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Horizontal Variation of Atmospheric Extinction/Backscatter Coefficients

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Horizontal Variation of Atmospheric Extinction Backscatter Coefficients

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This document describes the evaluation of aerosol data to determine the horizontal homogeneity of atmospheric extinction/backscatter coefficients, as it applies to the inversion technique for deducing extinction from lidar returns.
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INTRODUCTION

The development and evaluation of lidars capable of measuring atmospheric parameters, such as extinction and backscatter coefficients, depend upon various factors. These include the ability to quickly and accurately extract the coefficients from the returns of monostatic single-wavelength lidar systems by inverting the lidar equation governing the single-scattering phenomena. Two basic mathematical techniques have been suggested/used to simplify the inversion technique (reference 1). The first technique utilizes the fundamental assumptions that the extinction and backscatter coefficients are horizontally homogeneous, while the latter technique assumes that a power law relationship exists between the backscatter and extinction. This report addresses some aspects of the former category, or horizontal homogeneity of extinction and backscatter.
BACKGROUND

The basic equation for a pulsed monostatic single-wavelength lidar is

\[ P(r) = P_0 \frac{cT}{2} A \frac{\beta(r)}{r^2} \exp \left[ -2\sigma(r)dr \right] \tag{1} \]

where \( P(r) \) is the instantaneous received power at any instance from a scattering volume at range \( r \); \( P_0 \) the transmitted power; \( c \), the velocity of light; \( T \), the pulse duration; \( A \), the effective receiver aperture, and \( \beta(r) \) and \( \sigma(r) \) are, respectively, the backscatter and extinction coefficients of the atmosphere. If one defines \( S(r) \) as the "logarithmic range adjusted power," and \( S(r_0) \) as \( S(r) \) evaluated at \( r_0 \) (constant reference range), equation (1) becomes

\[ S(r) - S(r_0) = \ln \frac{\beta}{\beta_0} - 2 \int_{r_o}^{r} \sigma dr \tag{2} \]

where \( \beta_0 = \beta(r_0) \). The differential equation for (2) is

\[ \frac{dS}{dr} = \frac{1}{\beta} - \frac{d\beta}{dr} - 2\sigma \tag{3} \]

The solution for this equation when \( d\beta/dr \) does not equal zero requires knowing or assuming a relationship between \( \beta \) and \( \sigma \). However, if the atmosphere is homogeneous, then the \( d\beta/dr \) term of (3) equals zero and the extinction coefficient can be expressed directly in terms of the slope of the \( S(r) \) curve or

\[ \sigma = -\frac{1}{2} \frac{dS}{dr} \tag{4} \]

This equation is the basis for the slope inversion technique for inverting the lidar equation. The extinction coefficient is obtained by using the slope of the lidars received \( S(r) \) curve \( (dS/dr) \) as determined by a least squares straight line fit of the data over an interval where \( S(r) \) versus \( r \) appears to be nearly a straight line. By applying this assumption over a succession of small intervals, it is also assumed that a reasonable approximation to \( \sigma = \sigma(r) \) could be obtained for a notably inhomogeneous atmosphere (reference 1). The validity of this technique, however, depends upon the extent to which the atmosphere is horizontally homogeneous, i.e., \( df/dr \) and \( d\sigma/dr = 0 \), and under what conditions can horizontal homogeneity be assumed.

MEASUREMENTS

The spatial variability of the extinction and backscatter coefficients can be determined by measuring the particle size distribution along a given horizontal path and calculating the coefficients using MIE theory (reference 2). This technique was accomplished utilizing the NOSC aircraft (reference 3) and making aerosol size
distribution measurements along constant altitude radials over a 4-nmi path between 100 and 2500 feet. Aircraft altitude and position along the radials were determined by a Bonzer TRA-2500 radar altimeter and a Texas Instrument TI9100 LORAN-C receiver, respectively. On each radial flight, the aircraft was flown at the assigned altitude and the position of the aircraft along the radial recorded digitally. Figure 1 shows the typical aircraft flight pattern. All flights were made into and away from the prevailing winds.

