OPTIMIZATION OF THE ANTENNA PATTERN OF

A PHASED ARRAY WITH FAILED ELEMENTS

THESIS

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Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

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Bernard F. Collins II, B.S.

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Preface

The purpose of this study was to find some method of
optimizing the far-field pattern of a phased array with
failed elments. The topic was generated by Col. Larry
Sanborn, while he was at Space Division, who was concerned
with the survivability of a phased array antenna aboard a
satellite. The problem was further defined by Mr. John
McNamara of the Space Based Radar Branch, Rome Air
Development Center, Rome, New York.

At the beginning of the study, I hoped that any
optimization method found would be easily implemented, that
a black box could be built which would have as its imput the
coordimates of the failed element, and as its output the
new, optimized array current amplitude and phase
distribtion. The result of this quest is called the direct
method. However, the direct method is not yet ready to be
molded into a black box, for the study done was largely
theoretical. I believe the direct method has strong
potential and sl ould be modified for a practical array.

I would like to thank Dr. Andrew Terzuoli for patiently
answering my questions even when he was incredibly busy
(always), for helping me find either the information I
needed to solve a problem or the experts who might already
have the answer, and for his personal dedication; Lt Col
Zdzislaw Lewantowicz for teaching me all about gradien

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Abstract

Using a phased array antenna on a system can add dimensions of flexibility and control. By varying the phase and amplitude of the current exciting each element, the far field antenna pattern can be modified in such a way as to obtain a desired, optimal pattern. The purpose of this thesis is to develop a rapid compensation method for any element failures by varying the currents exciting the remaining elements. Although many methods for optimizing the pattern of a full array are available, this work is unique because it develops methods of finding the optimum pattern of an array which contains failed or missing elements. Optimization methods in the literature which were investigated include thinned array analysis, adaptive array analysis, eigenvalue methods, null displacement techniques, and assorted iterative techniques. Another method discussed in this thesis, the direct technique, originated from the need for a quick optimization method, especially when very large arrays are used.

Of the optimization methods in the literature, only the eigenvalue method and the gradient search iterative technique were successfully applied to this problem. The

direct method, a spin off from the eigenvalue method and the gradient search technique, has been shown to be relatively quick and accurate.

Recommendations for future study include the functional derivation describing the direct optimization method and including more practical aspects of phased arrays in the propagation model.

OPTIMIZATION OF THE ANTENNA PATTERN OF
A PHASED ARRAY WITH FAILED ELEMENTS

I. INTRODUCTION

Electrically steered phased array antennas are being
used for applications requiring a feasible, lightweight,
high gain antenna. The phased array antenna can scan its
main beam, reduce its sidelobes and improve its signal to
noise ratio by varying the amplitude and phase of the
current exciting each of the array's elements. These
capabilities make the phased array antenna ideal for use on
aircraft, ships, ground sites, and even satellites. A
phased array on a satellite however, has an unique problem
associated with it. Once the satellite is space-borne, the
phased array antenna is not maintainable. If for some
reason an element of the phased array no longer performs its
designed function, the antenna's far field pattern generally
degrades, i.e., the gain of the antenna decreases, the width
of the main beam widens, and the power of the peak sidelobes
increases. These degradations affect the ability of the
phased array antenna to perform its task, which directly
affects the usefulness of the dependent system. This study
develops a rapid compensation method for any element
failure, (or optimizes the far field pattern of an antenna

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with failed elements), thereby extending the effective usefulness of the phased array antenna and its dependent system.

Background

Designing a phased array antenna for an optimum pattern is an old problem. Techniques from thinne array analysis [11,17,18], adaptive array analysis [1,3,18:253-259], eigenvalue methods [2,15,16], null displacement analysis [4,7], and assorted iterative techniques [10,13,14] have been developed and used to design phased array antennas with patterns optimized according to some desired criteria. The variables for the design of these arrays include the position of each element and the amplitude and phase of the current exciting each element.

Optimizing the pattern of a phased array antenna with failed elements is a different problem. The study at hand takes an existing phased array antenna (possibly designed using one of the aforementioned techniques) which contains failed elements and compensates for the failures (re-optimizes) by varying the amplitude and phase of the current exciting each of the working elements. The position of each element is not a variable in this case, however the solution does depend on the position of the failed element.

Assumptions

The problem can be further defined by stating the

assumptions made:

A. The antenna is erected in a perfect environment, with no other bodies to distort the pattern or cause multipath distortions and no other forces, such as cosmic rays, to distort the current on the elements.

B. Pattern degradation will be caused by failed elements only. The elements will fail completely – there will not be any stuck phase shifters or partially working amplifiers.

C. When the elements fail, the mutual coupling will be as if the element never existed (i.e. matched load vs. parasitic load).

D. The elements will fail in clusters (due to space debris crashing into the array), rows (due to power supply or computer falure), or randomly (due to long-term individual element failures).

E. The exact position of the failed element is known. This assumption undoubtably pushes the limits of practicality, however, work is being done to make this assumption more realistic [6].

F. The phased array will be mounted on a satellite which will be at an altitude which requires a +/- 0.26 radians (+/- 15°) scan to cover the area of interest. The area of interest will be called the field of view (FOV). This assumption will ease the requirement of controlling all the sidelobes and also allow an interelement spacing of 0.7 of the wavelength, thereby reducing the effects of mutual coupling.

G. Although the work done should be extendable to a planar array, this thesis applies the theories to linear arrays only.

H. The pattern is defined to be optimized if the average power in the sidelobes of interest (within the FOV) is minimized and the power in the mainlobe is maximized (Fig 1).

Approach

Once the problem is defined, the steps to solving this problem are first to search the literature for work which

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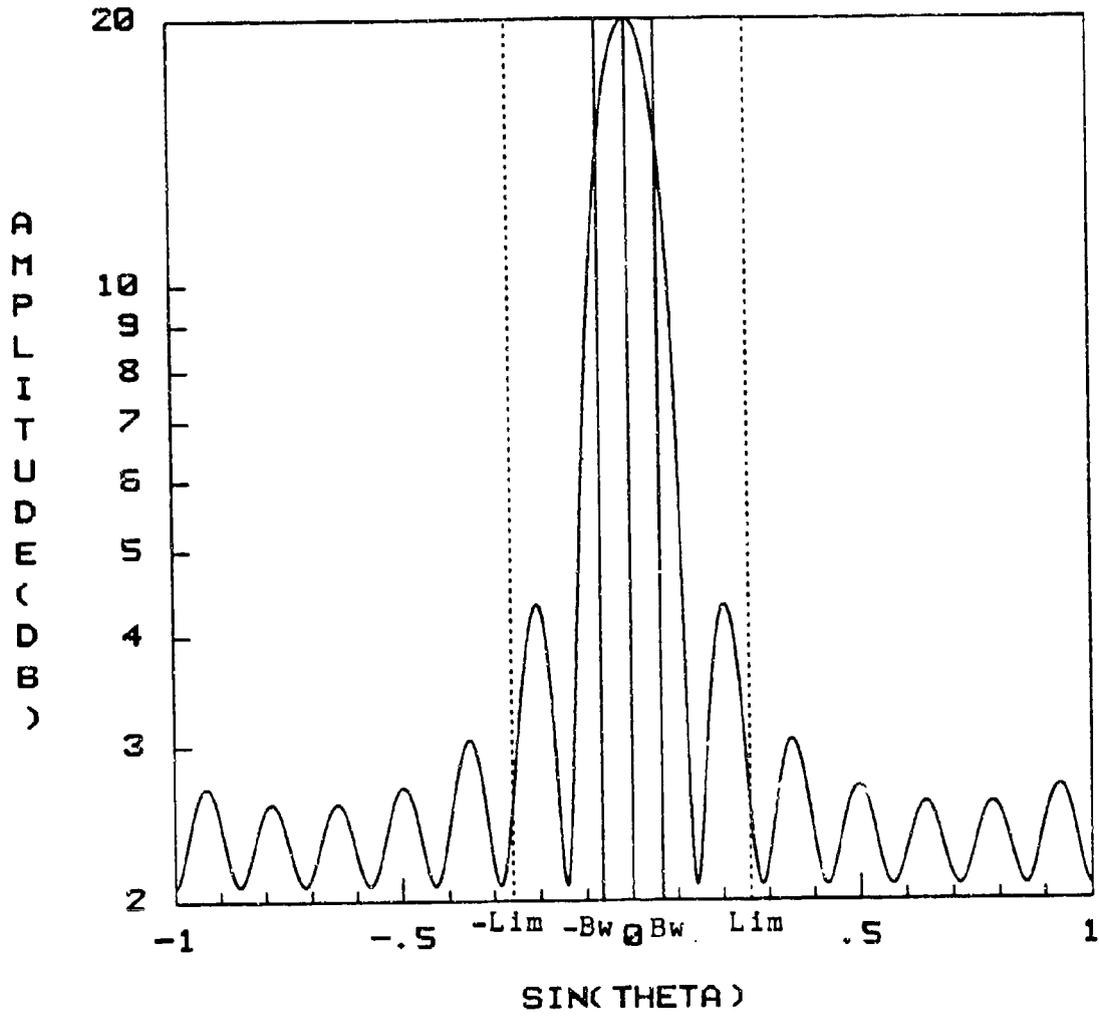
Fig 1.   Antenna Pattern Showing Field of View

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may be applicable to the problem, then to modify and apply the work to the problem and finally to determine whether or not the results are satisfactory.

Literature Search. Work which has been done and can be related to this study was found through database searches (DTIC, Analog), related indicies (Engineering Index, Science Citation Index), prominent journals (IEEE Antennas and Propagation), telephone conversations with experienced experts and meetings.

Apply Knowledge. The knowledge gained from the current literature was modified and applied to the subject problem. To begin with, the array was described as a simple linear array. If the technique under question can be applied to the linear array, the technique could then be modified and applied to a planar array. Much of the analysis was done on a VAX 11/780 using the $MATRIX_x$ software package and other ADA software packages written specifically for this study.

Survey Results. The results were analyzed to determine which technique best satisfied the objectives of the study.

Order of Presentation

This thesis is not presented in the traditional way. It does contain an introductory section, a developing section, and a concluding section, but the developing section is different from the usual. A normal developing section of a theoretical thesis has a chapter on existing

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theory, a chapter on new theory, and a chapter on application. Because the problem of optimizing the antenna pattern of a phased array with failed elements was attacked by surveying several theories, the developing section of this thesis consists of one chapter divided by the theories, yet each theory is further divided by a description, application, and results subsection.

II. Basic Array Analysis Techniques

A phased array antenna has many advantages over the more conventional reflector antenna, the most significant of which is the array's ability to modify its far field pattern by changing the complex weights of the currents exciting each of the elements. This chapter briefly describes the relations between the current amplitude distribution $[W(x)]$ and the far field pattern $[P(\theta)]$, specifically the Fourier transform and the z transform.

The far field pattern of a phased array antenna is the sum of the far field pattern of each of its elements. The phase of the signal received by an element is dependent on the distance between the elements and a reference element and the angle from which the signal is coming. For instance, Figure 2 shows a three element, linear array. The amplitude distribution is:

$$w_n = W(x) \; \delta(x - d_n) \qquad -1 <= n <= 1,$$

and the array's far field pattern is:

$$P(\theta) = P_{-1}w_{-1}\exp(jkd_{-1}\sin\theta) + P_0 w_0$$
$$+ P_1 w_1 \exp(jkd_1 \sin\theta)$$

$$= \sum_{n=-1}^{1} P_n w_n \exp(jkd_n \sin\theta) \qquad (1)$$
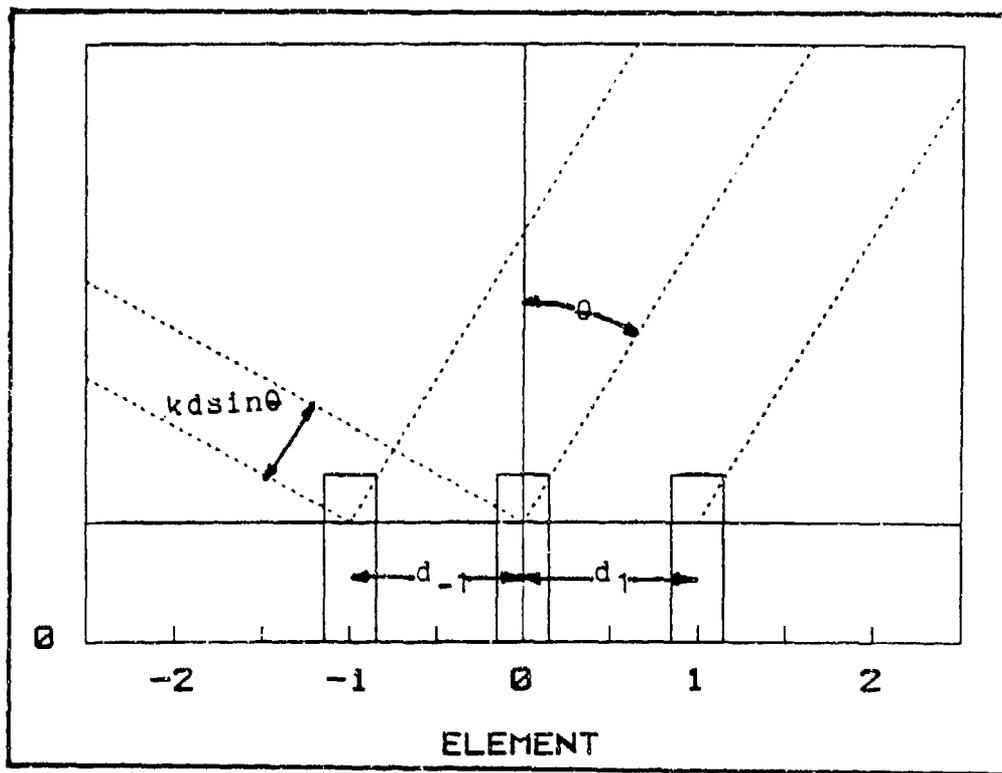
Fig 2. Three Element Linear Array

where

$$w_n = a_n \exp(j\emptyset_n)$$

is the amplitude and phase of the current exciting the nth element (w_n is also known as the complex weight), $kd_n \sin\theta$ is the relative phase shift (in radians) experienced by the signal at the nth element, and P_n is the far field pattern of the nth element. The constant, k ($=2\pi/\lambda$), is known as the wave number and converts distance in unit length to phase angle in radians. Equation 1 is valid regardless of the inter-element spacing. However, in most array applications, the elements are spaced by an equal amount

$(d_n = nd)$ – creating a periodic array. The remainder of this chapter will be concerned with periodic arrays with isotropic radiators $(P_n = 1)$ as elements.

Fourier Transform

For a periodic array with isotropic radiators as elements, equation 1 becomes:

$$P(u) = \sum_{n=-1}^{1} w_n \exp(jkdnu) \qquad (2)$$

where $u = \sin(\Theta)$ and d is the constant inter-element separation. Equation 2 is recognized as the Fourier series, therefore $W(x)$ and $P(u)$ can be described as a Fourier transform pair. For example, if $W(x)$ is uniformily distributed $(w_n = w)$ over a three element, periodic array, then:

$$
\begin{aligned}
P(u) &= \int_{x=-\infty}^{\infty} W(x) \exp(jkxu)\, dx \\
&= \int_{x=-\infty}^{\infty} w\, \text{rect}(x/Nd) \sum_{m=-\infty}^{\infty} \delta(x-nd) \exp(jkxu)\, dx \\
&= wNd\, Sa(pNdu/\lambda) * 1/d \sum_{m=-\infty}^{\infty} \delta(u-m\lambda/d) \\
&= wN \sum_{m=-\infty}^{\infty} Sa[(pNd/\lambda)(u-n\lambda/d)] \qquad (3)
\end{aligned}
$$

where

$$Sa(x) = \sin(x)/x.$$

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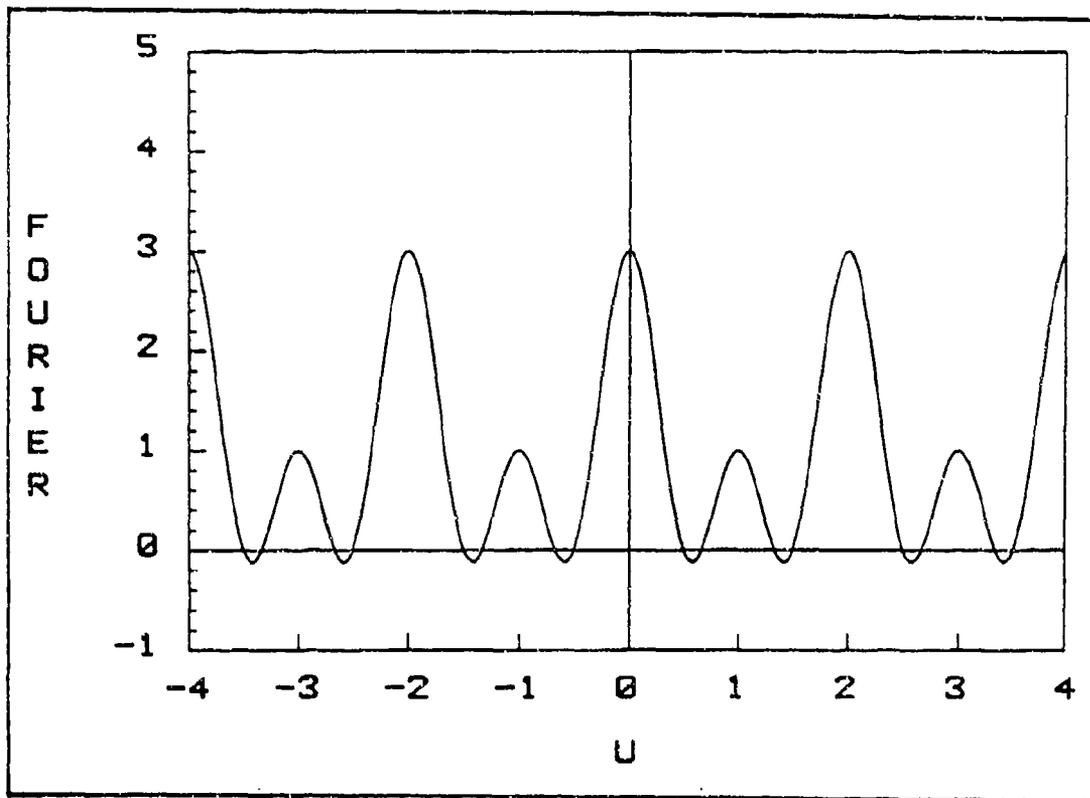
Fig 3.   Fourier Transform of Three Element Linear Array

Figure 3 illustrates the far field pattern which results
from equation 3. Note that the major lobes have a width of
λ/Nd and are separated by λ/d. The major lobe in the
visible range $(-1 < u < 1)$ is called the main lobe and the
remaining major lobes are called grating lobes. To scan the
pattern to a given angle, $u_o = \sin\theta_o$, all that is required
is to add kd_nu_o to the phase of each element:

$$P(u-u_o) = \sum_{n=-1}^{1} a_n \exp[j(\emptyset_n + kdnu_o)]\exp(jkdnu).$$

A grating lobe will move into the visible range if the

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inter-element separation is greater than one wavelength or
greater than one-half wavelength with a scan of $u_o = \pm 1$.

