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PROGRESS REPORT

ANALYTICAL AND EXPERIMENTAL INVESTIGATIONS OF THE DAMPING CHARACTERISTICS OF BOLTED AND WELDED STRUCTURAL CONNECTIONS FOR PLATES AND SHELLS

by

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Analytical and Experimental Investigations of the Damping Characteristics of Bolted and Welded Structural Connections for Plates and Shells.

Young S. Shin; Kilsoo Kim, and Jonathan C. Iverson

The design of the structural connections is important from the standpoint of reducing system vibration amplitudes or enhancing joint damping capacity. A state-of-the-art review was performed and a generic experimental model was developed and constructed. The test model consists of two concentric circular cylindrical shells and four vanes connected by groups of bolts. First, a finite element model was developed and analyzed, using the MSC/NASTRAN computer program. The first analysis was a modal survey of the model to investigate dynamic characteristics of the whole system. Secondly a frequency response analysis gave the input/output relationship between an input point (excitation) and an output point (response) in the frequency domain, which depends on the damping characteristics of the structure.
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PART I

A LITERATURE SURVEY ON
FRICITION DAMPING IN JOINTS

I. INTRODUCTION

Engineering structures sometimes require substantial damping to operate in acceptable dynamic response levels. In most engineering structures the major portion of damping occurs in joints. The structural joints which are of interest in this study are friction joints which transmit forces and moments through frictional contact. The interface in the joint slips and dissipates energy during the vibration of the structure.

Understanding damping characteristics of such a joint has been a significant research topic for a long time. The main objective of this literature review is to summarize and obtain a broad overview of previous and current works accomplished in this area. The list of references included in this article is intended to be representative not exhaustive.

The work discussed in this article falls into two main categories: (1) analytic studies progressively developed and related experiments; (2) experimental works for basic understanding of phenomena or for the development of experimental technique.

II. ANALYTICAL APPROACHES

The analytical approaches for understanding friction damping can be broadly classified in two as follows: macroslip approach and microslip approach. In the macroslip approach, the entire interface is assumed to be either slipping or stuck. The friction mechanism is assumed to be governed by some form of Coulomb’s law of dry friction. In the microslip approach, the extent of local slip between pairs of contacting points is determined. Therefore, using the microslip approach, a smoother transition from stick to total slip can be described. However, a relatively detailed analysis of the stress distribution at the interface is necessary.
II.1 Macroslip Analysis

II.1.1 Single-Degree-Of-Freedom System

Den Hartog [1] obtained the exact solution for steady state stick/slip motion for a single-degree-of-freedom system. The solution was limited to stick/slip motion which has at most two times intervals of lockup per cycle. His solution was pieced together analytically over each subinterval to construct one entire cycle of the periodic motion. He showed that for the forced vibration with dry friction only with the ratio $F/P$ smaller than $\pi/4$ the amplitude becomes infinitely large at resonance, where $F$ is the friction force and $P$ is the amplitude of the excitation (see Figure 1). Den Hartog's result was extended by Pratt and Williams [2] to the steady state relative motion of two masses with Coulomb friction contact with multiple lockups per cycle.

Levitan [3] analyzed the forced oscillation of a support-excited simple spring-mass system with Coulomb friction damper by using a Fourier series approach. His results are limited to continuously slipping motion. In this range he showed that the coefficients of the Fourier series representation of the response converges very fast and the true response curves are only slightly different from a pure harmonic response.

Caughey [4] analyzed the response of a single-degree-of-freedom system with bilinear hysteresis to sinusoidal excitation. Bilinear hysteresis behavior of a system represented the presence of Coulomb friction or elastoplastic behavior of the material in the system. He used the method of slowly varying parameters (or sometimes called the Kryloff and Bogoliuboff method), and showed that for such a system there exists a critical value of excitation above which unbounded resonance occurs. He also analyzed the response of the same system to random excitation [5]. He showed that hysteresis damping of this type may actually lead to larger displacements than would occur in the equivalent linear system. Iwan [6] used the same method as that of Caughey [4] in the analysis of the steady state response of a limited slip joint to sinusoidal excitation (see Figure 2). It was shown that the system may posses such features as disconnected response curves and jumps in the response depending on the strength of the nonlinearity, the level of excitation, and the amount of viscous damping.

