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THEORETICAL DEVELOPMENT FOR IDENTIFYING UNDERLYING INTERNAL PROCESSES

VOLUME 1

THE THEORY OF UNDERLYING INTERNAL PROCESSES

R. J. Wherry, Jr.

JOINT REPORT

Naval Aerospace Medical Research Laboratory
Pensacola, Florida

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Modern aviation weapon systems impose increasingly complex and highly demanding command/control and information processing requirements on aircrew personnel. Improved assessment methods and more complete knowledge of human performance capabilities and limitations in high-demand, multi-task environments are needed to better match the operator to the changing human roles in emerging aviation systems. The human engineering and human performance assessment and prediction technologies have, unfortunately, failed to keep pace with increasingly sophisticated airborne weapons systems currently being developed.

The paucity of scientifically-based knowledge concerning the underlying human perceptual, cognitive, and motor processes makes it impossible to confidently influence system design or to be able to predict human and/or system performance in complex situations. This lack of knowledge stems primarily from not having firmly established:

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22b TELEPHONE *(Include Area Code)*: (904) 452-3286

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ABSTRACT (Continued)

(a) the numbers of, the nature of, the underlying internal processes; (b) the distributions of time and accuracy capabilities for those processes; (c) the extent to which individual differences among those processes are stable across tasks which use those processes (d) the nature or identification of task factors which cause (or accompany) the invoking of some processes but not others; and (e) possible fatigue, recovery, and/or interference in internal processing brought about by repeated and/or competing demands on those processes.

Resolution of these basic problems is seen as central in elevating both human engineering design/evaluation and human performance assessment/prediction technologies to a more responsive level for the Navy's RDT&E system acquisition process and for meeting the Navy's personnel selection, assignment, and training requirements.

The more basic concepts for "The Theory of Underlying Internal Processes (UIPs)," presented here, are not new; they represent, in fact, the basis for the author's development of the Human Operator Simulator, which was originally conceived of in the late 1960's. However, the UIP theory's unique implications for a new methodology for the collection and analysis of human performance data remained unrecognized until 1982 when they were informally worked out and presented in a brief unpublished paper.

Development of the formalized UIP theory has been accomplished under the Navy's special focus program entitled **Augmentation of Human Factors Technology Efforts**, which has been jointly sponsored by the Engineering Psychology Programs of the Office of Naval Research and the Naval Air Systems Command.

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THEORETICAL DEVELOPMENT FOR IDENTIFYING UNDERLYING INTERNAL PROCESSES.

VOLUME 1

THE THEORY OF UNDERLYING INTERNAL PROCESSES

R. J. Wherry, Jr.*

Joint Report

Naval Aerospace Medical Research Laboratory
Naval Air Development Center
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Approved and Released by

Captain J. O. Houghton, MC USN
Commanding Officer
Naval Aerospace Medical
Research Laboratory

Approved and Released by

Captain E. J. Sturm
Commanding Officer
Naval Air Development Center

September 1986

SUMMARY PAGE

THE PROBLEM

Modern aviation weapon systems impose increasingly complex and highly demanding command/control and information processing requirements on aircrew personnel. Improved assessment methods and more complete knowledge of human performance capabilities and limitations in high-demand, multi-task environments are needed to better match the operator to the changing human roles in emerging aviation systems. The human engineering and human performance assessment and prediction technologies have, unfortunately, failed to keep pace with increasingly sophisticated airborne weapons systems currently being developed.

The paucity of scientifically-based knowledge concerning the underlying human perceptual, cognitive, and motor processes makes it impossible to confidently influence system design or to be able to predict human and/or system performance in complex situations. This lack of knowledge stems primarily from not having firmly established: (a) the numbers of, the nature of, the underlying internal processes; (b) the distributions of time and accuracy capabilities for those processes; (c) the extent to which individual differences among those processes are stable across tasks which use those processes; (d) the nature or identification of task factors which cause (or accompany) the invoking of some processes but not others; and (e) possible fatigue, recovery, and/or interference in internal processing brought about by repeated and/or competing demands on those processes.

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In addition to the Theory of Underlying Internal Processes, presented in this volume, several other significant methodological developments have arisen during this project. They are discussed in detail in the other volumes of this series and include:

Volume 2 - Modifications to Hierarchical Factor Analysis; Positive Manifold (POSMAN) Rotations.

Volume 3 - Random Sampling of Domain Variance (RSDV); A New Experimental Methodology.

Volume 4 - Task Domains of Naval Flight Officers (NFOs).

Volume 5 - Special Computer Applications in UIP/RSDV Studies.

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Development of the formalized UIP theory has been accomplished under the Navy's special focus program entitled **Augmentation of Human Factors Technology Efforts,** which has been jointly sponsored by the Engineering Psychology Programs of the Office of Naval Research and the Naval Air Systems Command. I am indebted to Mr. G. Malecki and Commander T. Jones, from those respective commands, for their support in this effort. I am also especially appreciative to Dr. N. Lane who reviewed much of the theoretical material and many of the statistical derivations presented in this document.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

1. INTRODUCTION

The theory of Underlying Internal Processes (UIPs) is basically a framework for discussing possible causes of good and poor human task performance and discovering what internal processes may underlie that behavior. The theory assumes speed and accuracy of task performance is a direct result of the speed and accuracy of the various internal processes being used during the performance of that particular task. The UIP theory requires no arbitrary distinction between various traditionally recognized process categories (e.g., sensory, perceptual, cognitive, memory, or motor). Nor is the theory directly concerned with either the "site" of, or the "mechanisms" needed for, the internal processes. It may well be that the UIP theory will be useful in addressing these concerns, but the theory is primarily concerned with identifying the number and nature of the different internal processes involved in the performance of various tasks.

Keywords: human factors engineering; airborne weapons systems; human information processing; aircrew personnel.

The concept of underlying internal processes, as used in the UIP theory, is meant to convey simply that certain sequential activities must be taking place internally for any task to be accomplished. No a *priori* hypotheses need be made about what those processes may be or what tasks will cause the invoking of which of those internal processes. UIPs are simply hypothetical "activity blocks" from which the accomplishment of parts of various human tasks may be said to have been constructed. As such, a given UIP is a sort of "micro-activity" which must transpire if parts of certain tasks are to be accomplished.

The reason for searching for the underlying processes is that an infinite number of possible tasks can be imagined, but it is unreasonable to contend that an infinite number of different basic processes must underlie all human behavior. In the realm of reading, to take one example, one could easily conceive of hundreds of thousands of different paragraphs each of which could be conceived of as a somewhat different task. It is totally unreasonable, however, to contend that some of the same processes are not being used in the reading of each of those different paragraphs. It is equally absurd to maintain that all processes involved in reading a book are different from those required for reading a paragraph or even a single sentence. Yet, it is obvious that these are very different tasks since the times to accomplish those separate tasks would be quite disparate. In the same fashion, the task of driving from Chicago to New York differs from the task of driving from Chicago to New Orleans, but many of the same activities must take place in both tasks.

Central to the UIP theory is also the concept of stable individual differences in the ability to perform certain underlying processes. Thus, using the earlier reading tasks example, those who are faster at reading a single statement would be expected to correspond closely with those who are faster at reading paragraphs and books.

The UIP theory, as might be expected, is also concerned not only with what different processes underlie various tasks, but also with how many times the different processes are used for a particular task. Indeed, the UIP theory maintains that the number of times certain processes must be used is highly dependent on the precise task being accomplished, and that each person accomplishing a task invokes the same processes the same, or at least a very similar, number of times. The major reason persons who are very fast at reading paragraphs are also very fast at reading books is that they possess relative rapid speeds for at least some of those processes which underlie all reading tasks.

The mere fact that an individual is very fast and accurate in performing certain tasks is, however, no guarantee that the same person will excel in the performance of other and different tasks. If there were but a single underlying process that was responsible for all behavior, then the theory would expect a very high correlation among all kinds of tasks. The UIP theory, however, maintains that there are likely to be many different and independent processes that underlie performance in almost any task. For example, even a simple choice-reaction task may involve sensory, perceptual, cognitive, memory and motor processes. The UIP theory would maintain that different persons could do well on even that simple task, but that their individual successes might well be attributable to different causes.

Identification of the underlying internal processes is based on the analysis of task-time data rather than task-accuracy data. The theory assumes that any individual's time to complete a given task must be an additive function of the times that person needs to invoke and utilize the underlying internal processes (UIPs) required to perform that task. Task-accuracy data, on the other hand, are likely to be a multiplicative function of the accuracy of the UIPs. Consequently, the analysis of the relationships among task times yields different results than an analysis of the relationships among task accuracies. The UIP theory capitalizes on these differences in a variety of ways.

The next section reviews the basic assumptions of the theory and provides detailed rationale justifying them. Section 3 describes the mathematical and statistical derivations based on the assumptions and shows the independent contributions being made by each UIP to the correlation between two task-times across a group of persons. This section also describes how a newly modified Hierarchical Factor Analysis method can be used to arrive at an estimate of each UIP's independent contribution to that correlation coefficient. Section 4 describes some additional mathematical manipulations to the separate UIP contributions to estimate both the variance in times to utilize a particular UIP and the number of times that process was being used in a given task. Knowledge of the variability in the time-to-use each UIP and the number of times various UIPs are used in a given task provide a potentially new and powerful method for the classification of tasks. Section 5 discusses how one might go about setting up a study to determine the UIPs for a particular task domain. The final section traces some of the major antecedents of the UIP theory and reviews some past theoretical approaches to identifying the causes or good and poor performance on various tasks.

2. ASSUMPTIONS OF THE UIP THEORY

A number of assumptions, which are central to the UIP theory, were included in the preceding discussion. In this section, those assumptions are reviewed and discussed in greater detail.

2.1 A LIMITED NUMBER OF PROCESSES

The first assumption is that a fairly limited number of UIPs exist that ***can be and are invoked as needed by humans to accomplish any specified task.*** The total number of existing UIPs is currently unknown; the theory merely states that there are not an tremendous number of them. One objective of the theory is to establish the means to discover how many there are and their nature (by determining which tasks cause them to be invoked). In performing tasks (or tests) in a given battery, usage of all possible UIPs probably will never be required. For a given battery, the theory can only address those UIPs **that** are required in common by at least two tasks in that battery. Prior knowledge of the exact number of commonly-required UIPs for a given battery is not needed since the theory describes how the number (P) of commonly-required processes may be established.

2.2 PERFORMING TASKS REQUIRES USING PROCESSES

The second assumption states that accomplishment of any task requires use of a sequence of some UIPs, but never all of them. Further, a given UIP may be invoked more than once during the accomplishment of a given task. Actually, the UIP theory is not concerned with the order in which various UIPs get invoked, but is concerned with the number of times each participating UIP is used.

2.3 USING PROCESSES REQUIRES TIME

The third assumption states that the usage of any given UIP requires some time for that UIP to be completed after it is invoked. This assumption merely states that each usage of any UIP requires the person to expend some amount of time. Regardless of what the underlying processes actually consist of (e.g., the firing of neurons, contraction of muscle fibers, etc.), even the shortest of these actions takes a real amount of time. In physiological processes, hundreds or even thousands of neurons or nerve cells might be involved in a given process. Typically, such processes may begin with small amounts of activity which gradually build to a peak. After some period of time, the activity begins to fade or diminish in its intensity and/or scope. Through these bursts of activity, "information" is internally proceeding from one location in the person to other locations in that person. The question of deciding precisely how long a given process takes, even if one had a faithful record of, say, when each neuron fired, need not concern us at this point. It is sufficient to state that if certain processes had not taken place, a given task would never have been completed. Information must have moved through the human's system, and probabilistically, at least, the information was located at one process site or another throughout the duration of the task. For purposes of discussion, we assume that a given UIP was the "most active" one whenever it contained the majority of the information.

2.4 PROCESSES ACTIVATED ONE AT A TIME

The fourth assumption maintains that the next UIP in sequence cannot be invoked until the preceding UIP has been completed. The theory is, again, unconcerned as to how one would recognize that a given UIP has been completed. It merely states that by whatever criterion is used to make that judgment (i.e., which is the "most active," when did it start being that way, and when did it cease being that way), only one UIP may be the most active one at any one time. This concept is sometimes difficult to comprehend because of the thousands of micro-events that must be happening within the brain at the same time.

An analogy, involving multiple actors performing a "mega-task," is helpful in understanding this assumption. Consider the mega-task to be having fifty persons leave a specific apartment house, drive a car to a particular supermarket, purchase a selected set of items, and return to their apartments. Even if all fifty persons started at the same time, some of those people will be able to get out of their apartments and into their cars faster than others. Some may drive faster to the supermarket. Some may have more difficulty finding a parking place and take longer in walking into the supermarket. Once in the store, the acquisition of the items to be purchased may consume more time for some specific items and for some specific people than for others. Getting through the check-out lines, even if there are several of them, will occur for different people at different times. Ultimately, however, all of the persons do return to their apartments with the required items. Despite the fact that no individual may have behaved exactly like any other individual, it is still possible to talk about the average times and variances in performing different required processes (e.g., leaving the apartment house, driving to the supermarket, parking, finding the items to purchase, paying for them, etc.).