Z Transform

The periodic array can also be analyzed with the z
transform. Equation 2 can be written as:

$$P(u) = \sum_{n=-1}^{1} w_n \exp(jkdu)^n$$

or letting

$$z = \exp(jkdu),$$

$$P(z) = \sum_{n=-1}^{1} w_n z^n \tag{4}$$

$$= \prod_{m=1}^{2} (z - z_m)$$

where z_m are the N-1 roots or zeros of equation 4. The
zeros can be plotted on a z-plot, a graph with its origin at
(0, 0j) and with an abscissa of real numbers and an ordinate
of imaginary numbers. For a periodic, three element linear
array with uniform distribution, the far field pattern can
be described by:

$$P(z) = (z - 0.5 + 0.866j)(z - 0.5 - 0.866j).$$

The resulting z-plot is shown in Figure 4, where the circle
shown is the unit circle. The far field radiation pattern
can be found by first transforming the argument of z to the

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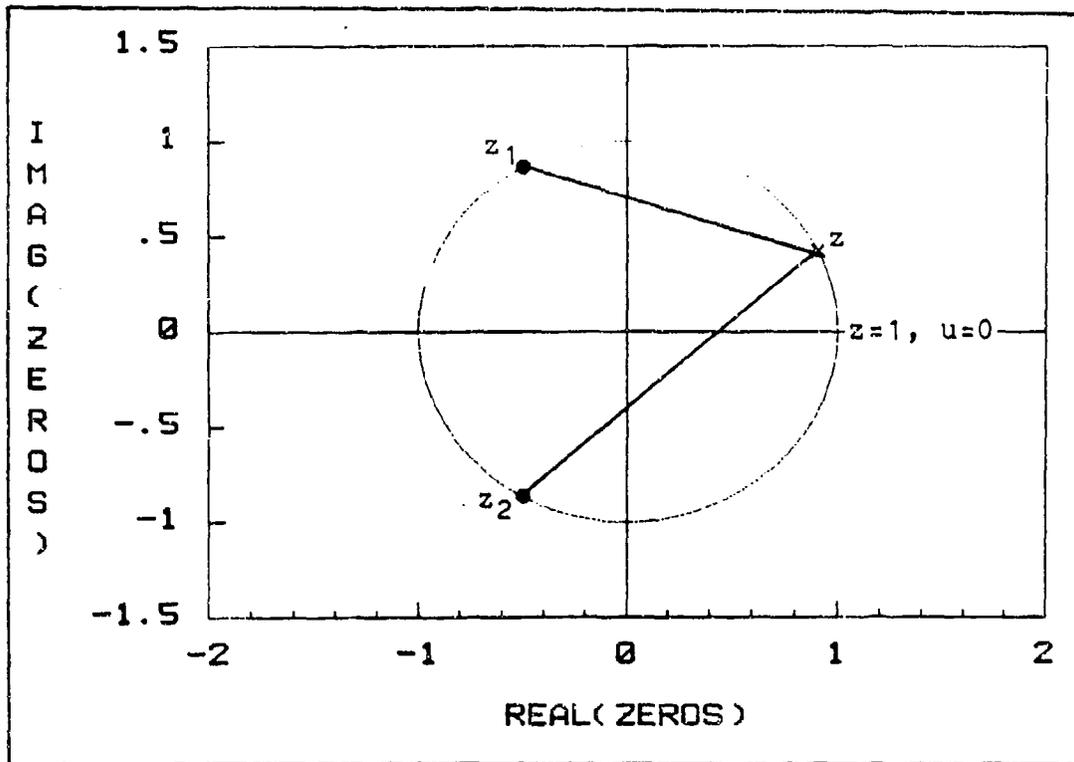
Fig 4. Z-plot of Three Element Linear Array

u axis:

$$u = \arg(z)/kd,$$

and then by calculating the magnitude of the normalized
radiation pattern as the product of the distance from the
point on the unit circle corresponding to $\arg(z)$ to each
root, normalized by the value of the product of the
distances from the point corresponding to $\arg(0)$ to each
root:

$$|P(z)| = \frac{|z-z_1| \; |z-z_2|}{|1-z_1| \; |1-z_2|}$$

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The visible range is $-kd < \arg(z) < kd$. If the pattern is scanned, z is defined as:

$$z = \exp[jkd(u-u_o)].$$

If an element of a periodic array has a failed element, the foregoing Fourier transform and z transform still hold, the difference is the amplitude distribution. Figure 5 shows the amplitude distribution, far field pattern and z-plot which result if the center element of a three element, periodic array is failed. For this simple example, when a failure occurs the power in the sidelobes increases to the level of the power in the main beam. Typically, the power in the sidelobes do increase, but not as dramatically.

ELEMENT

(a)
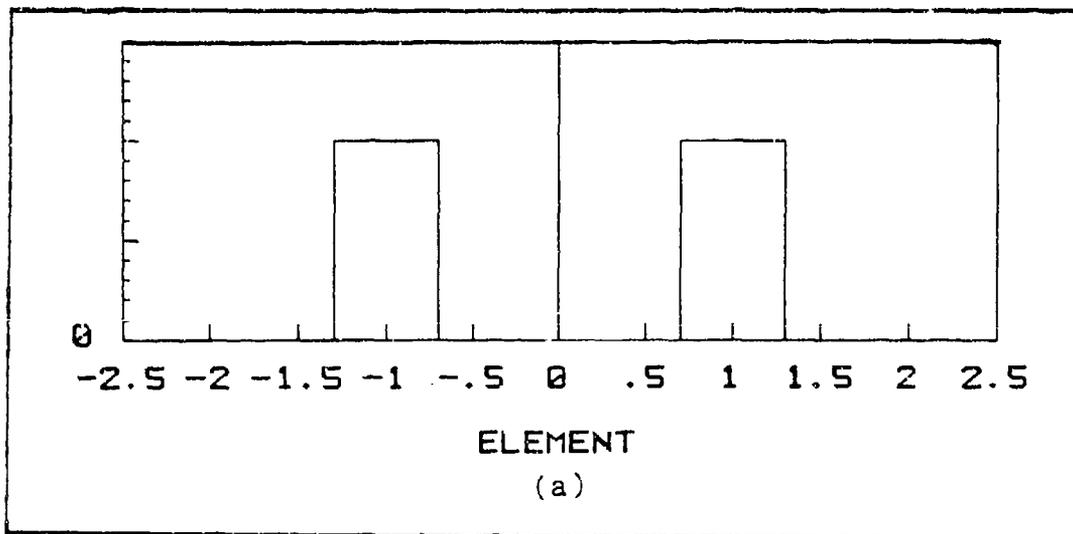
Fig 5. Three Element Linear Array with Center Element Failed: a) Amplitude Distribution
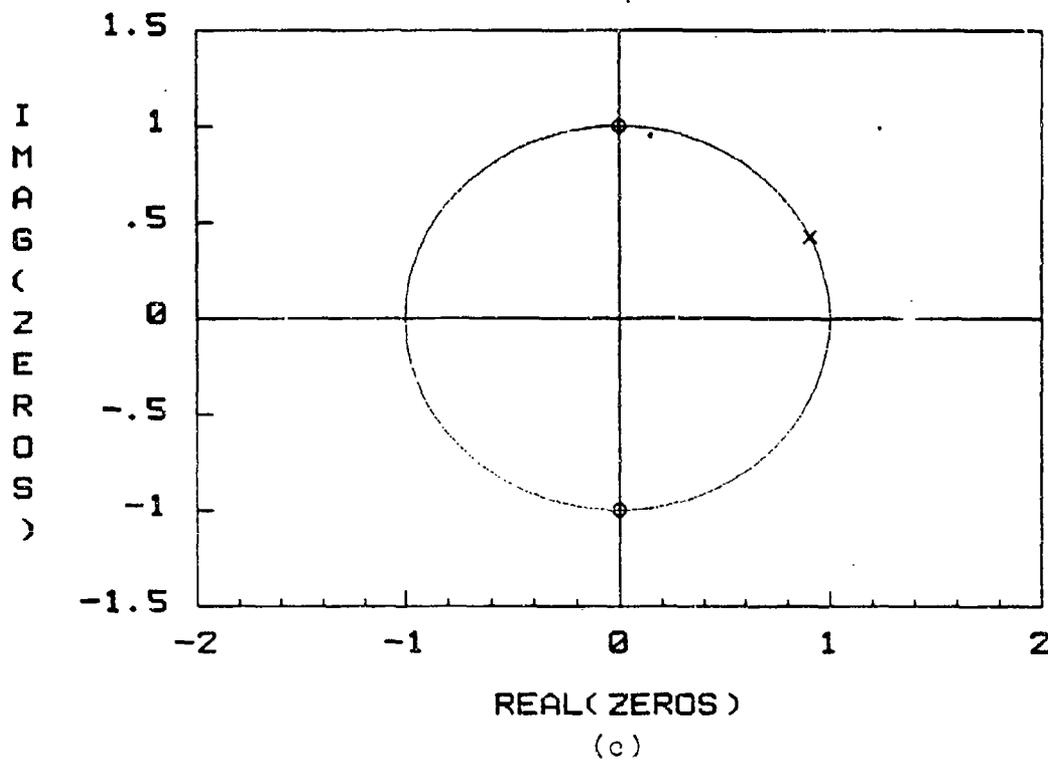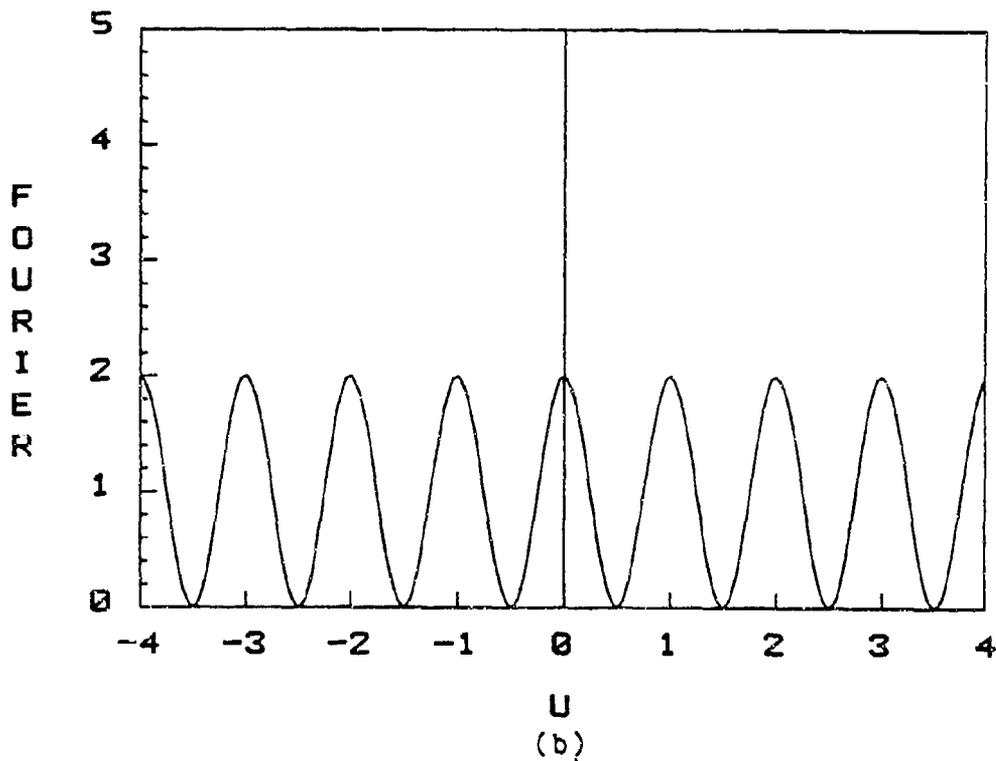
Fig 5 cont. Three Element Linear Array with Center Element
Failed: b) Fourier Transform, c) Z-plot

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III. Optimization Techniques

This section describes, in a more or less chronological order, the work analyzed and applied in order to complete this study. The theory behind each technique is first described in general, and then modified and applied to an array with failed elements, and finally the results of applying the theory are stated.

Thinned Array Analysis

A filled array is a periodic array which has its elements located at all integral oositions of a given inter-element separation over the entire aperture. Thinned array analysis is a method of strategically removing and repositioning elements of a filled array such that the far field pattern remains satisfactory. The work done in this area was investigated because an array with failed elements can be viewed as an accidentally thinned array.

Description. Thinned arrays are usually created using either a deterministic, statistic, or random method. One way to build a deterministically thinned array is to choose a continuous current distribution function which corresponds to the desired far field pattern and then sample the function with the given number o elements in such a way as to minimize the quantization error [17:212]. For small numbers of elements and linear arrays, this is a fairly simple process. For large number of elements or planar

arrays, it is more common to use a statistical method. Furthermore, it has been shown that the arrays designed using deterministic methods exhibit average properties similar in character and level to arrays designed using statistical methods [18:132]. A statistically thinned array, like the deterministically thinned array, is also designed by first choosing the appropriate current distribution function, however, the elements are then positioned according to the probabilities of the normalized current distribution function [17:219]. For instance, the peak of the amplitude distribution, which is usually in the center of the aperture, would have a probability of one. Therefore, an element would be placed in the center of the array. The probability that an element would be placed on the edge of the tapered distribution would be smaller. Statistically thinned arrays are sometimes equated to randomly thinned arrays [18:140]. However, for this paper, the difference must be distinct. A randomly thinned array is an antenna created by randomly removing elements, either intentionally or accidentally (see assumptions), from a filled array.

It is difficult to quantify the effects that thinning an array has on the pattern for the deterministic method. The other methods can be described with statistics, therefore the average pattern can be described. Thinning an array decreases the peak power of the main beam (assuming

additional current is not added to the remaining elements)
and increases the floor of the sidelobes. In other words,
the average power pattern can be modeled as:

$$|P(u)|^2 = |P_0(u)|^2 (1-1/N) + 1/N$$

where $P_0(u)$ is the desired array factor and N is the number
of elements in the statistically thinned array. The first
term on the right hand side of the equation shows the
average reduction in power and the second term shows the
average increase in the floor of the sidelobes. Further
analysis shows that the probability density function of the
power of the sidelobes reduces to a Rayleigh distribution:

$$p[N|P(u)|] = 2 |P(u)| \exp(-N |P(u)|^2)$$

where $N|P(u)|$ is the unnormalized amplitude of the complex
radiation pattern of an N element array.

Application. To illustrate the effects of thinning an
array, three patterns are produced from three arrays
designed with different amplitude and spatial tapers. For
comparison purposes, all the patterns are normalized. The
first array (Fig 6) is a uniformly distributed, filled,
linear array with 19 elements. The second array (Fig 7) is
similar to the first array except that the amplitude
distribution is triangular. The final array (Fig 8) is a
thinned array made of 15 equal amplitude elements spatially
tapered such that each element corresponds to equal areas

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Fig 6. Uniformily Distributed, Filled, Linear Array: a) Amplitude Distribution; b) Far Field Radiation Pattern
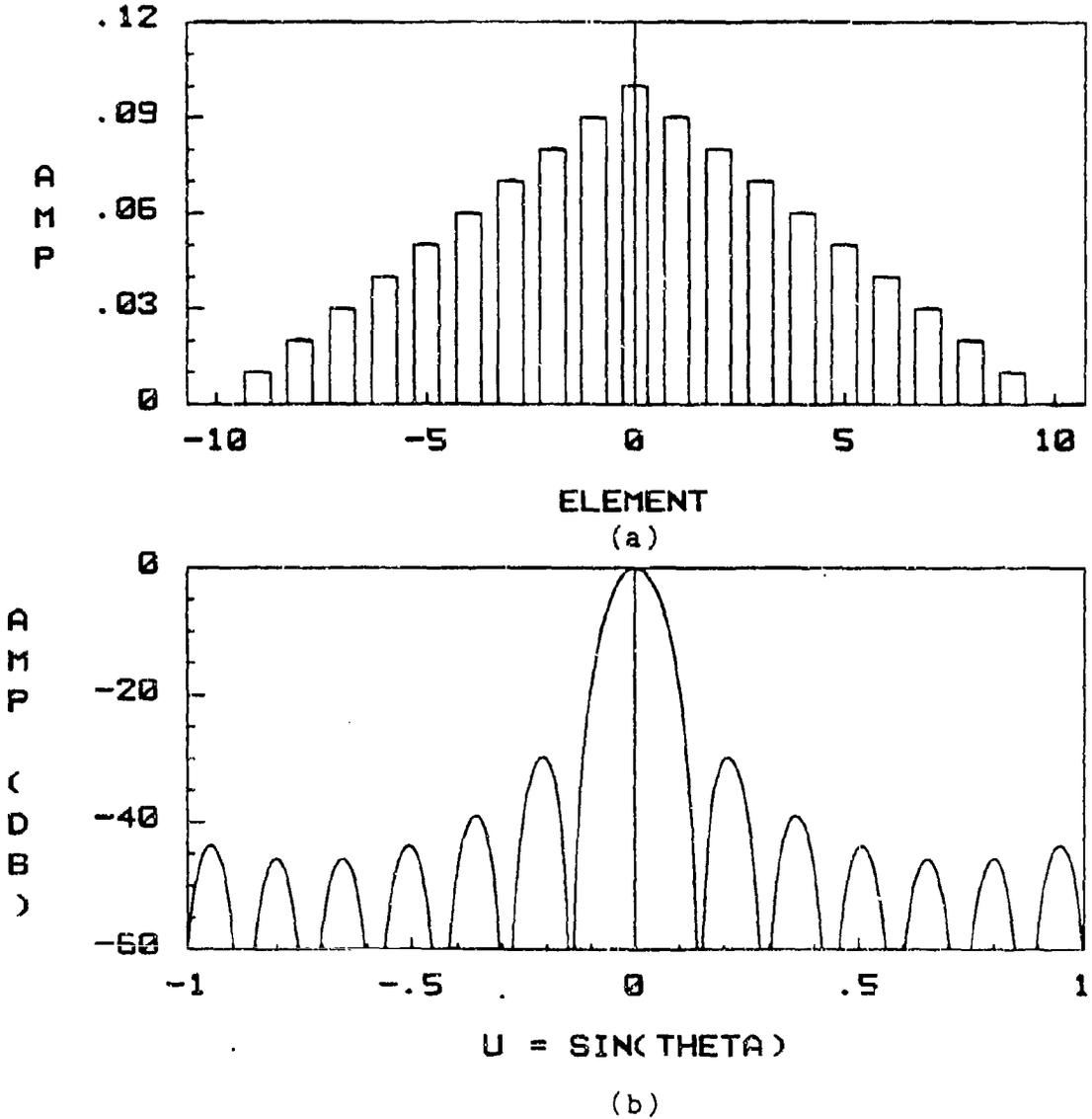
Fig 7. Triangular Amplitude Taper Distribution, Filled,
Linear Array: a) Amplitude Distribution, b) Far Field
Radiation Pattern

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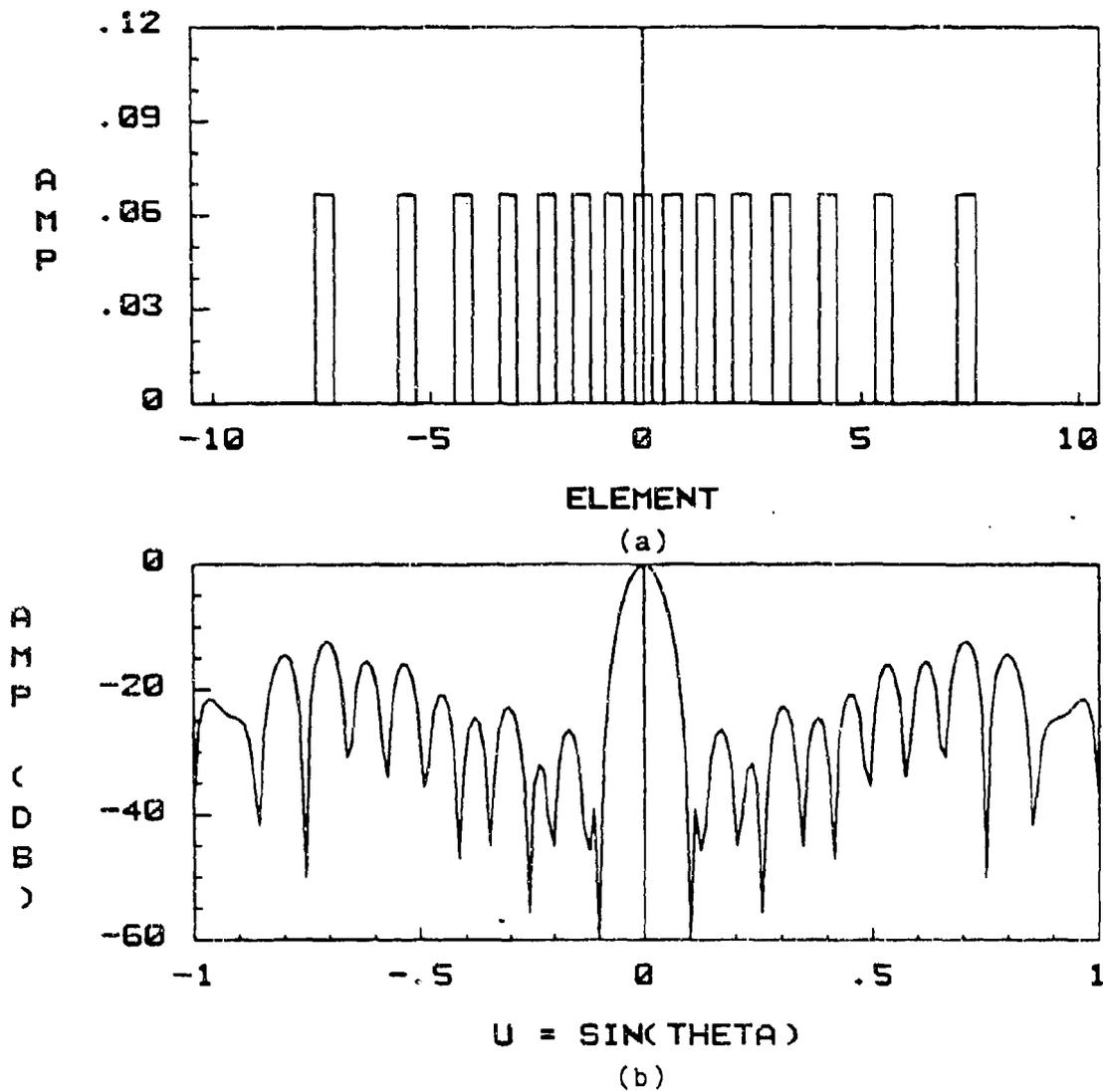
Fig 8.   Triangular Spatial Taper Distribution, Thinned,
Linear Array: a) Amplitude Distribution, b) Far Field
Radiation Pattern

under the triangular distribution. Note that there is a
practical limit to the positions the elements can be
placed. For instance, mutual coupling affects and actual
element size dictate the minimum separation.

Results. At first, it was hoped that a pattern similar
to Figure 8, a pattern with low sidelobes in the FOV, could
be obtained using thinned array analysis. However, thinned
array analysis is used to design phased array antennas and
could not be adapted to determine the necessary compensation
for any element failures. Thinned array analysis can be
used to determine where elements should be placed in order
to obtain a desired pattern, but in the given problem, the
position of the elements failed is not a variable.

The statistics developed to analyze the patterns of
thinned arrays are useful in determining what the pattern
would look like after element failures occur, but the goal
of this study is to improve the pattern.

