exponent combustion speed characteristics for solid propellants, we deduced a new pressure response function formula. This formula explains, in a reasonable way, phenomena associated with unstable combustion which exists in all solid propellants with positive, negative and zero pressure exponents.
2) In the same way which was the case with the forming of a model for steady state combustion, in the combustion of propellants, the phenomenon of molten binder agent covering oxidizer surface area should also be considered in the forming of a model for unstable combustion of solid propellants.

3) Using the unusually regular combustion of the condensed phase reaction and opposed gasification covered by molten binder agent to replace localized shut down is a precise and necessary method. This is true not only for steady states, but for non-steady states as well.

4) The model presented in this article is still awaiting further experimental testing and perfecting.

REFERENCES


THE ANALYSIS AND CALCULATION FOR THE DYNAMIC CHARACTERISTICS OF THE OMNI-AXIAL MOVABLE FLEXIBLE JOINT NOZZLE

YANG Shi-xue
SUMMARY

In this article, under simulated engine combustion chamber pressure conditions, we carry out an analysis of and calculations for the dynamics of the semi-axial rotation of flexible jet nozzles. The important contents include: analytical calculations of measured data; analytical calculations for the center of instantaneous oscillation of jet nozzles; and, calculations of the moments of force of oscillation. Besides this, we also calculated the parameters that follow: the angle of oscillation and angular velocity of jet nozzles; azimuth angles of oscillation; the length of operating tubes; eccentricity of thrust; and, axial and radial displacement of jet nozzles under differing pressures of contents, etc.

This method of calculation, after having appropriately resolved the problem of installing telemetry pickups, is still suitable for use with hot test beds and is also suitable for calculations and measurements under conditions of static oscillation of flexible jet nozzles.

1. Calculations of Adjustments in the Amounts of Displacement and Angles of Oscillation

(1) Selection of Measuring Equipment and Coordinate System

The apparatus for measuring simulated axial oscillation of flexible jet nozzles is as shown (Fig. 1). The flexible connector head connects respectively with the end ring and the lower flat plate. They and the container shell body together form a sealed high pressure vessel. To the bottom flat plate is firmly attached a rod (simulating the jet nozzle). Its axis line and the axis line of the flexible connector head are congruent. Its point of contact on the Z plate is the origin of the coordinate system O (At this time the content pressure is zero.) The axis OZ is then congruent with the axis line of the rod.

The top end of the rod and the axis line are installed at right angles to the flat plate, which is called the Z plate. The axial line of the displacement sensor Z, is congruent with the axis line of the rod. The direction and amount of thrust are controlled by actuator tubes 1 and 2, which are respectively positioned in planes OXYZ and OYZ. In this way a fixed coordinate system OX YZ...
is then established. Two flat plates are installed on the rod. They are respectively perpendicular to the X axis (called the X plate) and the Y axis (called the Y plate). Displacement sensors \( X, X, X, \) and \( Y, Y, \) are respectively perpendicular to the X plate and the Y plate. Moreover, lines extending from \( X, \) and \( Y, \) and \( X, \) and \( Y, \) connect respectively with two points on the Z axis. Their points of intersection, with coordinates in three dimensions, are \((O,0,-Z),(O,0,-Z)\) . An extension of the X axis line of the displacement sensor intersects with the \( OZY \) coordinate plane at point \((O,-Y,-Z)\) . The distance from the X plate and the Y plate to the coordinate point is \( R \) in both cases. The other two displacement sensors \( H, H, \) respectively measure the two actuator tubes fixed to rod points \( Q, \) and \( Q, \) in terms of the amount of axial displacement caused by elastic deformations of the vessel.

Besides this, the radius of measurement contact heads for displacement sensors \( X, X, X, Y, Y, \) is \( R \) in all cases. The radius of the measurement contact head for sensor \( Z \) may be selected at will. Let this value be \( R \) . The rest of the installation constants and structural constants are as shown in the Figure.

\[ F, \text{ (or } P, ), \text{ and } F, \text{ (or } P, ) \text{ are activating forces (or pressure differential) sensors.} \]

(2) Analysis of Displacement Sensor Measurement Data

Among the eight displacement sensors, except for the measurement data from \( H, H, \) which there is no need to correct, the measured data obtained from the other six sensors certainly does not perfectly reflect the amount of displacement in the oscillation of the rod. This is due to the reasons set out below.

(1) The measuring contact head of the sensors is a hemisphere. According to changes in the angle of oscillation, contact points experience displacement along the hemisphere, introducing an additional amount of displacement.

(2) The measurement contact heads for the various displacement sensors $X, X', X', Y, Y'$ and the contact points of the experimental measurement plates are at a distance from contact points on the rod axis line $X$, and from the symmetrical surface of the rod so that the distance varies with changes in the angle of oscillation.

(3) Because of the fact that the axis lines of $X, Y,$ and $Z$ rotate at the same time, their mutual influences cause plates $X$ and $Y$ to correspond to the slant which occurs in the sensors. This introduces an amount of non-displacement in the sensors.
(3) Adjustment Calculations for Angle of Oscillation and Amount of Displacement

Let the angles of rotation for the axis lines X, Y, and Z be \( \beta, \alpha, \theta \). In order to simplify analysis and calculations, we make use of the geometrical principle of superposition to make a two step calculation. After that, we superimpose. That is, first we make calculations for the situation in which \( \alpha \neq 0 \) and \( \beta \neq 0 \), with \( \theta = 0 \). Then, we make the calculations for the situation in which \( \alpha \neq 0, \beta \neq 0 \) and \( \theta \neq 0 \).

(1) Adjustment Calculations for the Situation in Which \( \alpha \neq 0, \beta \neq 0 \) and we assume \( \theta = 0 \).

There are four types of factors in omni-axial oscillation which cause adjustments to the displacements measured by the displacement sensors: (a) \( a > 0, \beta < 0 \), (b) \( a < 0, \beta < 0 \), (c) \( a < 0, \beta > 0 \), (d) \( a > 0, \beta > 0 \).

Equation 

1. Eccentric rotation of the axis line within the first apparent limit \( a > 0, \beta < 0 \).

If we assume that the cross section of a rod is a square \( 2R \) on a side, then, Fig. 2 is a cubic diagram of angle \( \alpha \) and angle \( \beta \) of the deflection. Fig. 3 is a horizontal projection diagram corresponding to it.

Definition: The plane which holds the displacement sensors \( X_i \) and \( Y_i \) (or \( X_i \) and \( Y_i \)) is the horizontal plane of test measurements. The same rod cross section used in it is represented by \( \square EFGH \).

If we assume that axis line \( Y \) rotates through angle \( \alpha \) and axis line \( X \) then rotates through angle \( \beta \), the change in the horizontal plane of test measurements is: square \( \square E,F,G,H, \) elongated square \( \square E',F',G',H', \) quadrilateral \( \square EFGH \). This corresponds to a change in the contact point of the experimental measurement head of the \( X \) displacement sensor: \( A_0 \rightarrow A_1 \rightarrow A_2 \).

At point \( A_0 \): \( a=0, \beta=0 \). \( X \) sensor measurements give us a displacement amount \( X_0 = 0, A_0 = R \).

At point \( A_1 \): \( a > 0, \beta = 0 \). The measured amount of displacement is \( X' \). The actual amount of displacement is \( X = 0, 0 \).

One of the additional amounts of displacement is:

\[
A_0 - A_0 = R / \cos a - R
\]
Fig. 2 Cubic Diagram of the Horizontal Test Measurement Plane When $a > 0, \beta < 0, \theta = 0$

Fig. 3 Plane Diagram of the Test Measurement Plane When $a > 0, \beta < 0, \theta = 0$
From the diagrams it is possible to see four amounts of additional displacement caused by the displacement of the contact points on the contact heads: \( A_iA_i' = O'A_i - O'A_i = R_i \cdot \cos \alpha - R \).

\[ X' = X + (R + R_i)(1/\cos \alpha - 1) \]  \hspace{1cm} (1)

At point \( A_i \): \( a > 0, \beta < 0 \). Let the numerical data measured by the sensor be \( X' \). From Fig. 2 or Fig. 3 it is easy to see that the value of the difference between \( X' \) and \( X \) is \( A_iA_i' \).