The people in the mega-task are analogous to the "distributed" information passing through parts of the human in a regular task. Statistically, the information as a whole, even though it may be distributed over hundreds of firing neurons, does continuously progress through the brain as a chain of activities. Such activities must be transpiring for any specified task to be completed. Finally, some of the required mega-task activities, such as taking an item off of a shelf and putting it into the shopping basket, have to be repeated several times in the course of the task. Similarly, in reading paragraphs, there must be many repetitions of activities such as moving the eyes and fixating the material to be read. While there may be variability in the time taken by this type of activity, there also must be an average time per movement and a variance in the times for that process.

The first four assumptions discussed lead inevitably to the *conclusion that the time required for accomplishing a given task will be equal to the sum of the products of the number of times a given UIP is invoked (because of that task) multiplied by the average time to use that particular UIP while that task was being done.*

2.5 EXISTENCE OF INDIVIDUAL DIFFERENCES AND VARIABILITY

The fifth assumption maintains that each human has access to the same basic processes, but the speed and accuracy of those processes may differ both from person to person as well as from time to time within a person. This merely states that the theory permits both inter- and intra-person variability in the effectiveness with which each of the UIPs get executed whenever they are invoked.

To continue with the mega-task analogy, the way the world is structured for the fifty persons in question, will, to a large degree, determine the times taken to accomplish various segments of the mega-task. The construction of the apartment house, the lengths and surfaces of the roads leading to the supermarket, the weather conditions, the type of cars owned, the size and layout of the supermarket, the number of check-out lines, etc., would all be factors affecting the average time for the fifty persons to accomplish the mega-task. In the same fashion, the way a given person's brain is structured, the "richness" and "strength" of connections among neurons, the presence and amount of certain biochemicals, and so forth, will all affect the speeds and accuracies of certain UIPs within a single human being.

The time to use a given UIP is one measure of the effectiveness with which a person can invoke and utilize a given UIP. Since the time to use a given UIP can vary from person to person, the theory concludes that we should expect variation in the times required by different persons to do the same tasks (even if they use exactly the same UIPs in exactly the same order). Because the time to use a given UIP may also vary within a person, the theory concludes that we must also expect some variation in the time taken by one individual to accomplish the same task at different times.

Of course, the UIP theory would also draw similar conclusions about the accuracy of responses in terms of the expected variability between and within persons. However, at this point we shall limit our interest strictly to means and variances in the times to complete various UIPs and tasks.

2.6 TIMES REQUIRED FOR PROCESSES ARE INDEPENDENT

The sixth assumption is that the average time it takes individuals to perform a given UIP is unrelated to the average time it takes those same individuals to perform any other UIP. This is, perhaps, the most difficult assumption to explain. The reader may question why the UIP theory requires this assumption. It seems equally reasonable that two different UIPs (e.g., process **a** and process **b**) might well exist, for which the average times to use those processes would be related across people. The problem, however, is that it would be impossible to ever distinguish this case (i.e., two separate, but related processes) from several cases in which the times for processes **a** and **b** are actually independent, but are caused to appear to be related by other causes such as those described in the sections that follow.

2.6.1 Sequentially Related by Another UIP

Two independent processes (**a** and **b**) may appear to be related because usage of another independent process (**c**) always either precedes or follows usage of **a** or **b**. An analogy here might be that the times to walk to the far sides of two rooms (**a** or **b**) might appear to be related if one must always traverse a certain corridor (**c**) before entering either of the two rooms. Even if the distances across **a**, **b**, and **c** are statistically independent of one another, the times (across a large sample of rooms in which the sizes of **a**, **b**, and **c** are randomly determined) to get to the far end of room **a** will have to be related to the time to get to the far end of **b**. Since some sensory processes may typically precede certain perceptual or cognitive processes, and some cognitive processes may typically precede certain motor processes, the concept of independent, but sequentially dependent, processes is certainly possible. Much evidence is being accumulated that indicates certain mental processes take place in the left brain while other kinds of processes take place in the right brain. The conditions and lengths of pathways that sensory, cognitive, and motor activities must follow could be the cause of apparent relatedness in the times to perform various kinds of tasks.

2.6.2 Genetically Related UIPs

Even if times to perform process a and process b were related across people, we desire an explanation to account for that relationship. One possible explanation is that a genetic process was responsible for the similarities in the structures or mechanisms that permit those two processes to take place. A relationship existing between two UIPs may have been caused by a genetic process which took place in the past but continues to exert its influence.

2.6.3 Experientially Related UIPs

Another explanation might be that the times to perform processes a and b are currently related because of similar amounts of previous training and/or practice in using those processes. Persons do become faster and better on some practiced tasks. Improvement may be due to quickened speeds through various UIPs needed by those practiced tasks. Structurally, the mechanisms required for various processes may well be changing as one practices a task. The possibility for "transfer of training" of some acquired skill from one task to another is certainly well established in the psychological literature. Had it not been for the similarity in the practicing of certain tasks (and, consequently, certain UIPs) in the past, the times required for performing those tasks (and those UIPs) might otherwise have been unrelated. In this sense, a and b currently appear as related because of training processes or experiences that occurred in the past.

2.6.4 Biochemically Related UIPs

Finally, the times to perform two different biological processes might well be related because certain biochemical agents are currently present, which control the overall speed that both process a and b can be accomplished. It is well established that introduction of some chemicals into the human's system can have an impact on the performance of certain tasks. Thus, processes a and b, whose usage times might otherwise be independent, may be currently related because of the presence of similar amounts of various biochemicals which have "invaded" the sites where those processes take place. The presence of certain biochemical agents in the past could also have caused relatively permanent structural changes to the mechanisms responsible for the accomplishment of various UIPs. Nor is it necessary that only externally introduced biochemicals be considered as the cause of the apparent relationships among certain UIPs. It may be that the production and dispersal of various biochemicals are taking place naturally and internally at all times. It is, of course, possible that the amount of various biochemical agents that affect the performance of two or more independent processes is, itself, a direct result of a genetic process. Hereditary factors have been established as being responsible for certain chemical imbalances in some persons.

Unfortunately, it is mathematically impossible to distinguish whether two process times appear related because they are "naturally" that way, because they have become that way, or because they only appear that way since we are unaware of the other things and processes which occur in close temporal proximity. Since it is impossible to make these determinations merely from examining the task-time data, the UIP theory assumes that usage times of separate UIPs are independent across people, and that, if they appear to be related, there must be some additional independent (past or present) process that is causing that apparent relationship. A process that is responsible for causing the relationship between the times to perform two or more otherwise independent processes will be referred to as a "higher-level" process. The UIP theory is also concerned with identifying the number and types of higher-level UIPs and which lower-level UIPs they may be affecting.

2.7 SAME NUMBER OF USAGES OF EACH UIP

The last assumption states that a given task determines the number of usages required for each UIP to be invoked during the task. This assumption may not be justified for highly complex and difficult mental tasks. Individuals may well differ in how they attempt to solve complex problems. Different persons may have acquired different learned procedures for attacking certain kinds of problems. Or a given person may have alternative procedures for attacking a given type of problem, and we may be unaware of which of those procedures is being followed while the person solves the problem. However, for simpler tasks, the theory maintains that the nature of the task itself defines the essential UIPs that must be used and the number of usages of each of them. For example, if a subject is asked to count the number of objects in a given picture, the actual number of objects and their relative locations should play a dominant role in determining the number of fixations needed to count them.

A given individual, however, might perform some visual task with, say, fewer than the average fixations required by the group as a whole. Whatever total time he spends in doing the UIPs that permit refixation of the eyes could be divided by the average number of fixations required by the group. The result of this division may make it appear that each fixation time is faster for that individual than it really was. This index, however, would accurately reflect the fact that this person does accomplish those processes more effectively than do others. It may also be that an individual who uses fewer fixations than the average, might, on the average, take somewhat longer to accomplish them. Thus, there may well be a compensatory tradeoff in the number of usages of a given UIP and the average time to use it. The theory is interested in establishing the average amount of time required for a given UIP to be used to accomplish a given amount of work.

The assumption that each person must be using the same UIPs for the same number of times is simply a convenient way to evaluate time used to accomplish a task that requires a certain amount of work to be done. An analogy might be in considering the time required for a person to fill an empty tank by carrying buckets of water from another tank, which is filled with water. If we are interested in how fast persons can move water, we could divide any person's total time taken to complete the task by the average number of bucketfuls required by the group as a whole. This should give us an appropriate index of effectiveness for that person as compared with the others. Why a person is faster than others (e.g., he uses a bigger bucket, he has a smaller bucket but works faster, etc.) is relatively unimportant to us. The theory seeks to not only identify the underlying processes invoked by various tasks, but also to establish the relative speeds with which individuals can utilize those processes.

2.8 TIME REQUIRED FOR AN INDIVIDUAL TO PERFORM A TASK

The UIP theory basically states that, for a task k, which requires the usage of underlying processes (all of which are required by at least one other task in the battery), the time required for individual i to perform task k will be

$$T_{ik} = \sum_{p=1}^{P} U_{pk} \, (T_{pi} + D_{pik}) \tag{2-1}$$

where

P = the total number of commonly used processes,

U_{pk} = the number of times process p is used during task k,

T_{pi} = the average time (across all tasks) for i to use process p,

D_{pik} = the average per-usage difference in time for i to use process p while performing task k from from average time of i to use that process across all tasks.

The derivations for this and other important UIP theory equations are found in Appendix A. The reader may wish to refer to that Appendix before proceeding to the next section.

3. IDENTIFYING THE UNDERLYING PROCESSES

3.1 THE RELATIONSHIP BETWEEN TASK TIMES

The UIP theory is concerned with how one may analyze the task times for a battery of tasks to discover (a) how many UIPs are common to more than one task in that battery, and (b) to determine the nature of the UIPs. The previous section stated that, based on the UIP theory's first four assumptions, the time required (T_{ik}) for individual i to accomplish some task k must be equal to the sum (across all P processes) of the products of the number of times each process gets used multiplied by the average time person i required to accomplish that process. Appendix A describes in detail the derivations of the equations discussed below. The reader may wish to refer to that Appendix for a better understanding of the derivational steps and assumptions made for the equations that follow. **It can be shown that the correlation between the times taken** to perform two tasks (k and m) should be:

$$r_{km} = \sum_{p=1}^{P} \left(\left(U_{pk}\, \sigma_{T_{p.}} / \sigma_{T_k} \right) \times \left(U_{pm}\, \sigma_{T_{p.}} / \sigma_{T_m} \right) \right) \tag{3-1}$$

where

- U_{pk} = number of uses of process p during task k,
- U_{pm} = number of uses of process p during task m,
- $\sigma_{T_{p.}}$ = the standard deviation of the average times taken by individuals to use process p across all tasks,
- σ_{T_k} = the standard deviation of task k times across individuals, and
- σ_{T_m} = the standard deviation of task m times across individuals.

3.2 THE CONTRIBUTION OF EACH UIP TO A CORRELATION

The above equation suggests that a matrix of correlations among task times should be able to be replicated by forming a matrix **F** with K rows (i.e., one for each task in the battery) and P columns (one for each underlying independent process used by two or more tasks) where the entry in cell k,p would be

$$f_{kp} = U_{pk} \, \sigma_{T_{p.}} / \sigma_{T_{k}} \; . \qquad (3\text{-}2)$$

Using the above entries, it can easily be shown that

$$r_{km} = \sum_{p=1}^{P} f_{kp} f_{mp} \; . \qquad (3\text{-}3)$$

Further, the entire correlation matrix should be able to be reproduced by multiplying matrix **F** by its transpose. That is, **R** = **F** x **F'** .

3.3 USING FACTOR ANALYSIS TO OBTAIN THE DESIRED VALUES

The formulation in the above equation suggests that a factor analysis of the correlation matrix, followed by an appropriate rotation of the factors found, should produce matrix **F**. Several clues exist for finding the "appropriate" rotation. First, when a particular process (e.g., process p) is not used in a particular task (e.g., task k), then U_{pk} must equal zero. Thus, the p,k cell in matrix **F** must have a value that does not differ significantly from zero since the standard deviations of both T_{pi} and T_{ik} must be positive. Second, if U_{pk} is not zero, it must be positive, since it is impossible for a process to be used a negative number of times. Thus, if any entry in matrix **F** is not zero, then it must be positive. Therefore, the appropriate rotation must be one which yields entries which are all either zero or positive. Such a factor structure is referred to as having both "simple structure" and "positive manifold." Wherry, Jr. (1985) has recently described modifications to the Hierarchical Factor Analysis (HFA) technique, which can objectively discover such structures when, and if, they exist.