Adaptive Array Analysis

Adaptive array anaylsis [18:253-269] encompasses the
methods of actively optimizing the pattern of a phased array
antenna based on information processed from the received
signal. Usually, in this case, a pattern will be optimized
if the signal to noise ratio is maximized. Therefore,
adaptive arrays are specially concerned with nulling out
jammers and other sources of unwanted signals in the

environment. Adaptive arrays also have the capability to compensate for normal variations in an element's position and complex current weight, which is the reason for investigating this technique.

Description. Two of the most notable methods for building adaptive arrays are the nulling tree method and the least mean square error (LMS) method.

An adaptive array with nulling tree hardware (Fig 9) has the ability to null out any type of signal, if the direction of the interference is known. To create and scan a null, the control hardware modifies the complex weights of one row of the nulling hardware. All the weights in the row are the same and each row creates only one null. The complex radiation pattern $(P(u))$ is the product of the original pattern $(P_0(u))$ and the patterns created by the nulling hardware $(P_1(u), \ldots, P_K(u))$ or

$$P(u) = (P_0(u)) (P_1(u)) \cdots (P_K(u))$$

where

$$P_0(u) = \sum_{m=1}^{N} w_m \exp(jkd_m u),$$

$$P_K(u) = \exp(jkdu) - \exp(jkdu_K),$$

$N =$ the number of elements,

$K =$ the number of rows of nulling hardware,

w_m = the complex weight of the element,

k = the wave number $(=2\pi/\lambda)$,

d_m = the distance from the center to the element,

$u = \sin(\Theta)$,

and

u_1 = the desired direction of the null.
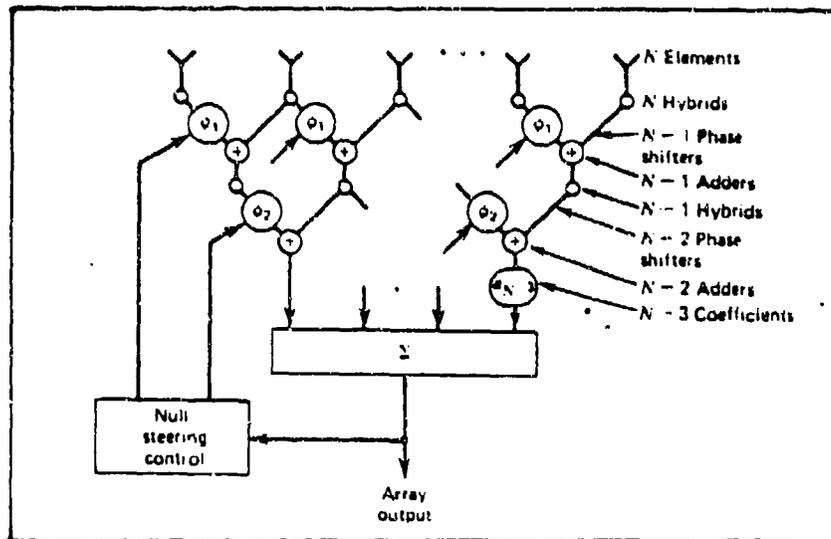
Fig 9. Adaptive Array with Nulling Tree Hardware [18:259]

As stated earlier, the designer must know, or must determine, where to put the null and must track the source of noise with the null in order to remain effective.

Unlike the nulling tree adaptive array, the adaptive array with the LMS hardware (Fig 10) can be used to null out unwanted signals regardless of their origin, but only if the
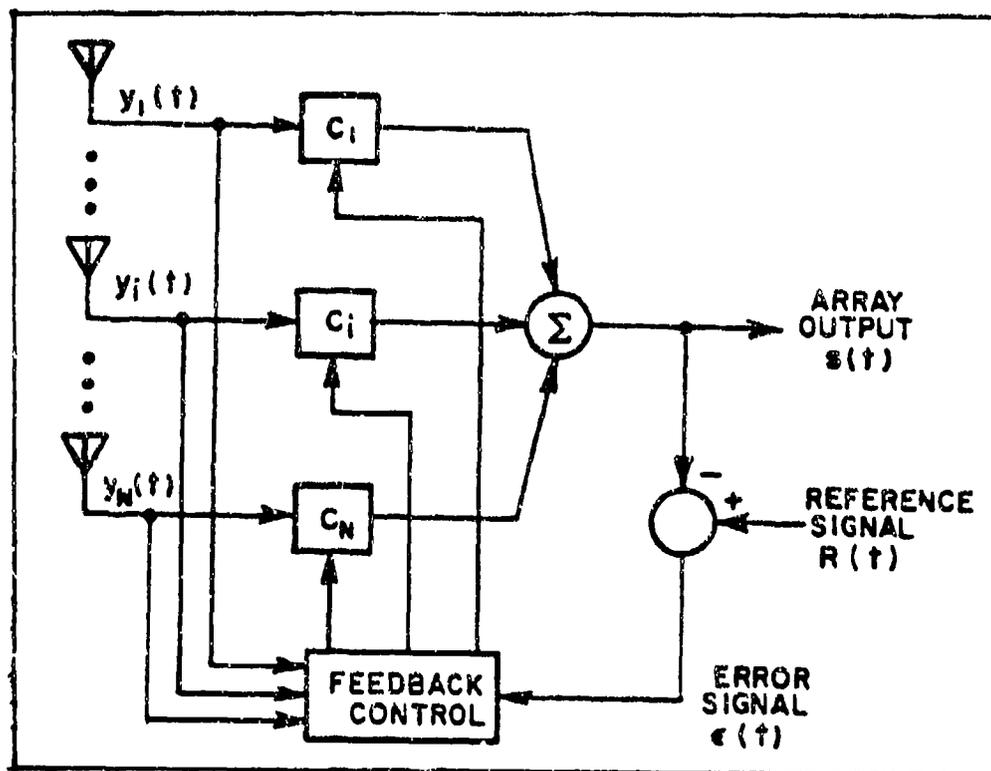
Fig 10.   Adaptive Array with LMS Hardware [3:3]

characteristics of the desired signal are known.   The mean

squared value of the error (e(t)) between the received

signal (s(t)) and the reference signal (R(t)) is minimized

by varying the complex weights of the array elements.   If

the reference signal is exactly the transmitted signal, then

the error signal would be jammer noise, background noise,

and other types of interference.   Therefore, minimizing the

error signal results in a reduction of the unwanted signals,

a reduction caused by nulls being placed in direction of the

interference.

The circuitry for the feedback control can be

determined by applying the LMS algorithm to the error

signal. The error signal is:

$$e(t) = R(t) - s(t)$$

$$= R(t) - \sum_{i=1}^{N} w_i y_i(t)$$

where w_i is the complex weight of each element and $y_i(t)$ is the received signal. The time-average value of $e^2(t)$ is:

$$e^2(t) = r^2(t) - 2 \sum_{i=1}^{N} w_i y_i(t) R(t)$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j y_i(t) y_j(t)$$

To minimize the mean square error, the complex weights can be varied according to the LMS algorithm:

$$dw_i/dt = -k \, d[e^2(t)]/dw_i$$

or

$$dw_i/dt = 2k y_i(t) e(t) \qquad (5)$$

Finally, the circuit (Fig 11) is derived from integrating equation 5:

$$w_i = w_{io} + 2k \int_0^t y_i(t) e(t) dt$$

Application. A 21 element, linear, nulling tree adaptive array was designed to determine the applicability of this technique to an array with failed elements. After
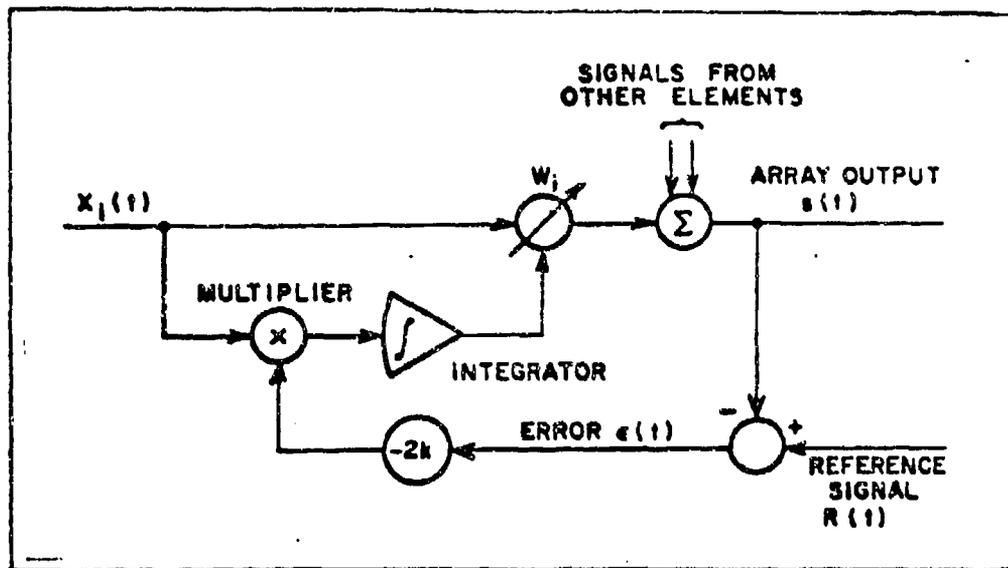
Fig 11. Feedback Control Circuitry [3:7]

element #7 was failed (the center element is #0) and the
antenna pattern was modeled (Fig 12), it was evident the
best angle for the null was close to the first sidelobe, but
not too close to the main beam (0.3 radians). The null
greatly reduced the sidelobe at this location, but at the
cost of decreased gain. To limit the affect of the null to
the gain, it is necessary to increase the power gain of the
row of nulling hardware. To improve the pattern even
further, another null could be placed at -0.3 radians.

The LMS algorithm was not applied to an adaptive array
with failed elements. Once again the work done by Compton
was used to illustrate the value of the algorithm. Figure
13 shows the pattern of a four element linear adaptive array
with and without interference. Note that the depth of the
null depends on the power of the interference. Compton also
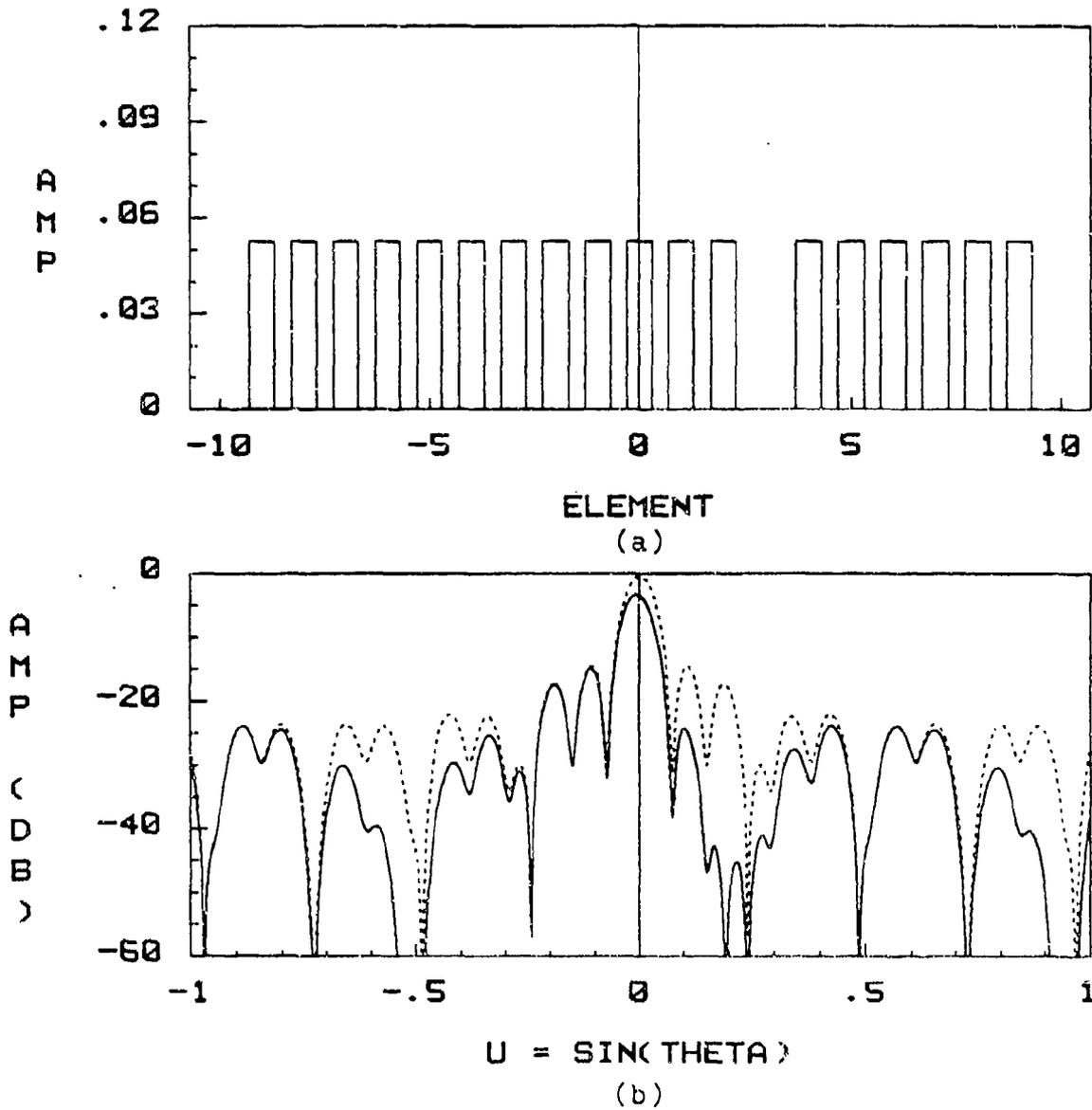
Fig 12. Array with Failed Element and Null at 0.3 Radians:
a) Amplitude Distribution; b) Far Field Pattern, Original
(---) vs Nulled (———)
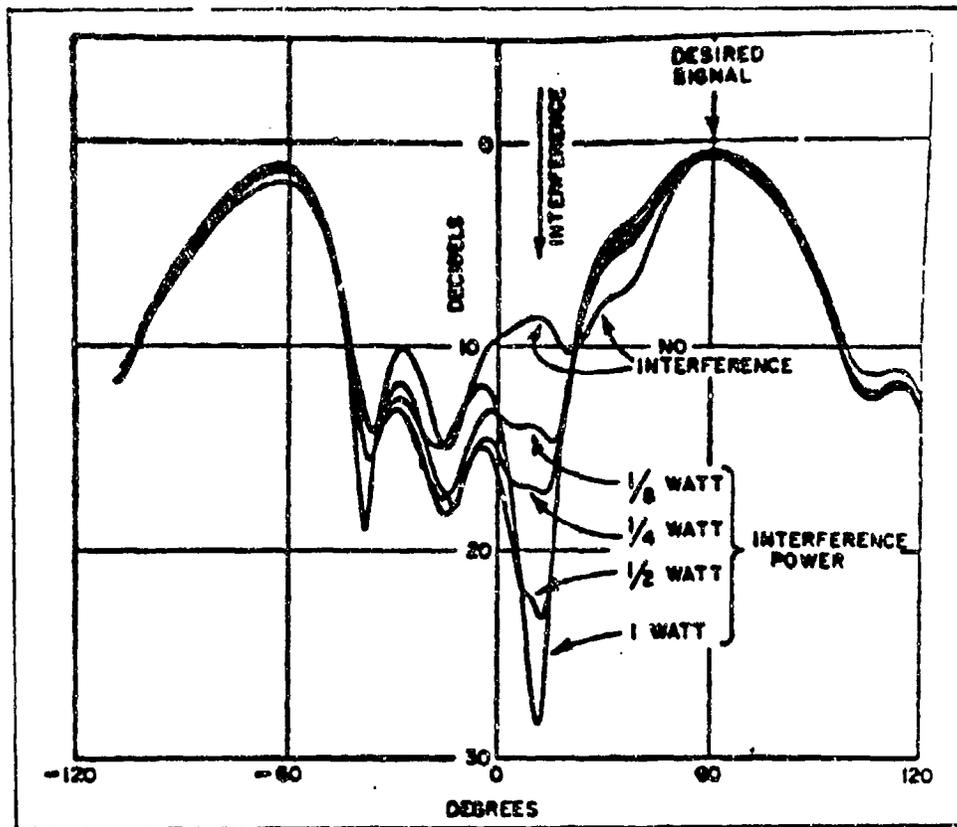
Fig 13. Far Field Pattern from a Four Element Linear LMS Adaptive Array with Interference [3:105]

investigated the use of interference for overall pattern shaping and concluded that the technique wouldn't work because the interference just caused a sidelobe to move over or caused sidelobes to develop elsewhere [3:117] and therefore, just improve a small portion of the pattern while degrading the overall pattern.

Results. The adaptive array techniques do have the potential of modifying an antenna pattern, although the pattern control is rather limited compared to what is required for the FOV, and therefore cannot be successfully used for this study. The nulling tree adaptive array is

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further unattractive because of the additional hardware required and because apriori knowledge required of where to place the null.

An optimization method similar to the LMS algorithm (called the steepest descent gradient) was used in a following section as an iterative technique. However, reduction of interference will not be the main goal there.

Eigenvalue Methods

Eigenvalue methods are methods which, thru the marvels of linear algebra, analytically arrive at a solution which will maximize or minimize a given function. The advantage of using these methods is they are relatively quick to come to a solution compared to the other techniques investigated.

Description. Two different eigenvalue methods were tried, one by Cheng [2] and the other by Shore [15].

Cheng's eigenvalue method is based on a theorem on the properties of a function of a matrix vector. The theorem says that given a function:

$$G = a^{T*}Aa \; / \; a^{T*}Ba$$

where a is an Nx1 column matrix, A and B are both Hermitian, NxN square matricies, and B is positive definite, then:

1. The roots (eigenvalues) of the characteristic equation are real.

2. The minimum eigenvalue and maximum eigenvalue represent the bounds of G:

$$\lambda_1 < G < \lambda_N.$$

3. G is maximized when:

$$Aa = \lambda_N Ba$$

or minimized when

$$Aa = \lambda_1 Ba.$$

Cheng applied the theorem to maximize the gain of an array:

$$G = \frac{s(u_0, \theta_0)}{1/(4) \int_0^{2\pi} d\theta \int_{-1}^{1} s(u,\theta) du} \qquad (6)$$

where

$$s(u,\theta) = |P(u)|^2 P_n,$$

P_n is the element power pattern ($=1$ for isotropic elements), and the other symbols are as before. Equation 6 can be written as:

$$G = W^{T*}AW / W^{T*}BW$$

where W is a column vector whose elements are the complex weights of the current distribution and A and B are Hermitian, square matricies with elements

$$A_{mn} = \exp[jk(d_n - d_m)u]$$

and

$$B_{mn} = Sa \ [k(d_n - d_m)]$$

respectively, and B is positive definite. To solve for W,
the A matrix is decomposed into

$$A = SS^{T*} \tag{7}$$

where S is an N element vector with elements $s_m = \exp(-jkd_m$
u). Finally, the optimum current distribution and the
maximum gain are:

$$W = B^{-1}S$$

and

$$G_{max} = S^{T*}B^{-1}S.$$

Shore's eigenvalue method relies on Lagrangian
multipliers to find the current distribution which gives the
optimized pattern. The objective of Shore's work was to
minimize the power within a spec fied interval of the
antenna pattern with constraints on the weight perturbations
required and the value of the look direction gain. The cost
function, which is the function to be minimized, is modeled
by:

$$G = (W-W_o)^{T*}(W-W_o) + u_1 W^{T*}CW.$$

The first term on the right hand side of the equation

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minimizes the weight perturbations and is the square of the change in the weights (W = vector of perturbed weights, W_o = vector of original weights) scaled by a factor (u_1) which increases or decreases the relative importance of minimizing the perturbations versus the minimizing the sidelobe sector. The second term corresponds to the power within the sidelobe sector which is to be minimized:

$$W^{T*}CW = 1/2e \int_{u_o-e}^{u_o+e} |P(u)|^2 \, du$$

where u_o and e are the center and the range of the sidelobe sector respectively and C is a Hermitian, Toeplitz matrix whose elements are:

$$C_{mn} = \exp[j(d_n-d_m)u_o] \, Sa[(d_n-d_m)e]$$

By taking the derivative of the cost function with respect to the complex weight, setting it equal to zero, and solving for W, the current distribution which corresponds to the optimum pattern is:

$$W = AW_o$$

where

$$A = (I + u_1C)^{-1}.$$

Shore constrained the cost function not only by minimizing the weight perturbations, but also by specifying the look

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direction gain. The look direction gain is defined as

$$g = S_O^{T*} W$$

where S is as before except u ($=u_O$) is the desired direction of the main beam. The solution constrained to a given gain is found by using the method of Lagrangian multipliers and is:

$$W = A[W_O - (\ (S^{T*} A W_O - g)/S^{T*} A S\ )S] \qquad (8)$$

As the relative importance of limiting the weight perturbations increases (i.e., u_1 goes to 0), W converges to W_O and as the relative importance of minimizing the power in the sidelobe sector increases (i.e., u_1 goes to infinity),

$$W = (C^{-1} S\ /\ S^{T*} C^{-1} S)\ g.$$

Both Cheng's method and Shore's method result in a straight forward equation to be solved and both can be solved for a general array. A trivial difference between the two methods is that Cheng is trying to maximize a function whereas Shore is trying to minimize a function.