The resonant response of an engine blade with a blade-to-ground damper was analyzed by Griffin [7] using an equivalent single-degree-of-freedom system (see Figure 3). The damper was characterized by two parameters: the force at which it slips and its stiffness. He showed that the optimum slip load which minimizes the resonant response can be determined when all the parameters are held constant. The effectiveness of the damper was increased as the damper became stiffer.

II.1.2 Multimode System

Earles and Williams [8] developed a method for the analysis of vibration in a system comprising a linear undamped multi-degree-of-freedom structure with a single frictional
damper and excited by any number of sinusoidal forces of the same frequency. He used cross receptance coefficients of the linear structure, which are the ratios of response of a point to an excitation at an another point, as the response characteristics of the linear structure. The frictional force was assumed sinusoidal at any frequency. A test model consisted of two parallel cantilever beams linked by a single frictional damper was tested by applying single sinusoidal forces. They compared the experimental results with the theoretical prediction and determined a parameter so as to bring in proximity the experimental and theoretical curves. They concluded that the dynamic coefficient of friction is linearly dependent on the slip amplitude.

The response of a simplified model of a shrouded compressor blade was analyzed using the same method described above to predict the optimum conditions [9]. The optimization criterion was the minimization of the blade response (stress or displacement). Experiments on a rectangular cross sectional beam having two fixed bars which contact two transducers were performed and good agreements between experimental and theoretical values of both the stress levels and deflection of the beam were obtained.

Dowell [10] generalized the results for a single-degree-of-freedom system with a damper to a multimode linear system with a spring-mass-dry friction damper attached. He analyzed the multimode system in rigorous mathematical detail by using component mode analysis method. The dry friction damper law was approximated by its first harmonic. The same method was applied to the analysis of forced response of a cantilever beam with a dry friction damper attached [11]. This was a simplified model of a turbine blade with a blade-to-ground damper. A series of experiment on a double cantilever member physical system was carried out by varying the frequency of excitation, forcing position location, and the stiffness of the damper support member.

Menq and Griffin [12] adopted Earles and Williams [8] idea which separates the problem into linear and nonlinear parts and uses receptance of linear structure. They developed a method which utilizes linear finite element analysis of the primary structure with no friction element to establish the relevant response characteristics of the structure. Their method also uses a more accurate calculation of the Fourier coefficients by a procedure originally presented by Caughey [4]. The accuracy of the method was acceptable compared with long time numerical solutions of the transient problem of a flexible structure with stiff damper as shown in Figure 4. However, the experimental results significantly deviated from theoretical predictions for conditions of high slip load or damper lockup due to the fact that the Coulomb friction element does not model the influence of microslip which is important at higher slip loads.

Muszynska and his co-workers [13,14] developed discrete models to characterize the dynamic response of a bladed disk system with dry friction dampers to harmonic excitation. They obtained results which illustrate the effects of various levels of frictional damping on tuned and mistuned systems. Two factors were found to be important on the friction damping: (a) the ratio of the friction force to the exciting force, and (b) the blade-to-blade phase angle of the exciting force. Another effect introduced by the frictional elements was the coupling of the system. Even ideally tuned bladed disks were subjected to mistuning effects.

II.2 Microslip Analysis
The origin of the microslip analysis can be found historically in the Mindlin's extension of Hertz's contact analysis between two spheres. He solved the contact problem between two spheres when tangential loads apply in addition to normal forces. Extensive works, both analytical and experimental, have been accomplished in the area of spherical contact analysis. However, the major application of microslip analysis on friction damping in joints can be found in contact analysis between flat surfaces.