These are three of the amounts of additional displacement caused by angle \( \beta \) on sensor \( X \). From Fig. 2 we obtain the set of relationships set out below:

- From \( R \Delta A_iA_iA_i' \) we get: \( A_iA_i' = A_iA_i' \cdot \tan \beta = \gamma \tan \beta \)
- From \( R \Delta A_iA_iA_i' \) we get: \( A_iA_i' = A_iA_i' \cdot \tan \alpha = \gamma \tan \alpha \)

(2)

(3)

Compared with equation (2), equation (3) has: \( r = \arctan (\tan \alpha \cdot \tan \beta) \) \hspace{1cm} (4)

Because \( A_iA_i' = \gamma \beta \), we also give consideration to satisfying the geometrically equivalent relationship, and we then have:

\[ X' - A_iA_i' = X \]  \hspace{1cm} (5)

After we now do some more analysis of the deflection angle \( \beta \), the four additional amounts of displacement which are induced in the displacements of the contact points on the measuring contact head of displacement sensor \( X \) can be seen from Fig. 4. The contact point moves from point \( A_i \) to point \( A_i' \), that is, it rotates through angle \( \angle A_iO'A_i = \gamma, O'A_i = R \cos \alpha \). Because of this, there is always a positive amount of increase:

\[ R \cos \alpha (Y \cos \gamma - 1) \]  \hspace{1cm} (6)

When one combines this with equations (1)(2)(5)(4)(6) one gets:

\[ X = X' - Y \gamma r - (R + R_i)(1/\cos \alpha - 1) - R_i \cos \alpha (1/\cos r - 1) \]  \hspace{1cm} (7)

Below, when we make adjustment calculations for the amount of displacement measured by the Y direction displacement sensor. We similarly have:
At point $B_0$:
$$Y = 0, \quad B_0 = R$$

At point $B_1$:
$$Y' = Y + (R + R_b)\left(\frac{1}{\cos \beta} - 1\right)$$

At point $B_2$:
$$B_2 = X_{gr}$$

If $X_{gr} < 0$, it satisfies the equivalent relationship

$$Y'' + B_2B_1 = Y'$$
$$Y' - X_{gr} = Y'$$

The additional amount of displacement caused by the angle $\alpha$ is:

$$R_0 \cos \beta (1/\cos \alpha - 1)$$

From the various equations (3)(9)(10) one then obtains:

$$Y = Y' - X_{gr} - (R + R_b)(1/\cos \beta - 1) - R_0 \cos \beta (1/\cos \alpha - 1)$$

(2) Calculations to Adjust the Amount of Displacement When the Angles $\alpha, \beta, \theta$ Are Simultaneously Not Zero

As far as an ideally flexible contact head is concerned, one has only the times when the azimuth angle of oscillation $\psi_0$ is 45 degrees and 225 degrees, making use of forces, the combined force of which crosses over the axis of the jet tube. In the case of oscillations with other azimuth angles, making use of forces the combined force of which does not cross the axis of the jet tube, adds an additional moment of force to the axis line of the jet tube. In this way, the jet tube rotates around the Z axis line, the angle of which is designated $\theta$. During omni-axial oscillation, it is only possible for the angle $\theta$ to be positive or negative. If one gives consideration to $\alpha, \beta$, the overall influencing factors can be grouped into six types of situations, that is,

$$
0^\circ = \psi_0 < 45^\circ \quad \theta < 0, (a > 0, \beta < 0) \\
45^\circ < \psi_0 < 90^\circ \quad \theta > 0, (a > 0, \beta < 0) \\
90^\circ < \psi_0 < 180^\circ \quad \theta > 0, (a < 0, \beta < 0) \\
180^\circ < \psi_0 < 225^\circ \quad \theta > 0, (a < 0, \beta > 0) \\
225^\circ < \psi_0 < 270^\circ \quad \theta < 0, (a < 0, \beta > 0) \\
270^\circ < \psi_0 < 360^\circ \quad \theta < 0, (a > 0, \beta > 0)
$$
Going through a practical derivation clearly shows that the six situations all possess the same type of expression. In this case, it is only necessary to do calculations for one situation.

$$0^\circ \leq \varphi < 45^\circ , a > 0, \beta < 0, \theta < 0, X > 0, Y > 0$$

\( \theta \) is the angle of rotation around the OZ coordinate axis. It is obvious that the horizontal test measurement surface rotates through angle \( \theta \) in the same way. Analysis of the front surface down to the mutual influences of the angles of deflection \( a \) and \( \beta \) cause the horizontal test measurement surface to give rise to deformations. After making adjustments, \( a \) and \( \beta \) are used independently to cause the horizontal test measurement surface to give rise to deformations in \( \varnothing E''F''G''H'' \) as in the diagram, and rotate it through the angle \( \theta \);

that is, becoming \( \varnothing E'''F'''G'''H''' \), as shown in Fig. 5.

From Fig. 5 one can see that the angle \( \theta \) causes the X sensor and Y sensor to respectively acquire an additional amount of displacement becoming \( A_1'A_1 \) and \( E_1'B_1 \).
Solving for \( A''A_s \), the D point coordinates are

\[
\begin{align*}
X_D &= X + O_D \cos \theta \\
Y_D &= Y - O_D \sin \theta
\end{align*}
\]

\[ A_O, O_D = O, D, \ X_A' = X + (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) = X + A_O, \]

\[
.D = A_O, O_D = (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right), \quad \text{also because} \quad DC = O_D \sin \theta < 0,
\]

therefore, the D point coordinate should be:

\[
\begin{align*}
X_D &= X + (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) \cos \theta \\
Y_D &= Y + (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) \sin \theta
\end{align*}
\]

(12)

Make \( H''E'' \) and \( H^*E^* \) cross at point J. Then, the equation for the straight line DJ is:

\[
Y = -\frac{1}{\tan \theta} (X - X_D) + Y_D
\]

(13)

Also one has:

\[
X_J = X' = X + (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right)
\]

(14)

Take equation (14) and substitute in equation (13):

\[
y_J = -\frac{1}{\tan \theta} \left[ (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) (1 - \cos \theta) \right] + Y + (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) \sin \theta
\]

(15)

From \( Rt: A''A_J \) we get:

\[
A''A_s = Y, \tan \theta
\]

(16)

\[ \therefore A''A_s = (R + R_s) \left( \frac{1}{\cos \alpha} - 1 \right) \left( \frac{1}{\cos \beta} - 1 \right) + Y, \tan \theta \]
From Fig. 5 we get the geometrical relationship

\[ A,O + A';A_s = A';O = X' \]
\[ A_s O = X', A';A_s = y, \tan \theta < 0 \]
\[ A_s A = A';A_s = A;A \]

Substituting the various algebraic quantities and simplifying, we get:

\[ X = X' - Y(\tan \theta + \tan \theta) - (R + R_s)(\frac{1}{\cos \alpha} - 1)\frac{1}{\cos \theta} - R, \cos \alpha(\frac{1}{\cos \theta} - 1) \]  \hspace{1cm} (17)

In solving \( R;B;B_s \), we use a process identical to that used in solving for \( A;A_s \), and we obtain an S point coordinate which is:

\[ \begin{align*}
X_s &= X - (R + R_s)(\frac{1}{\cos \beta} - 1)\sin \theta \\
Y_s &= Y + (R + R_s)(\frac{1}{\cos \beta} - 1)\cos \theta
\end{align*} \hspace{1cm} (18)\]