Application. The function which relates to the optimum pattern for this study was alluded to in the assumptions and is:

$$G = \int_{-Lim}^{-Bw} |P(u)|^2 du - s_1 \int_{-Bw}^{Bw} |P(u)|^2 du + \int_{Bw}^{Lim} |P(u)|^2 du$$

$$= \int_{-Lim}^{Lim} |P(u)|^2 du - (1+s_1)\int_{-Bw}^{Bw} |P(u)|^2 du$$

$$= \sum_{n=1}^{N} \sum_{m=1}^{N} w_n w_m^* H_{mn} \tag{9}$$

where s_1 is a scale factor which adjusts the relative importance of maximizing the power in the main lobe versus minimizing the power in the FOV, Bw is the one sided main lobe beam width ($= \sin(1/(0.7N)) * 0.5$), Lim is the bound of the FOV ($= \sin(0.26)$), H is a Hermitian, Toepliz matrix (Fig 14) with elements

$$H_{mn} = 2Lim\ Sa[k(d_n-d_m)Lim] - 2(1+s_1)Bw\ Sa[k(d_n-d_m)Bw] \tag{10}$$

and the remaining variables are as previously described. The H matrix will replace the A matrix in Cheng's method and the C matrix in Shore's method.

The difficulty in applying Cheng's method came with the decomposition (eqn 7). In order for the decomposition to work the H matrix must be separable into the product of a column matrix and its complex conjugate, but because the H matrix is the result of summing three different areas of the far field pattern, it is not separable as Cheng's A matrix was separated.

Shore's method; however, can be successfully applied. Figure 15 depicts the amplitude distribution resulting from

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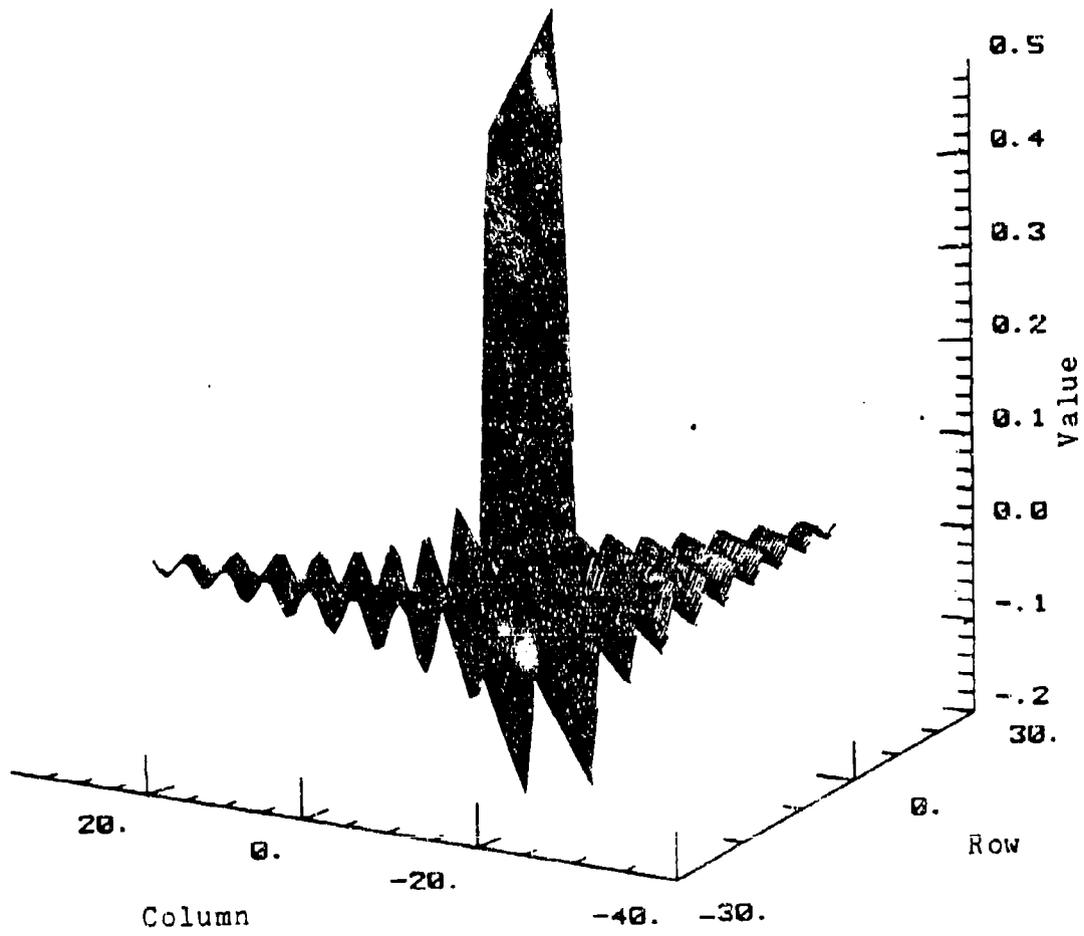
Fig 14.   Values of Elements of H Matrix

optimizing a 51 element linear array with unity scaling factor. Figure 16 shows the resulting far field power pattern compared to the pattern of an uniform distribution. Note that, as a result of the tapered amplitude distribution, the sidelobes in the FOV are reduced, but the width of the main beam is increased. If an element is failed, initially it was believed that the elements' corresponding complex weight could be set to zero. After the equation was applied however, a value was assigned to the failed element. To correct for this anomoly, the row and column of the H matrix corresponding to the failed element were set equal to zero. This changed the H matrix into a singular matrix, the inverse of which is does not exist. Finally, it was realized that the H matrix did not require a periodic inter-element separation. Because of this, the row and column of the H matrix corresponding to the failed element could simply be removed, reducing the matrix to an N-1 by N-1 square matrix. After appropriately reducing the orders of the remaining matricies, the model (eqn 8) was reapplied. Figures 15 - 20 show the amplitude distribution and the normalized antenna pattern of a 51 element linear array for five cases: a) optimum distribution, no failures; b) optimum distribution, one failure (near end: #21); c) optimum distribution, one failure (center: #0); d) optimum distribution, two failures (separated: #5, #-16); e) optimum distribution, ten failures
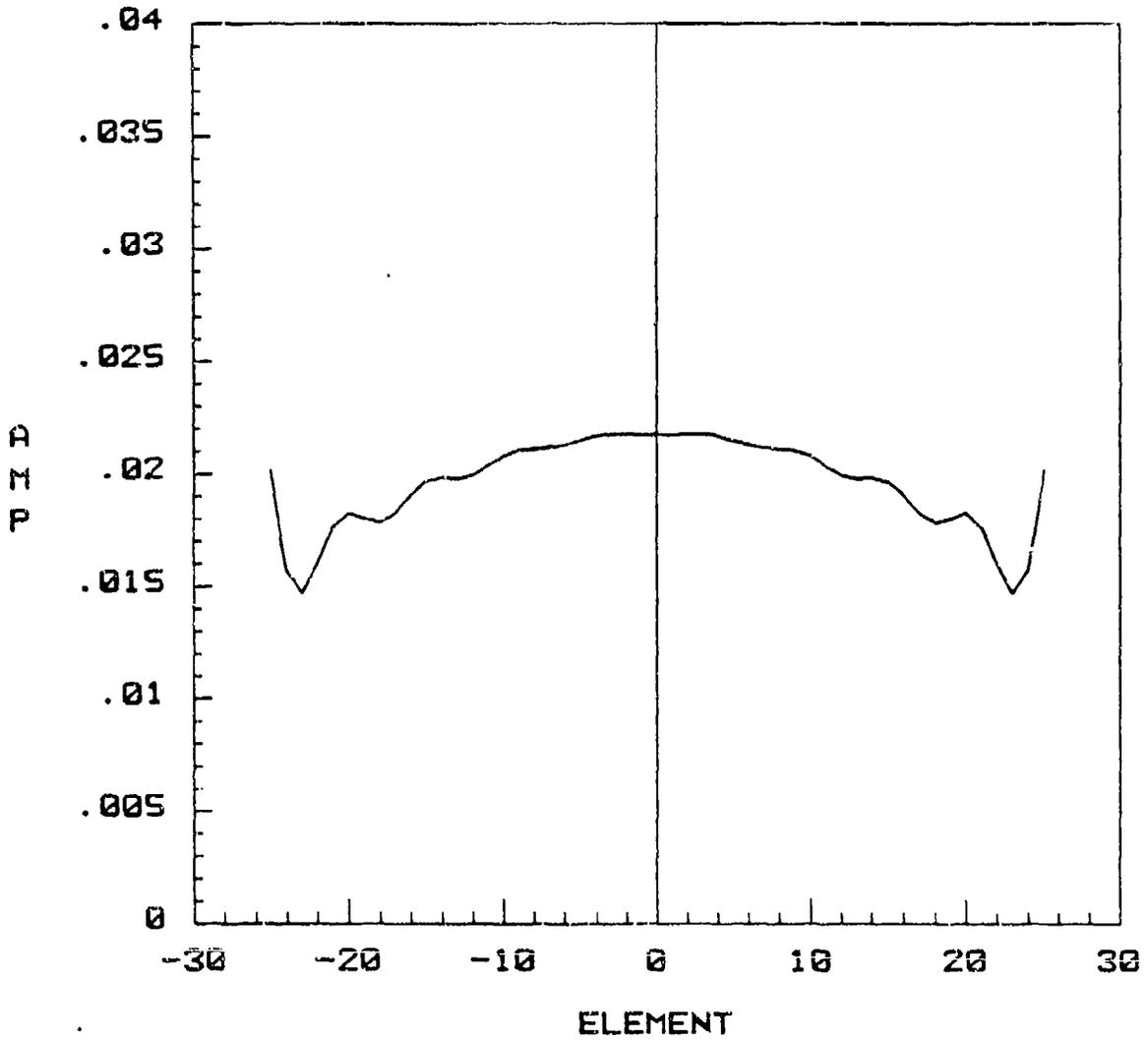
Fig 15. Optimum Amplitude Distribution for Filled Array
(Shore's Eigenvalue Method)
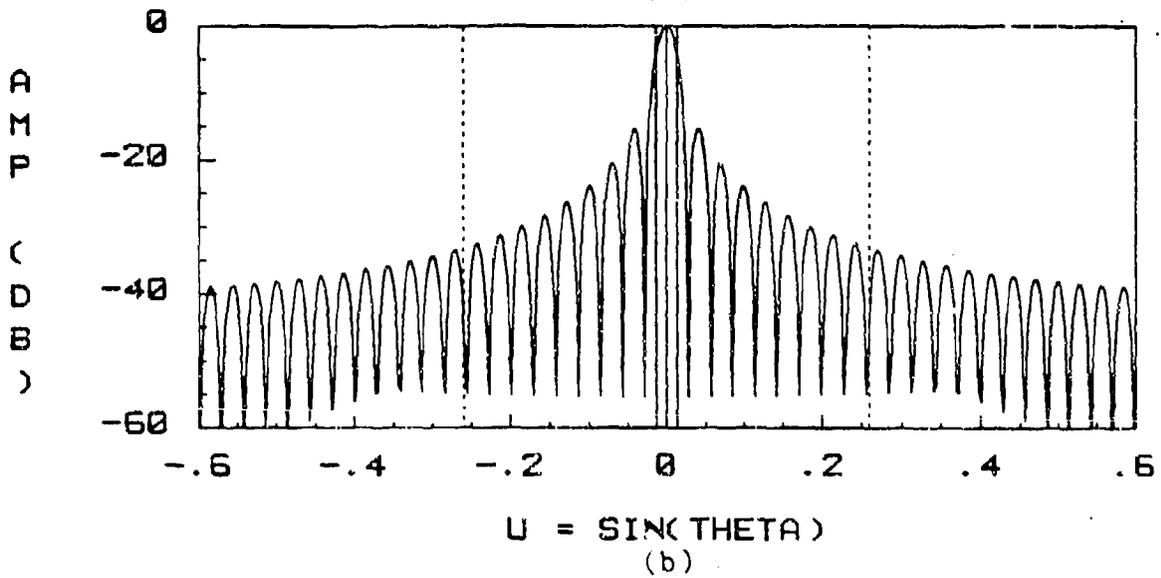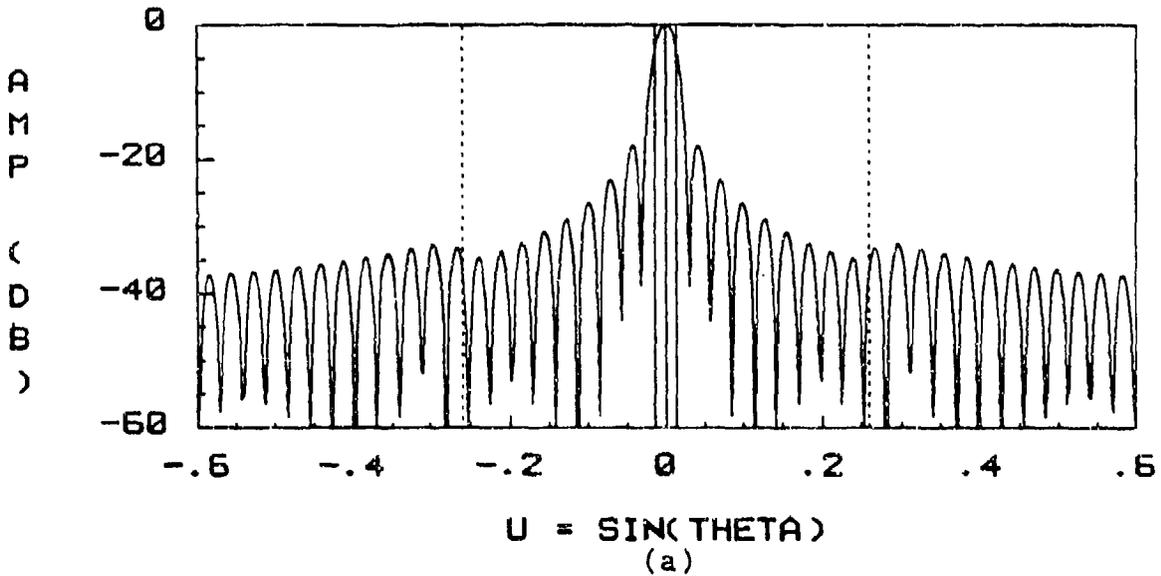
Fig 16. Far Field Pattern of: a) Optimum Distribution vs b) Uniform Distribution
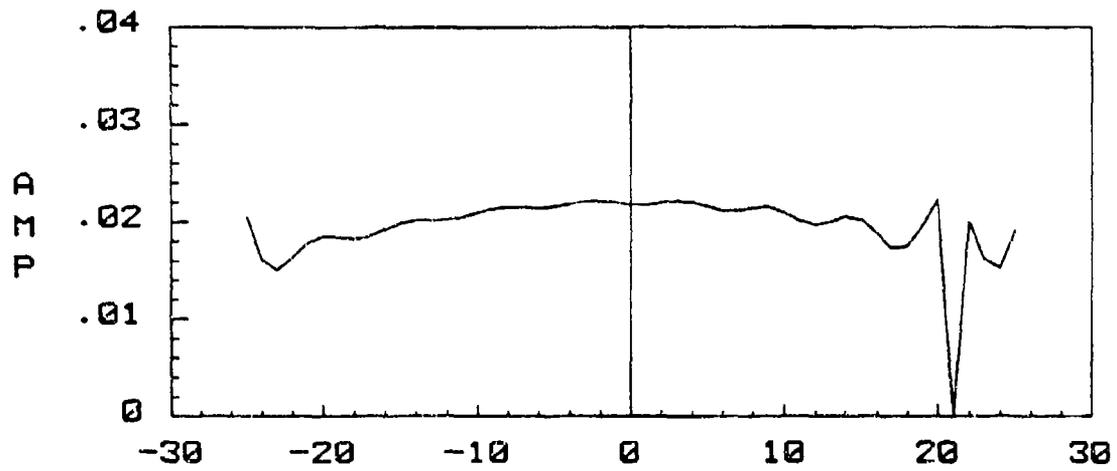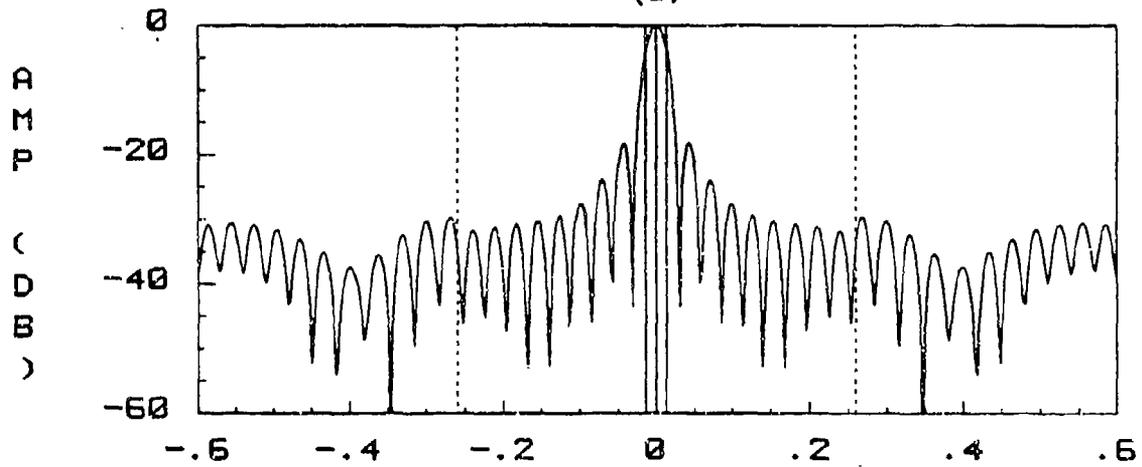
Fig 17. One Failure Near End: a) Optimum Distribution; b) Far Field Pattern
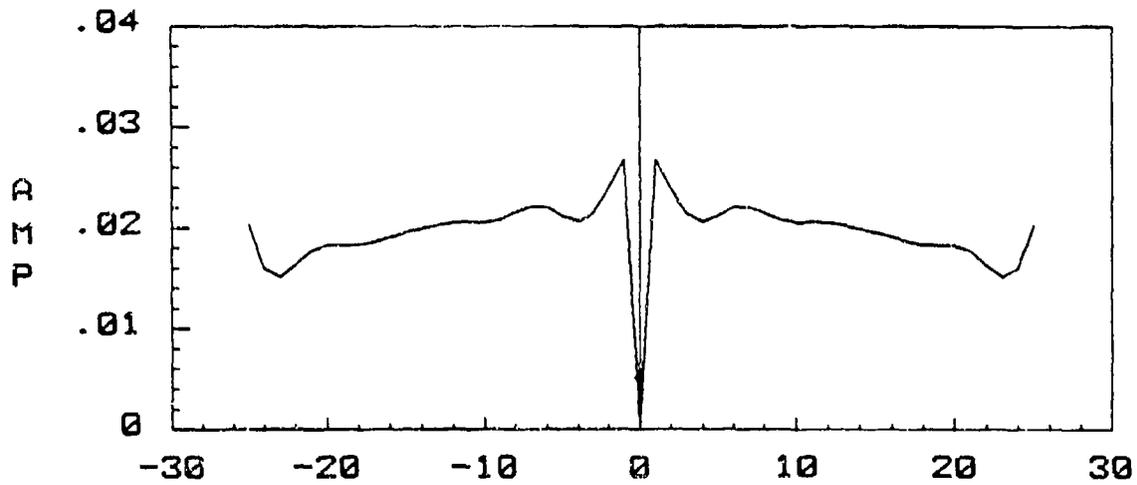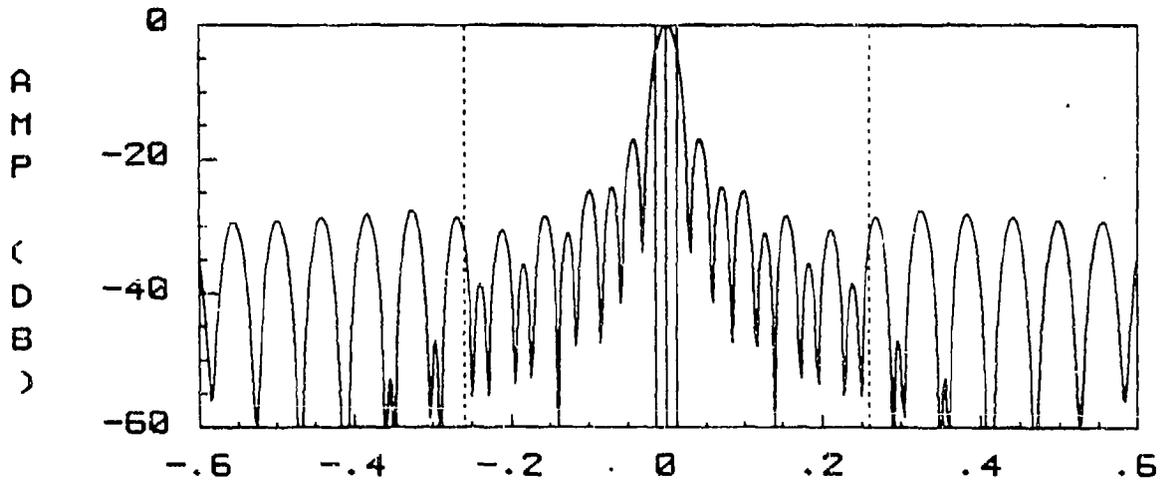
Fig 18. One Failure Center: a) Optimum Distribution; b) Far Field Pattern

