Quantum Limitations in Microdevices

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Quantum limitations, Microdevice, FET, Uncertainty relation, Frequency upper bound.
20. Abstract

Stemming from quantum mechanical uncertainty relations, the following expressions are obtained for maximum frequency, \( f_{\text{max}} \), and minimum switching time, \( \Delta t_{\text{min}} \), relevant to micro field-effect transistor devices.

\[
f_{\text{max}}^{-1} = \Delta t_{\text{min}} = \frac{2\xi_{C}c}{(m^{*}/m_{e})v_{\text{max}}^{2}}
\]

The speed of light is \( c \), \( \xi_{C} \) is the Compton wavelength, \( m^{*} \) is effective mass, \( m_{e} \) is electron mass and \( v_{\text{max}} \) is maximum electron speed. These limiting values differ roughly by factors of 10 from present-day measured extreme values.
I. INTRODUCTION

Recent advances in microprocess fabrication continue to reduce operational dimensions. Primary goals in the design of such devices seek to obtain maximum operational frequency and minimum switching time.

Sadler and Eastman have recently reported a switching time of 15.4 ps. Yamasaki et al. have reported on operational frequency of 60 GHz and further suggest that frequencies in excess of 100 GHz may be attained.

The primary circuit element of these devices is a metal-semiconductor field-effect transistor. In such field-effect transistors charge carriers are driven through a GaAs semiconductor from source to drain by an applied electric field. Modulation of the current is achieved by varying the voltage to the gate.

In this analysis we consider the degree to which the laws of quantum mechanics impose limitations on operational properties of such microdevices. The energy-time uncertainty relation employed in the present analysis as well as overall fundamental limitations in the operation of microdevices was discussed previously by Keyes.

II. ANALYSIS

Let the energy of an electron in the device be denoted by \( \langle E \rangle \).

Then under normal operating conditions,

\[
\langle E \rangle \leq \frac{1}{2} m^* v_{max}^2
\]  

where \( m^* \) is effective mass and \( v_{max} \) is maximum electron speed. Here we have assumed that the electron remains in a single band.
In many instances in quantum mechanics, uncertainty of a given parameter may be taken as a measure of the magnitude of the parameter. In the present study we direct our attention to electrons in the conduction band of an extrinsic semiconductor. We assume carrier densities less than \(10^{19} \text{ cm}^{-3}\) and impurity levels close to the band edge. These assumptions permit use of Maxwell-Boltzmann statistical factors and a simplified form of the density of states to estimate the expectation value of energy. Setting the zero of energy at the bottom of the conduction band we obtain

\[
\langle E^2 \rangle = \frac{10}{3} \langle E \rangle^2
\]

which gives

\[
(\Delta E)^2 = \frac{7}{3} \langle E \rangle^2
\]

Thus we may write

\[
(\Delta E)^2 = \langle E^2 \rangle - \langle E \rangle^2 = O(1) \langle E \rangle^2
\]

(2)

where \(O(1)\) denotes a numerical factor of the order of unity.

In this spirit we view \(\langle E \rangle\) in the preceding relation (1) as a measure of uncertainty in energy. Combining (1) so interpreted with the fundamental energy-time uncertainty relation,

\[
\Delta E \Delta t \geq \hbar
\]

(3)
gives

\[
\Delta E_{\text{max}} \Delta t \geq \Delta E \Delta t \geq \hbar
\]

(4)

\[
\Delta E_{\text{max}} = \frac{1}{2} m^* v^2_{\text{max}}
\]
It has been pointed out by Aharonov and Bohm \(^8\) that there is no reason to assume the validity of the energy-time uncertainty relation for \(\Delta t\) interpreted as a time interval of measurement. However, it has been shown by Keyes \(^5\) that for practical devices this is a reasonable assumption.

Returning to the mainstream of analysis, inverting (4) gives the inequality

\[
\Delta t \geq \frac{\hbar}{\Delta E_{\text{max}}} \tag{5}
\]

Present-day measured values indicate \(^{10,11}\) \(v_{\text{max}} = 4 \times 10^7 \text{ cm/s}\). Employing this value together with the effective electron mass for GaAs \(^{12}\) \(m^* = 0.067 m_e\), in (5), gives the following lower bound on switching times.

\[
\Delta t > 2.2 \times 10^{-14} \text{s} = 22 \text{ fs} = \Delta t_{\text{min}} \tag{6}
\]

An upper bound on frequencies may be obtained by setting \(f = (\Delta t)^{-1}\). We find

\[
f < 4.5 \times 10^{13} \text{ Hz} = 45 \text{ THz} = f_{\text{max}} \tag{7}
\]

We see that the limiting values (6) and (7) are roughly separated by factors of \(10^{23}\) from previously measured present-day extreme values. The results (6) and (7) may be combined to read (in cgs)

\[
f^{-1}_{\text{max}} = \Delta t_{\text{min}} = \frac{2\hbar c^2}{(m^*/m_e)v_{\text{max}}^2} = \frac{2.32}{(m^*/m_e)v_{\text{max}}^2} \tag{8}
\]
where \( c \) is the speed of light and \( \lambda_C \) is the Compton wavelength. As this relation stems from the uncertainty principle, it should be employed more as an estimate than as a precise result.

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References