Let \( E''F'' \) and \( E'F' \) cross at point Q. The straight line equation of SQ is

\[ Y = \tan \theta(X - X_s) + Y_s \]  \hspace{1cm} (19)

The Q point coordinate is:

\[ Y_0 = Y' = Y + (R + R_s)(\frac{1}{\cos \beta} - 1) \]  \hspace{1cm} (20)

The equations (17)(18) and substitute in equation (19):

\[ X_0 = \frac{1}{\tan \theta}(R + R_s)(\frac{1}{\cos \beta} - 1)(1 - \cos \theta) + X - (R + R_s)(\frac{1}{\cos \beta} - 1)\sin \theta \]  \hspace{1cm} (21)

Thus \( R;A;B;B_s \), it is possible to obtain \( B;B_s = X_0 \tan \theta \)

Or

\[ B;B_s = (R + R_s)(\frac{1}{\cos \beta} - 1)(1 - \frac{1}{\cos \theta}) + X_0 \tan \theta \]  \hspace{1cm} (22)

The geometrical relationship has the form:

\[ B_s O - B;B_s = B;O \]
: \dot{B}_1 \dot{B}_2 = X \dot{\theta} < 0 \quad \text{at the same time, after giving consideration to the superposition quantities for \( \dot{B}_1 \dot{B}_2 \), one then has:}

\[ Y = Y' - X(\dot{\theta}_1 - \dot{\theta}_2) - (R + R_1) \left( \frac{1}{\cos \beta_1} - 1 \right) \frac{1}{\cos \beta_2} - R_1 \cos \beta_1 \left( \frac{1}{\cos \beta_1} - 1 \right) \]  

Equations (17) and (23) are the equations for calculating the amount of displacement adjustment which is measured by the displacement sensors in the \( X \) and \( Y \) directions during omni-axial oscillation. The two are both functions, and, set up together, they can be solved for an equation through which the amount of displacement \((X', Y')\) can be calculated from the measured amount \((X'', Y'')\):

\[ X_{ji} = \frac{X''; Y''; (\dot{\theta}_1 + \dot{\theta}_2) - (R + R_1) \left( \frac{1}{\cos \beta_1} - 1 \right) \frac{1}{\cos \beta_2} - R_1 \cos \beta_1 \left( \frac{1}{\cos \beta_1} - 1 \right)}{1 - \dot{\theta}_1^2 \dot{\theta}_2^2} \]

\[ Y_{ji} = \frac{X''; Y''; (\dot{\theta}_1 - \dot{\theta}_2) - (R + R_1) \left( \frac{1}{\cos \beta_1} - 1 \right) \frac{1}{\cos \beta_2} - R_1 \cos \beta_1 \left( \frac{1}{\cos \beta_1} - 1 \right)}{1 - \dot{\theta}_1^2 \dot{\theta}_2^2} \]

In the equations \( j = 1, 2, i = 0, \ldots, n \)

\( R, R_1 \) — constants; \( a_i, \beta_i, r, \theta \) are calculated from the formulas given below.

(3) Calculation of Angle of Oscillation

From the definitions of the various angles of rotation one can have:

\[ \dot{\alpha}_1 = \frac{X_{ji} - X_{ji}}{Z_i - Z_j} = \frac{X''; X''; - (Y''; Y'';)(\dot{\alpha}_1 + \dot{\theta}_1)}{(1 - \dot{\theta}_1^2 \dot{\theta}_2^2)} \]

\[ \dot{\beta}_1 = \frac{Y_{ji} - Y_{ji}}{Z_i - Z_j} = \frac{Y''; Y''; + (X''; X'';)(\dot{\alpha}_1 + \dot{\theta}_1)}{(1 - \dot{\theta}_1^2 \dot{\theta}_2^2)} \]

Calculation of the Angle \( \psi \): From equation (17) it is possible to obtain:
Through dilution it is possible to obtain:

\[ X_{ii} = X_{ii} - (Y_{ii} + Y_{i})(tg_{ri} + tg_{r}) - (R + R_{i})\left( \frac{1}{\cos \alpha_{i}} - 1 \right) \left( \frac{1}{\cos \theta_{i}} - 1 \right) \]

From the definition:

\[ ig_{\theta_{i}} = \frac{X_{ii} - X_{ii}}{Y_{i}} = \frac{X_{ii} - X_{ii}}{2Y_{i}} - \frac{1}{2} tg_{r_{i}} \]

Equations (26)(27)(29) are a set of implicit transcendental equations. Using the iterative substitution method in a computer application, one solves for values of \( a_{i}, \beta_{i}, \theta_{i} \). After this, one then substitutes equations (24) and (25). It is then possible to solve for \( X_{ii} \) and \( Y_{ii} \) for the various amounts of displacement.

(4) Adjustment of the Numerical Data Measured By the Displacement Sensor \( Z \).

Definition: The angle of oscillation is the angle included between the geometrical center line of the jet tube and the \( OZ \) coordinate axis. It is expressed by the use of \( \theta \).

The numerical data recorded by the \( Z \) sensor, alone, due to the angle of slant on the \( Z \) plate, introduces an additional amount of displacement. Moreover, this angle of slant is always equal to the oscillation angle \( \theta \). In order to solve for this oscillation angle, one must first solve for the direction number of the geometrical center line

\[ A_{i} = X_{ii} - X_{ii} B_{i} = Y_{ii} - Y_{ii} C = Z_{i} - Z_{i} = Z_{i} \]

(30)
Oscillation angle $\theta$ : 
\[ \cos \theta = \frac{C}{\sqrt{A+B+C}} \]  
(31)

Adjustment calculation for $Z_{ij}$ : 
\[ Z_{ij} = Z_{ij} - R \left( \frac{1}{\cos \theta} - 1 \right) \]  
(32)

In a heat test bed situation, it is possible to make use of two displacement sensors $Z_{1}$ and $Z_{2}$ in order to replace sensor $Z_{i}$. $Z_{1}$ and $Z_{2}$ are installed symmetrically on the two sides of the jet tube in order to avoid ignition flame heat corrosion.

As Fig. 6 shows

Fig. 6 (a) Chart of Sensor Distribution (b) A Comparison of Sensors $Z_{1}$ and $Z_{2}$ to Sensor $Z_{i}$

It is easy to demonstrate that, between $Z_{1}$, $Z_{2}$, and $Z_{i}$, there is the relationship shown below:

\[ Z_{ij}' = \frac{Z_{ij} + Z_{ij}}{2} \]  
(33)

(5) Adjustment of the Coordinates of Actuator Tube Fixing Points to Control the Direction and Amount of Thrust

Due to the results of pressure contents and the forces used with actuator tubes, the displacement of the rod fixing point along the axis reaches above 5m. This not only produces an influence on the shock process of the actuator tubes. It is also related to the direction of the forces used and the force momentum. Therefore, it is necessary to make adjustments to the measured amounts. Most of the
amount of the change is along the direction of the axis. Therefore, the adjustment is approximately:

\[
\begin{align*}
Z_{o11} &= H_0 + H_{11} \\
Z_{o12} &= H_0 + H_{21}
\end{align*}
\]  

(34)

In the equations, \( H_0 \) is the coordinate of the rod fixing point. \( H_{11}, H_{21} \) are the values of the amounts measured.

(6) Calculations of the Motive Forces \( F_{u1} \) and \( F_{u2} \) Used to Control the Amount and Direction of Thrust \( P \).

As motive force one can choose to use the tensile pressure sensor contact measurements obtained as \( F_{u1} \) and \( F_{u2} \). Or, one can measure the pressure differential \( P \) on the two sides of the piston. The force used is then:

\[ F_{u1} = SP_{12} \]

In the equation, \( S \) is the area of the piston, \( i=1,2, i=1,2,...,n \).

