From previous studies of scattering from multiple gratings of resonant compliant tubes in water (R. P. Radlinski and M. M. Simon, J. Acoust. Soc. Am., vol. 72 (1982)), the excitation of noncompliant, antisymmetric structural modes by nearfield evanescent waves was found to severely degrade the reflectivity of closely packed gratings in the bandwidth of excitation of the compliant symmetric modes. Also, transmission resonances due to the spring-mass-spring configuration of the two gratings separated by a fluid mass diminished low frequency performance. In this paper, encapsulating the gratings in a low-stiffness elastomer is shown experimentally to have a minimum effect on single and widely separated gratings with respect to fluid but enhances the performance of closely packed gratings. Comparison of insertion loss performance with a high stiffness encapsulant indicates dramatic differences in bandwidth and frequency response. A mathematical model will be discussed and compared with the experimental data.
Scattering From Multiple Compliant Tube Gratings in A Viscoelastic Layer

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PREFACE

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SCATTERING FROM MULTIPLE COMPLIANT TUBE GRATINGS IN AN ELASTIC LAYER

INTRODUCTION

Compliant tubes are used as reflecting baffles. Their pressure release characteristics are the result of a net volume velocity associated with structural bending modes that are symmetric about the major and minor axes of the tubes. In this paper, the interactions between gratings of tubes encapsulated in two elastomers that have shear moduli which differ by two orders of magnitude will be investigated. The spacing between gratings are important because of evanescent waves which can excite noncompliant structural modes that degrade the performance of the baffle. Typically each grating is composed of a different sized tube to enhance the bandwidth of decoupling. Placing the grating in a suitable elastomer provides a means of minimizing the effects of the evanescent waves.

The first section of this presentation will compare experimental data of tubes in fluid versus tubes in elastic layers. The second part discusses a two dimensional theoretical model that includes propagation of both dilational and shear waves in the elastic layer with compliant tube inclusions. Comparisons of the calculation from the model with the experimental data are discussed in the third section.
To measure insertion loss, an unshaded twenty-four element array was used as the receiver to discriminate against diffraction around the gratings and to average residual evanescent waves. Diffraction was further reduced by positioning the array diagonally with respect to the rectangular panels. Initially the array was used to measure the incident field without the tubing. The gratings were then inserted between the source and receiver and the transmitted pressure relative to the incident pressure was determined. Insertion loss is defined here as 20 log (incident pressure/transmitted pressure). A large insertion loss implies the transmitted energy is small. For the configurations that will be considered, the grating spacing is always less than a wavelength in water, thus only one wave propagates outside the elastomer with a direction normal to the gratings. The other waves are evanescent in the sense that they propagate parallel to the grating but decay rapidly in a direction normal to the gratings.
The measured normal incidence insertion loss for a single grating to be used later in combined multiple gratings is presented to show the stiffness effect of an encapsulant as a function of the shear modulus. The frequency axis is normalized to the in-air resonance frequency \(f_1\) of the first compliant bending mode. The tubes are steel oval shells and have an aspect ratio of major to minor axis of about 7.5. The static compliance is about 200 times that of water. When the tubes are encapsulated in a low stiffness elastomer, the locations of insertion loss maxima do not change significantly but the depth decreases because of the damping factor. As the stiffness and damping increase, the character of the insertion loss changes dramatically. The additional stiffness on the tubes increases the effective resonance frequency of the structural modes. The stiffness also decreases the vibrational motion so that more energy tends to be transmitted through the layer. The increased insertion loss at about 5.5\(f_1\) is due to the second symmetric bending resonance. Both elastomers have a bulk modulus close to that of water.
The performance of the smaller tube grating to be used in combination with the larger tube grating is presented. The fundamental compliant mode of the smaller tubes ($f_2$) is equal to $1.8f_1$. Because mass loading is more significant for the wider spaced smaller tubes, the insertion loss maximum are lower in frequency than the in-air compliant resonance of the individual elements ($f_2$). With wider spaced gratings, the bandwidth of insertion loss is narrower. The performance changes with encapsulation are similar to that for the larger tube grating.
The two grating combination of the above single layers was measured for wide separation of $\lambda_1/13$ where $\lambda_1$ is the wavelength in water at the fundamental compliant structural mode of the larger tubes. The additional bandwidth obtained from the combination is significant. The transmission maximum at $\alpha$ is due to a resonance of the compliance of the gratings with the inner layer mass. The low stiffness elastomer has a minimum effect on the performance with respect to the unencapsulated tubes, but the high stiffness encapsulant again shifts the maximum bandwidth of insertion loss to higher frequencies and degrades performance.
For the closely spaced gratings of separation λ₁/82, the fundamental insertion loss bandwidth is severely degraded. This decrease in the fundamental bandwidth is caused by the nearfield excitation of the second noncompliant bending mode (S,A)₁ of the larger tubes and the enhanced motion of the radiating rigid body mode. Because the decrease in mass between the two gratings raises the resonance frequency of the first transmission resonance, increased insertion loss with respect to the widely spaced gratings is found at low frequencies. The low stiffness elastomer actually improves the performance bandwidth with respect to the gratings in fluid. Excitation of the antisymmetric mode of the smaller tubes is designated by (s,a)₁. The higher stiffness encapsulant again degrades performance.
A schematic representation of multiple compliant tube gratings embedded in a viscoelastic layer is shown for a normally incident plane wave. A two dimensional representation is used to describe the interaction. The portion of the incident wave which enters the layer undergoes multiple reflections at both the tube and elastic-fluid boundaries. Some of this energy is absorbed and some is transmitted through the layer. Because of bending and rigid body translational motion of the tubes, shear waves are generated even at normal incidence. With the symmetry in the grating arrangement, the solution reduces to satisfying the boundary conditions between the planes of symmetry. The boundary condition in the fluid at the symmetry planes is that the transverse velocity is zero. In the elastomeric material, the additional boundary condition is zero transverse shear at the symmetry planes.
FIELD EQUATIONS

