RESEARCH MEMORANDUM

THE RAND COST-PERFORMANCE MODEL FOR SETTING QUALIFICATION STANDARDS: PRELIMINARY COMMENTS

Laurie J. May
Paul W. Mayberry
The Rand Corporation has developed a model to determine cost-effective qualification standards for military occupational specialties. Since this model may potentially have an impact on enlistment policy, it is important to fully understand its assumptions and procedures. This research memorandum examines key elements of the model, discusses the impact they have on the outcome of the model, and identifies issues that might benefit from further analysis.
10 October 1986

MEMORANDUM FOR DISTRIBUTION LIST

Subj: Center for Naval Analyses Research Memorandum 86-200


1. Enclosure (1) is forwarded as a matter of possible interest.

2. The Job Performance Measurement Project is a joint-service effort to develop good measures of job performance, to relate these measures to recruit aptitude test scores, and to use this relationship along with cost data to determine enlistment standards. To this end the Rand Corporation has developed a cost-performance model and applied it to the Army Infantryman specialty.

3. Given the importance of the model outcomes to enlistment policy, it is essential to fully understand its assumptions and procedures. This Research Memorandum is intended to be the first in a series in which we attempt to evaluate and apply the model.

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Director, Manpower and Training Program
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Joint Service Job Performance Measurement Working Group (11 copies)
THE RAND COST-PERFORMANCE MODEL FOR SETTING QUALIFICATION STANDARDS: PRELIMINARY COMMENTS

Laurie J. May
Paul W. Mayberry

Marine Corps Operations Analysis Group
ABSTRACT

The Rand Corporation has developed a model to determine cost-effective qualification standards for military occupational specialties. Since this model may potentially have an impact on enlistment policy, it is important to fully understand its assumptions and procedures. This research memorandum examines key elements of the model, discusses the impact they have on the outcome of the model, and identifies issues that might benefit from further analysis.
EXECUTIVE SUMMARY

Service enlistment standards are generally defined in terms of educational level and scores on the Armed Services Vocational Aptitude Battery (ASVAB). In recent years, the services have been under increasing pressure to justify their enlistment standards against measures of job performance as opposed to the traditional criterion of training grades. In a pioneering effort, the Rand Corporation has developed a model that addresses the difficult problem of determining cost-effective standards. The model has been applied by Rand to estimate qualification standards for assigning Army recruits to the Infantry specialties. The resulting qualification standards were similar to those established by the Army and were generally viewed as intuitively reasonable.

Since the Rand approach could potentially affect enlistment policy, it is important that its assumptions and procedures be fully understood. This research memorandum examines key elements of the model, discusses the impact they have on the outcome of the model, and identifies issues that might benefit from further analysis.

The Rand model constrains performance or the number of accessions to some predetermined level and then evaluates the cost ramifications resulting from various standards. The a priori specification of performance was achieved indirectly by selecting a baseline group of accessions who were thought to have performed at an "acceptable level." The performance level of this baseline group may be operationalized in terms of two performance measures: one that reflects an individual's ability to remain in the service (retained man-months) and another that extends this retention measure by including an assessment of on-the-job performance (qualified man-months). One of these performance measures or the number of accessions must be constrained so that the solution of the cost-effective model can be determined.

The overall framework of the Rand model is general in the sense that specific model variants can be created by altering the constraints and ob-
jective function. By changing the performance constraint (that is, by fixing the level of retained man-months as opposed to qualified man-months or vice versa) and the corresponding model objective, the Rand Corporation, in effect, developed two versions of the model.

The Rand application of the model [1] was completed under a tight time schedule. As a result, the Rand authors did not have time to identify the variables that have the greatest effect on standards or to conduct sensitivity analyses. Because the model may be used in the Joint Service Job Performance Measurement/Enlistment Standards Project, the Center for Naval Analyses is embarking on a thorough evaluation of the model. This report reflects our current thinking about the model and provides an outline for further analyses.

OBSERVATIONS

- The selection of a baseline group of accessions can be critical to the outcome of the model. The sensitivity of results to the choice of a baseline group may vary with each application of the model and should be explored.

- The distribution of qualified man-months is essentially dichotomous. Potentially important differences among individuals are ignored. In this regard, the model treats individuals who barely pass the on-the-job performance test as equals to those persons who perform extremely well on the test. Accordingly, the model favors recruiting individuals who score just above the cutoff score on the on-the-job performance test because they are less expensive to recruit. The use of a continuous measure of performance should be investigated.

- Constraining one of the performance measures as opposed to the other has important implications regarding the model's objective function and real-world conditions within which the model operates. In one version of the model, retained man-months are constrained to a fixed level and optimal standards are determined by minimizing the total
cost per qualified man-month. This method is equivalent to valuing an additional qualified man-month at average cost, which is a questionable assumption. In the other model variant, qualified man-months are held constant and total cost is minimized. In this version of the model, retained man-months are allowed to vary, which, as noted by the Rand authors, may be an unrealistic condition. Further analysis of the impact of these constraints and objective functions, in addition to possible alternative approaches, is necessary.