Up to the present time, we have solved for the following parameters:

\( X_{11}, X_{u1}, X_{12}, X_{u2}, Y_{u1}, Y_{u2}, Z_{11}, Z_{u1}, Z_{12}, Z_{u2}, F_{11}, F_{u1}, \alpha, \alpha, \theta, \theta, \delta, \delta, A, A, C \)

and \( C \), as well as constants to be used in later calculations. It is not necessary to explain these for our purposes here.

2. Calculation of Center of Oscillation for Flexible Jet Tube

THE CONCEPT OF A CENTER OF OSCILLATION

As far as the concept of a center of oscillation for flexible jet tubes is concerned, both inside and outside China, there have been treatments in a number of technical reports. But, these reports have not found a clear and precise definition. Moreover, there have been several methods put forward which are obscure and unclear. Before making calculations of the center of oscillation, we plan to have a clear and precise concept to act as a theoretical guide for analysis and calculations.

What is called a center of oscilation must be understood in the following terms. In the case of rigid bodies, there are, of course plane oscillations taking place as well as instant oscillations (omni-axial oscillation). Both of these exist simply as a series of instantaneous axial rotations, and no instantaneous center of oscillation exists. Obviously, a jet tube is a rigid body, and its center of oscillation can change.
Flexible jet tubes also have a point which functions as their theoretical center of oscillation. If the design and manufacture of the flexible connector heads are both absolutely ideal, this leads to the flexible jet tube rotating around the same fixed point. What are called its instantaneous axes of rotation all pass through its theoretical center of oscillation.

Absolute idealization of flexible jet tubes does not exist. In reality, due to content pressure (or combustion chamber pressure) changes, flexible jet tubes are made to produce relatively large amounts of axial instability. Besides this, flexible connector heads do not have uniform distribution of material around their circumference, and, during oscillation, their acceptance of forces will not be entirely symmetrical.

It is also impossible that the deformations in the flexible connector heads, which are caused by this, should be symmetrical and uniform. This will also cause the instantaneous axis of rotation of flexible jet tubes not to pass through their theoretical center of oscillation. When flexible jet tubes undergo semi-axial oscillation under various types of content pressures, the series of instantaneous axes of rotation will form a cylinder-shaped included figure. This figure will be symmetrical around the center of oscillation which will be included in it. This is called the "oscillation center" for short. This is also nothing else except the actual oscillation center of the jet tube, the size of the geometrical dimensions of which reflect the quality characteristics and rigidity of the flexible connector heads.

If one is considering directly solving for the geometrical range of the included center of oscillation, it is necessary to have dense instantaneous axes of rotation. It is also necessary to have large amounts of experimental data. It is difficult to get to this point, and it is also not economical. On the basis of this special characteristic of jet tubes rotating around these fixed points, it is possible to reach the conclusion which follows. The instantaneous oscillation center of the geometrical center line of jet tubes is always a reduplication of this fixed point. This fixed point is what is called the oscillation center of the jet tube. Because of this, we then use the instantaneous oscillation center of the geometrical center line of a jet tube to replace the point center of oscillation of the jet tube. By means of
this method, we use a limited amount of experimental data to solve for the geometrical size range of the jet tube oscillation center. Moreover, this supplies a reference point for solving for the instantaneous moment of force of oscillation.

During oscillation, if the geometrical center lines cross each other, the results of the calculations discussed above are completely accurate. However, it is also possible for them to be mutually off and not cross. In such a case, one then takes their "average oscillation plane" and solves for the oscillation center. Obviously, the results of this are approximate. The degree of this approximation varies with and increases as the degree of graduation of the measured angle of oscillation is reduced. Experimentation clearly shows that, when the measured angle of oscillation is changed from 15 degrees to 1.5 degrees, the distance between two adjacent geometrical center lines drops from 0.5m to 0.2m, roughly speaking.

CALCULATION OF THE OSCILLATION CENTER OF FLEXIBLE JET TUBES

In order to simplify the discussion, let the geometrical center line of the jet tube be the $Z'$ axis. This is not the same as the axis of the coordinate system.

The $Z'$ axis equation is:

$$\frac{X-X_{1t}}{A_{1t}} = \frac{Y-Y_{1t}}{B_{1t}} = \frac{Z-Z_{1t}}{C_{1t}}$$  \hspace{1cm} (35)

The equation for the Z plate plane is

$$A_{0}X + B_{0}Y + C_{0}(Z-Z_{1t}) = 0$$  \hspace{1cm} (36)

The set of simultaneous equations (35) (36) gives us the coordinates for the $O_{0}$ point. The $O_{0}$ point coordinates are:

$$x_{0} = \frac{CZ_{1t} - A_{0}Q_{0} - B_{0}P_{0}}{A_{0}K_{0} + B_{0}L_{0} + C_{0}}$$  \hspace{1cm} (37)

$$x_{0} = K_{0}z_{0} + Q_{0}$$

$$y_{0} = L_{0}z_{0} + P_{0}$$

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In the equations

\[ K_i = A_i/C_i = X_{i1} - K_{i1}Z_{i1}, L_i = B_i/C_i = Y_{i1} - L_{i1}. \]

The \( O_i \) point is a point on the \( Z' \) axis. In the initial configuration, it is a duplication of the origin point of the coordinate system.

If we postulate two mutually adjacent instants, the plane at \( O_i \) which contains the instantaneous center of oscillation is \( \pi_{ii} \). We then have:

\[
(x-x_{ii})^2 + (y-y_{ii})^2 + (z-z_{ii})^2 = (x-x_{ii-1})^2 + (y-y_{ii-1})^2 + (z-z_{ii-1})^2
\]

After rearrangement, we get:

\[ \pi_{ii} : A_{ii}x + B_{ii}y + C_{ii}z = D_{ii} \]  \((53)\)

In the equations

\[
\begin{align*}
A_{ii} &= x_{ii} - x_{ii-1}, \\
B_{ii} &= y_{ii} - y_{ii-1}, \\
C_{ii} &= z_{ii} - z_{ii-1}, \\
D_{ii} &= (x_{ii} + y_{ii} + z_{ii} - x_{ii-1} - y_{ii-1} - z_{ii-1})/2
\end{align*}
\]

In order to solve for the instantaneous axis of rotation of jet tubes, one must still find a second point on the plane on which we find the center of oscillation for the time interval during the same period. Let this be the intersection line of the plane \( \pi_{ii}, \pi_{ii} \) and the plane \( \pi_{ii} \), and one then has the instantaneous axis of rotation. Because of the fact that a measurement error in the system is unavoidable, it is only when \( \pi_{ii} \perp \pi_{ii} \) that the degree of precision associated with the axis of rotation which we find is optimum. The \( O_i \) point is a fixed point on the jet tube axis line. Points which satisfy the condition \( \pi_{ii} \perp \pi_{ii} \) will vary with changes in the oscillation of the jet tube. The method for the fixing of coefficients is none other than an unceasing pursuit of these points.

If we take the plane \( \pi_{ii} \), and set \( \pi_{ii} \parallel \pi_{ii} \)

\[ \pi_{ii} : A_{ii}x + B_{ii}y + C_{ii}z = D_{ii} \]  \((53)\)

If we solve the simultaneous equations \((55)\) and \((53)\):

let the intersection point of \( \pi_{ii} \) and the \( Z' \) axis be \( E_{ii}(x_{iii}, y_{iii}, z_{iii}) \). When we solve, we get the result:
The coordinates for point \( E_0 \):

\[
\begin{align*}
z_0 &= \frac{D_{01} - A_{01}Q_{01} - B_{01}P_{01}}{A_{01}K_{01} + B_{01}L_{01} + C_{01}} = K_{01}D_{01} + Q_{01}, \\
x_0 &= K_{01}D_{01} + Q_{01}, \\
y_0 &= K_{01}D_{01} + Q_{01},
\end{align*}
\]

In the equations

\[
\begin{align*}
K_{01} &= 1/(A_{01}K_{01} + B_{01}L_{01} + C_{01}) \\
Q_{01} &= -(A_{01}Q_{01} + B_{01}P_{01})/(A_{01}K_{01} + B_{01}L_{01} + C_{01}) \\
K_{01} &= K_{01}K_{01} \\
Q_{01} &= K_{01}Q_{01} + Q_{01} \\
Q_{01} &= L_{01}Q_{01} + P_{01}
\end{align*}
\]

Let the intersection point of plane \( \Pi \) and axis \( Z \) be \( E_n(x_n, y_n, z_n) \).