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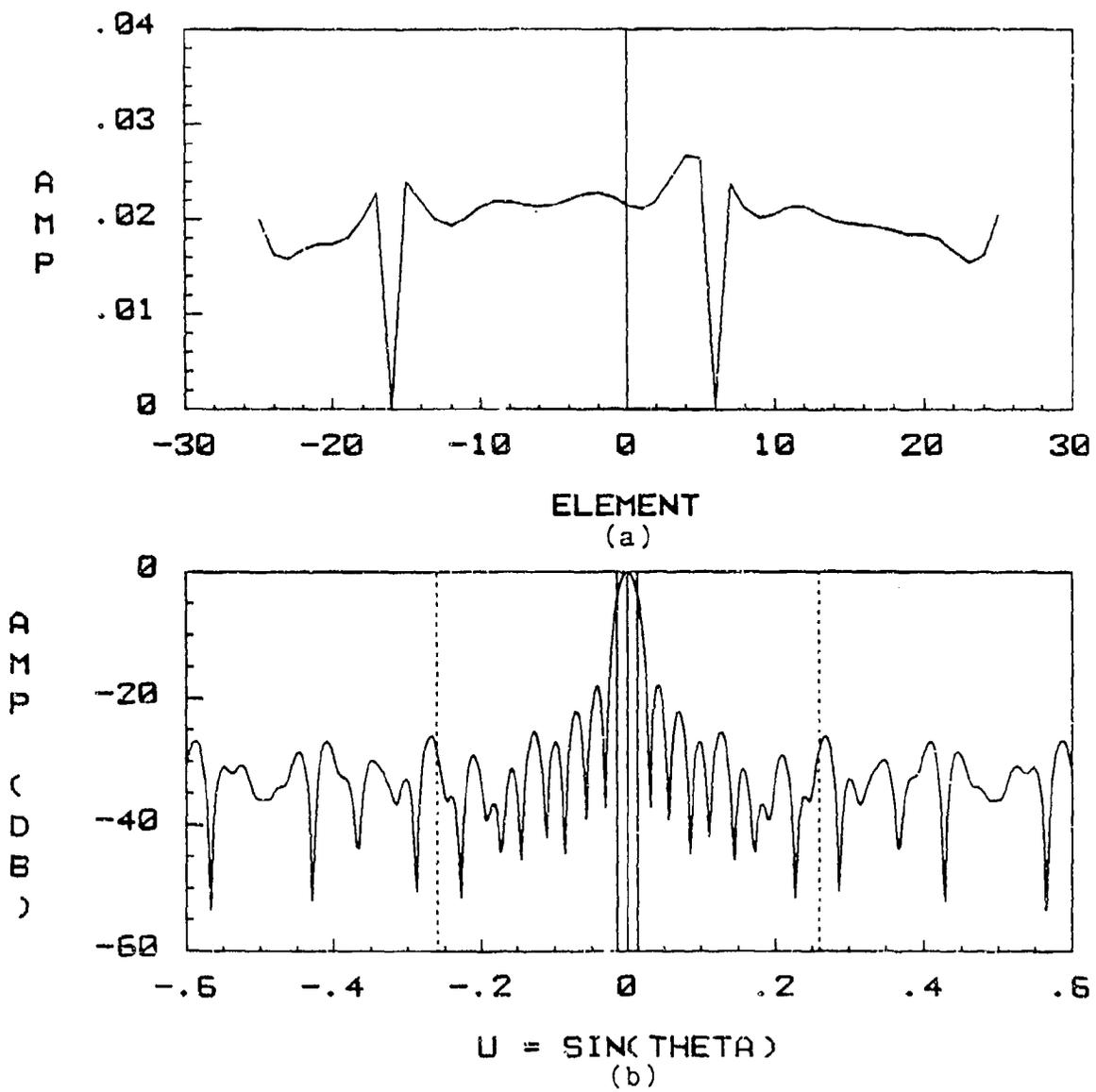
Fig 19. Two Failures Separated: a) Optimum Distribution;
b) Far Field Pattern
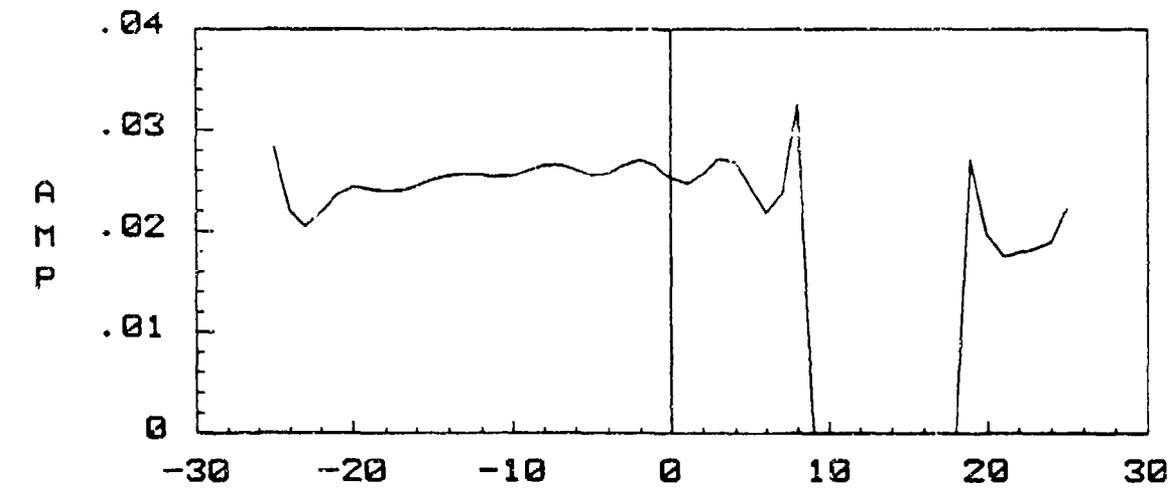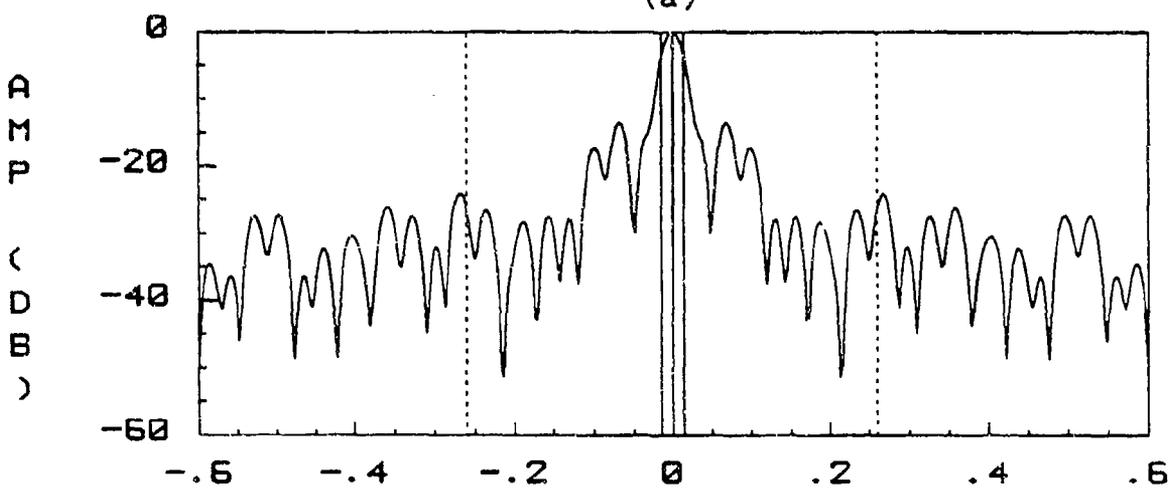
Fig 20. Ten Failures .Together: a) Optimum Distribution; b)
Far Field Pattern

TABLE I

Relative Cost Using Eigenvalue Method

Case	Pre-optimized	Optimized	Uniform
A	-	100.00	97.11
B	97.08	97.52	94.91
C	95.52	96.21	92.27
D	92.22	92.88	89.29
E	33.66	35.49	31.08

(together: #18 - #9). The cost values in Table I are normalized with respect to the cost of case A and are a relative value of "goodness" for the associated case. The cost is calculated by substituting the optimum distribution into equation 9 and solving for G.

Results. Shore's eigenvalue method provides a quick and accurate method of obtaining the optimum current distribution for an array with failed elements. For each of the cases, the initial distribution was the uniform distribution. Table I indicates that the improvement of the optimum distribution compared to the uniform distribution is more significant than the improvement of the optimum distribution compared to the pre-optimum distribution. Also, the improvement is more significant for failures of many elements compared to failures of only one or two elements.

Null Displacement Technique

Null displacement is a method of quickly optimizing an antenna pattern by modifying the positions of the pattern nulls. This method is useful because the angular position of the pattern's nulls plotted on a z-plot is the z-transform of the current distribution. Therefore, once the position of the pattern's nulls have been defined, the required amplitude and phase of the current exciting each element can be easily determined by using the inverse z-transform.

Description. The roots (zeros) of the z-transform of an array factor can be moved by varying the complex weight of the currents on each element. The zeros, if on the unit circle of the z-plot, represent a null in the antenna pattern. If the zero is not on the unit circle, the pattern dips at the location of the zero, but does not null (Fig 21). Also, the closer the zeros are to each other, the lower the peak power of the lobe between them. The nulls 4 through 7 exist twice within the visible range ($-1 < u < 1$, $-1.4\pi < \arg(z) < 1.4\pi$) of the antenna pattern because of the 0.7λ inter-element separation. Therefore, the cost function (eqn 9) is minimized if the zeros corresponding to the nulls beside the main beam were held constant and all the other zeros were moved into the FOV. Gaushell [7] has shown that the amplitude distribution which creates a desired pattern can be determined by first modifing the positions of the

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Fig 21. Correspondence of Far Field Pattern Nulls and z-plot Zeros

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zeros corresponding to the nulls to be moved and then by
taking the z-transform of the resulting polynomial to find
the required complex weights. In other words:

$$W_n = Z^{-1}[E(z) \; G(z)] \qquad (11)$$

where $G(z)$ is the z-transform corresponding to original
array coefficients (z_i):

$$G(z) = \prod_{i=1}^{N} (1-z_i z^{-1})$$

and $E(z)$ is the ratio of the new zero position (y_i) to old
zero positions (z_i):

$$E(z) = \prod_{i=1}^{N} (1-y_i z^{-1}) \; / \; (1-z_i z^{-1}).$$

Application. To apply the null displacement technique
to an array with failed elements, it was postulated that
equation 11 should include a factor $(E_o(z))$ to account for
the difference between the pattern generated by a full array
and the pattern generated a partial array:

$$W_n = Z^{-1} [E_o(z) \; E(z) \; G(z)].$$

To get an idea of what types of patterns were possible using
null displacement, it was necessary to determine the effects
that amplitude and phase perturbations had on the array

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pattern. Thus, the "root migration" of the array factor was plotted for several different perturbations. The root migration was created by plotting the z-plot of the array as amplitude or phase of the current of a given element were varied. The 0's in the plots indicate a zero and the X's in the plot indicate the position of the zeros as the amplitude of the current of the given element is made 1000 times the amplitude of the current of the remaining, unperturbed elements. Figure 22a shows that as the amplitude of the current is increased evenly for a pair of elements ($a_n = a_{-n}$), the zeros move towards the z-plot which would result from an array of just those two elements. As expected, the zeros move in an even fashion, always as conjugate pairs. If the current amplitude of a pair of elements is varied oddly ($a_n = -a_{-n}$). Figure 22b results. This shows that the array pattern will be even symmetric but the main lobe will be nulled out. Finally, if the current of only one element is varied (Fig 22c) the zeros temporarily travel off the unit circle as the current goes from zero to the value equal to the other currents, and then heads off toward infinity or zero. With all the zeros at the limits, the antenna pattern is that of an isotropic radiator.

The behavior of the zeros is not as apparent with phase perturbation as it is with amplitude perturbations. From Figure 23 these observations can be made:

1. The further out the phase perturbed element is, the greater its effect on the main lobe and near-in side lobes.

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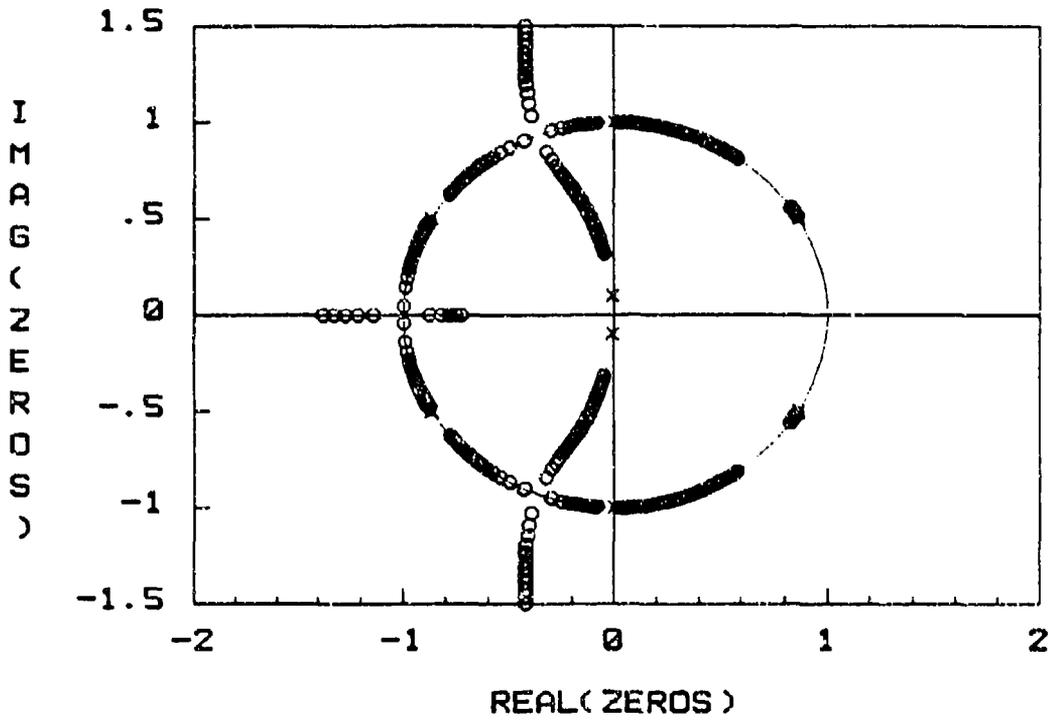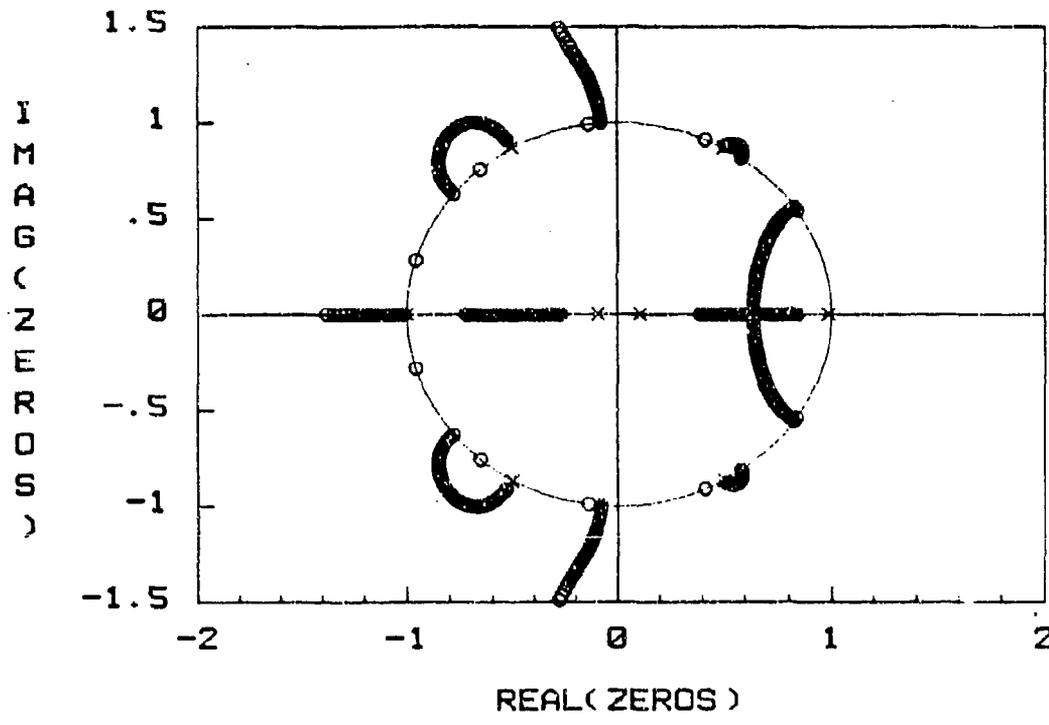
Fig 22. Zero Travel Due to Amplitude Perturbations of
Selected Elements of an 11 Element Array: a) Even (Elements
-3, 3); b) Odd (-3, 3)
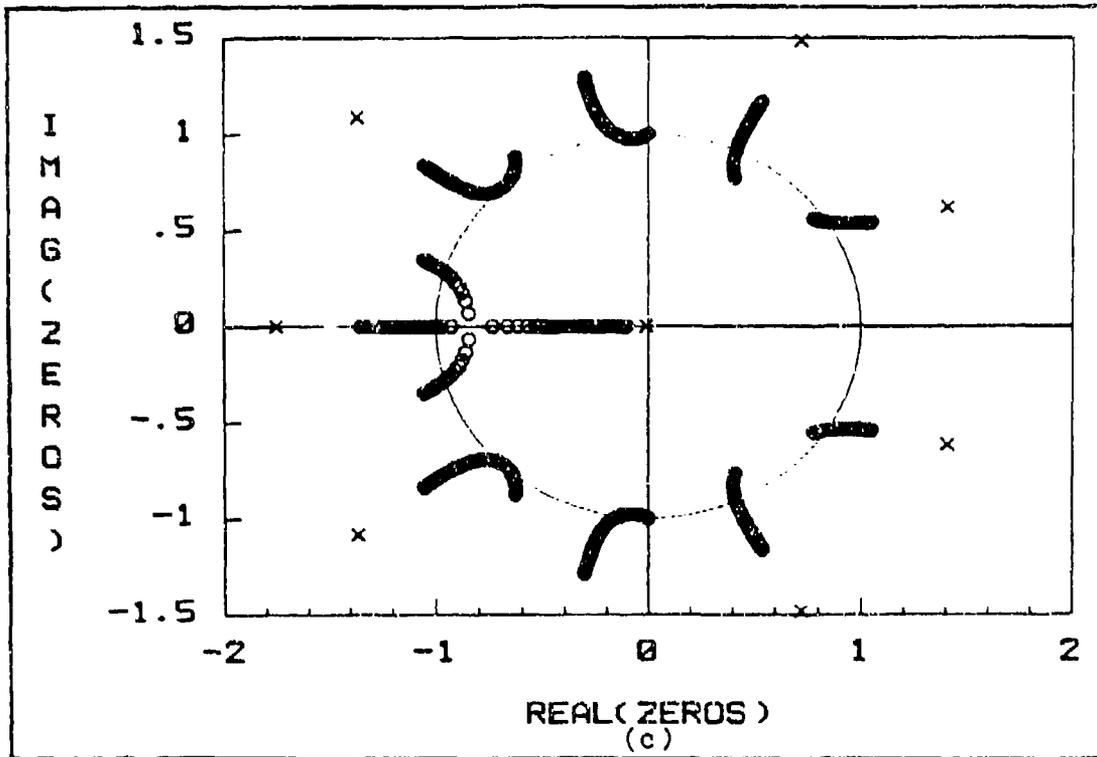
Fig 22 cont. Zero Travel Due to Amplitude Perturbations of
Selected Elements of an 11 Element Array: c) Single (4)

2. The pattern created by varying the phase of an element
    on one side of an array will be the flip side of the
    pattern created by varying the phase of the opposing
    element on the other side of the array.

3. Zeros from an even ($\emptyset_n = \emptyset_{-n}$) and single element
    phase perturbation are more constrained to a
    particular angle whereas zeros from an odd phase
    perturbation ($\emptyset_n = \emptyset_{-n}$) are more constrained to the
    unit circle.

4. The zeros have a complex conjugate only when all the
    phases are equal. If a zero starts at $z=0+1j$, its
    complex conjugate will be at $z=0-1j$. As the pane is
    varied, the zero at $z=0+1j$ will move towards $z=-1+0j$
    and the zero at $z=0-1j$ will move towards $z=1+0j$ or
    vice versa. This characteristic may improve one side
    of the pattern but will distort the other side.