- The process by which the replacements for ineligible individuals are distributed over the quality mix of the baseline-accessions group after higher standards are imposed has important ramifications on the determination of the optimal standard. From the limited data examined, the proportionally weighted redistribution process may be acceptable for AFQT ability categories, but may not apply to educational status. This area should be further researched.

- Recruiting-cost estimates were determined on a gross level only—high- versus low-quality recruits. Given that no finer gradations of recruiting costs were available and that low-quality recruits were assumed to be essentially cost-free, the cost of higher quality personnel is exaggerated and causes the model to favor recruiting low-quality enlistees. In addition, the recruiting-cost estimates are valid only for small changes in the force structure. Therefore, caution is required when using the model to deal with potentially significant changes in the mix of personnel.
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INTRODUCTION

Each year the military tests approximately one million applicants for enlistment; of these about one-quarter fail to meet the aptitude and educational standards. The aptitude standards for enlistment are defined in terms of scores on the Armed Services Vocational Aptitude Battery (ASVAB). In the late 1970s, the appropriateness of these standards was questioned because of problems found in the ASVAB score scale. Scores had been inadvertently inflated compared to their traditional meaning as the result of an error in test calibration. Accordingly, the operational standards used for enlistment were effectively lowered (by remaining at the same nominal level while test scores overestimated the aptitude of applicants) and large numbers of low-aptitude personnel entered the services.

The Army was most severely affected by this influx of low-aptitude accessions. In fact, before the test miscalibration, approximately 10 percent of Army recruits were thought to be from the lowest aptitude category (category IV) on the Armed Forces Qualification Test (AFQT), a composite of the ASVAB designed to measure general trainability. However, based on the accurately scaled scores, this percentage was about 50 percent in 1979 and 1980. The other services were also affected; from 12 to 31 percent of their recruits were in the lowest aptitude category. Thus, the services faced the critical task of determining the impact on manpower effectiveness of these less-qualified accessions. Congress mandated that the services develop defensible enlistment standards that directly link ASVAB scores to job performance.

Historically, the services have set aptitude standards primarily on the basis of the probability of success in training courses. Little was known about the empirical relationship between the ASVAB and job performance because of the lack of adequate on-the-job performance measures.

In conjunction with the Office of the Secretary of Defense, the Rand Corporation developed a general model to satisfy the congressional intent to link enlistment standards to measures of job performance. The Rand
Corporation chose to analyze the problem of determining standards with a cost-performance model. The model has been applied by Rand to estimate qualification standards for assigning Army recruits to the Infantry specialties. The resulting qualification standards were similar to those established by the Army and were generally viewed as intuitively reasonable.

The Rand model is a pioneering effort in the area of using job performance measures to evaluate qualification standards. Since this model may potentially have an impact on enlistment policy, it is important that its assumptions and procedures are fully understood. This paper identifies key assumptions made in the Rand model and discusses the impact of these assumptions on model outcome. The model and its application to selected Army specialties are described in two Rand publications [1,2].

The Rand application of the model [1] was completed under a tight time schedule. As a result, the Rand authors did not have time to identify the variables that have the greatest effect on standards or to conduct sensitivity analyses. Because the model may be used in the Joint Service Job Performance Measurement/Enlistment Standards Project, the Center for Naval Analyses is embarking on a thorough evaluation of the model. This research memorandum reflects current thinking about the model and provides an outline for further analyses.

THE MODEL

Required Input

The Rand model requires some input "upfront" before the successive stages can be exercised. These model requisites relate primarily to the performance measure to be used in the model. They are: defining the performance criterion, establishing the desired level of accession performance, and relating accession attributes to the performance criterion.
Define the Performance Criterion

The first requisite of the Rand model is to define the performance criterion. This is not a simple task, because no single measure of performance exists for any of the services. The model is indifferent as to the specific measure of performance used. From the perspective of the model, a variety of performance measures would serve the purpose equally well: promotion rate, supervisor rating of proficiency, final course grade in specialty training, hands-on job performance test score, etc. This liberty in the manner of defining performance means that the researcher is not restricted to applying the model in limited settings or circumstances.

The Rand group chose to measure performance as a combination of several surrogate performance measures; these include attrition during basic and advanced skill training, post-training attrition, and qualification on the Army Skill Qualification Test (SQT). In the two Rand reports [1,2] two composite performance criterion measures were defined: one that reflects only the ability to remain in the service (retained man-months) and another that extends this retention measure by including an assessment of job performance (qualified man-months). The rationale for using composite measures is that accessions must first pass through the initial screens of basic and advanced training before they even have the opportunity to make a contribution in their duty assignments. In addition, given that an individual completes training and stays in the service, some measure of job performance is necessary to determine the quality of the individual’s work contribution.

The retained man-months (RMM) criterion is a composite of passing rates for basic and advanced training and retention data. It does not include an evaluation of the quality of an individual’s performance; rather it is simply the individual’s ability to remain in the service. An individual contributes a retained man-month for each month he is in the service after completing basic and advanced training. Conversely, a qualified man-month (ωMM) does include an evaluation of the quality of an individual’s performance. It is computed as retained man-months times an individ-
ual's likelihood of passing the SQT. In other words, QMM is determined by successively refining a group of accessions through basic and advanced training, completion of the first term, and qualification on the SQT to arrive at the number of months contributed by successful job performers.