When we solve, we get the result:

Coordinates for point \( E_n \):

\[
\begin{align*}
\begin{cases}
 z_n = \frac{D_{n1} - A_{n1}Q_{n1} - B_{n1}P_{n1}}{A_{n1}K_{n1} + B_{n1}L_{n1} + C_{n1}} = L_nD_n + P_n, \\
x_n = L_nD_n + P_n, \\
y_n = L_nD_n + P_n,
\end{cases}
\end{align*}
\]

in the equations:

\[
\begin{align*}
L_n &= \frac{1}{A_{n1}K_{n1} + B_{n1}L_{n1} + C_{n1}}, \\
P_n &= \frac{-(A_{n1}Q_{n1} + B_{n1}P_{n1})}{A_{n1}K_{n1} + B_{n1}L_{n1} + C_{n1}}, \\
L_n &= K_{n1}L_{n1}, \\
P_n &= Q_{n1} + K_{n1}P_{n1}, \\
L_n &= L_{n1}, \\
P_n &= P_{n1} + L_{n1}P_{n1}.
\end{align*}
\]

The point \( K_nL_nP_nQ_n \) is the intersection point of line \( E_n \) and plane \( \Pi_n \) for the point \( E_n \). The point \( \Pi_0E_n = 0,E_n \).
After one takes equations (40) and (41) and substitutes them in an expansion arrangement, one has

$$U_sD_x^2 + V_sD_x + W_s = 0$$

in the equations

$$U_s = l_s^2 + l_s^1 + l_s^1 - K_s^2 - K_s^1$$
$$V_s = 2[K_s(x_{s1} - Q_{s1}) + K_s(y_{s1} - Q_{s1}) + K_s(z_{s1} - Q_{s1})$$
$$- L_s(x_{s1} - P_{s1}) - L_s(y_{s1} - P_{s1}) - L_s(z_{s1} - P_{s1})]$$
$$W_s = (x_{s1} - P_{s1})^1 + (y_{s1} - y_{s1})^1 + (z_{s1} - z_{s1})^1 - (x_{s1} - Q_{s1})^1 - (z_{s1} - Q_{s1})^1$$

Solving equation (42) we have:

$$D_{s1} = -V_s \pm \sqrt{V_s^2 - 4U_sW_s}$$

$$D_{s1}$$ is then the fixed coefficient we were solving for. There are two sets of solutions. However, corresponding to our actual problem, only the solution set in which $$z_{s1}$$ and $$z_{s1}$$ are simultaneously smaller than zero is applicable. Therefore, we take the two solution sets of 43 and substitute them respectively into equations 40 and 41. From the principles discussed above, we decide and make a selection. Because of this, after we make a decision, the solutions of 43 and 41 which we choose, then make it possible to solve for the plane $$\pi_{21}$$ on which is located the simultaneous oscillation center for \(A_{s1}, x + B_{s1}, y + C_{s1}, y = 0\)...

Moreover, $$\pi_{21}$$ is

$$(x - x_{s1})^1 + (y - y_{s1})^1 + (z - z_{s1})^1 = (x - x_{s1})^1 + (y - y_{s1}) + (z - z_{s1})$$

$$A_{s1}x + B_{s1}y + C_{s1}y = D_{s1}$$

(44)
in the equations

\[
\begin{align*}
A_{ii} &= x_{ii} - x_i, B_{ii} = y_{ii} - y_i, C_{ii} = z_{ii} - z_i, \\
D_{ii} &= (x_i + y_i + z_i - x_i - y_i - z_i)/2
\end{align*}
\]

The line of intersection between planes \( \pi_i \) and \( \pi_{ii} \) is the instantaneous axis of rotation we are solving for. Below, we also solve for the plane of oscillation of the geometrical center line \( Z' \) axis of the jet tube, and we let this plane be \( \pi_{ii} \).

During omni-axial oscillation of flexible jet tubes, the most general situation for the track of the \( Z' \) axis is a curved surface in space. If we take this curved surface and divide it into \( n \) partial curved surfaces, and, respectively use a plane to approximately replace their various oscillation planes \( \pi_{ii} \), it is obvious that, when the dissected areas \( n \) are increased in number, the level of the approximation is raised. We select the number \( n \) so as to satisfy the requirement for precision which happens to exist. In order to solve for the oscillation plane \( \pi_{ii} \), one first does the calculations set out below:

(i) The direction number \( N_i \) of the common perpendicular line between the jet tube axis line \( Z_{i-} \) and \( Z_i \) for two adjacent instants.

Make the direction numbers of the axis lines \( Z_{i-} \) and \( Z_i \) respectively \( N_{i-} \) and \( N_i \), then:

\[
N_i = (A_{i-} - A_i, B_{i-} - B_i, C_{i-} - C_i) = (A_{ii}, B_{ii}, C_{ii})
\]

in the equations

\[
A_{ii} = C(B_{i-} - B_i), B_{ii} = C(A_{i-} - A_i), C_{ii} = A_{i-} + B_i - A_B_{i-}.
\]

(ii) Make plane \( R_{i-ii} \) contain axis \( Z_{i-} \). And, make it parallel to \( N_i \). Then, the equation for \( R_{i-ii} \) is

\[
\begin{vmatrix}
 x - X_{i-ii} & y - Y_{i-ii} & z - Z_i \\
 A_{i-} & B_{i-} & C \\
 A_{ii} & B_{ii} & C_{ii}
\end{vmatrix} = 0
\]
Or it is:

\[ A_1 x + B_1 y + C_1 z = D_1 \]  

in the equations

\[
\begin{align*}
A_1 &= B_1 C_1 - B_1 C, \\
B_1 &= A_1 C - A_1 C_1, \\
C_1 &= A_1 B_1 - A_1 B, \\
D_1 &= A_1 X_{11} + B_1 Y_{11} + C_1 Z_1.
\end{align*}
\]

(iii) Make the plane \( R_{i1} \) contain the axis \( Z_{11} \), and make it parallel to \( N_{11} \). Then, the equation for \( R_{i1} \) is:

\[
\begin{vmatrix}
 x - X_{11} & y - Y_{11} & z - Z_1 \\
 A_{11} & B_{11} & C_{11} \\
 A_{12} & B_{12} & C_{12}
\end{vmatrix} = 0
\]

Or

\[ A_{11} x + B_{11} y + C_{11} z = D_{11} \]  

in the equations

\[
\begin{align*}
A_{11} &= B_1 C_1 - B_1 C, \\
B_{11} &= A_1 C - A_1 C_1, \\
C_{11} &= A_1 B_1 - A_1 B, \\
D_{11} &= A_1 X_{11} + B_1 Y_{11} + C_1 Z_1.
\end{align*}
\]

The intersection line between the planes \( R_{i1} \) and \( R_{i2} \) are the mutually perpendicular lines \( Z_{i1} \) and the \( Z_{i2} \) axis because of the fact that the direction number \( n_{i1} \) of the line of intersection is equal to the vector area \( (N_{11} \times N_{12}) \) of the direction number of \( Z_{i1} \) and the \( Z_{i2} \) axis.