4. FURTHER MANIPULATIONS OF THE HIERARCHICAL FACTOR LOADINGS

4.1 DEVELOPMENT OF THE "COVARIANCE" MATRIX

Assuming that the modified HFA technique is able to find a factor structure that shows both simple structure and positive manifold, then further interesting manipulations of matrix **F** can be accomplished. For example, each row of **F** can be multiplied by the standard deviation of the respective task. These new values can be placed into a **K** by **P** matrix called **C** (for "covariance"). The **k,p** cell should now contain simply the product of U_{pk} times the standard deviation of the values of T_{pi} across all individuals. That is

$$c_{kp} = \sigma_{T_k}(f_{kp})$$

$$= \sigma_{T_k}(U_{pk}\,\sigma_{T_{p\cdot}}/\sigma_{T_k}) = U_{pk}\,\sigma_{T_{p\cdot}} \;. \qquad (4\text{-}1)$$

As with the **F** matrix, all of the matrix **C** entries should be zero or positive. Matrix **C** values can be used in several interesting ways because those values are no longer influenced by the task time variances. For example, the ratio of any two entries from, say, column **p** should now yield the relative number of times process **p** is used in those two tasks. For example,

$$V_{km} = c_{kp}/c_{mp}$$

$$= (U_{pk}\,\sigma_{T_{p\cdot}})/(U_{pm}\,\sigma_{T_{p\cdot}}) = U_{pk}/U_{pm} \;. \qquad (4\text{-}2)$$

4.2 ESTIMATING THE STANDARD DEVIATIONS OF PROCESS TIMES

Conversely, if the number of usages of process **p** can be estimated (by knowing the nature of the task in which it is used), then an estimate of the standard deviation of the T_{pi} values can be obtained.

That is, each non-zero value in the **C** matrix must represent the product of the number of times that process was used multiplied by the standard deviation of that process' usage time. Thus, for task **k**,

$$\sigma_{T_{p\cdot}} = c_{kp}/U_{pk}$$

$$= U_{pk}\,\sigma_{T_{p\cdot}}/U_{pk} = \sigma_{T_{p\cdot}} \quad . \qquad (4\text{-}3)$$

For each process, as many estimates of $\sigma_{T_{p\cdot}}$ can be obtained as one has non-zero loadings on that factor. If, say, ten tasks load on a given factor, then ten independent estimates of the standard deviation of that process time can be obtained. To the extent that all ten of the estimates are highly similar, one can have confidence that both the nature of that process has been comprehended properly, and that the number of times it must be invoked by a given task has also been established correctly. The average of the ten estimates can then be used with confidence as the standard deviation of that process' usage time.

4.3 ESTIMATING THE AVERAGE TIME TO USE A PROCESS

The average time to use a process can be estimated in the following manner. First, matrix **C** is augmented with the column of mean task times. Next, the means, standard deviations, and correlations among the **P** columns (representing the processes) and the augmented column are determined. Finally, multiple correlation is accomplished, selecting all **P** of the process variables as predictors of the task time averages. This results in a prediction equation with **P** standard score "Beta" weights (**Bs**) which states that

$$\hat{z}_{T_k} = \sum_{p=1}^{P} B_p\, z_{c_{kp}} \quad . \qquad (4\text{-}4)$$

However, the predicted standard score of a average task time would be

$$\hat{z}_{T_k} = (\hat{T}_k - \bar{T}_{\cdot})/\sigma_{T_{\cdot}} \quad . \qquad (4\text{-}5)$$

Also,

$$z_{c_{kp}} = (c_{kp} - \bar{c}_{.p})/\sigma_{c_{.p}} \quad . \tag{4-6}$$

Since the means and standard deviations of the **c** values and the average task times can be computed (through the K tasks which represent the rows in the augmented matrix), Eqs. (4-5) and (4-6) can be substituted into Eq. (4-4). Then, multiplying both sides by $\sigma_{T_.}$ and adding $\bar{T}_.$ to both sides and rearranging terms yields

$$\hat{T}_k = \sum_{p=1}^{P}(B_p\sigma_{T_.}/\sigma_{c_{.p}})(c_{kp}-\bar{c}_{.p})+\bar{T} \quad . \tag{4-7}$$

Finally, letting $b_p = B_p\, \sigma_{T_.}/\sigma_{c_{.p}}$, we may show

$$\hat{T}_k = \sum_{p=1}^{P} b_p c_{kp} + \bar{T}_. - \sum_{p=1}^{P} b_p \bar{c}_{.p} \quad . \tag{4-8}$$

Now, since $c_{kp} = U_{kp}\, \sigma_{T_{.p}}$, then

$$\bar{c}_{.p} = \sum_{k=1}^{K} U_{kp}\sigma_{T_{.p}}/K \ , \tag{4-9}$$

But $\sigma_{T_{.p}}$ is a constant throughout the column, therefore,

$$\bar{c}_{.p} = \sigma_{T_{.p}} \sum_{k=1}^{K} U_{pk}/K = \sigma_{T_{.p}}\bar{U}_{.p} \quad . \tag{4-10}$$

It can also be shown (see Eq. (A7-10) in the Appendix) that

$$\bar{T}_. = \sum_{p=1}^{P} \bar{U}_{.p} T_p \quad . \tag{4-11}$$

Substituting Eqs. (4-10) and (4-11) into Eq. (4-8), we see that

$$\hat{T}_k = \sum_{p=1}^{P} b_p U_{kp}\sigma_{T_{.p}} + \sum_{p=1}^{P} \bar{U}_{.p} T_p + \sum_{p=1}^{P} b_p \sigma_{T_p} \bar{U}_{.p} \quad . \tag{4-12}$$

Now, assuming $b_p = T_p/\sigma_{T_p}$, and substituting it into Eq. (4-12)

$$\hat{T}_k = \sum_{p=1}^{P} T_p U_{kp}$$

$$+ \sum_{p=1}^{P} \bar{U}_{.p} T_p - \sum_{p=1}^{P} T_p \bar{U}_{.p} \qquad (4\text{-}13)$$

$$= \sum_{p=1}^{P} T_p U_{kp} \ .$$

Since we know that $T_k = \sum_{p=1}^{P} U_{kp}(T_p + D_{p.k})$, we also now know that

$$T_k = \hat{T}_k + \sum_{p=1}^{P} U_{kp} D_{p.k} \ . \qquad (4\text{-}14)$$

Subtracting $\hat{T}_k$ from both sides, we can show that

$$T_k - \hat{T}_k = \sum_{p=1}^{P} U_{kp} D_{p.k} \ . \qquad (4\text{-}15)$$

But $D_{p.k}$ is the interaction of process p during task k averaged across all individuals. If we sum the errors of prediction of the mean task times across all tasks, we can show that

$$\sum_{k=1}^{K} (T_k - \hat{T}_k) = \sum_{k=1}^{K} \sum_{p=1}^{P} U_{kp} D_{p.k} = 0 \ . \qquad (4\text{-}16)$$

This result supports the assumption made earlier that

$$b_p = B_p \ \sigma_{T_.} / \sigma_{C_{.p}} = T_p / \sigma_{T_p} \ . \qquad (4\text{-}17)$$

Since B_p, $\sigma_{T_.}$, and $\sigma_{C_{.p}}$ can be computed, and σ_{T_p} can be estimated by the earlier described method (see Eq. (4-3)), we can now show that an estimate of the average usage time for process p is

$$\hat{T}_p = (\sigma_{T_.} \ B_p) \ (\sigma_{T_p} / \sigma_{C_{.p}}) \ . \qquad (4\text{-}18)$$

4.4 ESTIMATING INDIVIDUALS' TIMES TO USE A GIVEN PROCESS

A given individual's time to use a given process can be estimated in exactly the same procedure as described above, except that individual's task time scores are augmented to matrix C instead of the mean task times. In this case,

$$\hat{T}_{ip} = (\sigma_{T_{i.}} B_{ip})\,(\sigma_{T_p}/\sigma_{c_{.p}}) \; . \qquad (4\text{-}19)$$

While it appears that a great deal of computation is needed to derive predictions for T_p and each person's T_{pi} value for each of the P processes, it should be pointed out the intercorrelations among the P predictor variables need be computed only once. For example, the intercorrelation of process **p** and process **q** is computed as

$$r_{c_p c_q} = \left(\sum_{k=1}^{K} c_{kp}c_{kq}/K - \bar{c}_{.p}\bar{c}_{.q}\right) \Big/ \sigma_{c_{.p}}\sigma_{c_{.q}} \; . \qquad (4\text{-}20)$$

The intercorrelations of the predictors can be stored in a **P** x **P** matrix named R_{cc}. The inverse of the R_{cc} matrix can then be obtained. Once the vector of intercorrelations between each predictor and any criterion is determined, the vector of P standard score beta weights can be obtained by the matrix operation of multiplying the inverse of R_{cc} by the vector of criterion correlations. Also, the final term in parentheses in Eqs. (4-18) and (4-19) is a constant. That is,

$$K_p = \sigma_{T_p}/\sigma_{c_{.p}} \; . \qquad (4\text{-}21)$$

Thus, the computations to obtain predictions of T_p and T_{ip} values are not overly complex or difficult.

4.5 USING THE DERIVED PROCESS VALUES AS TASK DESCRIPTORS

One of the enduring (and unrealized) goals of human engineering has been the development of a taxonomy of tasks. The hope has been that any task can be located within the taxonomy by progressively determining which subcategory a particular task falls into. That is, to build the taxonomy, one must first discover what the major categories of tasks should be. Having done this, subcategories are formed, followed by sub-subcategories, and so forth. The concept upon which a task taxonomy is founded is that every task can be said to belong to one, and only one, category, subcategory, etc. The UIP theory, however, suggests that this concept is erroneous. A given task may contain a requirement for using several different independent processes. Another task may well require some of those processes, but not all of them and, in addition, some other processes not required by the first task. Tasks, themselves, are simply too complex; they cannot be the basis of a taxonomy because an infinite variety of possible tasks exist. Underlying independent processes, if successfully identified, would provide a basis for both (a) enumerating which underlying processes are required by any given task as well as (b) quantifying the relative number of usages of each of those processes. If the actual average number of usages of some process can be estimated, then the average number of usages of that process can also be estimated for any given task in the battery. This section has shown that it is also possible to estimate the average time to use a given process as well as the standard deviation of those usage times. Finally, we have seen that individual capabilities in using some process can also be obtained from the methodology described in this section.

5. THE DESIGN OF STUDIES FOR TESTING THE UIP THEORY

5.1 UNIQUE DATA REQUIREMENTS; NUMBER OF TASKS TO STUDY

It must be remembered that the UIP theory utilizes factor analysis to discover the underlying common processes required by the analysis of task response times correlated across people. Thus, the input to the factor analysis portion is a K by K matrix of intercorrelations for the K tasks on which the data were collected. Since the number of factors extracted from this matrix should be equal to the number of underlying common processes, one would normally want to have at least twice as many tasks represented in the matrix as the number of processes one expects to find. Therefore, as a lower limit for the number of tasks to be studied, one should have at least 2 x P tasks. However, at the beginning of studying a task domain, we do not know how many underlying factors may be present. For example, if we desire to study a complex task such as one requiring the utilization of symbolic information from a tactical display screen, we may not know whether humans have separate visual processes for perceiving the size, color, shape, and orientation of various visually presented symbols. When we talk, here, about different visual processes, we are talking about the relative speeds with which an individual can discriminate visual features. If the relative speeds in discriminating size, shape, and orientation are virtually the same across all people, then the UIP theory would conclude that only a single process exists for discriminating those features, even though experimenters might make a logical distinction among what is being required of the subjects. Similarly, experimenters may believe that a separate process is required for the perception of color, while the UIP theory might find that several different processes are involved in the perception of different colored stimuli. The hierarchical factor analysis of the data might ultimately indicate the presence of separate "red", "blue", and "yellow" perceptual processes as well as a higher-level color process.

5.2 SPECIFIC HYPOTHESES NOT REQUIRED

The above discussion points out an important distinction about the UIP theory, that is, while the analysis of response time data by the methods being advocated here could be used to test various hypotheses about how many processes there are and what their nature may be, it is unnecessary to have formulated any such hypotheses. The UIP theory itself contains more global hypotheses about human performance in the form of the major assumptions of the theory. Thus, for example, one of the theory's global hypotheses concerns the existence of different and independent underlying internal processes. If more than one factor is found, then the hypothesis is confirmed. The theory, itself, is unconcerned with what specific processes may exist or what tasks might require the invoking of those processes.

5.3 UNIQUE DATA REQUIREMENTS; NUMBER OF SUBJECTS TO STUDY

If the intercorrelations of the task times are not predicated on the same subjects, then the ensuing factor analysis may yield strange and impossible results. Thus, each subject should be tested on each task being studied. Further, it is certainly desirable to have more subjects than tasks. The number of subjects in a study influence the probable error of a correlation coefficient. The standard deviation of a correlation coefficient of zero is $1/(N-1)^{.5}$. Thus, if the number of subjects studied was 145, the distribution of correlations found by chance alone (i.e., even when two tasks were, in reality, uncorrelated) would have a standard deviation of $1/(145-1)^{.5} = 1/(144)^{.5} = 1/12 = .085$. If the size of the sample of subjects were doubled to 290, then the standard deviation would be reduced to $1/(290-1)^{.5} = 1/(289)^{.5} = 1/17 = .0588$. Where this becomes important is during the extraction of factors. As each independent factor is extracted, statistical criteria are invoked to see if loadings on the next factor that would be extracted are significantly different from what might be expected by chance alone. If not, then the extraction of factors is stopped. Obviously, we would like to be able to find all the real underlying processes and this causes us to want to continue extracting factors. On the other hand, we do not wish to simply extract factors which are attributable to chance alone (i.e., measurement error). Consequently, the statistical criteria mentioned above are invoked. It should be obvious that the loadings on any factors found will be more stable (i.e., replicable) as the subject sample size is made larger. It should also be obvious that larger sample sizes should permit us to extract more factors that have real but fairly small loadings.