Throughout this paper the term "performance" will refer to either of these two performance criteria: retained man-months or qualified man-months. Although the Rand group chose to use these specific definitions of performance, the cost-performance model could equally well be applied to other definitions.

Establish Desired Level of Accession Performance

The second model requisite is the specification of a desired or acceptable level of performance from a cohort of accessions. A systematic method of determining the prescribed level at which performance should be fixed is necessary. Again, the manner in which this requisite is accomplished is not fixed a priori by the Rand model.

The Rand group accomplished this task indirectly by specifying a past group of accessions that were acceptable in terms of their "overall performance output." The calculated qualified man-months or retained man-months based on this group of accessions then serve as the baseline acceptable level of performance. Alternative standards and their associated costs are always compared to this baseline level of performance. Careful consideration must be given to the selection of this baseline group since potentially it can affect the outcome of the model.

One alternative to the selection of a past group of accessions, from which baseline performance is calculated, is to formalize the manning process currently used by all the services. All the services have developed manning documents that detail the force structure in terms of the number of people authorized at each skill level or rank. Through years of experience, policymakers have learned what general level of performance to expect from people at various ranks. The aggregate baseline performance level could
be determined by asking these job experts to rate the level of performance expected at each rank in an occupational specialty. The important point is that the Rand model does not restrict the specification of accession performance to the baseline-group technique.

Another possible alternative to specifying the necessary level of accession performance is to fix the number of accessions or retained man-months and allow qualified man-months to vary. However, this alternative runs the risk that aggregate performance may not be at the level necessary for the force to properly function. To avoid this problem, the minimum aggregate level of performance necessary for the force would have to be established and force performance constrained to at least this level.

Relate Accession Attributes to Performance Criterion

Another model requisite is the linkage of accession attributes (AFQT category, educational level, race, sex, aptitude composite score intervals, etc.) to the performance criterion. This is accomplished essentially by the regression of the performance criterion on the accession attributes. The model is indifferent to the specific attributes included in this regression and even to the regression technique (e.g., linear versus nonlinear) used. The Rand group used a logistic regression to determine the predicted probability of passing the SQT as a function of the Combat Arms aptitude score, high school status, AFQT, and time in the service. If the performance criterion is continuous, unlike the pass/fail distinction made for the SQT, a linear regression may be sufficient to fit the data. Again, the Rand model can handle a variety of accession attributes, regression techniques, and dichotomous or continuous score scales.

Estimation Stages of the Model

When the three requisites for the model have been completed, the Rand model examines several potential qualification standards for the specialty and determines their associated costs. This objective is accomplished in three stages.
Calculation of Baseline Performance

First, the model computes the level of performance for the selected accessions cohort. The level of performance calculated is called the baseline performance (expressed as retained man-months or qualified man-months, depending on which is constrained). The baseline performance level is held constant for the remaining stages of the model in order to evaluate different standards and associated costs.

Performance Resulting From Alternative Qualification Standards

In the second stage, the model examines alternative standards - that is, different cutoff scores on the aptitude composite for that specialty. A change in standards results in a change in the mix of AFQT categories and educational levels in order to meet the required level of performance (number of qualified man-months or retained man-months, whichever is fixed) as was determined in the first stage. This new mix of accessions is determined as follows:

- The baseline accession profile (a matrix of educational level by AFQT category for the baseline group) is stratified according to the possible alternative standards (e.g., 5-point intervals on the Combat Arms (CO) composite from 70 to 115). The effect of raising standards from 70 to 115 on CO is that the proportions of high school graduates and people in higher AFQT categories are higher, while their non-high school graduate and lower-AFQT-category counterparts are declared ineligible. This step defines the mix or quality of the accessions in terms of education and AFQT for the baseline group at each composite cutoff score.

- The model then determines the number of accessions necessary to maintain the required level of performance for each of the new standards or composite cutoff scores. This determination is based on the earlier analysis that related the components of the performance criterion to the accession attributes and is the same analysis conducted
in the first stage but with differing levels of standards.

• Finally, the model combines the stratified accessions profile, the required level of performance (if necessary), and a user-specified option regarding the proportion of accessions in each stratum for each new standard. The user can retain the original proportions of the matrix or alter them to reflect policy changes (e.g., for example, the user can specify no category IV non-high school graduates or can limit the total number of category IV to 10 percent).

Linkage of Performance and Associated Costs

In the third stage, the model computes the costs for each of the standards compared to the baseline standard. The model determines the change in aggregate accession costs by multiplying the change in the number of high school graduates from the baseline group by the per capita marginal costs associated with recruiting these personnel. Countering these aggregate recruiting costs are the savings associated with the lower attrition costs and higher level of performance for higher-ability personnel. Total costs are the sum of both recruiting and attrition costs.

MODEL ASSUMPTIONS

Specification of Baseline Performance

The term “baseline” as used in the Rand model refers to a variety of concepts: standards, an accessions group, and performance. These “baseline” concepts are not interchangeable, but they are directly related.

The baseline accessions group is that collection of personnel which has produced some “acceptable level” of performance output. The choice of this group is somewhat arbitrary. This determined level of performance reflected in the selection of a specific accessions group is defined as the baseline performance and is operationalized in terms of the number of qualified
man-months or retained man-months calculated by the Rand model. The selection of the baseline accessions group appears to be critical in determining the magnitude of the baseline performance.