The various simultaneous equations (35)(46)(47) necessarily make it possible to solve for the intersection point of the mutually perpendicular lines of the \( Z_{i1} \) and \( Z_{i2} \) axes. The reason for this is that the intersection line of \( R_{i1} \) and \( R_{i2} \) is located on \( R_{i1} \) on the one hand, and, therefore, crosses the \( Z_{i1} \) axis, but, on the other hand is also located on the plane \( R_{i2} \), and, therefore, must necessarily also cross \( Z_{i2} \), as is shown in Fig. 7.

(iv) In solving for the intersection point \( E_{i1} \) of the mutually perpendicular lines and the \( Z_{i1} \) axis, take the equation for the axis line \( Z_{i1} \), and change it to be of the form of a parametric equation.

From equation (35) we get:

\[
\begin{align*}
x &= A_{i1} t + X_{i1}, \\
y &= B_{i1} t + Y_{i1}, \\
z &= C t + Z_1.
\end{align*}
\]
If we take equation (48) and substitute in equation (47), and, we also make \( t = K \), we then have:

\[
K_{tt} = (D_{tt} - A_{tt} X_{tt+1} - B_{tt} Y_{tt+1} - C_{tt} Z_{tt+1})/(A_{tt} A_{tt+1} + B_{tt} B_{tt+1} + C_{tt} C)
\]

If we take \( K_{tt} \) and make iterative substitutions into equation (48), we then obtain coordinates for the intersection point \( E_{tt+1} \) of the mutually perpendicular lines:

\[
E_{tt+1} = \begin{cases} 
  x_{tt+1} = A_{tt+1} K_{tt} + X_{tt+1} \\
  y_{tt+1} = B_{tt+1} K_{tt} + Y_{tt+1} \\
  z_{tt+1} = C_{tt+1} K_{tt} + Z_{tt+1}
\end{cases}
\]

(v) In solving for the intersection point \( E \) of the mutually perpendicular lines with the \( Z \) axis:

Change the straight line equation of the axis \( Z \) to a parametric form:

\[
\begin{align*}
  x &= A_{tt} t + X_{tt} \\
  y &= B_{tt} t + Y_{tt} \\
  z &= C t + Z_{tt+1}
\end{align*}
\]
If we take equation (51) and substitute in equation (46), and we then make $t = K_{n-1}$, we have:

$$K_n = \frac{D_{n-1} - A_{n-1}X_{n-1} - B_{n-1}Y_{n-1} - C_{n-1}Z}{A_{n-1}A_n + B_{n-1}B_n + C_{n-1}C_n}$$  \hspace{1cm} (52)

If we take $K_{n-1} = t$ and make iterative substitutions into equation (51), we get coordinates for the intersection point $E_t$ of the mutually perpendicular lines and the axis $Z_t$:

$$E_t \left\{ \begin{array}{l}
x_{E_t} = A_tK_{n-1} + X_{n-1} \\
y_{E_t} = B_tK_{n-1} + Y_{n-1} \\
z_{E_t} = C_tK_{n-1} + Z_{n-1}
\end{array} \right.$$  \hspace{1cm} (53)

It is easy to see that the points $E_{n-1}$ and $E_t$ are certainly not the same geometrical points on the axis line of the jet tube.

(vi) Coordinates for the center point $E_{n-1}$ of the mutually perpendicular lines $E_{n-1}E_t$

Coordinates for the point $E_{n-1}$

$$E_{n-1} \left\{ \begin{array}{l}
x_{E_{n-1}} = (x_{E_{n-1}} + x_E)/2 \\
y_{E_{n-1}} = (y_{E_{n-1}} + y_E)/2 \\
z_{E_{n-1}} = (z_{E_{n-1}} + z_E)/2
\end{array} \right.$$  \hspace{1cm} (54)

From equations (45) and (54), it is possible to obtain the plane of oscillation $\pi_{n-1}$ from instant $t_{n-1}$ to instant $t_n$ of the geometrical center lines of the jet tube. That is,

$$A_{n-1}(x-x_{E_{n-1}}) + B_{n-1}(y-y_{E_{n-1}}) + C_{n-1}(z-z_{E_{n-1}}) = 0$$

or

$$A_{n-1}x + B_{n-1}y + C_{n-1}z = D_{n-1}$$

in the equations $A_{n-1}, B_{n-1}, C_{n-1}$ are the same as before.

$$D_{n-1} = A_{n-1}x_{E_{n-1}} + B_{n-1}y_{E_{n-1}} + C_{n-1}z_{E_{n-1}}.$$
The system of simultaneous equations (33), (44), (55)

\[
\begin{align*}
A_{11}x + B_{11}y + C_{11}z &= D_{11} \\
A_{12}x + B_{12}y + C_{12}z &= D_{12} \\
A_{13}x + B_{13}y + C_{13}z &= D_{13}
\end{align*}
\]

makes it possible to solve for the instantaneous center of oscillation of the geometrical center of a jet tube \( C(x_{\text{osc}}, y_{\text{osc}}, z_{\text{osc}}) \). This is also none other than a point on the center of oscillation of the jet tube. It is possible for this set of equations to be solved through the use of a computer. Therefore, we recognize that \( x_{\text{osc}}, y_{\text{osc}}, z_{\text{osc}} \) are already known. Later, we will make direct use of them. Fig. 8 is an illustrative diagram of the instantaneous axis of rotation of the jet tube and the instantaneous oscillation center of its geometrical center line.

![Diagram of instantaneous axis of rotation and instantaneous oscillation center of jet tube]

Fig. 8: (1) The Instantaneous Axis of Rotation and the Instantaneous Oscillation Center of its Geometrical Center Line. (2) The Intersection Point of \( x_n \) and \( z_n \) is the Instantaneous Axis of Rotation of the Jet Tube.
3. Calculations of Moments of Force

The systems of moments of force which we are solving for here point, under differing designated wave forms, to a selected series of instants (or, one might say, angles of oscillation) to figure out the overall instantaneous moment of force. The focus of this article is on solving for the moment of force–angle of oscillation function relationship \( M = f(\theta) \). From this, it is possible to obtain the maximum and minimum (algebraic values) overall moment of force \( M_{\text{max}} \) and \( M_{\text{min}} \), as well as its moment of force–angle of oscillation function relationship. Besides this, it is also possible, through the designation of different wave forms (normal sine waves, sawtooth waves, and square waves) to obtain the curve of the functional relationship \( M = f(\theta) \). As far as the elaborate separating out of elastic moments of force, asymmetrical moments of force, and moments of friction, and so on, is concerned, this article makes no additional discussion.

After one solves for the instantaneous center of oscillation, the major contradiction in a solution for the instantaneous moment of force becomes the problem of the point of action and the direction of a system of forces in a specified space. We chose to make use of the two methods of Euler transformations and four dimensional numerical transforms, and the relative difference in values of force arms which we obtain in the solutions is not greater than 0.8%. Even so, we only introduce one method here.

(1) The Point of Action Used to Control the Direction and Amount of Thrust

From Fig. 1 one can see that the actuator tube movement points are respectively \( G \), and \( G \), the points of action for motive forces \( F \), and \( F \). Moreover, the points \( G \), and \( G \) are connected to the rigidity provided by the rod. Therefore, the problem becomes one of solving for the spatial coordinates of points \( G \), and \( G \).

(1) Selection of a dynamic coordinate system
(a) Translation coordinate system:
The coordinate axis which corresponds to the fixed coordinate system $QXYZ$ is always maintained parallel. The origin point $G$ is the base on the rod center line for point $G_i$ (or $G_j$).

Fig. 9 The Transformation of a Dynamic Coordinate System From $GX'Y'Z'$ to $GX''Y''Z''$

(b) Rotary Coordinate System:

$GX'Y'Z'$ ---- the angles of rotation corresponding to the coordinate system $GX'Y'Z'$ are $\phi, \delta, \psi$, and their rotational order is as presented below: (See Fig. 9)

First, we rotate through angle $\phi$ around axis $GX'$, obtaining coordinate system $GX,Y,Z$. We also rotate through angle $\delta$ around axis $GY$, obtaining the coordinate system $GX,Y,Z$. Finally, we rotate through angle $\psi$ around axis $GZ$, obtaining coordinate system $GX''Y''Z''$. The angles $\phi, \delta, \psi$ are called Euler angles.

