### TITLE
Some Research Problems in Reliability Theory and in Limit Theory

### PERSONAL AUTHOR(S)
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### TYPE OF REPORT
Final

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DAAL03-86-K-0010

### ABSTRACT
Three papers were prepared:

In paper [1], functional Erdős–Rényi type laws were established. In a revised version, the importance of establishing the compactness condition in the large deviation principle is emphasized for such laws. In addition, some large deviation results are obtained in situations where the compactness condition does not hold.

In paper [2], how the dispersiveness of the mixing distribution carries over to the mixed model is qualified in terms of generalized convex functions. A sign change result is given which can be used as a diagnostic to determine if you’ve gone far enough when fitting a mixed model.

Finally, in paper [3], a strategy is given for analyzing mixtures of exponential distributions. The strategy is primarily based on graphical procedures.

### SUBJECT TERMS
- Reliability Theory
- Limit Theory
- Mixing Distributions
- Exponential Distributions
- Erdős–Rényi Type Laws

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FINAL REPORT

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NAME OF INSTITUTION: University of South Carolina

AUTHOR OF REPORT: James Lynch

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SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: James Lynch and Hsiu-Li Hsi. (Hsi is a graduate student who is writing computer programs for Lynch.)

OUTLINE OF RESEARCH FINDINGS: In this period, two papers were written ([1] and [2] above), one of which is under revision. A third paper is in preparation - a rough draft of which has already been written.

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In paper [2], how the dispersiveness of the mixing distribution carries over to the mixed model is qualified in terms of generalized convex functions. A sign change result is given which can be used as a diagnostic to determine "if you've gone far enough" when fitting a mixed model. Finally, a representation akin to the one for dilations is also given for balayages defined in terms of these generalized convex functions.

Finally, in paper [3], a strategy is given for analyzing mixtures of exponential distributions. The strategy is primarily based on graphical procedures. Plots of estimates the mean residual life against the failure rate of the equilibrium distribution are used to identify the two main components of the mixture. The justification of this is a characterization proved in the paper that a linear plot characterizes a mixture of two exponentials. (The use of estimates of the failure rate and of the mean residual life of the equilibrium distribution rather than that of the underlying distribution is to smooth the estimators.) The estimates of the main components are then used as initial values to obtain the maximum likelihood estimate for a mixture of two exponentials using the E-M algorithm. Gradient plots are then used to see if a higher order fit is needed and a heuristic based on the gradient plots is given to identify the components in the higher order fit when this is the case. Graphs of an assignment probability function (a function which gives the probability that an observation belongs to a particular component of the mixture) are then used to determine if the data is a mixed model or simply the effect of pooled data. Finally, this strategy is used to analyze two classical data sets.

CONTACT WITH ARMY LAB PERSONNEL: Spent June 23 and 24 at the Army Materials Technology Lab in Watertown, Massachusetts. The purpose of the visit was to discuss the types of statistical problems encountered at the lab. In particular, feasible component reliability models which can be used to incorporate prior coupon reliability information and some flaw models for ceramics were discussed. These models are topics of future research.
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