The baseline standard is an arbitrary decision because it is merely the cutoff score against which all other potential standards are compared. However, for the sake of consistency, the baseline standard was set at the value used to process the baseline accessions group into the specialty.

In applying the Rand model to the Army Infantry specialty, the cohort of 1977 accessions was established as the baseline group with a CO cutoff of 76 [1]. The proportions of the AFQT category and educational level for this baseline group were as follows:

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<th>IV</th>
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<td>NHS</td>
<td>.06</td>
<td>.06</td>
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The appropriateness of this particular cohort as the baseline group is questionable. Approximately 50 percent of the sample were category IV, and such a group is not likely to reflect the Army's definition today of the acceptable level of performance. The sensitivity of the resulting qualification standards to the choice of the baseline group should be further researched.

The Distribution of Qualified Man-Months

The concept of qualified man-months is central to the Rand model. The objective in one version of the model is to minimize the cost per qualified man-month; in the other version qualified man-months are constrained to a predetermined level, and total cost is minimized. Given this critical role
overall within the Rand model, it is essential that the variable be understood.

A qualified man-month is essentially a dichotomous variable; that is, performance is categorized as either acceptable or unacceptable. This fact is somewhat obscured by the use of the metric of QMM to measure performance and attrition jointly.

Figure 1 shows the bimodal distribution of the qualified man-month variable. During the latter part of the 1970s, approximately 30 percent of the Army Infantry did not qualify on the SQT. This percent represents the first mode of the distribution as those persons who have contributed zero qualified man-months to the Army. That is, regardless of the number of months these individuals have been in the service, even if they complete their term of enlistment, their effective contribution to the Army is zero if they do not pass the SQT. Of the remaining 70 percent of persons who pass the SQT, only 60 percent would complete their first term of enlistment. This 42 percent of persons who both passed the SQT and completed their term form the second mode of the distribution; they receive credit for the full 29 months of post-training service. (Army Infantrymen serve 3-year enlistment terms, in which the first 6 or 7 seven months is spent in training.) The remaining 28 percent of the personnel passed the SQT but at some point left the service. It is assumed, for illustration, that these individuals are uniformly distributed between the two modes because generally as length in service increases the probability of attrition decreases and the probability of passing the SQT increases.

Measuring qualified man-months as a dichotomous variable has limitations. First, the categorization of performance levels (i.e., pass or fail) is sensitive to the specific cutoff score used and the accuracy of the SQT. Changing the cutoff score could change the model results. Thus, it is extremely important that the performance criterion be an accurate reflection of true performance with a valid cutoff score. Second, measuring qualified man-months as a dichotomous variable compresses the spectrum of performance into two broad categories and ignores relative performance differ-
FIG. 1: THE BIMODAL DISTRIBUTION OF QUALIFIED MAN-MONTHS
ences. The performance of an individual who barely passes the SQT and remains in the service is treated as equal to the performance of an individual who does exceptionally well on the test and remains in the service. As a group, individuals who do well on the SQT are probably more expensive to recruit than individuals who just pass the test. Thus, the model favors recruiting individuals who score just above the cutoff level over individuals who do well because they are less expensive and are treated as performing as well as those who get high scores. The use of a continuous performance criterion could alleviate these difficulties.

Model Objective for Optimization

The problem of determining cost-effective standards can be modeled in a variety of ways. The framework used by the Rand Corporation to analyze this problem is general in the sense that specific model variants can be created by altering the constraints and the objective function. The Rand model presents two approaches for determining cost-effective standards: in one version of the model, retained man-months are constrained to a fixed level and the cost per qualified man-month is minimized; in the other version, qualified man-months are constrained and total cost is minimized. As an alternative to the Rand approaches, the problem of determining cost-effective standards could be analyzed using a net-benefit maximization approach. Table 1 summarizes the basic differences between the two Rand approaches and an alternative net-benefit maximization approach (ALT). The approaches are illustrated in figure 2.

The two Rand approaches (RAND 1 and RAND 2) used to determine cost-effective standards differ significantly. The first variant of the Rand model (RAND 1) seeks to find the standard that minimizes the total cost per qualified man-month. This variant requires that retained man-months are held fixed at a predetermined level. The optimal standard will be that score which yields the force with the lowest cost per qualified man-month and satisfies the predetermined target for retained man-months. This formulation of the qualification problem has the positive feature that retained man-months are held fixed; this is consistent with real-world constraints
placed on the military. However, the objective of minimizing the total cost per qualified man-month may not accurately reflect the military’s goal.

The second Rand approach (RAND 2) uses an alternative objective for determining optimal standards. In this version, the objective is to find the standard that minimizes total cost given qualified man-months are held fixed at a predetermined level. The optimal cutoff score under this approach is the score that yields the cheapest force that satisfies the predetermined target for qualified man-months. There are two main disadvantages of this approach: the desired level of force performance, measured as qualified man-months in this model, must be predetermined, and retained man-months are not constrained to be a fixed level.