![Aircraft flight pattern](image)

Figure 1: Aircraft constant altitude flight plan for obtaining the vertical structure of atmospheric aerosols

Aerosol instrumentation on board the aircraft consisted of the Particle Measuring Systems (PMS) ASSP-100 and OAP-200 aerosol spectrometers. The particle size range of the two spectrometers is 0.5 to 300 microns diameter. A complete aerosol spectrum over this size range is obtained every 8 seconds. With an aircraft speed of 53.6 m/sec (120 mph), this 8-second sample time per distribution represents a horizontal distance traveled of 429 meters. This, then, is the minimum resolvable scale size for horizontal homogeneity that can be observed in the aircraft aerosol data. By nature of the sampling characteristics of the ASSP 100, i.e., four separate size ranges, each taking 2 seconds to provide a complete distribution from 0.5 to 30 microns (OAP 200 covers 30 to 300 microns in a sample time of 2 seconds), it is assumed that horizontal homogeneity exists throughout the total 8-second sampling period or over the 429-meter sampling distance, i.e., particles that are sized in range...
1 during the first 2 seconds of the 8-second period are assumed to exist also during the next 6 seconds while the spectrometers ranges 2, 3, and 4 are sampled. At the end of the 8-second sampling period, all overlap within the size channels are discarded and the remaining data combined to form one single distribution from which the extinction and backscatter coefficients are calculated.

Aerosol data from two aircraft flights, 29 May 1981 and 17 June 1986, were used to investigate the horizontal variability of extinction and backscatter coefficients. Both flights were conducted over open-ocean conditions approximately 40 nmi southwest of San Diego. Eighteen constant-altitude radials were flown at differing altitudes (figure 1). Radials were made both within and below a marine stratus layer. Flight logs for both flights are included in Appendix A. Extinction and backscatter coefficients were subsequently calculated using MIE theory for each measured aerosol size distribution along the constant altitude radial, i.e., every 8 seconds. Calculated data include the extinction and backscatter coefficients for each 8-second time slice (each aerosol distribution measured along the horizontal radial) as well as the average coefficients over the entire horizontal radial. These data are presented in Appendices B and C for extinction and backscatter, respectively. Plotted are the calculated extinction and backscatter coefficients normalized to the average coefficient over the path as a function of waypoint position. Absolute horizontal homogeneity (\( \frac{da}{dr} = \frac{d\beta}{dr} = 0 \)) is represented by a normalized \( \frac{da}{dr} \) extinction or backscatter coefficient of one (1). Also plotted for each of the constant altitude radials is the aircraft altitude.

Normalized extinction coefficients varied from one waypoint to another (over a distance of 429 meters) by as much as a factor of two. The average standard deviation (SD) of extinction about the norm of one for all of the runs, both within and below clouds, was 0.37 and varied between the limits of 0.14 ≤ SD ≤ 0.65. Below the clouds (including flights just at the cloud base but including no cloud data), the average standard deviation of extinction increased to 0.43 and varied from 0.25 ≤ SD ≤ 0.65. Within the stratus clouds (no below cloud data), the average standard deviation decreased to 0.22 and varied from 0.14 ≤ SD ≤ 0.26. Shown in Figure 2 is a plot of the standard deviation of the normalized extinction values as a function of altitude. These data for normalized extinction indicate that the horizontal variability of extinction increased with altitude and was at a maximum at cloud base. Within the stratus layer, and particularly at the middle of the layer, the variability of \( \sigma \) was at a minimum. The fluctuations observed in the extinction and backscatter coefficients are not correlated to altitude changes. The cross-correlation coefficients between altitude, extinction, and backscatter for all radials was less than significant (<0.4).

Similar results are observed for the backscatter coefficients (Appendix C) where the SD of the normalized backscatter below the clouds (no cloud data) varied from 0.20 ≤ SD ≤ 0.84, and within the clouds varied from 0.18 ≤ SD ≤ 0.49. The vertical profile of the normalized backscatter SD (figure 3) showed increased horizontal variability with altitude and a peak occurring at the cloud base.
CONCLUSIONS

Aerosol size distribution measurements made by the NOSC airborne meteorological platform on constant-altitude radials for 9 May 1981 and 17 June 1986 indicate that for scale sizes of 429 meters (minimum resolvable sampling distance for the airborne aerosol spectrometers), the coefficients of atmospheric extinction and backscatter do show appreciable horizontal variation both below and within a stratus layer. The variability, however, is much less within the stratus layer than below. Since horizontal homogeneity is defined where $d\sigma/dr = d\beta/dr = 0$, horizontal homogeneity does not exist for these two sampled periods. In the presence of stratus clouds, this non-homogeneity can be assumed to be characteristic of the marine boundary layer.

Since horizontal homogeneity does not appear to exist (at least for scale sizes of 429 meters), and since $d\sigma/dr$ and $d\beta/dr$ does not equal zero, extinction values cannot be deduced from the slope of the lidar $S(r)$ curve as given by equation (4) (deduced from equation (3) when $d\beta/dr = 0$). Homogeneity within the scale size of 429 meters is still unknown. One possible way to investigate this would be to operate the PMS spectrometers on a single range (sampling time now 1 second) and repeat the described flight configuration. This would reduce the minimum resolvable scale size to 54 meters. However, this would limit the observable aerosol size range of the aerosol particles counters.