(2) Euler Angles and Their Transformation

(a) Solving for Euler Angles

In the calculations of Part One, we already obtained the result that the angles of rotation around the various axes of the fixed coordinate system $QXYZ$ by the axis line of the jet tube are $\beta, a, \theta$. We must solve for the Euler angles which correspond to each instant.

From Fig. 9 it is possible to obtain the transform angular velocity projection relationship for the $GX'Y'Z'$ coordinate system.
From Table 1 we get the following series of relating equations:

\[ \omega_\phi = \frac{d\beta}{dt} = \frac{d\varphi}{dt} + \sin \delta \cdot \frac{d\psi}{dt} \]
\[ \omega_\delta = \frac{d\alpha}{dt} = \cos \varphi \cdot \frac{d\delta}{dt} - \cos \delta \sin \psi \cdot \frac{d\varphi}{dt} \]
\[ \omega_\psi = \frac{d\theta}{dt} = \sin \varphi \cdot \frac{d\delta}{dt} + \cos \delta \cdot \cos \varphi \cdot \frac{d\psi}{dt} \]

\[ \begin{align*}
\beta &= \varphi + \sin \delta \cdot \psi, \\
\alpha &= \cos \varphi \cdot \delta - \cos \delta \cdot \sin \varphi \cdot \psi, \\
\theta &= \sin \varphi \cdot \delta + \cos \delta \cdot \cos \varphi \cdot \psi.
\end{align*} \]

(56)

Coordinate transformations are only related to the results of rotational movements and do not consider the process of their rotational movements. If one desired to solve for the corresponding angles in a certain situation, it is possible to recognize that one always begins the rotational movement from zero degrees and simultaneously uses one of the method of freezing the coefficients, recognizing the use of approximately uniform rotational speed. In this way, equation (56) can be seen as becoming a set of differential equations with constant coefficients and a zero integration constant, for purposes of integration. Because of this, one has:

\[ \begin{align*}
\beta &= \varphi + \sin \delta \cdot \psi, \\
\alpha &= \cos \varphi \cdot \delta - \cos \delta \cdot \sin \varphi \cdot \psi, \\
\theta &= \sin \varphi \cdot \delta + \cos \delta \cdot \cos \varphi \cdot \psi.
\end{align*} \]

(57)

In order to facilitate the carrying out of iterative substitution calculations, we change these to the form shown below:
The initial value for iterative substitution can be taken to be: 

Finally, we solve for the Euler angles: \( \psi, \varphi, \delta \).

(b) Euler Transformation

The purpose of the transformation is to carry out a solution for moments in a fixed coordinate system. On the basis of the transformation order which is given in Fig. 9, we obtain the results shown below:

Transformation of \( GX'Y'Z' \rightarrow GX,Y,Z \):

\[
A_1 = \begin{pmatrix}
\cos \varphi & -\sin \varphi & 0 \\
\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{pmatrix}
\]  

Transformation of \( GX,Y,Z \rightarrow GX,Y,Z \):

\[
A_2 = \begin{pmatrix}
\cos \delta & 0 & \sin \delta \\
0 & 1 & 0 \\
-\sin \delta & 0 & \cos \delta
\end{pmatrix}
\]  

Transformation of \( GX,Y,Z \rightarrow GX'Y'Z' \):

\[
A_3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \varphi & -\sin \varphi \\
0 & \sin \varphi & \cos \varphi
\end{pmatrix}
\]  

Hence, the transformation matrix from \( GX'Y'Z' \) to \( GX,Y,Z \) is \( A \). Then, we have...
\[
A_i = \begin{bmatrix}
\cos \varphi \cos \delta_i & -\sin \varphi \cos \delta_i & 0 \\
\sin \varphi \cos \psi + \cos \varphi \sin \delta_i \cdot \sin \varphi_i & \cos \varphi \cdot \cos \psi_i - \sin \varphi \sin \delta_i \sin \varphi_i & 0 \\
\sin \varphi_i \cdot \sin \varphi_i - \cos \psi \sin \delta_i \cdot \cos \varphi_i & \cos \varphi_i \sin \varphi_i + \sin \varphi_i \cdot \sin \delta_i \cos \varphi_i & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sin \delta_i \\
-\cos \delta_i \sin \varphi_i \\
\cos \delta_i \cos \varphi_i \\
\end{bmatrix}
\]

The transformation of point \( G_i \) in coordinate system \( GX'Y'Z' \):

\[
\begin{bmatrix}
x'_{G_i} \\
y'_{G_i} \\
z'_{G_i}
\end{bmatrix} = A_i \begin{bmatrix} R_i \\
0 \\
0 \end{bmatrix}
\]

The transformation of point \( G_{ii} \) in coordinate system \( GX'Y'Z' \):

\[
\begin{bmatrix}
x'_{G_{ii}} \\
y'_{G_{ii}} \\
z'_{G_{ii}}
\end{bmatrix} = A_i \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix}
\]

(3) Transformation of moving rod points \( G_i \) and \( G_{ii} \) in a fixed coordinate system

As far as the translation operations of rigid bodies are concerned, the amount of displacement of each point must be equivalent. Now, in order to solve for the amount of displacement of the origin point \( G \) of the translation coordinate system:

\[
GO = H_i \quad \therefore \quad (x-x_i) + (y-y_i) + (z-z_i) = 0.
\]

75
Solution of the simultaneous equations (35) and (65) allows us to obtain coordinates for the point $G$:

$$
\begin{align*}
  z_c &= (-V_c - \sqrt{V_c^2 - 4U_cW_c} / 2U_c) \\
  x_c &= K_1z_c + Q_1 \\
  y_c &= L_1z_c + P_1
\end{align*}
$$

in the equations

$$
\begin{align*}
  U_c &= K_1 + L_1 + \sqrt{K_1^2 - 1} + 2[W_1(Q_1 - x_{c1}) + L_1(P_1 - y_{c1}) - z_{c1}] \\
  W_c &= (Q_1 - x_{c1})^2 + (P_1 - y_{c1})^2 + z_{c1}^2
\end{align*}
$$

By putting equation (63) and equation (64) together respectively with equation (65), it is possible to obtain the transformation of points $G_i$ and $G_{ii}$ in the fixed coordinate system:

$$
\begin{align*}
  x_{c1} &= x_{cii} + x_{c1} \\
  y_{c1} &= y_{cii} + y_{c1} \\
  z_{c1} &= z_{cii} + z_{c1}
\end{align*}
$$

The direction of the forces acting to control the amount and direction of forces acting:

- Acting duplicate the axis line of the activator tube.
- Coordinates for the dynamic fulcrum and the fixed fulcrum, as well as for their directional cosine:

$$
\begin{align*}
  A_i &= x_{eii} - R_0 G_{ii} = y_{eii} + C_{ii} = z_{eii} - H_{eii} \\
  F_i &= A_i = x_{eii} + B_{ii} = y_{eii} - R_0 G_{ii} = z_{eii} - H_{eii}
\end{align*}
$$
Length of activator tube:

\[ l_1 = \sqrt{A_i^2 + B_i^2 + C_i^2} \]
\[ l_2 = \sqrt{A_i^2 + B_i^2 + C_i^2} \]  \hspace{1cm} (70)

Directional cosine of forces acting:

\[ \cos a_{ii} = A_i / L_i \]
\[ \cos \beta_{ii} = B_i / L_i \]
\[ \cos \gamma_{ii} = C_i / L_i \]

Also

\[ \cos a_{ii} = A_i / L_i \]
\[ \cos \beta_{ii} = B_i / L_i \]
\[ \cos \gamma_{ii} = C_i / L_i \]  \hspace{1cm} (71)