Alternatively, instead of using the Rand variants of the cost-performance model, cost-effective standards can be determined by using a net-benefit maximization approach. Under this method, the assumed objective of the military is to maximize the difference between performance and total cost given retained man-months are held fixed. The optimal standard under this approach is the score that yields the force that gives the highest level of performance (that is, the highest number of qualified man-months) relative to cost and also satisfies the retained man-month constraint.

### TABLE 1

**MAIN FEATURES OF THREE COST-PERFORMANCE MODELS FOR DETERMINING STANDARDS**

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<th>Model</th>
<th>Objective</th>
<th>Predetermined fixed factors</th>
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<td>Minimize total cost per qualified man-month</td>
<td>Retained man-months</td>
</tr>
<tr>
<td>RAND 2</td>
<td>Minimize total cost</td>
<td>Qualified man-months</td>
</tr>
<tr>
<td>ALT</td>
<td>Maximize the difference between benefit and cost</td>
<td>Retained man-months</td>
</tr>
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</table>
FIG. 2: SCHEMATIC OBJECTIVE FUNCTIONS
The two variants of the Rand model differ in how the objective of the military is modeled and what factors are held constant. Because of these differences, the model solutions have different interpretations and implications.

**RAND 1: Constraining Retained Man-Months**

By constraining the number of retained man-months and minimizing the total cost per qualified man-month, RAND 1 basically employs a variant of a net-benefits maximization approach. To illustrate the problem here, it is assumed that there are only two types of personnel; high quality \((h)\) and low quality \((l)\). Both the total cost and the number of qualified man-months depend on the number of retained months contributed by each group, which in turn depends on the ASVAB Combat Arms cutoff score. Mathematically, the problem can be written as follows:

\[
\begin{align*}
\text{minimize} & \quad \frac{C(M_h(s), M_l(s))}{Q(M_h(s), M_l(s))} \\
\text{subject to} & \quad M_h(s) + M_l(s) = RMM^*, \\
\end{align*}
\]

where

- \(C\) = total cost
- \(Q\) = number of qualified man-months
- \(M_h\) = number of retained man-months for high-quality recruits
- \(M_l\) = number of retained man-months for low-quality recruits
- \(s\) = ASVAB Combat Arms cutoff score
- \(RMM^*\) = fixed number of retained man-months.

As the Combat Arms cutoff score is varied, the mix of personnel changes, thus changing the total cost and the number of qualified man-months associated with a given standard. The retained man-month constraint can also be written as

\[
M_l(s) = RMM^* - M_h(s),
\]
and substituted into the objective function to eliminate the variable \( M_i \). The first-order conditions for the above problem are

\[
\frac{dC}{dM_h} \frac{dM_h}{ds} = \frac{dQ}{dM_h} \frac{dM_h}{ds} \frac{C}{Q},
\]

which can be written more simply as

\[
\frac{dC}{ds} = \frac{dQ}{ds} \frac{C}{Q}.
\]

Thus, the optimal cutoff score occurs when marginal cost \( \frac{dC}{ds} \) equals the marginal change in qualified man-months times average cost \( \frac{dQ}{ds} \frac{C}{Q} \).

In contrast, the traditional net-benefits maximization approach (ALT) entails maximizing the difference between the number of qualified man-months and total cost for a fixed value of RMM. This is equivalent to an objective of the highest number of qualified man-months relative to the cost. Mathematically, this problem can be written as follows:

\[
\text{maximize} \quad Q(M_h(s), M_i(s)) - C(M_h(s), M_i(s))
\]

\[
\text{subject to} \quad M_h(s) + M_i(s) = RMM^*.
\]

Substituting the constraint into the objective function and eliminating the variable \( M_i \), the first-order conditions are

\[
\frac{dQ}{dM_h} \frac{dM_h}{ds} = \frac{dC}{dM_h} \frac{dM_h}{ds}
\]

or

\[
\frac{dQ}{ds} = \frac{dC}{ds}.
\]
However, the traditional net-benefits maximization approach is difficult to apply in practice because it requires measuring qualified man-months in the same metric as costs. That is, \( \frac{dQ}{ds} \) and \( \frac{dC}{ds} \) must be in the same metric for the first-order conditions to have any practical meaning. To apply the traditional net-benefits approach, qualified man-months must be measured in terms of their dollar value.

In RAND 1, assigning a dollar amount to the value of a qualified man-month is avoided by minimizing the cost per qualified man-month rather than maximizing the difference between the value of qualified man-months and cost. Minimizing the cost per qualified man-month avoids the problem of explicitly quantifying the value of a qualified man-month.

However, minimizing the unit cost of a qualified man-month can be viewed as simply a variation of the traditional net-benefits approach, since an implicit dollar value is assigned to a qualified man-month. In minimizing the unit cost of a qualified man-month, it is implicitly assumed that the dollar value of a marginal change in qualified man-months equals the average cost of a qualified man-month \( (C/Q) \). Under this implicit assumption, the value of a marginal change in qualified man-months equals the change in qualified man-months \( (dQ/ds) \) multiplied by the dollar value \( (C/Q) \), which is \( \frac{dQ}{ds} \frac{C}{Q} \). The same result is obtained from the traditional net-benefits approach when qualified man-months are explicitly valued at average cost (that is, the same dollar assignment is made to the value of a qualified man-month). In the traditional net-benefits approach, the change in qualified man-months can be translated into dollar terms by multiplying by the dollar value of a qualified man-month. As shown by the first-order conditions (equations 4 and 7), the traditional net-benefits maximization problem (ALT) is equivalent to the unit-cost minimization problem (RAND 1) when the change in qualified man-months \( (dQ/ds) \) is translated into a dollar value by multiplying by the average cost of a qualified man-month \( (C/Q) \).