5. Zeros traveling due to phase perturbations always
    travel in loops (in other words, 2π rads = 0 rads).
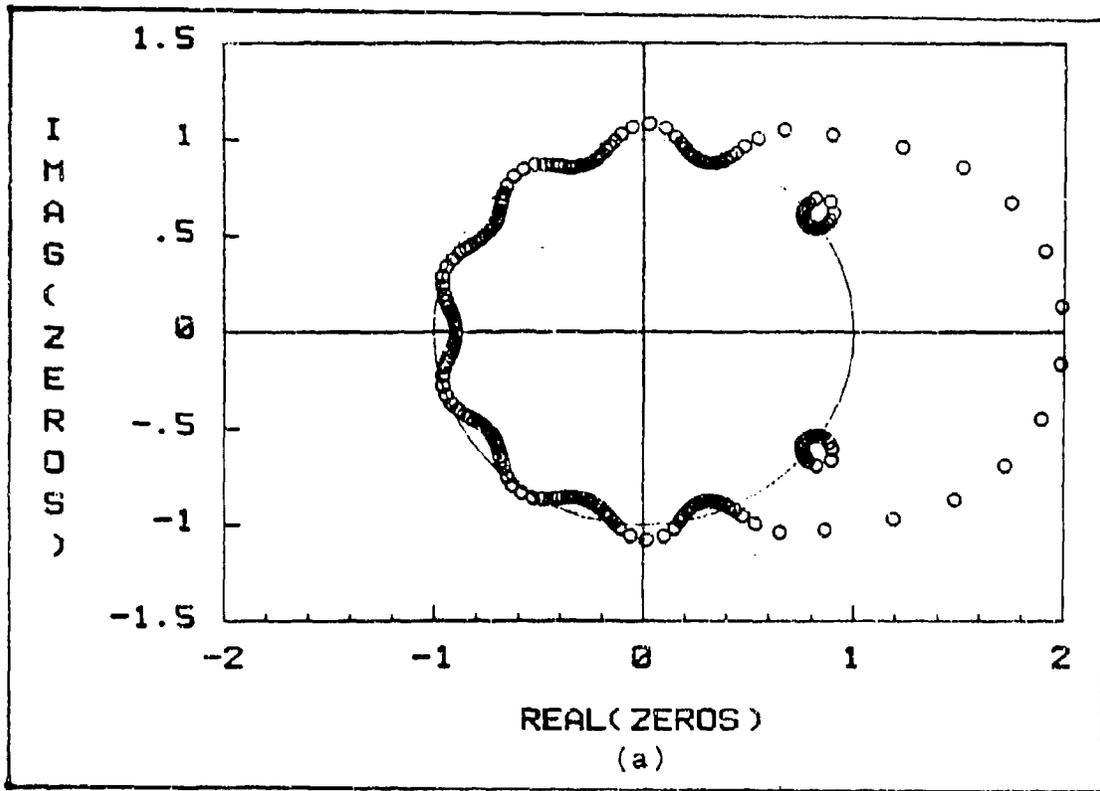
Fig 23. Zero Travel Due to Phase Perturbations of Selected
Elements of an 11 Element Array: a) Single (-5)

To further characterize the effects that a failed
element had on the array pattern, the root migration of a
partial array was compared to the root migration of a full
array. The exercise done for Figure 22a was repeated for an
array with a failed element. A comparison of Figure 22a
with Figure 24 shows that the full array is more capable of
producing a desired pattern than is the partial array,
indicated by the zeros nearer to the unit circle and
covering a greater portion of the unit circle. The
consequence of this exercise is that though the nulls may be
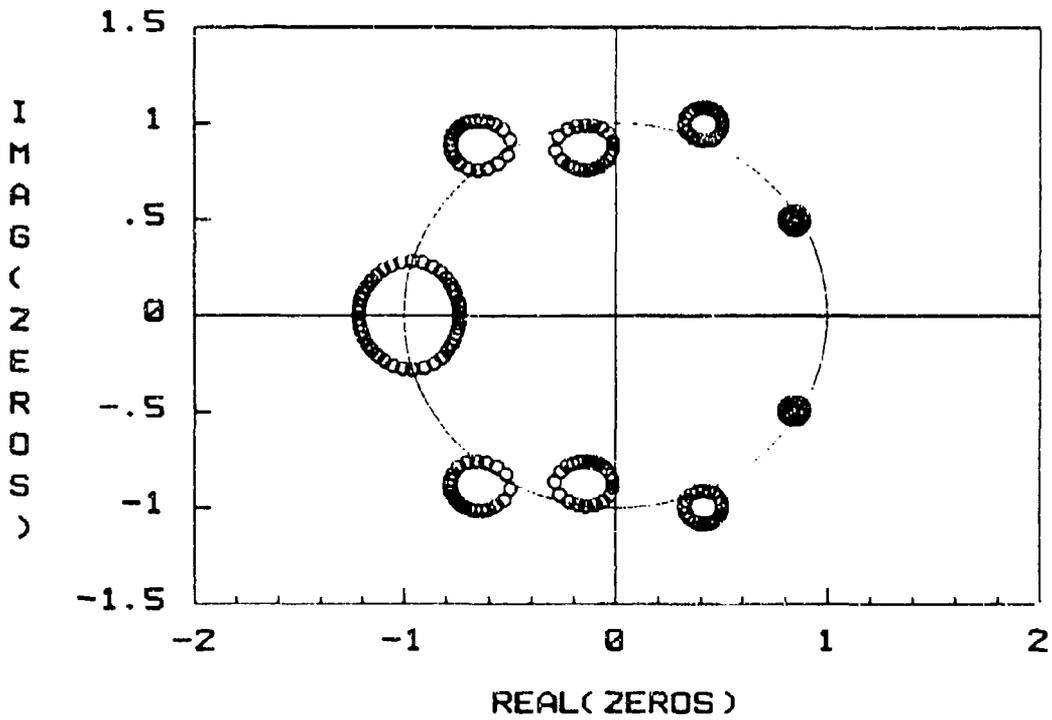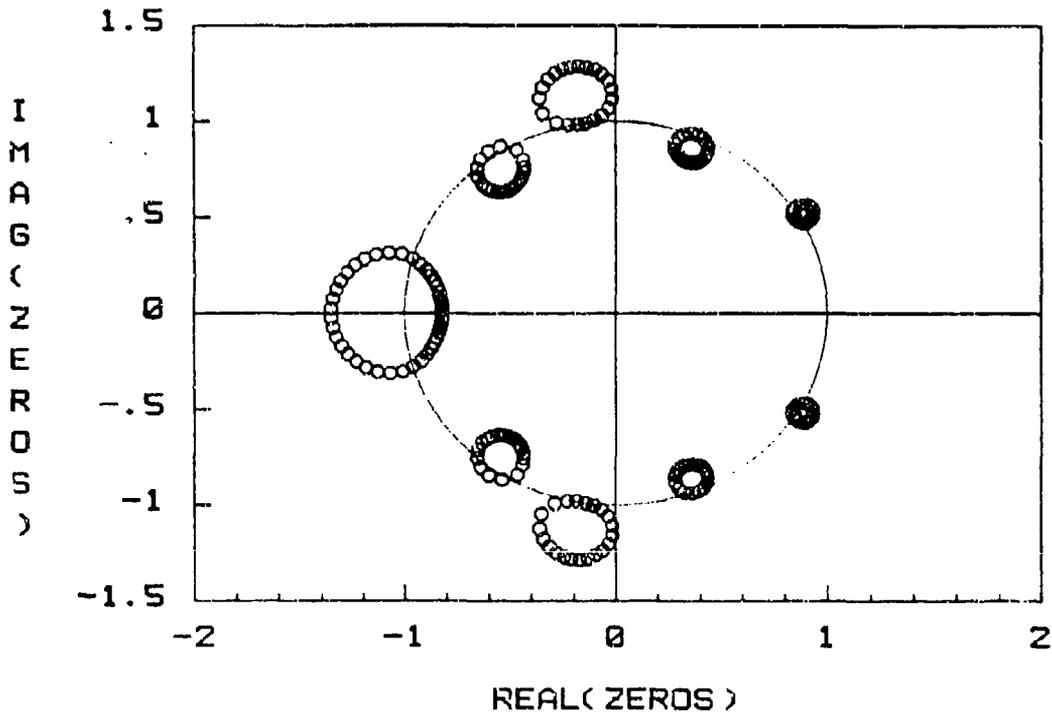
Fig 23 cont. Zero Travel Due to Phase Perturbations of
Selected Elements of an 11 Element Array: b) Single (-1);
c) Single (1)
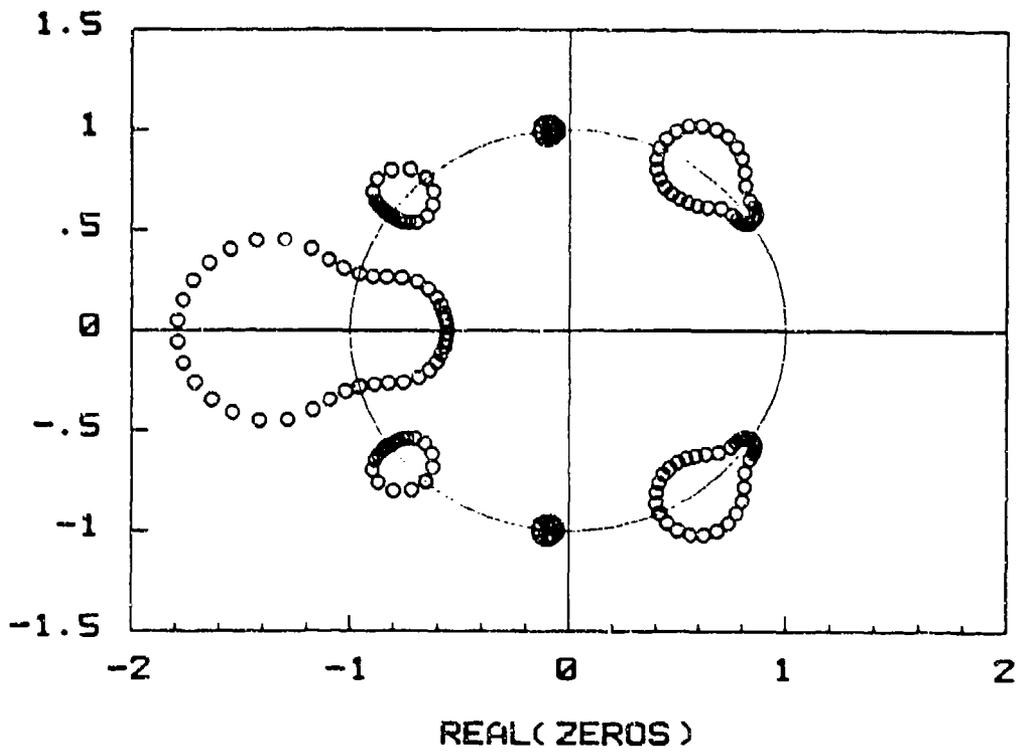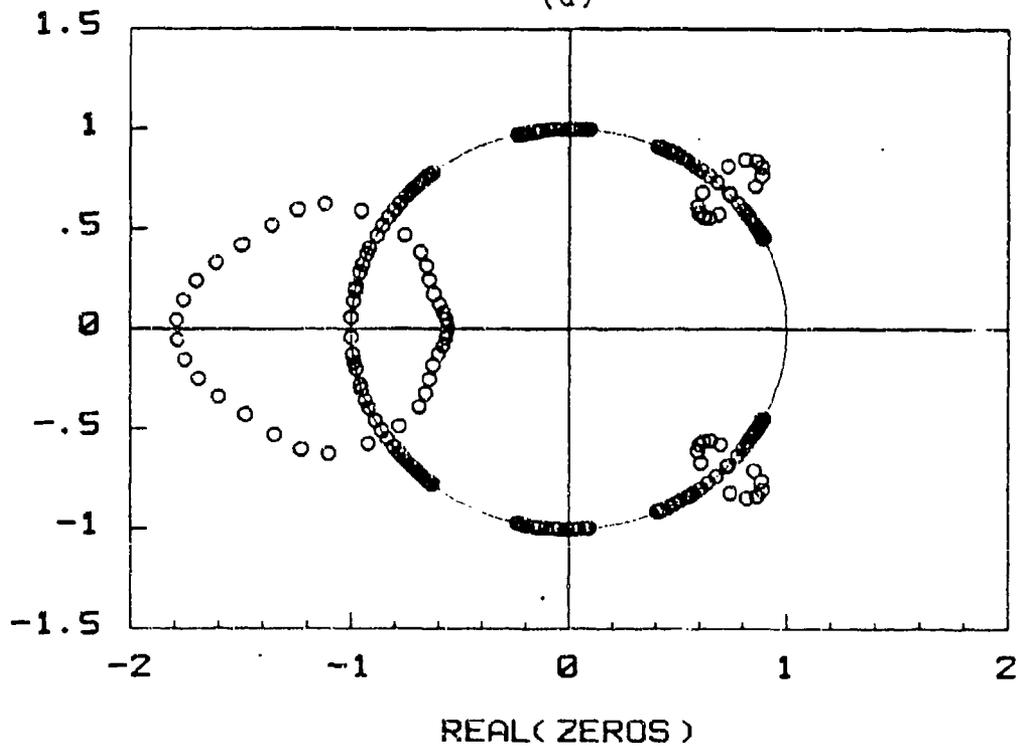
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Fig 23 cont. Zero Travel Due to Phase Perturbations of
Selected Elements of an 11 Element Array: d) Even (-3,1);
e) Odd (-3,3)
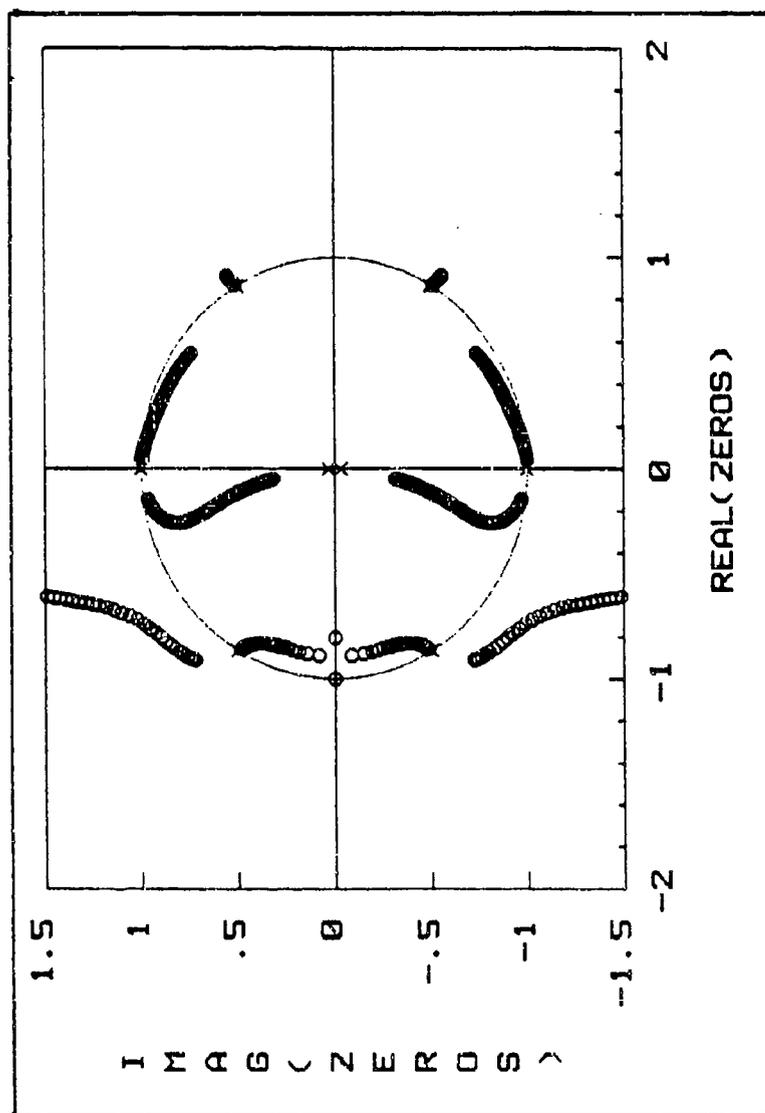
Fig 24. Zero Travel Due to Even Amplitude Perturbations of
Elements (-3, 3) of an 11 Element Array with a Failure

placed where desired, this would require a specific amplitude distribut'on - a distribution which may require the failed elements to carry current. In other words, there is a one-to-one mapping from the z-plot to the amplitude distribution through the z-transform.

Results. The null displacement technique may be used to optimize the pattern of an array by first determining all possible zero locations and then choosing the amplitude distribution which corresponds to the best null positioning or by first selecting the desired nulls and then with some type of iterative program, converge to the best set of available nulls. Either way, the advantage of this technique, which is its potential speed in arriving at a solution, is lost. This technique has an added disadvantage in that it is difficult to determine whether or not the null positions selected will actually create the best possible pattern.

Iterative Techniques

Iterative techniques are those which first approximate a solution, then compare the solution with some desired criteria, and then approximate a new solution to be compared until the solution matches the desired criteria. Of the iterative techniques investigated; most noteably minimax optimization, least mean square error optimization, and steepest descent gradient search optimization, the latter

appeared to be the most applicable to the given problem. The gradient search optimization technique can be used for arrays with arbitrary element positions and amplitude distributions. The technique has the added advantage in that the desired optimization criteria can be specified exactly.

Description. The gradient search optimization technique finds the minimum (or maximum) of a given function, called the cost function. Once the function is defined, the gradient, which points in the direction of the function's local steepest ascent (descent), is found by taking the partial derivative of the function with respect to the parameters of interest. To converge to the minimum, the optimization algorithm determines the initial gradient from a starting point, steps along the negative gradient direction, and checks the gradient after each step until the new gradient is orthogonal to the initial gradient. The algorithm then steps along the new gradient and so on until the gradient becomes zero, which characterizes a minimum or maximum. This process is shown graphically in Figure 25, where the ovals are the lines of equal cost, the arrows point in the negative gradient direction, and the center of the smallest oval is the minimum cost. To help the algorithm converge to the perpendicular intersection of two gradients, the step size is determined by:
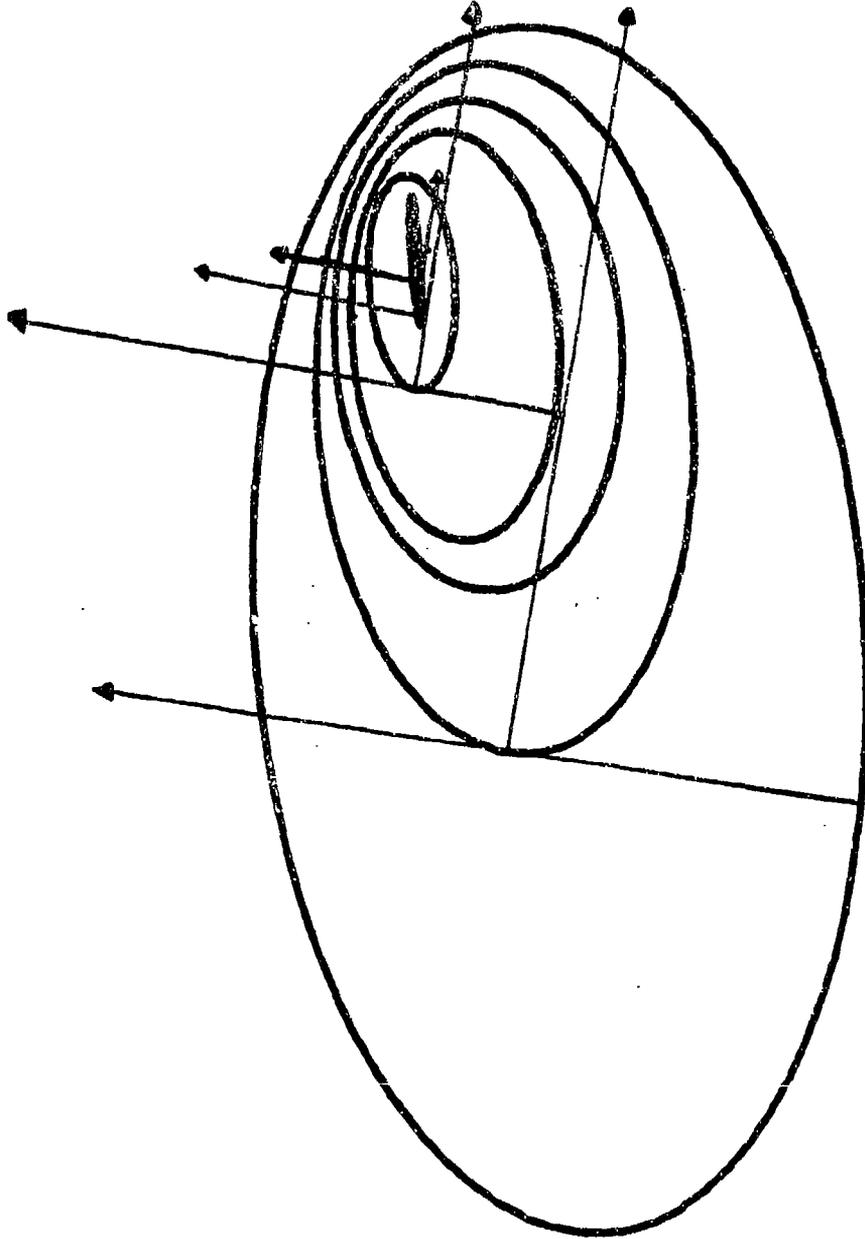
Fig 25. Gradient Search – Steepest Descent Optimization
Method

$$\text{new step} = \text{old step} \,[1 + 0.9 \cos(v)]$$

where v is the angle between the initial gradient vector and the new gradient vector. Inflection stationary points can be eliminated algorithmically. Although global optimality is not guaranteed, experience and intuition help assure that the minimum reached is optimal. This is especially true for problems where the parameter space is tightly constrained by physical limitations.