II.2.1 Spherical Contact Analysis

Mindlin [15] considered the contact between two spheres pressed into contact and analyzed the relative elastic displacement between spheres under the action of tangential loading. He assumed that Coulomb's law applies to the relative slip between two adjacent points on the mating surfaces. In a later paper [16], Mindlin and his co-workers extended the analysis to the case of an oscillating tangential load and developed an expression for the energy dissipation per cycle. They also performed experimental tests employing polished glass lenses and verified their theoretical results except that the energy dissipation per cycle varies as the square of the displacement where as the theory predicts a cube law.

Johnson [17] reported a series of careful experiments in which a hard steel ball on a hard flat steel surface have been subjected to cyclic tangential loadings. He showed that for small tangential forces the energy dissipation is proportional to the square of the sheere forces. However, when the shear force is greater than about half the slip load the energy loss increases rapidly. Goodman and his co-workers [18,19] also performed experiments in which a AISI 316 steel ball was subjected to a cyclic tangential displacement between two flat steel planes and verified good agreement between Mindlin's theory and experiments.

II.2.2 Approximate Analysis of Flat Contact

For the contact between flat surfaces approximate theories have been developed. Goodman and Klumpp [20] analyzed the frictional damping occuring at the surface between the two layers of a composite cantilever beam, as shown in Figure 5, during oscillatory bending. This approach began from the same assumption as that of Mindlin's theory, but it was necessary to make assumptions in regard to the stress distributions. They also performed experiments with a composite cantilever beam under uniform pressure and obtained results which were in satisfactory agreement with the analytic prediction.

Earles and Philpot [21] analyzed the energy dissipation at an axial double lap joint under partial slip and gross slip conditions. They showed that the energy dissipation was proportional to the cube of the load and inversely proportional to the coefficient of friction for the partial slip case. They also performed a series of experiments in which the frictional damping occuring at plane stainless steel contacts under oscillating tangential loads and obtained close agreement for the partial slip region and fair agreement in the gross slip region.
They performed experiments for different periods up to a maximum of $5 \times 10^6$ cycles to determine how surface developments affected the damping in an axial slip lap joint during repeated cycling.

Recently, Crawley et. al. [22] considered the application of dry friction damping to the space structure which is a truss structure made of tubular members. They considered a friction damping mechanism consists of segmented damping tubes placed end to end inside the tubular load carrying members. They analyzed the energy loss per cycle using a simple microslip Coulomb friction model. To validate the one-dimensional friction damping model, they performed a series of experiments. Since the coefficient of friction can be altered by contamination, surface preparation, and wear, they took the constants in the theoretical expression as free parameters in the least square curve fits. The resultant curve fits showed close agreement between theory and experiment.

Menq, Griffin and Bielak [23] developed a new microslip model which allows partial slip on the contact interface. The model consists of two elastic bars joined by an elastoplastic shear layer as shown in Figure 6. The friction force in the shear layer, $\tau$, per unit length is given by,

$$\tau = \begin{cases} k u, & |u| < \tau_m/k \\ \tau_m, & \text{otherwise} \end{cases}$$

where $\tau_m$ is the maximum shear stress for the occurance of local plastic deformation.

The system will deform elastically as long as the displacement at the end of the bar remains below the value $\tau_m/k$. As $P$ increases beyond this value a region of slip is generated and this region increases until the entire shear layer becomes plastic. The force vs. displacement of the microslip model under symmetric loading is shown in Figure 7.

They used the new microslip model in simulating the vibratory response of three sets of experiment [24]. In each case the microslip model could explain the experimental results that could not be explained well by the macroslip model (compare Figures 8, 9, and 10 which are the results on the system shown in Figure 4).