The implicit assumption that the dollar value of a qualified man-month equals average cost is unrealistic because the average cost of a qualified
man-month varies with the Combat Arms cutoff score. Over the range of the Combat Arms cutoff scores considered, the average cost curve is U-shaped. In addition, from the Rand report [1], it appears that \( dQ/ds \) is constant (or approximately constant). Thus, the assumption that the marginal dollar value of a qualified man-months equals average cost implies that the benefit value of an additional qualified man-month falls as the cutoff score rises and then, after some point, starts to rise as the cutoff score continues to rise. The implicit assumption that marginal benefit is initially falling and then starts to rise after some cutoff score is reached is unrealistic. If the specific expression for \( Q(s) \) and \( C(s) \) were available, integrating the second term in equation 4 would yield the expression for the implicit total-benefit curve. Without the expression for the total-benefit curve, it is difficult to analyze all the implications of the assumptions implicit in the RAND 1 model variation.

**RAND 2: Constraining Qualified Man-Months**

A second approach to determining cost-effective standards is presented in the appendix of [1], as well as in more recent work [2]. The approach used in RAND 2 is theoretically more appealing than RAND 1 because the enlistment objective of the military is more accurately modeled in this version of the model. In RAND 2, the number of qualified man-months is held constant and total cost \( C \) is minimized. There are, however, problems associated with this approach. First, the desired number of qualified man-months is not determined analytically. Instead, the desired performance level \( Q \) is assumed to equal the baseline number of qualified man-months. The model determines the mix of personnel that achieves this performance objective for the least cost. Second, force size (the number of retained man-months) is allowed to vary in this version of the model. This variation is inconsistent with the real-world constraint that end-strength level must be maintained.
The Proportionality Assumption

Raising or lowering the Combat Arms composite standard has a direct effect on the ability mix of recruits—that is, on the proportion of individuals in each AFQT category and educational level. As standards rise, fewer persons with lower aptitude and non-high school graduates are eligible for service. How are the replacements of these ineligible persons distributed across the other ability categories so that a constant force size or performance level can be maintained? The Rand model assumes that new persons enlisting after the new cutoff score is applied will be distributed in proportion to their previous shares among the ability mix categories. An example will help clarify this assumption.

Imagine that recruits in the baseline accessions group are distributed among the four AFQT categories as follows:

<table>
<thead>
<tr>
<th>AFQT category</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.15</td>
<td>.35</td>
<td>.30</td>
<td>.20</td>
</tr>
</tbody>
</table>

Assume that a higher Combat Arms cutoff score is applied to this cohort so that the proportions of persons in categories III and IV are reduced from 30 to 15 percent and from 20 to 5 percent, respectively. (The exact magnitude of this reduction is dependent on the relationship between AFQT and CO and is estimated in the second stage of the model.) The question is how this 30-percent of personnel lost from the two lowest aptitude categories will be distributed between categories I and II. The proportionality assumption distributes the 30 percent in proportion to the percentages of category Is and IIs of the ability mix before the new standard was applied. That is, categories I and II accounted for 50 percent of the baseline group (15 percent category I plus 35 percent category II) so that the 30 percent is distributed by adding $\frac{15}{50}$ of 30 to the category-I proportion and $\frac{35}{50}$ of 30 to the category-II proportion. Therefore, the resulting proportions for the ability mix after applying the higher standard are the following:
It is possible that recruiters would not proportionally replace category III and IV recruits with IIs and IIs. They could recruit only category-II replacements, given that higher-ability individuals are more difficult to recruit and that category IIs are the closest substitutes. If this were the case, the proportionality assumption would most likely overstate the effect of raising the Combat Arms cutoff score on the ability mix, and thus the optimal cutoff score generated by the model would probably be below the true optimum. This is a strong caveat if the proportionality assumption is not valid.

The data most applicable to this concern about the proportionality assumption come from the correction of the ASVAB misnoring problem. As a result of the preponderance of category IVs in the military, during FY 1980 and FY 1981, a concerted effort was made to properly identify category IVs and significantly reduce their number. In October 1980, corrected ASVAB forms were implemented to replace the miscalibrated versions. The tests were able to classify persons into their correct aptitude category, but the question remained as to what aptitude-level personnel the recruiters would enlist to compensate for the lower number of category IVs. While not truly representative of the single condition of changing the standard as applied in the Rand model, enlistments during this time frame reflect the general condition of having to replace a significant number of lower-aptitude recruits.