5.4 CHOOSING TASKS TO BE STUDIED

5.4.1 Selecting a Task Domain of Interest

A major contention of the UIP theory is that an infinite variety of human tasks can be imagined. Obviously, one cannot go out and collect data on every possible human task. It is equally obvious, however, that some tasks differ from others in only relatively minor ways. It is, therefore, advisable to perhaps start an investigation of UIPs with those required by some specific domain of tasks. For example, the use of visually presented symbolic information represents one domain of tasks that might be of interest to an investigator.

5.4.2 Determining the Variables of Interest

Having selected, say, the use of visually presented symbolic information as our task domain of interest, the important task variables should next be specified. With regard to what is found on a display screen, one might generically describe the things to which the operator must attend as being the various objects depicted on the screen and/or the various areas depicted on the screen. For example, at times an operator may only be interested in certain objects if they are in certain areas. With regard to the objects themselves, the operator may only be interested in objects that possess certain features. Various attributes of the objects themselves (e.g., location, size, color, orientation, shape, etc.) are typically used to symbolically code information about real-world objects' location, type of object, threat, allegiance, heading, etc. Another obvious attribute of objects depicted is the total number of objects on the screen at any one time.

5.4.3 Selecting the Levels of a Variable to Represent

"Levels of a variable" refer to the number of different states of a given attribute that will be represented in the study being planned.

In the Analysis of Variance (ANOVA) designs, because of requirements for independence of the variables (factors), the number of data cases in each cell must be proportional. This results in tremendous increases in total number of pieces of data to be collected as the number of factors and the number of levels within a factor increases. Typically, the number of levels of each variable investigated in ANOVA designs will be limited to two or three when the investigator desires to investigate a large number of variables. In a factorial experiment, for example, if one desired to investigate the same number of levels (L) in each of a number of main variables (V), the number of data cells required would be L^V. When only a few variables and a few levels in each are being studied, this is not a major problem. However, even eight variables with six levels in each would require over a million and a half data cells. Real-world tasks are usually far more complex and typically average more than eight variables and six different possible states on each variable. Thus, the idea of using ANOVA designs to study complex real-world tasks must be abandoned in favor of experimental designs that are far more efficient in their usage of data.

5.4.4 Using the RSDV Technique

The Random Sampling of Domain Variance (RSDV) technique was developed to overcome the limitations of ANOVA discussed above. It is particularly appropriate for the design of studies for the UIP theory. The background and rationale for the RSDV technique and its application is described in detail in Volume 3 of this series. A major purpose of the RSDV technique is to permit an experimenter to develop a random sample of real-world tasks so that empirical results obtained from data collected on that sample of tasks will generalize to the entire population of real-world tasks of interest to an investigator. To accomplish this, the RSDV technique requires the experimenter to first specify a statistical model of the real-world task domain of interest. All task and environmental variables that are believed to be operable in that task domain are listed. Next, any relationships among those variables must be estimated along with the probable distributions of various levels of each variable. Then, a computer is employed to generate random numbers to decide which specific levels of each variable will be represented in a given task based on the specified probabilities of those states occurring. This is analogous to a specification, by the experimenter, of what the complete population of that task domain is like in the real-world of interest to the experimenter, and, then, obtaining a random sample from that population of tasks. Having once specified the variables of interest and the probability of various states of those variables occurring, it is relatively easy to obtain, through the use of computers, an unbiased sample (of any desired size) of tasks from that specified task domain. The fact that the sample of tasks selected in this way must be an unbiased one is very important because it means that performance data gathered on that sample of tasks should be able to be generalized to the entire population of tasks described by the task domain specified by the experimenter. Unlike ANOVA designs, the RSDV technique has no limitation in either the number of variables to be studied or in the number of levels for each variable. Each task randomly selected in this way is, itself, specified by the vector of variables states which are randomly obtained in the computer.

5.5 CREATING THE SAMPLE OF TASKS TO BE STUDIED

Having selected a sample of tasks from the specified task domain of interest (by using the RSDV technique described above), the researcher must now create these sample task situations for the laboratory so that each person can perform each sample task. In this way, the individual differences in task response times (and accuracies) for the sample of people (on whom that performance data will be gathered) can be determined. In most modern human performance laboratories, computers are typically employed to simulate and control task situations and to collect data. It is possible, therefore, to directly input to the computer the state vectors for a particular randomly selected task and have the computer create those specific task situations on demand. A discussion of how this approach has been used for the task domain of "using visually presented symbolic information" is described in detail in Volume 4 of this series.

5.6 CHOOSING THE PARTICULAR SAMPLE OF PEOPLE TO STUDY

The sample of people should, of course, also be a random sample from the population of people to whom the experimenter wishes to generalize the results of the study. It should be recalled that the UIP theory allows the analyzer to determine the distribution of times required for the various UIPs that are discovered for a given task domain. To the extent that the sample of people are, in fact, a random sample of the population of people of interest, then the means, standard deviations, and shapes of the distributions of UIP usage times found for the persons sampled should provide excellent estimates of those same parameters for the population of interest.

5.7 COLLECTING AND ANALYZING THE DATA

Time and accuracy data should be collected on all persons in the subject sample for each task in the battery (i.e., the randomly sample domain) of tasks. Correlations among the task times are then computed and submitted for the modified (i.e., Positive Manifold) Hierarchical Factor Analysis. The rationale and procedural steps for this analysis is described in detail in Volume 2 of this series. Examination of the resulting factor loadings should reveal the nature of the underlying independent processes corresponding to each factor. The manipulations to the factor matrix described in section 4 can then be accomplished to determine the probable standard deviations of the separate process usage times and to derive the means and distributions of those process times.

6. HISTORICAL ANTECEDENTS OF THE UIP THEORY

6.1 OVERVIEW

Theorizing about the underlying processes that determine the behavior of humans and their performance on various tasks predates the discipline of psychology and, especially, the relatively modern concepts of collecting and analyzing data to verify one's theories. A major difficulty encountered by empiricists in researching this domain has been that the phenomena of interest (i.e., the mental events or underlying internal processes) were not directly observable. Nevertheless, over the years, two distinctly different data-analytic approaches to studying underlying mental processes emerged.

The earliest of these concentrated on determining the duration of the underlying processes by the analysis of differences in response times brought about by relatively minor variations in a task to be accomplished. The issue addressed in this approach is to what extent do any observed differences in response times correlate with the known (experimenter induced) manipulations of the task. This approach was begun by Donders (1868,1869), and, although it lost advocates for many years, it has recently gained new adherents through some clever and insightful changes and methodological extensions by Sternberg (1969).

The second major empirical approach to identifying basic mental/intellectual traits, skills, or processes is the factor analytic approach. The issue addressed in this approach deals with finding an explanation for why individual's scores on different tests are correlated. This approach had its start with the work of Spearman (1904,1927). Through the years, it gained many followers because of the outstanding pioneering work done by Thurstone (1935,1938), Thomson (1937), Burt (1941), Guilford (1971), and many others. The UIP theory can be thought of as both an integration and extension of both the response time and factor analytic approaches in that it uses a modified hierarchical factor analytic approach for the analysis of correlations of subjects response times across a fairly large battery of tasks.

A third important antecedent of the UIP theory is the various models of human tasks and processes. A model of any type of human process or human task is, in a broad sense, a theory about various underlying processes. With the advent of digital computers, it became possible to specify one's theories as dynamic simulation models and to repeatedly exercise those models under varying conditions to ascertain their implications. Historically, modeling of human performance appears to fall into two broad categories: (a) models of human processes, and (b) models of human tasks. Examples of the former would include models of such processes as short-term memory and retrieval, visual perception, and learning. Examples of tasks that have been modeled include tracking, visually searching for targets on a screen, monitoring of multiple displays, entry of data into computer terminals, etc. A model of a single process or a single task can, of course, be highly useful in discovering a theory's erroneous assumptions about how that process or task is performed. But the responsibilities of humans in real-world situations are never restricted to using a single process and rarely, for that matter, to performing a single task. What was needed to confront the challenge of simulating the performance of humans doing a wide variety of complex tasks in real-world situations were far more comprehensive models. The UIP theory maintains that all tasks are accomplished by the usage of a restricted number of underlying processes. Therefore, one approach to obtaining such a comprehensive model for human performance studies was to develop a number of independent process models that could be integrated together and invoked as needed to accomplish any real-world task. Wherry, Jr. (1969) first proposed using this approach. His general-purpose, Human Operator Simulator (HOS) program makes extensive use of additive process times for a variety of underlying, invokable processes. The fact that HOS has been found to yield sufficient valid results in a variety of complex simulations lends support to the basic assumptions of the UIP theory.

While there are certainly other historical antecedents to the UIP theory, which could be mentioned, the three appearing to have the most direct connections to it are:

(1) the analysis of response time data,

(2) the factor analysis of correlated scores, and

(3) the development of human performance models.

Somewhat expanded discussions of these three areas are presented in the following sections.

6.2 THE ANALYSIS OF RESPONSE TIME DATA

The Donders method (known as the "Subtraction method") and the Sternberg method (known as the "Additive-Factor method") are similar in several respects. First, both are primarily concerned with analysis of response times (rather than response accuracies). Secondly, they utilize systematic and relatively small changes to an experimental task to determine both the nature and extent of the effect(s) brought about by those changes. Third, they attempt to derive estimates of the times required by the underlying processes hypothesized by the experimenter. The Sternberg approach makes use of ANOVA designs and data analysis techniques which, of course, were unavailable to Donders in his era.

The Donders/Sternberg methods have been widely used to study such phenomena as the effect of the number of items (**N**) in a set of "target" stimuli on the response time required to determine if a single presented "probe" stimulus belongs, or does not belong to the target set. A basic hypothesis in this type of study is that to reject the probe as one of the targets, the subjects must utilize various retrieval and comparison processes **N** times. If the probe is one of the **N** target stimuli, one might expect that on the average the subject would only use various retrieval and comparison processes **(N+1)/2** times. In both cases (i.e., acceptance or rejection of the probe), one would expect an increase in **N** to be accompanied by an increase in response times. In general, these studies have supported the hypothesis that response times are linearly related to the number of **items** in the target set. Some of this research has indicated different slopes and intercepts for different persons.

Linear relationships between response times and number of items in the target set in these studies strongly suggest that at least some of the same underlying processes must be used repeatedly, and that, on the average, those processes have stable usage times. Different slopes for different individuals suggest that there are stable individual differences in the times to use those same processes. The different intercepts for different persons may also represent stable individual differences for various underlying processes needed in those tasks, but which are not processes directly needed for accepting or rejecting the probe as a target stimulus. Clearly, the results of these studies support many of the assumptions in the UIP theory.

The Donders method, while brilliant in its conception, was unfortunately rejected by most psychologists for nearly a century because of unsubstantiated claims made by respected introspectionists of that period that mental processes could not be additive. Donders had also believed that it was possible to create two tasks that differed only in that one task had an additional "inserted" process to be done, and therefore, any additional response time could be attributed to that additional process. This particular concept was widely criticized because of the contention that insertion of any new process might well influence how the other required processes would be accomplished. Critics reasoned that it would be impossible to distinguish between required time for the new process and possible changes in duration of the other required processes. When dealing with only two alternative tasks at a time, or only in average differences in response times, then this criticism is quite valid. However, the UIP theory requires that data be collected on a large variety of tasks from the same domain. If the "additional" process occurs in more than one task in the battery, then the factor analysis of the correlated tasks times should be able to isolate the additional "common" process and separate any time components required for it from time components required for the other processes.

Both the Donders and Sternberg approaches, while elegant, tend to suffer from restricting their data to be analyzed to a relatively few tasks, which differ in relatively minor ways. With the availability of ANOVA designs, Sternberg is able to co-vary several situational (i.e., task) factors at the same time. Adoption of ANOVA designs and its data analysis methodology by the Sternberg approach, while making it more understandable and acceptable to other researchers, unfortunately still limits both the number of factors and number of levels within those factors that can be effectively investigated in any one study. The UIP theory, on the other hand, has abandoned the ANOVA approach in favor of the Random Sampling of Domain Variance (RSDV) methodology. This not only permits more factors and levels within factors to be investigated more efficiently, but has the added advantage of being more generalizable to real-world tasks of interest.

While the typical tasks studied using the Donders and Sternberg approaches are very interesting and do yield significant insight into what underlying processes may exist, and what those process usage times may be, it is usually very difficult, if not impossible, to find any real-world tasks like those typically found in their studies. If one accepts the criticism of Donders' work (i.e., that introduction of any new process into a task may change both the nature of the task as well as the response times for the other task-required processes), then one must abandon hope that results from studying the relatively simple, typical laboratory tasks will ever generalize very well to the complex tasks in the real-world.