A more complex slip mechanism in which the normal load and the associated friction slip load can vary dynamically during the cycle were considered by Menq, Griffin, and Bielak. They analyzed a single-degree-of freedom system with a friction damper where the contact pressure was assumed to be a linear function of the displacement using a macroslip model [25]. The new microslip model developed in [23] was used in the analysis of the vibratory response of shrouded fan stages [26]. Since the vibratory motion causes the normal load on the friction interfaces to vary dynamically, both of the effects of microslip and variable normal load were included in this analysis. The effect of the variable normal load for small value of initial normal load can be seen in the 'overlapping' of the response curves at the lower frequencies, shown in Figure 11. The effects of microslip are shown more apparently for higher initial normal loads; the system locks up at a much higher value of normal load than that of a conventional Coulomb model of friction and the microslip model predicts much lower response in the tight joint regime due to energy dissipation.
III. EXPERIMENTAL WORK

Crema et. al. [27] carried out a series of tests on specimens with rivet joints. Specimens were made of aluminum alloys and three types of joints have been tested: lap joint; butt joint with single strap; and butt joint with double strap. The damping coefficient was measured by means of frequency sweep and decay transient. However, their results do not give any contribution to the knowledge of joint damping.

It is well recognized that an optimum joint clamping force exists for maximum energy dissipation due to interfacial slip in joint [28]. Beards and Williams [29] constructed a test frame consists of a rectangular frame with a solid steel bar bolted diagonally to it. The frequency response of the structure was measured by exciting one corner of the frame at a constant sinusoidal force and detecting the response by an accelerometer at one of the other corners of the frame. They analyzed the response of the frame using the same method developed by Earles and Williams [8]. They showed that a useful increase in the damping in a structure can be achieved by fastening joints tightly to prohibit translational slip but not tightly enough to prohibit rotational slip.

Beards and Imam [30] showed that plate type structures can be damped by friction by using laminated plate correctly fastened to allow controlled interfacial slip during vibration. Jezquel [31] presented a general formulation for cases when slip of plate occurs at the clamped boundary by inplane forces during harmonic excitation. The application of this method to circular plates gave a structural damping factor of the same intensity as those measured in bolted and riveted structures.

Beards and Woodwat [32] performed experimental work to investigate the effect of controlled friction damping in joints on the frequency response of a frame excited by a harmonic force. They compared the frequency response around the second mode of vibration of the frame for various clamping forces at joints. It was shown that the maximum reduction in frame response of 21 dB could be obtained. It was also shown that a reduction in peak response of 10 dB occurred due only to microslip in the joints.

In turbine engines, vibration damping due to dry friction can occur at shroud interfaces of blades and the platform of turbine blades fitted with platform dampers. Srinivasan and Cutts [33] studied the effect of these two sources of damping on blade vibration both experimentally and analytically. In the study of damping due to rubbing at shroud interfaces, the macroslip analysis predicted an abrupt transition from a region of no friction damping to the macroslip region. However, the test results indicated a smoother transition indicating a region for partial or microslip conditions.

Lagnese and Koester [34] tried to investigate the effect of friction damping on turbine blade dynamic response by means of a modal testing technique using impact pendulum. They also performed sine sweep tests of the damped blade response. However, the results could not be compared with those from impact tests due to the large difference in the excitation levels.

Some surface damage is inevitable in joints designed to dissipate vibrational energy by friction. To reduce the damaging effect of fretting, some work has been done on the use of surface preparations. Beards and Neroutsopoulos [35] performed vibrational tests on joints
with grounded interfaces and three different electro-discharge-machined (EDM) interfaces up to 10 x 10^6 cycles. It was shown that EDM joints have higher stiffness and possess up to 100% more damping capacity than ground joints.

Miller [36] applied an approximate analytical method to the case of plane harmonic shear waves normally incident on a Coulomb friction boundary between two halfspaces. Later Miller and Tran [37] extended the approximate method of analysis to the problems of determining the reflection, refraction, and absorption of obliquely incident planar harmonic P or SV waves at a frictional interface between dissimilar solids. Gaul [38] represented the nonlinear behavior of joints by equivalent linearized models and calculated the characteristics of transmission, reflection, and energy dissipation of flexural waves in the jointed beams. Recently, Trudell et. al. [39] reported that they measured the damping factors of four structural joints using a sine-pulse technique. The damping information was extracted from the changes that occur in certain characteristics of stress waves as a result of its passage through the joint. Details of the technique was not reported since they did not present the details of the preparatory processing of the response signals.