Table 2 presents the proportion of accessions for each service by AFQT category for the FY 1980–1981 period. Only the Army and the Marine Corps were severely affected by the ASVAB misnoring in terms of enlisting significant proportions of category IVs, although all services experienced increases. The last column of the table shows the drop in proportion of category IVs from FY 1980 to FY 1981. These differences were 20.7 percent and 14.6 percent for the Army and Marine Corps, respectively. The propor-
TABLE 2

PERCENTAGE OF ACCESSIONS TO SERVICES BY AFQT CATEGORY

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Expect.</td>
<td>Actual</td>
<td>Expect.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AF</td>
<td>34.9</td>
<td>38.6</td>
<td>36.1</td>
<td>52.4</td>
<td>51.8</td>
<td>54.2</td>
<td>12.6</td>
</tr>
<tr>
<td>Army</td>
<td>12.8</td>
<td>21.4</td>
<td>18.8</td>
<td>31.3</td>
<td>43.9</td>
<td>46.0</td>
<td>55.5</td>
</tr>
<tr>
<td>Navy</td>
<td>32.9</td>
<td>35.0</td>
<td>35.3</td>
<td>47.0</td>
<td>50.8</td>
<td>50.4</td>
<td>20.1</td>
</tr>
<tr>
<td>USMC</td>
<td>23.0</td>
<td>29.1</td>
<td>27.8</td>
<td>46.4</td>
<td>54.9</td>
<td>56.2</td>
<td>30.6</td>
</tr>
<tr>
<td>DOD</td>
<td>23.4</td>
<td>30.1</td>
<td>28.6</td>
<td>41.2</td>
<td>49.0</td>
<td>50.4</td>
<td>35.3</td>
</tr>
</tbody>
</table>

Note: These numbers are the respective percentage of accessions for each service for the four AFQT categories. The expected percentage (Expect.) is based on the proportionality assumption as applied to the difference (Diff.) between category-IV percentages for the two fiscal years 1980 and 1981.
tionality assumption was applied to the relative proportions of categories I through III for FY 1980 to determine an expected proportion. These expected proportions were compared to the actual proportions accessed for FY 1981.

For category I and II accessions, the Army did slightly better than expected under the proportionality assumption by enlisting 21.4 percent versus an expected 18.8 percent of Is and IIs. The Marine Corps essentially reproduced the expected percent by the actual percent of accessions. For category III recruits, the Army was constrained to do slightly worse than the expected percent, and the Marine Corps almost met the expected percent.

Further examination of another data set of Army applicants, showed that for this period the quality of applicants also increased. When the aptitude mix was broken down in terms of education, the improvement was shown to be attributable largely to the influx of much better non-high school graduates; the graduates remained in essentially the same AFQT proportions across the two fiscal years. In fact, the non-high school graduate category Is and IIs over-proportionally applied to the Army. The implication is that the proportionality assumption is appropriate for the aggregate level of AFQT categories but not for the joint consideration of AFQT and educational level. This statement must be tempered by the fact that it is based on applicants and not accessions, who compose the baseline group used to estimate the baseline level of performance.

In summary, the proportionality assumption is reasonable for predicting redistribution of accessions from lower- into higher-ability categories, based on these limited data. The process may become more complex as it is applied to educational levels as well as AFQT ability categories.
Recruiting Costs

For the purpose of estimating recruiting costs, individuals are divided into two broad groups: high- and low-quality accessions. The model currently assumes that low-quality recruits are cost-free to the services and thus have a marginal cost of zero. This implies that enlisting low-quality recruits requires no effort by production recruiters or support personnel. This assumption could potentially cause the cost of recruiting high-quality recruits to be overestimated. Exaggerating the cost of high-quality personnel causes the model to favor recruiting low-quality personnel.

The cost of recruiting additional enlisted personnel was investigated using two separate recruiting approaches. Additional high-quality recruits are assumed to be obtained by increasing the amount of enlistment bonuses or by increasing the number of recruiters. For both approaches, the same general formulation is used to determine the marginal recruiting cost.

The recruiting-cost-estimation procedure is based on elasticity measures. An elasticity equals the percentage of change in the dependent variable (high-quality enlistments) given a 1-percent change in an independent variable (bonuses or recruiters).

Mathematically, the recruiter elasticity is expressed as follows:

\[ \eta = \frac{dH(R)}{dR} \frac{R}{H(R)} \]

where

\[ \eta = \text{the recruiter elasticity} \]
\[ H = \text{the number of high-quality recruits} \]
\[ R = \text{the number of recruiters}. \]

Integrating the above differential equation at some base level of recruiters, \( R^0 \), yields
The "current" number of high-quality enlistments \(H(R)\) and recruiters \(R\) can be expressed as deviations from the base levels as follows:

\[
H(R) = H(R^o) + \Delta H
\]

and

\[
R = R^o + \Delta R
\]

Substituting the above expressions for \(H(R)\) and \(R\) in equation 9 yields

\[
\frac{H(R^o) + \Delta H}{H(R^o)} = \left(\frac{R^o + \Delta R}{R^o}\right)^\eta .
\]

Rearranging the above relationship, the change in the number of recruiters needed to attract \(\Delta H\) new high-quality recruits is expressed as follows:

\[
\Delta R = R^o[(1 + \Delta H/H(R^o))^{1/\eta} - 1] .
\]