6.3 THE FACTOR ANALYSIS OF CORRELATED SCORES

6.3.1 Spearman's General Intelligence Factor

The approach of using factor analysis to determine the number and kind of independent "traits" that may underlie human behavior is certainly not new. Indeed, factor analysis was invented specifically for the analysis of mental tests. Spearman (1904) was one of the earliest investigators to recognize that scores on different mental tasks were related across people. That is, persons who did well on one mental (or intellectual) task also tended to do well on other such tasks. He reasoned that some stable individual differences, of importance to all mental tests, must exist among people for these results to occur. Some underlying trait or skill or process must be responsible for making some people better (and/or faster) in their performance of the tasks. Spearman believed, at least initially, that a single factor was responsible for those differences, and he named this factor "general intelligence". Despite the fact that the assumption of a single factor was ultimately rejected, the work of Spearman (1927) set the stage for the enduring interest of British and American psychology in mental testing and in the factor analysis of abilities.

6.3.2 Thurstone's Multiple Factor Analysis

As more data were collected over the years on different types of mental tasks, Spearman's contention of a single factor became increasingly untenable. Thurstone (1931) introduced his Multiple Factor Analysis technique (using the Centroid approach) and subsequently (1938B, 1941) showed convincing evidence for the existence of multiple independent factors, which must underlie the mental domain. There can be little doubt that Thurstone's early work changed, not only how factor analysis was to be done, but also how all future factor analysts conceived of mental abilities.

6.3.3 Guilford's "Structure-of-Intellect" Theory

Guilford and Hoepfner (1971), in the culmination of a twenty-year research effort, also showed ample evidence of many underlying independent factors which must be responsible for the differences in how humans perform various kinds of mental and intellectual tasks. Guilford's work was an attempt to test his "Structure-of-Intellect" (SI) theory, which contained 120 cells based on a three-way classification of mental tasks. The three major classes dealt with six mental "operations" categories (i.e., cognition, memory, divergent production, convergent production, and evaluation), four "contents" or areas of information categories on which the operations took place (i.e., figural, symbolic, semantic, and behavioral), and six "product" categories (i.e., units, classes, relations, systems, transformations, and implications). Thus, Guilford's theory postulated, from the outset, a minimum of 120 different mental abilities. But he believed that there were separate subcategories for visual, auditory, and kinesthetic operations. Therefore, the SI theory actually suggested a possibility of 360 separate abilities. Guilford recognized that his SI model was not hierarchical in nature, and, because of this, he never sought to determine if higher-level factors existed or how much variance they might account for. Although he was a strong advocate of and used factor analysis as his major analytic tool, he doubted that all of the abilities could be segregated by normal factor analysis of traditional mental tests. He stated (page 19-20), *"The fact that they (the postulated abilities) habitually operate together in various mixtures in ordinary mental functioning has been the reason for the difficulty of recognizing them by direct observation or even by ordinary laboratory procedures."* Instead, Guilford believed that special tests must be constructed to prove the existence of his postulated abilities, and he spent twenty years doing that and factor analyzing various batteries that tested his hypotheses. While Guilford's work is remarkable and provides us with many insights into possible underlying processes, it never came to grips with the problem of generalizing the results found with his highly specialized tests to real-world tasks. The UIP theory, on the other hand, does not hypothesize to what the UIPs may be. It starts, instead, by investigating real-world task domains to discover what underlying internal processes are apparently needed by that domain. This represents a marked departure from both the traditional mental testing approach. Whatever the findings of this new approach turn out to be, the result of UIP studies designed using the RSDV technique should be immediately helpful in understanding the nature of the specific underlying processes used by real people in real-world tasks.

6.3.4 The Traditional Emphasis on Response Accuracy

The vast majority of past factor analytic studies have attempted to analyze the relationships among items and/or tests (that were scored by considering the number or percentage of items on the tests which the subjects answered correctly, rather than on the amount of time taken to complete the items or the tests. It may well be that the failure of a single underlying process used in a given task is solely responsible for an erroneous answer to a given test item or mental task. Further, whether a given process fails or not, may be highly probabilistic. That is, sometimes when the process is used, it works accurately (and produces an appropriate output); at other times, that same process may produce an inappropriate output. When an error occurs because of the failure of some internal process, its effect may well be carried along as inputs to the subsequent processing stages, which must occur. Consider, for example, what happens when a single error is made in mentally "carrying" a number during the multiplication of two three-digit numbers. The single failure of this one process during the multiplying task causes the entire response to be scored as a wrong answer. Yet the subject who makes an error on that particular problem may have used exactly the same number of the same processes as did another subject who answered the problem correctly.

Process times, according to the UIP theory, are additive during the solution of a problem or the performance of a task. But process accuracies are obviously not additive. If any process being used on a given task produces an incorrect output, the resulting erroneous internal information may be carried along in the human's processing system and the final answer to that item (or final output to that task) will be wrong, regardless of how accurate that person may be in performing all of the other subsequently required processes. Further, the failure of more than one process to produce accurate outputs is usually not appropriately reflected in a person's score when "right" or "wrong" are the only two categories used to mark items. If an individual's probable accuracy in using any one of the UIPs is also independent of the probable accuracy for other UIPs, then, assuming that any error goes undetected by the subject, the probability of responding correctly on a given task (p_{ki}) should be

$$A_{ki} = \prod_{p=1}^{P} A_{pi}^{U<pk>} \qquad (6\text{-}1)$$

where

A_{ki} = probability of individual i doing task k accurately,

$\prod_{p=1}^{P}$ = the multiplication operator,

A_{pi} = probability of i doing process p accurately, and

U_{pk} = the number of times process p is used on task k.

The above equation clearly shows that accuracy in performing a task is multiplicative across independent processes, rather than additive. It also clearly shows that the number of usages of a given process must be employed as an exponent of a person's average task usage accuracy while it is employed as a multiplier for their average task usage time.

It is, of course, true that if (over a large number of similar items) one averages (or sums) the total number of "right" answers, one should obtain an indication of the joint probability of a given subject's UIPs to function sufficiently accurately for that subject to perform those types of items. But even these total scores may not be particularly helpful in diagnosing which specific underlying processes were actually responsible for the errors which did occur.

Indeed, it is precisely these sorts of issues that have created the debates about the need for "culture free" tests. If, for example, one of the required underlying processes in a series of tasks cannot be accomplished accurately for one subgroup in the population, then it does seem highly inappropriate to generalize that one process' average inaccuracy to other underlying processes needed to perform those items. Suppose a group of subjects is required to perform a test, composed solely of fairly simple problems in using logic and drawing inferences. Suppose, further, that all of the items are written in a language unfamiliar to those subjects. The fact that none of those subjects can correctly answer these items should not be taken as an indication that they cannot accurately perform the processes required for logically drawing inferences.

It may be argued that most tests given are timed tests and, therefore, the scores derived by counting the number of correct answers are greatly influenced by the speed by which the underlying processes can be used. Thus, the individual differences reflected by those scores must be influenced by the speed of processing times. This, of course, is true, but it does three things to the data that are undesirable. First, it confounds process speed with process accuracy so that the investigator cannot subsequently separate these aspects of performance. Secondly, it gives equal credit for all items answered correctly, regardless of whether a given item or task required more or different processes and took more time or not. Third, it forces the data to obscure the effects of separate process durations. One would expect that the shorter one's process times are, the more tasks or items that person should be able to complete. For example, even if subjects made no errors and even if each item or task took exactly the same amount of time per subject, the number of items correctly answered would have to be divided by the total time to complete the test to obtain an estimate of $\sum_{p=1}^{P} U_{pk} T_{ip}$. Without doing this, there is no way to equate process times on one test with process times on another test.

6.3.5 Responding Without Performing Required Processes

An issue **that** arises when scoring tests, regardless of whether they are scored on the basis of "correctness of response" or "time to respond", is whether the subject really attempted to use the appropriate processes. It is always possible that an answer given may have been the result of guessing. Many widely used testing situations (including those studies employing the Sternberg paradigm) employ multiple-choice formats. A subject may guess the correct answer and make the correct response without ever going through the processes **that** should have been used. Similarly, a subject could respond to a test item, whether multiple-choice or not, without ever accomplishing the requisite underlying processes. Indeed, if a subject knows that he does not possess one or more of the requisite processes, (or if the subject becomes aware of the fact that he is running out of time) he may well abandon "appropriate" processing of the task(s) and attempt to guess the correct answers. In many cases, such behavior could be detected by greatly reduced task response times in conjunction with an increase in incorrect answers. This suggests that self-paced tasks may be more helpful than experimenter-paced tasks in determining how subjects typically would prefer to perform a task.

6.3.6 Need to Consider Both Time and Accuracy

It is obvious that merely noting that a given person is very fast at doing some task does not indicate the level of accuracy that person is exhibiting on that task. Similarly, even if one can establish a person's average time to perform each separate underlying independent process, that alone will not indicate how accurate each of those processes are for that individual. But, by using the analysis of task-time approach advocated earlier, it should lead to an identification of what the underlying processes are for each task and the relative number of times each of the required processes had to be used.

6.3.7 The Effects of Practicing a Task

Practicing a given task has at least three known different effects on task performance. Typically, subjects who practice a task over an extended period of time tend to (a) show a reduction in task time, (b) show a reduction in errors made, and (c) show less variability in task performance. Some investigators have favored using highly practiced subjects while others have used subjects as they are found (i.e., with whatever skills and knowledge they possess). The strategy chosen depends upon the interests of the researchers. If one is interested in determining the "upper bounds" of "good" human performance, then clearly the extensive training of subjects in performing tasks of interest would be indicated prior to collecting data. However, data derived from such studies may only be appropriate for generalizing to other similarly highly trained subjects. Further, the training until subjects stabilize their performance may permit a greater percent of the variance in a laboratory study to be explained, but, again, this may not accurately reflect the amounts of variability typically found in the real-world and, thus, may not generalize to real-world situations of interest.

With complex tasks, a fourth possible difference in subjects may emerge with practice of that task; the processes required to accomplish the task may be different. In the course of practicing a complex task, a subject may discover new strategies of which he was unaware in earlier stages of training. Thus, it is possible that part of the reduction in response time results from the dropping of unneeded processes and less optimal strategies. Secondly, as procedures are learned better and better, there is less likelihood of a subject using some process at an inappropriate time (e.g., before the situation really dictates its use). For example, if a subject "prematurely" determines that he is ready to start a different phase of solving some problem, he may have to return to an earlier phase when he discovers that he does not yet have sufficient information to solve that problem. This type of behavior could lead to more usages of some underlying processes by a novice than by a trained individual.

The only apparent guideline that seems reasonable with regard to how much practice subjects should be given is that subjects should have the same distribution of skills and training as those in the real-world to which the researcher wishes to generalize. Regardless of where this guideline leads the researcher, the collection and ultimate comparison of both time and accuracy data seems warranted and advisable.

6.4 COMPREHENSIVE HUMAN OPERATOR SIMULATIONS

It was mentioned earlier that the Human Operator Simulator (HOS) was predicated on the basic assumptions of the UIP theory. HOS is a highly sophisticated computer program used to simulate a human's behavior in a complex system and environment. It utilizes an English-like Human Operator Procedures (HOPROC) language to indicate various learned procedures, which are assumed to be part of the "long-term" memory of the human to be simulated. The model of the human, which is resident to the HOS program (i.e., not user furnished), includes strategies for dynamically deciding what the simulated operator will attend to next. When a given "learned" procedure is being worked on, the appropriate statement in that procedure is "recalled" and "deciphered" by what in HOS are referred to as "statement handlers." These modules permit the simulated operator to determine what "micro-processes" need to be invoked in order to attempt to satisfy the procedural requirements.

Among the various micro-processes currently modeled in HOS are **"Short-Term Memory"** (for attempting recall of new information acquired during the simulation); **"Information Absorption"** (for simulating the acquisition (perception) of new information from various displays and controls in the simulated operator's workstation); **"Anatomy Movement"** (for simulating the limb movements required to reach for, grasp, and manipulate controls, and for head and eye movements and eye fixations to enable the absorption of simulated visually presented information); **"Decision Making"** (for simulating the comparison of two or more values or states and for deciding among alternatives); and several other micro-process models. Each micro-process model, when invoked, must determine the outcome of using that process, including the amount of time used by the process. In this way, the simulation of the operator and the operator's system can be accomplished in a dynamic fashion.

Many of the modeled underlying internal processes (i.e., the micro-processes) in HOS utilize various constant time charges for invoking those processes. Since a given task may invoke the same micro-processes a number of times, it can be seen that, in HOS, task completion times are a function of both the number of uses of each process and the average time charged for each use of those processes. These constants in the micro-process usage time equations can be altered prior to the start of a simulation run to determine the effects of individual differences. The time charges for anatomy movement processes operate in a slightly different way than other micro-processes. In anatomy movements, time charges are a function of both the individual being simulated and the particular movements to be made. For example, the time to reach from one position to another is a function of the distance to be reached as well as the speed of reaching for that individual. The absorption of visual information can also be a function of the individual as well as a function of the type and size of the displayed information.