IV. CONCLUDING REMARKS

The experimental works, especially those performed by Beards and his co-workers, on controlled friction damping in joints showed that large increase in damping is possible by controlling the clamping force in joint and that optimal clamping force exists under which a joint dissipates maximum vibrational energy. However, the practical problems are usually concerned with minimizing the response of the system rather than maximizing the energy dissipation. These conditions are not necessarily compatible, since the clamping force which maximizes the energy dissipation depends on the level of excitation.

Usually clamping forces are very high for bolted joints, and both the microslip and macroslip can occur. Therefore, the macroslip analysis is not sufficiently accurate for many joints, especially near lockup. It seems that the current trend in analytical approaches is the development of a new model which exhibits the three main characteristics of such contact problems: elastic deformation when the input force is small, followed by a region of partial slip, and lastly-total slip of the joint for very large input forces.
(a) Single degree-of-freedom system with Coulomb friction.

(b) Amplitude diagram

Figure 1. Forced vibration with friction damping only [1].
Figure 2. A limited slip system [6].
(a) A blade to ground damper

(b) Equivalent one degree-of-freedom system

Figure 3. A model of a blade with a BG damper [7]
Figure 4. A flexible structure with friction damping at the tips of platform [12].

Figure 5. A uniform pressure joint [20].
Figure 6. A microslip model [23].

Figure 7. Force vs. Displacement of a microslip model under symmetric loading [23].
Figure 8. Vibration response of a beam with platform from experiments [24].

Figure 9. Response predicted by using a macroslip friction model [24].
Figure 10. Response predicted by using a microslip friction model [24].

Figure 11. Response of a shrouded blade tip predicted by using a microslip friction model with variable normal load [26].
V. REFERENCES


I. INTRODUCTION

A test model was constructed at the Naval Postgraduate School, which consists of two circular cylindrical shells and four vanes connected by bolts, as shown in Figure 12. This structure will be tested both in air and in water. Before doing these tests, the test model is analyzed using the MSC/NASTRAN computer program. MSC/NASTRAN is a large scale general purpose computer program which solves a wide variety of engineering problems by the finite element method. The first analysis is a modal survey of the model to investigate the dynamic characteristics of the whole system. The second analysis is the frequency response analysis which gives the input-output relationship between an input point (excitation point) and an output point (response point) in frequency domain, which depends on the damping characteristics of the structure.

II. MODAL ANALYSIS

The test model is discretized into 240 finite elements as shown in Figure 13. These elements are defined by 304 grid points shown in the figure. The structure is allowed free motion in the 3-dimensional space and the total degrees-of-freedom of the whole system is 1760. The geometry and mechanical properties of each finite element is described in this analysis using a quadrilateral element called CQUAD4 in MSC/NASTRAN.

The material properties used in this analysis are as follows:

(A) The inner cylindrical shell and four vanes (Material = Bronze)
The modal analysis, which is an eigenvalue problem in mathematics, is solved using the modified Givens method. The Givens method is a transformation method, where the matrix of an eigenvalue problem is first transformed to a tridiagonal matrix by a techniques of Givens and then eigenvalues are extracted from the triangular matrix. This method is particularly effective for vibration modes when all, or a substantial fraction, of the eigenvalues are desired.

The analysis indicates that there are 61 modes of free vibration in the frequency range from 0 Hz to 500 Hz including 6 rigid body modes. The mode shapes of the 9 lowest deformation modes are shown in Figure 14. The natural frequency of the first deformation mode of free vibration is 27.4 Hz. The outer cylindrical shell and vanes vibrate with very small vibration of the inner shell. The first large vibration of the inner cylindrical shell occurs at 72.5 Hz with the 6th deformation mode shape shown in Figure 14.