(2) Projections of the forces acting on the various coordinate axes:

\[ F_{x_1} = F_{i_1} \cos a_{i_1} \]
\[ F_{y_1} = F_{i_1} \cos \beta_{i_1} \]
\[ F_{z_1} = F_{i_1} \cos \gamma_{i_1} \]

Also

\[ F_{x_1} = F_{i_1} \cos a_{i_1} \]
\[ F_{y_1} = F_{i_1} \cos \beta_{i_1} \]
\[ F_{z_1} = F_{i_1} \cos \gamma_{i_1} \]  \hspace{1cm} (72)

(3) Matrix Calculations

(1) Solve for the matrix of the instantaneous oscillation center of the geometrical center line of the jet tube

(a) Radius vector of the instantaneous oscillation center of the forces acting

\[ r_{ii} = \overrightarrow{CG_{ii}} \] and \[ r_{ii} = \overrightarrow{CG_{ii}} \] or

\[ r_{x_1} = (x_{c_1} - x_{c_1}) / 1000 \]
\[ r_{y_1} = (y_{c_1} - y_{c_1}) / 1000 \]
\[ r_{z_1} = (z_{c_1} - z_{c_1}) / 1000 \]

Also

\[ r_{x_1} = (x_{c_1} - x_{c_1}) / 1000 \]
\[ r_{y_1} = (y_{c_1} - y_{c_1}) / 1000 \]
\[ r_{z_1} = (z_{c_1} - z_{c_1}) / 1000 \]  \hspace{1cm} (73)

(b) Overall moment of force—Main moment and its relative moments

Projections of the main moment on the various coordinate axes:
\[ M_r = \sum_{i} r_{i} F_{i} \]
\[ M_r = \sum_{i} (r_{i} F_{i} - r_{i} F_{i}) = (r_{i} F_{i} + r_{i} F_{i}) \]
\[ M_r = \sum_{i} r_{i} F_{i} - \sum_{i} r_{i} F_{i} = r_{i} F_{i} \]

\[ M_r = \sum_{i} r_{i} F_{i} - \sum_{i} r_{i} F_{i} = r_{i} F_{i} \]

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\[ M_r = \sum_{i} r_{i} F_{i} - \sum_{i} r_{i} F_{i} = r_{i} F_{i} \]

\[ \mu - \theta - \phi - \theta_{\mu} \]

\[ A_{\mu} = B_{\mu}, C_{\mu} = C_{\mu}, B_{\mu} = B_{\mu}, A_{\mu} = A_{\mu}, C_{\mu} = C_{\mu}, B_{\mu} = B_{\mu} - B_{\mu} A_{\mu} \]

\[ \text{Const.} = \frac{M_{\mu} A_{\mu} + M_{\mu} B_{\mu} + M_{\mu} C_{\mu}}{\sqrt{M_{\mu}^2 + M_{\mu}^2 + M_{\mu}^2}} \]
From equations (73) and (74), we can then solve for the moment of force - $M_x$. of the instantaneous axis of rotation

$$M_x = |M_x| \cos \theta_0.$$  \hspace{1cm} (73)

The corresponding specific amount of force is:

$$M_{x,s} = |M_x|/v.$$  \hspace{1cm} (74)

2. The Instantaneous Axis of Rotation

We have the instantaneous axis of rotation $x$ originating at the center which we already solved for $x_{n-1}, y_{n-1}$ and $A_{n-1}, B_{n-1}, C_{n-1}$, it is then possible to solve for the equation of the instantaneous axis of rotation:

$$\frac{x-x_{n-1}}{A_{n-1}} = \frac{y-y_{n-1}}{B_{n-1}} = \frac{z-z_{n-1}}{C_{n-1}}.$$  \hspace{1cm} (75)

An attempt is made through the horizontal projection diagram of the instantaneous axis of rotation which appears in the $X-Y$ diagram, by finding the center of oscillation of the $x$ and $y$, we obtain its projection equation:

$$y = K_{x,y} x + b,$$  \hspace{1cm} (76)

$$K_{x,y} = -B_{n-1}/A_{n-1}, b = y_{n-1} - K_{x,y} x_{n-1}.$$  \hspace{1cm} (76a)

If the original center line of jet tube is $\psi_x$ then:

$$\psi_x = -B_{n-1}/A_{n-1}.$$  \hspace{1cm} (76b)

$\psi_x$ is an angle of rotation for a range of 0-360 degrees. Then it is possible to precisely determine the size of $\psi_x$. 

79
(3) Calculation of Eccentric Thrust

Jet tubes, due to the effects of additional designated oscillations and contained pressures, are capable of causing jet tube axis lines to give rise to radial displacement, creating thrust eccentricity. This is an important technical target of flexible connector heads.

In initial configuration, the jet tube axis line equation is:

\[ \frac{x-x_0}{A} = \frac{y-y_0}{B} = \frac{z-z_0}{C} \]  

in the equations:

- \( X_0, Y_0, Z_0 \) --- are the coordinates of a certain point on the \( OZ \) axis.
- \( A, B, C \) --- is the directional number of the \( OZ \) axis, which is always, for \( X_0 = Y_0 = 0, A_0 = B_0 = 0, \) \( Z_0 \) equal to certain constants. If we let the thrust eccentricity be \( D \), and we solve equations (35) and (34), we then have:

\[ D = \frac{|x-x_0, y-y_0, z-z_0|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|Y_1A_1 - X_1B_1|}{\sqrt{B^2 + A^2}} \]  

(35)

(4) Angular Velocity of Jet Tube Oscillation

By the sine function we already obtained for the angles of rotation around the various axes:

\[ \beta_t = f(K \cdot \Delta T), a_t = f(K \cdot \Delta T), \theta = f(K \cdot \Delta T) \quad K = 0, 1, \ldots n \]

\( \Delta T \) --- equivalent sampling intervals.

\[ \omega_t = (\beta_t - \beta_{t-1}) / \Delta T \]
\[ \omega_t = (\theta_t - \theta_{t-1}) / \Delta T \]  

\[ \omega_t = (a_t - a_{t-1}) / \Delta T \]  

(36)
The overall angular velocity \( \omega \) is:

\[
\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}
\]  

(87)

5) Stroke of activator tubes

In the third section, we already solved for the instantaneous lengths of activator tubes \( L_x \) and \( L_y \) with the zero position length of the activator tube as \( L_0 \). From this, the stroke of the activator tubes is:

\[
\begin{align*}
\Delta L_x &= L_x - L_0 \\
\Delta L_y &= L_y - L_0
\end{align*}
\]  

(88)

5) Spatial displacements caused by the effects of axial displacement compensation and contained jet tube pressures

This compensation is automatically carried out when the command signal is zero. After that, one carries out measurements and calculations of its spatial displacement. This is also another important technical parameter of flexible connector heads.

With jet tubes in their initial configuration (pressure zero, command zero) the electrical zero value of the mechanical apparatus and the sensors should satisfy:

\[
X_n - X_i \Rightarrow 0; Y_n - Y_i \Rightarrow 0; Z_n = 0
\]

After carrying out compensation for the effects of contained pressure, this should satisfy:

\[
|X_n - X_i| \leq 0.1; |Y_n - Y_i| \leq 0.1
\]

We then have:

Axial displacement:

\[
\Delta Z = Z_n - Z_i
\]

Radial displacement:

\[
\begin{align*}
\Delta X &= X_n - X_i \\
\Delta Y &= Y_n - Y_i
\end{align*}
\]  

(89)
Note: Units in all the equations and calculations are as shown below:

Length: The radial dimension use meters as a unit. All other units are mm.

Angles and angular velocities: these respectively use degrees and degrees per second as units.

Force and pressure: these respectively use kilograms and kilograms per square cm as units.

Moments of force: kilograms/meter.

Time: seconds.

REFERENCES


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