In conclusion, these results indicate that the slope technique for deducing extinction and backscatter from lidar returns does not appear to be a valid inversion technique. One must, therefore, determine or use a known relationship between $\beta$ and $\sigma$, coupled with the known boundary conditions, before the lidar equation can be solved for extinction.
Figure 2. Standard deviation of extinction as a function of altitude.
Figure 3. Standard deviation of backscatter as a function of altitude.
REFERENCES


*NOSC TNs are working documents intended for internal use only.*
APPENDIX A

Aircraft Flight Logs for 29 May 1981 and 17 June 1986
**Table 1. Aircraft Flight Logs for 29 May 1981 and 17 Jun 1986**

<table>
<thead>
<tr>
<th>Date</th>
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APPENDIX B

Horizontal Variation of Normalized Extinction Coefficients
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

LEGEND

ALT

Altitude (ft)

Waypoints Along 4 nmi Path (15 Waypoints)

Normalized Extinction

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Legend:
- - - EXT
- - - ALT

Normalized Extinction

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Normalized Extinction

Waypoints Along 4nmi Path (15 Waypoints)

Altitude (ft)

LEGEND
- - EXT
- - ALT
HORIZONTAL VARIATION OF EXTINCTION

29 May 1981

Normalized Extinction

Altitude (ft)

1200 1160 1160 1140 1120 1100 1080 1080 1040 1020 1000

Waypoints Along 4nm Path
(15 Waypoints)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Legend

EXT ALT
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Normalized Extinction

Waypoints Along 4nmi Path (15 Waypoints)

Altitude (ft)

Legend
- EXT
- ALT
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Normalized Extinction

Waypoints Along 4nmi Path (15 Waypoints)

Altitude (ft)

Legend
--- EXT
--- ALT

0 0.5 1 1.5 2 2.5 3
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

1300 1320 1340 1360 1380 1400 1420 1440 1460 1480 1500
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Normalized Extinction

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF EXTINCTION
29 May 1981

LEGEND

--- EXT

--- ALT

Normalized Extinction

Waypoints Along 4nmi Path (15 Waypoints)

Altitude (ft)
HORIZONTAL VARIATION OF EXTINCTION
17 June 1986

Normalized Extinction

Waypoints Along 4nmi Path
(12 Waypoints)
HORIZONTAL VARIATION OF EXTINCTION
17 June 1986

Normalized Extinction

Waypoints Along 4nmi Path
(19 Waypoints)

LEGEND
--- EXT
--- ALT

Altitude (ft)
HORIZONTAL VARIATION OF EXTINCTION

Altitude (ft)

Normalized Extinction

Waypoints Along 4 nmi Path
(19 Waypoints)

17 June 1986
HORIZONTAL VARIATION OF EXTINCTION
17 June 1986

Waypoints Along 4nm Path
(19 Waypoints)

Normalized Extinction

Altitude (ft)

Legend

EXT ALT
APPENDIX C

Horizontal Variation of Normalized Backscatter Coefficients
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

Normalized Backscatter

Altitude (ft)

LEGEND

BSCAT

ALT

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

Normalized Backscatter

Waypoints Along 4nmi Path (15 Waypoints)

 Legend
--- BSCAT
--- ALT

Altitude (ft)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

LEGEND
--- BSCAT
--- ALT

Normalized Backscatter

Altitude (ft)

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

LEGEND

--- BSCAT
--- ALT

Normalized Backscatter

Waypoints Along 4nmi Path
(15 Waypoints)

Altitude (ft)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

Normalized Backscatter

Legend:
- BSCAT
- ALT

Waypoints Along 4nmi Path (15 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

LEGEND
--- BSCAT
--- ALT

Normalized Backscatter

Waypoints Along 4nmi Path
(15 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
29 May 1981

Normalized Backscatter

Waypoints Along 4nmi Path
(15 Waypoints)

Altitude (ft)

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</table>

Legend
- BSCAT
- ALT
HORIZONTAL VARIATION OF BACKSCATTER
17 June 1986

LEGEND

- BSCAT

- ALT

Normalized Backscatter

Altitude (ft)

Waypoints Along 4nmi Path
(12 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
17 June 1986

Waypoints Along 4nmi Path
(18 Waypoints)

Normalized Backscatter

Altitude (ft)

LEGEND

BSCAT

ALT
HORIZONTAL VARIATION OF BACKSCATTER
17 June 1986

Normilized Backscatter

Altitude (ft)

Waypoints Along 4nmi Path (19 Waypoints)
HORIZONTAL VARIATION OF BACKSCATTER
17 June 1986

Normalized Backscatter

Waypoints Along 4nmi Path
(19 Waypoints)
END
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