Application. The gradient search algorithm was programmed with the ADA programming language for a VAX 11/780. The array used was an 101 element linear array. The cost function which is to be optimized is described in equation 9. The gradient vectors are:

$$\partial G / \partial a_k = \sum_{m=1}^{N} 2\, a_m \cos(\phi_k - \phi_m)\, H_{km}$$

and

$$\partial G / \partial \phi_k = \sum_{m=1}^{N} -2\, a_k\, a_m \sin(\phi_k - \phi_m)\, Hkm$$

where a_m and ϕ_m are the amplitude and phase, respectively, of the current exciting element m. The optimization algorithm was initially run on a linear array with no failures using amplitude only perturbations. The resulting distribution (Fig 26) approached the ideal described by
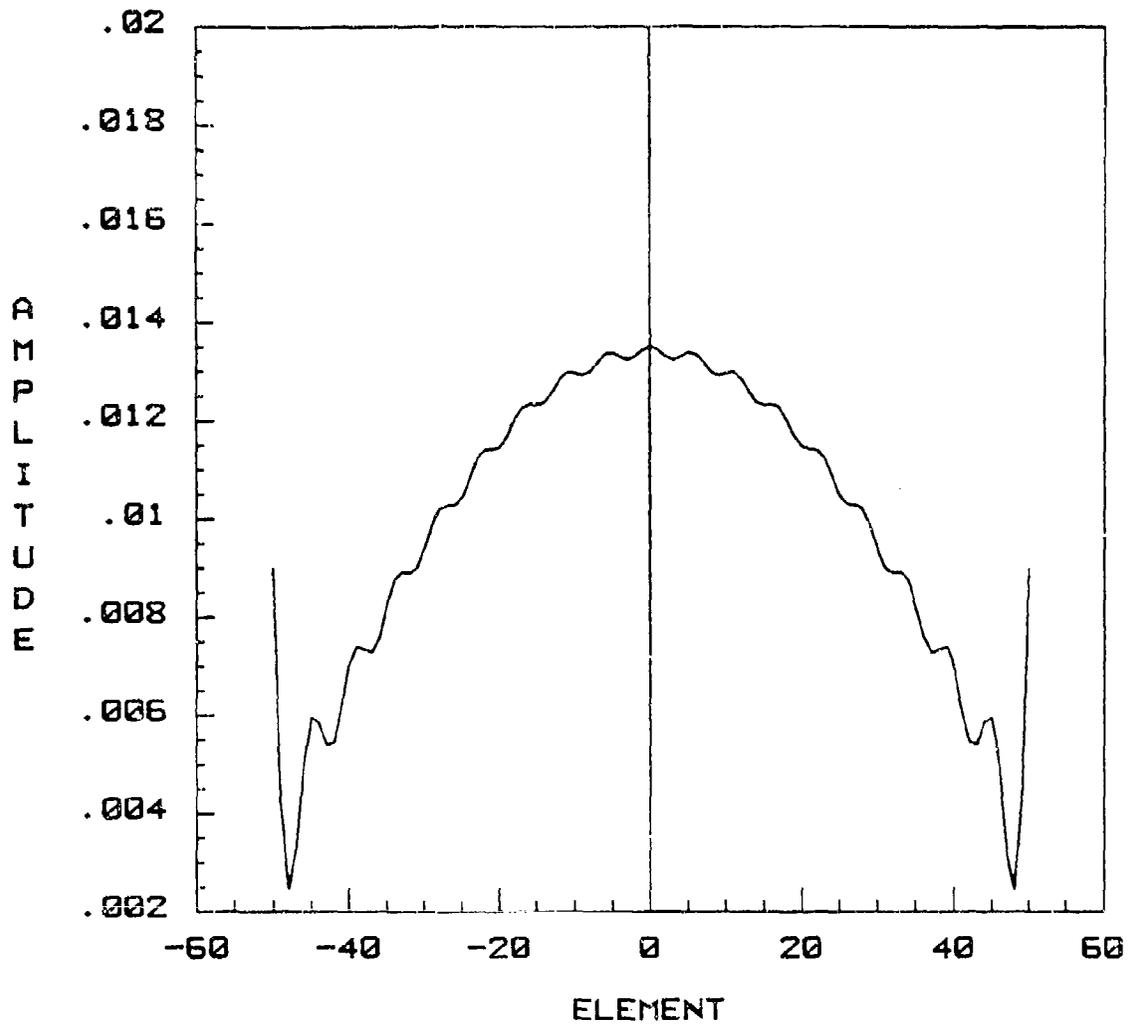
Fig 26. Optimum Amplitude Distribution for Filled Array
(Gradient Search Method)
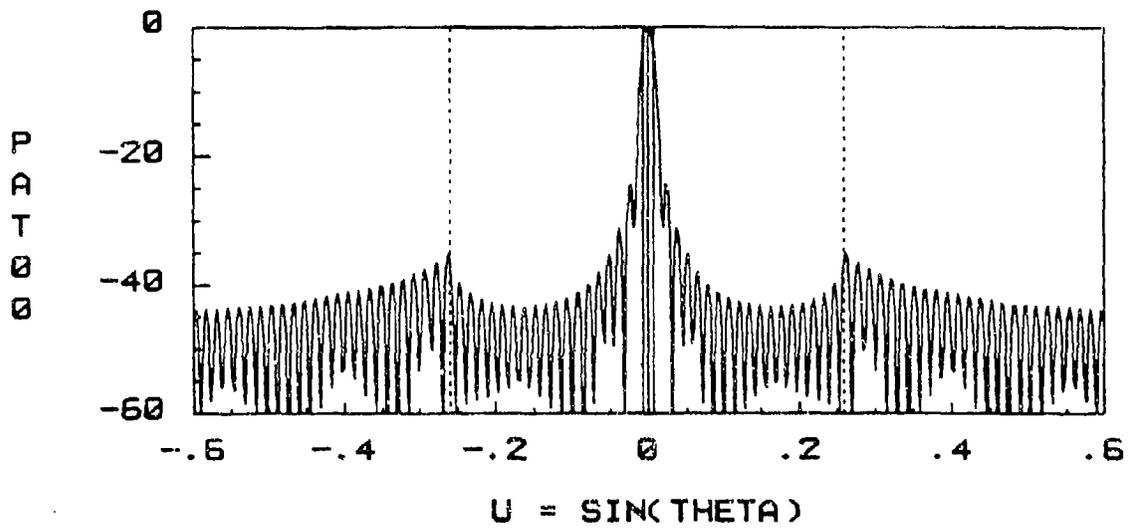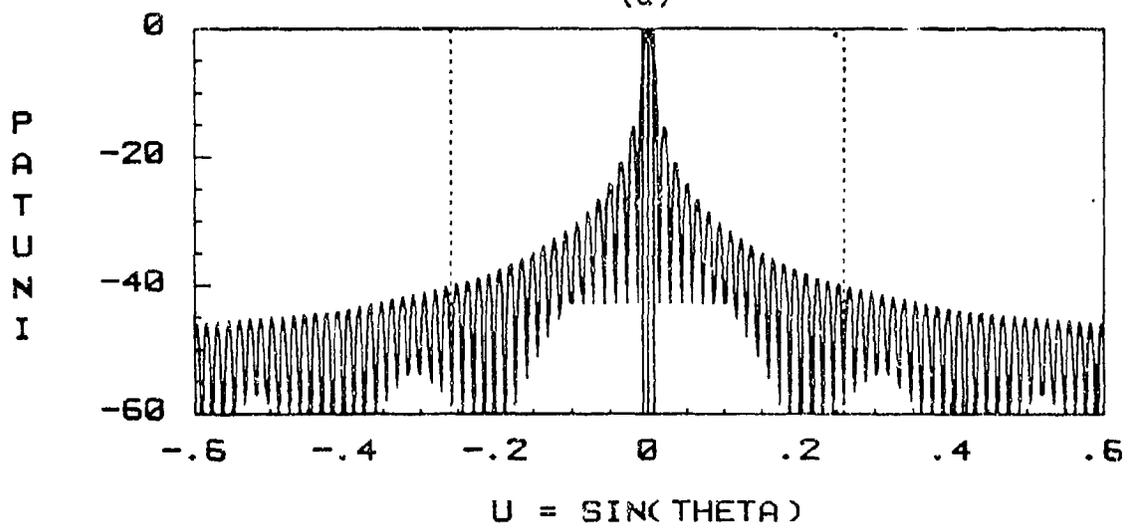
Fig 27. Far Field Pat ern of: a) Optimum Distribution vs b) Uniform Distribution
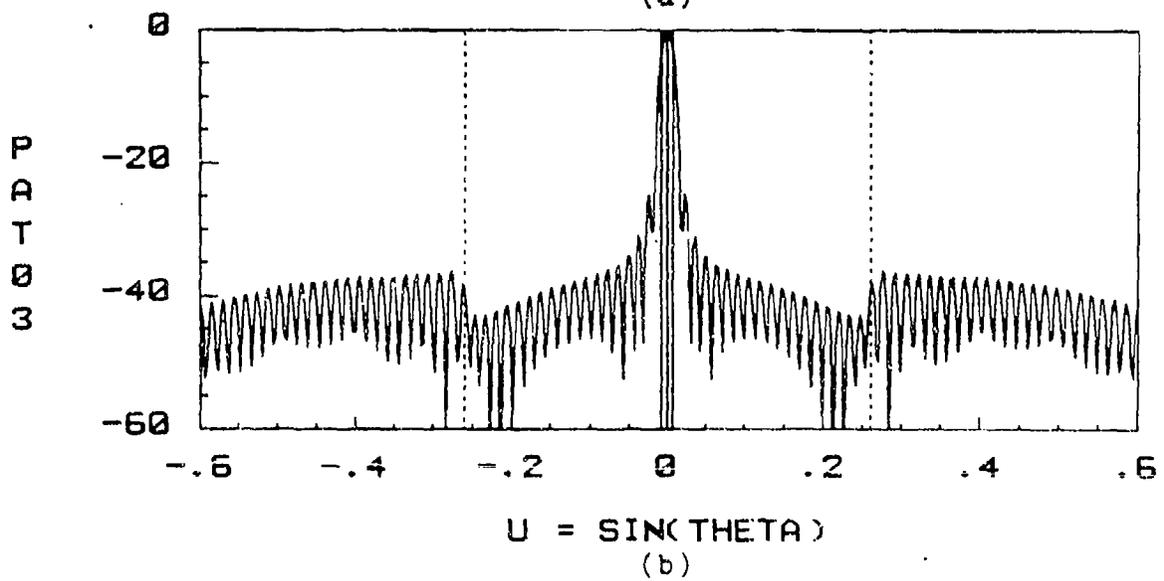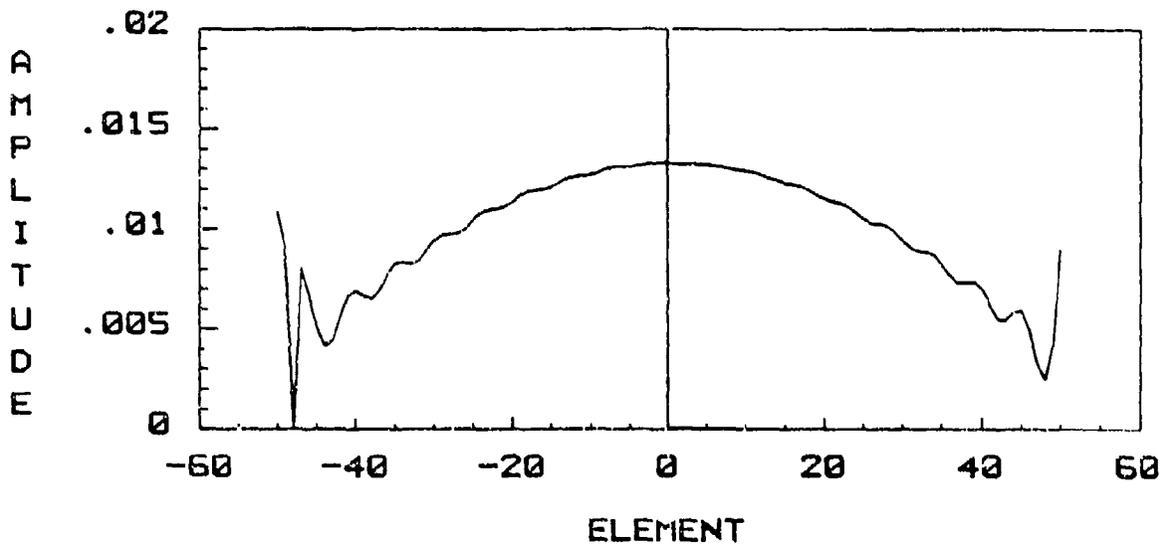
Fig 28.   One Failure Near End: a) Optimum Distribution; b)
Far Field Pattern
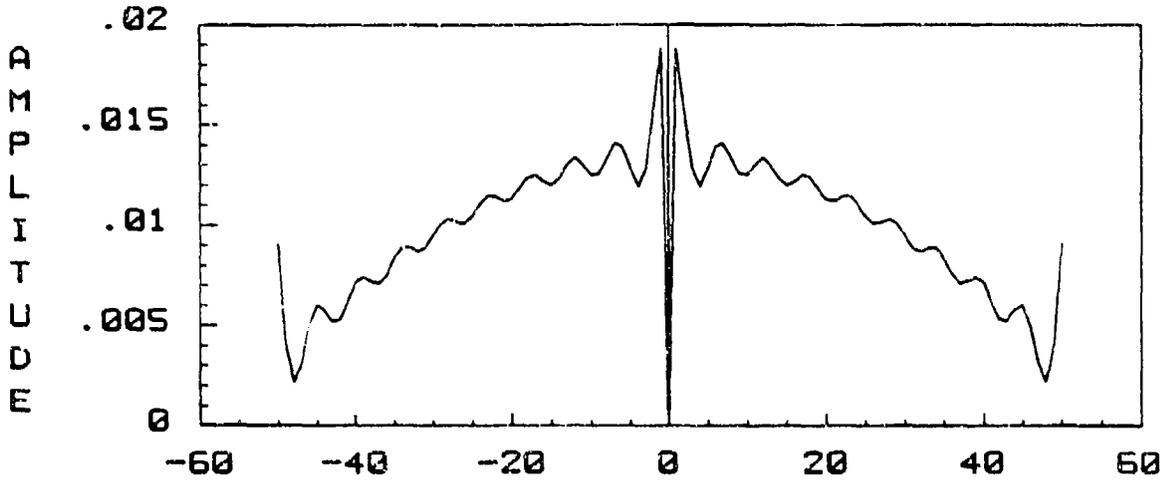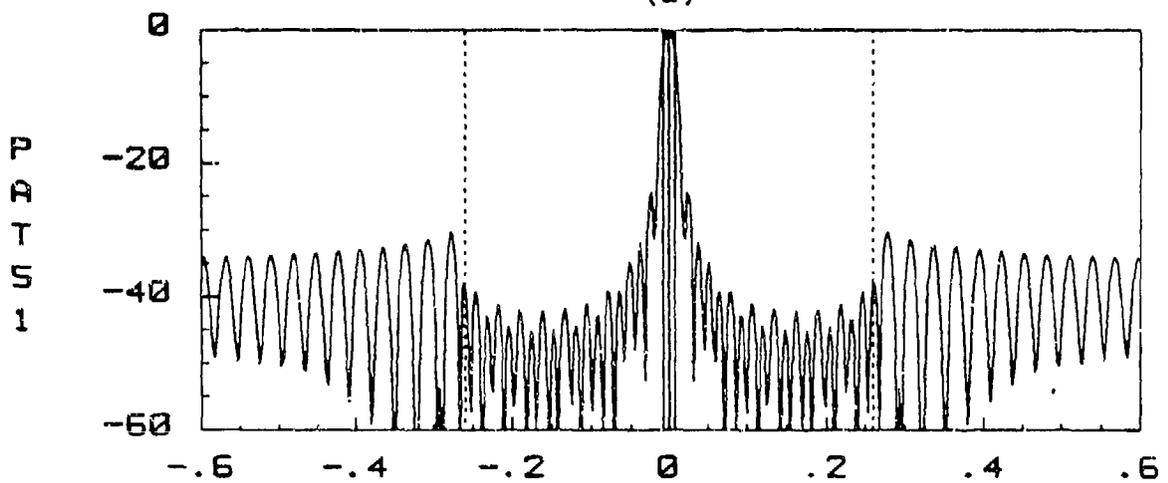
Fig 29.   One Failure Center: a) Optimum Distribution; b)
Far Field Pattern
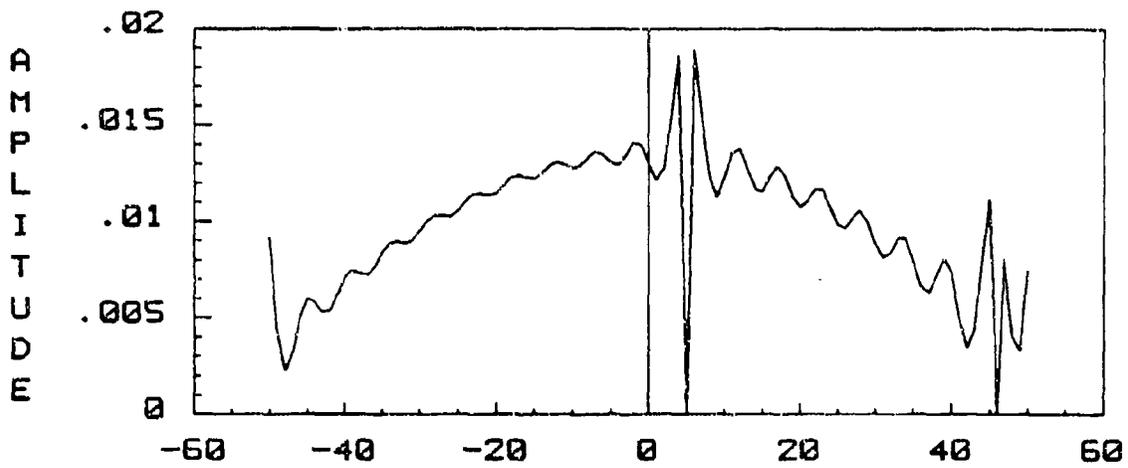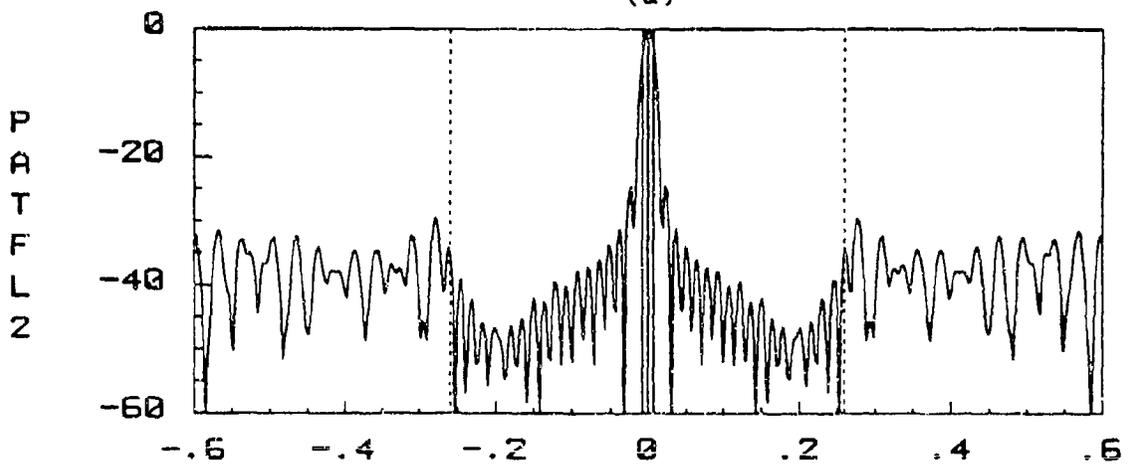
Fig 30. Two Failures Separated: a) Optimum Distribution;
b) Far Field Pattern
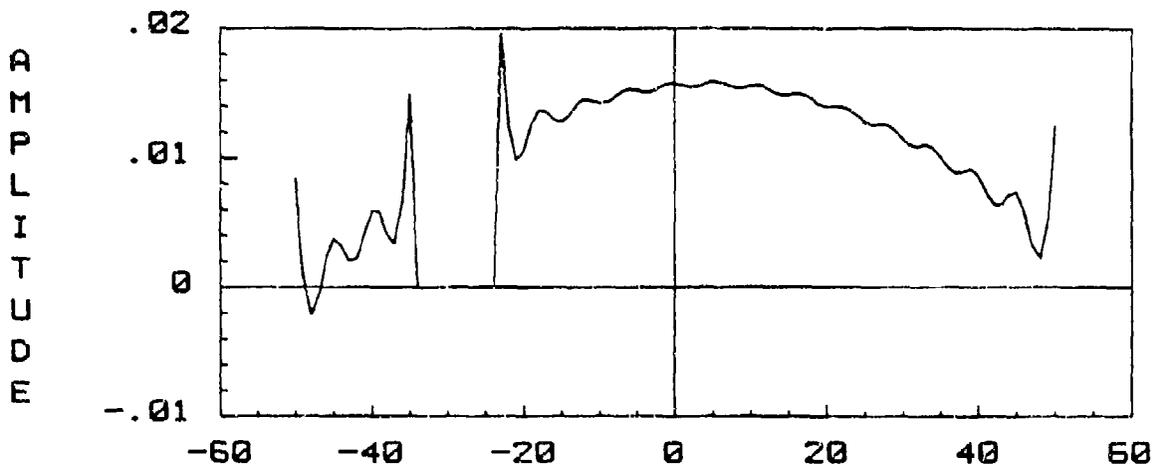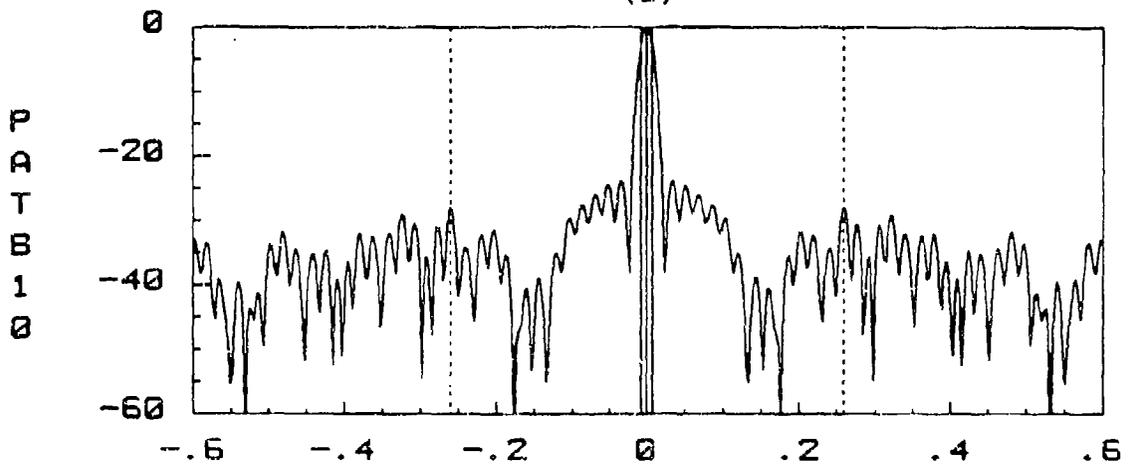
Fig 31. Ten Failures Together: a) Optimum Distribution; b) Far Field Pattern

TABLE II

Relative Cost Using Gradient Search Technique

Case	Pre-optimized	Optimized	Uniform
A	-	100.00	94.55
B	99.22	99.42	94.08
C	97.92	98.51	92.47
D	97.17	98.03	92.02
E	76.10	76.79	70.98

Dolph [5]. Figure 27 shows the improved pattern versus the pattern for an array with a uniform distribution. The algorithm was run again for an array with failed elements, failures similar to those used previously with Shore's eigenvalue method application. The results are shown in Figures 26 - 31. Table II displays the relative cost of the amplitude distributions before optimization (i.e., Figure 26 with the missing elements), after optimization, and without optimizing (uniform distribution with failed elements). It is important to note that each pattern is normalized. Because of the normalization, the maximum current on the optimized array can be much greater than the current on the uniform array. This may be an improper condition, for if one element is capable of handling a greater current, then all elements should be equally capable. After the algorithm

was modified to constrain the amplitude of the current on each individual element, the tests were repeated. In each case the algorithm stepped up to the constraint boundary - in order to minimize the cost function, all elements should be at their maximum current handling capabilities. Only in the case of limited total power available should amplitude tapering (as a form of power management) be used.