III. FREQUENCY RESPONSE ANALYSIS

The same finite element model that used in modal analysis, Figure 13, is used in the frequency response analysis. The input point is Grid 108, which is a point in a vane parallel to the -Y axis of the global coordinates as shown in Figure 15. This point is excited with a unit amplitude sinusoidal force in the direction normal to the plate and the response (acceleration in the direction normal to the plate) of four grid points (Grids 108, 105, 182, and 264) on vanes and the triaxial response of a grid point on the outer cylindrical shell (Grid 121, which is a grid point about midway between the Grids 117 and 126) are obtained.

The frequency responses (magnitude in logarithmic scale and phase) in the frequency range from 5 Hz to 100 Hz are shown in Figure 16 for the 1 % modal damping ratio for the structure. Since the frequency response function depends on the damping characteristics of the structure, when the modal damping ratio is reduced to 0.5 %, the frequency response has more sensitive variation at resonance frequencies as shown in Figure 17.
Figure 12. The test model. (All 1/4” plates)
Figure 13. The finite element model for analysis.

Figure 14. (a) The 1st mode. $f = 27.4$ Hz

(a) The mode shapes of free vibration.
The 2nd mode, $f = 30.9$ Hz

The 3rd mode, $f = 35.9$ Hz

Figure 14. The mode shapes of free vibration.
(d) The 4th mode. $f = 37.8$ Hz

(e) The 5th mode. $f = 37.8$ Hz

Figure 14. The mode shapes of free vibration.
(f) The 6th mode. $f = 72.5$ Hz

(g) The 7th mode. $f = 80.3$ Hz

Figure 14. The mode shapes of free vibration.
(h) The 8th mode. \( f = 82.3 \) Hz

(i) The 9th mode. \( f = 93.4 \) Hz

Figure 14. The mode shapes of free vibration.
Figure 15. The input and output grid points.
(a) Magnitude and phase of response at Grid 108 in the normal direction.

Figure 16. The frequency response curves. $\zeta = 1 \%$. 
(b) Magnitude and phase of response at Grid 105 in the normal direction.

Figure 16. The frequency response curves. $\zeta = 1\%$. 
(c) Magnitude and phase of response at Grid 182 in the normal direction.

Figure 16. The frequency response curves. $\zeta = 1\%$. 
(d) Magnitude and phase of response at Grid 264 in the normal direction.

Figure 16. The frequency response curves. $\zeta = 1\%$. 

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(e) Magnitude and phase of response at Grid 121 in the tangential direction.

Figure 16. The frequency response curves. \( \zeta = 1 \% \).
(f) Magnitude and phase of response at Grid 121 in the normal direction.

Figure 16. The frequency response curves. $\zeta = 1\%$. 
(g) Magnitude and phase of response at Grid 121 in the axial direction.

Figure 16. The frequency response curves. $\zeta = 1\%$. 
(a) Magnitude and phase of response at Grid 108 in the normal direction.

Figure 17. The frequency response curves. $\zeta = 0.5 \%$. 
(b) Magnitude and phase of response at Grid 105 in the normal direction.

Figure 17. The frequency response curves. $\zeta = 0.5\%$. 

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(c) Magnitude and phase of response at Grid 182 in the normal direction.

Figure 17. The frequency response curves. $\zeta = 0.5 \%$. 
(d) Magnitude and phase of response at Grid 264 in the normal direction.

Figure 17. The frequency response curves. $\zeta = 0.5 \%$. 
(e) Magnitude and phase of response at Grid 121 in the tangential direction.

Figure 17. The frequency response curves. $\zeta = 0.5 \%$. 

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(f) Magnitude and phase of response at Grid 121 in the normal direction.

Figure 17. The frequency response curves. $\zeta = 0.5 \%$. 

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(g) Magnitude and phase of response at Grid 121 in the axial direction.

Figure.17. The frequency response curves. $\zeta = 0.5\%$. 

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</tbody>
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