The constants for the "average human operator" in the micro-process time equations in HOS were, for the most part, gleaned from the comparison and reanalysis of studies that have been published in the literature. Some of the time constants in HOS were also estimated by employing some modifications to the Donders' "subtraction method" on selected laboratory data. Unfortunately, relatively few researchers, especially when the early versions of HOS were being developed in the early 1970s, had been concerned with determining the times required for using various sensory, cognitive, memory, or motor processes or, for that matter, even identifying what the underlying perceptual and cognitive processes might be. It is very encouraging to see the expanded interest from researchers in discovering what the underlying internal processes may be and, especially, to note the resurgence of the interest in determining the times required for using those processes. The need for comprehensive human performance simulation models **that** can accurately predict the likely performance of different kinds of operators in complex systems is obvious and need not be defended here. But such models cannot be totally realized until we become much more confident in our knowledge of what the underlying internal processes really are and how they work together to allow different humans to accomplish all kinds of real-world tasks. It is hoped that the UIP theory and its associated methodologies will be instrumental in (a) discovering what those processes are (so that the appropriate micro-process models can be constructed), (b) determining what tasks will cause them to be invoked (so that appropriate "statement handlers" can be written), and **(c) determining the shapes of the distributions** of time charges for those processes (so that individual differences can be more accurately reflected in human performance models).

APPENDIX A.

DERIVATION OF EQUATIONS FOR THE UIP THEORY

A1. DEFINITION OF SYMBOLOGY TO BE USED

The following sections derive many equations for the UIP theory. It is helpful to understand the symbology used for tasks, individuals, processes, and usages of those processes. The most frequently used symbols are discussed below:

A1.1 TASKS

We assume that the battery consists of K tasks. We number the tasks 1, 2, 3, ... , k, ... , m, ... , K.

A1.2 INDIVIDUALS

We assume there are N individuals and each performs all of the tasks. We number the individuals 1, 2, 3, ... , i, ... , j, ... , N.

A1.3 PROCESSES

We assume that P different underlying internal processes are used by the individuals in performing the tasks in the battery. We number the processes 1, 2, 3, ... , p, ... , q, ... , P.

A1.4 USAGES OF A GIVEN PROCESS

The number of usages of process p by individual i during task k is symbolized U_{pik}. We number the usages 1, 2, ... , u, ... , U_{pik} .

A1.5 SUBSCRIPTS

The letters, $_{pik}$, shown in the above statement are subscripts representing process p, individual i, and task k, respectively.

A1.6 THE "SUM" OPERATOR

The capital Greek letter "sigma", Σ, is used to indicate a sequence of values to be added together. In the derivations that follow, the expression $\sum_{u=1}^{U} X_u$ should be read as meaning, "Starting with $u = 1$ and ending with $u = U_{pik}$, sum the X values indicated." In the same fashion, $\sum_{k=1}^{K}$ would indicate summing across all tasks, $\sum_{p=1}^{P}$ would indicate summing across all processes, and $\sum_{i=1}^{N}$ would indicate summing across all individuals.

A1.7 THE "MULTIPLY" OPERATOR

The Greek letter "pi", Π, is used to indicate a series of values to be multiplied together. The expression $\prod_{p=1}^{P} X_p$ should be read as meaning, "Starting with $p = 1$ and ending with $p = P$, obtain the product of the X values indicated."

A1.8 EXPONENTIATION

In the derivations that follow, X^2 would indicate the square of X, while $X^{.5}$ would indicate the square root of X .

A1.9 MEANS AND STANDARD DEVIATIONS OF TIMES TO USE A PROCESS

The time required for usage u during task k by individual i for process p is symbolized T_{piku} . We express the average (mean) usage time for process p by individual i during task k as T_{pik} . The average time (across all usages and all tasks) individual i takes to utilize process p is symbolized as T_{pi} , and the average time, across all tasks, usages, and persons, to use process p is symbolized as T_p .

The standard deviation, a measure of variability of a set of numbers around the mean of those numbers, is symbolized as σ . The subscripts associated with a σ indicate the appropriate mean around which the variability is being measured. The variance of a set of numbers is always the square of the standard deviation. The symbol σ^2 is used to indicate the variance.

A2. THE NUMBER OF USAGES OF A GIVEN PROCESS DURING A TASK

In accomplishing task **k**, individual **i** may have to use process **p**. We define the number of usages of that process as

$$U_{pik} = \text{usages of process } \mathbf{p} \text{ by } \mathbf{i} \text{ during task } \mathbf{k}. \tag{A2-1}$$

The average number (across all persons) of usages of process **p** during task **k** would be found to be

$$U_{pk} = \sum_{i=1}^{N} U_{pik}/N \ . \tag{A2-2}$$

If persons differed in how often they used process **p** during task **k**, the variance in those usages would be

$$\sigma^2_{U_{pk}} = \left(\sum_{i=1}^{N} (U_{pik} - U_{pk})^2\right)/N \ , \tag{A2-3}$$

and the standard deviation of these usages would be

$$\sigma_{U_{pk}} = (\sigma^2_{U_{pk}})^{.5} \ . \tag{A2-4}$$

However, we may not know whether, during a given task, a certain individual uses a given process more or less frequently than the average person does. Further, the very nature of the task may not only dictate what processes must be used, but also the number of times it must be invoked by each person. Therefore, we assume that individual **i** uses process **p** during task **k** the same number of times as all other individuals. That is, we assume

$$U_{pik} = U_{pk} \ . \tag{A2-5}$$

A3. THE MEAN AND VARIANCE OF A PERSON'S TIME TO USE A GIVEN PROCESS DURING A GIVEN TASK

We symbolize the amount of time required to complete process p by individual i during task k on usage u as T_{piku}. That is

$$T_{piku} = \text{usage u time for process p by i on task k .} \qquad \text{(A3-1)}$$

The average or mean time for an individual to use a process during a task can be computed as

$$T_{pik} = \sum_{u=1}^{U} T_{piku}/U_{pk} . \qquad \text{(A3-2)}$$

Multiplying both sides of Eq. (A3-2) by U_{pk} we obtain

$$U_{pk}\, T_{pik} = \sum_{u=1}^{U} T_{piku} . \qquad \text{(A3-3)}$$

The variance of those process p usage times for individual i during task k would be

$$\sigma^2_{T_{pik.}} = \sum_{u=1}^{U} (T_{piku} - T_{pik})^2/U_{pk} . \qquad \text{(A3-4)}$$

The standard deviation of those same times is simply the square root of the variance $\sigma^2_{T_{pik.}}$. That is

$$\sigma_{T_{pik.}} = (\sigma^2_{T_{pik.}})^{.5} . \qquad \text{(A3-5)}$$

Knowing the mean and standard deviation of the times required for individual i to use process p during task k, we can express each separate usage time as a standard score. That is

$$z_{piku} = (T_{piku} - T_{pik})/\sigma_{T_{pik.}} \quad . \tag{A3-6}$$

The use of standard scores will be helpful in later derivations because it can easily be shown that

$$\sum_{u=1}^{U} z_{piku} = 0 \text{ , and} \tag{A3-6.1}$$

$$\sum_{u=1}^{U} z^2_{piku} = U_{pk} \ . \tag{A3-6.2}$$

Multiplying both sides of Eq. (A3-6) by $\sigma_{T_{pik.}}$ and adding T_{pik} to both sides,

$$T_{piku} = \sigma_{T_{pik.}} z_{piku} + T_{pik} \ . \tag{A3-7}$$

A4. THE MEAN AND VARIANCE OF A PERSON'S TIME TO USE A GIVEN PROCESS ACROSS ALL TASKS

In performing a battery of K tasks, individual **i** may have to use process **p** on one or more of those tasks. The mean process **p** usage time across all usages and tasks for person **i** will be

$$T_{pi} = \sum_{k=1}^{K} \left(\sum_{u=1}^{U} T_{piku} \right) / \sum_{k=1}^{K} U_{pk}. \qquad \text{(A4-1)}$$

Substituting Eq. (A3-3) into (A4-1), we obtain

$$T_{pi} = \sum_{k=1}^{K} (U_{pk} T_{pik}) / \sum_{k=1}^{K} U_{pk}. \qquad \text{(A4-2)}$$

For individual **i** , a difference between the average process **p** usage time during task k (i.e., T_{pik}) and the average process **p** usage time across all K tasks (i.e., T_{pi}) may exist. This difference, or deviation, between those values is

$$D_{pik} = T_{pik} - T_{pi} . \qquad \text{(A4-2.1)}$$

We may calculate the variance of these differences for individual **i** as

$$\sigma^2_{T_{pi.}} = \sum_{k=1}^{K} U_{pik} \, D^2_{pik} / \sum_{k=1}^{K} U_{pk} . \qquad \text{(A4-3)}$$

Taking the square root of Eq. (A4-3) we obtain

$$\sigma_{T_{pi.}} = (\sigma^2_{T_{pi.}})^{.5} . \qquad \text{(A4-4)}$$

Again, we may define, for individual **i**, a standard score for the difference between the mean process **p** usage time during task k and the mean across all tasks as

$$z_{pik} = (T_{pik} - T_{pi})/\sigma_{T_{pi.}} \quad . \tag{A4-5}$$

With these standard scores, it can also be shown that

$$\sum_{k=1}^{K} U_{pk} z_{pik} = 0, \text{ and} \tag{A4-5.1}$$

$$\sum_{k=1}^{K} U_{pk} z_{pik}^2 = \sum_{k=1}^{K} U_{pk} \quad . \tag{A4-5.2}$$

Multiplying both sides of Eq. (A4-5) by $\sigma_{T_{pi.}}$ and adding T_{pi} to both sides yields

$$T_{pik} = \sigma_{T_{pi.}} z_{pik} + T_{pi} \quad . \tag{A4-6}$$

Another way of thinking about individual i's time to use a process p during task k is to say that T_{pik} is composed of two parts. That is

$$T_{pik} = T_{pi} + D_{pik} \tag{A4-7}$$

where

D_{pik} = an interaction between process p, individual i, and task k.

Now, substituting Eq. (A4-7) into Eq. (A4-2) it can be seen that

$$\begin{aligned} T_{pi} &= \sum_{k=1}^{K}(U_{pk}(T_{pi} + D_{pik}))/\sum_{k=1}^{K} U_{pk} \\ &= (\sum_{k=1}^{K} U_{pk}T_{pi} + \sum_{k=1}^{K} U_{pk}D_{pik})/\sum_{k=1}^{K} U_{pk} \quad . \end{aligned} \tag{A4-8}$$

But since T_{pi} is a constant for individual i and process p, then

$$\begin{aligned} T_{pi} &= T_{pi}(\sum_{k=1}^{K} U_{pk}/\sum_{k=1}^{K} U_{pk}) \\ &\quad + \sum_{k=1}^{K} U_{pk}D_{pik}/\sum_{k=1}^{K} U_{pk} \\ &= T_{pi} + \sum_{k=1}^{K} U_{pk}D_{pik}/\sum_{k=1}^{K} U_{pk} \quad . \end{aligned} \tag{A4-9}$$

Thus, it can be shown that

$$\sum_{k=1}^{K} U_{pk} D_{pik} = 0 \ . \qquad \text{(A4-10)}$$

If Eq. (A4-10) is true, then we can also show that

$$\sum_{i=1}^{N} \sum_{k=1}^{K} U_{pk} D_{pik}/N = \sum_{i=1}^{N} (0)/N = 0 \ . \qquad \text{(A4-11)}$$

But it is also true that rearranging the order in which the summing is accomplished would yield

$$\sum_{i=1}^{N} \sum_{k=1}^{K} U_{pk} D_{pik}/N = \sum_{k=1}^{K} U_{pk} \sum_{i=1}^{N} D_{pik}/N \qquad \text{(A4-12)}$$

$$= \sum_{k=1}^{K} U_{pk} D_{p.k} = 0 \ .$$

Finally, we may also show that

$$\sum_{p=1}^{P} \sum_{k=1}^{K} U_{pk} D_{p.k} = 0 \text{ , and} \qquad \text{(A4-13)}$$

$$\sum_{p=1}^{P} \sum_{i=1}^{N} \sum_{k=1}^{K} U_{pk} D_{pik} = 0 \ . \qquad \text{(A4-14)}$$

A5. THE MEAN AND VARIANCE OF THE T_{pi} VALUES

The overall mean time for using process **p** across all usages, all tasks, and all persons would be given by

$$T_p = \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{u=1}^{U} T_{piku} / (\sum_{k=1}^{K} U_{pk}) N \ . \qquad \text{(A5-1)}$$

Substituting Eq. (A3.3) for $\sum_{u=1}^{U} T_{piku}$ in the above equation gives

$$T_p = \sum_{i=1}^{N} (\sum_{k=1}^{K} U_{pk} T_{pik} / \sum_{k=1}^{K} U_{pk}) N \ . \qquad \text{(A5-1.1)}$$