INCIDENT AND REFLECTED PRESSURE WAVES

\[ P = e^{ik_0x} + \sum_{n=0}^{\infty} R_n e^{-ik_nx} \cos \alpha_n y \]

\[ k_n^2 = k_0^2 - \alpha_n^2; \alpha_n = \frac{n\pi}{d} \]

DISPLACEMENT POTENTIALS FOR DILATATIONAL & SHEAR WAVES

\[ \phi_j = \sum_{m=0}^{\infty} \left\{ A_m e^{ik_mp_x} + B_m e^{-ik_mp_x} \right\} \cos \alpha_m (y-c_j) \]

\[ A_j = \sum_{m=0}^{\infty} \left\{ C_m e^{ik_ms_x} + D_m e^{-ik_ms_x} \right\} \sin \alpha_m (y-c_j) \]

\[ kp^2 = k_p^2 - \alpha_m^2; \quad k_{ms}^2 = k_s^2 - \alpha_m^2; \alpha_m = \frac{m\pi}{d-c_j} \]

\[ k_p, k_s \text{ complex dilatational wavenumber} \]

\[ k_s \text{ complex shear wavenumber} \]

TRANSMITTED PRESSURE WAVES

\[ P_T = \sum_{n=0}^{\infty} T_n e^{ik_nx} \cos \alpha_n y \]