Optimizing the antenna pattern using phase only perturbations has been done by Voges and Butler [18] and Shore [16]. For this study, the same gradient search technique was used for phase only optimization as was used for amplitude only optimization. The result of running cases a through e with phase only perturbations is that the cost function was optimized when the phases of all the elements were the same. The major reason for this is that the area of interest is relatively large and is even-symmetric, but from the root locus analysis, it was determined that phase perturbations cause odd symmetric changes in array patterns. This means that an improvement on one side of the pattern will cause a degradation on the other side of the pattern. If the area of interest wasn't so large, optimization may occur [16] using non-zero phase shifts.

Results. The gradient search technique can be used to optimize a phased array with failed elements. In fact the solutions derived appear to be similar to the solutions

obtained with Shore's eigenvalue method. The reason for the
difference is that Shore was interested in minimizing the
amplitude perturbations, whereas the gradient search
optimization method was run with a constraint on the total
current only. This technique did show that if the current
carrying capabilty of each element is constrained, the
optimum distribution would be an uniform distribution, but
if the total current available was constrained, the optimum
distribution would approach the Dolph-Chebyshev
distribution. This technique also showed that for the case
of a complete element failure, phase perturbations would not
improve the pattern.

Direct Technique

The direct technique is a method which quickly arrives
at the optimum distribution for a partial array and is a
consequence of the results of the gradient search technique
and Shore's eigenvalue technique.

Description. The direct method originated while trying
to find an even quicker optimization method. When
optimizing a very large array, with tens of thousands of
elements, the gradient search technique and the eigenvalue
technique may take a prohibitively long time. Inititally,
it was assumed that the compensation for failures involved
mostly the elements in the proximity of the failure and as a
result, only those elements needed to be included in the
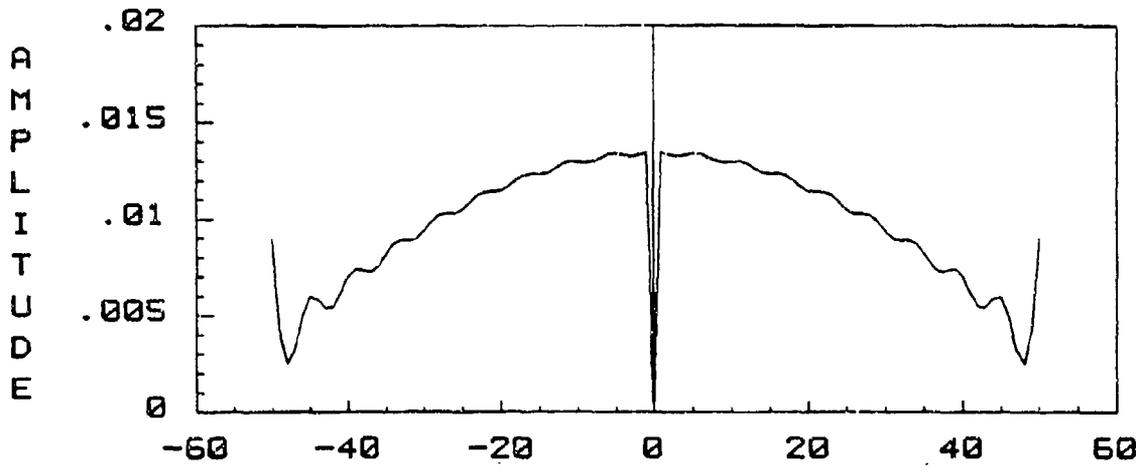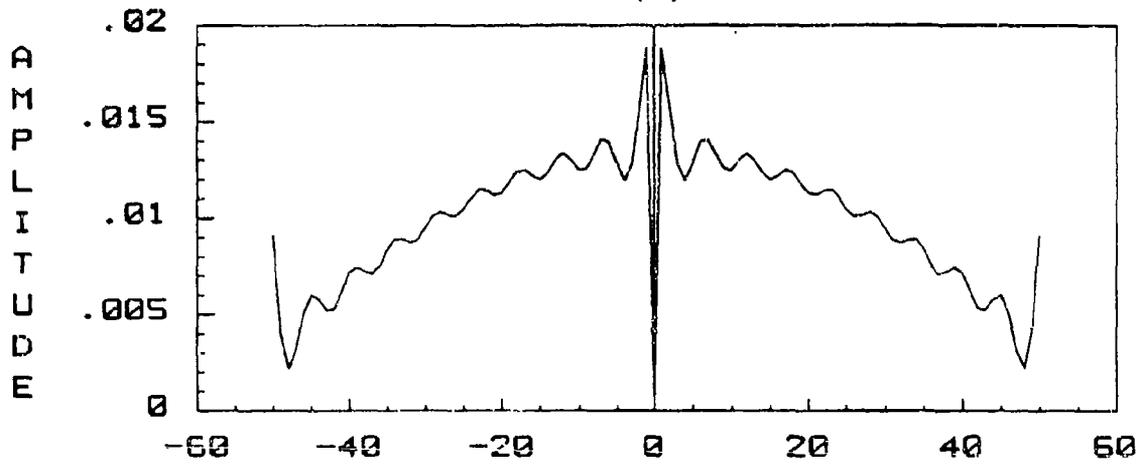
Fig 32. Amplitude Distribution: a) Before and b) After
Optimization

optimization analysis. To determine the required compensation (comp), the difference between the pre-optimized distribution and the optimized distribution (Fig 32) for an 101 element array with the center element failed was determined and is shown in Figure 33a. The compensation was initially believed to be a Sa function resulting from the inverse Fourier transform of what would undoubtably be the optimum pattern, a rect function centered at u=0 and with width 2Bw, and zero elsewhere in the FOV. When this proved incorrect, curve fitting programs were used. These proved fruitless and after some sleuthing, it was determined that the compensation is a scaled version of the center row of the H matrix (Fig 33b). In general, a failed element can be compensated for by simply adding a scaled version of the corresponding row from the H matrix to the remaining elements:

$$W_{opt} = W_{fail} + C,$$

where the elements of C are:

$$C_m = s\, H_{fm}, \text{ with } C_f = 0,$$

where C is a vector corresponding to the required compensation for each element, f is the position index of the failed element, m is the position index of the remaining elements, and s is the scaling which is proportional to the original amplitude of the failed element.

Application. The direct method can be applied by first

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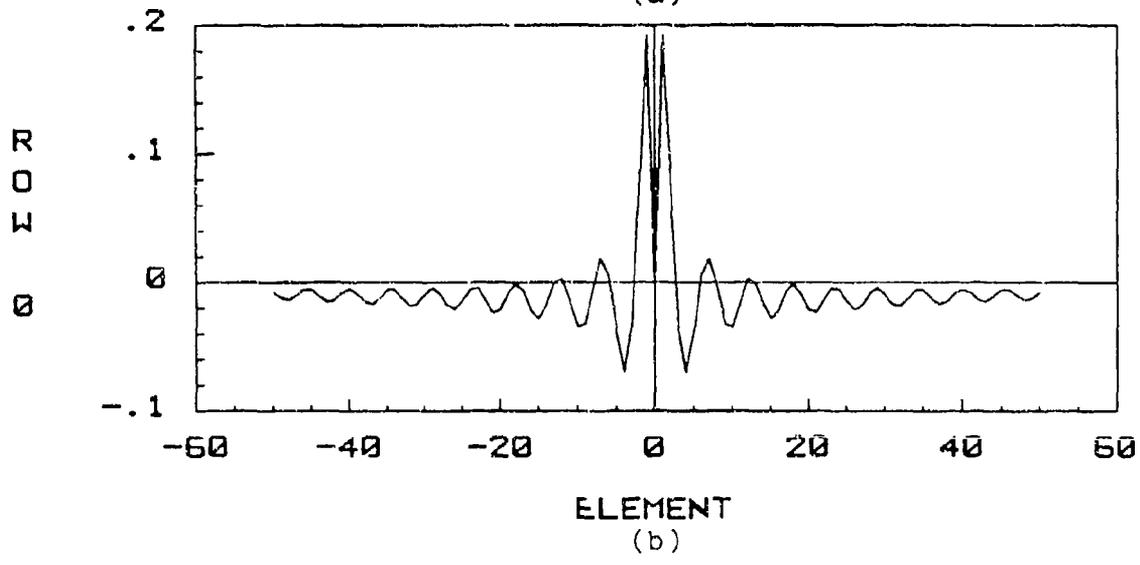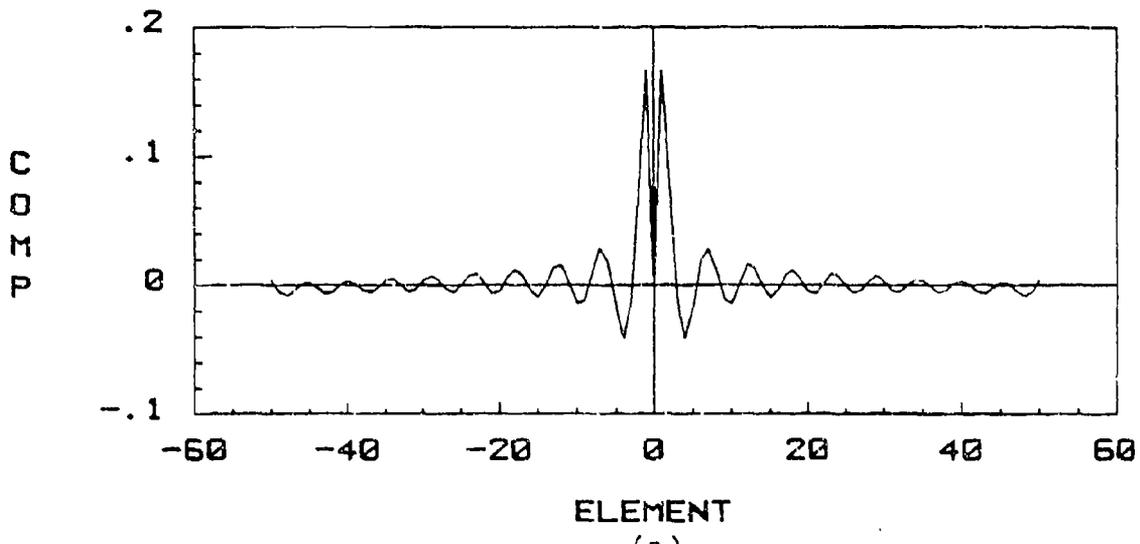
Fig 33. Compensation Required vs Row 0 of H Matrix

determining the optimum distribution for a full array, and then, when a failure occurs, simply calculating the required compensation using equation 10 (H_{mn}). The scaling factor will have to be pre-determined by running the gradient search program for each case. Once determined, it is then multiplied by the compensation. The scaled compensation can then be added to the pre-optimized distribution to arrive at the optimum distribution. Figure 34 shows the difference between the amplitude distributions generated by the direct method and the gradient search method.

Results. The direct optimization method is very quick and simple to implement. Once the location of the failed element is known, the power management system calculates the compensation required to re-optimize the pattern, accesses the scaling factor from a table, multiplies the compensation by the scaling factor, and then redistributes the available power accordingly by adding the scaled compensation to the currents of the remaining elements.
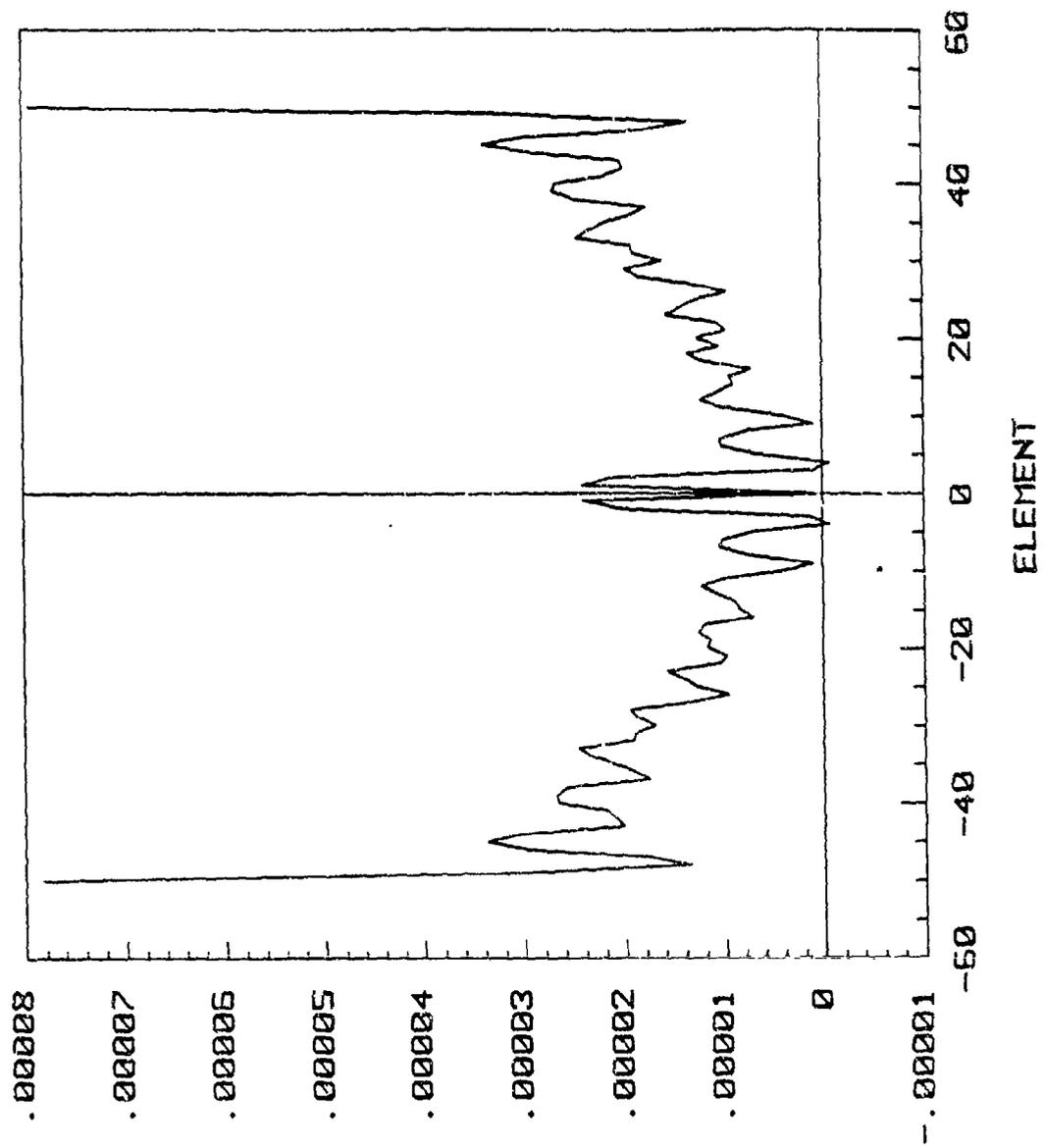
Fig 34. Error of Approximation

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IV. Conclusions and Recommendations

Using a phased array antenna on a system can add
dimensions of flexibility and control. By varying the phase
and amplitude of the current exciting each element, the far
field antenna pattern can be modified in such a way as to
obtain a desired, optimal pattern. The optimal pattern for
this study was defined as a pattern with minimal sidelobe
power in a +/- 15° field of view and maximal main lobe power
within a given beam width. Although many methods for
optimizing the pattern of a full array are available, this
work is unique because it shows methods of finding the
optimum pattern of an array which contains failed or missing
elements. Optimization methods in the literature which were
investigated include thinned array analysis, adaptive array
analysis, eigenvalue methods, null displacement techniques,
and assorted iterative techniques. Another method discussed
in this thesis, the direct technique, originated from the
need for a quick optimization method, especially when very
large arrays are used.

Conclusions

Of the optimization methods in the literature, only
Shore's eigenvalue method and the gradient search iterative
technique were successfully applied to this problem. Of the
two techniques, Shore's eigenvalue technique is quicker and
more accurate (i.e., absolute convergence to the solution);

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however, the gradient search technique is more flexible (i.e., easier to change or add any constraints). For an array with many elements neither technique is quick. The direct method, a spin off from the solutions arrived at from the eigenvalue method and the gradient search technique, has been shown to be relatively quick and accurate. The direct method is based on the fact that the amplitude perturbations required to compensate for a failed element can be described by a single equation (from the H matrix).

Recommendations

There are two types of recommendations for future study to be made: the first type involves improving the direct optimization technique and the second type involves considering more practical aspects of phased arrays.

The direct technique was developed through experimentation with computer simulations. It is suggested that the resulting failure compensation and scaling factor can be derived analytically. The equations netted through such efforts should be flexible enough to include practical constraints which are necessary for a realistic array.

The first practical recommendation is that the complete optimization technique should converge to an amplitude distribution which limits the antenna's Q-factor and maximizes the antenna's signal to noise ratio. It is well known that the Dolph-Chebyshev distribution, as was found in

the study, has an impractically high Q-factor. It has been suggested [8,9] that a Taylor n̄ distribution provides the optimum distribution for a practical array. The Taylor n̄ distribution maintains a low Q-factor and controls the n sidelobes near the main beam. The second practical recommendation is to consider more realistic failures. For instance, it is possible that the element may fail only partially - that the phase shifter gets stuck at a certian phase or that the amplitude merely becomes reduced instead of being totally knocked out. Also, a failed element may not become a matched load, as was assumed for the foregoing study, but may become a short or open, thereby making mutual coupling a more significant player in the problem. The third practical recommendation is that it is necessary to determine the effect which the final solution has on the atenna's operational characteristics such as frequency tolerance (bandwidth), scanning ability, and power efficiency. The final recommendation is to apply this work to a planar array with other than isotropically radiating elements.

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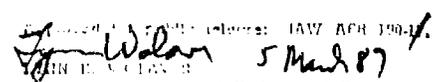
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19. Abstract

Using a phased array antenna on a system can add dimensions of flexibility and control. By varying the phase and amplitude of the current exciting each element, the far field antenna pattern can be modified in such a way as to obtain a desired, optimal pattern. The purpose of this thesis is to develop a rapid compensation method for any element failures by varying the currents exciting the remaining elements. Although many methods for optimizing the pattern of a full array are available, this work is unique because it develops methods of finding the optimum pattern of an array which contains failed or missing elements. Optimization methods in the literature which were investigated include thinned array analysis, adaptive array analysis, eigenvalue methods, null displacement techniques, and assorted iterative techniques. Another method discussed in this thesis, the direct technique, originated from the need for a quick optimization method, especially when very large arrays are used.

Of the optimization methods in the literature, only the eigenvalue method and the gradient search iterative technique were successfully applied to this problem. The direct method, a spin off from the eigenvalue method and the gradient search technique, has been shown to be relatively quick and accurate.

Recommendations for future study include the functional derivation describing the direct optimization method and including more practical aspects of phased arrays in the propagation model.