Substituting Eq. (A4-2) for the part in parentheses in the above equation gives

$$T_p = \sum_{i=1}^{N} T_{pi} / N \ . \qquad \text{(A5-2)}$$

The difference between T_{pi} and T_p is a deviation score

$$t_{pi} = T_{pi} - T_p \ , \qquad \text{(A5-2.1)}$$

and the variance of the T_{pi} values around T_p would be given by

$$\sigma^2_{T_{p.}} = \sum_{i=1}^{N} \sum_{k=1}^{K} U_{pk} \ t^2_{pi} / \sum_{k=1}^{K} U_{pk} N \ . \qquad \text{(A5-3)}$$

The standard deviation is given by

$$\sigma_{T_{p.}} = (\sigma^2_{T_{p.}})^{.5} \ . \qquad \text{(A5-4)}$$

A standard score to represent T_{pi} would be

$$z_{pi} = (T_{pi} - T_p) / \sigma_{T_{p.}} \ . \qquad \text{(A5-5)}$$

Multiplying both sides by $\sigma_{T_{p.}}$ and adding T_p to both sides yields

$$T_{pi} = \sigma_{T_{p.}} z_{pi} + T_p . \tag{A5-6}$$

Using Eqs. (A4-6), and (A5-6) we may show that the average time to use process p by a given person i during task k is

$$T_{pik} = \sigma_{T_{pi.}} z_{pik} + \sigma_{T_{p.}} z_{pi} + T_p . \tag{A5-7}$$

A6. THE TIME REQUIRED FOR AN INDIVIDUAL TO PERFORM A TASK

The UIP theory holds that the time to perform a task is the sum across all processes of the times to use the various underlying internal processes each time those processes are used. That is

$$T_{ik} = \sum_{p=1}^{P} \sum_{u=1}^{U} T_{piku} \ . \qquad \text{(A6-1)}$$

But Eq. (A3-3) may be substituted for $\sum_{u=1}^{U} T_{piku}$ to show that

$$T_{ik} = \sum_{p=1}^{P} U_{pk} \ T_{pik} \ . \qquad \text{(A6-2)}$$

Now, substituting Eq. (A5-7) for T_{pik} it may be seen that

$$T_{ik} = \sum_{p=1}^{P} U_{pk} (\sigma_{T_{pi.}} \ z_{pik} + T_{pi}) \ . \qquad \text{(A6-3)}$$

While z_{pik} shows up as a standard score in Eq. (A6-3), those values would sum to zero only if we summed them across all tasks. The value $\sigma_{pi.} z_{pik}$ is equal to $T_{pik} - T_{pi}$. To represent this difference, we used D_{pik}, and we can compute its mean and variance across all N persons as follows. That is, back in Eq. (A4-7) we defined

$$D_{pik} = T_{pik} - T_{pi} \ . \qquad \text{(A6-4)}$$

The average of these scores across all persons would be

$$D_{p.k} = \sum_{i=1}^{N} D_{pik}/N \ . \qquad \text{(A6-5.1)}$$

The variance and standard deviation of these values can be shown to be

$$\sigma^2_{D_{p.k}} = \sum_{i=1}^{N}(D_{pik} - D_{p.k})^2/N \; . \qquad \text{(A6-5.2)}$$

$$\sigma_{D_{p.k}} = (\sigma^2_{D_{p.k}})^{.5} \; . \qquad \text{(A6-5.3)}$$

We may also express the difference between T_{pik} and T_{pi} as a standard score because

$$z_{D_{pik}} = (D_{pik} - D_{p.k})/\sigma_{D_{p.k}} \; , \text{ and} \qquad \text{(A6-5.4)}$$

$$D_{pik} = \sigma_{D_{p.k}} z_{D_{pik}} + D_{p.k} \; . \qquad \text{(A6-5.5)}$$

Since, by Eq. (A4-5), it can be shown that D_{pik} equals $\sigma_{T_{pi.}} z_{D_{pik}}$, we may substitute Eq. (A6-5.5) into Eq. (A6-3) to show that

$$T_{ik} = \sum_{p=1}^{P} U_{pk}(\sigma_{D_{p.k}} z_{D_{pik}} + D_{p.k} + T_{pi}) \; . \qquad \text{(A6-6)}$$

Finally, substituting Eq. (A5-7) for T_{pi}, we see

$$T_{ik} = \sum_{p=1}^{P} U_{pk}(\sigma_{D_{p.k}} z_{D_{pik}} \qquad \text{(A6-7)}$$

$$+ D_{p.k} + \sigma_{T_{p.}} z_{T_{pi}} + T_p) \; .$$

Multiplying as indicated we obtain the time for person i to do task k. That is

$$T_{ik} = \sum_{p=1}^{P}(U_{pk}\sigma_{D_{p.k}} z_{D_{pik}} + U_{pk}D_{p.k}$$

$$+ U_{pk}\sigma_{T_{p.}} z_{T_{pi}} + U_{pk}T_p) \; . \qquad \text{(A6-8)}$$

A7. THE MEAN TASK TIME FOR A GIVEN TASK

To obtain the average task time, we merely sum Eq. (A6-8) across all persons and divide by **N** . Thus,

$$T_k = \sum_{i=1}^{N} T_{ki}/N \text{ , or}$$

$$T_k = \sum_{p=1}^{P}(U_{pk}\sigma_{D_{p.k}} z_{D_{pik}} + U_{pk}D_{p.k}$$

$$+ U_{pk}\sigma_{T_{p.}} z_{pi} + U_{pk}T_p) / N . \qquad \text{(A7-1)}$$

All the **z** scores in Eq. (A7-1) are true standard scores with reference to summing across all the persons. That is, if we were to sum them across all **N** persons they would sum to zero. Appropriately summing the terms in Eq. (A7-1) will show that

$$T_k = \sum_{p=1}^{P} (U_{pk}(\sigma_{D_{p.k}} \sum_{i=1}^{N} z_{D_{pik}} /N$$

$$+ D_{p.k} + T_p + \sigma_{T_{p.}} \sum_{i=1}^{N} z_{pi}/N) . \qquad \text{(A7-2)}$$

Each of the final terms **that**, sum standard scores across all persons will equal zero, and those terms will disappear. Removing all terms from Eq. (A7-2) **that** equal zero, we see

$$T_k = \sum_{p=1}^{P} U_{pk}(D_{p.k} + T_p) . \qquad \text{(A7-3)}$$

Even though individuals may differ in the average time it takes them to use a given process, those differences also do not directly **affect** mean task time. Now $D_{p.k}$ does **affect** mean task time, but $D_{p.k}$ is simply the average (across all the persons) of the difference between the average time it takes person **i** to use process **p** during task **k** (i.e., T_{pik}) and the average time it takes that same person to use that same process across all tasks (i.e., T_{pi}). Several explanations can be advanced as to what D_{pik} (and hence $D_{p.k}$) may represent. First, what $D_{p.k}$ may be is an increment (or decrement) in time to use process **p**, which might be attributable to various stages of "fatigue" or "recovery". It might also represent the average increment (or decrement) in processing time brought about by some "differential depth of processing" requirement attributable to the nature of task **k**. Finally, it may merely represent some moment-to-moment variability in the time to use process **p**.

It is also interesting to determine the average of all task means ($\bar{T}_.$) . This derivation is shown below. It is known, from Eq. (A7-1) that

$$T_k = \sum_{i=1}^{N} T_{ki}/N .$$

Thus, the mean of all task times will be

$$\bar{T}_. = \sum_{k=1}^{K} T_k/K = \sum_{k=1}^{K}\sum_{i=1}^{N} T_{ki}/N\,K . \qquad \text{(A7-4)}$$

Now substituting Eq. (A6-2) for T_{ik} , we see that

$$\bar{T}_. = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{p=1}^{P} U_{pk}T_{pik} \;/\; N\,K \qquad \text{(A7-5)}$$

And substituting Eq. (A4-7) for T_{pik} , we see that

$$\bar{T}_. = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{p=1}^{P} U_{pk}(T_{pi} + D_{pik})/\;N\,K . \qquad \text{(A7-6)}$$

Rearranging the summations, it is equally true that

$$\bar{T}_. = \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{k=1}^{K} U_{pk}(T_{pi} + D_{pik})/\;N\,K \qquad \text{(A7-7)}$$

$$= \sum_{p=1}^{P}\sum_{i=1}^{N}\Big(\sum_{k=1}^{K} U_{pk}T_{pi} + \sum_{k=1}^{K} U_{pk}D_{pik}\Big)/NK .$$

But we know from Eq. (A4-10) that the second term in the numerator is equal to zero. Therefore,

$$\bar{T}_{.} = \sum_{p=1}^{P} \sum_{i=1}^{N} T_{pi} \left(\sum_{k=1}^{K} U_{pk} \right) / K N . \qquad (A7-8)$$

But $\sum_{k=1}^{K} U_{pk}/K$ is a constant and represents the average number of times process p is used for each of the K tasks. That is

$$\bar{U}_{p.} = \sum_{k=1}^{K} U_{pk}/K . \qquad (A7-9)$$

Therefore,

$$\bar{T}_{.} = \sum_{p=1}^{P} \bar{U}_{p.} \sum_{i=1}^{N} T_{pi}/N .$$

Finally, however, we know from Eq. (A5.2) that $T_p = \sum_{i=1}^{N} T_{pi}/N$.

Thus,

$$\bar{T}_{.} = \sum_{p=1}^{P} \bar{U}_{p.} T_p . \qquad (A7-10)$$

A8. THE VARIANCE IN TIMES TO PERFORM A GIVEN TASK

We may represent the deviation between the time to perform task k by individual i and the average task time for the entire group as

$$t_{ik} = T_{ik} - T_k \; . \qquad \text{(A8-1)}$$

Substituting Eq. (A6-8) for T_{ik} and Eq. (A7-3) for T_k, it can be shown that

$$t_{ik} = \sum_{p=1}^{P} (U_{pk}(\sigma_{D_{p.k}} z_{D_{pik}} + \sigma_{T_{p.}} z_{pi})) \; . \qquad \text{(A8-2)}$$

Eq. (A8-2) is particularly interesting because by Eq. (A6-5.5) could show that

$$\sigma_{D_{p.k}} z_{D_{pik}} = D_{pik} - D_{p.k} \; ,$$

and by Eq. (A5-6) we could also show

$$\sigma_{T_{p.}} z_{pi} = T_{pi} - T_p \; .$$

Both expressions to the right of the equal signs in the two prior equations are, themselves,deviation scores. That is, while D_{pik} is the deviation between T_{pik} and T_{pi} , $D_{p.k}$ is the average of those deviations. Thus, $D_{pik} - D_{p.k}$ must, itself, be a deviation score. To indicate that this is a deviation of some deviations, we use the symbol d_{pik} . That is

$$d_{pik} = D_{pik} - D_{p.k} \; . \qquad \text{(A8-3)}$$

And, by Eq. (A5-2.1) we had already defined

$$t_{pi} = T_{pi} - T_p \ . \tag{A8-3.1}$$

Both of these deviation scores represent a difference between this individual and the group as a whole. The second (i.e., t_{pi}) represents an abiding difference between individual i and the group across all tasks in the battery. From that standpoint, t_{pi} represents a more stable difference. The other (i.e., d_{pik} if it in fact exists) starts, first, as a difference between the amount of time individual i required to use process p during task k over that which he required on the average across all tasks. However, beyond that, it also must be different from the average of all persons' task k deviation from their own separate average times to use process p. Thus, if on a particular task, all persons' process p usage time increased or decreased by a constant amount, then no d_{pik} would exist.