VIEWGRAPH 7

The total pressure in the fluid above the elastomer is the sum of the incident plane wave and an infinite series of reflected waves. \( R_n \) represents an unknown scattering coefficient. The wavenumber of the \( n \)-th reflected wave, namely \( k_n \), is real when \( k_0 \), the fluid wavenumber, is greater than the grating parameter \( \alpha_n \). In this case, the reflected wave propagates away from the grating into the fluid. For \( k_0 \) less than \( \alpha_n \), \( k_n \) is imaginary and an evanescent wave propagates nearly parallel to the grating and decays exponentially away from the grating. In the assumed linear viscoelastic layer, the scalar and vector displacement potentials for the dilatational and shear waves are given as infinite series of inhomogeneous waves traveling in both directions. The dilatational and shear wavenumbers \( k_p \) and \( k_s \) are assumed complex. The constants \( c_j \) and \( c_j' \) are determined.
by the geometry. \( A_m, B_m, C_m, \) and \( D_m \) are scattering coefficients determined by the boundary conditions. In the viscoelastic interstice between tubes, the transverse velocity and transverse shear stress are assumed zero at the tube-elastomer abutment. Of particular interest in this presentation is the calculation of the transmitted pressure. For grating spacing less than a wavelength in the fluid, the transmitted pressure in the field is determined by \( T_0 \) which is the scattering coefficient of the single transmitted propagating wave.
The plates are assumed to obey thin plate theory. The compliant symmetric modes of the compliant tube whose motion results in a net volume displacement are approximated by a sum of cosines and hyperbolic cosines. The designation \((s,s)\) implies symmetric about the major and minor axes. Mode shaped \((s,s)_0\) is the fundamental and \((s,s)_1\) is the second symmetric bending mode.
The modes that are symmetric about the minor axis but antisymmetric about the major axis \((s,a)\) include the rigid body mode \((s,a)_0\) and the "banana" shaped mode \((s,a)_1\). These modes are described mathematically with guided end conditions or zero slope and zero transverse shear at the ends. The \((s,a)_1\) mode was shown in the experimental data to be excited by evanescent waves for closely packed gratings and it degrades insertion loss performance.
The boundary conditions to be satisfied at the fluid-elastomer and elastic-elastic interfaces are those common for plane strain problems. The continuity of tangential displacement at the plate-elastomer boundary is derived by assuming inextentional motion of the plate. A system of linear equations is formed by matching the boundary conditions. Solutions of the simultaneous equations arising from truncation of the infinite sets are used to determine the scattering coefficients.
A comparison of measurements and calculations of insertion loss are shown for both the larger tubes in a dense packing and the smaller tubes in a sparse packing. Both gratings have a periodicity of \( d \). The Young's moduli of the materials were measured with a Bruel and Kjaer apparatus where a strip is coated on a metal bar and the resonances of the uncoated and coated bar are compared to obtain the complex-valued material properties. No measurement was available for the bulk or dilatational modulus of the materials but in this frequency range, the calculations were not sensitive to say a 50% variation in the bulk modulus. A bulk modulus similar to water was assumed for these materials. As seen in the comparisons, for both cases excellent agreement is found for the single gratings.
With this comparison, the differences in insertion loss performance due to spacing between gratings is dramatically apparent. Again agreement between the predicted and measured insertion loss for both cases is good. The low frequency performance is better for the closely packed gratings because the transmission resonance seen at about 0.7f₁ for the widely spaced gratings shifts to about 0.9f₁.
Although good agreement is seen between the experimental data and predictions, it was found that the calculations were sensitive to small variations of the thickness of the thin covering of the asymmetric layer configuration. With a very thin layer of the polyurethane on one side of the tubing, the additional stiffness effects can be mitigated. Neither configuration would perform satisfactorily as a decoupler but might be more appropriately used as a anechoic absorptive material.
The large oscillations in insertion loss in the undamped material are due to thickness resonances of the elastic layer from the shear waves generated by the motion of the tubes. The damping in the viscoelastic material converts these shear waves to heat and thus smooths the response of the insertion loss curves.
MEASUREMENTS AND CALCULATIONS FOR DOUBLE GRATINGS IN HIGH STIFFNESS POLYURETHANE (20,000 psi, $\delta = 0.5$)

Viewgraph 15

Differences between insertion loss performance between the closely packed and widely spaced gratings are more significant for the higher stiffened polyurethane encapsulant. The difference between calculations and experimental data for the higher stiffness material may be partially due to uncertainties in determination of the shear modulus. In the rubber-glass transition region of this polymer, small uncertainties in temperatures result in significant uncertainties in determination of the material properties. When these measurements were performed at the NUSC Dodge Pond facility, a temperature gradient was measured across the panel depth.
CONCLUSIONS

Good agreement has been found between predictions of an analytical model and measurement of insertion loss from multiple gratings of compliant tubes in a viscoelastic layer immersed in water. For single gratings, low shear modulus materials were shown to have minimum effect with respect to in fluid measurements; however, high shear modulus polymers both raise the frequencies and decrease the quality factor of the array resonances. With increased stiffness of the encapsulant, more energy is transmitted through the layer. The loss factor of the material was shown to minimize the effects of layer thickness resonances. For multiple gratings in close proximity, the low stiffness encapsulant was shown to mitigate the near field excitations of the antisymmetric guided modes and thereby improve baffling performance.
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