A similar expression for the marginal cost of recruiting is derived for the bonus method. However, for this method a cash bonus is viewed as being the same as a pay increase. Thus, the wage elasticity (the effect of a 1-percent change in wages on high-quality enlistments), rather than the bonus elasticity, is used as the basis for the recruiting-cost calculations. However, evidence from the civilian sector suggests that the behavioral response of employees to bonuses differs from their response to a pay increase. Thus, it is likely that the effect of a raise in pay differs from the effect of a bonus on enlistments. The bonus-recruiting-cost estimates could be improved if it were possible to directly estimate the bonus elasticity.
The recruiter- and wage-elasticity estimates come from previous works on enlistment supply. In the previous works, an exponential functional form was estimated, which has constant elasticities. The estimated elasticities are accurate only in measuring the percentage of change in enlistments given a 1-percent change from a base case that is actually on the estimated supply curve. For equation 13 to hold true, \( H(R^0) \) and \( R^0 \) must correspond to a point on the estimated supply curve. In the Rand model, \( R^0 \) is estimated from current data and \( H^0 \) is measured as the total number of high-quality recruits for FY 1981. Thus, the baseline measures for \( R \) and \( H \) do not necessarily define a point on the estimated supply curve. Equation 13 will generate an accurate estimate of the marginal recruiting effort only if the base data and the elasticity estimates come from the same supply curve.

The error that occurs when an inconsistent base-data point is used to calculate marginal recruiting costs is best illustrated with an example. Suppose that the estimate for the recruiter elasticity (\( \eta \)) is 0.5 and that \( H=100 \) and \( R=50 \) define a point on the estimated supply curve. A recruiter elasticity of 0.5 implies that as one moves along the supply curve a 1-percent change in the number of recruiters results in a 0.5-percent change in the number of high-quality enlistments. Therefore, to attract 101 high-quality recruits (a 1-percent change), the number of recruiters must rise by 2 percent. In addition, suppose it is estimated from a different data source that when \( H=100, R=30 \). The point \( R=30, H=100 \) is not on the estimated supply curve. Using 100 and 50 as the base measures for \( H \) and \( R \), the change in the number of recruiters needed to attract an additional high-quality recruit is 1.0 (that is, the point \( H=101 \) and \( R=51 \) define another point on the estimated supply curve). Using 100 and 30 as the base measures for \( H \) and \( R \), the estimate for the change in the number of recruiters needed to attract an additional high-quality recruit is 0.60. (The point \( H=101, R=30.6 \) is not on the estimated supply curve.) Using data points that are not on the estimated supply curve as base values for \( R \) and \( H \) yields inaccurate estimates of the marginal recruiting effort. If the base data point is close to a point on the estimated supply curve the error will be relatively small. However, if the base data point is not close to a point on the estimated supply curve, this method may generate very poor estimates of the
recruiting cost. In addition, the accuracy of these estimates deteriorates as the marginal change increases. The proximity of the base data point to a point on the estimated supply curve cannot be determined from the information given in the Rand publications.

Irrespective of the estimation method used, the recruiting-cost estimates are valid only for small changes in the force structure. The Rand model, however, considers large changes in the mix of personnel. Estimating the marginal cost of recruiting high-quality personnel, given that the number of high-quality enlistments doubles, requires extrapolation outside the range of the current data. Estimates outside the range of the data are most likely poor. Therefore, caution is required when applying the model to potentially significant changes in the mix of personnel.

**OBSERVATIONS**

- The selection of a baseline group of accessions can be critical to the outcome of the model. The sensitivity of results to the choice of a baseline group may vary with each application of the model and should be explored.

- The distribution of qualified man-months is essentially dichotomous. Potentially important differences among individuals are ignored. In this regard, the model treats individuals who barely pass the on-the-job performance test as equals to those persons who perform extremely well on the test. Accordingly, the model favors recruiting individuals who score just above the cutoff score on the on-the-job performance test because they are less expensive to recruit. The use of a continuous measure of performance should be investigated.

- Constraining one of the performance measures as opposed to the other has important implications regarding the model’s objective functions and real-world conditions within which the model operates. In one version of the model, retained man-months are constrained to a fixed level and optimal standards are determined by minimizing the total
cost per qualified man-month. This method is equivalent to valuing an additional qualified man-month at average cost, which is a questionable assumption. In the other model variant, qualified man-months are held constant and total cost is minimized. In this version of the model, retained man-months are allowed to vary, which, as noted by the Rand authors, may be an unrealistic condition. Further analysis of the impact of these constraints and objective functions, in addition to possible alternative approaches, is necessary.

- The process by which the replacements for ineligible individuals are distributed over the quality mix of the baseline-accessions group after higher standards are imposed has important ramifications on the determination of the optimal standard. From the limited data examined, the proportionally weighted redistribution process may be acceptable for AFQT ability categories, but may not apply to educational status. This area should be further researched.

- Recruiting-cost estimates were determined on a gross level only—high-versus low-quality recruits. Given that no finer gradations of recruiting costs were available and that low-quality recruits were assumed to be essentially cost-free, the cost of higher quality personnel is exaggerated and causes the model to favor recruiting low-quality enlistees. In addition, the recruiting-cost estimates are valid only for small changes in the force structure. Therefore, caution is required when using the model to deal with potentially significant changes in the mix of personnel.
REFERENCES