We may express t_{ik} in terms of those deviation scores. That is, by substituting the above known equivalents

$$t_{ik} = \sum_{p=1}^{P}(U_{pk}(t_{pi} + d_{pik})) \ . \tag{A8-4}$$

Equation (A8-4) is also interesting from the standpoint that any process p **that** is not used during task k will have a U_{pk} of zero. Let us examine two different processes (e.g., p and q) which have U values different from zero. In this case,

$$t_{ik} = U_{pk}(t_{pi} + d_{pik}) + U_{qk}(t_{qi} + d_{qik}) \ ,$$

$$= U_{pk}t_{pi} + U_{pk}d_{pik} + U_{qk}t_{qi} + U_{qk}d_{qik} \ .$$

The square of t_{ik} when averaged across all N persons, gives the variance of the time to perform task k. That is,

$$\sigma^2_{T_k} = \sum_{i=1}^{N} t^2_{ik}/N \ . \tag{A8-5}$$

In our example where only two processes are being used, the task time variance would produce ten terms and would be

$$
\begin{aligned}
\sigma_{T_k} = {} & U_{pk}^2 \quad (\sum_{i=1}^{N} t_{pi}^2/N) \\
& + U_{pk}^2 \quad 2(\sum_{i=1}^{N} t_{pi} d_{pik}/N) \\
& + U_{pk}U_{qk} \ 2(\sum_{i=1}^{N} t_{pi} t_{qi}/N) \\
& + U_{pk}U_{qk} \ 2(\sum_{i=1}^{N} t_{pi} d_{qik}/N) \\
& + U_{pk}^2 \quad (\sum_{i=1}^{N} d_{pik}^2/N) \\
& + U_{pk}U_{qk} \ 2(\sum_{i=1}^{N} d_{pik} t_{qi}/N) \\
& + U_{pk}U_{qk} \ 2(\sum_{i=1}^{N} d_{pik} d_{qik}/N) \\
& + U_{qk}^2 \quad (\sum_{i=1}^{N} t_{qi}^2/N) \\
& + U_{qk}^2 \quad 2(\sum_{i=1}^{N} t_{qi} d_{qik}/N) \\
& + U_{qk}^2 \quad (\sum_{i=1}^{N} d_{qik}^2/N) \ .
\end{aligned}
$$

Four of the ten terms produced contain variances:

$$\sum_{i=1}^{N} t_{pi}^2/N = \sigma_{T_{p\cdot}}^2 \ ,$$

$$\sum_{i=1}^{N} t_{qi}^2/N = \sigma_{T_{q\cdot}}^2 \ ,$$

$$\sum_{i=1}^{N} d_{pik}^2/N = \sigma_{t_{p\cdot k}}^2 \ ,$$

$$\sum_{i=1}^{N} d_{qik}^2/N = \sigma_{t_{q\cdot k}}^2 \ .$$

All of the other $\sum_{i=1}^{N}$ terms represent covariances. For example, the covariance between T_{pi} and D_{qik} is

$$C_{T_{pi} D_{qik}} = \sum_{i=1}^{N} t_{pi} d_{qik} / N .$$

However, it is known that any covariance is equal to the product of the respective standard deviations times the correlation of that set of numbers. That is

$$C_{T_{pi} D_{qik}} = \sigma_{T_{p.}} \sigma_{D_{q.k}} r_{T_{pi} D_{qik}} .$$

The UIP theory assumes that all such r terms have a value of zero. That is, the T_{pi} and D_{pik} values are independent of each other and independent of the T_{qi} and D_{qik} values. Thus, the sum of their products across all persons will be zero.

Using these assumptions, Eq. (A8-5) becomes

$$\sigma^2_{T_k} = \sum_{p=1}^{P} U^2_{pk} (\sigma^2_{T_{p.}} + \sigma^2_{D_{p.k}}) . \qquad \text{(A8-6)}$$

A9. THE CORRELATION AMONG TASK TIMES

Prior to deriving the equation for the correlation between the times to perform two different tasks, let us summarize what we have established about task times. In doing this, we will follow the convention we started in preceding sections of considering T_{pik} as being composed of two distinct and independent components: T_{pi} and D_{pik} . First, we know from Eqs. (A6-5.5) and (A6-6) that the time required for person i to perform task k would be

$$T_{ik} = \sum_{p=1}^{P} U_{pk}(T_{pi} + D_{pik}) \ . \tag{A9-1}$$

Secondly, by Eq. (A7-3), we know that the mean time for all persons to perform task k would be

$$T_{k} = \sum_{p=1}^{P} U_{pk}(T_{p} + D_{p.k}) \ . \tag{A9.2}$$

By Eq. (A8-4) we know that

$$t_{ik} = \sum_{p=1}^{P} U_{pk}(t_{pi} + d_{pik}) \ . \tag{A9-3}$$

Finally, we have shown (in Eq. (A8-6)) that the variance in task performance times for task k is

$$\sigma^2_{T_k} = \sum_{p=1}^{P} U^2_{pk} \ (\sigma^2_{T_{p.}} + \sigma^2_{D_{p.k}}) \ . \tag{A9-4}$$

We may note that in all of these four equations, each value is composed of the sum of separate components from each of the possible P processes. However, we may also note that if the average usage of any given process is zero (i.e., $U_{pk} = 0$), then that process cannot contribute to any of the four measures shown above (because U_{pk} is always used as a multiplier and its product will be zero). Now the correlation between any two tasks, say, k and m, would be

$$r_{km} = \sum_{i=1}^{N} t_{ik} t_{im} / N \, \sigma_{T_k} \sigma_{T_m} \; . \tag{A9-5}$$

The covariance term, $\sum_{i=1}^{N}(t_{ik}t_{im})/N$, in Eq. (A9-5) can be symbolized as C_{km} . That is

$$C_{km} = \sum_{i=1}^{N} t_{ik} t_{im} / N \; . \tag{A9-6}$$

The value of t_{im} will follow the same form as t_{ik} as shown in Eq. (A9-3), and will be

$$t_{im} = \sum_{p=1}^{P} U_{pm} (t_{pi} + d_{pim}) \; . \tag{A9-7}$$

Multiplying t_{ik} by t_{im} and summing through all N persons, and then dividing the sum by N, we may obtain the covariance term value. As in our derivation of Eq. (A8-6), we may better appreciate what products will occur by considering that we have two terms for each process to be represented in each of the two tasks. Let us consider two processes (p and q) for each of the two tasks. That is,

$$t_{ik} = U_{pk} t_{pi} + U_{pk} d_{pik} + U_{qk} t_{qi} + U_{qk} d_{qik} \; , \tag{A9-7.1}$$

and

$$t_{im} = U_{pm} t_{pi} + U_{pm} d_{pim} + U_{qm} t_{qi} + U_{qm} d_{qim} . \tag{A9-7.2}$$

Multiplying Eqs. (A9-7.1) and (A9-7.2) together will create sixteen terms, which when summed across all N persons and averaged, will be as follows:

(1) $U_{pk}U_{pm} \sum_{i=1}^{N} t_{pi}^2/N$

(2) $U_{pk}U_{pm} \sum_{i=1}^{N} t_{pi}d_{pim}/N$

(3) $U_{pk}U_{qm} \sum_{i=1}^{N} t_{pi}t_{qi}/N$

(4) $U_{pk}U_{qm} \sum_{i=1}^{N} t_{pi}d_{qim}/N$

(5) $U_{pk}U_{pm} \sum_{i=1}^{N} d_{pik}t_{pi}/N$

(6) $U_{pk}U_{pm} \sum_{i=1}^{N} d_{pik}d_{pim}/N$

(7) $U_{pk}U_{qm} \sum_{i=1}^{N} d_{pik}t_{qi}/N$

(8) $U_{pk}U_{qm} \sum_{i=1}^{N} d_{pik}d_{qim}/N$

(9) $U_{qk}U_{pm} \sum_{i=1}^{N} t_{qi}t_{pi}/N$

(10) $U_{qk}U_{pm} \sum_{i=1}^{N} t_{qi}d_{pim}/N$

(11) $U_{qk}U_{qm} \sum_{i=1}^{N} t_{qi}^2/N$

(12) $U_{qk}U_{qm} \sum_{i=1}^{N} t_{qi}d_{qim}/N$

(13) $U_{qk}U_{pm} \sum_{i=1}^{N} d_{qik}t_{pi}/N$

(14) $U_{qk}U_{pm} \sum_{i=1}^{N} d_{qik}d_{pim}/N$

(15) $U_{qk}U_{qm} \sum_{i=1}^{N} d_{qik}t_{qi}/N$

(16) $U_{qk}U_{qm} \sum_{i=1}^{N} d_{qik}d_{qim}/N$.

In the derivation of Eq. (A8-6), we had already assumed that certain individual processing times during a given task were unrelated. Similarly, some individual's processing times would be unrelated from one task to another. The UIP theory makes three main assumptions about the lack of relationship between certain processing times. First, no average individual processing times for one process (e.g., the T_{pi} values) can be related to any other process' average individual processing times (e.g., the T_{qi} values). This is the assumption that leads to the statement that $r_{T_{pi} T_{qi}} = 0$.

Second, no average individual processing times (e.g., the T_{pi} or T_{qi} values) can be related to any individual's deviation from an average processing time on any task (e.g., the D_{pik}, D_{pim}, D_{qim}, or D_{qik} values).

Third, no individual's deviation from his average processing time on one task (e.g., the D_{pik} or D_{qik} values) can be related to each other or to an individual's deviation from his average processing time on some other task (e.g., the D_{pim} or D_{qim} values).

With these assumptions made, only two possible terms (i.e., term i and term ii) can remain in Eq. (A9-5). The first term represents the cross-products of the usages of process **p** on tasks **k** and **m** multiplied by variance of the average process **p** usage time for each person around the overall average process **p** usage time. The other term is identical in form except that it is for process **q**. In this example, we had assumed that only processes **p** and **q** were used in both tasks. Had other processes also been used in both tasks, similar terms would have also shown up for them. If all the processes had been used, we can anticipate that

$$C_{km} = \sum_{p=1}^{P} (U_{pk} U_{pm} \sigma^2_{T_{p.}}) . \tag{A9-8}$$

However, this equation will also work for all cases since, for any process **p**, if U_{pk} or U_{pm} is zero (i.e., that process is not used, then that term will disappear from the covariance term.

Substituting Eq. (A9-8) into (A9-6) and (A9-6) into (A9-5), the correlation between any two task times (e.g., task **k** and task **m**) will be

$$r_{km} = \sum_{p=1}^{P} (U_{pk} U_{pm} \sigma^2_{T_{p.}}) / \sigma_{T_k} \sigma_{T_m} . \tag{A9-9}$$

A10. TASK TIME CORRELATIONS AND FACTOR ANALYSIS

We recall that $\sigma_{T_{p.}}$ is the square root of $\sigma^2_{T_{p.}}$, and, therefore, we could express the $\sigma^2_{T_{p.}}$ term in Eq. (A9-9) as $\sigma_{T_{p.}}$ times $\sigma_{T_{p.}}$. This suggests that we could create a matrix **F** having **P** columns (i.e., one for each of the underlying processes) and K rows (one for each task in the battery). If in row **k** and column **p**, we **could place a value for cell** f_{kp}, we would want it to be equal to U_{pk} $\sigma_{T_{p.}}/\sigma_{T_k}$. That is,

$$f_{kp} = U_{pk}\,\sigma_{T_{p.}}/\sigma_{T_k}\text{, and} \tag{A10-1}$$

$$f_{mp} = U_{pm}\,\sigma_{T_{p.}}/\sigma_{T_m}\,. \tag{A10-2}$$

Note that if we summed (across all columns) the products of these respective entries in row **k** and row **m**, we would obtain

$$\sum_{p=1}^{P} f_{kp}f_{mp} = \sum_{p=1}^{P}(U_{pk}\,\sigma_{T_{p.}}/\sigma_{T_k}$$

$$\times\ U_{pm}\,\sigma_{T_{p.}}/\sigma_{T_m})\,, \tag{A10-3}$$

$$= \sum_{p=1}^{P}(U_{pk}U_{pm}\sigma^2_{T_{p.}}/\sigma_{T_k}\sigma_{T_m}).$$

But this is precisely what Eq. (A9-8) is equal to. Thus,

$$\sum_{p=1}^{P} f_{kp}f_{mp} = r_{km}\,. \tag{A10-4}$$

Those familiar with factor analysis will readily recognize Eq. (A10-4) as the traditional model for factor analysis. It states that, after factor analyzing the correlation matrix, the factor matrix should be able to be used to reconstruct all of the correlation coefficients from that matrix. The rows in a factor matrix correspond to the tests while the columns correspond to independent factors.

The independent factors found in doing a factor analysis must somehow correspond to "underlying internal processes" **that** a person uses to prosecute various tasks. This, of course, means that the underlying processes, at least in regard to the time required to use them, must be independent as well. This was the assumption made earlier when we stated tnat $r_{T_{pi} T_{qi}} = 0$.

One of the major issues in factor analysis is deciding how to rotate the obtained factors to their most meaningful structure. Various mathematical schemes for rotating the extracted factors have been developed. For example, Kaiser (1959) developed the Normalized Varimax Rotation method to achieve "simple structure." It is the most widely used rotation method among psychologists. Of particular interest, in deciding how the factors obtained from factor analyzing the intercorrelations of task times, is the composition of the entries suggested earlier for each cell of the **F** matrix. We indicated that its value should be, for row **k** column **p**,

$$f_{kp} = U_{pk} \sigma_{T_{p.}} / \sigma_{T_k} .$$

We can immediately see that $\sigma_{T_{p.}}$ and σ_{T_k} must be positive (since it is impossible for a standard deviation to be negative). Since U_{pk} is the average number of times process **p** is used during task **k**, it follows that U_{pk} must be zero or positive. This requires that every entry in the factor matrix, if its factors have been rotated to correspond with the underlying independent processes, must be zero or positive. The Varimax method can sometimes arrive at such a structure. Usually, however, when it does obtain all positive loadings, there are very few loadings in the factor matrix **that** are not significantly different from zero. Often, this problem is caused by the presence of higher-level (general and/or subgeneral) factors, and the Varimax method cannot handle such factors. Hierarchical Factor Analysis (HFA) (see Wherry, Sr. (1984)) was developed in the late 1960s to overcome this difficulty. However, the original HFA technique, which could achieve the desired near-zero loadings sought by "simple structure", might also, at times, result in some small, but significant, negative loadings. Recently, Wherry, Jr. (1985) has developed a new mathematical criterion for achieving both "simple structure" and "positive manifold", the type of factor loading structure demanded by the theory of underlying internal processes.

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